CIVE 3331 - ENVIRONMENTAL ENGINEERING
Spring 2003

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Purpose: Lecture #15 CIVE3331

Plume Modeling

Point sources are a major contribution to air pollution (½). Plume models are used to estimate the impact of emissions on surrounding environment (much like the DO Sag model in water). The most advanced models account for various atmospheric processes, chemical reactions, and detailed flow physics. These models require immense expertise to run (let alone use properly), and are beyond the scope of this course.

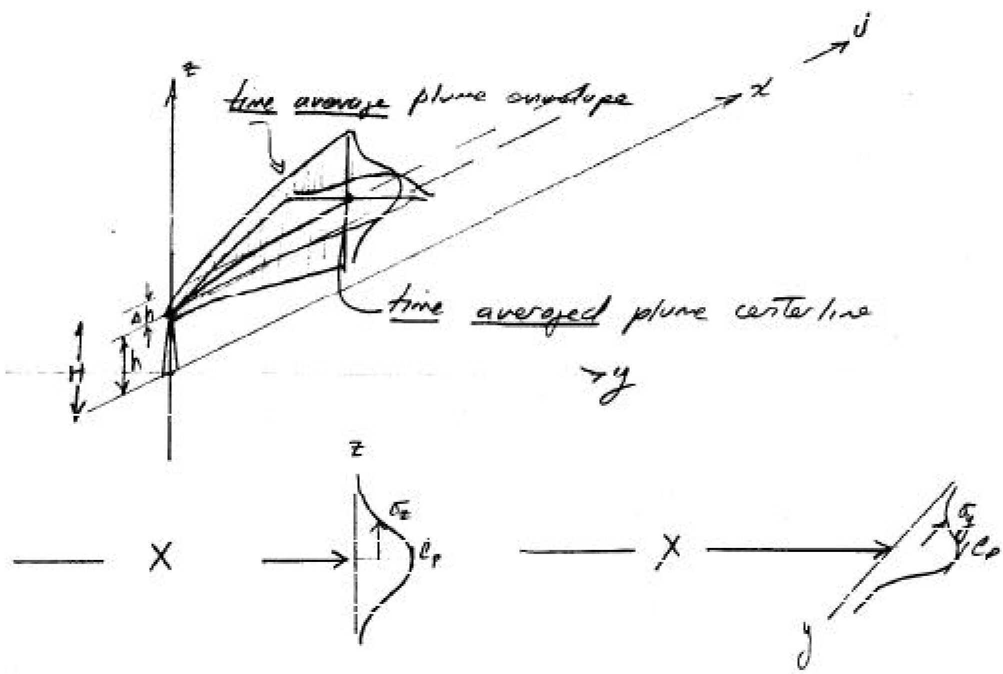

Figure 1 Gaussian Plume Model - Definition Sketch

H = effective stack height
h= physical stack height
Δh = plume rise

A simple approach that captures a lot of behavior is gaussian plume modeling.

The model assumes constant emissions rate, constant wind speed and direction, uniform in elevation. It assumes that a plane can approximate terrain.

The single source equation for such a situation is

$$c(x,y,z) = \frac{\dot{M}}{\pi U_H \sigma_y \sigma_z} \exp\left(\frac{-(H-z)^2}{2\sigma_z^2}\right) \exp\left(\frac{-(y)^2}{2\sigma_y^2}\right)$$

M = emissions rate of the pollutant (mass/time); x = distance downwind; y = distance crosswind; z = elevation; σ_y = crosswind (transverse-y) dispersion coefficient; σ_z = elevation (transverse-z) dispersion coefficient; U_H = effective stack height windspeed.

The following equation is used to relate windspeeds and effective stack height.

$$\frac{U_H}{U_a} = \left(\frac{H}{z_a}\right)^p$$

p is based on terrain and atmospheric stability classifications. The dispersion coefficients are also based on atmospheric stability classifications. Numerical values are obtained by table lookup or curve fits.

Determining peak concentrations is easiest by spreadsheet calculations to plot a profile of concentration versus distance from the equation. Alternatively the dimensionless graph (pg 417) can be used to estimate peak concentration and location.

Additional modifications to the gaussian model include:

Plume rise corrections

Temperature inversion corrections

Terrain corrections

Gaussian models can also be integrated in space to represent line and area sources.

Document History:

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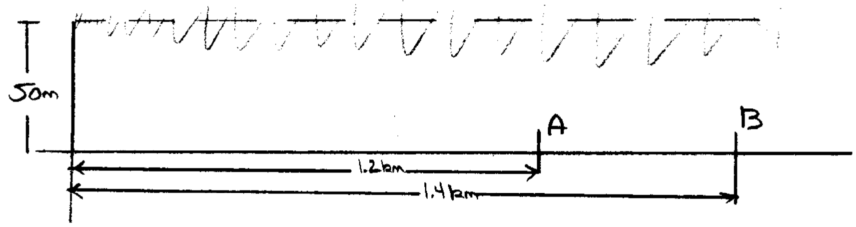
Theodore G. Cleveland	Created	January 24,2003	CIVE3331_Lecture_005.PDF

7.20



Step ① determine atmospheric stability classification
Class D — overcast conditions

a) Use chart pg 417.
$x = 1.0$ km. $C_{max}\left(\frac{UH}{Q}\right) = 5 \cdot 10^{-5}$
location Ⓐ will have greater pollution level.

b) Clear sky, night class F (or maybe E) $U < 5$ m/s
x moves away from stack
$x \approx 2.0 - 3.5$ km

c) Location Ⓑ will have greater pollution level than location Ⓐ under conditions in part (b).

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(7.47)

example illustrates

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tion, and (b) effect of

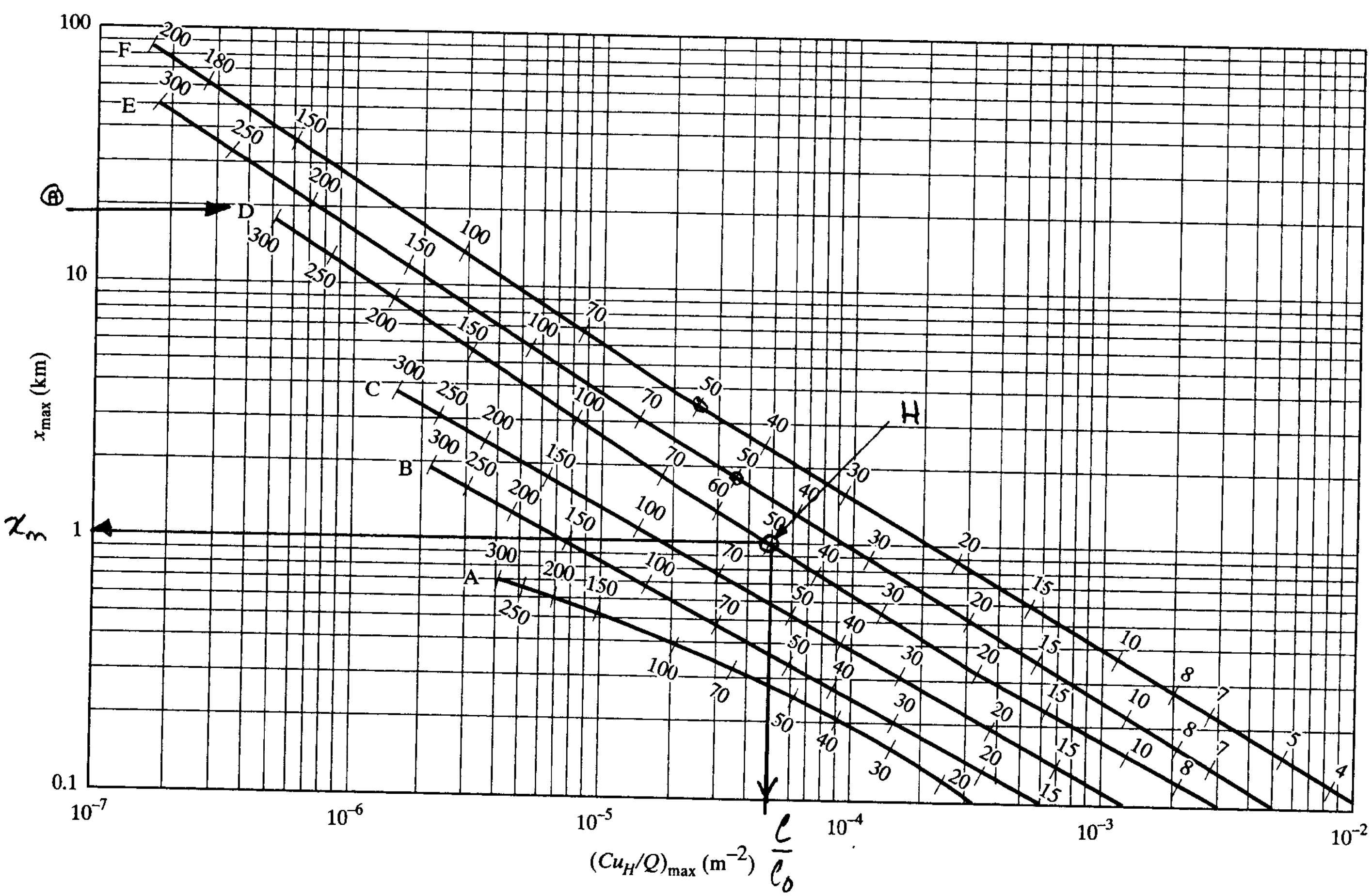
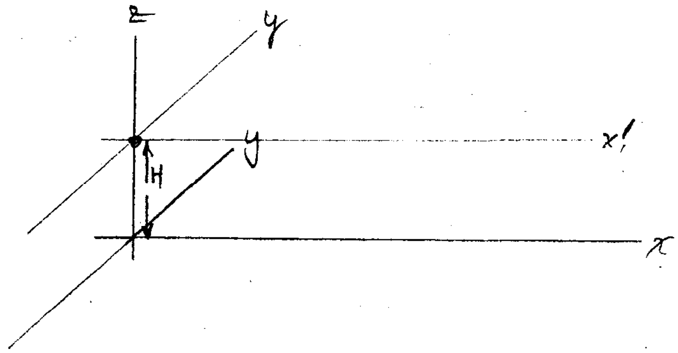


FIGURE 7.50 To determine the downwind concentration peak, enter the graph at the appropriate stability classification and effective stack height (numbers on the graph in meters) and then move across to find the distance to the peak, and down to find a parameter from which the peak concentration can be found (Turner, 1970).

Gaussian Dispersion Model - Steady State, Usual Air assumptions



Point source in x', y, z

$$c(x', y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\exp\left(-\frac{1}{2}\frac{z^2}{\sigma_z^2}\right)$$

Shift axis to x, y, z

$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right)$$

Now add ground level reflection

$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\left[\exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H)^2}{\sigma_z^2}\right)\right]$$

Now add mixing-layer reflection(s)

$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\left\{\left[\exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H)^2}{\sigma_z^2}\right)\right] + \sum_{\substack{i=-\infty \\ i \neq 0}}^{\infty}\left(\exp\left(-\frac{1}{2}\frac{(z-H+2iL)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H+2iL)^2}{\sigma_z^2}\right)\right)\right\}$$

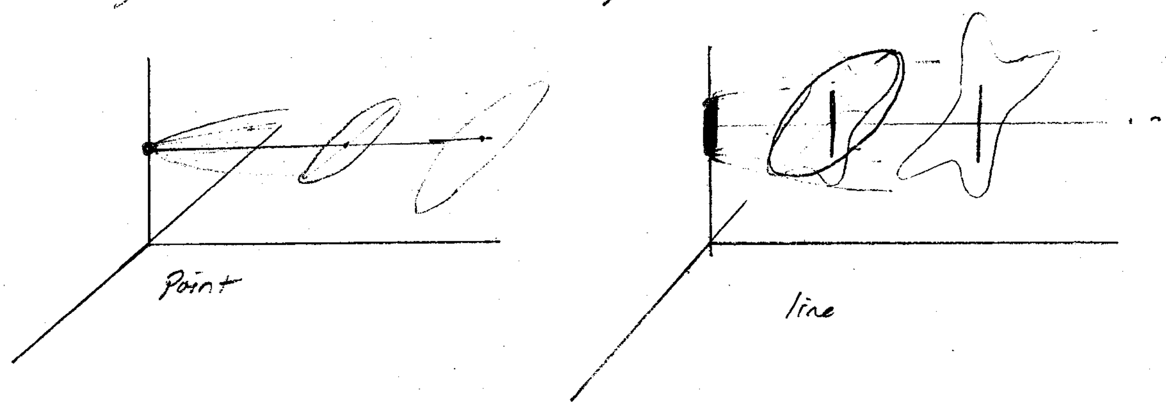
or more compactly:

mixing height (length)

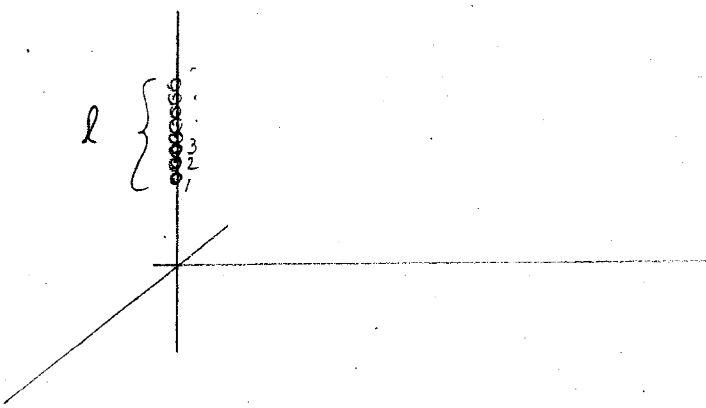
$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right) * \sum_{i=-\infty}^{\infty}\left[\exp\left(-\frac{1}{2}\frac{(z-H+2iL)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H+2iL)^2}{\sigma_z^2}\right)\right]$$

Line Source concept –

assume (for your thesis) that source behaves like a finite line segment instead of a point

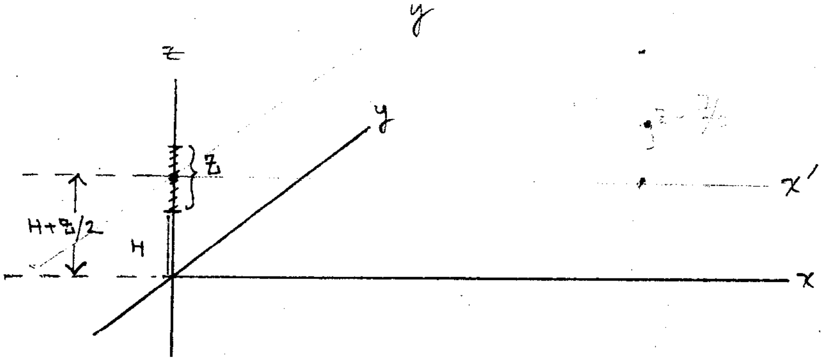


mathematically we "sum up" the contribution of an infinite number of point sources, each with contribution dC



$$C(x,y,z) = C_1(\ ) + C_2(\ ) + C_3(\ ) + \cdots$$
$$= \int_l dC(x,y,z)$$

Line Source Model



line source in x', y, z system

$$dc = \frac{dQ\,dz}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left\{\exp\left(-\frac{z^2}{2\sigma_z^2}\right)\right\}$$

$$dQ = \left(\frac{Q}{Z}\right) \qquad K = \frac{dQ}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$

$$c = \int_{-Z/2}^{Z/2} dC = \underbrace{K \int_{z-Z/2}^{z+Z/2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) dz}_{\text{examine this integral}}$$

$$= \int_0^{z+Z/2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) dz \; - \int_0^{z-Z/2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) dz$$

let $\beta = \frac{z}{\sqrt{2}\sigma_z}$ $\quad \frac{d\beta}{dz} = \frac{1}{\sqrt{2}\sigma_z}$

$\therefore \sqrt{2}\sigma_z \, d\beta = dz$

$z = 0, \; \beta = 0$

$z = \frac{Z}{2}, \; \beta = \frac{z+Z}{2\sqrt{2}\sigma_z}$

$$\int_0^{z+\frac{Z}{2\sqrt{2}\sigma_z}} \exp(-\beta^2)\sqrt{2}\sigma_z \, d\beta$$

$$\left[\sqrt{2}\sigma_z \cdot \frac{\sqrt{\pi}}{2} \cdot \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{\frac{Z}{2\sqrt{2}\sigma_z}} \exp(-\beta^2)\, d\beta}_{\operatorname{erf}\left(\frac{z+Z}{2\sqrt{2}\sigma_z}\right)}\right] - \left[\sqrt{2}\sigma_z \cdot \frac{\sqrt{\pi}}{2} \cdot \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{z-\frac{Z}{2\sqrt{2}\sigma_z}} \exp(-\beta^2)\, d\beta}_{\cancel{-}\,+\,\operatorname{erf}\left(\frac{z-Z}{2\sqrt{2}\sigma_z}\right)}\right]$$

1mg/sec

$Z = 10m \quad dQ = \frac{1mg/s}{10m} = .1mg/s/m$

Collect terms & clean up model

$$C = \frac{Q}{Z} \cdot \frac{1}{\cancel{\sqrt{2\pi}} \cdot \sqrt{2\pi}\, u \sigma_y \cancel{\sigma_z}} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left[\frac{\cancel{\sqrt{2}\sigma_z} \cdot \cancel{\sqrt{\pi}}}{2} \operatorname{erf}\left(\frac{z + \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right) \underset{-}{\cancel{+}} \frac{\cancel{\sqrt{2}}\cancel{\sigma_z}\cancel{\sqrt{\pi}}}{2} \operatorname{erf}\left(\frac{z - \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right)\right]$$

$$C = \frac{Q}{Z} \cdot \frac{1}{\sqrt{2\pi}\, u \sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left[\frac{1}{2} \operatorname{erf}\left(\frac{z + \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right) \underset{-}{\cancel{+}} \frac{1}{2} \operatorname{erf}\left(\frac{z - \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right)\right]$$

Now shift back to x, y, z system

$$c(x,y,z) = \frac{Q}{Z} \cdot \frac{1}{\sqrt{2\pi}\, u \sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left(\frac{1}{2}\right)\left(\operatorname{erf}\left(\frac{z + \frac{Z}{2} - H}{\sqrt{2}\,\sigma_z}\right) \underset{-}{\cancel{+}} \operatorname{erf}\left(\frac{z - \frac{Z}{2} - H}{\sqrt{2}\,\sigma_z}\right)\right)$$

Then will need to add reflections, each plane will produce 4 terms.

Something like:

$$\sum_{i=-\infty}^{\infty} \frac{1}{2}\left[\operatorname{erf}\left(\frac{z + \frac{Z}{2} - H + 2iL}{\sqrt{2}\,\sigma_z}\right) \underset{-}{\cancel{+}} \operatorname{erf}\left(\frac{z - \frac{Z}{2} - H + 2iL}{\sqrt{2}\,\sigma_z}\right)\right] + \frac{1}{2}\left[\operatorname{erf}\left(\frac{z + \frac{Z}{2} + H + 2iL}{\sqrt{2}\,\sigma_z}\right) \underset{-}{\cancel{+}} \operatorname{erf}\left(\frac{z - \frac{Z}{2} + H + 2iL}{\sqrt{2}\,\sigma_z}\right)\right]$$

→ You will need to check the mathematics, best tool is Mathematica — it can handle the symbol manipulation —

Also test that with $\frac{Q}{Zu} = 1$ then $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c(x,y,z)\, dz\, dy = 1$

i from -2 to 2 should be equivalent to the 4 terms used in your model.

Excel has erf(z) built in, but it does not handle negative arguments correctly or large arguments correctly.

Check with Dr. Rafai, get her opinion on adding this component — I think it is relatively easy ~~— it Z has to be less to fit~~