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Computational Design of a nature-inspired architectural structure using the concepts of self-similar and random fractals

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ABSTRACT

This paper aims to explore the scope of applying the concept of fractal geometry in the field of architecture and construction. There are mainly two different types of fractals – self-similar fractal and random fractal. In this paper, both types of fractals are used to design a nature-inspired architectural structure with the strategy of exploring the potency of fractal geometry as a geometric framework that can offer new structural forms. Based on the mathematical formulations of self-similar fractal shape and random fractal shape, tree-inspired branching supports and natural terrain inspired unsmooth crinkled roof are modeled using the algorithms of Iterated Function System and Midpoint Displacement (Diamond Square Algorithm) method respectively. Fractal dimensions are calculated to assess the visual complexity of the roof surface and branching supports. Finite element analysis is performed to assess the structural strength of the model with respect to changing of fractal dimensions.

1. Introduction

1.1. Background and problem statement

Architects, designers and engineers sometimes take inspiration from nature's forms to develop their designs and use different geometric systems as frameworks to replicate the complex or abstract forms of nature. Many of nature's forms are highly complex and not easy to model using Euclidean, linear or other regular geometric systems. Fractal geometry is one of the youngest geometric concepts that can easily and quickly model the complex forms of many natural objects and phenomena by using some simple algorithms. Hence, fractal geometry is one of the most suitable choices for designing nature-inspired forms in architecture. Fractal geometry was first systematically developed and categorized as a new geometric concept by Benoit Mandelbrot in the 1970s. It is commonly characterized by the properties of self-similar repetitions and unending irregularity, and mathematically defined by the shapes which are mainly fractional dimensional and whose Hausdorff dimensions exceed their topological dimensions [31].

Soon after its early development in the beginning of 1980s, the notion of fractal geometry has been applied in exploring and modeling nonlinear and complex shapes in a list of different disciplines - ranging from science [41,48,53] to engineering [11,27,28] and medicine [29] to arts [55]. Its application also encompasses the field of architecture, but mainly limited to the visual analysis of building forms [8,39,42] and cities [5]. However, in the field of construction, so far, the application of fractal geometry as a framework is rarer as compared to the other fields, especially by means of its application in designing structural shapes. [1,2]) are probably one of the initial researchers, who proposed new structural forms using the mathematical principle of fractal geometry. Later, some other researchers applied fractal geometry for designing new configurations of planar trusses [43,44] and analyzed their mechanical behavior [14,43,44]. In the field of physics and material science, some investigators [17,57,58] used the property of fractal's self-similar repetition to design ultra-light hierarchical lattice trusses with high strength. In addition, some researchers [45,49,54] proposed new free-forms and complex shapes for shell structures using the principle of fractal geometry.

However, most of the previous proposals for developing fractal-based structures do not interpret the impact of fractal dimension on the structural behavior. As far as it is known, except for the efforts taken by Asayama and Mae [1,2] and Rian and Sassone [43,44], there are a very few systematic investigations and interpretations of the fractal dimension of the fractal-based structural shapes corresponding to their mechanical behavior. In addition, so far, all of these

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applications are based on the fractal's main property of self-similarity and these types of fractals are known as self-similar fractals. But, fractal geometry has another, yet important property, which is frequently abundant in nature. It is its continuous irregularity and randomness which gives an abstract impression of the whole into parts. These types of fractals are known as random fractals or statistically self-similar fractals. Natural terrains, real tree branches and crinkled paper are some common examples of random fractals. So far, it is extremely rare to find any published papers that deal with the concept of random fractals for modeling structural forms and analyzing their structural behavior with respect to the factor of fractal dimension as a measure and parameter of irregularity or randomness.

Accordingly, this paper aims to apply both the notions of self-similar and random fractals as a conceptual framework to design a canopy structure, as a case and a benchmark, in order to demonstrate the structural behavior of fractal forms in contrast to the regular forms. Here, the canopy structure has been inspired by the self-similar and random forms of nature that are fractal-like. In particular, the self-similar branching pattern of trees and the irregular random surface of a natural land terrain or a crinkled paper are considered only as the geometric references. The reason for selecting these two natural examples is explained in details in the design concept section (Section 3.1).

In the recent years, the mathematical concept of fractal geometry has been deliberately and systematically applied in designing computational models of tree-like structures by several researchers based on self-similar bifurcation [9] and reciprocal scheme [47]. Nonetheless, in this context, Van Tonder [52] argued that the computational model of a tree-like figure generated by using self-similar bifurcation is too simplified; while, in contrast, a simple hand-made classic illustration of a tree or various degrees of randomization on a computational tree-model display high degree of complexity and looks natural. Structurally, a natural tree tackles complicated scheme of different loads, while the man-made tree-like structure, which is a simplified form of a real tree, is preferred to tackle simple load systems such as a roof load. Hence, we have intentionally avoided mimicking the true natural form of a tree; instead, we opted for its simplistic form and decided to find the possible structural advantage of such a simple configuration of a branching structure.

Fractal as a geometric framework has also been used in designing some canopies in architecture. Sakai et al. [59] has applied the Sierpinski gasket, a canonical example of fractals, as a geometric framework to design a perforated roof surface for a canopy structure inspired by the tree foliage that filters sunlight and allows air circulation, thus giving a pleasant shadow as compared to a hard shadow cast by solid shed. Cox and D'Antonio [10] applied fractal concept for designing an unsmooth canopy surface for acoustic reasons. They believe that the fractal surface as a canopy can diffuse sound. However, both of these applications are limited to dealing with the energy and acoustic aspects and do not deal with the structural side of fractal applications in canopy design.

In our study, unlike branching columns, we intend to make the canopy roof truly complex by adding some factor of randomness to express the true nature. In the case of such approaches in dealing with the complex forms in design, Terzidis [60] concerns that the non-strategic use of fractals and other complex geometries as a mathematical tool for computational design may sometimes mislead our design intuition. Besides, according to Zappulla [56], fractals including other mathematical concepts do not teach us how to design, but they can certainly help us to improve the way we design. In this paper, nevertheless, we have strategically applied the concept of fractals as a geometric tool for the early design process. Here, its role is not to lead the conceptual design progression, but to act as an aid to improve the design process and lead the geometric modeling process only, dealing mainly with the structural aspects.

1.2. Paper outline

The first part of this paper is the introduction part which has discussed the motivation of this study and stated the principal research problem.

The second part is the theoretical framework and mathematical formulations which will briefly explain the mathematics of fractal geometry, the concept of self-similar fractals and random fractals, the Iterated Function System and the Midpoint Displacement Method as the methods for constructing fractals, and the calculations of fractal dimensions of the resulted fractal shapes.

The third part will deal with the geometric modeling of the proposed structure starting with the design concept. As stated before, the geometric model of a branching support will be constructed using the mathematical formulation of self-similar fractals and the process of Iterated Function System (IFS), which was systematically developed by Barnsley [3] using contraction mapping method. IFS is one of the most common and easiest processes for modeling fractal shapes, and hence, we see several architectural examples where IFS has been used as a main generative tool and as an analytical tool. Among them, Bovill [8] was one of the initial researchers who methodically discussed the potential relationship between fractal geometry and architectural design, and emphasized the role of IFS in analyzing and modeling architectural forms. James Harris [19] explored the possible fusion of fractal geometry and architectural form with a new direction with the advent of computers. He found architectural possibilities in the work of Michael Barnsley's Iterated Function System and believed that the fractals generated with this method had a direct correlation with a diverse array of structures found in nature. Van Loocke [50,51] applied IFS to produce polygon-based and origami-based fractals that are useful as a geometric framework for designing architectural forms. However, all of their approaches to using IFS cover the discussion about the application of IFS in developing or analyzing architectural designs and decorations, but not the structural forms from a mechanical point of view. Ostwald [40], Joye [26] and Van Loocke (2009) critically reviewed some of these attempts and argued that the IFS application for analyzing or designing architectural forms in terms of connecting fractals with architecture are sometimes inappropriate, superficial, trivial and non-mathematical. For example, Bovill [8], Eaton [13] and Sala [46] quantitatively demonstrated that the floor plan of Palmer House designed by F. L. Wright displays fractal characters, but in fact, Joye [26] and Eaton [13] later clarified that it has no connection with true fractal geometry, even it does not show any fractal-like feature.

It is important to make clear that the IFS does not necessarily produce fractals all the time, but also generates shapes that are non-fractal yet complex and self-similar. As an another example, the façade of Federation Square Building in Melbourne was claimed as an example of fractal geometry developed by using IFS [61]. However, in fact, this façade pattern is not a mathematical fractal, but a so-called trivial fractal [25,40,43]. In our study, our approach of using IFS for producing a branching pattern is mathematical and the end product is a mathematical fractal within the finitely iterated state. Here, IFS has been used as a geometric tool for a parametric design process. In the field of structural design, Asayama and Mae [2], Rian and Sassone [43], Rian et al. [45], Vyzantiadou et al. [54] and Stotz et al. [49] have used IFS for modeling new structural forms and explore the mechanical characteristics of IFS-generated fractal-based structures as a venture. Their approaches focused on using IFS as a geometrical tool for developing efficient forms of structures. However, in our present study, we have explored the role and impact of IFS not only for modeling a structural form but also for manipulating its geometric form as well as its structural behavior as a scheme for finding optimized structure.

Apart from IFS, the Midpoint Displacement Method will be applied for generating the geometric model of a crinkled surface for the canopy roof design using the mathematical formulation of random fractals and inspired by the randomly irregular surface of a natural terrain. Later, both of these branching support and roof models will be combined to construct the final model of the canopy structure. The final geometric model will be transformed into a parametric model to understand the changing structural behavior with the varying geometric variables.

Therefore, the fourth part will perform structural analysis on the varied forms of the structure, and find the best forms. The final part will discuss the results of the finite element analysis and conclude the study with the possibility of further applications, feasibility, limitations and research scopes of fractal geometry in the field of architecture and construction.

2. A brief mathematical background of fractal geometry

A non-strict definition of a fractal is a shape or an image that contains the exact or approximate copies of itself at an infinitely different range of scales; which means, it is self-similar at every magnifying level. However, its mathematical definition is more precise. According to Robert L. Devany's [12] mathematical definition, a fractal is a subset of R^n (an n-dimensional metric space) which is perfectly or approximately self-similar and whose Hausdorff dimension exceeds its topological dimension. In simpler description, fractal shapes are fractional dimensional and fall in between two successive integer dimensional objects, i.e., $0 < D < 1$, or $1 < D < 2$, or $2 < D < 3$, where D is a fractal dimension. There are mainly two different types of fractals, one is perfectly self-similar fractal, and another is the random fractal or statistically self-similar fractal.

2.1. Self-similar fractals

A self-similar fractal is a set which is a union of self-similar sets that lie in the Metric space, more precisely, in the Hausdorff Metric space. Based on the *contraction mapping theory* introduced by M. F. Barnsley [3], a fractal set is an *attractor* when it is the resulting figure at the limit state obtained from a set of affine transformations 'f_i', i = 1 to k, applied infinite times. An example, with just one transformation 'f', ($k = 3$) is shown in 'Fig. 1'. A fractal is a unique non-empty compact set. The resulting figure can be obtained by the *union* of all self-similar subsets.

When a fractal is an *attractor* 'Δ' which is a *union* set of all the identical infinite subsets that are the scaled copies of the initial set, then it can be represented as,

$$\Delta \bigcup_{n=0}^{\infty} \Delta_n \tag{1}$$

i.e.,

$$\Delta = \Delta_1 \cup \Delta_2 \cup \Delta_3 \cup \ldots \ \ldots \cup \Delta_n \cup \Delta_0 \tag{2}$$

where,

$$\Delta_n = f_1(\Delta_{n-1}) \cup f_2(\Delta_{n-1}) \cup \ldots \ ..f_m(\Delta_{n-1}) \cup \Delta_0$$

where, n is the number iterations and k is the number of self-similar copies produced at each iteration. When Δ_1, Δ_2, ….., Δ_n, … are contraction sets of Δ_0, that are scaled by using the *contractivity factor* λ_i and transformed by using an *affine transformation* function f_i, such that,

$$\Delta_0 \subset \Delta_1 \subset \Delta_2 \subset \ldots \subset \Delta_{n-1} \subset \Delta_n \subset \ldots \tag{4}$$

then, they form a perfect self-similar fractal set.

2.1.1. Iterated function system (IFS)

Fractals are formed by the repetition of the original shape after the geometric transformations in the first step, and then repeating this process iteratively in the next steps for infinite times. This process produces a fractal figure, which, in many cases, is unpredictable. However, in 1981, based on the Hutchinson's operator [23], Barnsley developed a system, known as the *Iterated Function System* (IFS) that can predict the end result of a fractal formation in a deterministic way [3]. In the Barnsley's concept of IFS, it is mainly noticed that the final outcome of the fractal figure is not defined by the initial shape. Instead, it is defined by *affine transformations* that can be regarded as the true 'initial condition'. In 'Fig. 6', it is shown that a triangle and a hut shape separately converge into a Sierpinski triangle after a few iterations after keeping the *affine transformation* rules same for both the initial shapes.

Barnsley's IFS is a simple but powerful function system that produces fractal figures in a deterministic way. It is shown in the 'Fig. 2'

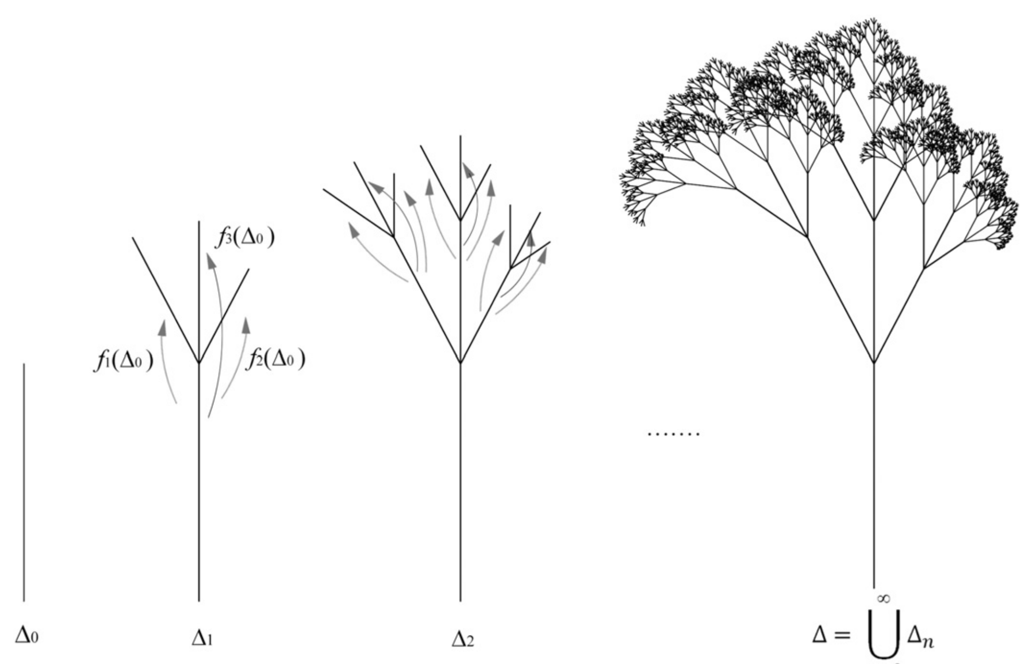


Fig. 1. Fractal as a union of perfectly self-similar subsets after *affine transformation*.

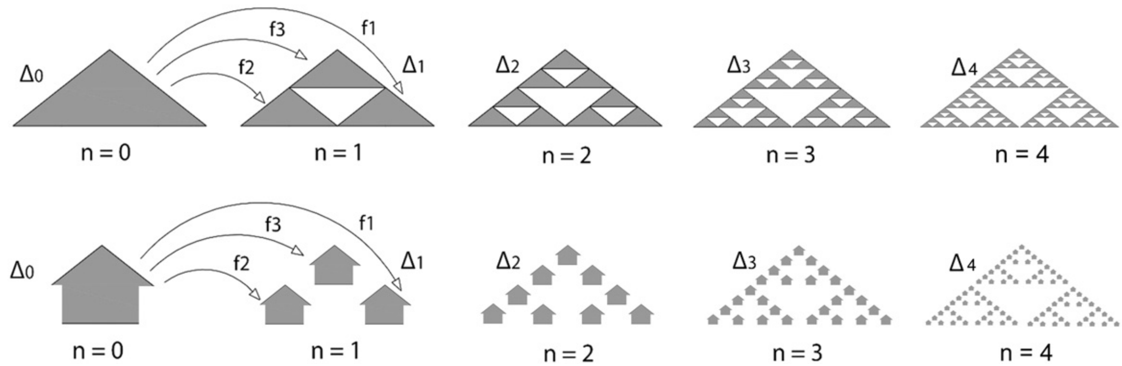


Fig. 2. A convergent sequence of the Sierpinski triangle by using contraction mapping and IFS. Top: The Initial shape is a triangle; Bottom – Initial shape is a hut.

that the main key operator of the IFS is a set of *affine transformations* that lead the final outcome towards an *attractor* '*A*'. The IFS works recursively using the *Hutchinson operator* as shown in the 'Fig. 3' to construct a self-similar fractal.

The figure resulted at the finite number of iterations (n) is,

$$A_n = \bigcup_{i=1}^{m} f_i(A_{n-1}) \quad (5)$$

Therefore, from the properties of IFS, we notice the importance of *affine transformations* 'f_i' which is, in fact, the main key determinant for the *attractor* '*A*'. The two-dimensional *affine transformation* in the Euclidean XY plane is represented as,

$$f = \lambda[\mu][r]\{x\} + \{\delta\} = \lambda\begin{bmatrix}\mu_1 & 0\\ 0 & \mu_2\end{bmatrix}\begin{bmatrix}cos\theta & -sin\theta\\ sin\theta & cos\theta\end{bmatrix}\begin{Bmatrix}x\\ y\end{Bmatrix} + \begin{Bmatrix}\delta_x\\ \delta_y\end{Bmatrix} \quad (7)$$

where $\mu_1 = -1$ and $\mu_2 = 1$ are the reflections along Y-axis, $\mu_1 = 1$ and $\mu_2 = -1$ are the reflections along X-axis, θ is the angle of rotation, and δ_x and δ_y are the displacements along X-axis and Y-axis respectively.

2.1.2. Hausdorff dimension

Hausdorff dimension is a mathematical measure that is mostly accepted for evaluating the fractal dimension of self-similar fractals. Based on the Barnsley's *contraction mapping* theory (1988), the Hausdorff dimension of a fractal is linked to the *contractivity* factors by the following relation:

$$\sum_{i=1}^{k} \lambda_i^D = 1 \quad (9)$$

where λ_i is a *contractivity* factor of transformation f_i and k is the number of transformations.

For example, the fractal tree, shown in 'Fig. 2', produces three self-similar copies of itself at each iteration. The *contractivity* factors of the left branch, right branch and the central branch are 0.6, 0.6 and 0.4

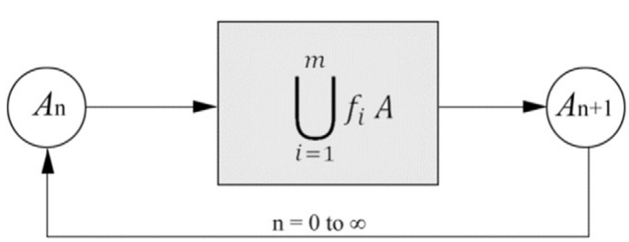


Fig. 3. A schematic diagram that shows how IFS recursively calls output as input iteratively using the function of *Hutchinson operator*.

respectively. Its Hausdorff dimension D can be calculated by following way by using 'Eq. (9)',
i.e.,

$$(0.6\,)^D + (0.4)^D + (0.6)^D = 1$$

Using Newton's method, we get,

$$D \approx 1.86$$

2.2. Random fractals

In nature, there are plenty of objects that have impressions of self-similarity, but they are not strictly self-similar, although they may exhibit fractal behavior. This type of fractal is known as statistically self-similar fractal or random fractal, displaying high visual complexity. Random fractal repeats a pattern stochastically, yet a part of it sometimes gives a near impression of the whole. Coastlines, mountain terrain and trees are such examples. A part of tree branches give an impression of the whole tree, although they are not exactly similar in reality. Because of its random characteristics, this type of fractals can be generated by using stochastic rule, such as Gaussian Midpoint Displacement Method.

2.2.1. Gaussian midpoint displacement method

In the Gaussian Midpoint Displacement Method, at the first step, the midpoint of a straight line is vertically displaced by a random value taken from a Gaussian distribution (Fig. 4b). This random displacement (σ_0) is a scaled value (λ) of the base length (L_0), i.e., $\sigma_0 = \lambda L_0$. The scaling factor (λ) which is related to the Hurst exponent (H) that varies in between 0 to 1 in the case of the one-dimensional fractal curve. The relation between the scaling factor (λ) and the Hurst exponent is as follows:

$$\lambda = 2^{-H} \quad (10)$$

The elevated midpoint is then connected with the endpoints, producing two new lines. In the second step, the midpoints of both the new lines are again displaced by a random value which is the scaled values of the length of these two lines. In The next steps, this process is continued for each new line ad infinitum.

From Eq. (10) we can see that when the value of Hurst exponent gets higher, the scaling factor λ gets smaller, i.e., the fluctuations of the values of midpoint displacements get smaller and the fractal curve becomes less noisy. And, when H becomes 0.5, the distribution generated becomes the same as Brown noise because of its relation to the Brownian motion [8].

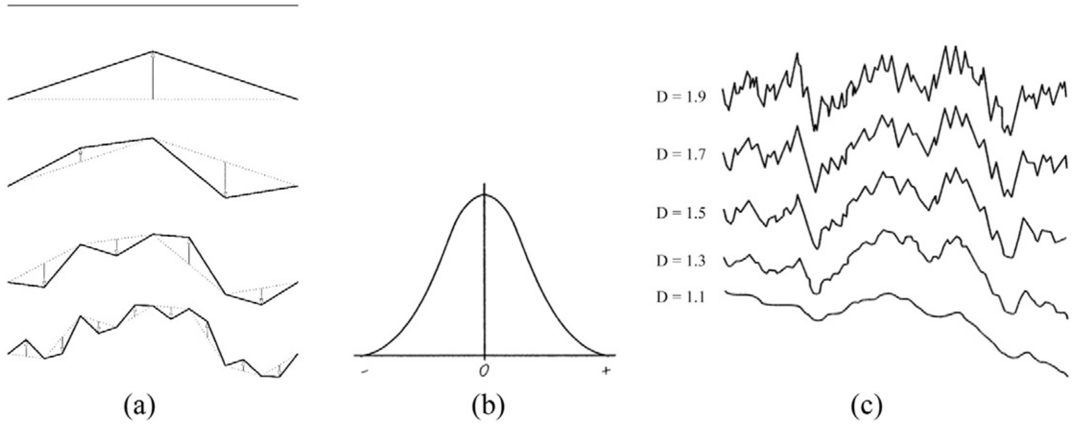


Fig. 4. (a) Unpredictable random fractal curve using the Gaussian Midpoint Displacement method. (b) A Gaussian distribution centered at zero; (c) Fractal noise generated using the different values of fractal dimensions as a parametric input. [8].

2.2.2. Roughness and fractal dimension

The primary feature of a random fractal is its roughness or noise, and this characteristic is measured by fractal dimension. Higher the roughness or noise of a shape, the higher is its fractal dimension. In the case of random fractal curves constructed by using Gaussian Midpoint Displacement method, we noticed that the factor of the Hurst exponent H results in roughness or noise of the curve which is a property that can be measured by fractal dimension. Hence, there is a relation between the Hurst exponent H and the fractal dimension D of the resultant fractal curve [4], which is,

$$\text{For the one-dimensional fractal curve,} \quad D = 2\text{–}H \quad ;0 < H < 1 \tag{11}$$

$$\text{Therefore, the scaling factor,} \quad \lambda = 2^{D-2} \tag{12}$$

It means, in the case of random fractal curves produced by Gaussian Midpoint Displacement Method, we can control its noise using the Eq. (12) and by deciding the value of fractal dimension D as an input value before the curve construction. 'Fig. 4c' shows that how the fractal dimension controls the noise of the fractal curve, keeping the deviation (σ) as unchanged ($-1 \le \sigma \le 1$). This method is commonly used to generate a silhouette of a distant mountain range. In the three-dimensional space, this method can produce a noisy fractal surface or the topology of a natural terrain as a two-dimensional counterpart of the one-dimensional random fractal curve.

3. Geometric modeling

3.1. Design concept

There is a long chronological history about the structures of branching columns that support flat roofs inspired by the structural appearance of natural trees [43,44]. Branching structures exhibit a close relationship between the force flows and their shapes. It is a functional combination of the roof construction and supporting structures. The advantage of the tree-like branching system is to have short distances from the loading points to the supports. Branching structures are usually referred to as tree-like supports. However, their action cannot be compared with that of a natural tree. While the branches of a tree are under bending stress, bending forces are systematically avoided in technically constructed tree-like structures [38]. The inner structure of the tree-like columns represents a type of framework that is unique in the construction industry. It is not a truss with a triangular structure which would brace the structure, yet it allows articulation of joints between the truss elements and prevents bending even under alternating loads. In the tree-like column, therefore, the individual elements must be rigidly connected at the joints. A tree-like column is particularly well suited for only the main load scheme for which it is optimized. All other loading conditions will cause bending stresses within the structure.

In our design, the branching structures will be designed to support a wide-span roof so that we can obtain a large-span indoor space. Commonly, most of the existing canopy structures with branching supports have flat roofs. From an architectural point of view, the flat is commonly an impression of a man-made object, while the branching structure is an abstract impression of nature. Keeping the natural image as the visual demand of our design concept, we aim to design the roof which will not be flat, but irregular so that it can give an impression of nature. Falk et al. [62] proposed a non-flat roof for such tree-column supported canopy structure after hanging model experiments (Fig. 5a). Their roof is a folded plate origami roof supported by optimally designed branching columns (Fig. 5b). However, the folded roof in their proposal does not resemble with any natural object - their design intuition might not be mimicking nature. There are few built examples of canopy structures, such as at ION Orchard in Singapore designed by Benoy in 2009 (Fig. 5c) and at WestendGate in Frankfurt designed by Heinrich Rohlfing GmbH and Sternwede-Niedermehnen in 2010 (Fig. 5d), where both the columns and roof forms give natural impressions. However, in these built examples, the mathematical concept of fractal geometry has not been employed as a design tool. In our proposal, we aim to apply the principle of random fractals for designing a canopy structure whose roof will be randomly folded.

3.2. Branching column: A self-similar fractal

3.2.1. Computational Modeling: IFS Method

The branching structure is designed to act as a structural support. There are different methods to simulate branching figures. Here, the Barnsley's *Iterated Function System*, which is based on the *contraction mapping* method, has been used to generate a parametric model of a branching structure. It starts with a trunk represented as a vertical line 'T_0' which is replicated into four copies that are 'B_1', 'B_2', 'B_3' and 'B_4' after its affine transformations f_1, f_2, f_3 and f_4 respectively. According to Barnsley's method, a fractal branching is an *attractor* 'T' which can be mathematically expressed as,

$$T = \bigcup_{n=0}^{\infty} T_n \tag{13}$$

i.e.,

$$T = T_1 \cup T_2 \cup T_3 \cup \ldots\ldots \cup T_n \cup T_0$$

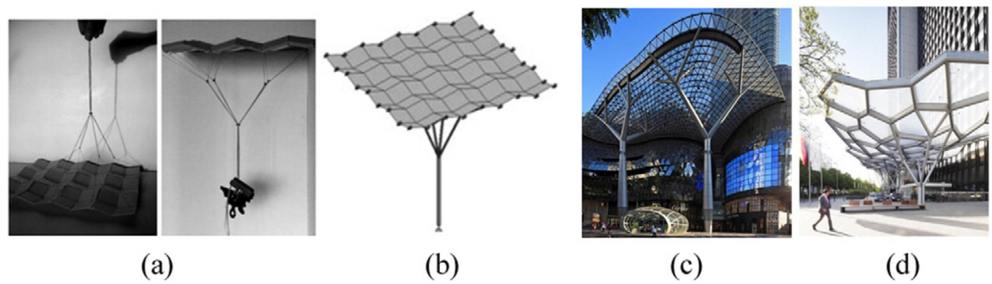


Fig. 5. (a) Thread hanging model for designing an optimized canopy structure [16], (b) Canopy structure with folded plates origami roof [16], (c) Canopy structure at ION Orchard in Singapore designed by Benoy in 2009, (D) Canopy structure at WestendGate in Frankfurt designed by Heinrich Rohlfing GmbH and Sternwede-Niedermehnen in 2010.

where,

$$\mathrm{T}_1 = \mathrm{B}_1 \cup \mathrm{B}_2 \cup \mathrm{B}_3 \cup \mathrm{B}_4 \cup \mathrm{T}_0 = f_1(\mathrm{T}_0) \cup f_2(\mathrm{T}_0) \cup f_3(\mathrm{T}_0) \cup f_4(\mathrm{T}_0) \cup \mathrm{T}_0 \quad (14)$$

And

$$\mathrm{T}_\mathrm{n} = f_1(\mathrm{T}_{n-1}) \cup f_2(\mathrm{T}_{n-1}) \cup f_3(\mathrm{T}_{n-1}) \cup f_4(\mathrm{T}_{n-1}) \cup \mathrm{T}_0$$

where, n is the number iterations. When T_1, T_2, ….., T_n, … are contraction sets of T_0, that are scaled by *contractivity factor* λ_i and transformed by *affine transformation* function f_i, such that,

$$T_0 \subset T_1 \subset T_2 \subset \ldots \subset T_{n-1} \subset T_n \subset \ldots \quad (15)$$

then, they form a perfect self-similar fractal set.

In the case of constructing the fractal branching, following function of IFS is applied that consists of four different three-dimensional *affine transformations* (f_1, f_2, f_3 and f_4) of T_0 shown in 'Fig. 6'.

$$f_i = \lambda_i \begin{bmatrix} cos\alpha & -sin\alpha & 0 \\ sin\alpha & cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\beta & 0 & sin\beta \\ 0 & 1 & 0 \\ -sin\beta & 0 & cos\beta \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \end{bmatrix}; \; i = 1 \text{ to } 4. \quad (16)$$

In the above IFS function, there are several variables which control the whole appearance of the branching structure. In the 'Fig. 6', the trunk height is set 6 m and its IFS variables are set as follows,

However, the values of variables shown in 'Table 1' are not fixed. They are changeable which transform the geometric model of the branching structure as a parametric model.

3.2.2. Hausdorff dimension calculation

According to Barnsley's method, the Hausdorff dimension D of the fractal branching is depending on the *contractivity* factors based on the following relation,

$$\sum_{i=1}^{4} \lambda_i^D = 1 \quad (17)$$

where, λ_i is a *contractivity* factor of transformation f_i of the branch B_i.

For proposed branching structure,

$$(\lambda_1)^D + (\lambda_2)^D + (\lambda_3)^D + (\lambda_4)^D = 1$$

In the 'Fig. 8',

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.6$$

Using Newton's method, we get,

$$D \approx 2.714$$

3.3. Crinkled roof surface: A random fractal

In the proposed canopy structure, the roof form is made crinkled inspired by the morphology of natural terrain, which is an example of nature's random fractals, sometimes known as the 'fractal landscape'. The selection of crinkled surface over the flat surface for designing the roof, as mentioned before, is because of the apparent higher stiffness of the crinkled surface than that of a flat surface since folds in the crinkled surface act as self-stiffeners. Crumpling of a piece of paper is a relevant and ubiquitous example of a stress-induced morphological transformation in thin sheets [6]. As shown in 'Fig. 7', if a flat piece of thin paper is placed at the top center of a bottle, then the two sides of the sheet will bend down. But, if that paper after being crumpled and

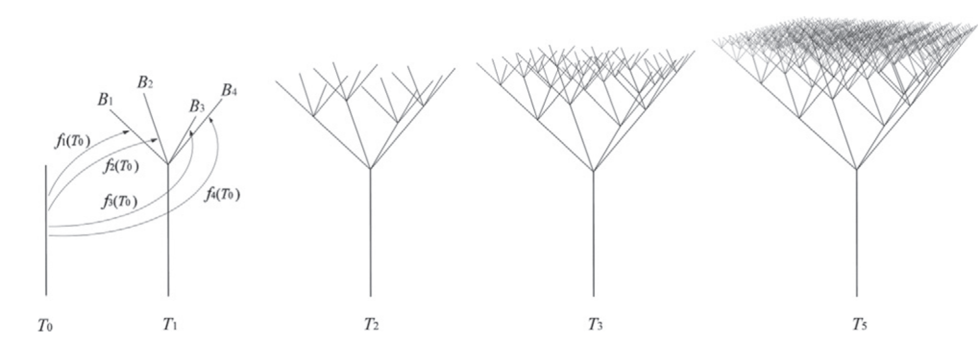


Fig. 6. A convergent sequence of fractal branching tree in three-dimensional space.

Table 1
Design Variables.

IFS functions	Contractivity	Rotation		Displacement	
		XY plane	XZ plane	XY plane	XZ plane
f	λ	α	β	δ_x (m)	δ_y (m)
f_1	0.6	-45^o	45^o	0	6
f_2	0.6	-135^o	45^o	0	6
f_3	0.6	45^o	45^o	0	6
f_3	0.6	135^o	45^o	0	6

unfolded is placed at the same position on the bottle, then the crinkled version does not noticeably bend down. It, rather, stays almost non-deformed, because ridges of crinkles, i.e., its random folds after crumpling act as stiffeners. Furthermore, the crinkled paper sheet can carry the load to some extent with a negligible deformation. Each crumpling produces a unique pattern of random folds or crinkles. Therefore, countless crinkle patterns can be obtained by the countless crumpling of a thin piece of paper.

Physically it is easy to crumple a sheet of paper infinite times and then obtain infinitely different random patterns of the crinkled surface. But, geometrically, it is difficult to simulate crinkled surface especially by using Euclidean or other regular geometric system and without any algorithmic process. Several methods have been developed that are able to simulate the realistic models of the crinkled surface which are mainly based on the random fold lines due to crumpling [6,7,37]. It is noticeable at a quick glance that the surface of the crinkled sheet after crumpling resembles with the surface of the fractal-like natural terrain, although the pattern of random fold lines in both the examples is slightly different to each other. While the geometric shape of crumpled paper is mainly related to the fold lines, the geometric shape of natural land terrain is mainly associated with the height maps. However, in the end, both the morphologies are similar to each other from a particular scale of view. Nakamura and Asayama [36] applied an Algorithm of Land Erosion to model a non-smooth random form of a space structure to replicate the nature's land terrain topology and assess the mechanical behavior of such nature-inspired unconventional structure. However, in this study, we will use the Diamond Square Algorithm, a three-dimensional counterpart of the Gaussian Midpoint Displacement Method, which is mainly popular in simulating the natural terrain, for generating an irregular fractal surface.

3.3.1. Computational Modeling: Diamond Square Algorithm Method

The Diamond Square (DS) Algorithm is a stochastic algorithm that results in a random surface with fractal behavior. The DS algorithm, which is the three-dimensional counterpart of Gaussian Midpoint Displacement Method discussed in the Section 2.2.2, is popularly useful for simulating realistic natural terrains. The concept of DS Algorithm, first introduced by Fournier et al. [18] and later simplified by Martz [32], has been adopted here for modeling a crinkled surface in the following way.

(i) Initial step:

In the initial step, a large empty two-dimensional square grid is constructed whose array size is equal to $2^n + 1$, where n is the number of iteration. The four corners of the square grid are assigned with some random height values as the first seed for the operation. In the 'Fig. 8', as a demonstration, we have assumed 5×5 square grids where the corners are 'A', 'B′, 'C′ and 'D' and their height values are A, B, C, and D respectively marked as red dots.

(ii) Diamond step:

In the 'diamond step', the center point ('E') of the square which is obtained by intersecting the two diagonals ('AC' and 'BD'), is vertically displaced by a value which is the average value of the heights of four corners added with a random value that ranges from $-\Delta$ to Δ. Thus, the displacement of 'E' is,

$$E = \frac{A+B+C+D}{4} + rand(\Delta)$$

This displacement of the center point 'E' transforms the flat square into a pyramid. At this stage, E is a new value, i.e., a new seed, and that is why it is shown in red dot while the corner heights are shown in blue dots as the previous values in the 'Fig. 8'.

(iii) Square step:

In this step, when the grid has multiple squares, then the height of the center of each diamond is assigned by a value which is the average height of its corners with the addition of a random perturbation in a much similar way as in 'diamond step'. In this step, one thing is important to notice that the diamonds, which are at the edges of the square grid, have three corners while the diamonds inside the array have four corners. The calculated height values of the peaks, i.e., center points of the edged diamonds are (Fig. 8c).

$$F = \frac{A+B+E+E}{4} + \mathrm{rand}(\Delta) \qquad G = \frac{B+C+E+E}{4} + rand(\Delta)$$

$$H = \frac{C+D+E+E}{4} + rand(\Delta) \qquad I = \frac{D+A+E+E}{4} + rand(\Delta)$$

The calculated height values of the peaks, i.e., center points of the internal diamonds are (Fig. 8d),

$$N = \frac{E+J+F+K}{4} + rand(\Delta) \qquad O = \frac{E+K+G+L}{4} + rand(\Delta)$$

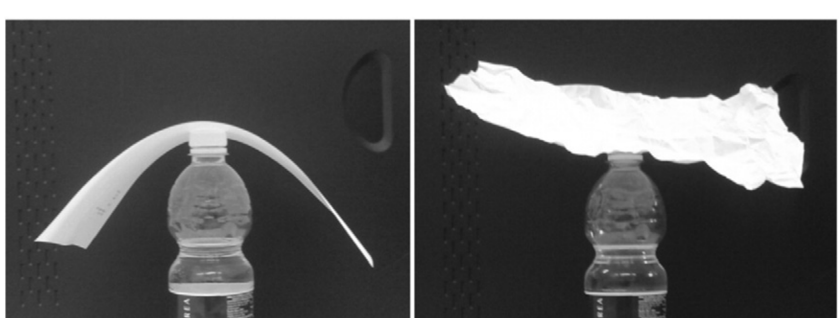


Fig. 7. Left - A piece of flat paper on a bottle; Right – A piece of crinkled paper on the top of a bottle.

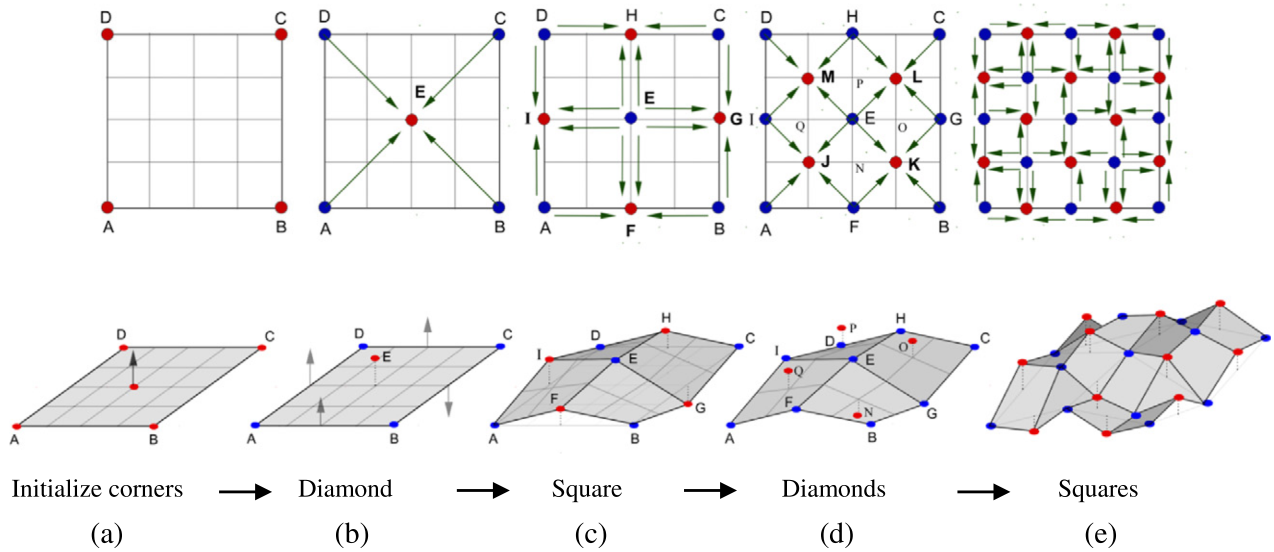


Fig. 8. The method DS Algorithm that produces fractal terrain. Red dots are the new vertices while the blue dots represent previous vertices. Top and middle - Plan views; Bottom – Isometric views.

$$P = \frac{E+L+H+M}{4} + rand(\Delta) \qquad Q = \frac{E+M+I+J}{4} + rand(\Delta)$$

(iv) Recursive step:

In the 'recursive step', the 'diamond step' is repeated to each square that was created in the 'square step', then the 'square step' is repeated to each diamond that was created in the 'diamond step', and continue this process recursively until the grid becomes sufficiently dense and gives an impression of a crinkled surface.

'Fig. 9' shows the transformation of a flat surface into a natural terrain-like or a randomly folded crinkled-like surface after increasing the number of iterations using the process of DS Algorithm.

3.3.2. Fractal dimension of the crinkled surface

In the DS Algorithm, the range of random perturbation is a key factor that controls the noise of the surface generated. This range is based on the value of roughness coefficient which is referred to the Hurst exponent H. If the Hurst exponent H is 1.0, then the random perturbation is multiplied by a value that ranges from −1.0 and 1.0. At each new iteration, the value of H is reduced by a scaling factor λ in such a way that,

$$\lambda = 2^{-H}$$

Therefore, if the height scale or z-deviation is δ, then the random value of the displacement in the first iteration is,

$$\Delta_1 = \lambda.\delta = 2^{(-H)}.\delta$$

i.e., at nth iteration,

$$\Delta_n = \lambda_n.\delta = 2^{(-nH)}.\delta \tag{18}$$

According to Barnsley et al. [4], there is a relation between the Hurst exponent H and the fractal dimension D of the resulting surface produced by the Gaussian random midpoint displacement process, which is,

$$D = 3 - H \tag{19}$$

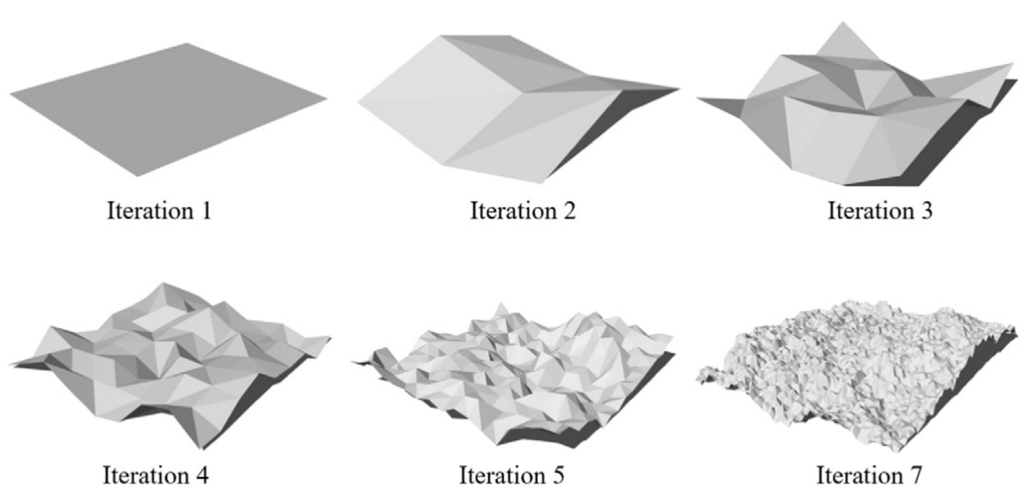


Fig. 9. Natural terrain-like crinkled surface generation from a flat surface using DS Algorithm.

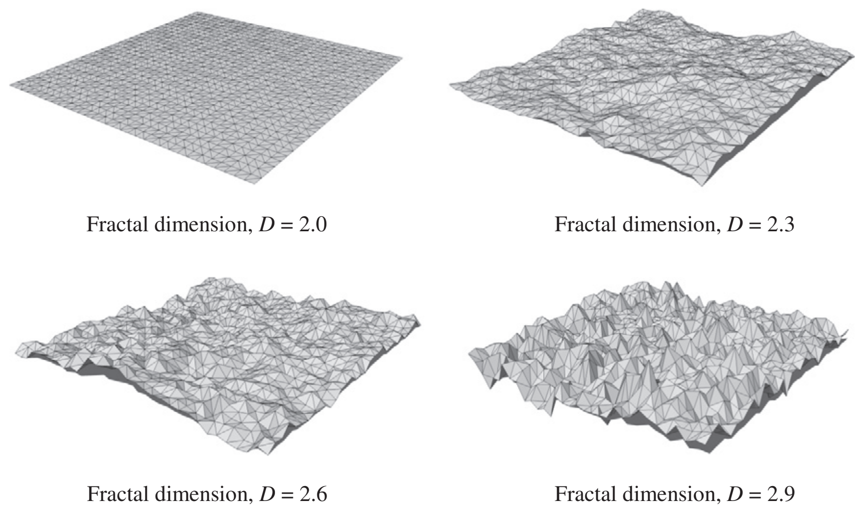


Fig. 10. Noises of a surface resulted after different fractal dimensions.

i.e.,

$$\lambda = 2^{D-3} \tag{20}$$

From the above equation, if H gets larger, the scaling factor λ gets smaller and so the fractal dimension D.

Hence, the DS Algorithm can be easily scripted by any programming language based on the above process and formulations for modeling a wrinkled surface for a free-form roof design. In this algorithm, the first input as a variable is the iteration number n, which gives the size of the square grid, i.e., the size of the proposed roof surface. The second input is the roughness coefficient or the Hurst exponent which decides the range of the random number, i.e., the range of the maximum and minimum heights of vertices of the proposed roof surface. The last input is the fractal dimension D, which decides the roughness or noise of the surface model. 'Fig. 10' shows an example of the crinkled surface whose noises appear as a result of the changing of fractal dimension value, a geometric variable input. It illustrates that the higher the fractal dimension of a surface, the higher is the noise and crinkliness of the surface.

3.4. Final form of the canopy structure

One important objective of this study is to describe and provide one level of structural testing and define a benchmark for a non-flat crinkled roof whose design is inspired by the topological appearance of a natural (random) terrain and motivated by the self-stiffening property of crinkled paper sheet. The design of crinkled roof surface is modeled by directly adopting the crinkled surface generated in the previous section using the DS Algorithm. To support the crinkled roof efficiently, some key points on its surface are required as joints where the column tops will be connected. If we insert a set of vertical columns connecting those key points, then these columns will occupy a large indoor space inside. Therefore, to minimize the number of supporting members by keeping the key supporting points under the roof surface same, we will use branching columns. Branching support is useful for its minimal path principle in which the distributed loads are transferred through the branches and transferred towards their trunk. The tree supports are modeled here by adopting the three-dimensional branching trees modeled in the previous section using the process of IFS. In this canopy design, the crinkled surface has been placed at such a height that all the end branches either penetrate through the surface or intersect the surface or, at least, touch the surface. After such strategic intersection, all the branches that overlap the surface are trimmed and split. Thus, the roof surface divides the branches into two groups – one group is above the surface and another group is under the surface (Fig. 11b). We consider the second group, which is under the surface, as the structural supports, whereas the first group, which is above the surface is discarded (Fig. 11c). In this situation, now all the end points of the end branches are connected to the surface at the intersection points. This has been done parametrically which means, whenever the roof surface randomly changes its topology, automatically we will get the second group of branches as the resultant structural supports.

In fact, the whole model is made as a parametric, keeping the architectural and structural aspects in mind. For example, in the case of branching supports, the vertical angles influence the structure by expanding its topmost area; while the iteration number influences the number of roof-branch joints at the roof surface, thus controlling the load distribution of roof. Similarly, the *contractivity* factor (λ) and horizontal angle also influence the aesthetic appearance as well as the structural behavior of the branching structure. On the other hand, the crinkled roof also influences the structure by changing its values of the fractal dimension and z-limit factor (δ). As a result, the final model has several variables in total which could produce infinite design

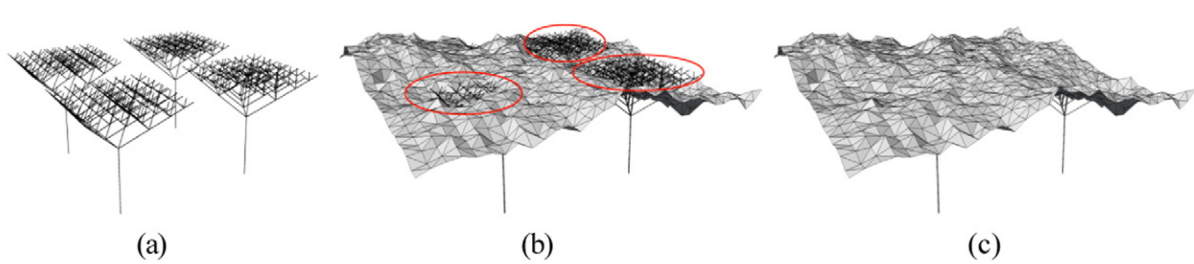


Fig. 11. (a) Independent branching structure (b) Wrinkled roof, which is placed at the centre of the overall branching configuration at a certain height, overlaps the branches separating some branches above the surface and some are under the surface, (c) the upper branches are trimmed out from the intersection points in between the surface and branches.

Table 2
Design Variables.

Branching Structure	
Design Variables	Parameters
Initial height, h_0	5.5 m (fixed)
Contractivity factor or scale, *s*	0.6 (fixed)
Vertical angle, θ_V	35° to 50° (variable)
Horizontal angle, θ_H	45° (fixed)
Iteration number, *n*	4 (fixed)
Crinkled Roof Surface	
Variables	Parameters
Size, s	17 m (fixed)
Z-Limit, δ	1.0 to 2.0 (variable)
Fractal dimension, *D*	2.0 to 3.0 (variable)
Iteration number, *n*	4 (fixed)

variations of the main model. To find a practical and efficient form, the values of some variables have been fixed and the parameters of the remaining variables have been defined in such a way that they can generate feasible design possibilities, but not beyond the impractical models. The variables and their values and parameters are shown in 'Table 2'.

Based on the above unrestricted variables, different design variations of a single unit (main canopy structure is composed of four units with four branching supports) of the main canopy structure are shown in 'Fig. 12'. 'Fig. 12, Top' shows different design possibilities due to the variations of branching supports influenced by the changing of vertical angles and *contractivity* factors, keeping the parameters of roof surface constant. 'Fig. 12, Bottom' shows the different design variations of roof surface influenced by the changing of deviation (δ) and the fractal dimension (*D*), while the parameters of branching supports are constant.

However, to design an initial model of the main canopy structure, we have defined the values of different variables mentioned in 'Table 3', which results in an architectural outcome shown in 'Fig. 13'.

4. Structural analysis

The geometric complexity of roof surface is achieved by the heights of its mesh vertices that are present in an array of a square grid pattern. Hence, the height of each vertex plays a pivotal role to offer such complexity of surface. In the case of structural optimization of shell structures or similar smooth or flat roof canopy structures, the height of each vertex is kept variable, and optimization process controls each height as a variable to obtain the optimal shape with regard to some defined fitness functions and structural conditions. Computational morphogenesis on a NURBS based (or non-NURBS based) grid surface generally results in a smooth surface as an optimized form of the roof. Usually, smooth surface offers better distribution of forces-flows, and that is why computational morphogenesis indicates the smooth surface as a preferable choice. However, since, in our example, the architectural design goal is to obtain a natural terrain like irregular roof form and the structural goal is to study the mechanical behavior of fractal-based random surfaces, we will maintain the unsmooth fractal character of the roof surface during the form-finding process. For this, unlike other common examples, we do not take all the heights of the points as the individual variables; instead, we keep only two variables that are deviation or Z-limit factor δ and the fractal dimension factor *D*. These two factors will maintain the fractal characters of the roof surface, and to stick with this idea, we have restricted the parameter in between 1.0 and 2.0 for z-limit factor and 2.0 and 3.0 for fractal dimension factor. The Z-limit value ranging from 0.0 to 0.5 usually produces nearly flat and approximately smooth surfaces.

For the structural analysis of the fractal-based canopy structure, its geometric model has been transformed into a finite element model by assuming the following consideration:

- All the duplicate lines of branching supports are removed. Each line is considered as a beam element.
- All beam elements are hollow steel tube. The diameters of all trunks are 20 cm, all first tier branches are 15 cm and the remaining branches are 8 cm. The thickness of all the tubes is 5 mm.

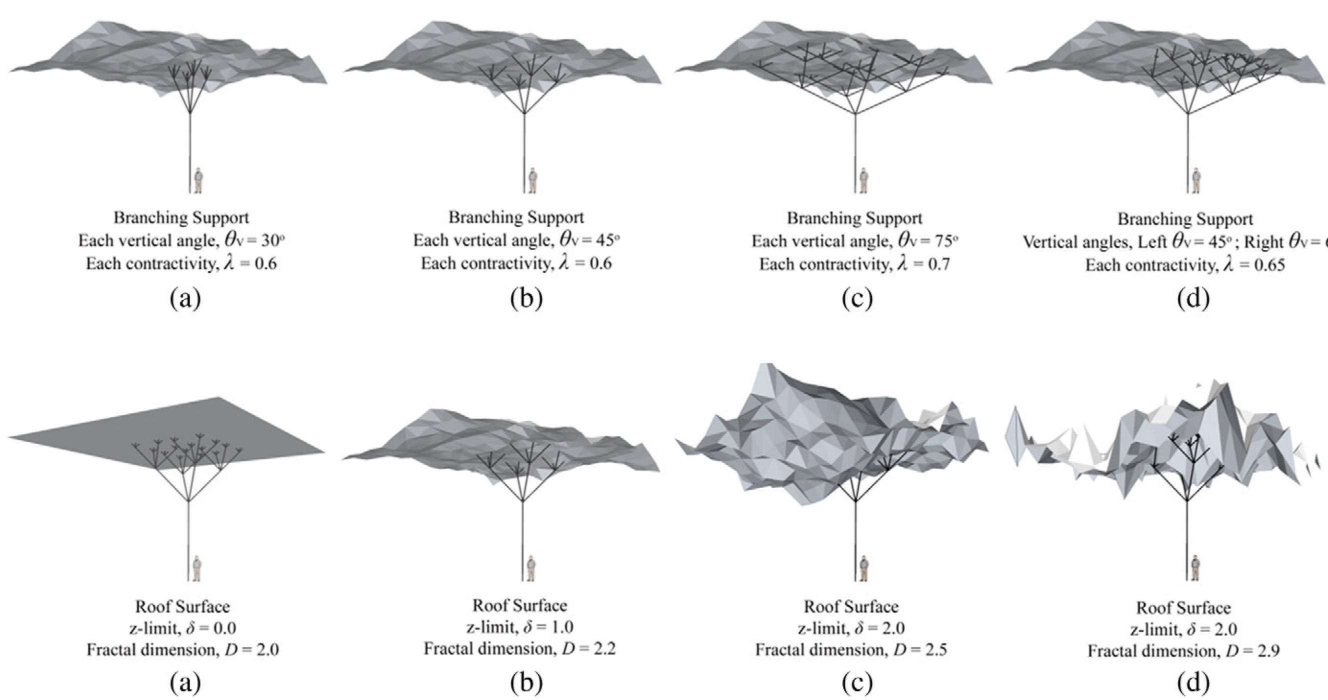


Fig. 12. Design variations of the pavilion unit; Top – The design variations of branching supports with the changing of vertical angles and *contractivity* values; Bottom – The design variations of roof surface from flat to random and noisy with the changing of z-limits and fractal dimensions.

Table 3
Design Variables.

Branching Structure	
Design Variables	Parameters
Initial height, h_0	5.5 m
Contractivity factor or scale, s	0.6
Vertical angle, θ_V	45°
Horizontal angle, θ_H	45°
Iteration number, n	4
Crinkled Roof Surface	
Variables	Parameters
Size, s	17 m
Z-Limit, δ	1.5
Fractal dimension, D	2.5
Iteration number, n	4

- All the duplicate connecting points, i.e., overlapped structural nodes are removed. Each connecting node is considered as a structural welded joint.
- The bottom of the trunk is vertically and horizontally restrained.
- The roof surface is a triangular mesh and all the duplicate mesh units are discarded. Each mesh unit is a shell element.
- Shell elements are made of cross laminated timber (Young's modulus 800 KN/cm^2, shear modulus 400 KN/cm^2, yield stress 0.7 KN/cm^2) and their thickness is 5 cm.
- The shell and the branching tubes are connected by screws via steel plates at the intersection joints.

For investigating the influence of fractal dimension of the roof's surface on its structural strength, we set up a 'parametric toolbox' which can instantly give structural feedbacks after finite element analysis with the change in design variables such as fractal dimension. The strategy is to find the appropriate form that provides maximum strength. In this 'parametric toolbox', there is a 'generative tool' which automatically generates the geometric model of the main structure connected with some design variables that control the shape of the model. In the 'parametric toolbox', there is another tool, named as 'FEM solver tool' that will perform the finite element analysis of the parametric model of the structure. These two tools are connected in such a way that they can interact with each other promptly. For this strategy, we have used the *Grasshopper*, a parametric software that is embedded in *Rhinoceros 3D* which is a NURBS-based modeling and visualizing software. In the *Grasshopper*, as the 'generative tool', we have used *GhPython* component which is used for scripting codes of different algorithms. We have two such components, one is for generating the model of branching structure where we have written an IFS code while the another is for generating the model of crinkled roof surface where we have written DS Algorithm. Then, in the *Grasshopper* platform, we have used *karamba* as the 'FEM solver tool'. *Karamba* has preprocessing tools that transform the geometric models into finite element models. The analysis and post-processing tools of the *karamba* analyze the structure and offer a set of results that include the displacements, stresses, axial forces, and so on. The advantage of the *karamba* is its instant feedback with the changing variables of the parametric model.

5. Results and discussion: fractal dimension vs stiffness

Before the structural analysis of the main canopy structure, a finite element linear static analysis was performed on a smaller prototype of the main roof that is centrally supported by a space truss. This trial is a mock-up of a piece of paper which is centrally placed on the top of a bottle shown in 'Fig. 9a'. As we see in the 'Fig. 9', the flat paper sheet is not stiff enough by its own weight and its two sides bend down; but, after crumpling the paper, its crinkled shape becomes stiffer and do not bent down as before, but acts as a cantilever, as the crinkled paper is self-stiffened by its random folds. Therefore, through the finite element analysis, we expect to see similar mechanical behavior in prototype flat roof and then its crinkled version. In order to confirm this prediction, we modeled three different roof surfaces; one is flat having z-limits, $\delta = 0$ and fractal dimension, $D = 2.0$, second is crinkled having deviation, $\delta = 1.0$ and fractal dimension, $D = 2.5$, and the third is further crinkled surface having deviation, $\delta = 1.5$ and fractal dimension, $D = 2.9$. The results obtained after performing FEM analysis on these models using the 'parametric toolbox' are shown in 'Table 4'. In the 'Table 4', it is noticed that when the roof is flat then its deformation is large, and it bends down at its edges quite similar to the flat paper sheet. The maximum displacement is 264 mm. Soon after the shape of the flat roof is crinkled with fractal dimension 2.5, it does not bend down; instead, it stays almost in the same position. The maximum displacement, in this case, falls down to 38 mm from the previous 264 mm of a flat roof. But, when the crinkled surface is further folded with the fractal dimension of 2.9, then no significant change is seen. In this condition, the maximum displacement is 32 mm. The similar behavior we also notice when a paper sheet is further crumpled, then the stiffness gets a little higher, although the difference is not much as compared to the less

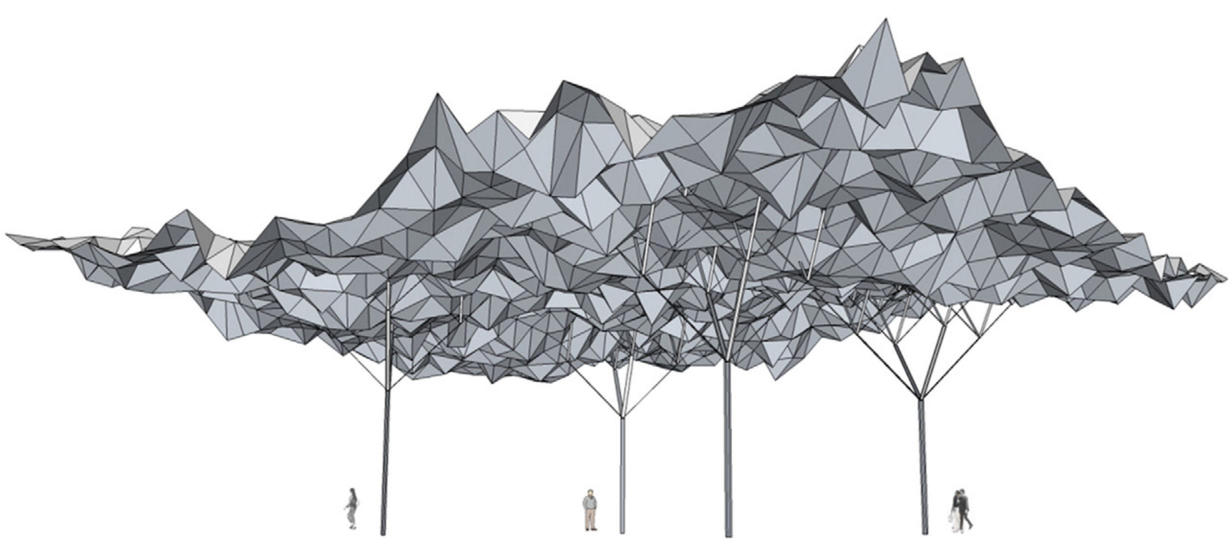
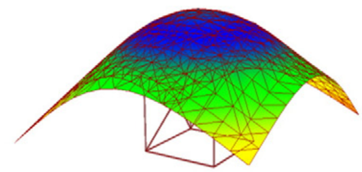
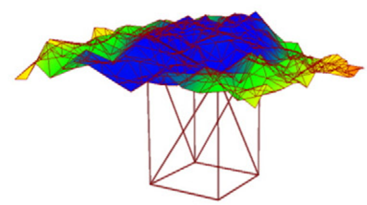
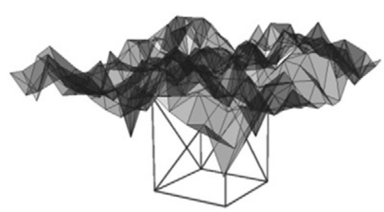
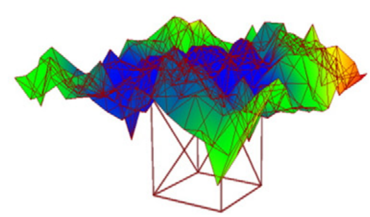


Fig. 13. A wireframe geometric model of the fractal-based canopy structure.

Table 4
Deformations of a roof due to self-weight when its flat surface is transformed into crinkled surface.

Fractal dimension D	Deviation (Z-Limit) δ	Before deformation	After deformation (Scale multiplier 30)	Max. vertical disp. dz (mm)
2.0	0.0			264
2.5	1.0			38
2.9	1.0			32

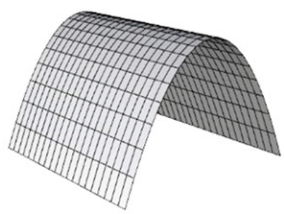
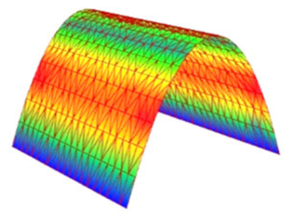
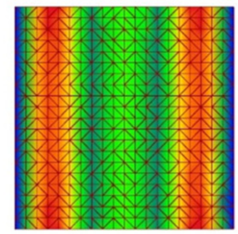
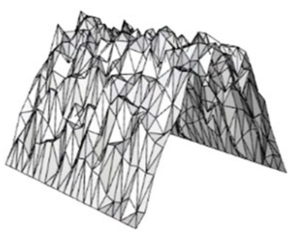
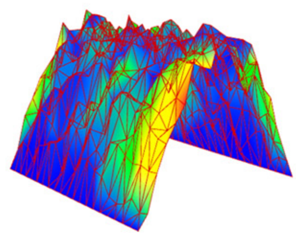
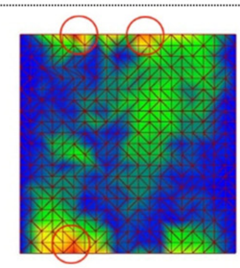
crumpled paper. The only major difference occurs between flat paper and crinkled paper. These results confirm our hypothesis that the crinkled surface acts as a self-stiffened structure which can be practically useful for designing large-span canopy roof.

Another test has been performed to further explore the impact of crinkling on a shell structure. A barrel vault, an example of a smooth shell structure, has been geometrically exploited and transformed into a crinkled vault using fractal dimension as a geometric variable. 'Table 5' shows the geometric models of a barrel vault and its crinkled form (fractal dimension 2.5), and also displays their deformations under gravity and mesh loads after the finite element analysis. The maximum displacement is noticed in the crinkled shell (57 mm) which is much higher than that of the barrel vault. But this maximum displacement of crinkled shell takes place only in few parts of both the ends. But, rest of the structure has no significant deformation, and as a whole, its deformation is much less than that of the smooth barrel vault. In the case of a barrel vault, the maximum deformation (10 mm – 12 mm) occurs along the large area of longitudinal strips at the both sides, and as whole, it deforms more than the crinkled vault. Therefore, it can be stated that the crinkling of a smooth surface may it be flat or a shell structure, is likely to display better structural performance, especially in terms of self-stiffness and less overall deformation. This structural testing of the shell structure will not be directly useful in our canopy design, instead, it has only been tested to see the mechanical behavior crinkled surface even in different modes of structures.

Based on the FEA results of single unit demonstration shown in 'Table 4', it can be predicted that the roof of our main canopy structure will act the similar way as the crinkled paper and the roof of prototype unit structure. For this purpose, at present, we only consider the gravity load and mesh load of the roof and check its maximum displacement as a measure of stiffness, as we know that the stiffness is directly proportional to the acting forces and inversely proportional to the displacements. Our curiosity is to see whether the maximum displacement of the roof gets smaller when the fractal dimension as a measure of the crinkliness of the roof surface are increased; although, the displacement is also affected by the vertical angle of the branching supports. At this moment, we have fixed the branching vertical angle as 45°, horizontal angle as 45°, trunk height as 5.5 m, *contactivity* factor as 0.6 and iteration number as 5. For the roof model, we have fixed the size as 17 m and z-limit as 1.5. So, the only variable is the fractal dimension that ranges between 2.0 to 3.0. The 'parametric toolbox' has analyzed the structure for different fractal dimensions which are shown in the 'Table 4' in which red circles indicate the positions of maximum displacements.

From the 'Table 6' it can be seen that when the fractal dimension of the roof surface gets larger, the displacement under gravity and mesh loads gets smaller. When the roof is flat, i.e., fractal dimension is 2.0,

Table 5
The deformation of the smooth barrel vault and its crinkled version under the gravity and mesh loads (1 KN/m^2) with the changing of fractal dimensions, while z-limit is constant (1.0).

Fractal dim.	Before deformation	After deformation (Scale multiplier 10)		Max. vertical disp.
D	Isometric view	Isometric view	Top view	dz (mm)
2.0				12
2.5				57


+ 10 mm ------------------------------► – 60 mm

which is an integer value, the displacement is relatively very high. As soon as the surface starts getting crinkled ($D > 0$ and $\delta > 1$), the maximum displacement sharply falls. Although, as seen in the Table, the value of maximum displacement is getting smaller with the increase in fractal dimension, it may not always be true. Because of the factor of randomness, the repeated input of same fractal dimension results in the different topology of the roof surface by maintaining the degree of roughness, hence delivering varying stiffness. For understanding the influence of fractal dimension on the structural strength of crinkled roof, we repeated the same value of fractal dimension two times, and get two different maximum displacement behaviors as shown in 'Fig. 14' are obtained.

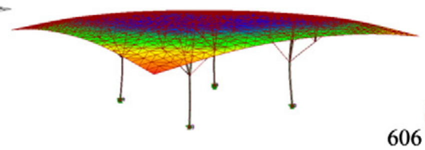
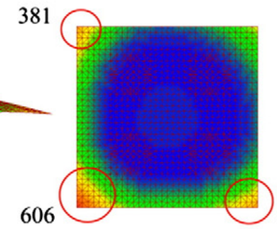
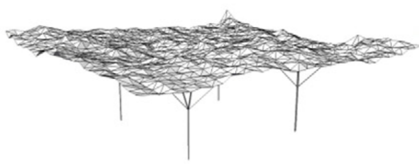
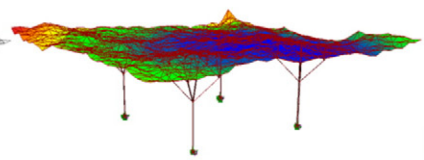
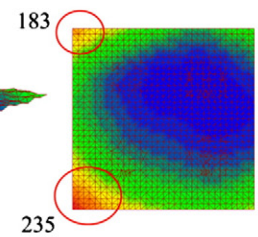
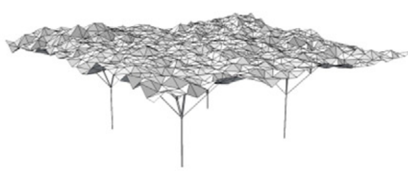
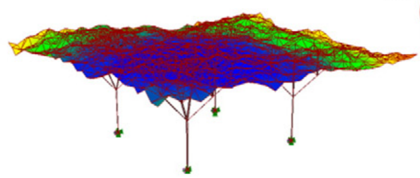
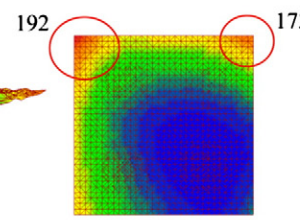
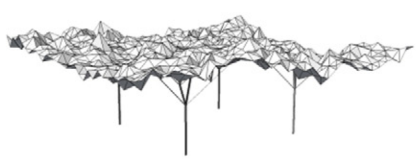
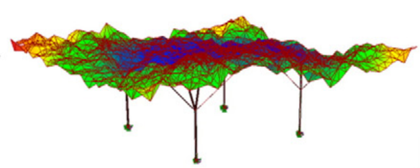
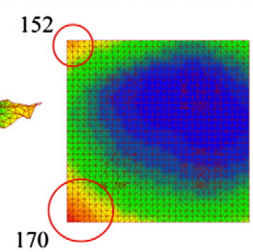
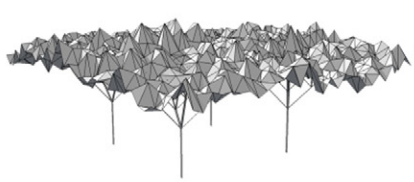
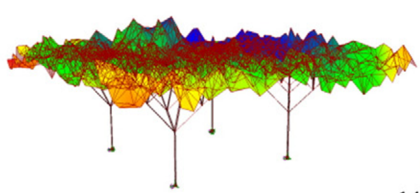
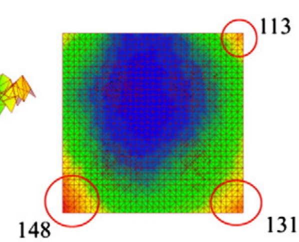
In the 'Fig. 14', it is observed that in each case, the maximum displacement sharply falls as soon as fractal dimension becomes more than 2.0 to 2.1, and from 2.1 to 2.9, its displacement is significantly different to each other as compared to the maximum displacement of flat surface ($D = 2.0$), fluctuating in between 70 mm to 150 mm. These fluctuations are completely different from each other in both the trials. It represents that as soon as the roof is crinkled from its flat surface, it becomes self-stiffened due to its random folds. However, further folding makes the surface more crinkled but does not necessarily increase its stiffness, because stiffness is further dependent on the surface topology.

Besides, increasing the fractal dimension increases the roughness of the surface, hence increases the weight of the roof. 'Fig. 15' shows the relationship between the fractal dimension and the weight of the roof. This relation is helpful to find an optimized shape of the roof with regard to its minimum weight but maximum stiffness. As we see from the 'Fig. 14', the roof surface having $D = 2.1$ is stiff enough as compared to the flat roof, while 'Fig. 15' shows that the roof surface with $D = 2.1$ is the lightest roof structure among the other roofs having $D > 2.1$. On the contrary, the roof form should be such that it can express the appearance natural terrain, neither nearly flat nor too rough. In this condition, we can select the shape which has a fractal dimension in between 2.3 to 2.5. In this range, the roofs are lighter and stiffer enough while maintaining the natural expression by its appearance.

6. Conclusion

This paper has attempted to apply the concept of fractal geometry from its theory level of mathematics to applied fields of architecture and construction. While the fractal geometry's main property of self-similarity has been recently and quite extensively studied in the application and analysis of architectural designs and structures, it's another important property of randomness has not been specifically studied so far to find the scope and potency of its application in the same fields. Accordingly, this paper has mainly focused on finding an opportunity to use the concept of random fractals in designing a wide-span crinkled canopy structure inspired by the random form of a natural land terrain. We have started our study with the transformation of the mathematical formulations of self-similar and random fractals into an algorithm that has aided to develop a computational model followed by its parametric digital model for designing a crinkled canopy structure. Finite element analysis has been performed on this model to assess its structural behavior, thus explored the relation between the factor of irregularity, i.e., the fractal dimension and the structural strength. Besides, it has also analyzed the relation between the fractal dimension of the roof form and its weight. Finite element analysis has been used here as a first-hand assessment of structural testing which is helpful in the early stage of conceptual design development. A brief structural analysis in this study has confirmed the structural feasibility and the enough strength of such an irregular canopy structure. Finite element analysis has shown that, as compared to a flat roof, a crinkled roof exhibits a unique self-stiffened quality which encouraged us to develop this

Table 6
The displacements of the roof under the gravity and mesh loads (1 KN/m^2) with the changing of fractal dimensions, while z-limit is constant (1.5).

Fractal dimension *D*	Before deformation – Isometric view	After deformation (multiplier factor = 10) – Isometric view	After deformation (multiplier factor = 10) – Top view	Maximum displacement (mm)
2.0				606
2.3				235
2.5				192
2.7				170
2.9				148

crinkled canopy structure. However, apart from the finite element analysis, some other rigorous alternative analysis methods can also be used for evaluating the structural performance of such an unconventional form in details as a part of an extension of this research.

Due to the effect of randomness and changing fractal dimension, an infinite number of unpredictable form variations of the canopy roof can be obtained. Therefore, it is not easy to find the most suitable one in terms of high strength and less weight out of these uncountable

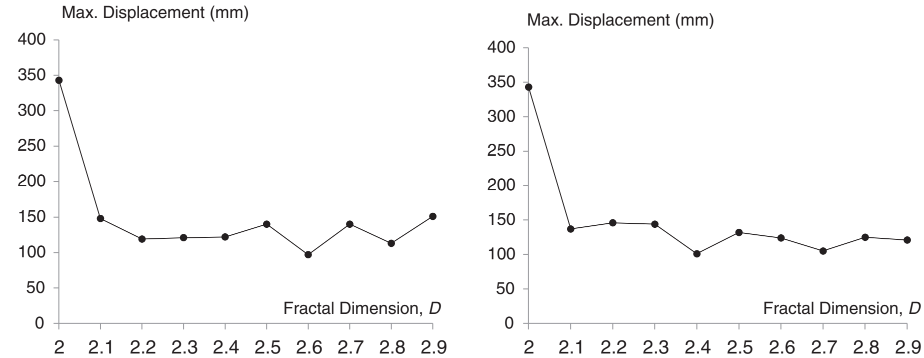


Fig. 14. Fractal dimension Vs. *maximum* displacement graphs for two different trials.

variations. For this reason, as a further research, a computational optimization process based on automatic search algorithms (such as Genetic Algorithm) can be applied to find the optimal form of such a structure.

This type of canopy structure can be used as a pavilion or a shed shelter for multifunctional events. In rain-less hot areas, this type of irregular canopy can be advantageous because the wrinkles of the roof will cast self-shadows on the top surface, thus, they may reduce the roof temperature as compared to a flat canopy roof. Nevertheless, in fact, there are some practical disadvantages and challenges associated with such a system. For example:

- The practical construction of such a crinkled canopy is a crucial challenge. Assembling of different flat panels could be one possible way to fabricate such a discontinuous roof form. But this type of assembly is a complex process because all the panels are completely dissimilar to each other, and therefore, connecting them at their edges by maintaining the random z-coordinates and the jointing angles is a tiringly difficult task. The robotic fabrication may reduce this difficulty if it is constructed in a smaller scale.
- The structural stability against heavy winds or earthquake could be an issue of serious concern. In this study, our structural analysis has considered only the gravitational and mesh loads but not other environmental and imposed loads.
- Another practically important problem of such an irregular canopy surface is its depressed crinkles which will collect water or snow in rains or in snowfall, and as a result, the canopy roof will become heavier and its roofing material can be damaged by collected water or snow if that remains for long hours.
- The difficulty of cleaning and regular maintenance of this type of crinkled canopies may also be a significant practical problem.

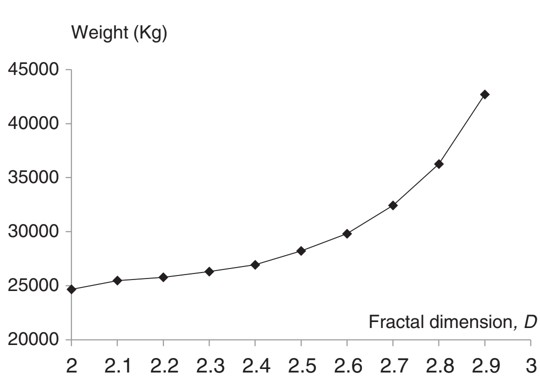


Fig. 15. Fractal dimension vs. roof weight graphs.

The practical issues addressed above have not been attempted to be solved in this study. Tackling these real-world challenges can be a subject of further study for improving the design of such a complex structure, and thus, can ensure a new opportunity to develop structurally feasible and practically constructible structures that are crinkled and randomly fractal.

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