CHAPTER 2

Section 2-1

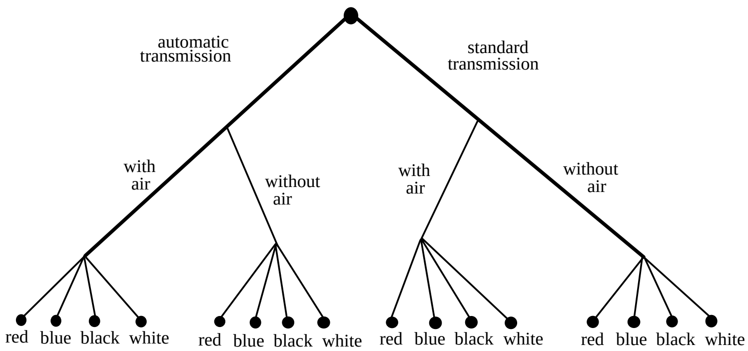
2-2. Let e and o denote a bit in error and not in error (o denotes okay), respectively.

$$S = \left\{ \begin{matrix} eeee, eoee, oeee, ooee, \\ eeeo, eoeo, oeeo, ooeo, \\ eeoe, eooe, oeoe, oooe, \\ eeoo, eooo, oeoo, oooo \end{matrix} \right\}$$

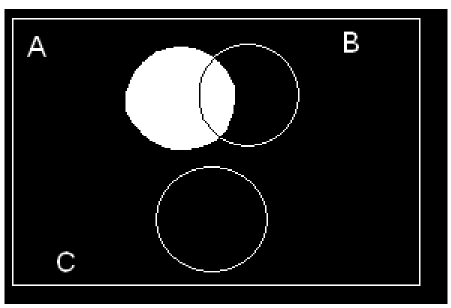
2-3. Let a denote an acceptable power supply.
Let f, m, and c denote a power supply that has a functional, minor, or cosmetic error, respectively.

$$S = \{a, f, m, c\}$$

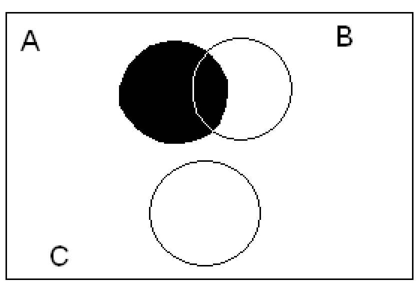
2-14.



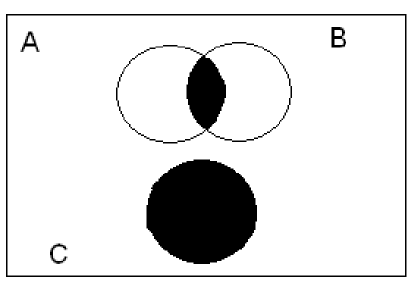
2-20. a)



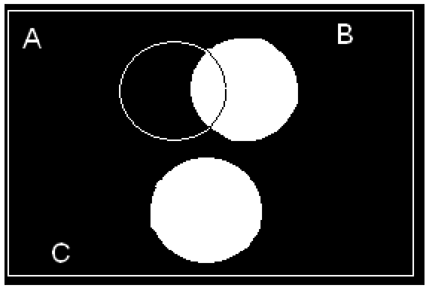
b)



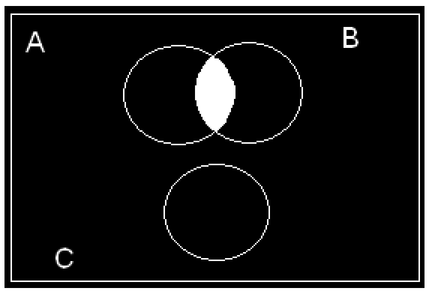
c)



d)



e)



2-23. Let *d* and *o* denote a distorted bit and one that is not distorted (*o* denotes okay), respectively.

a) $S = \left\{ \begin{matrix} dddd, dodd, oddd, oodd, \\ dddo, dodo, oddo, oodo, \\ ddod, dood, odod, oood, \\ ddoo, dooo, odoo, oooo \end{matrix} \right\}$

b) No, for example $A_1 \cap A_2 = \{dddd, dddo, ddod, ddoo\}$

c) $A_1 = \left\{ \begin{matrix} dddd, dodd, \\ dddo, dodo \\ ddod, dood \\ ddoo, dooo \end{matrix} \right\}$

d) $A_1' = \left\{ \begin{matrix} oddd, oodd, \\ oddo, oodo, \\ odod, oood, \\ odoo, oooo \end{matrix} \right\}$

e) $A_1 \cap A_2 \cap A_3 \cap A_4 = \{dddd\}$

f) $(A_1 \cap A_2) \cup (A_3 \cap A_4) = \{dddd, dodd, dddo, oddd, ddod, oodd, ddoo\}$

2-29. a) $A' = \{x \mid x \geq 72.5\}$
b) $B' = \{x \mid x \leq 52.5\}$
c) $A \cap B = \{x \mid 52.5 < x < 72.5\}$
d) $A \cup B = \{x \mid x > 0\}$

2-37. From the multiplication rule, $3 \times 4 \times 3 \times 4 = 144$

2-38. From equation 2-1, the answer is 10! = 3,628,800

2-39. From the multiplication rule and equation 2-1, the answer is 5!5! = 14,400

2-40. From equation 2-3, $\frac{7!}{3!4!} = 35$ sequences are possible

2-42. a) If the chips are of different types, then every arrangement of 5 locations selected from the 12 results in a different layout. Therefore, $P_5^{12} = \frac{12!}{7!} = 95{,}040$ layouts are possible.

b) If the chips are of the same type, then every subset of 5 locations chosen from the 12 results in a different layout. Therefore, $\binom{12}{5} = \frac{12!}{5!7!} = 792$ layouts are possible.

Section 2-2

2-54. All outcomes are equally likely
a) P(A) = 2/5
b) P(B) = 3/5
c) P(A') = 3/5
d) P(A∪B) = 1
e) P(A∩B) = P(∅)= 0

2-61. The sample space is {0, +2, +3, and +4}.
(a) The event that a cell has at least one of the positive nickel charged options is {+2, +3, and +4}. The probability is 0.35 + 0.33 + 0.15 = 0.83.
(b) The event that a cell is not composed of a positive nickel charge greater than +3 is {0, +2, and +3}. The probability is 0.17 + 0.35 + 0.33 = 0.85.

2-67. a) P(A) = 30/100 = 0.30
b) P(B) = 77/100 = 0.77
c) P(A') = 1 – 0.30 = 0.70
d) P(A∩B) = 22/100 = 0.22
e) P(A∪B) = 85/100 = 0.85
f) P(A'∪B) =92/100 = 0.92

2-69. a) Because E and E' are mutually exclusive events and $E \cup E'$ = S
1 = P(S) = P($E \cup E'$) = P(E) + P(E'). Therefore, P(E') = 1 - P(E)
b) Because S and ∅ are mutually exclusive events with S = $S \cup \varnothing$
P(S) = P(S) + P(∅). Therefore, P(∅) = 0
c) Now, B = $A \cup (A' \cap B)$ and the events A and $A' \cap B$ are mutually exclusive. Therefore,
P(B) = P(A) + P($A' \cap B$). Because P($A' \cap B$) $\geq$ 0 , P(B) $\geq$ P(A).

Section 2-3

2-74. a) P(A') = 1- P(A) = 0.7
b) P ($A \cup B$) = P(A) + P(B) - P($A \cap B$) = 0.3+0.2 - 0.1 = 0.4

c) P($A' \cap B$) + P($A \cap B$) = P(B). Therefore, P($A' \cap B$) = 0.2 - 0.1 = 0.1
d) P(A) = P($A \cap B$) + P($A \cap B'$). Therefore, P($A \cap B'$) = 0.3 - 0.1 = 0.2
e) P(($A \cup B$)') = 1 - P($A \cup B$) = 1 - 0.4 = 0.6
f) P($A' \cup B$) = P(A') + P(B) - P($A' \cap B$) = 0.7 + 0.2 - 0.1 = 0.8

2.77. a) 70/100 = 0.70
b) (79+86-70)/100 = 0.95
c) No, P($A \cap B$) $\neq 0$

2-81. a) P(unsatisfactory) = (5 + 10 – 2)/130 = 13/130
b) P(both criteria satisfactory) = 117/130 = 0.90, No

2-86. P(A) = (1685 + 3733 + 1403)/8493 = 0.8031, P(B) = (170 + 2 + 443 + 14 + 29 + 60 + 46 + 3)/8493 = 0.0903,
P(A ∩ B) = (170 + 443 + 60)/8493 = 0.0792, P(A ∩ B′) = (1515+3290+1343)/8493 = 0.7239
a) P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.8031 + 0.0903 – 0.0792 = 0.8142
b) P(A ∪ B′) = P(A) + P(B′) – P(A ∩ B′) = 0.8031 + (1 – 0.0903) – 0.7239 = 0.9889
c) P(A′ ∪ B′) = 1 – P(A ∩ B) = 1 – 0.0792 = 0.9208

Section 2-4

2-88. (a) $P(A) = \dfrac{7+32}{100} = 0.39$

(b) $P(B) = \dfrac{13+7}{100} = 0.2$

(c) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{7/100}{20/100} = 0.35$

(d) $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{7/100}{39/100} = 0.1795$

2-93. Let *A* denote the event that autolysis is high and let *B* denote the event that putrefaction is high. The total number of experiments is 100.

(a) $P(B' \mid A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{18/100}{(14+18)/100} = 0.5625$

(b) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{14/100}{(14+59)/100} = 0.1918$

(c) $P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{9/100}{(18+9)/100} = 0.333$

2-95. a) 20/100
b) 19/99
c) (20/100)(19/99) = 0.038
d) If the chips were replaced, the probability would be (20/100) = 0.2

Section 2-5

2-106.

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ &= P(A|B)P(B) + P(A|B')P(B') \\ &= (0.2)(0.8) + (0.3)(0.2) \\ &= 0.16 + 0.06 = 0.22 \end{aligned}$$

2-112. a)P = 0.13×0.73=0.0949
b)P = $0.87 \times (0.27+0.17)$=0.3828

2-113. Let A and B denote the event that the first and second part selected has excessive shrinkage, respectively.
a) P(B)= $P(B|A)P(A) + P(B|A')P(A')$
= (4/24)(5/25) + (5/24)(20/25) = 0.20
b) Let C denote the event that the third part selected has excessive shrinkage.

$$P(C) = P(C|A\cap B)P(A\cap B) + P(C|A\cap B')P(A\cap B')$$
$$+P(C|A'\cap B)P(A'\cap B) + P(C|A'\cap B')P(A'\cap B')$$
$$= \frac{3}{23}\left(\frac{4}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{20}{24}\right)\left(\frac{5}{25}\right) + \frac{4}{23}\left(\frac{5}{24}\right)\left(\frac{20}{25}\right) + \frac{5}{23}\left(\frac{19}{24}\right)\left(\frac{20}{25}\right)$$
$$= 0.20$$

Section 2-6

2-124. If A and B are mutually exclusive, then $P(A\cap B) = 0$ and P(A)P(B) = 0.04.
Therefore, A and B are not independent.

2-134. (a) $P = \dfrac{10^6}{10^{16}} = 10^{-10}$

(b) $P = 0.25 \times (\dfrac{1}{12}) = 0.020833$

2-135. Let A denote the event that a sample is produced in cavity one of the mold.
a) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) = (\frac{1}{8})^5 = 0.00003$

b) Let B_i be the event that all five samples are produced in cavity i. Because the B's are mutually exclusive, $P(B_1 \cup B_2 \cup ... \cup B_8) = P(B_1) + P(B_2) + ... + P(B_8)$

From part a., $P(B_i) = (\frac{1}{8})^5$. Therefore, the answer is $8(\frac{1}{8})^5 = 0.00024$

c) By independence, $P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5') = (\frac{1}{8})^4(\frac{7}{8})$. The number of sequences in which four out of five samples are from cavity one is 5. Therefore, the answer is $5(\frac{1}{8})^4(\frac{7}{8}) = 0.00107$.

2-141. P(A) = (3*5*3*5)/(4*3*5*3*5) = 0.25, P(B) = (4*3*4*3*5)/(4*3*5*3*5) = 0.8,
$P(A \cap B)$ = (3*4*3*5) /(4*3*5*3*5) = 0.2
Because P(A)*P(B) = (0.25)(0.8) = 0.2 = $P(A \cap B)$, A and B are independent.

Section 2-7

2-143.
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$
$$= \frac{0.4 \times 0.8}{0.4 \times 0.8 + 0.2 \times 0.2} = 0.89$$

2-147. Let G denote a product that received a good review. Let H, M, and P denote products that were high, moderate, and poor performers, respectively.
a)

$$\begin{aligned} P(G) &= P(G|H)P(H) + P(G|M)P(M) + P(G|P)P(P) \\ &= 0.95(0.40) + 0.60(0.35) + 0.10(0.25) \\ &= 0.615 \end{aligned}$$

b) Using the result from part a.,

$$P(H|G) = \frac{P(G|H)P(H)}{P(G)} = \frac{0.95(0.40)}{0.615} = 0.618$$

c) $P(H|G') = \frac{P(G'|H)P(H)}{P(G')} = \frac{0.05(0.40)}{1-0.615} = 0.052$

2-153. Denote as follows: A = affiliate site, S = search site, B =blue, G =green

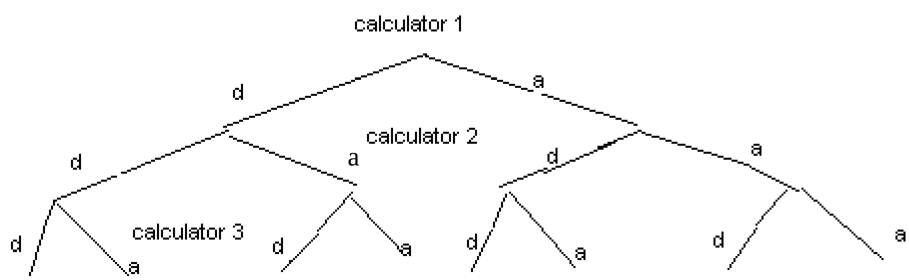
$$\begin{aligned} P(S|B) &= \frac{P(B|S)P(S)}{P(B|S)P(S)+P(B|A)P(A)} \\ &= \frac{(0.4)(0.7)}{(0.4)(0.7)+(0.8)(0.3)} \\ &= 0.5 \end{aligned}$$

Section 2-8

2-154. Continuous: a, c, d, f, h, i; Discrete: b, e, and g

Supplemental Exercises

2-156. Let "d" denote a defective calculator and let "a" denote an acceptable calculator



a) $S = \{ddd, add, dda, ada, dad, aad, daa, aaa\}$

b) $A = \{ddd, dda, dad, daa\}$

c) $B = \{ddd, dda, add, ada\}$

d) $A \cap B = \{ddd, dda\}$

e) $B \cup C = \{ddd, dda, add, ada, dad, aad\}$

2-161. (a) P(the first one selected is not ionized)=20/100=0.2

(b) P(the second is not ionized given the first one was ionized) =20/99=0.202

(c) P(both are ionized)

= P(the first one selected is ionized) × P(the second is ionized given the first one was ionized)

= (80/100)×(79/99)=0.638

(d) If samples selected were replaced prior to the next selection,

P(the second is not ionized given the first one was ionized) =20/100=0.2.

The event of the first selection and the event of the second selection are independent.

2-164. Let U denote the event that the user has improperly followed installation instructions.

Let C denote the event that the incoming call is a complaint.
Let P denote the event that the incoming call is a request to purchase more products.
Let R denote the event that the incoming call is a request for information.
a) P(U|C)P(C) = (0.75)(0.03) = 0.0225
b) P(P|R)P(R) = (0.50)(0.25) = 0.125

2-165. (a) $P = 1-(1-0.002)^{100} = 0.18143$

(b) $P = C_3^1(0.998^2)0.002 = 0.005976$

(c) $P = 1-[(1-0.002)^{100}]^{10} = 0.86494$

2-167. Let A_i denote the event that the ith readback is successful. By independence,
$P(A'_1 \cap A'_2 \cap A'_3) = P(A'_1)P(A'_2)P(A'_3) = (0.02)^3 = 0.000008.$

2-180. P(Possess) = 0.95(0.99) +(0.05)(0.90) = 0.9855

2-186. a) By independence, $0.15^5 = 7.59 \times 10^{-5}$
b) Let A_i denote the events that the machine is idle at the time of your ith request. Using independence, the requested probability is

$$P(A_1 A_2 A_3 A_4 A'_5 \quad or \quad A_1 A_2 A_3 A'_4 A_5 \quad or \quad A_1 A_2 A'_3 A_4 A_5 \quad or \quad A_1 A'_2 A_3 A_4 A_5 \quad or \quad A'_1 A_2 A_3 A_4 A_5)$$
$$= 5(0.15^4)(0.85^1)$$
$$= 0.000000215$$

c) As in part b, the probability of 3 of the events is

$$P(A_1 A_2 A_3 A'_4 A'_5 \quad or \quad A_1 A_2 A'_3 A_4 A'_5 \quad or \quad A_1 A_2 A'_3 A'_4 A_5 \quad or \quad A_1 A'_2 A_3 A_4 A'_5 \quad or \quad A_1 A'_2 A_3 A'_4 A_5 or$$
$$A_1 A'_2 A'_3 A_4 A_5 \quad or \quad A'_1 A_2 A_3 A_4 A'_5 \quad or \quad A'_1 A_2 A_3 A'_4 A_5 \quad or \quad A'_1 A_2 A'_3 A_4 A_5 \quad or \quad A'_1 A'_2 A_3 A_4 A_5)$$
$$= 10(0.15^3)(0.85^2)$$
$$= 0.0244$$

For the probability of at least 3, add answer parts a) and b) to the above to obtain the requested probability. Therefore, the answer is 0.0000759 + 0.0022 + 0.0244 = 0.0267

2-194. (a) P=(24/36)(23/35)(22/34)(21/33)(20/32)(19/31)=0.069
(b) P=1-0.069=0.931

2-196. (a) $P(A) = \dfrac{5+25+30+7+20}{1000} = 0.087$

(b) $P(A \cap B) = \dfrac{25+7}{1000} = 0.032$

(c) $P(A \cup B) = 1 - \dfrac{800}{1000} = 0.20$

(d) $P(A' \cap B) = \dfrac{63+35+15}{1000} = 0.113$

(e) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.032}{(25+63+15+7+35)/1000} = 0.2207$

(f) $P = \dfrac{5}{1000} = 0.005$

CHAPTER 3

Section 3-1

3-4. The range of X is $\{0,1,2,3,4,5\}$

3-7. The range of X is conveniently modeled as all nonnegative integers. That is, the range of X is $\{0,1,2,\ldots\}$

3-10. The possible totals for two orders are 1/8 + 1/8 = 1/4, 1/8 + 1/4 = 3/8, 1/8 + 3/8 = 1/2, 1/4 + 1/4 = 1/2, 1/4 + 3/8 = 5/8, 3/8 + 3/8 = 6/8.
Therefore the range of X is $\left\{\frac{1}{4},\frac{3}{8},\frac{1}{2},\frac{5}{8},\frac{6}{8}\right\}$

Section 3-2

3-14.

$$f_X(0) = P(X = 0) = 1/6 + 1/6 = 1/3$$
$$f_X(1.5) = P(X = 1.5) = 1/3$$
$$f_X(2) = 1/6$$
$$f_X(3) = 1/6$$

a) $P(X = 1.5) = 1/3$
b) $P(0.5 < X < 2.7) = P(X = 1.5) + P(X = 2) = 1/3 + 1/6 = 1/2$
c) $P(X > 3) = 0$
d) $P(0 \le X < 2) = P(X = 0) + P(X = 1.5) = 1/3 + 1/3 = 2/3$
e) $P(X = 0 \text{ or } X = 2) = 1/3 + 1/6 = 1/2$

3-21. X = number of wafers that pass
$P(X=0) = (0.2)^3 = 0.008$
$P(X=1) = 3(0.2)^2(0.8) = 0.096$
$P(X=2) = 3(0.2)(0.8)^2 = 0.384$
$P(X=3) = (0.8)^3 = 0.512$

3-22. X: the number of computers that vote for a left roll when a right roll is appropriate.
p=0.0001.
$P(X=0)=(1-p)^4=0.9999^4=0.9996$
$P(X=1)=4*(1-p)^3p=4*0.9999^3 0.0001=0.0003999$
$P(X=2)=C_4^2(1-p)^2p^2=5.999*10^{-8}$
$P(X=3)=C_4^3(1-p)^1p^3=3.9996*10^{-12}$
$P(X=4)=C_4^0(1-p)^0p^4=1*10^{-16}$

Section 3-3

3-35.

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.008, & 0 \le x < 1 \\ 0.104, & 1 \le x < 2 \\ 0.488, & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

.

$$f(0) = 0.2^3 = 0.008,$$
$$f(1) = 3(0.2)(0.2)(0.8) = 0.096,$$
$$f(2) = 3(0.2)(0.8)(0.8) = 0.384,$$
$$f(3) = (0.8)^3 = 0.512,$$

3-36.

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.9996, & 0 \le x < 1 \\ 0.9999, & 1 \le x < 3 \\ 0.99999, & 3 \le x < 4 \\ 1, & 4 \le x \end{cases}$$

$$f(0) = 0.9999^4 = 0.9996,$$
$$f(1) = 4(0.9999^3)(0.0001) = 0.0003999,$$
$$f(2) = 5.999 * 10^{-8},$$
$$f(3) = 3.9996 * 10^{-12},$$
$$f(4) = 1 * 10^{-16}$$

3-40. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f(1) = 0.7, f(4) = 0.2, f(7) = 0.1
a) $P(X \le 4) = 0.9$
b) $P(X > 7) = 0$
c) $P(X \le 5) = 0.9$
d) $P(X>4) = 0.1$
e) $P(X\le 2) = 0.7$

3-42. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
pmf: f1/8) = 0.2, f(1/4) = 0.7, f(3/8) = 0.1
a) $P(X\le 1/18) = 0$
b) $P(X\le 1/4) = 0.9$
c) $P(X\le 5/16) = 0.9$
d) $P(X>1/4) = 0.1$
e) $P(X\le 1/2) = 1$

Section 3-4

3- 48. Mean and Variance for random variable in exercise 3-14

$$\mu = E(X) = 0f(0) + 1.5f(1.5) + 2f(2) + 3f(3)$$
$$= 0(1/3) + 1.5(1/3) + 2(1/6) + 3(1/6) = 1.333$$
$$V(X) = 0^2 f(0) + 1.5^2 f(1) + 2^2 f(2) + 3^2 f(3) - \mu^2$$
$$= 0(1/3) + 2.25(1/3) + 4(1/6) + 9(1/6) - 1.333^2 = 1.139$$

3-56. (a) F(0)=0.17

Nickel Charge: X	CDF
0	0.17
2	0.17+0.35=0.52
3	0.17+0.35+0.33=0.85
4	0.17+0.35+0.33+0.15=1

(b)E(X) = 0*0.17+2*0.35+3*0.33+4*0.15=2.29

$$V(X) = \sum_{i=1}^{4} f(x_i)(x_i - \mu)^2 = 1.5259$$

3-57. X = number of computers that vote for a left roll when a right roll is appropriate.
μ = E(X)=0*f(0)+1*f(1)+2*f(2)+3*f(3)+4*f(4)
= 0+0.0003999+2*5.999*10^{-8}+3*3.9996*10^{-12}+4*1*10^{-16}= 0.0004

$$V(X)=\sum_{i=1}^{5} f(x_i)(x_i - \mu)^2 = 0.00039996$$

Section 3-5

3-66. X=(1/100)Y, Y = 15, 16, 17, 18, 19.

$$E(X) = (1/100)\ E(Y) = \frac{1}{100}\left(\frac{15+19}{2}\right) = 0.17 \text{ mm}$$

$$V(X) = \left(\frac{1}{100}\right)^2 \left[\frac{(19-15+1)^2 - 1}{12}\right] = 0.0002 \text{ mm}^2$$

3-69. a = 675, b = 700
a) μ = E(X) = (a+b)/2= 687.5
V(X) = [(b – a +1)^2 – 1]/12= 56.25

b) a = 75, b = 100
μ = E(X) = (a+b)/2 = 87.5
V(X) = [(b – a + 1)^2 – 1]/12= 56.25
The range of values is the same, so the mean shifts by the difference in the two minimums (or maximums) whereas the variance does not change.

3-71. The range of Y is 0, 5, 10, ..., 45, E(X) = (0+9)/2 = 4.5
E(Y) = 0(1/10)+5(1/10)+...+45(1/10)
= 5[0(0.1) +1(0.1)+ ... +9(0.1)]
= 5E(X)
= 5(4.5)
= 22.5
V(X) = 8.25, V(Y) = 5^2(8.25) = 206.25, σ_Y = 14.36

Section 3-6

3-87. Let X denote the number of mornings the light is green.

$$a) \quad P(X=1) = \binom{5}{1} 0.2^1 0.8^4 = 0.410$$

$$b) \quad P(X=4) = \binom{20}{4} 0.2^4 0.8^{16} = 0.218$$

$$c) \quad P(X>4) = 1 - P(X \le 4) = 1 - 0.630 = 0.370$$

3-92. E(X) = 20 (0.01) = 0.2
V(X) = 20 (0.01) (0.99) = 0.198

$\mu_X + 3\sigma_X = 0.2 + 3\sqrt{0.198} = 1.53$

a) X is binomial with n = 20 and p = 0.01

$$P(X > 1.53) = P(X \geq 2) = 1 - P(X \leq 1)$$
$$= 1 - \left[\binom{20}{0}0.01^0 0.99^{20} + \binom{20}{1}0.01^1 0.99^{19}\right] = 0.0169$$

b) X is binomial with n = 20 and p = 0.04

$$P(X > 1) = 1 - P(X \leq 1)$$
$$= 1 - \left[\binom{20}{0}0.04^0 0.96^{20} + \binom{20}{1}0.04^1 0.96^{19}\right] = 0.1897$$

c) Let Y denote the number of times X exceeds 1 in the next five samples.
Then, Y is binomial with n = 5 and p = 0.190 from part b.

$$P(Y \geq 1) = 1 - P(Y = 0) = 1 - \left[\binom{5}{0}0.190^0 0.810^5\right] = 0.651$$

The probability is 0.651 that at least one sample from the next five will contain more than one defective

3-93. Let X denote the passengers with tickets that do not show up for the flight.
Then, X is binomial with n = 125 and p = 0.1.

$$a) P(X \geq 5) = 1 - P(X \leq 4)$$
$$= 1 - \left[\begin{array}{l}\binom{125}{0}0.1^0(0.9)^{125} + \binom{125}{1}0.1^1(0.9)^{124} + \binom{125}{2}0.1^2(0.9)^{123} \\ + \binom{125}{3}0.1^3(0.9)^{122} + \binom{125}{4}0.1^4(0.9)^{121}\end{array}\right]$$
$$= 0.9961$$
$$b) P(X > 5) = 1 - P(X \leq 5) = 0.9886$$

Section 3-7

3-103. Let X denote the number of trials to obtain the first successful alignment.
Then X is a geometric random variable with p = 0.8

a) $P(X = 4) = (1-0.8)^3 0.8 = 0.2^3 0.8 = 0.0064$

b) $P(X \leq 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$
$$= (1-0.8)^0 0.8 + (1-0.8)^1 0.8 + (1-0.8)^2 0.8 + (1-0.8)^3 0.8$$
$$= 0.8 + 0.2(0.8) + 0.2^2(0.8) + 0.2^3 0.8 = 0.9984$$

c) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - [P(X = 1) + P(X = 2) + P(X = 3)]$
$$= 1 - [(1-0.8)^0 0.8 + (1-0.8)^1 0.8 + (1-0.8)^2 0.8]$$
$$= 1 - [0.8 + 0.2(0.8) + 0.2^2(0.8)] = 1 - 0.992 = 0.008$$

3-108. X = number of attempts before the hacker selects a user password.

(a) $p = 9900/36^6 = 0.0000045$

$\mu = E(X) = 1/p = 219877$

$V(X) = (1-p)/p^2 = 4.938 \times 10^{10}$

$\sigma = \sqrt{V(X)} = 222222$

(b) $p = 100/36^3 = 0.00214$

$\mu = E(X) = 1/p = 467$

$V(X) = (1-p)/p^2 = 217892.39$

$\sigma = \sqrt{V(X)} = 466.78$

Based on the answers to (a) and (b) above, it is clearly more secure to use a 6 character password.

3-111. Let X denote the number of transactions until all computers have failed.
Then, X is negative binomial random variable with $p = 10^{-8}$ and $r = 3$.
a) $E(X) = 3 \times 10^8$
b) $V(X) = [3(1-10^{-8}0]/(10^{-16}) = 3.0 \times 10^{16}$

Section 3-8

3-122. Let X denote the number of cards in the sample that are defective.
a)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \frac{\binom{20}{0}\binom{120}{20}}{\binom{140}{20}} = \frac{\frac{120!}{20!100!}}{\frac{140!}{20!120!}} = 0.0356$$

$$P(X \geq 1) = 1 - 0.0356 = 0.9644$$

b)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \frac{\binom{5}{0}\binom{135}{20}}{\binom{140}{20}} = \frac{\frac{135!}{20!115!}}{\frac{140!}{20!120!}} = \frac{135!120!}{115!140!} = 0.4571$$

$$P(X \geq 1) = 1 - 0.4571 = 0.5429$$

3-124. Let X denote the count of the numbers in the state's sample that match those in the player's sample.
Then, X has a hypergeometric distribution with N = 40, n = 6, and K = 6.

a) $P(X = 6) = \frac{\binom{6}{6}\binom{34}{0}}{\binom{40}{6}} = \left(\frac{40!}{6!34!}\right)^{-1} = 2.61 \times 10^{-7}$

b) $P(X = 5) = \frac{\binom{6}{5}\binom{34}{1}}{\binom{40}{6}} = \frac{6 \times 34}{\binom{40}{6}} = 5.31 \times 10^{-5}$

c) $P(X = 4) = \frac{\binom{6}{4}\binom{34}{2}}{\binom{40}{6}} = 0.00219$

d) Let Y denote the number of weeks needed to match all six numbers.

Then, Y has a geometric distribution with $p = \frac{1}{3{,}838{,}380}$ and

$E(Y) = 1/p = 3{,}838{,}380$ weeks. This is more than 738 centuries!

3-125. Let X denote the number of blades in the sample that are dull.
a)

$$P(X \geq 1) = 1 - P(X = 0)$$

$$P(X = 0) = \frac{\binom{10}{0}\binom{38}{5}}{\binom{48}{5}} = \frac{\frac{38!}{5!33!}}{\frac{48!}{5!43!}} = \frac{38!43!}{48!33!} = 0.2931$$

$$P(X \geq 1) = 1 - P(X = 0) = 0.7069$$

b) Let Y denote the number of days needed to replace the assembly.

P(Y = 3) = $0.2931^2(0.7069) = 0.0607$

c) On the first day, $P(X=0) = \frac{\binom{2}{0}\binom{46}{5}}{\binom{48}{5}} = \frac{\frac{46!}{5!41!}}{\frac{48!}{5!43!}} = \frac{46!43!}{48!41!} = 0.8005$

On the second day, $P(X=0) = \frac{\binom{6}{0}\binom{42}{5}}{\binom{48}{5}} = \frac{\frac{42!}{5!37!}}{\frac{48!}{5!43!}} = \frac{42!43!}{48!37!} = 0.4968$

On the third day, P(X = 0) = 0.2931 from part a). Therefore,
P(Y = 3) = 0.8005(0.4968)(1-0.2931) = 0.2811.

Section 3-9

3-133. λ=1, Poisson distribution. $f(x) = e^{-\lambda}\lambda^x/x!$

(a) $P(X\geq 2) = 0.264$

(b) In order that $P(X\geq 1) = 1-P(X=0) = 1-e^{-\lambda}$ exceed 0.95, we need λ=3.
Therefore 3*16=48 cubic light years of space must be studied.

3-134. (a) λ=14.4, $P(X=0)=6*10^{-7}$

(b) λ=14.4/5=2.88, P(X=0)=0.056

(c) λ=14.4*7*28.35/225=12.7

$P(X\geq 1)$=0.999997

(d) $P(X\geq 28.8) = 1-P(X \leq 28) = 0.00046$. Unusual.

3-136. a) Let X denote the number of flaws in one square meter of cloth. Then, X is a Poisson random variable with λ = 0.1. $P(X=2) = \frac{e^{-0.1}(0.1)^2}{2!} = 0.0045$

b) Let Y denote the number of flaws in 10 square meters of cloth. Then, Y is a Poisson random variable with λ = 1. $P(Y=1) = \frac{e^{-1}1^1}{1!} = e^{-1} = 0.3679$

c) Let W denote the number of flaws in 20 square meters of cloth. Then, W is a Poisson random variable with λ = 2. $P(W=0) = e^{-2} = 0.1353$

d) $$\begin{aligned} P(Y \geq 2) &= 1 - P(Y \leq 1) = 1 - P(Y=0) - P(Y=1) \\ &= 1 - e^{-1} - e^{-1} \\ &= 0.2642 \end{aligned}$$

Supplemental Exercises

3-152. Geometric with p=0.1

(a) $f(x)=(1-p)^{x-1}p=0.9^{(x-1)}0.1$

(b) $P(X=5) = 0.9^4*0.1=0.0656$

(c) μ=E(X)= 1/p=10

(d) $P(X\leq 10)$=0.651

3-155. Let X denote the number of calls that are answered in 30 seconds or less.
Then, X is a binomial random variable with p = 0.75.

a) P(X = 9) = $\binom{10}{9}(0.75)^9(0.25)^1 = 0.1877$

b) $P(X \geq 16)$ = P(X=16) +P(X=17) + P(X=18) + P(X=19) + P(X=20)

$$= \binom{20}{16}(0.75)^{16}(0.25)^4 + \binom{20}{17}(0.75)^{17}(0.25)^3 + \binom{20}{18}(0.75)^{18}(0.25)^2$$
$$+ \binom{20}{19}(0.75)^{19}(0.25)^1 + \binom{20}{20}(0.75)^{20}(0.25)^0 = 0.4148$$

c) E(X) = 20(0.75) = 15

3-157. Let W denote the number of calls needed to obtain two answers in less than 30 seconds. Then, W has a negative binomial distribution with p = 0.75.

a) P(W=6) = $\binom{5}{1}(0.25)^4(0.75)^2 = 0.0110$

b) E(W) = r/p = 2/0.75 = 8/3

3-167.
$$f_X(0) = (0.1)(0.7) + (0.3)(0.3) = 0.16$$
$$f_X(1) = (0.1)(0.7) + (0.4)(0.3) = 0.19$$
$$f_X(2) = (0.2)(0.7) + (0.2)(0.3) = 0.20$$
$$f_X(3) = (0.4)(0.7) + (0.1)(0.3) = 0.31$$
$$f_X(4) = (0.2)(0.7) + (0)(0.3) = 0.14$$

3-168. a) $P(X \le 3) = 0.2 + 0.4 = 0.6$
b) $P(X > 2.5) = 0.4 + 0.3 + 0.1 = 0.8$
c) $P(2.7 < X < 5.1) = 0.4 + 0.3 = 0.7$
d) E(X) = 2(0.2) + 3(0.4) + 5(0.3) + 8(0.1) = 3.9
e) $V(X) = 2^2(0.2) + 3^2(0.4) + 5^2(0.3) + 8^2(0.1) - (3.9)^2 = 3.09$

3-169.

x	2	5.7	6.5	8.5
f(x)	0.2	0.3	0.3	0.2

3-170. Let X and Y denote the number of bolts in the sample from supplier 1 and 2, respectively.
Then, X is a hypergeometric random variable with N = 100, n = 4, and K = 30.
Also, Y is a hypergeometric random variable with N = 100, n = 4, and K = 70.
a) P(X=4 or Y=4) = P(X = 4) + P(Y = 4)

$$= \frac{\binom{30}{4}\binom{70}{0}}{\binom{100}{4}} + \frac{\binom{30}{0}\binom{70}{4}}{\binom{100}{4}}$$
$$= 0.2408$$

b) P[(X=3 and Y=1) or (Y=3 and X = 1)]= $= \frac{\binom{30}{3}\binom{70}{1} + \binom{30}{1}\binom{70}{3}}{\binom{100}{4}} = 0.4913$

3-171. Let X denote the number of errors in a sector. Then, X is a Poisson random variable with $\lambda = 0.32768$.
a) $P(X>1) = 1 - P(X\le 1) = 1 - e^{-0.32768} - e^{-0.32768}(0.32768) = 0.0433$
b) Let Y denote the number of sectors until an error is found.

Then, Y is a geometric random variable and $P = P(X \geq 1) = 1 - P(X=0) = 1 - e^{-0.32768} = 0.2794$
$E(Y) = 1/p = 3.58$

3-176. a) Let X denote the number of flaws in 50 panels.
Then, X is a Poisson random variable with $\lambda = 50(0.02) = 1$.
$P(X = 0) = e^{-1} = 0.3679$.
b) Let Y denote the number of flaws in one panel.
$P(Y \geq 1) = 1 - P(Y=0) = 1 - e^{-0.02} = 0.0198$.
Let W denote the number of panels that need to be inspected before a flaw is found.
Then W is a geometric random variable with p = 0.0198.
E(W) = 1/0.0198 = 50.51 panels.
c) $P(Y \geq 1) = 1 - P(Y = 0) = 1 - e^{-0.02} = 0.0198$
Let V denote the number of panels with 1 or more flaws.
Then V is a binomial random variable with n = 50 and p = 0.0198

$$P(V \leq 2) = \binom{50}{0} 0.0198^0 (.9802)^{50} + \binom{50}{1} 0.0198^1 (0.9802)^{49}$$
$$+ \binom{50}{2} 0.0198^2 (0.9802)^{48} = 0.9234$$

CHAPTER 4

Section 4-2

4-8. a) $P(X > 3000) = \int_{3000}^{\infty} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_{3000}^{\infty} = e^{-3} = 0.05$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233$

c) $P(X < 1000) = \int_{0}^{1000} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_{0}^{1000} = 1 - e^{-1} = 0.6321$

d) $P(X < x) = \int_{0}^{x} \frac{e^{\frac{-x}{1000}}}{1000} dx = -e^{\frac{-x}{1000}}\Big|_{0}^{x} = 1 - e^{-x/1000} = 0.10$.

Then, $e^{-x/1000} = 0.9$, and x = –1000 ln 0.9 = 105.36.

4-9. a) $P(X > 50) = \int_{50}^{50.25} 2.0 dx = 2x\Big|_{50}^{50.25} = 0.5$

b) $P(X > x) = 0.90 = \int_{x}^{50.25} 2.0 dx = 2x\Big|_{x}^{50.25} = 100.5 - 2x$

Then, 2x = 99.6 and x = 49.8.

4-11. a) P(X < 2.25 or X > 2.75) = P(X < 2.25) + P(X > 2.75) because the two events are mutually exclusive. Then, P(X < 2.25) = 0 and

$$P(X > 2.75) = \int_{2.75}^{2.8} 2 dx = 2(0.05) = 0.10.$$

b) If the probability density function is centered at 2.55 meters, then $f_X(x) = 2$ for 2.3 < x < 2.8 and all rods will meet specifications.

Section 4-3

4-20. Now, $f(x) = \frac{e^{-x/1000}}{1000}$ for 0 < x and

$$F_X(x) = \frac{1}{1000}\int_0^x e^{-x/1000} dy = -e^{-y/1000}\Big|_0^x = 1 - e^{-x/1000} \text{ for } 0 < x.$$

Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/1000}, & x > 0 \end{cases}$

$P(X>3000) = 1 - P(X \le 3000) = 1 - F(3000) = e^{-3000/1000} = 0.5$

4-22. Now, $f(x)=\frac{e^{-x/10}}{10}$ for 0 < x and

$$F_X(x)=1/10\int_0^x e^{-x/10}dx=-e^{-x/10}\Big|_0^x=1-e^{-x/10}$$

for 0 < x.

Then, $F_X(x)=\begin{cases}0, & x\le 0\\ 1-e^{-x/10}, & x>0\end{cases}$

a) P(X<60) = F(60) =1 - e^{-6} = 1 - 0.002479 = 0.9975

b) $1/10\int_{15}^{30} e^{-x/10}dx = e^{-1.5}-e^{-3}=0.173343$

c) $P(X_1>40)+P(X_1<40 \text{ and } X_2>40)= e^{-4}+(1-e^{-4})\,e^{-4}=0.0363$

d) P(15<X<30) = F(30)-F(15) = $e^{-1.5}-e^{-3}=0.173343$

4-26. $f_X(x)=\begin{cases}0.25, & -2<x<1\\ 0.5, & 1\le x<1.5\end{cases}$

Section 4-4

4-28. $E(X)=\int_0^4 0.125x^2dx=0.125\frac{x^3}{3}\Big|_0^4=2.6667$

$$V(X)=\int_0^4 0.125x(x-\tfrac{8}{3})^2dx=0.125\int_0^4(x^3-\tfrac{16}{3}x^2+\tfrac{64}{9}x)dx$$

$$=0.125(\tfrac{x^4}{4}-\tfrac{16}{3}\tfrac{x^3}{3}+\tfrac{64}{9}\cdot\tfrac{1}{2}x^2)\Big|_0^4=0.88889$$

4-34. a)

$$E(X)=\int_{1200}^{1210}x0.1dx=0.05x^2\Big|_{1200}^{1210}=1205$$

$$V(X)=\int_{1200}^{1210}(x-1205)^2 0.1dx=0.1\frac{(x-1205)^3}{3}\Big|_{1200}^{1210}=8.333$$

$$\textit{Therefore,}\quad \sigma_x=\sqrt{V(X)}=2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1195<X<1205)=P(1200<X<1205)=\int_{1200}^{1205}0.1dx=0.1x\Big|_{1200}^{1205}=0.5$$

4-37. a) $E(X)=\int_5^\infty x10e^{-10(x-5)}dx$.

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -xe^{-10(x-5)}\Big|_5^{\infty} + \int_5^{\infty} e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Bigg|_5^{\infty} = 5.1$$

Now, $V(X) = \int_5^{\infty} (x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2 e^{-10(x-5)}\Big|_5^{\infty} + 2\int_5^{\infty}(x-5.1)e^{-10(x-5)}dx$. From the definition of E(X) the integral above is recognized to equal 0.

Therefore, $V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$

<u>Section 4-5</u>

4-41. a) The distribution of X is $f(x) = 10$ for $0.95 < x < 1.05$. Now,

$$F_X(x) = \begin{cases} 0, & x < 0.95 \\ 10x - 9.5, & 0.95 \le x < 1.05 \\ 1, & 1.05 \le x \end{cases}$$

b) $P(X > 1.02) = 1 - P(X \le 1.02) = 1 - F_X(1.02) = 0.3$

c) If P(X > x) = 0.90, then 1 – F(X) = 0.90 and F(X) = 0.10. Therefore, 10x - 9.5 = 0.10 and x = 0.96.

d) E(X) = (1.05 + 0.95)/2 = 1.00 and V(X) = $\frac{(1.05-0.95)^2}{12} = 0.00083$

4-42. $E(X) = \frac{(1.5+2.2)}{2} = 1.85\,\text{min}$

$$V(X) = \frac{(2.2-1.5)^2}{12} = 0.0408\,\text{min}^2$$

b) $P(X < 2) = \int_{1.5}^{2} \frac{1}{(2.2-1.5)}dx = \int_{1.5}^{2}(1/0.7)dx = (1/0.7)x\big|_{1.5}^{2} = (1/0.7)(0.5) = 0.7143$

c.) $F(X) = \int_{1.5}^{x} \frac{1}{(2.2-1.5)}dy = \int_{1.5}^{x}(1/0.7)dy = (1/0.7)y\big|_{1.5}^{x}$ for 1.5 < x < 2.2. Therefore,

$$F(x) = \begin{cases} 0, & x < 1.5 \\ (1/0.7)x - 2.14, & 1.5 \le x < 2.2 \\ 1, & 2.2 \le x \end{cases}$$

4-48. (a) Let X denote the measured voltage.

So the probability mass function is $P(X = x) = \frac{1}{6}, \quad for \quad x = 247,...,253$

(b) E(X)=250

Var(X)= $\frac{(253-247+1)^2-1}{12} = 4$

Section 4-6

4-50. a) P(–1 < Z < 1) = P(Z < 1) – P(Z > 1)
= 0.84134 – (1 – 0.84134)
= 0.68268

b) P(–2 < Z < 2) = P(Z < 2) – [1 – P(Z < 2)]
= 0.9545

c) P(–3 < Z < 3) = P(Z < 3) – [1 – P(Z < 3)]
= 0.9973

d) P(Z > 3) = 1 – P(Z < 3)
= 0.00135

e) P(0 < Z < 1) = P(Z < 1) – P(Z < 0)
= 0.84134 – 0.5 = 0.34134

4-51. a) P(Z < 1.28) = 0.90
b) P(Z < 0) = 0.5
c) If P(Z > z) = 0.1, then P(Z < z) = 0.90 and z = 1.28
d) If P(Z > z) = 0.9, then P(Z < z) = 0.10 and z = –1.28
e) P(–1.24 < Z < z) = P(Z < z) – P(Z < –1.24)
= P(Z < z) – 0.10749.
Therefore, P(Z < z) = 0.8 + 0.10749 = 0.90749 and z = 1.33

4-55. a) P(X < 11) = $P\left(Z < \frac{11-5}{4}\right)$
= P(Z < 1.5)
= 0.93319

b) P(X > 0) = $P\left(Z > \frac{0-5}{4}\right)$
= P(Z > –1.25)
= 1 – P(Z < –1.25)
= 0.89435

c) P(3 < X < 7) = $P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right)$
= P(–0.5 < Z < 0.5)
= P(Z < 0.5) – P(Z < –0.5)
= 0.38292

d) P(–2 < X < 9) = $P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right)$
= P(–1.75 < Z < 1)
= P(Z < 1) – P(Z < –1.75)]
= 0.80128

e) P(2 < X < 8) = $P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right)$
=P(–0.75 < Z < 0.75)
= P(Z < 0.75) – P(Z < –0.75)
= 0.54674

4-57. a) P(X < 6250) = $P\left(Z < \frac{6250-6000}{100}\right)$
= P(Z < 2.5)
= 0.99379

b) P(5800 < X < 5900) = $P\left(\frac{5800-6000}{100} < Z < \frac{5900-6000}{100}\right)$
=P(–2 < Z < –1)
= P(Z <– 1) – P(Z < –2)
= 0.13591

c) P(X > x) = $P\left(Z > \frac{x-6000}{100}\right)$ = 0.95.

Therefore, $\frac{x-6000}{100}$ = –1.65 and x = 5835.

4-60. Let X denote the cholesterol level.
X ~ N(159.2, σ^2)

(a) $P(X < 200) = \Phi(\frac{200-159.2}{\sigma}) = 0.841$

$$\frac{200-159.2}{\sigma} = \Phi^{-1}(0.841)$$

$$\sigma = \frac{200-159.2}{\Phi^{-1}(0.841)} = 40.8582$$

(b) $\Phi^{-1}(0.25)\times 40.8528 + 159.2 = 131.6452$

$\Phi^{-1}(0.75)\times 40.8528 + 159.2 = 186.7548$

(c) $\Phi^{-1}(0.9)\times 40.8528 + 159.2 = 211.5550$

(d) $\Phi(2) - \Phi(1) = 0.1359$

(e) $1 - \Phi(2) = 0.0228$

(f) $\Phi(1) = 0.8413$

4-70. a) P(X > 0.0026) = $P\left(Z > \frac{0.0026-0.002}{0.0004}\right)$
= P(Z > 1.5)
= 1-P(Z < 1.5)
= 0.06681.

b) P(0.0014 < X < 0.0026) = $P\left(\frac{0.0014-0.002}{0.0004} < Z < \frac{0.0026-0.002}{0.0004}\right)$

$= P(-1.5 < Z < 1.5)$
$= 0.86638.$

c) $P(0.0014 < X < 0.0026) = P\left(\frac{0.0014 - 0.002}{\sigma} < Z < \frac{0.0026 - 0.002}{\sigma}\right)$

$= P\left(\frac{-0.0006}{\sigma} < Z < \frac{0.0006}{\sigma}\right).$

Therefore, $P\left(Z < \frac{0.0006}{\sigma}\right) = 0.9975$. Therefore, $\frac{0.0006}{\sigma} = 2.81$ and $\sigma = 0.000214$.

4-71. a) $P(X > 13) = P\left(Z > \frac{13-12}{0.5}\right) = P(Z > 2) = 0.02275$

b) If $P(X < 13) = 0.999$, then $P\left(Z < \frac{13-12}{\sigma}\right) = 0.999$.

Therefore, $1/\sigma = 3.09$ and $\sigma = 1/3.09 = 0.324$.

c) If $P(X < 13) = 0.999$, then $P\left(Z < \frac{13-\mu}{0.5}\right) = 0.999$.

Therefore, $\frac{13-\mu}{0.5} = 3.09$ and $\mu = 11.455$

<u>Section 4-7</u>

4-78. a) $P(X < 4) = \sum_{i=0}^{3} \frac{e^{-6}6^i}{i!} = 0.1512$

b) X is approximately $X \sim N(6,6)$

Then, $P(X < 4) \cong P\left(Z < \frac{4-6}{\sqrt{6}}\right) = P(Z < -0.82) = 0.206108$

If a continuity correction were used the following result is obtained.

$$P(X < 4) = P(X \le 3) \cong P\left(Z \le \frac{3+0.5-6}{\sqrt{6}}\right) = P(Z \le -1.02) = 0.1539$$

c) $P(8 < X < 12) \cong P\left(\frac{8-6}{\sqrt{6}} < Z < \frac{12-6}{\sqrt{6}}\right) = P(0.82 < Z < 2.45) = 0.1990$

If a continuity correction were used the following result is obtained.

$$P(8 < X < 12) = P(9 \le X \le 11) \cong P\left(\frac{9-0.5-6}{\sqrt{6}} \le Z \le \frac{11+0.5-6}{\sqrt{6}}\right)$$

$$\cong P(1.02 < Z < 2.25) = 0.1416$$

4-83. Let X denote the number of original components that fail during the useful life of the product. Then, X is a binomial random variable with p = 0.001 and n = 5000. Also, E(X) = 5000 (0.001) = 5 and V(X) = 5000(0.001)(0.999) = 4.995.

$$P(X \ge 10) \cong P\left(Z \ge \frac{9.5-5}{\sqrt{4.995}}\right) = P(Z \ge 2.01) = 1 - P(Z < 2.01) = 1 - 0.978 = 0.022$$

4-86. X is the number of minor errors on a test pattern of 1000 pages of text. X is a Poisson random variable with a mean of 0.4 per page

a) The numbers of errors per page are random variables. The assumption that the occurrence of an event in a subinterval in a Poisson process is independent of events in other subintervals implies that the numbers of events in disjoint intervals are independent. The pages are disjoint intervals and the consequently the error counts per page are independent.

b) $P(X = 0) = \dfrac{e^{-0.4} 0.4^0}{0!} = 0.670$

$P(X \ge 1) = 1 - P(X = 0) = 1 - 0.670 = 0.330$

The mean number of pages with one or more errors is 1000(0.330) = 330 pages

c) Let Y be the number of pages with errors.

$$P(Y > 350) \cong P\left(Z \ge \frac{350.5-330}{\sqrt{1000(0.330)(0.670)}}\right) = P(Z \ge 1.38) = 1 - P(Z < 1.38)$$
$$= 1 - 0.9162 = 0.0838$$

Section 4-8

4-95. Let X denote the time until the first call. Then, X is exponential and $\lambda = \frac{1}{E(X)} = \frac{1}{15}$ calls/minute.

a) $P(X > 30) = \int_{30}^{\infty} \frac{1}{15} e^{-\frac{x}{15}} dx = -e^{-\frac{x}{15}} \Big|_{30}^{\infty} = e^{-2} = 0.1353$

b) The probability of at least one call in a 10-minute interval equals one minus the probability of zero calls in a 10-minute interval and that is P(X > 10).

$P(X > 10) = -e^{-\frac{x}{15}} \Big|_{10}^{\infty} = e^{-2/3} = 0.5134$.

Therefore, the answer is 1- 0.5134 = 0.4866. Alternatively, the requested probability is equal to P(X < 10) = 0.4866.

c) $P(5 < X < 10) = -e^{-\frac{x}{15}} \Big|_{5}^{10} = e^{-1/3} - e^{-2/3} = 0.2031$

d) P(X < x) = 0.90 and $P(X < x) = -e^{-\frac{t}{15}} \Big|_{0}^{x} = 1 - e^{-x/15} = 0.90$. Therefore, x = 34.54 minutes.

4-99. Let X denote the time until the arrival of a taxi. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.1$ arrivals/ minute.

a) P(X > 60) = $\int_{60}^{\infty} 0.1 e^{-0.1x} dx = -e^{-0.1x} \Big|_{60}^{\infty} = e^{-6} = 0.0025$

b) P(X < 10) = $\int_{0}^{10} 0.1 e^{-0.1x} dx = -e^{-0.1x} \Big|_{0}^{10} = 1 - e^{-1} = 0.6321$

c) P(X > x) = $\int_x^{\infty} 0.1e^{-0.1t}dt = -e^{-0.1t}\Big|_x^{\infty} = e^{-0.1x} = 0.1$ and x = 23.03 minutes.

d) P(X < x) = 0.9 implies that P(X > x) = 0.1. Therefore, this answer is the same as part c).

e) P(X < x) = $-e^{-0.1t}\Big|_0^{x} = 1 - e^{-0.1x} = 0.5$ and x = 6.93 minutes.

4-104. Let Y denote the number of arrivals in one hour. If the time between arrivals is exponential, then the count of arrivals is a Poisson random variable and $\lambda = 1$ arrival per hour.

a) P(Y > 3) = $1 - P(Y \le 3) = 1 - \left[\frac{e^{-1}1^0}{0!} + \frac{e^{-1}1^1}{1!} + \frac{e^{-1}1^2}{2!} + \frac{e^{-1}1^3}{3!}\right] = 0.01899$

b) From part a), P(Y > 3) = 0.01899. Let W denote the number of one-hour intervals out of 30 that contain more than 3 arrivals. By the memoryless property of a Poisson process, W is a binomial random variable with n = 30 and p = 0.01899.

P(W = 0) = $\binom{30}{0}0.01899^0(1-0.01899)^{30} = 0.5626$

c) Let X denote the time between arrivals. Then, X is an exponential random variable with $\lambda = 1$ arrivals per hour. P(X > x) = 0.1 and $P(X > x) = \int_x^{\infty} 1e^{-1t}dt = -e^{-1t}\Big|_x^{\infty} = e^{-1x} = 0.1$. Therefore, x = 2.3 hours.

4-157. a) P(X > 90.3) + P(X < 89.7)

$$= P\left(Z > \frac{90.3 - 90.2}{0.1}\right) + P\left(Z < \frac{89.7 - 90.2}{0.1}\right)$$

= P(Z > 1) + P(Z < −5)
= 1 − P(Z < 1) + P(Z < −5)
=1 − 0.84134 +0
= 0.15866.

Therefore, the answer is 0.15866.

b) The process mean should be set at the center of the specifications; that is, at μ = 90.0.

c) P(89.7 < X < 90.3) = $P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right)$

= P(−3 < Z < 3) = 0.9973.

The yield is 100*0.9973 = 99.73%

d) P(89.7 < X < 90.3) = $P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right)$

= P(−3 < Z < 3)
= 0.9973.

P(X=10) = $(0.9973)^{10}$ = 0.9733

e) Let *Y* represent the number of cases out of the sample of 10 that are between 89.7 and 90.3 ml. Then Y follows a binomial distribution with *n*=10 and *p*=0.9973. Thus, E(*Y*)= 9.973 or 10.

4-159. E(X) = 1000(0.2) = 200 and V(X) = 1000(0.2)(0.8) = 160

a) $P(X > 225) = P(X \ge 226) \cong 1 - P(Z \le \frac{225.5-200}{\sqrt{160}}) = 1 - P(Z \le 2.02) = 1 - 0.9783 = 0.0217$

b)
$P(175 \le X \le 225) \cong P(\frac{174.5-200}{\sqrt{160}} \le Z \le \frac{225.5-200}{\sqrt{160}}) = P(-2.02 \le Z \le 2.02)$
$= 0.9783 - 0.0217 = .9566$

c) If P($X > x$) = 0.01, then $P\left(Z > \dfrac{x-200}{\sqrt{160}}\right) = 0.01$.

Therefore, $\frac{x-200}{\sqrt{160}} = 2.33$ and $x = 229.5$

4-160. The time to failure (in hours) for a laser in a cytometry machine is modeled by an exponential distribution with 0.00004.

a) $P(X > 20{,}000) = \int_{20000}^{\infty} 0.00004e^{-0.00004x}dx = -e^{-0.00004x}\Big|_{20000}^{\infty} = e^{-0.8} = 0.4493$

b) $P(X < 30{,}000) = \int_{30000}^{\infty} 0.00004e^{-0.00004x}dx = -e^{-0.00004x}\Big|_{0}^{30000} = 1 - e^{-1.2} = 0.6988$

c)

$$P(20{,}000 < X < 30{,}000) = \int_{20000}^{30000} 0.00004e^{-0.00004x}dx$$
$$= -e^{-0.00004x}\Big|_{20000}^{30000} = e^{-0.8} - e^{-1.2} = 0.1481$$

4-165. a) $P(X < 2.5) = \int_{2}^{2.5} (0.5x-1)dx = \left(0.5\dfrac{x^2}{2} - x\right)\Big|_{2}^{2.5} = 0.0625$

b) $P(X > 3) = \int_{3}^{4} (0.5x-1)dx = 0.5\frac{x^2}{2} - x\Big|_{3}^{4} = 0.75$

c) $P(2.5 < X < 3.5) = \int_{2.5}^{3.5} (0.5x-1)dx = 0.5\frac{x^2}{2} - x\Big|_{2.5}^{3.5} = 0.5$

d) $F(x) = \int_{2}^{x} (0.5t-1)dt = 0.5\frac{t^2}{2} - t\Big|_{2}^{x} = \frac{x^2}{4} - x + 1$. Then,

$$F(x) = \begin{cases} 0, & x < 2 \\ \frac{x^2}{4} - x + 1, & 2 \le x < 4 \\ 1, & 4 \le x \end{cases}$$

e) $E(X) = \int_{2}^{4} x(0.5x-1)dx = 0.5\frac{x^3}{3} - \frac{x^2}{2}\Big|_{2}^{4} = \frac{32}{3} - 8 - (\frac{4}{3} - 2) = \frac{10}{3}$

$$V(X)=\int_2^4 (x-\tfrac{10}{3})^2(0.5x-1)dx=\int_2^4 (x^2-\tfrac{20}{3}x+\tfrac{100}{9})(0.5x-1)dx$$
$$=\int_2^4 (0.5x^3-\tfrac{13}{3}x^2+\tfrac{110}{9}x-\tfrac{100}{9})dx=\tfrac{x^4}{8}-\tfrac{13}{9}x^3+\tfrac{55}{9}x^2-\tfrac{100}{9}x\Big|_2^4$$
$$=0.2222$$

4-176. Let X denote the life.

a) $P(X<5800)=P(Z<\frac{5800-7000}{600})=P(Z<-2)=1-P(Z\le 2)=0.023$

b) If P(X > x) = 0.9, then P(Z < $\frac{x-7000}{600}$) = -1.28. Consequently, $\frac{x-7000}{600}$ = -1.28 and x = 6232 hours.

c) If P(X > 10,000) = 0.99, then P(Z > $\frac{10{,}000-\mu}{600}$) = 0.99. Therefore, $\frac{10{,}000-\mu}{600}$ = -2.33 and $\mu = 11{,}398$.

d) The probability a product lasts more than 10000 hours is $[P(X>10000)]^3$, by independence.
If $[P(X>10000)]^3$ = 0.99, then P(X > 10000) = 0.9967.
Then, P(X > 10000) = $P(Z>\frac{10000-\mu}{600})=0.9967$. Therefore, $\frac{10000-\mu}{600}$ = -2.72 and $\mu = 11{,}632$ hours.

4-180. Using the normal approximation to the binomial with X being the number of people who will be seated. Then X ~Bin(200, 0.9).

a) P(X ≤ 185) $=P\left(\frac{X-np}{\sqrt{np(1-p)}}\le\frac{185.5-180}{\sqrt{200(0.9)(0.1)}}\right)=P(Z\le 1.30)=0.9032$

b)
$$P(X<185)$$
$$\approx P(X\le 184.5)=P\left(\frac{X-np}{\sqrt{np(1-p)}}\ge\frac{184.5-180}{\sqrt{200(0.9)(0.1)}}\right)=P(Z\le 1.06)=0.8554$$

c) P(X ≤ 185) ≅ 0.95,
Successively trying various values of n: The number of reservations taken could be reduced to about 198.

n	Z_o	Probability P(Z < Z_0)
190	3.51	0.999776
195	2.39	0.9915758
198	**1.73**	**0.9581849**

CHAPTER 5

Section 5-1

5-2. Let R denote the range of (X,Y). Because

$$\sum_R f(x,y) = c(2+3+4+3+4+5+4+5+6) = 1,$$ 36c = 1, and c = 1/36

a) $P(X = 1, Y < 4) = f_{XY}(1,1) + f_{XY}(1,2) + f_{XY}(1,3) = \frac{1}{36}(2+3+4) = 1/4$

b) $P(X = 1)$ is the same as part (a) = 1/4

c) $P(Y = 2) = f_{XY}(1,2) + f_{XY}(2,2) + f_{XY}(3,2) = \frac{1}{36}(3+4+5) = 1/3$

d) $P(X < 2, Y < 2) = f_{XY}(1,1) = \frac{1}{36}(2) = 1/18$

e)

$$E(X) = 1[f_{XY}(1,1) + f_{XY}(1,2) + f_{XY}(1,3)] + 2[f_{XY}(2,1) + f_{XY}(2,2) + f_{XY}(2,3)]$$
$$+ 3[f_{XY}(3,1) + f_{XY}(3,2) + f_{XY}(3,3)]$$
$$= \left(1 \times \tfrac{9}{36}\right) + \left(2 \times \tfrac{12}{36}\right) + \left(3 \times \tfrac{15}{36}\right) = 13/6 = 2.167$$

$$V(X) = (1 - \tfrac{13}{6})^2 \tfrac{9}{36} + (2 - \tfrac{13}{6})^2 \tfrac{12}{36} + (3 - \tfrac{13}{6})^2 \tfrac{15}{36} = 0.639$$

$$E(Y) = 2.167$$

$$V(Y) = 0.639$$

f) marginal distribution of *X*

x	$f_X(x) = f_{XY}(x,1) + f_{XY}(x,2) + f_{XY}(x,3)$
1	1/4
2	1/3
3	5/12

g) $f_{Y|X}(y) = \dfrac{f_{XY}(1,y)}{f_X(1)}$

y	$f_{Y\|X}(y)$
1	(2/36)/(1/4)=2/9
2	(3/36)/(1/4)=1/3
3	(4/36)/(1/4)=4/9

h) $f_{X|Y}(x) = \dfrac{f_{XY}(x,2)}{f_Y(2)}$ and $f_Y(2) = f_{XY}(1,2) + f_{XY}(2,2) + f_{XY}(3,2) = \frac{12}{36} = 1/3$

x	$f_{X\|Y}(x)$
1	(3/36)/(1/3)=1/4
2	(4/36)/(1/3)=1/3
3	(5/36)/(1/3)=5/12

i) $E(Y|X=1) = 1(2/9)+2(1/3)+3(4/9) = 20/9$

j) Since $f_{XY}(x,y) \neq f_X(x)f_Y(y)$, X and Y are not independent.

5-7. a) The range of (X,Y) is $X \geq 0,\ Y \geq 0\ and\ X+Y \leq 4$.

Here *X* and *Y* denote the number of defective items found with inspection device 1 and 2, respectively.

	x=0	x=1	x=2	x=3	x=4
y=0	1.94×10^{-19}	1.10×10^{-16}	2.35×10^{-14}	2.22×10^{-12}	7.88×10^{-11}
y=1	2.59×10^{-16}	1.47×10^{-13}	3.12×10^{-11}	2.95×10^{-9}	1.05×10^{-7}
y=2	1.29×10^{-13}	7.31×10^{-11}	1.56×10^{-8}	1.47×10^{-6}	5.22×10^{-5}
y=3	2.86×10^{-11}	1.62×10^{-8}	3.45×10^{-6}	3.26×10^{-4}	0.0116
y=4	2.37×10^{-9}	1.35×10^{-6}	2.86×10^{-4}	0.0271	0.961

$$f(x,y) = \left[\binom{4}{x}(0.993)^x(0.007)^{4-x}\right]\left[\binom{4}{y}(0.997)^y(0.003)^{4-y}\right]$$

For x = 1,2,3,4 and y = 1,2,3,4

b)

$$f(x,y) = \left[\binom{4}{x}(0.993)^x(0.007)^{4-x}\right] \text{ for } x = 1,2,3,4$$

	x=0	x=1	x=2	x=3	x=4
f(x)	2.40×10^{-9}	1.36×10^{-6}	2.899×10^{-4}	0.0274	0.972

c) Because X has a binomial distribution $E(X) = n(p) = 4*(0.993)=3.972$

d) $f_{Y|2}(y) = \dfrac{f_{XY}(2,y)}{f_X(2)} = f(y)$, $f_x(2) = 2.899 \times 10^{-4}$

y	$f_{Y\|1}(y)=f(y)$
0	8.1×10^{-11}
1	1.08×10^{-7}
2	5.37×10^{-5}
3	0.0119
4	0.988

e) E(Y|X=2) = E(Y)= $n(p)$= 4(0.997)=3.988

f) V(Y|X=2) = V(Y)=$n(p)(1-p)$=4(0.997)(0.003)=0.0120

g) Yes, *X* and *Y* are independent.

5-12. Let X, Y, and Z denote the number of bits with high, moderate, and low distortion. Then, the joint distribution of X, Y, and Z is multinomial with n =3 and

$p_1 = 0.01,\ p_2 = 0.04,\ and\ p_3 = 0.95$.

a)

$$P(X=2, Y=1) = P(X=2, Y=1, Z=0)$$

$$= \frac{3!}{2!1!0!}0.01^2 0.04^1 0.95^0 = 1.2 \times 10^{-5}$$

b) $P(X=0,Y=0,Z=3)=\frac{3!}{0!0!3!}0.01^0 0.04^0 0.95^3=0.8574$

c) X has a binomial distribution with n = 3 and p = 0.01. Then, E(X) = 3(0.01) = 0.03 and V(X) = 3(0.01)(0.99) = 0.0297.

d) First find $P(X\mid Y=2)$

$$P(Y=2)=P(X=1,Y=2,Z=0)+P(X=0,Y=2,Z=1)$$

$$=\frac{3!}{1!2!0!}0.01(0.04)^2 0.95^0+\frac{3!}{0!2!1!}0.01^0(0.04)^2 0.95^1=0.0046$$

$$P(X=0\mid Y=2)=\frac{P(X=0,Y=2)}{P(Y=2)}=\left(\frac{3!}{0!2!1!}0.01^0 0.04^2 0.95^1\right)\Big/0.004608=0.98958$$

$$P(X=1\mid Y=2)=\frac{P(X=1,Y=2)}{P(Y=2)}=\left(\frac{3!}{1!2!1!}0.01^1 0.04^2 0.95^0\right)\Big/0.004608=0.01042$$

$$E(X\mid Y=2)=0(0.98958)+1(0.01042)=0.01042$$

$$V(X\mid Y=2)=E(X^2)-(E(X))^2=0.01042-(0.01042)^2=0.01031$$

5-13. Determine c such that $c\int_0^3\int_0^3 xydxdy=c\int_0^3 y\left.\frac{x^2}{2}\right|_0^3 dy=c(4.5\left.\frac{y^2}{2}\right|_0^3)=\frac{81}{4}c.$

Therefore, $c = 4/81$.

a) $P(X<2,Y<3)=\frac{4}{81}\int_0^3\int_0^2 xydxdy=\frac{4}{81}(2)\int_0^3 ydy=\frac{4}{81}(2)(\frac{9}{2})=0.4444$

b) P(X < 2.5) = P(X < 2.5, Y < 3) because the range of Y is from 0 to 3.

$$P(X<2.5,Y<3)=\frac{4}{81}\int_0^3\int_0^{2.5} xydxdy=\frac{4}{81}(3.125)\int_0^3 ydy=\frac{4}{81}(3.125)\frac{9}{2}=0.6944$$

c) $P(1<Y<2.5)=\frac{4}{81}\int_1^{2.5}\int_0^3 xydxdy=\frac{4}{81}(4.5)\int_1^{2.5} ydy=\frac{18}{81}\left.\frac{y^2}{2}\right|_1^{2.5}=0.5833$

d) $P(X>1.8,1<Y<2.5)=\frac{4}{81}\int_1^{2.5}\int_{1.8}^3 xydxdy=\frac{4}{81}(2.88)\int_1^{2.5} ydy=\frac{4}{81}(2.88)\frac{(2.5^2-1)}{2}=0.3733$

e) $E(X)=\frac{4}{81}\int_0^3\int_0^3 x^2ydxdy=\frac{4}{81}\int_0^3 9ydy=\frac{4}{9}\left.\frac{y^2}{2}\right|_0^3=2$

f) $P(X<0,Y<4)=\frac{4}{81}\int_0^4\int_0^0 xydxdy=0\int_0^4 ydy=0$

g) $f_X(x)=\int_0^3 f_{XY}(x,y)dy=x\frac{4}{81}\int_0^3 ydy=\frac{4}{81}x(4.5)=\frac{2x}{9}$ for $0<x<3$.

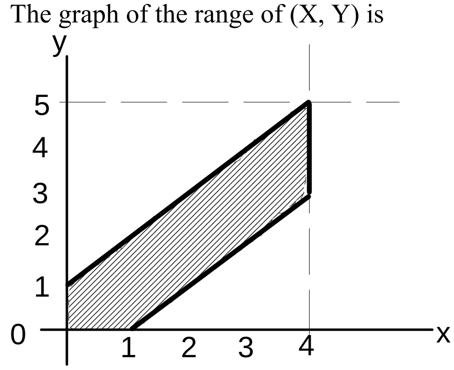
h) $f_{Y|1.5}(y) = \dfrac{f_{XY}(1.5, y)}{f_X(1.5)} = \dfrac{\frac{4}{81} y(1.5)}{\frac{2}{9}(1.5)} = \frac{2}{9} y$ for $0 < y < 3$.

i) $E(Y|X=1.5) = \int_0^3 y\left(\frac{2}{9} y\right) dy = \frac{2}{9}\int_0^3 y^2 dy = \left.\frac{2y^3}{27}\right|_0^3 = 2$

j) $P(Y < 2 \mid X = 1.5) = f_{Y|1.5}(y) = \int_0^2 \frac{2}{9} y dy = \left.\frac{1}{9} y^2\right|_0^2 = \frac{4}{9} - 0 = \frac{4}{9}$

k) $f_{X|2}(x) = \dfrac{f_{XY}(x,2)}{f_Y(2)} = \dfrac{\frac{4}{81} x(2)}{\frac{2}{9}(2)} = \frac{2}{9} x$ for $0 < x < 3$.

5-19. The graph of the range of (X, Y) is



$$\int_0^1 \int_0^{x+1} c\,dy\,dx + \int_1^4 \int_{x-1}^{x+1} c\,dy\,dx = 1$$

$$= c\int_0^1 (x+1)dx + 2c\int_1^4 dx$$

$$= \tfrac{3}{2} c + 6c = 7.5c = 1$$

Therefore, c = 1/7.5=2/15

a) $P(X < 0.5, Y < 0.5) = \int_0^{0.5}\int_0^{0.5} \frac{1}{7.5} dy\,dx = \frac{1}{30}$

b) $P(X < 0.5) = \int_0^{0.5}\int_0^{x+1} \frac{1}{7.5} dy\,dx = \frac{1}{7.5}\int_0^{0.5} (x+1)dx = \frac{2}{15}\left(\frac{5}{8}\right) = \frac{1}{12}$

c)

$$E(X) = \int_0^1 \int_0^{x+1} \tfrac{x}{7.5} dy\,dx + \int_1^4 \int_{x-1}^{x+1} \tfrac{x}{7.5} dy\,dx$$

$$= \tfrac{1}{7.5}\int_0^1 (x^2 + x)dx + \tfrac{2}{7.5}\int_1^4 (x)dx = \tfrac{12}{15}\left(\tfrac{5}{6}\right) + \tfrac{2}{7.5}(7.5) = \tfrac{19}{9}$$

d)

$$E(Y) = \tfrac{1}{7.5}\int_0^1\int_0^{x+1} y\,dy\,dx + \tfrac{1}{7.5}\int_1^4\int_{x-1}^{x+1} y\,dy\,dx$$
$$= \tfrac{1}{7.5}\int_0^1 \tfrac{(x+1)^2}{2}dx + \tfrac{1}{7.5}\int_1^4 \tfrac{(x+1)^2-(x-1)^2}{2}dx$$
$$= \tfrac{1}{15}\int_0^1 (x^2+2x+1)dx + \tfrac{1}{15}\int_1^4 4x\,dx$$
$$= \tfrac{1}{15}\left(\tfrac{7}{3}\right) + \tfrac{1}{15}(30) = \tfrac{97}{45}$$

e)

$$f(x) = \int_0^{x+1} \frac{1}{7.5}dy = \left(\frac{x+1}{7.5}\right) \quad \text{for} \quad 0 < x < 1,$$
$$f(x) = \int_{x-1}^{x+1} \frac{1}{7.5}dy = \left(\frac{x+1-(x-1)}{7.5}\right) = \frac{2}{7.5} \quad \text{for} \quad 1 < x < 4$$

f)

$$f_{Y|X=1}(y) = \frac{f_{XY}(1,y)}{f_X(1)} = \frac{1/7.5}{2/7.5} = 0.5$$
$$f_{Y|X=1}(y) = 0.5 \quad \text{for } 0 < y < 2$$

g) $E(Y \mid X = 1) = \int_0^2 \frac{y}{2}dy = \left.\frac{y^2}{4}\right|_0^2 = 1$

h) $P(Y < 0.5 \mid X = 1) = \int_0^{0.5} 0.5dy = 0.5y\Big|_0^{0.5} = 0.25$

5-21. $\mu = 3.2$, $\lambda = 1/3.2$

$$P(X > 5, Y > 5) = (1/10.24)\int_5^\infty\int_5^\infty e^{-\frac{x}{3.2}-\frac{y}{3.2}}dydx = 3.2\int_5^\infty e^{-\frac{x}{3.2}}\left(e^{-\frac{5}{3.2}}\right)dx$$
$$= \left(e^{-\frac{5}{3.2}}\right)\left(e^{-\frac{5}{3.2}}\right) = 0.0439$$

$$P(X > 10, Y > 10) = (1/10.24)\int_{10}^\infty\int_{10}^\infty e^{-\frac{x}{3.2}-\frac{y}{3.2}}dydx = 3.2\int_{10}^\infty e^{-\frac{x}{3.2}}\left(e^{-\frac{10}{3.2}}\right)dx$$
$$= \left(e^{-\frac{10}{3.2}}\right)\left(e^{-\frac{10}{3.2}}\right) = 0.0019$$

b) Let X denote the number of orders in a 5-minute interval. Then X is a Poisson random variable with $\lambda = 5/3.2 = 1.5625$.

$$P(X = 2) = \frac{e^{-1.5625}(1.5625)^2}{2!} = 0.256$$

For both systems, $P(X=2)P(Y=2)=0.256^2=0.0655$

c) The joint probability distribution is not necessary because the two processes are independent and we can just multiply the probabilities.

5-28. a) Let X denote the grams of luminescent ink. Then,

$P(X<1.14)=P(Z<\frac{1.14-1.2}{0.3})=P(Z<-2)=0.022750$.

Let Y denote the number of bulbs in the sample of 25 that have less than 1.14 grams. Then, by independence, Y has a binomial distribution with n = 25 and p = 0.022750. Therefore, the answer is $P(Y\geq 1)=1-P(Y=0)=\binom{25}{0}0.02275^0(0.97725)^{25}=1-0.5625=0.4375$.

b)

$$P(Y\leq 5)=P(Y=0)+P(Y=1)+P(Y=2)+(P(Y=3)+P(Y=4)+P(Y=5)$$
$$=\binom{25}{0}0.02275^0(0.97725)^{25}+\binom{25}{1}0.02275^1(0.97725)^{24}+\binom{25}{2}0.02275^2(0.97725)^{23}$$
$$+\binom{25}{3}0.02275^3(0.97725)^{22}+\binom{25}{4}0.02275^4(0.97725)^{21}+\binom{25}{5}0.02275^5(0.97725)^{20}.$$
$$=0.5625+0.3274+0.09146+0.01632+0.002090+0.0002043=0.99997\cong 1$$

c) $P(Y=0)=\binom{25}{0}0.02275^0(0.97725)^{25}=0.5625$

d) The lamps are normally and independently distributed, therefore, the probabilities can be multiplied.

Section 5-2

5-29. E(X) = 1(3/8)+2(1/2)+4(1/8)=15/8 = 1.875
E(Y) = 3(1/8)+4(1/4)+5(1/2)+6(1/8)=37/8 = 4.625

$$\text{E(XY)}=[1\times 3\times(1/8)]+[1\times 4\times(1/4)]+[2\times 5\times(1/2)]+[4\times 6\times(1/8)]$$
$$=75/8=9.375$$
$$\sigma_{XY}=E(XY)-E(X)E(Y)=9.375-(1.875)(4.625)=0.703125$$

$V(X) = 1^2(3/8)+ 2^2(1/2) +4^2(1/8)-(15/8)^2 = 0.8594$
$V(Y) = 3^2(1/8)+ 4^2(1/4)+ 5^2(1/2) +6^2(1/8)-(37/8)^2 = 0.7344$

$$\rho_{XY}=\frac{\sigma_{XY}}{\sigma_X\sigma_Y}=\frac{0.703125}{\sqrt{(0.8594)(0.7344)}}=0.8851$$

5-34.

Transaction	Frequency	Selects(X)	Updates(Y)	Inserts(Z)
New Order	43	23	11	12
Payment	44	4.2	3	1
Order Status	4	11.4	0	0
Delivery	5	130	120	0
Stock Level	4	0	0	0

Mean Value		18.694	12.05	5.6

(a) COV(X,Y) = E(XY)-E(X)E(Y) = 23*11*0.43 + 4.2*3*0.44 + 11.4*0*0.04 + 130*120*0.05 + 0*0*0.04 - 18.694*12.05=669.0713

(b) V(X)=735.9644, V(Y)=630.7875; Corr(X,Y)=cov(X,Y)/(V(X)*V(Y))$^{0.5}$ = 0.9820

(c) COV(X,Z)=23*12*0.43+4.2*1*0.44+0-18.694*5.6 = 15.8416

(d) V(Z)=31; Corr(X,Z)=0.1049

5-35. From Exercise 5-19, $c = 8/81$, $E(X) = 12/5$, and $E(Y) = 8/5$

$$E(XY) = \frac{8}{81}\int_0^3\int_0^x xy(xy)dydx = \frac{8}{81}\int_0^3\int_0^x x^2y^2dydx = \frac{8}{81}\int_0^3 \frac{x^3}{3}x^2dx = \frac{8}{81}\int_0^3 \frac{x^5}{3}dx$$

$$= \left(\frac{8}{81}\right)\left(\frac{3^6}{18}\right) = 4$$

$$\sigma_{xy} = 4 - \left(\frac{12}{5}\right)\left(\frac{8}{5}\right) = 0.16$$

$$E(X^2) = 6 \qquad E(Y^2) = 3$$

$$V(x) = 0.24, \qquad V(Y) = 0.44$$

$$\rho = \frac{0.16}{\sqrt{0.24}\sqrt{0.44}} = 0.4924$$

5-39. $$E(X) = -1(1/4) + 1(1/4) = 0$$

$$E(Y) = -1(1/4) + 1(1/4) = 0$$

$$E(XY) = [-1\times 0\times(1/4)] + [-1\times 0\times(1/4)] + [1\times 0\times(1/4)] + [0\times 1\times(1/4)] = 0$$

V(X) = 1/2
V(Y) = 1/2

$$\sigma_{XY} = 0 - (0)(0) = 0$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{0}{\sqrt{1/2}\sqrt{1/2}} = 0$$

The correlation is zero, but *X* and *Y* are not independent, since, for example, if y = 0, X must be –1 or 1.

5-40. If X and Y are independent, then $f_{XY}(x,y) = f_X(x)f_Y(y)$ and the range of (X, Y) is rectangular. Therefore,

$$E(XY) = \iint xyf_X(x)f_Y(y)dxdy = \int xf_X(x)dx\int yf_Y(y)dy = E(X)E(Y)$$

hence $\sigma_{XY}=0$

Section 5-3

5-43. a) percentage of slabs classified as high = p_1 = 0.05
percentage of slabs classified as medium = p_2 = 0.85
percentage of slabs classified as low = p_3 = 0.10

b) X is the number of voids independently classified as high $X \geq 0$
Y is the number of voids independently classified as medium $Y \geq 0$
Z is the number of with a low number of voids and $Z \geq 0$ and $X+Y+Z = 20$

c) p_1 is the percentage of slabs classified as high.

d) $E(X)=np_1 = 20(0.05) = 1$
$V(X)=np_1(1-p_1)= 20(0.05)(0.95) = 0.95$

e) $P(X=1, Y=17, Z=3) = 0$ Because the point $(1,17,3) \neq 20$ is not in the range of (X,Y,Z).

f)

$$P(X \leq 1, Y=17, Z=3) = P(X=0, Y=17, Z=3) + P(X=1, Y=17, Z=3)$$

$$= \frac{20!}{0!17!3!} 0.05^0 0.85^{17} 0.10^3 + 0 = 0.07195$$

Because the point $(1,17,3) \neq 20$ is not in the range of (X, Y, Z).

g) Because X is binomial, $P(X \leq 1) = \binom{20}{0} 0.05^0 0.95^{20} + \binom{20}{1} 0.05^1 0.95^{19} = 0.7358$

h) Because X is binomial E(Y) = np = 20(0.85) = 17

i) The probability is 0 because x+y+z>20

j) $P(X=2 \mid Y=17) = \dfrac{P(X=2, Y=17)}{P(Y=17)}$. Now, because x+y+z = 20,

P(X=2, Y=17) = P(X=2, Y=17, Z=1) = $\dfrac{20!}{2!17!1!} 0.05^2 0.85^{17} 0.10^1 = 0.0540$

$$P(X=2 \mid Y=17) = \frac{P(X=2, Y=17)}{P(Y=17)} = \frac{0.0540}{0.2428} = 0.2224$$

k)

$$E(X \mid Y=17) = 0\left(\frac{P(X=0, Y=17)}{P(Y=17)}\right) + 1\left(\frac{P(X=1, Y=17)}{P(Y=17)}\right) + 2\left(\frac{P(X=2, Y=17)}{P(Y=17)}\right) + 3\left(\frac{P(X=3, Y=17)}{P(Y=17)}\right)$$

$$E(X \mid Y=17) = 0\left(\frac{0.07195}{0.2428}\right) + 1\left(\frac{0.1079}{0.2428}\right) + 2\left(\frac{0.05396}{0.2428}\right) + 3\left(\frac{0.008994}{0.2428}\right) = 1$$

5-45. a) The probability distribution is multinomial because the result of each trial (a dropped oven) results in either a major, minor or no defect with probability 0.6, 0.3 and 0.1 respectively. Also, the trials are independent

b) Let X, Y, and Z denote the number of ovens in the sample of four with major, minor, and no defects, respectively.

$$P(X=2, Y=2, Z=0) = \frac{4!}{2!2!0!} 0.6^2 0.3^2 0.1^0 = 0.1944$$

c) $P(X=0,Y=0,Z=4)=\frac{4!}{0!0!4!}0.6^0 0.3^0 0.1^4 = 0.0001$

d) $f_{XY}(x,y)=\sum_R f_{XYZ}(x,y,z)$ where R is the set of values for z such that x+y+z = 4. That is, R consists of the single value z = 4-x-y and

$$f_{XY}(x,y)=\frac{4!}{x!y!(4-x-y)!}0.6^x 0.3^y 0.1^{4-x-y} \quad for\ x+y\le 4.$$

e) $E(X)=np_1=4(0.6)=2.4$

f) $E(Y)=np_2=4(0.3)=1.2$

g) $P(X=2\,|\,Y=2)=\frac{P(X=2,Y=2)}{P(Y=2)}=\frac{0.1944}{0.2646}=0.7347$

$P(Y=2)=\binom{4}{2}0.3^2 0.7^4 = 0.2646$ from the binomial marginal distribution of *Y*

h) Not possible, *x*+*y*+*z* = 4, the probability is zero.

i) $P(X\,|\,Y=2)=P(X=0\,|\,Y=2),P(X=1\,|\,Y=2),P(X=2\,|\,Y=2)$

$$P(X=0\,|\,Y=2)=\frac{P(X=0,Y=2)}{P(Y=2)}=\left(\frac{4!}{0!2!2!}0.6^0 0.3^2 0.1^2\right)\Big/0.2646=0.0204$$

$$P(X=1\,|\,Y=2)=\frac{P(X=1,Y=2)}{P(Y=2)}=\left(\frac{4!}{1!2!1!}0.6^1 0.3^2 0.1^1\right)\Big/0.2646=0.2449$$

$$P(X=2\,|\,Y=2)=\frac{P(X=2,Y=2)}{P(Y=2)}=\left(\frac{4!}{2!2!0!}0.6^2 0.3^2 0.1^0\right)\Big/0.2646=0.7347$$

j) E(X|Y=2) = 0(0.0204)+1(0.2449)+2(0.7347) = 1.7143

5-49. Because $\rho = 0$ and X and Y are normally distributed, X and Y are independent. Therefore, μ_X = 0.1 mm, σ_X=0.00031 mm, μ_Y = 0.23 mm, σ_Y=0.00017 mm

Probability X is within specification limits is

$$P(0.099535<X<0.100465)=P\left(\frac{0.099535-0.1}{0.00031}<Z<\frac{0.100465-0.1}{0.00031}\right)$$
$$=P(-1.5<Z<1.5)=P(Z<1.5)-P(Z<-1.5)=0.8664$$

Probability that Y is within specification limits is

$$P(0.22966<X<0.23034)=P\left(\frac{0.22966-0.23}{0.00017}<Z<\frac{0.23034-0.23}{0.00017}\right)$$
$$=P(-2<Z<2)=P(Z<2)-P(Z<-2)=0.9545$$

Probability that a randomly selected lamp is within specification limits is (0.8664)(0.9594) = 0.8270

5-51.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)dxdy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\frac{1}{2\pi\sigma_X\sigma_Y} e^{-\frac{1}{2}\left[\frac{(x-\mu_X)^2}{\sigma_X{}^2}+\frac{(y-\mu_Y)^2}{\sigma_Y{}^2}\right]}\right]dxdy =$$

$$\int_{-\infty}^{\infty}\left[\frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{1}{2}\left[\frac{(x-\mu_X)^2}{\sigma_X{}^2}\right]}\right]dx \int_{-\infty}^{\infty}\left[\frac{1}{\sqrt{2\pi}\sigma_Y} e^{-\frac{1}{2}\left[\frac{(y-\mu_Y)^2}{\sigma_Y{}^2}\right]}\right]dy$$

and each of the last two integrals is recognized as the integral of a normal probability density function from $-\infty$ to ∞. That is, each integral equals one. Since $f_{XY}(x,y) = f(x)f(y)$ then *X* and *Y* are independent.

Section 5-4

5-54. a) E(2X + 3Y) = 2(0) + 3(10) = 30
b) V(2X + 3Y) = 4V(X) + 9V(Y) = 97
c) 2X + 3Y is normally distributed with mean 30 and variance 97. Therefore,

$$P(2X+3Y<30) = P(Z<\tfrac{30-30}{\sqrt{97}}) = P(Z<0) = 0.5$$

d) $P(2X+3Y<40) = P(Z<\tfrac{40-30}{\sqrt{97}}) = P(Z<1.02) = 0.8461$

5-59. a) *X*~N(0.1, 0.00031) and *Y*~N(0.23, 0.00017) Let T denote the total thickness.
Then, T = X + Y and E(T) = 0.33 mm,

V(T) = $0.00031^2 + 0.00017^2 = 1.25x10^{-7} mm^2$, and $\sigma_T = 0.000354$ mm.

$$P(T<0.2337) = P\left(Z<\frac{0.2337-0.33}{0.000354}\right) = P(Z<-272) \cong 0$$

b)

$$P(T>0.2405) = P\left(Z>\frac{0.2405-0.33}{0.000345}\right) = P(Z>-253) = 1-P(Z<253) \cong 1$$

5-63. Let $\overline{X}$ denote the average thickness of 10 wafers. Then, E($\overline{X}$) = 10 and V($\overline{X}$) = 0.1.
a) $P(9<\overline{X}<11) = P(\frac{9-10}{\sqrt{0.1}}<Z<\frac{11-10}{\sqrt{0.1}}) = P(-3.16<Z<3.16) = 0.998$.
The answer is 1 – 0.998 = 0.002
b) $P(\overline{X}>11) = 0.01$ *and* $\sigma_{\overline{X}} = 1/\sqrt{n}$.
Therefore, $P(\overline{X}>11) = P(Z>\frac{11-10}{1/\sqrt{n}}) = 0.01$, $\frac{11-10}{1/\sqrt{n}}$ = 2.33 and n = 5.43 which is rounded up to 6.

c) $P(\overline{X}>11) = 0.0005$ *and* $\sigma_{\overline{X}} = \sigma/\sqrt{10}$.
Therefore, $P(\overline{X}>11) = P(Z>\frac{11-10}{\sigma/\sqrt{10}}) = 0.0005$, $\frac{11-10}{\sigma/\sqrt{10}}$ = 3.29

$$\sigma = \sqrt{10}/3.29 = 0.9612$$

5-64. X~N(160, 900)
a) Let Y = $X_1 + X_2 + ... + X_{25}$, E(Y) = 25E(X) = 4000, V(Y) = $25^2(900)$ = 22500

P(Y > 4300) =

$$P\left(Z > \frac{4300-4000}{\sqrt{22500}}\right) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.9773 = 0.0227$$

b) c) P(Y > x) = 0.0001 implies that $P\left(Z > \frac{x-4000}{\sqrt{22500}}\right) = 0.0001.$

Then $\frac{x-4000}{150}$ = 3.72 and $x = 4558$

Section 5-5

5-71. a) If $y = x^2$, then $x = \sqrt{y}$ for $x \ge 0$ and $y \ge 0$. Thus, $f_Y(y) = f_X(\sqrt{y})\frac{1}{2}y^{-\frac{1}{2}} = \frac{e^{-\sqrt{y}}}{2\sqrt{y}}$ for y > 0.

b) If $y = x^{1/2}$, then $x = y^2$ for $x \ge 0$ and $y \ge 0$. Thus, $f_Y(y) = f_X(y^2)2y = 2ye^{-y^2}$ for y > 0.

c) If y = ln x, then $x = e^y$ for $x \ge 0$. Thus, $f_Y(y) = f_X(e^y)e^y = e^y e^{-e^y} = e^{y-e^y}$ for $-\infty < y < \infty$.

5-73. If $y = e^x$, then x = ln y for $1 \le x \le 2$ and $e^1 \le y \le e^2$. Thus, $f_Y(y) = f_X(\ln y)\frac{1}{y} = \frac{1}{y}$ for $1 \le \ln y \le 2$. That is, $f_Y(y) = \frac{1}{y}$ for $e \le y \le e^2$.

Supplemental Exercises

5-75. The sum of $\sum_x \sum_y f(x,y) = 1$, $\left(1/4\right) + \left(1/8\right) + \left(1/8\right) + \left(1/4\right) + \left(1/4\right) = 1$

and $f_{XY}(x,y) \ge 0$

a) $P(X < 0.5, Y < 1.5) = f_{XY}(0,1) + f_{XY}(0,0) = 1/8 + 1/4 = 3/8$.

b) $P(X \le 1) = f_{XY}(0,0) + f_{XY}(0,1) + f_{XY}(1,0) + f_{XY}(1,1) = 3/4$

c) $P(Y < 1.5) = f_{XY}(0,0) + f_{XY}(0,1) + f_{XY}(1,0) + f_{XY}(1,1) = 3/4$

d) $P(X > 0.5, Y < 1.5) = f_{XY}(1,0) + f_{XY}(1,1) = 3/8$

e) E(X) = 0(3/8) + 1(3/8) + 2(1/4) = 7/8.
V(X) = 0^2(3/8) + 1^2(3/8) + 2^2(1/4) - $7/8^2$ =39/64
E(Y) = 1(3/8) + 0(3/8) + 2(1/4) = 7/8.
V(Y) = 1^2(3/8) + 0^2(3/8) + 2^2(1/4) - $7/8^2$ =39/64

f) $f_X(x) = \sum_y f_{XY}(x,y)$ *and* $f_X(0) = 3/8,\ f_X(1) = 3/8,\ f_X(2) = 1/4$.

g) $f_{Y|1}(y) = \frac{f_{XY}(1,y)}{f_X(1)}$ *and* $f_{Y|1}(0) = \frac{1/8}{3/8} = 1/3,\ \ f_{Y|1}(1) = \frac{1/4}{3/8} = 2/3$.

h) $E(Y \mid X = 1) = \sum_{x=1} y f_{Y|X=1}(y) = 0(1/3) + 1(2/3) = 2/3$

i) As is discussed after Example 5-19, because the range of (X, Y) is not rectangular, X and Y are not independent.

j) E(XY) = 1.25, E(X) = E(Y)= 0.875 V(X) = V(Y) = 0.6094
COV(X,Y)=E(XY)-E(X)E(Y)= 1.25-0.875^2=0.4844

$$\rho_{XY} = \frac{0.4844}{\sqrt{0.6094}\sqrt{0.6094}} = 0.7949$$

5-76. $P(X = 2, Y = 4, Z = 14) = \frac{20!}{2!4!14!} 0.10^2 0.20^4 0.70^{14} = 0.0631$

b) $P(X = 0) = 0.10^0 0.90^{20} = 0.1216$

c) $E(X) = np_1 = 20(0.10) = 2$

$V(X) = np_1(1 - p_1) = 20(0.10)(0.9) = 1.8$

d) $f_{X|Z=z}(X \mid Z = 19) \frac{f_{XZ}(x,z)}{f_Z(z)}$

$$f_{XZ}(xz) = \frac{20!}{x!z!(20-x-z)!} 0.1^x 0.2^{20-x-z} 0.7^z$$

$$f_Z(z) = \frac{20!}{z!(20-z)!} 0.3^{20-z} 0.7^z$$

$$f_{X|Z=z}(X \mid Z = 19) \frac{f_{XZ}(x,z)}{f_Z(z)} = \frac{(20-z)!}{x!(20-x-z)!} \frac{0.1^x 0.2^{20-x-z}}{0.3^{20-z}} = \frac{(20-z)!}{x!(20-x-z)!} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{20-x-z}$$

Therefore, X is a binomial random variable with n=20-z and p=1/3. When z=19,

$f_{X|19}(0) = \frac{2}{3}$ and $f_{X|19}(1) = \frac{1}{3}$.

e) $E(X \mid Z = 19) = 0\left(\frac{2}{3}\right) + 1\left(\frac{1}{3}\right) = \frac{1}{3}$

5-78. Let X, Y, and Z denote the number of calls answered in two rings or less, three or four rings, and five rings or more, respectively.

a) $P(X = 8, Y = 1, Z = 1) = \frac{10!}{8!1!1!} 0.7^8 0.25^1 0.05^1 = 0.0649$

b) Let W denote the number of calls answered in four rings or less. Then, W is a binomial random variable with n = 10 and p = 0.95.

Therefore, P(W = 10) = $\binom{10}{10} 0.95^{10} 0.05^0 = 0.5987$.

c) E(W) = 10(0.95) = 9.5.

d) $f_{Z|8}(z) = \frac{f_{XZ}(8,z)}{f_X(8)}$ and $f_{XZ}(x,z) = \frac{10!}{x!z!(10-x-z)!} 0.70^x 0.25^{(10-x-z)} 0.05^z$ for

$x + z \le 10$ *and* $0 \le x, 0 \le z$. Then,

$$f_{Z|8}(z) = \frac{\frac{10!}{8!z!(2-z)!} 0.70^8 0.25^{(2-z)} 0.05^z}{\frac{10!}{8!2!} 0.70^8 0.30^2} = \frac{2!}{z!(2-z)!} \left(\frac{0.25}{0.30}\right)^{2-z} \left(\frac{0.05}{0.30}\right)^z$$

for $0 \le z \le 2$. That is Z is binomial with n =2 and p = 0.05/0.30 = 1/6.

e) E(Z) given X = 8 is 2(1/6) = 1/3.
f) Because the conditional distribution of Z given X = 8 does not equal the marginal distribution of Z, X and Z are not independent.

5-82. a) Let $X_1, X_2, \ldots, X_6$ denote the lifetimes of the six components, respectively. Because of independence,

$$P(X_1 > 5000, X_2 > 5000, \ldots, X_6 > 5000) = P(X_1 > 5000)P(X_2 > 5000)\ldots P(X_6 > 5000)$$

If X is exponentially distributed with mean θ, then $\lambda = \frac{1}{\theta}$ and

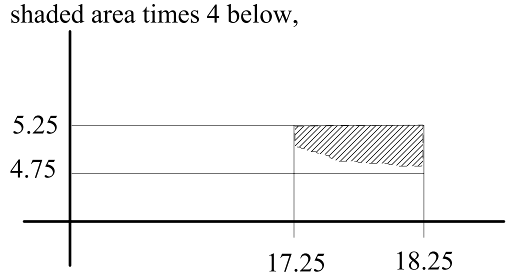
$$P(X > x) = \int_x^\infty \tfrac{1}{\theta}\, e^{-t/\theta} dt = -e^{-t/\theta}\Big|_x^\infty = e^{-x/\theta}$$. Therefore, the answer is

$$e^{-5/8}e^{-0.5}e^{-0.5}e^{-0.25}e^{-0.25}e^{-0.2} = e^{-2.325} = 0.0978.$$

b) The probability that at least one component lifetime exceeds 25,000 hours is the same as 1 minus the probability that none of the component lifetimes exceed 25,000 hours. Thus,
1-P(X_a<25,000, X_2<25,000, …, X_6<25,000)=1-P(X_1<25,000)…P(X_6<25,000)
=1-(1-$e^{-25/8}$)(1-$e^{-2.5}$)(1-$e^{-2.5}$)(1-$e^{-1.25}$)(1-$e^{-1.25}$)(1-e^{-1})=1-.2592=0.7408

5-85. a)

Because $\int_{17.75}^{18.25}\int_{4.75}^{5.25} c\,dy\,dx = 0.25c,$ c = 4. The area of a panel is XY and P(XY > 90) is the shaded area times 4 below,



That is, $\int_{17.75}^{18.25}\int_{90/x}^{5.25} 4\,dy\,dx = 4\int_{17.75}^{18.25} 5.25 - \frac{90}{x}\,dx = 4(5.25x - 90\ln x\Big|_{17.75}^{18.25}) = 0.499$

b) The perimeter of a panel is 2X+2Y and we want P(2X+2Y >46)

$$\int_{17.75}^{18.25}\int_{23-x}^{5.25} 4\,dy\,dx = 4\int_{17.75}^{18.25} 5.25 - (23-x)dx$$

$$= 4\int_{17.75}^{18.25}(-17.75 + x)dx = 4(-17.75x + \frac{x^2}{2}\Big|_{17.75}^{18.25}) = 0.5$$

5-86. a) Let X denote the weight of a piece of candy and X~N(0.1, 0.01). Each package has 16 candies, then P is the total weight of the package with 16 pieces and E(P) = 16(0.1)=1.6 ounces and V(P) = $16^2(0.01^2)$=0.0256 ounces^2

b) $P(P < 1.6) = P(Z < \frac{1.6-1.6}{0.16}) = P(Z < 0) = 0.5$.

c) Let Y equal the total weight of the package with 17 pieces, E(Y) = 17(0.1)=1.7 ounces and $V(Y) = 17^2(0.01^2)=0.0289$ ounces^2

$$P(Y < 1.6) = P(Z < \tfrac{1.6-1.7}{\sqrt{0.0289}}) = P(Z < -0.59) = 0.2776.$$

5-92. Let T denote the total thickness. Then, $T = X_1 + X_2 + X_3$ and

a) E(T) = 0.5+1+1.5 =3 mm

$V(T)=V(X_1)+V(X_2)+V(X_3)+2Cov(X_1X_2)+2Cov(X_2X_3)+2Cov(X_1X_3)=0.01+0.04+0.09+2(0.014)+2(0.03)+2(0.009)=0.246mm^2$

where $Cov(XY)=\rho\sigma_X\sigma_Y$

b) $P(T < 1.5) = P\left(Z < \dfrac{1.5-3}{0.246}\right) = P(Z < -6.10) \cong 0$

5-93. Let X and Y denote the percentage returns for security one and two respectively.
If ½ of the total dollars is invested in each then ½X+ ½Y is the percentage return.
E(½X+ ½Y) = 0.05 (or 5 if given in terms of percent)
V(½X+ ½Y) = 1/4 V(X)+1/4V(Y)+2(1/2)(1/2)Cov(X,Y)
where $Cov(XY)=\rho\sigma_X\sigma_Y=-0.5(2)(4) = -4$
V(½X+ ½Y) = 1/4(4)+1/4(6)-2 = 3
Also, E(X) = 5 and V(X) = 4. Therefore, the strategy that splits between the securities has a lower standard deviation of percentage return than investing 2million in the first security.

CHAPTER 6

Section 6-1

6-14. Sample average:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{19.56}{9} = 2.173 \quad mm$$

Sample variance:

$$\sum_{i=1}^{9} x_i = 19.56$$

$$\sum_{i=1}^{9} x_i^2 = 45.953$$

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1} = \frac{45.953 - \frac{(19.56)^2}{9}}{9-1} = \frac{3.443}{8}$$

$$= 0.4303 \quad (mm)^2$$

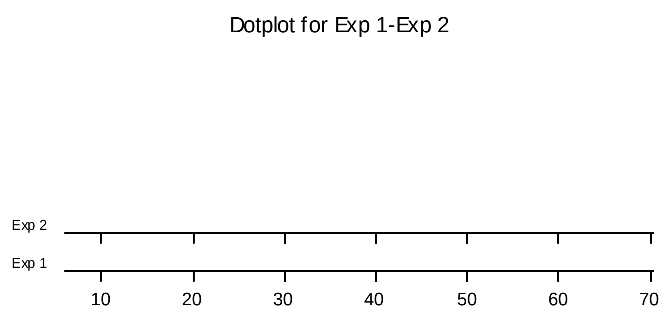
Sample standard deviation:

$$s = \sqrt{0.4303} = 0.6560 \quad mm$$

Dot Diagram

```
.     .            .     .  .         .  .         ..
-------+---------+---------+---------+---------+---------crack length
     1.40      1.75      2.10      2.45      2.80      3.15
```

6-16. Dot Diagram of CRT data in exercise 6-5 (Data were rounded for the plot)



The data are centered a lot lower in the second experiment. The lower CRT resolution reduces the visual accommodation.

6-19. a) $\bar{x} = 65.86$ °F

$s = 12.16$ °F

Dot Diagram

```
                                                            :  :
       .        .    .   .  . .   . :  . : .  . : . . :  . .  : :  . . . .  . .
-+---------+---------+---------+---------+---------+-----temp
30        40        50        60        70        80
```

b) Removing the smallest observation (31), the sample mean and standard deviation become

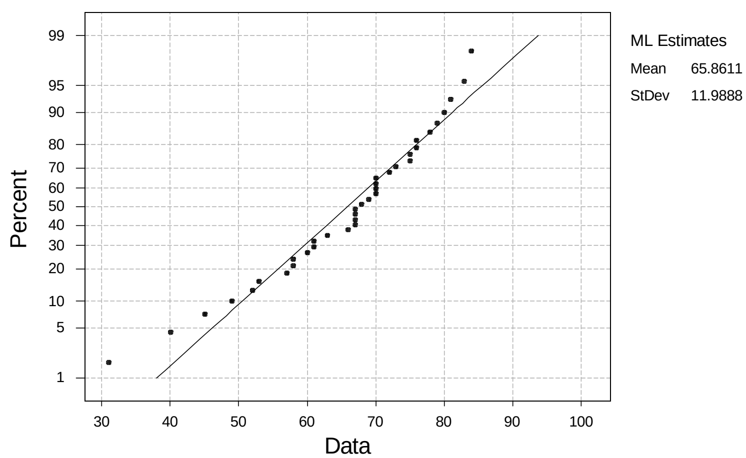
$\bar{x} = 66.86$ °F

s = 10.74 °F

<u>Section 6-6</u>

6-77.

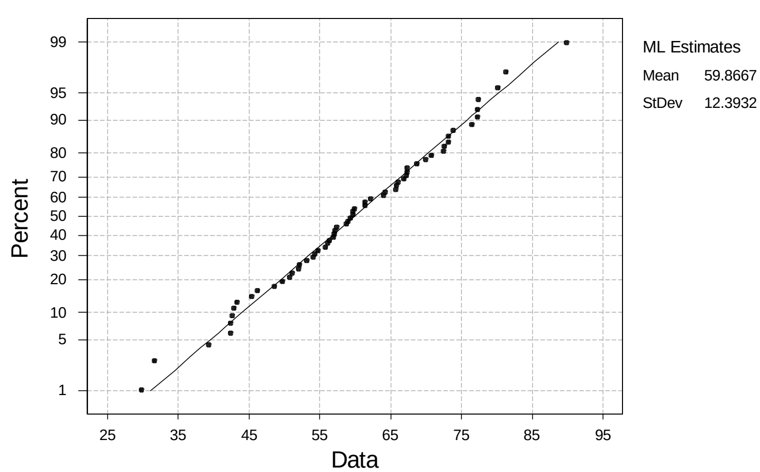
Normal Probability Plot for temperature
Data from exercise 6-13



The data appear to be approximately normally distributed. However, there are some departures from the line at the ends of the distribution.

6-80. The data appear to be normally distributed. Nearly all of the data points fall very close to, or on the line.

Normal Probability Plot for concentration
Data from exercise 6-24



6-82. Yes, it is possible to obtain an estimate of the mean from the 50th percentile value of the normal probability plot. The fiftieth percentile point is the point at which the sample mean should equal the population mean and 50% of the data would be above the value and 50% below. An estimate of the standard deviation would be to subtract the 50th percentile from the 84th percentile These values are based on the values from the z-table that could be used to estimate the standard deviation.

CHAPTER 7

Section 7-2

7-3.

$$P(1.009 \le \overline{X} \le 1.012) = P\left(\frac{1.009-1.01}{0.003/\sqrt{9}} \le \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \le \frac{1.012-1.01}{0.003/\sqrt{9}}\right)$$
$$= P(-1 \le Z \le 2) = P(Z \le 2) - P(Z \le -1)$$
$$= 0.9772 - 0.1586 = 0.8186$$

7-9. $\sigma^2 = 25$

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$
$$n = \left(\frac{\sigma}{\sigma_{\overline{X}}}\right)^2 = \left(\frac{5}{1.5}\right)^2 = 11.11 \sim 12$$

7-10. Let $Y = \overline{X} - 6$

$$\mu_X = \frac{a+b}{2} = \frac{(0+1)}{2} = \tfrac{1}{2}$$
$$\mu_{\overline{X}} = \mu_X$$
$$\sigma_X^2 = \frac{(b-a)^2}{12} = \tfrac{1}{12}$$
$$\sigma_{\overline{X}}^2 = \frac{\sigma^2}{n} = \frac{\tfrac{1}{12}}{12} = \frac{1}{144}$$
$$\sigma_{\overline{X}} = \tfrac{1}{12}$$
$$\mu_Y = \tfrac{1}{2} - 6 = -5\tfrac{1}{2}$$
$$\sigma_Y^2 = \tfrac{1}{144}$$

$Y = \overline{X} - 6 \sim N(-5\tfrac{1}{2}, \tfrac{1}{144})$, approximately, using the central limit theorem.

7-12.

$\mu_X = 8.2$ minutes	n = 49
$\sigma_X = 1.5$ minutes	$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{1.5}{\sqrt{49}} = 0.2143$
	$\mu_{\overline{X}} = \mu_X = 8.2$ mins

Using the central limit theorem, $\overline{X}$ is approximately normally distributed.

a) $P(\overline{X} < 10) = P(Z < \frac{10-8.2}{0.2143}) = P(Z < 8.4) = 1$

b) $P(5 < \overline{X} < 10) = P(\frac{5-8.2}{0.2143} < Z < \frac{10-8.2}{0.2143})$

$= P(Z < 8.4) - P(Z < -14.932) = 1 - 0 = 1$

c) $P(\overline{X} < 6) = P(Z < \frac{6-8.2}{0.2143}) = P(Z < -10.27) = 0$

7-14. If $\mu_B = \mu_A$, then $\overline{X}_B - \overline{X}_A$ is approximately normal with mean 0 and variance $\frac{\sigma_B^2}{25} + \frac{\sigma_A^2}{25} = 20.48$.

Then, $P(\overline{X}_B - \overline{X}_A > 3.5) = P(Z > \frac{3.5-0}{\sqrt{20.48}}) = P(Z > 0.773) = 0.2196$

The probability that $\overline{X}_B$ exceeds $\overline{X}_A$ by 3.5 or more is not that unusual when μ_B *and* μ_A are equal. Therefore, there is not strong evidence that μ_B is greater than μ_A.

Section 7-3

7-20. $E\left(\overline{X}_1\right) = E\left(\frac{\sum_{i=1}^{2n} X_i}{2n}\right) = \frac{1}{2n} E\left(\sum_{i=1}^{2n} X_i\right) = \frac{1}{2n}(2n\mu) = \mu$

$E\left(\overline{X}_2\right) = E\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n}(n\mu) = \mu$, $\overline{X}_1$ and $\overline{X}_2$ are unbiased estimators of μ.

The variances are $V\left(\overline{X}_1\right) = \frac{\sigma^2}{2n}$ and $V\left(\overline{X}_2\right) = \frac{\sigma^2}{n}$; compare the MSE (variance in this case),

$$\frac{MSE(\hat{\Theta}_1)}{MSE(\hat{\Theta}_2)} = \frac{\sigma^2/2n}{\sigma^2/n} = \frac{n}{2n} = \frac{1}{2}$$

Since both estimators are unbiased, examination of the variances would conclude that $\overline{X}_1$ is the "better" estimator with the smaller variance.

7-21. $E\left(\hat{\Theta}_1\right) = \frac{1}{7}\left[E(X_1) + E(X_2) + \cdots + E(X_7)\right] = \frac{1}{7}(7E(X)) = \frac{1}{7}(7\mu) = \mu$

$E\left(\hat{\Theta}_2\right) = \frac{1}{2}\left[E(2X_1) + E(X_6) + E(X_7)\right] = \frac{1}{2}[2\mu - \mu + \mu] = \mu$

a) Both $\hat{\Theta}_1$ and $\hat{\Theta}_2$ are unbiased estimates of μ since the expected values of these statistics are equivalent to the true mean, μ.

b) $V\left(\hat{\Theta}_1\right) = V\left[\frac{X_1 + X_2 + ... + X_7}{7}\right] = \frac{1}{7^2}\left(V(X_1) + V(X_2) + \cdots + V(X_7)\right) = \frac{1}{49}(7\sigma^2) = \frac{1}{7}\sigma^2$

$V(\hat{\Theta}_1) = \frac{\sigma^2}{7}$

$V\left(\hat{\Theta}_2\right) = V\left[\frac{2X_1 - X_6 + X_4}{2}\right] = \frac{1}{2^2}\left(V(2X_1) + V(X_6) + V(X_4)\right) = \frac{1}{4}(4V(X_1) + V(X_6) + V(X_4))$

$= \frac{1}{4}\left(4\sigma^2 + \sigma^2 + \sigma^2\right)$

$$= \frac{1}{4}(6\sigma^2)$$

$$V(\hat{\Theta}_2) = \frac{3\sigma^2}{2}$$

Since both estimators are unbiased, the variances can be compared to select the better estimator. The variance of $\hat{\Theta}_1$ is smaller than that of $\hat{\Theta}_2$, $\hat{\Theta}_1$ is the better estimator.

7-23. $E(\hat{\Theta}_1) = \theta \qquad E(\hat{\Theta}_2) = \theta / 2$

$$Bias = E(\hat{\Theta}_2) - \theta$$

$$= \frac{\theta}{2} - \theta = -\frac{\theta}{2}$$

$V(\hat{\Theta}_1) = 10 \qquad V(\hat{\Theta}_2) = 4$

For unbiasedness, use $\hat{\Theta}_1$ since it is the only unbiased estimator.
As for minimum variance and efficiency we have:

Relative Efficiency $= \dfrac{(V(\hat{\Theta}_1) + Bias^2)_1}{(V(\hat{\Theta}_2) + Bias^2)_2}$ where bias for θ_1 is 0.

Thus,

Relative Efficiency $= \dfrac{(10+0)}{\left(4 + \left(\frac{-\theta}{2}\right)^2\right)} = \dfrac{40}{\left(16+\theta^2\right)}$

If the relative efficiency is less than or equal to 1, $\hat{\Theta}_1$ is the better estimator.

Use $\hat{\Theta}_1$, when $\dfrac{40}{(16+\theta^2)} \le 1$

$$40 \le (16+\theta^2)$$
$$24 \le \theta^2$$
$$\theta \le -4.899 \text{ or } \theta \ge 4.899$$

If $-4.899 < \theta < 4.899$ then use $\hat{\Theta}_2$.

For unbiasedness, use $\hat{\Theta}_1$. For efficiency, use $\hat{\Theta}_1$ when $\theta \le -4.899$ or $\theta \ge 4.899$ and use $\hat{\Theta}_2$ when $-4.899 < \theta < 4.899$.

7-26. Show that $\dfrac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n}$ is a biased estimator of σ^2:

a)

$$E\left(\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n}\right)$$

$$= \frac{1}{n}E\left(\sum_{i=1}^{n}\left(X_i - n\overline{X}\right)^2\right) = \frac{1}{n}\left(\sum_{i=1}^{n}E\left(X_i^2\right) - nE\left(\overline{X}^2\right)\right) = \frac{1}{n}\left(\sum_{i=1}^{n}\left(\mu^2 + \sigma^2\right) - n\left(\mu^2 + \frac{\sigma^2}{n}\right)\right)$$

$$= \frac{1}{n}\left(n\mu^2 + n\sigma^2 - n\mu^2 - \sigma^2\right) = \frac{1}{n}((n-1)\sigma^2) = \sigma^2 - \frac{\sigma^2}{n}$$

$\therefore \dfrac{\sum(X_i - \overline{X})^2}{n}$ is a biased estimator of σ^2.

b) Bias $= E\left[\dfrac{\sum(X_i^2 - n\overline{X})^2}{n}\right] - \sigma^2 = \sigma^2 - \dfrac{\sigma^2}{n} - \sigma^2 = -\dfrac{\sigma^2}{n}$

c) Bias decreases as n increases.

7-29. **Descriptive Statistics**

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Oxide Thickness	24	423.33	424.00	423.36	9.08	1.85

a) The mean oxide thickness, as estimated by Minitab from the sample, is 423.33 Angstroms.
b) Standard deviation for the population can be estimated by the sample standard deviation, or 9.08 Angstroms.
c) The standard error of the mean is 1.85 Angstroms.
d) Our estimate for the median is 424 Angstroms.
e) Seven of the measurements exceed 430 Angstroms, so our estimate of the proportion requested is 7/24 = 0.2917

7-31. a) $E(\overline{X}_1 - \overline{X}_2) = E(\overline{X}_1) - E(\overline{X}_2) = \mu_1 - \mu_2$

b) $s.e. = \sqrt{V(\overline{X}_1 - \overline{X}_2)} = \sqrt{V(\overline{X}_1) + V(\overline{X}_2) + 2COV(\overline{X}_1, \overline{X}_2)} = \sqrt{\dfrac{\sigma_1^{\;2}}{n_1} + \dfrac{\sigma_2^{\;2}}{n_2}}$

This standard error could be estimated by using the estimates for the standard deviations of populations 1 and 2.

c)

$$E(S_p^{\;2}) = E\left(\frac{(n_1-1)\cdot S_1^{\;2} + (n_2-1)\cdot S_2^{\;2}}{n_1+n_2-2}\right) = \frac{1}{n_1+n_2-2}\left[(n_1-1)E(S_1^{\;2}) + (n_2-1)\cdot E(S_2^{\;2})\right] =$$

$$= \frac{1}{n_1+n_2-2}\left[(n_1-1)\cdot\sigma_1^{\;2} + (n_2-1)\cdot\sigma_2^{\;2})\right] = \frac{n_1+n_2-2}{n_1+n_2-2}\sigma^2 = \sigma^2$$

7-32. a) $E(\hat{\mu}) = E(\alpha\overline{X}_1 + (1-\alpha)\overline{X}_2) = \alpha E(\overline{X}_1) + (1-\alpha)E(\overline{X}_2) = \alpha\mu + (1-\alpha)\mu = \mu$

b)

$$s.e.(\hat{\mu}) = \sqrt{V(\alpha\overline{X}_1 + (1-\alpha)\overline{X}_2)} = \sqrt{\alpha^2 V(\overline{X}_1) + (1-\alpha)^2 V(\overline{X}_2)}$$

$$= \sqrt{\alpha^2\frac{\sigma_1^{\;2}}{n_1} + (1-\alpha)^2\frac{\sigma_2^{\;2}}{n_2}} = \sqrt{\alpha^2\frac{\sigma_1^{\;2}}{n_1} + (1-\alpha)^2 a\frac{\sigma_1^{\;2}}{n_2}}$$

$$= \sigma_1\sqrt{\frac{\alpha^2 n_2 + (1-\alpha)^2 a n_1}{n_1 n_2}}$$

c) The value of alpha that minimizes the standard error is:

$$\alpha = \frac{an_1}{n_2 + an_1}$$

d) With $a = 4$ and $n_1=2n_2$, the value of alpha to choose is 8/9. The arbitrary value of α=0.5 is too small and will result in a larger standard error. With α=8/9 the standard error is

$$s.e.(\hat{\mu}) = \sigma_1\sqrt{\frac{(8/9)^2 n_2 + (1/9)^2 8n_2}{2{n_2}^2}} = \frac{0.667\sigma_1}{\sqrt{n_2}}$$

If α=0.5 the standard error is

$$s.e.(\hat{\mu}) = \sigma_1\sqrt{\frac{(0.5)^2 n_2 + (0.5)^2 8n_2}{2{n_2}^2}} = \frac{1.0607\sigma_1}{\sqrt{n_2}}$$

7-33. a) $E(\frac{X_1}{n_1} - \frac{X_2}{n_2}) = \frac{1}{n_1}E(X_1) - \frac{1}{n_2}E(X_2) = \frac{1}{n_1}n_1 p_1 - \frac{1}{n_2}n_2 p_2 = p_1 - p_2 = E(p_1 - p_2)$

b) $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$

c) An estimate of the standard error could be obtained substituting $\frac{X_1}{n_1}$ for p_1 and $\frac{X_2}{n_2}$ for p_2 in the equation shown in (b).

d) Our estimate of the difference in proportions is 0.01

e) The estimated standard error is 0.0413

<u>Supplemental Exercises</u>

7-60. $E(a\overline{X}_1 + (1-a)\overline{X}_2 = a\mu + (1-a)\mu = \mu$

$$V(\overline{X}) = V[a\overline{X}_1 + (1-a)\overline{X}_2]$$
$$= a^2V(\overline{X}_1) + (1-a)^2V(\overline{X}_2) = a^2(\tfrac{\sigma^2}{n_1}) + (1-2a+a^2)(\tfrac{\sigma^2}{n_2})$$
$$= \frac{a^2\sigma^2}{n_1} + \frac{\sigma^2}{n_2} - \frac{2a\sigma^2}{n_2} + \frac{a^2\sigma^2}{n_2} = (n_2a^2 + n_1 - 2n_1a + n_1a^2)(\frac{\sigma^2}{n_1n_2})$$
$$\frac{\partial V(\overline{X})}{\partial a} = (\frac{\sigma^2}{n_1n_2})(2n_2a - 2n_1 + 2n_1a) \equiv 0$$
$$0 = 2n_2a - 2n_1 + 2n_1a$$
$$2a(n_2 + n_1) = 2n_1$$
$$a(n_2 + n_1) = n_1$$
$$a = \frac{n_1}{n_2 + n_1}$$

CHAPTER 8

Section 8-1

8-1 a) The confidence level for $\bar{x} - 2.14\sigma/\sqrt{n} \le \mu \le \bar{x} + 2.14\sigma/\sqrt{n}$ is determined by the value of z_0 which is 2.14. From Table III, Φ(2.14) = P(Z<2.14) = 0.9838 and the confidence level is 2(0.9838-0.5) = 96.76%.

b) The confidence level for $\bar{x} - 2.49\sigma/\sqrt{n} \le \mu \le \bar{x} + 2.49\sigma/\sqrt{n}$ is determined by the by the value of z_0 which is 2.14. From Table III, Φ(2.49) = P(Z<2.49) = 0.9936 and the confidence level is is 2(0.9936-0.5) = 98.72%.

c) The confidence level for $\bar{x} - 1.85\sigma/\sqrt{n} \le \mu \le \bar{x} + 1.85\sigma/\sqrt{n}$ is determined by the by the value of z_0 which is 2.14. From Table III, Φ(1.85) = P(Z<1.85) = 0.9678 and the confidence level is 93.56%.

8-13 a) 99% Two-sided CI on the true mean piston ring diameter

For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$, and $\bar{x} = 74.036$, $\sigma = 0.001$, n=15

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$74.036 - 2.58\left(\frac{0.001}{\sqrt{15}}\right) \le \mu \le 74.036 + 2.58\left(\frac{0.001}{\sqrt{15}}\right)$$

$$74.0353 \le \mu \le 74.0367$$

b) 99% One-sided CI on the true mean piston ring diameter

For $\alpha = 0.01$, $z_{\alpha} = z_{0.01} = 2.33$ and $\bar{x} = 74.036$, $\sigma = 0.001$, n=15

$$\bar{x} - z_{0.01}\frac{\sigma}{\sqrt{n}} \le \mu$$

$$74.036 - 2.33\left(\frac{0.001}{\sqrt{15}}\right) \le \mu$$

$$74.0354 \le \mu$$

The lower bound of the one sided confidence interval is less than the lower bound of the two-sided confidence. This is because the Type I probability of 99% one sided confidence interval (or $\alpha = 0.01$) in the left tail (or in the lower bound) is greater than Type I probability of 99% two-sided confidence interval (or $\alpha/2 = 0.005$) in the left tail.

8-15 a) 95% two sided CI on the mean compressive strength

$z_{\alpha/2} = z_{0.025} = 1.96$, and $\bar{x} = 3250$, $\sigma^2 = 1000$, n=12

$$\bar{x} - z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.025}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 1.96\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 1.96\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3232.11 \le \mu \le 3267.89$$

b) 99% Two-sided CI on the true mean compressive strength

$z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$3250 - 2.58\left(\frac{31.62}{\sqrt{12}}\right) \le \mu \le 3250 + 2.58\left(\frac{31.62}{\sqrt{12}}\right)$$

$$3226.4 \le \mu \le 3273.6$$

The 99% CI is wider than the 95% CI

8-21 a) 99% two sided CI on the mean temperature

$z_{\alpha/2} = z_{0.005} = 2.57$, and $\bar{x} = 13.77$, $\sigma = 0.5$, $n=11$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$13.77 - 2.57\left(\frac{0.5}{\sqrt{11}}\right) \le \mu \le 13.77 + 2.57\left(\frac{0.5}{\sqrt{11}}\right)$$

$$13.383 \le \mu \le 14.157$$

b) 95% lower-confidence bound on the mean temperature

For $\alpha = 0.05$, $z_\alpha = z_{0.05} = 1.65$ and $\bar{x} = 13.77$, $\sigma = 0.5$, $n = 11$

$$\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu$$

$$13.77 - 1.65\left(\frac{0.5}{\sqrt{11}}\right) \le \mu$$

$$13.521 \le \mu$$

c) 95% confidence that the error of estimating the mean temperature for wheat grown is less than 2 degrees Celsius.

For $\alpha = 0.05$, $z_{\alpha/2} = z_{0.025} = 1.96$, and $\sigma = 0.5$, $E = 2$

$$n = \left(\frac{z_{a/2}\sigma}{E}\right)^2 = \left(\frac{1.96(0.5)}{2}\right)^2 = 0.2401$$

Always round up to the next number, therefore $n = 1$.

d) Set the width to 1.5 degrees Celsius with $\sigma = 0.5$, $z_{0.025} = 1.96$ solve for n.

$1/2 \text{ width} = (1.96)(0.5)/\sqrt{n} = 0.75$

$0.98 = 0.75\sqrt{n}$

$$n = \left(\frac{0.98}{0.75}\right)^2 = 1.707$$

Therefore, n = 2.

Section 8-2

8-27 95% confidence interval on mean tire life

$$n = 16 \quad \bar{x} = 60{,}139.7 \quad s = 3645.94 \quad t_{0.025,15} = 2.131$$

$$\bar{x} - t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,15}\left(\frac{s}{\sqrt{n}}\right)$$

$$60139.7 - 2.131\left(\frac{3645.94}{\sqrt{16}}\right) \le \mu \le 60139.7 + 2.131\left(\frac{3645.94}{\sqrt{16}}\right)$$

$$58197.33 \le \mu \le 62082.07$$

8-29 $\bar{x} = 1.10$ $s = 0.015$ $n = 25$

95% CI on the mean volume of syrup dispensed

For $\alpha = 0.05$ and n = 25, $t_{\alpha/2,n-1} = t_{0.025,24} = 2.064$

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$1.10 - 2.064\left(\frac{0.015}{\sqrt{25}}\right) \le \mu \le 1.10 + 2.064\left(\frac{0.015}{\sqrt{25}}\right)$$

$$1.094 \le \mu \le 1.106$$

8-31 99% upper confidence interval on mean SBP

$$n = 14 \quad \bar{x} = 118.3 \quad s = 9.9 \quad t_{0.01,13} = 2.650$$

$$\mu \le \bar{x} + t_{0.005,13}\left(\frac{s}{\sqrt{n}}\right)$$

$$\mu \le 118.3 + 2.650\left(\frac{9.9}{\sqrt{14}}\right)$$

$$\mu \le 125.312$$

Section 8-3

8-46 95% two sided confidence interval for σ

$$n = 10 \quad s = 4.8$$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,9} = 19.02 \text{ and } \chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,9} = 2.70$$

$$\frac{9(4.8)^2}{19.02} \le \sigma^2 \le \frac{9(4.8)^2}{2.70}$$

$$10.90 \le \sigma^2 \le 76.80$$

$$3.30 < \sigma < 8.76$$

8-47 95% confidence interval for σ: given n = 51, s = 0.37

First find the confidence interval for σ^2 :

For α = 0.05 and n = 51, $\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,50} = 71.42$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,50} = 32.36$

$$\frac{50(0.37)^2}{71.42} \le \sigma^2 \le \frac{50(0.37)^2}{32.36}$$

$$0.096 \le \sigma^2 \le 0.2115$$

Taking the square root of the endpoints of this interval we obtain,

$$0.31 < \sigma < 0.46$$

8-48 95% confidence interval for σ

$$n = 17 \quad s = 0.09$$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,16} = 28.85 \text{ and } \chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,16} = 6.91$$

$$\frac{16(0.09)^2}{28.85} \le \sigma^2 \le \frac{16(0.09)^2}{6.91}$$

$$0.0045 \le \sigma^2 \le 0.0188$$

$$0.067 < \sigma < 0.137$$

Supplemental Exercises

8-83 Where $\alpha_1 + \alpha_2 = \alpha$. Let $\alpha = 0.05$

Interval for $\alpha_1 = \alpha_2 = \alpha/2 = 0.025$

The confidence level for $\bar{x} - 1.96\sigma/\sqrt{n} \le \mu \le \bar{x} + 1.96\sigma/\sqrt{n}$ is determined by the by the value of z_0 which is 1.96. From Table III, we find Φ(1.96) = P(Z<1.96) = 0.975 and the confidence level is 95%.

Interval for $\alpha_1 = 0.01, \quad \alpha_2 = 0.04$

The confidence interval is $\bar{x} - 2.33\sigma/\sqrt{n} \le \mu \le \bar{x} + 1.75\sigma/\sqrt{n}$, the confidence level is the same because $\alpha = 0.05$. The symmetric interval does not affect the level of significance; however, it does affect the length. The symmetric interval is shorter in length.

8-85 $\mu = 50, \quad \sigma^2 = 5$

a) For $n = 16$ find $P(s^2 \ge 7.44)$ or $P(s^2 \le 2.56)$

$$P(S^2 \geq 7.44) = P\left(\chi^2_{15} \geq \frac{15(7.44)}{5^2}\right) = 0.05 \leq P\left(\chi^2_{15} \geq 22.32\right) \leq 0.10$$

Using Minitab $P(S^2 \geq 7.44)$ =0.0997

$$P(S^2 \leq 2.56) = P\left(\chi^2_{15} \leq \frac{15(2.56)}{5}\right) = 0.05 \leq P\left(\chi^2_{15} \leq 7.68\right) \leq 0.10$$

Using Minitab $P(S^2 \leq 2.56)$ =0.064

b) For $n = 30$ find $P(S^2 \geq 7.44)$ or $P(S^2 \leq 2.56)$

$$P(S^2 \geq 7.44) = P\left(\chi^2_{29} \geq \frac{29(7.44)}{5}\right) = 0.025 \leq P\left(\chi^2_{29} \geq 43.15\right) \leq 0.05$$

Using Minitab $P(S^2 \geq 7.44)$ = 0.044

$$P(S^2 \leq 2.56) = P\left(\chi^2_{29} \leq \frac{29(2.56)}{5}\right) = 0.01 \leq P\left(\chi^2_{29} \leq 14.85\right) \leq 0.025$$

Using Minitab $P(S^2 \leq 2.56)$ = 0.014.

c) For $n = 71$ $P(s^2 \geq 7.44)$ or $P(s^2 \leq 2.56)$

$$P(S^2 \geq 7.44) = P\left(\chi^2_{70} \geq \frac{70(7.44)}{5}\right) = 0.005 \leq P\left(\chi^2_{70} \geq 104.16\right) \leq 0.01$$

Using Minitab $P(S^2 \geq 7.44)$ =0.0051

$$P(S^2 \leq 2.56) = P\left(\chi^2_{70} \leq \frac{70(2.56)}{5}\right) = P\left(\chi^2_{70} \leq 35.84\right) \leq 0.005$$

Using Minitab $P(S^2 \leq 2.56) < 0.001$

d) The probabilities get smaller as n increases. As n increases, the sample variance should approach the population variance; therefore, the likelihood of obtaining a sample variance much larger than the population variance will decrease.

e) The probabilities get smaller as n increases. As n increases, the sample variance should approach the population variance; therefore, the likelihood of obtaining a sample variance much smaller than the population variance will decrease.

8-87 a) The probability plot shows that the data appear to be normally distributed. Therefore, there is no evidence conclude that the comprehensive strength data are normally distributed.

b) 99% lower confidence bound on the mean $\vec{x} = 25.12$, s $= 8.42$, $n = 9$ $t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \le \mu$$

$$25.12 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \le \mu$$

$$16.99 \le \mu$$

The lower bound on the 99% confidence interval shows that the mean comprehensive strength is most likely be greater than 16.99 Megapascals.

c) 98% two-sided confidence interval on the mean $\vec{x} = 25.12, \quad s = 8.42, \quad n = 9 \quad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right)$$

$$25.12 - 2.896\left(\frac{8.42}{\sqrt{9}}\right) \le \mu \le 25.12 + 2.896\left(\frac{8.42}{\sqrt{9}}\right)$$

$$16.99 \le \mu \le 33.25$$

The bounds on the 98% two-sided confidence interval shows that the mean comprehensive strength will most likely be greater than 16.99 Megapascals and less than 33.25 Megapascals. The lower bound of the 99% one sided CI is the same as the lower bound of the 98% two-sided CI (this is because of the value of α)

d) 99% one-sided upper bound on the confidence interval on σ^2 comprehensive strength

$$s = 8.42, \quad s^2 = 70.90 \quad \chi^2_{0.99,8} = 1.65$$

$$\sigma^2 \le \frac{8(8.42)^2}{1.65}$$

$$\sigma^2 \le 343.74$$

The upper bound on the 99% confidence interval on the variance shows that the variance of the comprehensive strength is most likely less than 343.74 Megapascals^2.

e) 98% two-sided confidence interval on σ^2 of comprehensive strength

$$s = 8.42, \quad s^2 = 70.90 \quad \chi^2_{0.01,9} = 20.09 \quad \chi^2_{0.99,8} = 1.65$$

$$\frac{8(8.42)^2}{20.09} \le \sigma^2 \le \frac{8(8.42)^2}{1.65}$$

$$28.23 \le \sigma^2 \le 343.74$$

The bounds on the 98% two-sided confidence-interval on the variance shows that the variance of the comprehensive strength is most likely less than 343.74 Megapascals^2 and greater than 28.23 Megapascals^2.

The upper bound of the 99% one-sided CI is the same as the upper bound of the 98% two-sided CI because value of α for the one-sided example is one-half the value for the two-sided example.

f) 98% two-sided confidence interval on the mean $\vec{x} = 23, \quad s = 6.31, \quad n = 9 \quad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right)$$

$$23 - 2.896\left(\frac{6.31}{\sqrt{9}}\right) \le \mu \le 23 + 2.896\left(\frac{6.31}{\sqrt{9}}\right)$$

$$16.91 \le \mu \le 29.09$$

98% two-sided confidence interval on σ^2 comprehensive strength

$$s = 6.31, \quad s^2 = 39.8 \quad \chi^2_{0.01,9} = 20.09 \quad \chi^2_{0.99,8} = 1.65$$

$$\frac{8(39.8)}{20.09} \le \sigma^2 \le \frac{8(39.8)}{1.65}$$

$$15.85 \le \sigma^2 \le 192.97$$

Fixing the mistake decreased the values of the sample mean and the sample standard deviation. Because the sample standard deviation was decreased the widths of the confidence intervals were also decreased.

g) The exercise provides s = 8.41 (instead of the sample variance). A 98% two-sided confidence interval on the mean $\bar{x} = 25, \quad s = 8.41, \quad n = 9 \quad t_{0.01,8} = 2.896$

$$\bar{x} - t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.01,8}\left(\frac{s}{\sqrt{n}}\right)$$

$$25 - 2.896\left(\frac{8.41}{\sqrt{9}}\right) \le \mu \le 25 + 2.896\left(\frac{8.41}{\sqrt{9}}\right)$$

$$16.88 \le \mu \le 33.12$$

98% two-sided confidence interval on σ^2 of comprehensive strength

$$s = 8.41, \quad s^2 = 70.73 \quad \chi^2_{0.01,9} = 20.09 \quad \chi^2_{0.99,8} = 1.65$$

$$\frac{8(8.41)^2}{20.09} \le \sigma^2 \le \frac{8(8.41)^2}{1.65}$$

$$28.16 \le \sigma^2 \le 342.94$$

Fixing the mistake did not affect the sample mean or the sample standard deviation. They are very close to the original values. The widths of the confidence intervals are also very similar.

h) When a mistaken value is near the sample mean, the mistake will not affect the sample mean, standard deviation or confidence intervals greatly. However, when the mistake is not near the sample mean, the value can greatly affect the sample mean, standard deviation and confidence intervals. The farther from the mean, the greater is the effect.

8-88

With σ = 8, the 95% confidence interval on the mean has length of at most 5; the error is then E = 2.5.

a) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 8^2 = \left(\frac{1.96}{2.5}\right)^2 64 = 39.34 = 40$

b) $n = \left(\frac{z_{0.025}}{2.5}\right)^2 6^2 = \left(\frac{1.96}{2.5}\right)^2 36 = 22.13 = 23$

As the standard deviation decreases, with all other values held constant, the sample size necessary to maintain the acceptable level of confidence and the length of the interval, decreases.

8-99 a) The data appear to follow a normal distribution based on the normal probability plot since the data fall along a straight line.

b) It is important to check for normality of the distribution underlying the sample data since the confidence intervals to be constructed should have the assumption of normality for the results to be reliable (especially since the sample size is less than 30 and the central limit theorem does not apply).

c) 95% confidence interval for the mean

$n = 11 \quad \bar{x} = 22.73 \quad s = 6.33 \quad t_{0.025,10} = 2.228$

$$\bar{x} - t_{0.025,10}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,10}\left(\frac{s}{\sqrt{n}}\right)$$

$$22.73 - 2.228\left(\frac{6.33}{\sqrt{11}}\right) \le \mu \le 22.73 + 2.228\left(\frac{6.33}{\sqrt{11}}\right)$$

$$18.478 \le \mu \le 26.982$$

d) As with part b, to construct a confidence interval on the variance, the normality assumption must hold for the results to be reliable.

e) 95% confidence interval for variance

$n = 11 \quad s = 6.33$

$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,10} = 20.48$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,10} = 3.25$

$$\frac{10(6.33)^2}{20.48} \le \sigma^2 \le \frac{10(6.33)^2}{3.25}$$

$$19.565 \le \sigma^2 \le 123.289$$

$$P(0.930 < p < 0.970) = P\left(Z < \frac{(0.970 - 0.950)}{\sqrt{\frac{0.950(0.050)}{1000}}}\right) - P\left(Z < \frac{(0.930 - 0.950)}{\sqrt{\frac{0.950(0.050)}{1000}}}\right)$$

$$= P\left(Z < \frac{0.02}{0.006892}\right) - P\left(Z < \frac{-0.02}{0.006892}\right) = P(Z < 2.90) - P(Z < -2.90) = 0.9963$$

CHAPTER 9

Section 9-1

9-1 a) $H_0: \mu = 25, H_1: \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0: \sigma > 10, H_1: \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0: \bar{x} = 50, H_1: \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0: p = 0.1, H_1: p = 0.3$ No, the values in the null and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0: s = 30, H_1: s > 30$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-5 a) α = P(reject H_0 when H_0 is true)

$= P(\bar{X} \leq 11.5 \text{ when } \mu = 12) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{11.5-12}{0.5/\sqrt{4}}\right) = P(Z \leq -2)$

= 0.02275.

The probability of rejecting the null hypothesis when it is true is 0.02275.

b) β = P(accept H_0 when μ = 11.25) = $P(\bar{X} > 11.5 \mid \mu = 11.25)$

$= P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.25}{0.5/\sqrt{4}}\right) = P(Z > 1.0)$

= 1 − P(Z ≤ 1.0) = 1 − 0.84134 = 0.15866

The probability of accepting the null hypothesis when it is false is 0.15866.

c) β = P(accept H_0 when μ = 11.25) =

$= P(\bar{X} > 11.5 \mid \mu = 11.5) = P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{11.5-11.5}{0.5/\sqrt{4}}\right)$

= P(Z > 0) = 1 − P(Z ≤ 0) = 1 − 0.5 = 0.5

The probability of accepting the null hypothesis when it is false is 0.5

9-13 δ =103-100=3

δ >0 then $\beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$, where σ =2

a) $\beta = P(98.69 < \bar{X} < 101.31 \mid \mu=103) = P(-6.47<Z<-2.54) = 0.0055$

b) $\beta = P(98.25 < \bar{X} < 101.75 \mid \mu=103) = P(-5.31<Z<-1.40) = 0.0808$

a) As n increases, β decreases

9-15 a) $\alpha = P(\bar{X} > 185 \text{ when } \mu = 175)$

$= P\left(\frac{\overline{X}-175}{20/\sqrt{10}} > \frac{185-175}{20/\sqrt{10}}\right)$

= P(Z > 1.58)

= 1 − P(Z ≤ 1.58)

= 1 − 0.94295 = 0.05705

b) β = P($\overline{X}$ ≤ 185 when μ = 185)

$= P\left(\frac{\overline{X}-185}{20/\sqrt{10}} \le \frac{185-185}{20/\sqrt{10}}\right)$

= P(Z ≤ 0) = 0.5

c) β = P($\overline{X}$ ≤ 185 when μ = 195)

$= P\left(\frac{\overline{X}-195}{20/\sqrt{10}} \le \frac{185-195}{20/\sqrt{10}}\right)$

= P(Z ≤ −1.58) = 0.05705

9-23 P-value=2(1 - $\Phi(|Z_0|)$) where $z_0 = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$

a) $\overline{x}$ = 5.2 then $z_0 = \frac{5.2-5}{.25/\sqrt{8}} = 2.26$

P-value = 2(1- $\Phi(2.26)) = 2(1-0.988089) = 0.0238$

b) $\overline{x}$ = 4.7 then $z_0 = \frac{4.7-5}{.25/\sqrt{8}} = -3.39$

P-value = 2(1- $\Phi(3.39)) = 2(1-0.99965) = 0.0007$

c) $\overline{x}$ = 5.1 then $z_0 = \frac{5.1-5}{.25/\sqrt{8}} = 1.1313$

P-value = 2(1- $\Phi(1.1313)) = 2(1-0.870762) = 0.2585$

Section 9-2

9-30 a) α = 0.01, then a = $z_{\alpha/2}$ = 2.57 and b = - $z_{\alpha/2}$ = -2.57

b) α = 0.05, then a = $z_{\alpha/2}$ = 1.96 and b = - $z_{\alpha/2}$ = -1.96

c) α = 0.1, then a = $z_{\alpha/2}$ = 1.65 and b = - $z_{\alpha/2}$ = -1.65

9-40 a) 1) The parameter of interest is the true mean water temperature, μ.

2) H_0 : μ = 100

3) H_1 : μ > 100

4) $z_0 = \frac{\overline{x}-\mu}{\sigma/\sqrt{n}}$

5) Reject H_0 if $z_0 > z_\alpha$ where α = 0.05 and $z_{0.05}$ = 1.65

6) $\overline{x} = 98$, σ = 2

$$z_0 = \frac{98-100}{2/\sqrt{9}} = -3.0$$

7) Because -3.0 < 1.65 fail to reject H_0. The water temperature is not significantly greater than 100 at $\alpha = 0.05$.

b) P-value = $1-\Phi(-3.0) = 1-0.00135 = 0.99865$

c) $\beta = \Phi\left(z_{0.05} + \frac{100-104}{2/\sqrt{9}}\right)$

$= \Phi(1.65 + -6)$

$= \Phi(-4.35) \cong 0$

9-42 a) 1) The parameter of interest is the true mean melting point, μ.

2) $H_0 : \mu = 155$

3) $H_1 : \mu \neq 155$

4) $z_0 = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

5) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $\alpha = 0.01$ and $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $\alpha = 0.01$ and $z_{0.005} = 2.58$

6) $\bar{x} = 154.2$, $\sigma = 1.5$

$$z_0 = \frac{154.2-155}{1.5/\sqrt{10}} = -1.69$$

7) Because –1.69 > -2.58 fail to reject the null hypothesis. There is not sufficient evidence to support the claim the mean melting point differs from 155 °F at $\alpha = 0.01$.

b) P-value = 2*P(Z <- 1.69) =2* 0.045514 = 0.091028

c)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(155-150)\sqrt{10}}{1.5}\right) - \Phi\left(-2.58 - \frac{(155-150)\sqrt{10}}{1.5}\right)$$

$= \Phi(-7.96) - \Phi(-13.12) = 0 - 0 = 0$

d)

$$n = \frac{(z_{\alpha/2} + z_\beta)^2\sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2\sigma^2}{(150-155)^2} = \frac{(2.58+1.29)^2(1.5)^2}{(5)^2} = 1.35,$$

$n \cong 2$.

Section 9-3

9-58 a. 1) The parameter of interest is the true mean interior temperature life, μ.

2) $H_0 : \mu = 22.5$

3) $H_1 : \mu \neq 22.5$

4) $t_0 = \frac{\bar{x}-\mu}{s/\sqrt{n}}$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{\alpha/2,n-1} = 2.776$ for n = 5

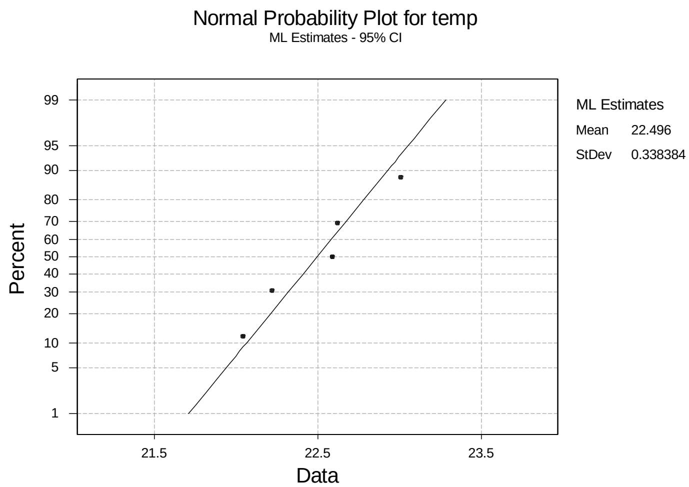
6) $\bar{x} = 22.496$, $s = 0.378$, $n = 5$

$$t_0 = \frac{22.496 - 22.5}{0.378/\sqrt{5}} = -0.00237$$

7) Because –0.00237 >- 2.776 we cannot reject the null hypothesis. There is not sufficient evidence to conclude that the true mean interior temperature is not equal to 22.5 °C at $\alpha = 0.05$.

Also, 2*0.4 < *P*-value < 2* 0.5. That is, 0.8 < *P*-value < 1.0

b) The points on the normal probability plot fall along the line. Therefore, the normality assumption is reasonable.



c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|22.75 - 22.5|}{0.378} = 0.66$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, $d = 0.66$, and n = 5, we obtain $\beta \cong 0.8$ and power of 1–0.8 = 0.2.

d) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|22.75 - 22.5|}{0.378} = 0.66$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, $d = 0.66$, and $\beta \cong 0.1$ (Power=0.9), n = 40

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right)$$

$$22.496 - 2.776\left(\frac{0.378}{\sqrt{5}}\right) \le \mu \le 22.496 + 2.776\left(\frac{0.378}{\sqrt{5}}\right)$$

$$22.027 \le \mu \le 22.965$$

We cannot conclude that the mean interior temperature differs from 22.5 because the value is included in the confidence interval.

9-59 a. 1) The parameter of interest is the true mean female body temperature, μ.

2) $H_0 : \mu = 98.6$

3) $H_1 : \mu \ne 98.6$

4) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

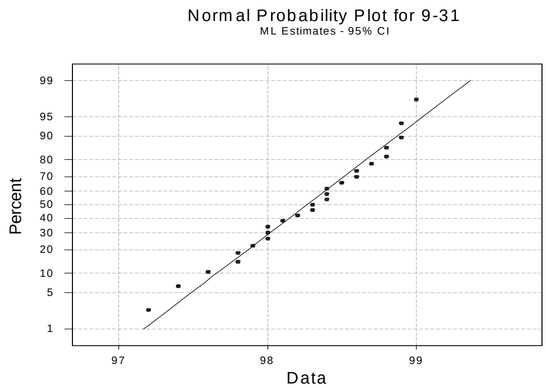
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{\alpha/2,n-1} = 2.064$ for n = 25

6) $\bar{x} = 98.264$, s = 0.4821, n = 25

$$t_0 = \frac{98.264 - 98.6}{0.4821/\sqrt{25}} = -3.48$$

7) Because 3.48 > 2.064 reject the null hypothesis. There is sufficient evidence to conclude that the true mean female body temperature is not equal to 98.6 °F at $\alpha = 0.05$.

P-value = 2* 0.001 = 0.002



b) Data appear to be normally distributed.

c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|98 - 98.6|}{0.4821} = 1.24$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 1.24, and n = 25, obtain $\beta \cong 0$ and power of $1 - 0 \cong 1$.

d) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|98.2 - 98.6|}{0.4821} = 0.83$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 0.83, and $\beta \cong 0.1$ (Power=0.9), n =20

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,24}\left(\frac{s}{\sqrt{n}}\right)$$

$$98.264 - 2.064\left(\frac{0.4821}{\sqrt{25}}\right) \le \mu \le 98.264 + 2.064\left(\frac{0.4821}{\sqrt{25}}\right)$$

$$98.065 \le \mu \le 98.463$$

Conclude that the mean female body temperature differs from 98.6 because the value is not included inside the confidence interval.

9-61 a)

1) The parameter of interest is the true mean sodium content, μ.

2) $H_0 : \mu = 130$

3) $H_1 : \mu \neq 130$

4) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{\alpha/2,n-1} = 2.093$ for n = 20

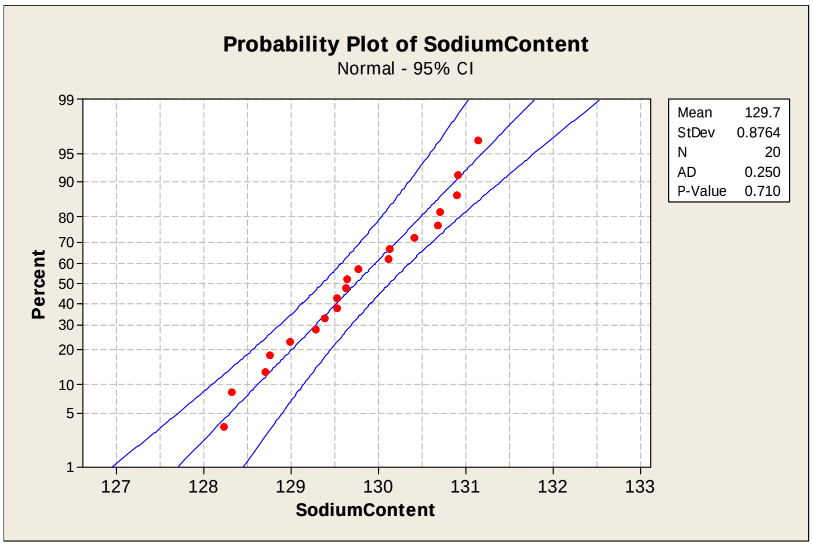
6) $\bar{x} = 129.747$, s = 0.876 n = 20

$$t_0 = \frac{129.747 - 130}{0.876/\sqrt{20}} = -1.291$$

7) Because 1.291< 2.093 we fail to reject the null hypothesis. There is not sufficient evidence that the true mean sodium content is different from 130mg at $\alpha = 0.05$.

From the *t* table (Table V) the t_0 value is between the values of 0.1 and 0.25 with 19 degrees of freedom. Therefore, 2(0.1) < P-value < 2(0.25) and 0.2 < P-value < 0.5.

b) The assumption of normality appears to be reasonable.



c) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|130.5 - 130|}{0.876} = 0.571$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 0.57, and n = 20, we obtain $\beta \cong 0.3$ and the power of 1 – 0.30 = 0.70

d) $d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|130.1 - 130|}{0.876} = 0.114$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 0.11, and $\beta \cong 0.25$ (Power=0.75), the sample sizes do not extend to the point *d* = 0.114 and $\beta = 0.25$. We can conclude that n > 100

e) 95% two sided confidence interval

$$\bar{x} - t_{0.025,29}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,29}\left(\frac{s}{\sqrt{n}}\right)$$

$$129.747 - 2.093\left(\frac{0.876}{\sqrt{20}}\right) \le \mu \le 129.747 + 2.093\left(\frac{0.876}{\sqrt{20}}\right)$$

$$129.337 \le \mu \le 130.157$$

There is no evidence that the mean differs from 130 because that value is inside the confidence interval. The result is the same as part (a).

9-64 a)

1) The parameter of interest is the true mean sodium content, μ.

2) $H_0 : \mu = 300$
3) $H_1 : \mu > 300$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{\alpha,n-1} = 1.943$ for n = 7
6) $\bar{x} = 315$, s = 16 n=7

$$t_0 = \frac{315 - 300}{16/\sqrt{7}} = 2.48$$

7) Because 2.48>1.943 reject the null hypothesis and conclude that there is sufficient evidence that the leg strength exceeds 300 watts at $\alpha = 0.05$.

The P-value is between .01 and .025

b) d = $\dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|305 - 300|}{16} = 0.3125$

Using the OC curve, Chart VII g) for $\alpha = 0.05$, d = 0.3125, and n = 7,
$\beta \cong 0.9$ and power = 1–0.9 = 0.1.

c) If $1 - \beta > 0.9$ then $\beta < 0.1$ and n is approximately 100

d) Lower confidence bound is $\bar{x} - t_{\alpha,n-1}\left(\dfrac{s}{\sqrt{n}}\right) = 303.2 < \mu$

Because 300 is not include in the interval, reject the null hypothesis

9-67 In order to use a t statistic in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean current, μ.
2) $H_0 : \mu = 300$
3) $H_1 : \mu > 300$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,9} = 1.833$ for n = 10
6) $n = 10 \quad \bar{x} = 317.2 \quad s = 15.7$

$$t_0 = \frac{317.2 - 300}{15.7/\sqrt{10}} = 3.46$$

7) Because 3.46 > 1.833 reject the null hypothesis. There is sufficient evidence to indicate that the true mean current is greater than 300 microamps at $\alpha = 0.05$. The $0.0025 < P\text{-value} < 0.005$

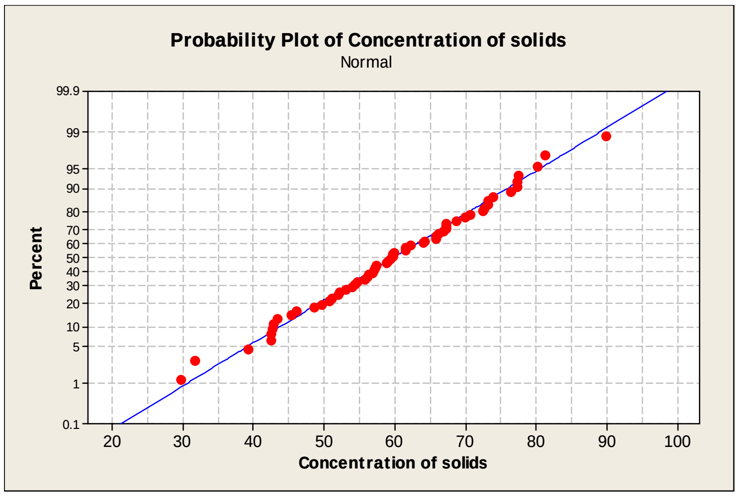
9-70 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.
1) The parameter of interest is the true mean concentration of suspended solids, μ.
2) $H_0 : \mu = 55$
3) $H_1 : \mu \neq 55$
4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{0.025,59} = 2.000$ for n = 60
6) $\bar{x} = 59.87$ s = 12.50 n = 60

$$t_0 = \frac{59.87 - 55}{12.50/\sqrt{60}} = 3.018$$

7) Because 3.018 > 2.000, reject the null hypothesis. There is sufficient evidence to conclude that the true mean concentration of suspended solids is not equal to 55 at $\alpha = 0.05$.

From Table V the t_0 value is between the values of 0.001 and 0.0025 with 59 degrees of freedom. Therefore, $2*0.001 <$ P-value $< 2*\ 0.0025$ and $0.002 <$ P-value < 0.005. Minitab gives a P-value of 0.0038.

b) From the normal probability plot, the normality assumption seems reasonable:



d) $d = \frac{|50-55|}{12.50} = 0.4$, n = 60 so, from the OC Chart VII e) for $\alpha = 0.05$, d= 0.4 and n=60 obtain $\beta \cong 0.2$.
Therefore, the power = 1 - 0.2 = 0.8.

e) From the same OC chart, and for the specified power, we would need approximately 75 observations.

$$d = \frac{|50-55|}{12.50} = 0.4$$

Using the OC Chart VII e) for $\alpha = 0.05$, d = 0.4, and $\beta \cong 0.10$ so that the power = 0.90, n = 75

Section 9-4

9-77

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true standard deviation of performance time σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = 0.75^2$

3) H_1 : $\sigma^2 > 0.75^2$

4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 > \chi^2_{\alpha,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.05,16} = 26.30$

6) n = 17, s = 0.09

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{16(0.09)^2}{.75^2} = 0.23$$

7) Because 0.23 < 26.30 fail to reject H_0. There is insufficient evidence to conclude that the true variance of performance time content exceeds 0.75^2 at $\alpha = 0.05$.

Because χ_0^2 =0.23 the *P*-value > 0.995

b) The 95% one sided confidence interval given below includes the value 0.75. Therefore, we are not be able to conclude that the standard deviation is greater than 0.75.

$$\frac{16(.09)^2}{26.3} \le \sigma^2$$
$$0.07 \le \sigma$$

9-78

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true measurement standard deviation σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) H_0 : $\sigma^2 = .01^2$

3) H_1 : $\sigma^2 \ne .01^2$

4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.975,14} = 5.63$ or $\chi_0^2 > \chi^2_{\alpha,2,n-1}$ where $\alpha = 0.05$ and $\chi^2_{0.025,14} = 26.12$ for n = 15

6) n = 15, s = 0.0083

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{14(.0083)^2}{.01^2} = 9.6446$$

7) Because 5.63 < 9.64 < 26.12 fail to reject H_0

0.1 < *P*-value/2 < 0.5. Therefore, 0.2 < *P*-value < 1

b) The 95% confidence interval includes the value 0.01. Therefore, there is not enough evidence to reject the null hypothesis.

$$\frac{14(.0083)^2}{26.12} \le \sigma^2 \le \frac{14(.0083)^2}{5.63}$$
$$0.00607 \le \sigma^2 \le 0.013$$

9-81

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the standard deviation of tire life, σ. However, the answer can be found by performing a hypothesis test on σ^2.

2) $H_0: \sigma^2 = 4000^2$

3) $H_1: \sigma^2 < 4000^2$

4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi_{1-\alpha,n-1}^2$ where $\alpha = 0.05$ and $\chi_{0.95,15}^2 = 7.26$ for n = 16

6) n = 16, $s^2 = (3645.94)^2$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{15(3645.94)^2}{4000^2} = 12.46$$

7) Because 12.46 > 7.26 fail to reject H_0. There is not sufficient evidence to conclude the true standard deviation of tire life is less than 4000 km at $\alpha = 0.05$.

P-value = $P(\chi^2 < 12.46)$ for 15 degrees of freedom. Thus, 0.5 < 1-P-value < 0.9 and 0.1 < P-value < 0.5

b) The 95% one sided confidence interval below includes the value 4000. Therefore, we are not able to conclude that the variance is less than 4000^2.

$$\sigma^2 \le \frac{15(3645.94)^2}{7.26} = 27464625$$

$$\sigma \le 5240$$

Supplemental Exercises

9-128 $\sigma = 8,\ \delta = 204 - 200 = 4,\ \frac{\alpha}{2} = 0.025,\ z_{0.025} = 1.96.$

a) n = 20: $\beta = \Phi\left(1.96 - \frac{4\sqrt{20}}{8}\right) = \Phi(-0.28) = 1 - \Phi(0.28) = 1 - 0.61026 = 0.38974$

Therefore, power = 1 – β = 0.61026

b) n = 50: $\beta = \Phi\left(1.96 - \frac{4\sqrt{50}}{8}\right) = \Phi(-2.58) = 1 - \Phi(2.58) = 1 - 0.99506 = 0.00494$

Therefore, power = 1 – β = 0.995

c) n = 100: $\beta = \Phi\left(1.96 - \frac{4\sqrt{100}}{8}\right) = \Phi(-3.04) = 1 - \Phi(3.04) = 1 - 0.99882 = 0.00118$

Therefore, power = 1 – β = 0.9988

d) As sample size increases, and all other values are held constant, the power increases because the variance of the sample mean decreases. Consequently, the probability of a Type II error decreases, which implies the power increases.

9-143 a)

1) The parameter of interest is the true mean percent protein, μ.

2) $H_0 : \mu = 80$

3) $H_1 : \mu > 80$

4) $t_0 = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

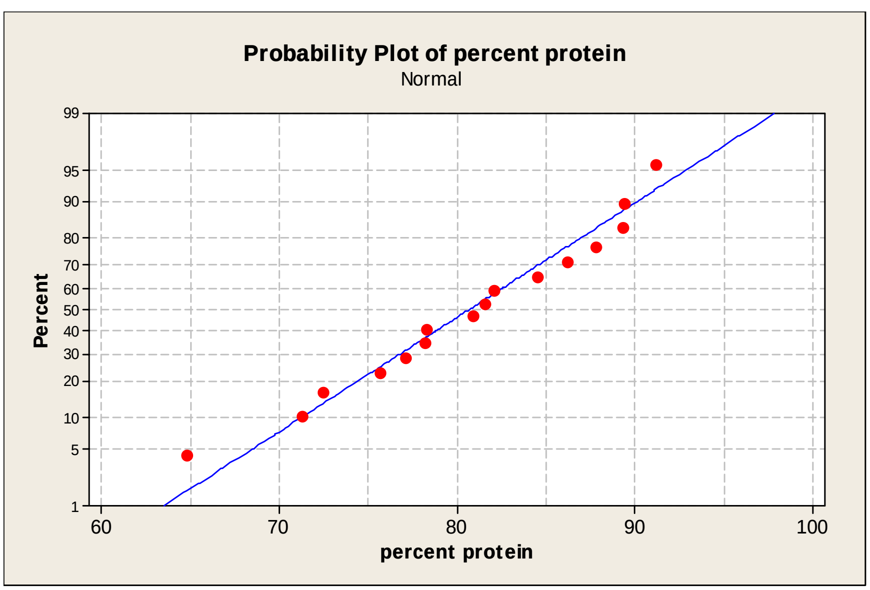
5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{0.05,15} = 1.753$ for $\alpha = 0.05$

6) $\bar{x} = 80.68$ $s = 7.38$ $n = 16$

$$t_0 = \frac{80.68 - 80}{7.38/\sqrt{16}} = 0.37$$

7) Because 0.37 < 1.753 fail to reject the null hypothesis. There is not sufficient evidence to indicate that the true mean percent protein is greater than 80 at α = 0.05.

b) From the normal probability plot, the normality assumption seems reasonable:



c) From Table V, 0.25 < P-value < 0.4

9-144

a) In order to use the χ^2 statistic in hypothesis testing and confidence interval construction, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true variance of tissue assay, σ^2.

2) H_0 : $\sigma^2 = 0.6$

3) H_1 : $\sigma^2 \neq 0.6$

4) $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) Reject H_0 if $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$ where $\alpha = 0.01$ and $\chi^2_{0.995,11} = 2.60$ or $\chi_0^2 > \chi^2_{\alpha/2,n-1}$ where $\alpha = 0.01$ and $\chi^2_{0.005,11} = 26.76$ for n = 12

6) n = 12, s = 0.758

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{11(0.758)^2}{0.6} = 10.53$$

7) Because 2.6 <10.53 < 26.76 we fail to reject H_0. There is not sufficient evidence to conclude the true variance of tissue assay is significantly different from 0.6 at $\alpha = 0.01$.

b) 0.1 < P-value/2 < 0.5, so that 0.2 < P-value < 1

c) 99% confidence interval for σ, first find the confidence interval for σ^2

For $\alpha = 0.05$ and n = 12, $\chi^2_{0.995,11} = 2.60$ and $\chi^2_{0.005,11} = 26.76$

$$\frac{11(0.758)^2}{26.76} \le \sigma^2 \le \frac{11(0.758)^2}{2.60}$$

$$0.236 \le \sigma^2 \le 2.43$$

$$0.486 \le \sigma \le 1.559$$

Because 0.6 falls within the 99% confidence bound there is not sufficient evidence to conclude that the population variance differs from 0.6

9-146 a)

1) The parameter of interest is the true mean of cut-on wave length, μ.

2) H_0 : $\mu = 6.5$

3) H_1 : $\mu \neq 6.5$

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$. Since no value of α is given, we will assume that $\alpha = 0.05$. So $t_{\alpha/2,n-1} = 2.228$

6) $\bar{x} = 6.55$, s = 0.35 n=11

$$t_0 = \frac{6.55 - 6.5}{0.35/\sqrt{11}} = 0.47$$

7) Because 0.47< 2.228, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true mean of cut-on wave length differs from 6.5 at $\alpha = 0.05$.

b) From Table V the t_0 value is found between the values of 0.25 and 0.4 with 10 degrees of freedom, so 0.5 < *P*-value < 0.8

c) $$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|6.25 - 6.5|}{0.35} = 0.71$$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, d = 0.71, and $1 - \beta > 0.95$ ($\beta < 0.05$). We find that n should be at least 30.

d) $$d = \frac{\delta}{\sigma} = \frac{|\mu - \mu_0|}{\sigma} = \frac{|6.95 - 6.5|}{0.35} = 1.28$$

Using the OC curve, Chart VII e) for $\alpha = 0.05$, n =11, d = 1.28, we find $\beta \approx 0.1$

9-148 a) 1) the parameter of interest is the standard deviation, σ

2) $H_0 : \sigma^2 = 400$

3) $H_1 : \sigma^2 < 400$

4) The test statistic is: $\chi_0^2 = \dfrac{(n-1)s^2}{\sigma^2}$

5) No value of α is given, so that no critical value is given. We will calculate the P-value.

6) n = 10, s = 15.7

$$\chi_0^2 = \frac{9(15.7)^2}{400} = 5.546$$

P-value = $P(\chi^2 < 5.546)$; $0.1 < P - \text{value} < 0.5$

7) The P-value is greater than a common significance level α (such as 0.05). Therefore, we fail to reject the null hypothesis. There is insufficient evidence to support the claim that the standard deviation is less than 20 microamps.

b) 7) n = 51, s = 20

$$\chi_0^2 = \frac{50(15.7)^2}{400} = 30.81$$

P-value = $P(\chi^2 < 30.81)$; $0.01 < P - \text{value} < 0.025$

The P-value is less than 0.05. Therefore we reject the null hypothesis and conclude that the standard deviation is significantly less than 20 microamps.

c) Increasing the sample size increases the test statistic χ_0^2 and therefore decreases the P-value, providing more evidence against the null hypothesis.