ECONOMICS OF WATER RESOURCES PLANNING

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McGRAW-HILL BOOK COMPANY
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Panama Rio de Janeiro Singapore Sydney Toronto

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Library of Congress Catalog Card Number 79-115146

32263

234567890 MAMM 798765432

This book was set in Modern by The Maple Press Company, and printed on permanent paper and bound by The Maple Press Company. The designer was Edward A. Butler; the drawings were done by John Cordes, J. & R. Technical Services, Inc. The editors were B. J. Clark and M. E. Margolies. Sally Ellyson supervised production.

PREFACE

In a world where an expanding population and an even more rapidly expanding urban-industrial development are intensifying the pressures for a better planned water resources management program, the engineer involved in water resources planning recognizes he must maintain a planning methodology capable of producing a viable resource development program. At the same time, he wonders how he can do it. How can he better structure his design to meet current human needs? How can he make his design flexible enough to accommodate future human needs, the nature of which he can scarcely anticipate? How can he devise a management system for adequately sensing changes in human need as they occur and quickly adjusting management policy and even system design as is necessary?

The hydrologic cycle is the vast natural water resources system. Water falls on the earth, travels downward, over, or under the surface of the ground, reaches the ocean, and returns to the atmosphere through evaporation induced by solar energy. But nature picks its own times and places. The water resources planner seeks to modify the natural cycle by structural measures that force the movement of water to times and places better meeting known human needs. He also seeks to modify, by non-structural measures, the activities of man, so as to better conform to known movement patterns. He considers a flood control channel and flood plain management. He considers water supply systems to existing cities and the development of new cities closer to available supplies. The best management selects the best possible combination of measures.

The concept of the optimum program is changing as men recognize the wisdom, if not the necessity, of taking additional factors into consideration. Pressures from many groups and disciplines have contributed to the expanding awareness of relevant considerations. Business management has long been concerned with decision making to maximize returns to the firm. Engineering economy provides the procedures for the cost analysis of alternatives for the purpose of finding the least-cost approach, irrespective of viewpoint. Studies in microeconomics examine benefits produced as well as costs incurred, and provide rules for maximizing benefits minus costs as a step in optimization to enhance the welfare of the general public. The other social sciences provide further insight into how the welfare of man, as an individual being and as part of a group, can be improved. The biological sciences extend the analysis to all life systems. The institutions responsible for actual resource development draw from all of these sources in formulating their plans, and improvise to supplement the available procedures where action cannot wait for research. As the state of the art now stands, the concepts and procedures required for planning are scattered through the literature of many disciplines, some in journal publications but many in otherwise unpublished research reports and conference proceedings. The purpose of this book is to consolidate into a single volume the basic economic concepts required in water resources planning.

In one companion volume, "Water Resources Engineering,"[1] Linsley and Franzini present the basic physical system and survey the available structural measures for engineered water resources development. In another volume, "Water Resources Systems Engineering,"[2] Hall and Dracup present the procedures for analysis of how water resources systems may be designed to function together as a whole to better achieve specific objectives. In this book, we are seeking to examine how relevant objectives can be specified as well as the reasoning needed to apply rather abstract concepts of social welfare to specific design choices.

The reader should approach the material contained in this book from the viewpoint of developing a philosophy of planning. The detailed procedural steps as presented are intended to illustrate basic concepts, rather than to finalize a method to be followed by rote. These concepts and the written material describing them have been presented by the authors in teaching courses on water resources planning and have been applied by the authors in their planning experience.

The material in this book has been used as the basic text for a one-year course sequence dealing with the economic, social, and institutional issues involved in water resources management. Parts 1, 2, and 6 plus Chapters 8 and 9 can be adopted to a one-semester senior or one-semester first-year graduate course in public works economics (appropriate within programs in transportation, air pollution, and civil engineering management as well as programs in water resources). The remaining portions can then be covered in a second-semester course for those specifically interested in water resources. Although the material is covered in a manner

1 Ray K. Linsley and Joseph B. Franzini, "Water Resources Engineering" (New York: McGraw-Hill Book Company, 1964).

2 Warren A. Hall and John A. Dracup, "Water Resources Systems Engineering" (New York: McGraw-Hill Book Company, 1970).

requiring no specific prerequisites, owing to the diversity of background among students interested in resource development, some background in one or more of the areas of engineering economy, sophomore microeconomics, hydrology, water resources engineering, and systems analysis may add depth to the understanding of selected sections.

The authors gratefully acknowledge the contributions made indirectly to the book through discussion with numerous colleagues. Particular thanks are extended to Professors Ray K. Linsley and Eugene L. Grant of Stanford University and Dr. Charles W. Howe of Resources for the Future for their review of and contribution to various parts of the manuscript. Mrs. Betty Bradshaw and Mrs. Alice Taylor spent many hours typing preliminary drafts, and Miss Pat Miller typed much of the final manuscript.

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ECONOMICS OF WATER RESOURCES PLANNING

1
ENGINEERING ECONOMY

Engineering economy is the science of applying economic criteria to select the best of a group of alternative engineering designs. A design if implemented will produce a time pattern of consequences which must be predicted, evaluated, and compared. The method of comparison might more appropriately be called decision economics since the principles may just as well be used to select among the choices available to other disciplines. For example, the same type of analysis could be called business economics, education economics, or medical economics depending on the skills needed to define the choices. Although the approach emphasizes comparison in economic units, it also includes identification, for comparison to the fullest extent possible, of those consequences which do not relate to economic goals or cannot be expressed in money terms.

Arthur M. Wellington pioneered modern engineering economics through its application to the analysis of alternative railway locations in 1877.[1] Wellington was prompted by the neglect of economic factors in selecting railway location at a time when capital investment in railroads exceeded that in all manufacturing endeavors. The approach has been more thoroughly developed and applied to many other kinds of choices through the years, and current techniques are presented in a number of recent works.[2]

[1] Arthur M. Wellington, "The Economic Theory of the Location of Railways," 1st ed. (New York: John Wiley & Sons, Inc., 1877).

[2] For example, see Eugene L. Grant and Grant Ireson, "Principles of Engineering Economy," 5th ed. (New York: The Ronald Press Company, 1970); and E. P. DeGarmo, "Engineering Economy," 4th ed. (New York: The Macmillan Company, 1967).

The principles and techniques of engineering economics are not always clearly understood nor correctly applied in water resources planning. In separate books, Eckstein and McKean examined federal practices for analyzing water resources projects and suggested numerous revisions.[1] Hirshleifer, DeHaven, and Milliman received two large water resources projects, one in New York and the other in California, and found serious conceptual errors in official economic feasibility studies.[2] Lee produced similar findings in an examination of procedures used in analyzing water projects on the local government level.[3] These failures demonstrate the need for the water resources planner to thoroughly understand the principles and techniques of engineering economics. Part 1 reviews the basic principles of engineering economics as applied to public works in general and water resources development more specifically and presents the mathematics required in their application. Part 2 builds on this foundation by examining some of the more knotty conceptual problems in economic analysis.

[1] Otto Eckstein, "Water Resource Development: The Economics of Project Evaluation" (Cambridge, Mass.: Harvard University Press, 1958); Roland McKean, "Efficiency in Government through Systems Analysis" (New York: John Wiley & Sons, Inc., 1958).

[2] Jack Hirshleifer, James C. DeHaven, and Jerome W. Milliman, "Water Supply: Economics, Technology, and Policy" (Chicago: The University of Chicago Press, 1960).

[3] Robert R. Lee, "Local Government Public Works Decision-Making" (Stanford, Calif.: Stanford University Institute in Engineering Economic Systems, 1964).

CHAPTER ONE

PRINCIPLES OF ENGINEERING ECONOMICS

The principles of engineering economics guide the structuring of alternatives so they may be compared to determine which should be selected. The evaluation process requires prediction of the consequences expected to result from picking the alternative, estimation of the magnitude of each consequence, and conversion of each consequence magnitude into commensurable units. The purpose of this chapter is to review the conceptual problems and basic principles involved in the process.

1-1 EQUIVALENCE OF KIND The major obstacles to expressing the consequences of alternative courses of action in commensurable units are differences in kind and differences in time. The two differences may be illustrated through the example of comparing two alternative irrigation projects. One project provides irrigation water for peaches. The second provides water for cotton. Construction of the first project will produce x tons of peaches. Construction of the second will produce y bales of cotton. If the two projects can be constructed for equal cost, selection depends on whether x tons of peaches or y bales of cotton is more valuable.

As long as the two outputs are expressed in these diverse units, the projects cannot be compared. Only when common units are used is comparison possible. The first step in economic analysis must be to find a common value unit. One might use tons of peaches. A farmer selling y bales of cotton might receive the same price as if he had sold y' tons of peaches. The decision could then be based on whether x or y' was the larger. One might use bales of cotton as the common unit, express the value of x tons of peaches as x' bales of cotton, and make the decision. As a third approach, farmers grow both peaches and cotton to buy bread for their families. A farmer could sell x tons of peaches and buy x'' loaves of bread. A farmer could sell y bales of cotton and buy y'' loaves of bread. The decision could be based on a comparison between x'' and y''.

However, such approaches are uncommon because society has established a system of units for comparing relative value. Tons of peaches, bales of cotton, and loaves of bread may all be evaluated in monetary units. The use of monetary units in economy studies is based solely on convenience and does not imply a materialistic approach of considering only monetary profit while ignoring the many values of life, health, and happiness which can not be expressed in money terms. Handling intangible values will be discussed later (Sec. 1-6).

The simple fact is that diverse values are understood by more people when expressed in monetary terms than when any other kind of unit is used. Far more people can visualize worth in dollars than in tons of peaches, bales of cotton, or loaves of bread. The proper approach for comparing the two irrigation projects is to convert both tons of peaches and bales of cotton into dollars, compare the dollar totals, and (provided intangible values do not indicate otherwise) select the project producing the greater total.

1-2 EQUIVALENCE OF TIME An irrigation project will provide water for many years. In evaluating the example project, should peaches produced this year be reckoned as having equal value to those expected to be produced 30 years from now? Most people would be more inclined to invest a dollar to produce 5 lb of peaches now than to invest the dollar to produce 5 lb of peaches 30 years from now.

An earlier realization of investment returns is desirable for the investor because it gives him greater flexibility for future action. If the returns are needed for consumption, they become available with less waiting. If they are to be reinvested, an earlier reinvestment will speed subsequent returns and result in a more rapid expansion of capital. To fail to differentiate returns by date is to say all economic expansion rates are equally desirable.

In order to make realistic investment decisions, each monetary value must be identified by both amount and time. Amounts at different times should not be directly compared or combined. They are not in common units. Amounts in different time periods may be made equivalent by multiplying future amounts by a factor becoming progressively smaller into the more distant future. The discount rate is the time rate of decrease in this factor expressed in percent per time period. An investment of a dollar at an annual rate of return of 5 percent would yield $1.05 a year hence. Similarly, $1.05 available a year from now is equivalent to $1.00 now when discounted at 5 percent.

The discount rate used has a great influence on the project selected. Future benefits and costs receive less weight with higher, and more weight with lower, discount rates. High discount rates favor projects with little initial investment, while low discount rates favor capital intensive projects. Determination of the proper discount rate for water resources planning is discussed in detail in Chap. 6.

1-3 WHOSE VIEWPOINT? Monetary value depends on the viewpoint taken in the evaluation. The grower who produces a ton of peaches will equate value with sale price. The community will add to this the gains accruing to food processors, farm workers, farm suppliers, and other individuals who profit indirectly. However, from the national viewpoint, committing resources to one community to grow more peaches may deny investment capital to another. Furthermore, peaches grown in different communities are competing goods.

The above description thus pinpoints the three viewpoints possible in an engineering economy study.[1]

1 That of the group sponsoring or financing the project. Consider only consequences affecting this group.
2 That of all the people in a specific area such as a state, county, or special district. Consider only consequences affecting those living in this defined area.
3 That of the entire nation. Consider all consequences to whomsoever they may accrue.

Viewpoint 1 is based on the premise that the sponsoring group should promote its own welfare. It is a legitimate viewpoint for private enterprise, but one of the primary justifications for action by government is to avoid the adverse consequences of individuals' putting personal above public welfare. Therefore, there appears to be no justification for a public agency's taking anything less than the public viewpoint. When a planning group ignores conflicting viewpoints, a higher level of government must bring about any adjustments necessary to protect the public interest.

[1] Eugene L. Grant and Grant Ireson, "Principles of Engineering Economy," 4th ed. (New York: The Ronald Press Company, 1960), p. 445. For a parallel discussion, see Tillo Kuhn, "Public Enterprise Economics and Transport Problems" (Berkeley: University of California Press, 1962), pp. 13–16.

Practical realities may restrict the freedom of a local government to take the national viewpoint.[1] First, the cost of tracing the consequences of proposed alternatives beyond its jurisdiction may be excessive. Secondly, a local government is subject to much political pressure from the taxpayers who support it but little from those living outside its jurisdiction. The tendency is to ignore these outside consequences. Higher levels of government must be responsible for making sure local planners adequately consider project consequences occurring in other areas.

Viewpoint 3 should, in principle, be taken by every level of government to maximize aggregate national welfare in the long run. Where federal programs, such as the reclamation of the arid West or the economic development of Appalachia, are designed to achieve regional goals, project consequences should be evaluated from both the national and regional viewpoints. Regional interests may try to influence federal agencies to select projects producing regional benefit, where they must repay only a fraction of project costs.[2] The decision maker needs to know if such a project can be justified from the national viewpoint and weigh the national sacrifice required to achieve local goals.

1-4 SUNK COST The justification for following a course of action depends on the events occurring with it being better than those occurring without it, by an amount exceeding its implementation cost. An engineering economy study need analyze only differences between alternatives and differences between resulting consequences. All costs and benefits unaffected by which alternative is chosen should be disregarded. Obviously, past events have already occurred and cannot be retracted by future action. Past expenditure, or sunk costs, are past events and thus should have no influence on deciding among alternatives except as they affect future cash flows.

Despite their economic irrelevance, sunk costs have often been allowed to influence decisions for two main reasons. The decision makers may have a psychological, political, or even a legal commitment to continue a past policy so that past efforts are not wasted. Secondly, accounting records indicating an undepreciated book value for assets having no economic worth may restrict freedom to make new investment. However, in no case are past mistakes a legitimate excuse for continuing a policy which cannot be justified by future benefits.

[1] Roland N. McKean, Costs and Benefits from Different Viewpoints, "Public Expenditure Decisions in the Urban Community" (Washington: Resources for the Future, 1962), pp. 148–151.

[2] See Kuhn, *op. cit.*, p. 18. Kuhn would have the decision-making authority set at the highest level so that the broadest view of the public interest is observed. McKean, "Costs and Benefits from Different Viewpoints," p. 147, sees central planning as too often leading "to planning of the people, by the few, and for the few."

The sunk-cost principle is illustrated in the following example. Suppose $5 million have been spent on a hydropower installation ultimately costing $10 million. A steam plant costing an estimated $3 million is subsequently found to be capable of supplying the same energy. Which facility should be selected, assuming all other future costs to be the same? The $5 million already spent on the hydropower facility is a sunk cost, hence is irrelevant. Since the cost of the steam plant is less than the remaining cost of the hydropower installation, the steam plant should be selected. Continuing the initial project is not in the economic interest of the public.

1-5 INCREMENTAL COST According to the incremental-cost principle, the change in benefits and the change in costs resulting from a given decision determine the merit of that decision. Each project segment should be judged on its own merits. The decision to enlarge a project should be justified by the enlargement's increasing benefits more than it increases cost.

For illustration, consider a 10,000 acre-ft reservoir which a city has determined to build for $1 million. Before construction begins, increasing the storage to 20,000 acre-ft and the cost to $1,500,000 is found to achieve $600,000 in downstream flood control benefits. The incorrect average-cost approach would preclude flood control on the basis that half the storage means half the cost, and $750,000 exceeds $600,000. The correct incremental-cost approach would include flood control because the additional expenditure of $500,000 is exceeded by the benefit of $600,000.

By the same token, an element costing $50,000 but producing only $20,000 in benefits should not be justified by inclusion in a large project with costs of $2 million and benefits of $3 million. The maximum net benefit is achieved with that element excluded.

1-6 INTANGIBLE VALUES Even though an economy study seeks to evaluate all consequences in commensurable monetary units, many values defy such quantification. Unique or extremely rare items such as species of plant or animal life or sights of unusual beauty have no acknowledged money value. Neither have direct effects on human beings physically through loss of health or life, emotionally through loss of national prestige or personal integrity, or psychologically through environmental changes. Nor do monetary values serve to measure the achievement of such extra economic goals as income redistribution, increased economic stability, or improved environmental quality (Sec. 5-1). Each value which cannot be expressed in monetary terms is called an *intangible* or *irreducible*.

Inability to express a value in economic units does not necessarily preclude evaluation in other units. All intangible values should be quantified as precisely as possible. Vague statements on threat to human life are not nearly as helpful as a precise statement on the number of lives expected to be lost. In weighing whether a given sacrifice in economic value is worthwhile to achieve a goal, the decision maker should have access to the best possible information on the nature of the intangible consequences as well as the magnitude of the economic consequences.

1-7 PREDICTIVE UNCERTAINTY Because economic analysis compares future consequences of engineering alternatives, the reliability of each conclusion depends on the ability to predict future events. A project may only appear to be economically feasible because of incorrect predictions. No matter how much data or experience one has, predicting the future is inherently uncertain.

Uncertainty with respect to water resources project evaluation has been described by McKean as "inherent in the nature of things and is not necessarily evidence of lazy or careless estimation."[1] He gives five classifications:

1 *Uncertainty about objectives.* Even though planning objectives as currently conceived may be perfectly clear, future developments may significantly alter social goals.
2 *Uncertainty about constraints on the system.* It is computationally infeasible to plan all economic decisions simultaneously. A particular analysis must be performed in the context of constraints imposed by outside events. The price of steel may be taken as given in an economy study without attempting to determine an optimum price through industry analysis. However, future developments may produce unpredictable price changes.
3 *Uncertainty about public response.* Even though a thorough analysis may indicate the need for project-produced services, public inertia against learning new ways or psychological commitment to established procedures will affect their use in an often unpredictable manner.
4 *Uncertainty about technological change.* Even though a project currently produces a needed output at low cost, innovations or technological changes may cause the output to be no longer needed or introduce an even less costly production process.
5 *Uncertainty about the chance element in recurring events.* Even when the probability of occurrence of random events can be established statisti-

[1] Roland N. McKean, "Efficiency in Government through Systems Analysis" (New York: John Wiley & Sons, Inc. 1958), pp. 65–68.

cally, the precise time of occurrence (of flood peaks, for example) is never known in advance. Furthermore, the many random elements in any system cumulatively increase overall uncertainty.

Widely used approaches to treating uncertainty include (1) applying preselected percentages to increase costs or reduce benefits, (2) limiting the period of analysis, or (3) adding a risk increment to the discount rate. However, because each of these approaches requires selection of a numerical factor without providing any help on how a specific value is to be selected, Eckstein has well argued:

> These crude adjustments are intellectually not very satisfying and one should try to derive better adjustments from explicit objective functions and from the probabilistic nature of benefits.[1]

A more satisfactory approach is to recognize explicitly that project effects should not be predicted as single fixed values but rather as variables having some probability distribution of possible values. A more detailed description of specific approaches is found following Sec. 8-16, and decision-theory techniques are presented in a number of works by other authors.[2]

1-8 PLANNING HORIZONS The planning horizon is the most distant future time considered in an engineering economy study. The inherent uncertainty of predicting the more distant future favors short planning periods, but the need for analysis of the long-run effects of plans to meet immediate needs favors a longer period. Actually, four different periods of time must be considered in any economic analysis: (1) the economic life, (2) the physical life, (3) the period of analysis, and (4) the construction horizon.

The *economic life* ends when the incremental benefits from continued use no longer exceed the incremental cost of continued operation. Economic life is usually shorter for such project elements as pumps and canal linings than for a water resources project as a whole.

The *physical life* ends when a facility can no longer physically perform its intended function. While the economic life never exceeds the physical life, it may be shorter because of obsolescence and changing demands for services. As an example, electric generation by nuclear power may become

1 Otto Eckstein, A Survey of the Theory of Public Expenditure Criteria, in National Bureau of Economic Research, "Public Finances: Needs, Sources, and Utilization" (Princeton, N.J.: Princeton University Press, 1961), p. 470.

2 See H. Chernoff and L. Moses, "Elementary Decision Theory" (New York: John Wiley & Sons, Inc., 1954).

so inexpensive as to make electric generation by fossil fuels uneconomical while such plants still function perfectly well.

The *period of analysis* is the length of time over which project consequences occurring are included in a particular study. The period of analysis for comparing alternative project designs has the project economic life as its upper limit but may be shortened arbitrarily to exclude the highly uncertain events of the very distant future.

The *construction horizon* is reached when the constructed facilities are no longer expected to satisfy the future demands. For example, the water supply alternatives for a community may be studied for a period of analysis of 40 years even though the original facilities may be planned large enough to supply the water usage predicted for only 20 years. The longer period of analysis helps integrate present action into the long-run solution. The shorter construction horizon adds flexibility to deal with unforeseen changes.

Regular maintenance and periodic replacement of worn parts may extend the life of a water resources project almost indefinitely, but a period of analysis of 50 or 100 years is generally used.[1] The optimum construction horizon for individual project components is often a shorter period and may be determined by economic analysis (Sec. 9-10). For example, tunnels may be economically built to maximum capacity because of the high cost of subsequent enlargement, whereas channels may be economically enlarged in 10- or 20-year stages.

When alternative schemes of water resources development are being compared, all must be evaluated over the same period of analysis. If a short economic life causes some alternatives to require periodic replacement, the most common assumption is that each cost will be repeated in a fixed cycle over a series of economic lives until the total project life is reached. However, this assumption should not be used automatically without considering, with respect to the cost or desirability of cyclic replacement, the effects of differential inflation (Sec. 9-8), the development of new production techniques through technological advance, and the changing nature of demand with time. Uncertainty with respect to any of these tends to favor short-lived alternatives.

If the period of analysis is not an even multiple of element lives, an adjustment must be made through a negative cash flow or salvage value equal to the value of the element at the end of the period of analysis. A refined value estimate is seldom warranted because of the relatively small present worth and the difficulty of predicting cash flows in the distant

[1] The President's Water Resources Council, *Policies, Standards, and Procedures in the Formulation, Evaluation, and Review of Plans for Use and Development of Water and Related Land Resources*, 87th Cong. 2d Sess., Sen. Doc. 97, 1962.

future. Straight-line depreciation may be used for a quick estimate of the value of unused life as

$$S = \left(1 - \frac{X}{L}\right)K \tag{1-1}$$

where X is the years of unused life, L is the years of total life, and K is the initial value (Ex. 1-1).

EXAMPLE 1-1

A certain type of pump is estimated to require replacement every 20 years and is to be used in a project where the economy study is based on a 50-year period of analysis. What salvage value should be used if the initial cost is \$15,000?

The third pump will be installed in year 40 and thus will have 10 years of useful life remaining at the end of the period of analysis. Thus, $X = 10$ years, $L = 20$ years, and $K = \$15{,}000$. From Eq. (1-1),

$$S = \left(1 - \frac{10}{20}\right)\$15{,}000 = \$7{,}500$$

1-9 STRUCTURING ALTERNATIVES Recognition of the full spectrum of potential alternatives for analysis is of paramount importance if the most efficient course of action is not to be omitted at the outset. All reasonable possibilities should be considered. The analyst must be imaginative in defining courses of action which will attain designated objectives. One of the most useful treatises on structuring and handling alternatives is found in the pioneering work of E. L. Grant[1] and is used as a basis for summarizing this chapter with the following points:

1 All alternatives physically capable of achieving the design objective should be clearly defined. One alternative is to "do nothing" if none of the other proposals is economically feasible. Limitations of time and funds often prevent a complete analysis of all alternatives. Before extending the study, the costs of additional information must be compared with the potential savings from better project selection.

2 The physical consequences of each alternative should be identified and evaluated in money units. Benefits and costs which cannot be evaluated in monetary terms should be explicitly identified.

3 The difference between alternatives should be the basis for comparison. Sunk costs are irrelevant in choosing between alternatives except as

[1] Eugene L. Grant, Concepts and Applications of Engineering Economy, in "Special Report 56, Economic Analysis in Highway Programming, Location, and Design" (Washington: Highway Research Board, 1960), pp. 8–14.

they affect the future. Allocated costs or average costs should not be used in economy studies; incremental or marginal costs should be used. Each separable increment of investment must return at least an equal increment of benefits in order to be justified.

4 Weight should be given to differences in intangibles as well as to differences in market consequences when comparing alternatives. Arbitrary monetary values should not be placed on intangibles since they distort the economic analysis. Economic analysis should not be ignored even if decisions must be based largely on intangibles. The decision maker should be aware of the cost of achieving other values when projects are justified on extraeconomic grounds.

5 The alternatives should be compared on a uniform basis. Such values as discount rates, period of analysis, and unit costs must be the same.

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PROBLEMS

1-1 Costs and revenues for a particular project having alternate possible levels of investment have been estimated on an equivalent basis and found to be

Cost:	39	83	117	155	194
Revenue:	100	150	175	185	190

Which project level should be selected?

1-2 To develop a new water supply, an industry will have to spend \$1 million. The resulting increased production is predicted to increase net income to the company from sales by \$900,000. Associated economic development will benefit the community by \$400,000 and other nearby communities in the same state by \$250,000. However,

$500,000 of the increased state income represents transfers from other states. The river on which the industry is located flows into another country. The new industrial development is expected to deteriorate water quality sufficiently to cause $250,000 worth of damage downstream from the border.

a Would the project be economically justified from the viewpoint of the industry?

b The community?

c The state?

d The nation?

e What kinds of intangible factors might be weighed by each of the four viewpoints?

f Should the project be built from the overall viewpoint?

1-3 A community has spent $50,000 developing a new well and has not yet obtained water. The geological consultant estimates another $50,000 will be required to guarantee a good supply but admits sufficient water may be obtained after spending only $10,000. As an alternative a spring exists several miles away from which an equivalent supply could be pumped for $40,000. What course of action would you recommend and why?

CHAPTER TWO

MATHEMATICS OF ECONOMIC ANALYSIS

Formulating the Analysis

Economic analysis is performed in a series of steps. Each alternative must be explicitly defined and the resulting physical consequences must be predicted. A monetary value must be placed on each physical consequence. A discount rate must be selected and applied to convert the predicted time stream of monetary values into an equivalent single number. Only then can the alternatives be directly compared. Each step is developed in the following pages.

2-1 DEFINING THE ALTERNATIVES An engineering alternative is a course of action physically capable of achieving the design objective. Structural alternatives (a dam, for example) characteristically involve a large first cost for project construction to produce benefits throughout the project life. Nonstructural alternatives (flood-plain zoning, for example) involve benefits and costs which are both fairly well distributed over project life. A properly defined alternative must be specified by the engineer with sufficient clarity for its economic and intangible consequences to be evaluated and its nature understood by those responsible for the final selection. Properly defined alternatives are an evidence of clear thinking and a necessity for adequate consequence prediction.

A properly formulated set of engineering alternatives includes all possibilities for action (including taking no action at all) which have a realistic chance of proving optimum. Special care is necessary to include nonstructural alternatives with which engineers may be less familiar. The alternatives are called *mutually exclusive* if only one of a set can be selected. Alternatives may be mutually exclusive because of conflicting space requirements, limited financial resources, limited resource inputs (water, for example), or limited demand or need for resulting output. At other times, it may be practical to implement two or more of the alternatives.

2-2 PHYSICAL CONSEQUENCES Each engineering alternative will if implemented produce a series of physical consequences occurring at various times into the future. For example, a project built to irrigate peaches, tomatoes, and alfalfa will produce a number of results. The project will have to be constructed. After construction is completed, the project will have to be maintained. Certain elements may wear out and require periodic replacement. Each such cost-associated event needs to be predicted by nature and date.

The water delivered by the project will be used to irrigate peaches, tomatoes, and alfalfa. The first year water is delivered, the acreage and increased yield of each crop can be used to predict a project output of X tons of peaches, Y tons of tomatoes, and Z tons of alfalfa. In a similar manner, X, Y, and Z may be predicted for each subsequent year of project life. The outputs can be expected to increase steadily in the early years as more and more land is irrigated. Later, they may be expected to fluctuate with changing weather and other factors which influence crop yield.

2-3 CASH FLOW DIAGRAM Having identified the physical consequences of each alternative, it is necessary to decide which ones are relevant to the analysis. Some may not be because of the viewpoint taken in the study, a neutral effect which is neither desirable nor undesirable, a tenuous connection to the project, their small magnitude, or some other reason. Other consequences may be dropped from further evaluation because they are identical for each alternative and an economy study is concerned only with differences (incremental costs). The relevant consequences can be separated into two groups. Some can be assigned a reasonable monetary value. The others may have some monetary value but also require supplemental determination of the intangible factors (Sec. 1-6).

The assignment of a monetary value to physical consequences is a very complicated process having many ramifications which will be discussed throughout the remainder of the book. However, for the time being we will assume that meaningful monetary values can be assigned to the major project consequences. Economic analysis becomes a less reliable guide to decision making as more consequences fall in the intangible class. For the sample project, the cost of installation, the cost of maintenance in each year, replacement cost for each short-lived item, and the benefit resulting from the increased yield of each crop in each year would have to be determined.

The graphic presentation of each value plotted by time is called a *cash flow diagram*. The standard representation for a cash flow diagram is that receipts (benefits) are represented by arrows pointing upward, while costs are represented by arrows pointing downward. Arrows pointing toward the centerline indicate cash flows which may be taken either way in a general diagram (see Fig. 2-2). The length of the arrow is made proportional to the cost or benefit. The horizontal axis denotes time. For convenience in analysis and with little loss in accuracy for long-lived projects, all cash flows during a year are by convention combined into lump sums occurring at the end of the year. Figure 2-1 is a cash flow diagram which might be predicted for our hypothetical irrigation project. Annual benefits and costs will not in fact be constant every year but will vary around average values in an almost random fashion with crop production and maintenance needs. However, only expected average values are normally predicted in advance, even though the random component could conceivably be introduced through simulation (Sec. 20-10). Drawing of the cash flow diagram can be greatly simplified by use of envelope curves as a substitute for the many arrows.

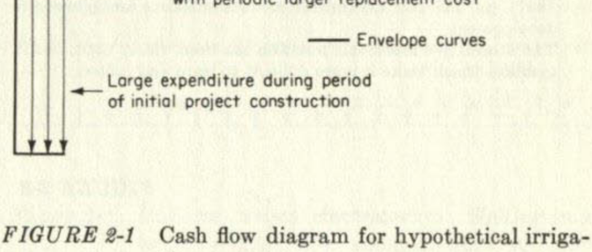

FIGURE 2-1 Cash flow diagram for hypothetical irrigation project.

Discounting Factors[1]

2-4 SINGLE-PAYMENT FACTORS In applying discounting to convert cash flows to a single number suitable for use in comparing alternatives, the basic objective is to convert a value at one date to an equivalent value at another date. Two single-payment factors are available for this purpose (Fig. 2-2).

Single-payment Compound-amount Factor The single-payment compound-amount factor indicates the number of dollars which will have accumulated after N years for every dollar initially invested at a rate of return of i percent. The functional notation is $(F/P,i\%,N)$,[2] where F implies a future and P a present amount. If one were to deposit P dollars initially, after 1 year

$$F = P(1 + i) \tag{2-1}$$

Each year the amount must again be multiplied by $1 + i$ to account for that year's interest; therefore after N years

$$F = P(1 + i)^N \tag{2-2}$$

The desired factor becomes

$$\left(\frac{F}{P}, i\%,N\right) = (1 + i)^N = \frac{F}{P} \tag{2-3}$$

Single-payment Present-worth Factor The single-payment present-worth factor indicates the number of dollars one must initially invest at i percent to have \$1 after N years. It will be abbreviated by $(P/F,i\%,N)$. The factor is the inverse of the previous factor, or

$$\left(\frac{P}{F}, i\%,N\right) = \frac{1}{(1 + i)^N} = \frac{P}{F} \tag{2-4}$$

[1] Standardized notation for discounting formulas has been suggested by the Ad Hoc Committee for Study of Standardization of Engineering Economy Notation, *Eng. Economist*, vol. 12 (Summer, 1967), pp. 253–263. Committee recommendations are followed for the most part in the subsequent development.

[2] The alternative mnemonic notation has been widely used, but it creates a group of rather artificial symbols which make it more difficult to learn and follow.

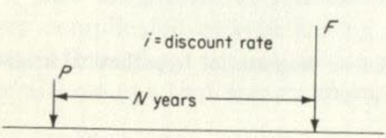

FIGURE 2-2 Single-payment factors. (a) Single-payment compound-amount factor $= F/P$; (b) single-payment present-worth factor $= P/F$.

Interest Tables Selected values of both single-payment factors are presented in Table A in the Appendix. When a discounting factor is needed for a combination of N and i not found in the tables, an approximate value may be found by interpolation. The error from interpolation becomes increasingly severe with higher discount rates. For precise values or values outside the range covered by the tables, one must substitute values for i and N in the appropriate formula.

2-5 UNIFORM-ANNUAL-SERIES FACTORS

All discounting problems can be solved by applying the two single-payment factors. However, additional factors can be developed to greatly reduce the required work. As an example, one may take the irrigation project of Fig. 2-1. If it were to produce crops having equal value for each of 50 years, fifty separate single-payment present-worth factors would have to be applied to find the present worth of this uniform annual cash flow. The task is made much shorter by developing uniform-annual-series factors.

Uniform-annual-series factors indicate equivalence between the value at an earlier date, P, and equal amounts A at the end of each of the N years or between the N equal values of A and an accumulated amount F (Fig. 2-3).

Sinking-fund Factor The sinking-fund factor indicates the number of dollars one must invest in uniform amounts at i percent interest at the end of each of N years to accumulate \$1. The functional notation is $(A/F,i\%,N)$. If one were to apply the single-payment compound-amount factor individually to each of the N values of A in Fig. 2-3 and sum the

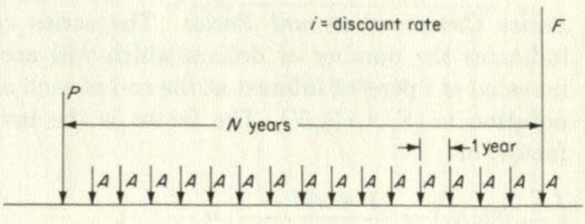

Figure 2-3 Uniform series factors. (*a*) Sinking-fund factor = A/F; (*b*) compound-amount factor = F/A; (*c*) capital-recovery factor = A/P; (*d*) present-worth factor = P/A.

results to obtain F, the result would be

$$F = A[1 + (1 + i) + (1 + i)^2 + \cdots + (1 + i)^{N-1}] \tag{2-5}$$

where the last value of A accumulates no interest because it is withdrawn immediately upon deposit and the first value of A accumulates interest for $N - 1$ years. Multiplying both sides of Eq. (2-5) by $1 + i$ gives

$$(1 + i)F = A[(1 + i) + (1 + i)^2 + (1 + i)^3 + \cdots + (1 + i)^N] \tag{2-6}$$

The relationship may now be converted from a series to an explicit expression through term-by-term subtraction of Eq. (2-5) from Eq. (2-6) to give

$$iF = A[(1 + i)^N - 1] \tag{2-7}$$

The desired factor becomes

$$\left(\frac{A}{F}, i\%, N\right) = \frac{i}{(1 + i)^N - 1} = \frac{A}{F} \tag{2-8}$$

Capital-recovery Factor The capital-recovery factor indicates the number of dollars one can withdraw in equal amounts at the end of each of N years if \$1 is initially deposited at i percent interest. The functional notation is $(A/P,i\%,N)$. Because

$$\frac{A}{P} = \frac{A}{F}\frac{F}{P} \tag{2-9}$$

One may substitute Eqs. (2-8) and (2-3) in Eq. (2-9) to get

$$\left(\frac{A}{P}, i\%, N\right) = \frac{i(1 + i)^N}{(1 + i)^N - 1} = \frac{A}{P} \tag{2-10}$$

Series Compound-amount Factor The series compound-amount factor indicates the number of dollars which will accumulate if exactly \$1 is invested at i percent interest at the end of each of N years. The functional notation is $(F/A,i\%,N)$. The factor is the inverse of the sinking-fund factor, or

$$\left(\frac{F}{A}, i\%, N\right) = \frac{(1 + i)^N - 1}{i} = \frac{F}{A} \tag{2-11}$$

Series Present-worth Factor The series present-worth factor indicates the number of dollars one must initially invest at i percent interest to withdraw \$1 at the end of each of N years. The factor $(P/A,i\%,N)$ is the inverse of the capital-recovery factor or

$$\left(\frac{P}{A}, i\%, N\right) = \frac{(1+i)^N - 1}{i(1+i)^N} = \frac{P}{A} \tag{2-12}$$

Interest Tables Values for all four uniform-annual-series factors for selected values of i and N are tabulated in Table A in the Appendix.

2-6 UNIFORM-GRADIENT-SERIES FACTORS The uniform-annual-series factors can be applied to an equal cash flow in each year. Often cash flows will not be equal but will follow some definite pattern. The simplest pattern is the uniformly increasing gradient series, a series in which the cash flow increases by some constant amount between each pair of years.

Uniform-gradient-series Present-worth Factor The uniform-gradient-series present-worth factor indicates the number of dollars one must initially invest at i percent interest to withdraw \$1 one year later, \$2 two years later, to N dollars N years later. The functional notation is $(P/G,i\%,N)$.

If one were to apply the single-payment compound-amount factor individually to each value, beginning with the last, in the gradient series of Fig. 2-4 and sum to obtain the accumulated amount just after the last deposit, the result would be

$$F = G[N + (N-1)(1+i) + \cdots + 2(1+i)^{N-2} + (1+i)^{N-1}] \tag{2-13}$$

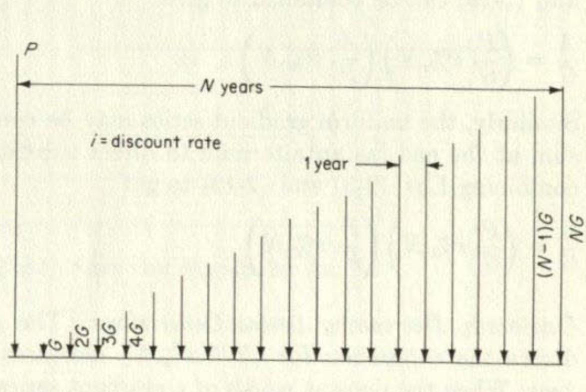

FIGURE 2-4 Gradient-series present-worth factor $= P/G$.

Multiplying both sides by $1 + i$ gives

$$(1 + i)F = G[N(1 + i) + (N - 1)(1 + i)^2 + \cdots + 2(1 + i)^{N-1} + (1 + i)^N] \tag{2-14}$$

Term-by-term subtraction of Eq. (2-13) from Eq. (2-14) gives

$$iF = G\{-N + (1 + i) + \cdots + (1 + i)^N\} \tag{2-15}$$

Multiplication of both sides by $1 + i$ gives

$$(1 + i)iF = G[-N(1 + i) + (1 + i)^2 + \cdots + (1 + i)^{N+1}] \tag{2-16}$$

Term-by-term subtraction of Eq. (2-15) from Eq. (2-16) then gives

$$i^2F = G[N - N(1 + i) - (1 + i) + (1 + i)^{N+1}] \tag{2-17}$$

Rearranged, Eq. (2-17) becomes

$$\frac{F}{G} = \frac{(1 + i)^{N+1} - (1 + Ni + i)}{i^2} \tag{2-18}$$

When Eq. (2-18) is combined with Eq. (2-4) to convert from F to P, the final result is

$$\left(\frac{P}{G}, i\%, N\right) = \frac{(1 + i)^{N+1} - (1 + Ni + i)}{i^2(1 + i)^N} = \frac{P}{G} \tag{2-19}$$

Interest Tables Values of the uniform-gradient-series present-worth factor for selected values of i and N are tabulated in Table B in the Appendix.

Conversion from Present Worth Whenever the uniform gradient series needs to be converted to an equivalent uniform annual series, Eqs. (2-10) and (2-19) can be combined to give

$$\frac{A}{G} = \left(\frac{P}{G}, i\%, N\right)\left(\frac{A}{P}, i\%, N\right) \tag{2-20}$$

Similarly, the uniform gradient series may be converted to a single lump sum at the end [as an alternate to direct substitution in Eq. (2-18)] by combining Eqs. (2-3) and (2-19) to get

$$\frac{F}{G} = \left(\frac{P}{G}, i\%, N\right)\left(\frac{F}{P}, i\%, N\right) \tag{2-21}$$

Uniformly Decreasing Series Conversions The gradient series used to derive the expression for $(P/G,i\%,N)$ increases in value from year to year. When the present worth of a gradient series that decreases in value from year to year is needed, it may be determined by subtracting a uniformly increasing gradient series from a uniform annual series in the manner shown in Ex. 2-1.

EXAMPLE 2-1

An individual invested the following amounts of money at 4 percent interest. How much would he have at the end of year 25?

Year	*Investment*	*Year*	*Investment*	*Year*	*Investment*
1	5	8	40	15	25
2	10	9	45	16	20
3	15	10	50	17	15
4	20	11	45	18	10
5	25	12	40	19	5
6	30	13	35	20–25	0
7	35	14	30		

The present worth of the pyramid-shaped series can be found by subdividing it into three portions to which factors from the tables can be directly applied. (Cf. Fig. 2-5.)

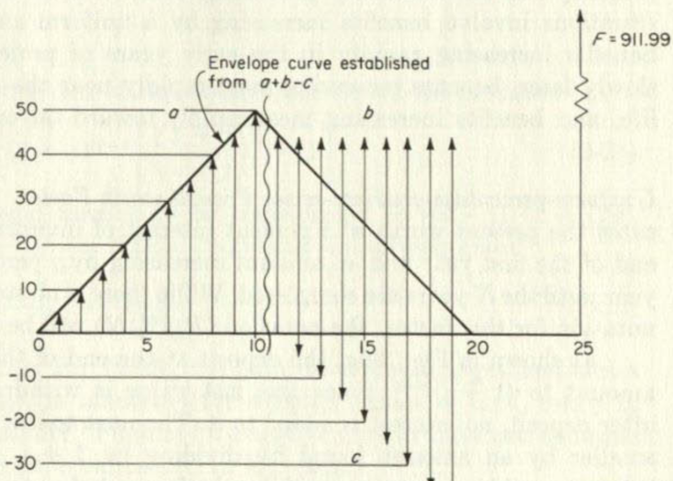

FIGURE 2-5 Cash flow diagram for Ex. 2-1.

1. Present worth of series 5, 10, . . . , 45, 50 in years 1 through 10.

$$5\left(\frac{P}{G}, 4\%, 10\right) = 5(41.99225) = \$209.96$$

2. Plus present worth of series of 45 per year in years 11 through 19,

$$45\left(\frac{P}{A}, 4\%, 9\right)\left(\frac{P}{F}, 4\%, 10\right) = 45(7.43533)(0.67556) = \$226.04$$

3. Minus present worth of series 5, 10, . . . , 35, 40 in years 12 through 19,

$$5\left(\frac{P}{G}, 4\%, 8\right)\left(\frac{P}{F}, 4\%, 11\right) = 5(28.91333)(0.64958) = -\$93.92$$

The three values sum to a \$342.08 present worth, which can be converted to a value in year 25 by

$$342.08\left(\frac{F}{P}, 4\%, 25\right) = (342.08)(2.66584) = \$911.99$$

2-7 *NONUNIFORM-GRADIENT-SERIES FACTORS* Project planning often requires determination of the present worth of some monotonically but not uniformly increasing time stream of benefits. Typical situations involve benefits increasing by a uniform annual percentage, benefits increasing rapidly in the early years of project life but more slowly later, benefits increasing most rapidly near the middle of project life, and benefits increasing most rapidly toward the end of project life.

Uniform-percentage-gradient-series Present-worth Factor This factor indicates the present worth at i percent interest of investment of \$1 at the end of the first year and an amount increasing by j percent from year to year until the N years are completed. While there is no standard functional notation for this factor, the notation $(P_j, i\%, N)$ will be used.

As shown in Fig. 2-6a, the deposit at the end of the last year would amount to $(1 + j)^{N-1}$. Since this last value is withdrawn immediately after deposit, no interest is added to it. The next to last deposit would be smaller by an amount found by dividing by $1 + j$, but accumulated interest would increase its worth by the factor $1 + i$. Summing each term backward through the series of Fig. 2-6 gives

$$F = (1 + j)^{N-1}\left[1 + \frac{1 + i}{1 + j} + \cdots + \left(\frac{1 + i}{1 + j}\right)^{N-1}\right] \tag{2-22}$$

Multiplication of both sides by $(1 + i)/(1 + j)$ gives

$$\frac{1 + i}{1 + j}F = (1 + j)^{N-1}\left[\frac{1 + i}{1 + j} + \left(\frac{1 + i}{1 + j}\right)^2 + \cdots + \left(\frac{1 + i}{1 + j}\right)^N\right] \tag{2-23}$$

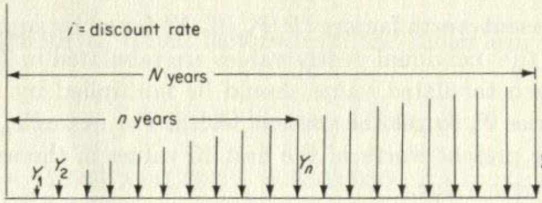

FIGURE 2-6 Nonuniform-gradient-series present-worth factors. (*a*) Percentage-gradient-series present-worth factor $(P_j,i\%,N)$; j = growth rate, $Y_1 = 1.00$, $Y_n = (1+j)^{n-1}$. (*b*) Accelerated-growth-curve present-worth factor $(P/F_a,i\%,N)$; $N = 50$ years, $F_a = Y_N$, $Y_n = [\ln (n+1)/3.93183]F_a$. (*c*) Normal-growth-curve present-worth factor $(P/F_n,i\%,N)$; $N = 50$ years, $F_n = Y_N$, $Y_n = (0.0012n^2 - 0.000016n^3)F_n$. (*d*) Deferred-growth-curve present-worth factor $(P/F_d,i\%,N)$; $N = 50$ years, $F_d = Y_N$, $Y_n = (0.0004n^2)F_d$.

Term-by-term subtraction of Eq. (2-22) from Eq. (2-23) produces

$$\left(\frac{1+i}{1+j} - 1\right) F = (1+j)^{N-1}\left[\left(\frac{1+i}{1+j}\right)^N - 1\right] \tag{2-24}$$

wherein the left-hand term may be transformed to $[(i-j)/(1+j)]F$. Substituting $P(1+i)^N$ for F [Eq. (2-2)] and simplifying gives

$$(P_j,i\%,N) = \frac{(1+i)^N - (1+j)^N}{(i-j)(1+i)^N} \tag{2-25}$$

The right-hand side of Eq. (2-25), the uniform-percentage-gradient-series present-worth factor, is tabulated for selected values of i, N, and j in Table B in the Appendix.[1] Factors for negative growth rates can be found by substituting negative values of j in Eq. (2-25).

Accelerated-growth Present-worth Factor This factor indicates the present worth at i percent interest of an annual investment pattern in which deposits begin by increasing very rapidly, but increase at a progressively slower rate in later years. Such a series is represented by an equation suggested by the Corps of Engineers[2] and indicated in Fig. 2-6*b*. The

[1] In the special case where $i = j$, Eq. (2-25) is indeterminate, but $(P_j,i\%,N) = N/(1+j)$.

[2] U.S. Army Corps of Engineers, *Eng. Manual* EM 1120-2-118, (Washington, June, 1960), app. 2, change 1.

present-worth factors $(P/F_a,i\%,N)$ found by summing the present worths of the individual yearly values are tabulated in Table C in the Appendix. Each tabulated value should be multiplied by the fiftieth value in the series F_a to get the present worth. For example, the 10-year factor gives the present worth of the first 10 values in the series as a multiple of the fiftieth value.

Normal-growth Present-worth Factor This factor indicates the present worth at i percent interest of a series of deposits which increase at a progressively faster rate until the midpoint of project life and then increase progressively more slowly through the later years. Its typical use would be in finding the present worth of a benefit series realized in an area where development is expected to be most rapid 20 to 25 years after project construction. Such a series is represented by a curve suggested by the Corps of Engineers[1] and depicted in Fig. 2-6*c*. The present worth factors $(P/F_n,i\%,N)$ are found in Table C in the Appendix.

Deferred-growth Present-worth Factor This factor indicates the present worth at i percent interest of a series of deposits which increase slowly throughout most of the project life only to increase very rapidly in the last few years. Such a series is represented by another curve suggested by the Corps of Engineers[2] and shown in Fig. 2-6*d*. The present-worth factors $(P/F_d,i\%,N)$ are also found in Table C in the Appendix.

2-8 OTHER CASH FLOW PATTERNS Cash flow patterns for economy studies are based on projected future events, cannot be known with any real certainty, and thus can normally be approximated with sufficient accuracy by one of the above patterns. Sometimes it may be better to approximate a future cash flow pattern by using different gradients over different time periods as illustrated in Ex. 2-2.

EXAMPLE 2-2

A particular water resources project produces benefits which amount to $12,000 in year 1 and increase on a uniform gradient to $120,000 in year 10. Thereafter, they increase on another uniform gradient of $5,000 per year to $200,000 in year 26, at which point they remain constant at $200,000 each year until the end of project life in year 50. What is the present worth of these benefits at a 4 percent discount rate?

The present worth of the given benefit series can be found by sub-

[1] *Ibid.*
[2] *Ibid.*

dividing it into four portions to which factors from the tables can be directly applied.

1. Present worth of 12, 24, . . . , 108, 120 in years 1 through 10

$$12{,}000\left(\frac{P}{G}, 4\%, 10\right) = 12{,}000 \times 41.99225 = \$503{,}900$$

2. Present worth of 120 per year in years 11 through 25

$$120{,}000\left(\frac{P}{A}, 4\%, 15\right)\left(\frac{P}{F}, 4\%, 10\right) = 120{,}000 \times 11.11839 \times 0.67556 = \$901{,}400$$

3. Present worth of 5, 10, . . . , 70, 75 in years 11 through 25

$$5{,}000\left(\frac{P}{G}, 4\%, 15\right)\left(\frac{P}{F}, 4\%, 10\right) = 5{,}000 \times 80.85389 \times 0.67556 = \$273{,}100$$

4. Present worth of 200 per year in years 26 through 50

$$200{,}000\left(\frac{A}{P}, 4\%, 25\right)\left(\frac{P}{F}, 4\%, 25\right) = 200{,}000 \times 15.62208 \times 0.37512 = \$1{,}172{,}000$$

The total present worth is the sum of these four values, or \$2,850,400.

If use of more complicated series is justified, an exact solution may be obtained by individual application of single-payment factors. For approximate results, a graphic solution may be used.[1]

Discounting Techniques

The procedure in which discounting factors may be systematically applied to compare alternatives (either different projects or different sizes of the same project) is called a *discounting technique*. The four conceptually correct discounting techniques are (1) the present-worth method, (2) the rate-of-return method, (3) the benefit-cost ratio method, and (4) the annual-cost method. Each method, if used correctly, leads to the same evaluation of the relative merit. However, each has advantages and disadvantages.

[1] George E. Ribble, Graphical Methods for Discounting Future Benefits in Feasibility Studies, *Civil Eng.*, vol. 35, no. 11 (November, 1965), pp. 86–87.

2-9 PRESENT-WORTH METHOD The present-worth method selects the project with the largest present worth PW of the discounted algebraic sum of benefits minus costs over its life.

$$PW = \sum_{t=1}^{n} \left(\frac{P}{F}, i\%, t\right) (B_t - C_t) \tag{2-26}$$

where C_t is the cost and B_t the benefit in the subscripted year, n is the period of analysis in years, and i is discount rate. When the annual net benefits $B = B_t - C_t$ are constant over the project life except for the initial first cost K, the formula may be simplified to

$$PW = -K + B\left(\frac{P}{A}, i\%, n\right) \tag{2-27}$$

When the net benefits vary according to some regular gradient, the appropriate gradient factor should be used.

Calculation of present worth from a cash flow diagram is a purely mechanical process. However, certain rules must be followed in comparing the calculated present worths to make correct choices.

RULE 1 *Figure all present worths to the same time base.* Whether or not alternatives are to be initiated at the same time, each present worth must be discounted to the same base year (1970, for example) because sums of money at different times are different economic goods.

RULE 2 *Figure all present worths by using the same discount rate.* Whether or not alternatives are to be financed from the same funds, each must be discounted at the same rate if the result is to be an index of intrinsic project merit.

RULE 3 *Base all present worths on the same period of analysis.* Whether or not alternatives have a common economic life, the comparison must be based on a service provided over a common period of time. This may be done either by evaluating the cost of extending the service past the termination of the shorter-lived alternatives or by calculating the value of the unused life of the longer-lived alternatives (Ex. 1-1).

RULE 4 *Calculate the present worth of each alternative. Choose all alternatives having a positive present worth. Reject the rest.* This ends the procedure if no sets of mutually exclusive alternatives are involved. The choice among alternatives in such a set is made by Rule 5.

RULE 5 *Choose the alternative in a set of mutually exclusive alternatives having the greatest present worth.*

RULE 6 *If the alternatives in the set of mutually exclusive alternatives have benefits which cannot be quantified but are approximately equal, choose the alternative having least cost.*

A single example based on the two mutually exclusive alternative water supply projects described in Table 2-1 will be used to illustrate all four discounting techniques. Project A provides an initial investment large enough to meet the demands for water for 40 years, and project B uses investment in two stages to meet the same demand. The present worths are calculated to be

$$\begin{aligned}
PW \text{ of } A &= -\$40{,}000{,}000 - \$160{,}000\left(\frac{P}{A}, 5\%, 40\right) \\
&\qquad\qquad + \$2{,}500{,}000\left(\frac{P}{A}, 5\%, 40\right) \\
&= -\$40{,}000{,}000 - \$160{,}000(17.159) + \$2{,}500{,}000(17.159) \\
&= \$153{,}000 \\
PW \text{ of } B &= -\$25{,}000{,}000 - \$30{,}000{,}000\left(\frac{P}{F}, 5\%, 20\right) \\
&\quad - \$100{,}000\left(\frac{P}{A}, 5\%, 20\right) - \$220{,}000\left(\frac{P}{A}, 5\%, 20\right)\left(\frac{P}{F}, 5\%, 20\right) \\
&\qquad\qquad + \$2{,}500{,}000\left(\frac{P}{A}, 5\%, 40\right) \\
&= -\$25{,}000{,}000 - \$30{,}000{,}000(0.377) - \$100{,}000(12.462) \\
&\qquad - \$220{,}000(12.462)(0.377) + \$2{,}500{,}000(17.159) \\
&= \$4{,}308{,}000
\end{aligned}$$

Therefore we should choose *project B* since its present worth is greater. If the rule of analyzing only differences were strictly applied, the equal annual benefits could be deleted from the evaluation of each alternative to provide the same conclusion with less work.

Project B would appear even more favorable were an adjustment made to account for the economic life of the second stage lasting 20 years past the period of analysis. The adjustment according to Eq. (1-1) would add a \$15

TABLE 2-1 Data for Sample Problem

	Project A	Project B
Construction cost	\$40,000,000	\$25,000,000, 1st stage \$30,000,000, 2d stage
Operations and maintenance	\$160,000 per year for 40 years	\$100,000 per year for 1st 20 years \$220,000 per year for 2d 20 years
Economic life	40 years	40 years for each stage
Period of analysis	40 years	40 years
Annual benefits	\$2,500,000	\$2,500,000
Discount rate	5 percent	5 percent

million salvage value in year 40 or \$15 million $(P/F,5\%,40)$ = \$2,130,000 to the present worth.

If the time value of money is neglected, A seems preferable to B because of its smaller total cost. However, at a 5 percent discount rate, B is definitely preferable. Sensitivity analysis shows the cost of B's second stage could increase to \$41 million, and B would still be preferable! This example dramatically illustrates the desirability of postponing costs until further investment is actually needed so as to free capital for alternative productive investment.

Capitalized worth is defined as the present worth of perpetual service. The present worth may be converted to a capitalized worth by assuming an equivalent reinvestment at the end of each economic life and multiplying by the ratio of the capital-recovery factor to the discount rate. The multiplier is close to 1 with long lives or high discount rates. Appropriate discount factors may be used to estimate capitalized worth where cash flows for reinvestment are expected to differ from those for initial investment. The decision rules used for present worth also apply for capitalized worth.

2-10 RATE-OF-RETURN METHOD The rate of return is the discount rate at which the present worth as defined by Eq. (2-26) equals zero as found by trial and error. Other decision rules apply when comparing alternatives by the rate-of-return method.

RULE 1 *Compare all alternatives over the same period of analysis.* Rates of return over different economic lives cannot be meaningfully compared because investment opportunity for the returns from the shorter-lived alternatives must be considered in determining whether capital should remain committed to the longer-lived alternative.

RULE 2 *Calculate the rate of return for each alternative. Choose all alternatives having a rate of return exceeding the minimum acceptable value. Reject the rest.* If sets of mutually exclusive alternatives are involved, proceed to Rule 3.

RULE 3 *Rank the alternatives in the set of mutually exclusive alternatives in order of increasing cost. Calculate the rate of return on the incremental cost and incremental benefits of the next alternative above the least costly alternative. Choose the more costly alternative if the incremental rate of return exceeds the minimum acceptable discount rate. Otherwise choose the less costly alternative. Continue the analysis by considering the alternatives in order of increased costliness, the alternative on the less costly side of each increment being the most costly project chosen thus far.*

The rate-of-return method will not lead to the same decisions as the present-worth method unless the incremental analysis of Rule 3 is used in place of selecting the mutually exclusive alternative with the highest rate of return. The rate-of-return method must be applied with caution because more than one rate of return exists when annual costs exceed annual benefits in years after annual benefits first exceed annual costs, but Heebink has shown that the rate-of-return method using Rule 3 still gives consistent answers even when dual solutions exist.[1] The water resources planner needs to be alert to this problem in comparing stage construction or nonstructural alternatives by the rate-of-return method.

In the example of two alternative water supply projects, each has been found to have a positive present worth when discounted at 5 percent and thus must have a rate of return exceeding the minimum acceptable value. Therefore, the difference between alternatives ($A - B$ in Table 2-2) is used to compute the incremental rate of return as directed by Rule 3.

The procedure is to assume discount rates until the present worth, or

$$PW = \$15{,}000{,}000 - \$30{,}000{,}000\left(\frac{P}{F}, i\%, 20\right) + \$60{,}000\left(\frac{P}{A}, i\%, 20\right) - \$60{,}000\left(\frac{P}{A}, i\%, 20\right)\left(\frac{P}{F}, i\%, 20\right)$$

equals zero. For $i = 5\%$, $PW = \$4{,}155{,}000$ indicates the trial discount rate to be too high and the extra cost of A over B to be not justified at a minimum acceptable rate of return of 5 percent. Therefore, project B is chosen. Had the present worth at 5 percent been negative, the incremental rate would have been greater than 5 percent. A complete solution provides

[1] David Heebink. "A Critique of Compound-Interest Models Used in Decision-making for Capital Budgets," Ph.D. thesis, Stanford University, Stanford, Calif., 1960, app. B, pp. 87–94.

TABLE 2-2 Incremental Data for Sample Problem

IN MILLIONS OF DOLLARS

	Project A	Project B	$A - B$
First cost	40.0	25.0, first stage	+15.0
		30.0, second stage	−30.0
Operations and maintenance	0.16 per yr	0.10 per yr, first 20 yr	+0.06 per yr, first 20 yr
		0.22 per yr, second 20 yr	−0.06 per yr, second 20 yr
Benefits	2.5 per yr	2.5 per yr	

an incremental rate of return of 3.39 percent and indicates project B to be favored only as long as the minimum acceptable rate of return exceeds 3.39 percent.

2-11 BENEFIT-COST RATIO METHOD The benefit-cost ratio PW_b/PW_c is the present worth of the benefits PW_b divided by the present worth of the costs PW_c. Annual values can alternatively be used without affecting the ratio. The present worth PW_b of annual benefits B_t is

$$PW_b = \sum_{t=1}^{n} \left(\frac{P}{F}, i\%, t\right) B_t \tag{2-28}$$

The present worth PW_c of the costs C_t is

$$PW_c = \sum_{t=1}^{n} \left(\frac{P}{F}, i\%, t\right) C_t \tag{2-29}$$

Series discounting factors may be used in either summation as appropriate.

The decision on whether particular cash flows should be considered costs or negative benefits is sometimes arbitrary (Sec. 8-4) and affects the benefit-cost ratio. While it does not affect project selection by the procedure described below, it is important to recognize that the best project has the greatest net benefits, not the largest benefit-cost ratio. Several authors have suggested that the benefit-cost ratio method leads to different decisions than the other techniques do.[1] However, this conflict only occurs when the incremental-cost principle of Rule 4 is neglected.

Four rules must be followed to apply the method correctly.

RULE 1 *Figure all benefit-cost ratios by using the same discount rate.*

RULE 2 *Compare all alternatives over the same period of analysis.*

RULE 3 *Calculate the benefit-cost ratio for each alternative. Choose all alternatives having a benefit-cost ratio exceeding unity. Reject the rest.* If sets of mutually exclusive alternatives are involved, proceed to Rule 4.

RULE 4 *Rank the alternatives in the set of mutually exclusive alternatives in order of increasing cost. Calculate the benefit-cost ratio by using the incremental cost and incremental benefit of the next alternative above the least costly alternatives. Choose the more costly alternative if the incremental benefit-cost ratio exceeds unity. Otherwise, choose the less costly alternative. Continue the analysis by considering the alternatives in order of increased costliness, the alternative on the less costly side of each increment being the most costly project chosen thus far.*

[1] Roland N. McKean, "Efficiency in Government through Systems Analysis" (New York: John Wiley and Sons, Inc., 1958), pp. 108–112; and Otto Eckstein, "Water Resource Development: The Economics of Project Evaluation" (Cambridge, Mass.: Harvard University Press, 1958), pp. 53–54.

From our previous calculations on our sample problem, we know that each project has a positive present worth at a 5 percent discount rate; therefore, each project has a benefit-cost ratio greater than 1 and Rule 3 is met. As with the rate-of-return method, differences between alternatives (Table 2-2) are taken to see if the incremental costs are justified. The incremental net cost found in Sec. 2-10 when coupled with the zero incremental benefit indicates a zero incremental benefit-cost ratio. Therefore, project *B* is chosen.

While project *B* has the higher overall benefit-cost ratio (1.11 instead of 1.00), the preferred project sometimes has a lower one. This may be illustrated by considering a project whose benefits equal 3 and whose costs equal 1 and which has an increment of investing an additional 4 to increase benefits to 10. The smaller project has a benefit-cost ratio of 3, while the larger one has a ratio of 2. Because the incremental ratio is 1.75, the larger investment should be chosen even though it has a smaller benefit-cost ratio.

2-12 ANNUAL-COST METHOD The annual-cost method converts all benefits and costs into equivalent uniform annual figures. Decision rules resemble those for the present-worth method because each annual cost is a present worth times a constant capital-recovery factor.

RULE 1 *Figure all annual costs by using the same discount rate.*
RULE 2 *Base all annual costs on the same period of analysis.*
RULE 3 *Calculate the net annual benefit of each alternative. Choose all alternatives having a positive net annual benefit. Reject the rest.* If sets of mutually exclusive alternatives are involved, proceed to Rule 4.
RULE 4 *Choose the alternative in a set of mutually exclusive alternatives, having the greatest net annual benefit.*

TABLE 2-3 Summation of Annual-cost Method

	Project *A*	Project *B*
Present worth of benefits	\$42,898,000	\$42,898,000
Present worth of costs	42,745,000	38,590,000
Net present worth	153,000	4,308,000
Capital-recovery factor $(A/P,5\%,40)$	0.05828	0.05828
Annual benefits	\$ 2,500,000	\$ 2,500,000
Annual cost	2,491,200	2,249,000
Net annual worth	8,800	251,000

RULE 5 *If the alternatives in the set of mutually exclusive alternatives have benefits which cannot be quantified but are approximately equal, choose the alternative having the least annual cost.*

Since the present worths for projects A and B are calculated in Sec. 2-9, they may be multiplied by the appropriate capital-recovery factor to get the equivalent annual figures shown in Table 2-3. Rule 4 says to choose project B as having the greater annual worth. Since the benefits are the same for each project, Rule 5 could be used to find the project accomplishing this benefit at least annual cost.

2-13 EVALUATION OF DISCOUNTING TECHNIQUES Each of the four discounting methods will when used correctly select the same project, given the same data. However, each technique has advantages and disadvantages associated with ease of calculation or presentation and understanding of the results. These need to be considered in selecting the method to apply in a given analysis.

Because it does not require an additional set of computations to apply the incremental-cost principle, the present-worth technique has been described as "simpler, safer, easier, and more direct."[1] Others have said this method is "logically prior to others, and we recommend its use."[2] The simple, direct expression of net present worth is conceptually straightforward and easily presented. However, one is working with larger numbers which may be harder to visualize and lead more frequently to numerical errors. Furthermore, the present-worth method cannot be used to rank projects in order of economic desirability unless all require equal investment.

The rate-of-return technique has been recommended because it does not require a preselected discount rate, rates of return are intuitively meaningful to many investors, and the resulting rates can be compared with those for many other types of investment.[3] On the other hand, it has been criticized (1) as giving ambiguous answers because of dual solutions, (2) because of the necessity of calculating incremental rate of return for interdependent projects, (3) the danger of people's accepting overall as contrasted with incremental rates of return as indicators of rank, and (4) the complexity of the required trial-and-error solutions.[4] Some have gone

[1] H. Bierman and S. Smidt, "The Capital Budgeting Decision" (New York: The MacMillan Company, 1960), p. 46.

[2] Jack Hirshleifer, James C. DeHaven, and Jerome W. Milliman, "Water Supply: Economics, Technology and Policy" (Chicago: The University of Chicago Press, 1960), p. 152.

[3] C. H. Oglesby and Eugene L. Grant, Economic Analysis—the Fundamental Approach to Decisions in Highway Planning and Design, *Highway Res. Board Proc.*, vol. 37 (1958), pp. 48–49.

[4] Bierman and Smidt, *loc. cit.*

so far as to suggest the technique *never* be used.[1] However, the cited advantages are important enough to make the rate-of-return method a valuable analytic tool.

The benefit-cost ratio method is almost universally used by federal and state water resource agencies and can be expected to remain in this position into the indefinite future. Moreover, Krutilla and Eckstein[2] base their analysis on benefit-cost methods, and Marglin's work shows it to be consistent with economic theory.[3] On the other hand, the use of the benefit-cost ratio without applying the required incremental benefit-cost analysis can lead to serious errors. Interdependent projects cannot be ranked according to their benefit-cost ratios because each enlargement must pass the *incremental* benefit-cost ratio test. Nevertheless, the fact remains that the benefit-cost ratio method can lead to the same results as other correct discounting techniques.

The annual-cost method uses constant multiples of the present-worth method and has the same advantages and disadvantages (except for the use of smaller numbers). However, the annual cost is sometimes preferred because more people are accustomed to thinking in terms of annual costs than of present worths.

Which method should be used? The answer depends primarily on the purpose of the analysis. Where benefits cannot be evaluated, it is not possible to use benefit-cost or rate-of-return techniques. Costs alone must be compared by using the present-worth or annual-cost method. There are more calculations for the rate-of-return or benefit-cost ratio methods and more opportunities for errors of interpretation, but computational work is never more than a minor part of the total analysis.

Other Approaches

2-14 UNRELIABLE TECHNIQUES Of the many other decision criteria in use, which do not give consistent, reliable results, the three most commonly found in the analysis of water resources projects are urgency ratings, standards, and least total costs.

The urgency-rating technique rates proposals on their postponability, those being least postponable getting priority. Since this method is highly subjective, the selection process tends toward a political content because

1 Hirshleifer et al., *op. cit.*, p. 156.

2 John V. Krutilla and Otto Eckstein, "Multiple Purpose River Development" (Baltimore: The Johns Hopkins Press, 1958), pp. 76–77.

3 Stephen A. Marglin, Objectives of Water-resource Development, in Arthur Maass et al., "Design of Water-resource Systems" (Cambridge, Mass.: Harvard University Press, 1962), pp. 17–87.

no firm figures are available to assess relative merit. If projects are truly nonpostponable, this will be reflected in the efficiency calculations.

Standards are expressions of minimum acceptable project quality often made prior to, and thus without the benefit of, economic analysis. Engineers are familiar with standards for structural design, water quality, street widths, design freeboard, etc. No matter what standard is used, it should be based on economic analysis unless intangible factors can be demonstrated to be overriding. Unfortunately, standards which reflect the ultimate goals of professional groups rather than the relative needs of the local community are sometimes taken as a valid representation of community needs to the neglect of other important services. No standard can be achieved without cost, and costs incurred should be commensurate with utility achieved. Standards are a poor substitute for a searching appraisal to obtain a balanced level of public services.

Least-cost methods are used when the benefits are estimated to be the same. Two common variations are (1) the least-total-cost method and (2) the least-total-annual-cost method. The least-total-cost method merely sums the estimated investment, operations, and maintenance costs over the life of the project and thus obviously ignores the timing of costs required by the discounting concept. The least-total-annual-cost method adds an interest cost to the total cost. Those using this method confuse financial analysis with economic analysis by including interest as a cost without determining time equivalence by discounting specific cash flows.

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PROBLEMS

2-1 A writer on the subject of the determination of the costs of public hydroelectric power projects included the following items as costs: (1) interest on the first cost of the project; (2) depreciation by the straight-line method based on the estimated life of the project; (3) an annual deposit in an amortization sinking fund sufficient to amount to the first cost of the project at the end of 50 years (or at the end of the life of the project if that should be less than 50 years); (4) where money is borrowed, the annual disbursements for bond interest and bond repayment; (5) all actual annual disbursements for operation and maintenance of the project.

Do you believe that annual cost should properly be considered as the sum of these items? Explain your answer.[1]

2-2 A project to be evaluated at a 4.25 percent discount rate cost $1 million and has a $20,000 annual cost. Project benefits are expected to be $20,000 in the first year, increase to $100,000 in the fiftieth year following an accelerated growth curve, remain constant at $100,000 annually until the ninetieth year, and then decline on a uniform gradient to nothing in the hundredth year. What is the benefit-cost ratio?

2-3 An industry which requires 10 percent return on its capital has an opportunity to invest in a business estimated to be profitable for 10 years. Alternative levels of investment and alternative net annual returns by level of investment are:

Investment	*Annual benefits*	*Annual O & M cost*
$1,000	$160	$10
1,500	265	15
2,000	340	20
2,500	445	25
3,000	535	30
3,500	610	35
4,000	665	40

a How large an investment should be made?

b How large would the minimum attractive rate of return of the industry have to be to prevent any of the above investments from being made?

c What minimum attractive rate of return would lead to a decision to invest $4,000?

2-4 A certain project has a first cost of $100,000 and an annual maintenance cost of $2,500 each year over a 50-year life. Benefits realized increase from $4,000 in the year immediately after construction to $10,000 in the last year of project life.

a At 4 percent interest, what is the annual project cost?

[1] Problem taken from Eugene L. Grant and Grant Ireson, "Principles of Engineering Economy," 4th ed., rev. ptg. (New York: The Ronald Press Company, 1964), p. 435.

b At 4 percent interest and with a straight-line gradient, what is the annual project benefit?
c What is the benefit-cost ratio?
d What is the annual benefit if benefits increase in an accelerated growth pattern?
e What is the annual benefit if benefits increase in a deferred growth pattern?
f What is the internal rate of return of the project using a straight-line gradient?

2-5 An investor has $20,000 and the four investment opportunities described below:

	Initial cost	*Net cash proceeds in year 1*	*2*	*3*
A	$10,000	$10,000	$1,000	$1,000
B	10,000	4,400	4,400	4,400
C	10,000	2,000	3,000	9,000
D	10,000	1,000	2,000	12,000

a In which two projects should a private investor invest his money if he uses the rate-of-return method?
b In which two projects should a public agency invest its money if it uses a social discount rate of 3 percent?
c What should the private investor do if he has no alternative investments this year, but starting next year (year 1), he can invest his money at a guaranteed return of 20 percent?

2-6 The three alternatives described below are available for supplying a community water supply for the next 50 years when all economic lives as well as the period of analysis terminates.

	Project A	*Project B*	*Project C*
Construction cost			
Year 0	$20,000,000	$10,000,000	$15,000,000
Year 20	0	10,000,000	12,000,000
Year 35	0	10,000,000	0
O and M cost			
Years 1–20	70,000	40,000	60,000
Years 21–35	80,000	70,000	80,000
Years 36–50	90,000	90,000	90,000

Using a 4.5 percent discount rate where applicable, compare the projects by:
a The present-worth method
b The rate-of-return method
c The benefit-cost ratio method
d The annual-cost method

2-7 A decision must be made between two alternative investments which perform equally well. Investment *A* has a life of 5 years, first cost of \$2,000, annual maintenance cost of \$25, and salvage value of \$250. Investment *B* has a life of 10 years, first cost of \$4,000, annual maintenance cost of \$30, and salvage value of \$1,000.

a Which alternative is to be preferred at a minimum acceptable rate of return of 8 percent?

b Investment *A* employs a scarce material which is expected to increase greatly in price during the next 5 years. How much would the cost of replacing *A* in 5 years have to be in current dollars to make *B* more economical in the present decision? Assume all other costs are unchanged.

2-8 Already \$20,000 has been spent on a \$200,000 project when it is learned that a research breakthrough may soon develop a substitute having a cost of \$135,000. The substitute has an annual operations and maintenance cost of \$4,000 instead of \$5,000 with construction in the originally planned manner. Annual benefits are projected to follow an accelerated growth curve from 0 to \$50,000 in year 50. The discount rate is 6 percent, and the period of analysis is 50 years.

a Compute the benefit-cost ratio for the project as initially conceived.

b Compute the benefit-cost ratio for implementing the substitute project if it could be done immediately.

c If the breakthrough is delayed 5 years and an interim measure is to be considered, what is the maximum uniform annual cost one could afford to pay to achieve the benefit during the intervening period rather than build the initial project? Neglect the salvage value of the substitute at the end of the period of analysis.

2-9 Two mutually exclusive investment alternatives which provide the identical service may be described as follows:

	First cost	*Annual cost*	*Salvage value*	*Life*
A	\$10,000	\$2,000	\$1,000	10
B	25,000	1,500	5,000	20

Based on a minimum attractive rate of return of 5 percent:

a Which alternative has the lower annual cost?

b What is the incremental annual cost of going from the less to the more expensive alternative?

c Select the optimum alternative by the present-worth method.

d What is the rate of return on the incremental investment of *B*?

e What first cost of replacing *A* after 10 years would make the two alternatives equivalent, assuming none of the other costs change?

2

MICROECONOMICS AND EFFICIENT RESOURCE ALLOCATION

An economy study assigns a value to each predicted physical consequence of each alternative and proceeds through a series of mathematical operations to condense these values into a scalar index of aggregate worth. A complex set of alternatives is reduced to a group of numbers which can be ranked in order of magnitude for deciding relative merit. The assignment of value is thus the critical step in the procedure. If done improperly, no meaningful conclusion can result from the calculations described in Chap. 2.

Part 2 seeks to provide the framework for evaluating these physical effects. Chapter 3 presents the competitive market under conditions of pure competition as providing the framework for establishing economic value. Chapter 4 shows how values once assigned can be analyzed to indicate an optimum or economically efficient design. Chapter 5 discusses how goals other than economic efficiency can be introduced to achieve desirable social objectives. Chapter 6 presents the implications of these objectives on discount rate selection. Microeconomics provides the tools for designing projects which in the aggregate will allocate to best use the total supply of available resources.

CHAPTER THREE

PRICE THEORY AND RESOURCE ALLOCATION

Introduction to Microeconomics

Price exerts a major influence on individual decisions of whether or not to use a particular economic good, and these many little decisions aggregate to allocate resources by use. In an economy based on the private ownership of property, or what may be called *capitalism* or the *free enterprise system*, economic forces interact to determine price. Under ideal conditions (pure competition), economic forces produce a first-order approximation of a normative system. Thus, analysis of these forces can be used to provide the values needed in engineering economy studies (Sec. 2-3). Price theory provides the framework for systematic study of these forces. It provides a foundation for production theory, the study of how a firm should operate to maximize profits. The result is an analogy indicating how a public works project should be designed to maximize benefits. The study of price theory guides the decision on whether a particular market price is a fair measure of true public worth for use in an economy study. It provides the tools needed for generating a shadow price for use where the market price is not fair or where none has been established. Price theory provides the analytic framework for establishing benefits and costs.

3-1 THE MARKET ECONOMY Because price theory analyzes the activities of individual participants in a market economy, it is a microeconomic approach. Study of the cumulative effect of all the many individual decisions on the national economy is a macroeconomic approach. The normative framework traditionally used for establishing value in economy studies is based on the principles of microeconomics under the assumption that water resources projects represent too small a portion of the total national productive capital for individual design decisions to have significant macroeconomic effects. The initial assumption is a macroeconomic setting of a stable economy and full resource employment.

A free enterprise economy reacts to economic decisions of individuals. Within limits, consumers are free to choose from a variety of goods and services, enterprisers are free to produce what they desire, and resource owners are free to sell to whatever buyer may be found. Voluntary exchanges occur in the marketplace whenever it is mutually advantageous to participants. Although profits are made as enterprisers correctly anticipate consumer demands and produce efficiently, losses occur if opposite conditions hold.

The market provides a link between consumers and producers and permits the exchange of goods and services. Some are geographically small; others are worldwide. A market may have few or many buyers and sellers. One product or many products may be offered. Government control may override economic forces in certain instances. In a market system, prices are the basic signals that direct production and distribution. To the degree that the goods exchanged are owned by many individuals free to buy and sell as they wish, prices are determined by impersonal market forces.

Cash flows within a market system can be classified by the use of Fig. 3-1. The owners of productive resources (landowners, laborers, and capitalists) sell them to enterprisers (firms) in the productive resources market. The money the resource owners receive is spent to buy the prod-

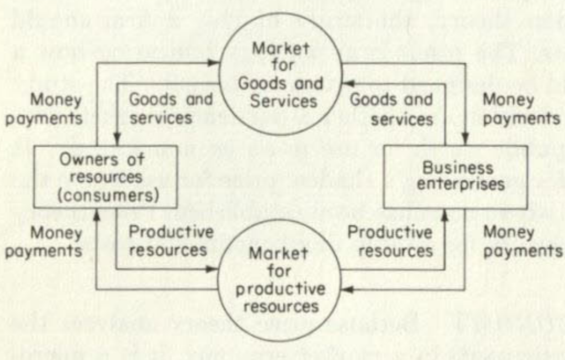

FIGURE 3-1 Model of a free enterprise economy (spending and production).

ucts of the enterprisers in the market for goods and services. In turn, the money that the enterprisers receive from consumers is used to buy additional productive resources. Thus, consumers and enterprisers operate in both the productive resources market and the goods and services market. This simplified model of a free enterprise economy neglects government action, interfirm transactions, and income from gifts or charity. It does not distinguish transactions occurring in the market or private sector of the economy from those occurring in the governmental or public sector.

A free enterprise system determines *what, how,* and *for whom* in the following manner:[1]

1 What is to be produced is determined by the dollar votes of consumers (the demand) cast each day for commodities purchased in the marketplace.
2 How things are to be produced is determined as individual firms are required to adopt the most efficient (least costly) methods of production to stay in business.
3 For whom things are produced is determined by the number of marketplace votes (income) an individual has. Incomes are determined as supply and demand in markets for productive services set wage rates, profits, land rents, and interest payments.

The absence of a costly regulatory structure is one of the greatest strengths of the market system. No central planning authority makes the myriad of economic decisions necessary to supply the goods and services needed for a city such as Chicago. Yet, instead of chaos, an order exists which supplies the variety of food, clothing, sundries, and entertainment that a cosmopolitan city demands. As Samuelson has stated:

> A competitive system is an elaborate mechanism for unconscious coordination through a system of prices and markets, a communication device for pooling the knowledge and actions of millions of diverse individuals. Without a central intelligence, it solves one of the most complex problems imaginable, involving thousands of unknown variables and relations. Nobody designed it, it just evolved, and like human nature, it is changing; but at least it meets the first test of any social organization—it is able to survive.[2]

How does the price system achieve an efficient allocation of productive resources? Allocations must be made at three levels: among industries, among firms in each industry, and within each firm. If an industry's products are in great demand, they can be sold for high prices. The

[1] Much of this paragraph is condensed from Paul A. Samuelson, "Economics: An Introductory Analysis," 5th ed. (New York: McGraw-Hill Book Company, 1962), pp. 41–42.
[2] *Ibid.*, pp. 38–39.

industry is able to pay high prices for productive resources and bid them away from industries whose products are less highly valued by consumers. Firms within an industry which produce a given output at a lower cost can pay more for productive resources and expand relative to inefficient firms. Lastly, the individual enterpriser strives to produce a given product in the least expensive manner from the cheapest combination of productive resources. According to Stigler:

> A competitive enterprise system allocates resources with maximum efficiency. If resources are used where they obtain the highest rates of renumeration, if they are employed efficiently in these industries, and if they produce the commodities that consumers most desire, output is as large as possible.[1]

Ebenstein says,

> The economy justification of competition is that it keeps everybody—worker, businessman, investor—on his toes, constantly alert to changes in the market, and constantly on the outlook for ways to increase his efficiency and thereby improve his chances in the market. By increasing his own efficiency, the individual worker or entrepreneur proportionately increases the efficiency and productivity of the whole market. Better products, lower prices, better services and ultimately higher living standards for all result from the constant incentive to keep up with, and if possible outdo, one's competitors.[2]

3-2 PURE COMPETITION A market economy will automatically maximize production from a given set of resources and thus be economically efficient under the conditions of pure competition. Competition as defined in economics does not necessarily denote rivalry. In fact, under pure competition, there is no rivalry among individual sellers or buyers. The conditions necessary for pure competition include:[3]

1 Consumers must be consistent and independent. A consistent consumer gets more satisfaction from a larger amount of a given commodity than from a smaller amount. The satisfaction gained by one consumer must be independent of purchases by others.

2 Producers must operate with the goal of profit maximization. The

[1] George J. Stigler, "The Theory of Price" (New York: The Macmillan Company, 1961), p. 9.

[2] William Ebenstein, "Today's Isms," 4th ed. (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1964), p. 166.

[3] Otto Eckstein, "Water Resource Development: The Economics of Project Evalutaion" (Cambridge, Mass.: Harvard University Press, 1958), pp. 25–30.

production processes of the firms must be independent so that one firm's costs are not borne by others.

3 The transactions by each buyer or seller must be too small in relation to the market to affect prices paid or received.

4 No price regulation or rationing or other artificial constraints by government, labor, business, or other institutions are placed on the demand and supply of goods and resources or on their prices.

5 Goods and services and resources must be mobile. This requires free entry by firms into any industry and goods and labor free to move from one local market to another to seek the best price.

6 Buyers and sellers must be aware of prices throughout the economy. When buyers and sellers receive such information instantaneously, we have what is known as *perfect competition*. The closest approximation to this condition is on the New York Stock Exchange where information on stock prices is transmitted continuously to all parts of the nation.

7 Commodities must be sufficiently divisible so that sellers can withhold all or part of the product from individual buyers who do not pay the market price.

8 The existing income distribution must be considered equitable for the dollar votes of the individual participants to be weighted equally.

9 All resources must be fully employed. When unemployment persists, prices do not reflect opportunity costs or returns from the viewpoint of the nation.

Even though pure competition does not exist in real markets, the model provides an ideal for judging the efficiency of actual markets and guidelines to help develop criteria for establishing value when its conditions are not met.

3-3 MARKET DEMAND Experience tells us that people will buy less at higher prices provided income, tastes, and prices of substitutes remain constant. Obversely, people buy more at lower prices. The demand for a good is the quantity per unit time that people within a defined area will buy as a function of all possible prices, all other factors remaining constant.

One way to indicate demand is by a demand schedule, a list of the different quantities of a good that people will take within a particular time period at various prices. A hypothetical demand schedule for Idaho potatoes is shown by the first two columns of Table 3-1. A demand curve is the plot of the demand schedule. The vertical axis indicates the price per unit, and the horizontal axis indicates the quantity of the good pur-

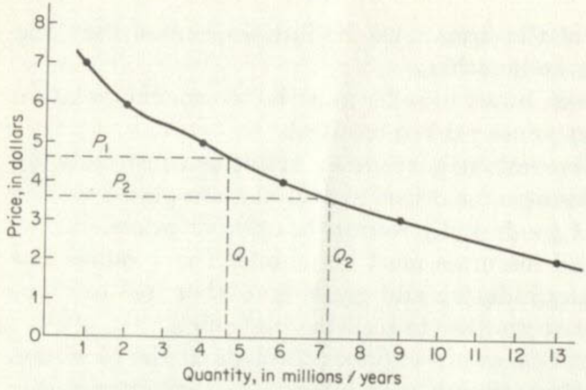

FIGURE 3-2 Hypothetical demand curve for Idaho potatoes, given for 100-lb sacks.

chased per unit time. Figure 3-2 is a hypothetical demand curve for Idaho potatoes.

The demand curve slopes downward to the right because lower prices increase sales, a principle designated as the *law of downward-sloping demand*. Two reasons for this increase are that (1) lowered prices attract new buyers and (2) lowered prices induce extra purchases by former users. Lower water prices would cause some to abandon more expensive alternate sources of supply and become new buyers. Old buyers would use more. When water is very expensive, a person only buys enough to drink. As the price lowers progressively, he buys some for personal cleanliness, then for household cleaning, and finally for yard watering.

It is important to distinguish between movement along a given

TABLE 3-1 Supply and Demand Schedules for Idaho Potatoes, given for 100-lb sacks

Price, in dollars	Quantity demanded, in millions per year	Quantity supplied, in millions per year	Price tendency	Market conditions
7	1	10	Fall	Surplus
6	2	9	Fall	Surplus
5	4	8	Fall	Surplus
4	6	6	Neutral	Equilibrium
3	9	4	Rise	Shortage
2	13	1	Rise	Shortage

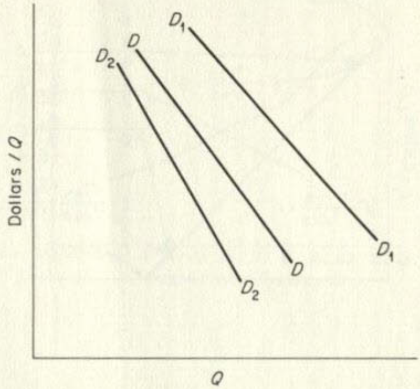

FIGURE 3-3 **Shifts in demand curves (caused by changes in demand).**

demand curve and a shift of the demand curve caused by a change in demand. Movement along a demand curve occurs with a change in the price of the good. Shifting of the demand curve is caused by changes in (1) consumer preferences, (2) the number of consumers, (3) consumer incomes, (4) the prices of related goods, and (5) the range of goods available.[1] When price changes from P_1 to P_2, we should have movement along the demand curve as the quantity of potatoes purchased changed from Q_1 to Q_2 (Fig. 3-2). If there were an increase in consumer preferences for Idaho potatoes, more consumers, greater consumer incomes, higher prices for Maine potatoes, or fewer alternative foods available for purchase, the demand curve would shift from DD to D_1D_1 (Fig. 3-3). Opposite changes would shift the demand curve to D_2D_2.

3-4 PRICE ELASTICITY One of the most important relationships expressed by a demand curve is the change in sales resulting from a given change in price. This change could be measured by the slope of the demand curve, but the general usefulness of the answer is limited by its units (sacks per dollar in Fig. 3-2). A different slope on a curve plotted in different units (bushels per cent) would indicate the identical relationship between price and demand. Economists avoid this difficulty in units by use of the price elasticity of demand defined as

$$E = \frac{-\Delta Q/Q}{\Delta P/P} \quad \text{or} \quad -\frac{\Delta Q}{\Delta P}\frac{P}{Q} \tag{3-1}$$

The negative sign is introduced because Q increases as P decreases.

[1] Richard H. Leftwich, "The Price System and Resource Allocation" (New York: Holt, Rinehart and Winston, Inc., 1964), p. 27.

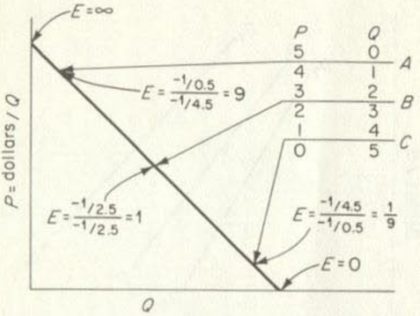

FIGURE 3-4 Price elasticity of demand.

Computation of the price elasticity of demand is illustrated by Fig. 3-4. Applying Eq. (3-1) to points A, B, and C, respectively, gives elasticities of 9, 1, and $\frac{1}{9}$. Even though the demand curve at all three points has the same slope, the price elasticity of demand varies from infinity along the vertical axis to zero along the horizontal axis.

A value of infinity for E indicates a perfectly elastic good which no one at all will buy if the price is raised. It is represented by a horizontal demand curve. Goods become perfectly elastic at the price that they are completely priced out of the market.

As the price is reduced, elasticity drops. Eventually it reaches unity, and the good is no longer said to be elastic. This point would provide the supplier the largest gross revenue; the PQ product is a maximum. Until the elasticity reaches unity, additional sales more than offset lower prices and revenue increases. If the price is reduced past the point of unit elasticity, sales no longer increase fast enough to offset the lowering price and revenue declines. The good is said to be inelastic. A value of zero for E indicates a perfectly inelastic good or one for which price has no effect on demand. It is represented by a vertical demand curve. Goods become perfectly inelastic as the price becomes too low to remain a factor determining the amount purchased. The same good is inelastic at low prices and elastic at high prices.

3-5 MARKET SUPPLY On the sellers' side of the market, the supply schedule and supply curve indicate the amounts that producers are willing to sell at various prices, other things being equal. The first and third column of Table 3-1 show the supply schedule for Idaho potatoes, and Fig. 3-5 is the corresponding supply curve.

The supply curve slopes upward to the right since old sellers will produce more goods for sale and new sellers will enter the market as the

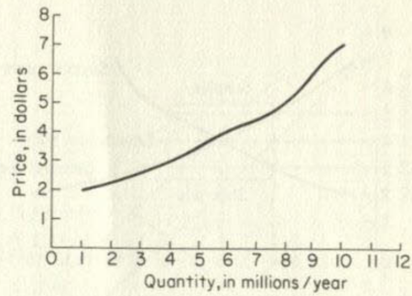

FIGURE 3-5 Hypothetical supply curve for Idaho potatoes, given for 100-lb sacks.

price increases. The supply curve shifts (in contrast with movement along the curve) with a change in the supply produced when the price remains unchanged. Shifts to the right may be caused by technological advance, favorable production conditions, or lower prices for the input factors of production. Shifts to the left are caused by opposite conditions.

3-6 MARKET PRICE DETERMINATION The demand curve and the supply curve combine to establish the equilibrium market price. The combined demand and supply schedules in Table 3-1 illustrate the tendency toward an equilibrium price.

If the initial price is above $4 per 100 lb, more will be supplied than the quantity demanded. Potatoes will be in oversupply, and sellers will cut prices in order to sell their crops. Also, if the initial price is below $4 per 100 lb, less will be supplied than the quantity demanded and consumers will bid the price up. Only at the equilibrium price of $4 per 100 lb will the demand equal the supply. Figure 3-6 shows the same result graphically. This equilibrium price is the minimum under conditions of pure competition that each individual buyer must pay for each 100 lb of potatoes purchased and the maximum that each farmer can receive for each 100 lb of potatoes sold.

3-7 RESULTS OF SHIFTS IN DEMAND AND SUPPLY If the demand for potatoes increases while the supply remains fixed, Fig. 3-7 shows how a shift in demand from DD to D_1D_1 causes an increase in price from P to P_1. This happens because at P there is now a shortage of potatoes, the price will be bid up, and sellers will be induced to place more on the market. Just the opposite happens with a decrease in demand: a surplus creates pressure to lower prices.

When the supply curve shifts and the demand curve is fixed, equilib-

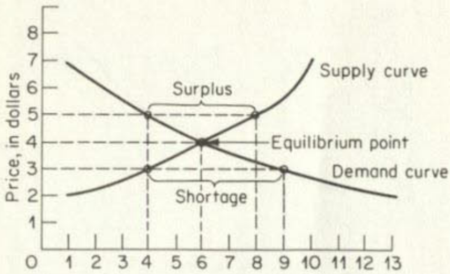

FIGURE 3-6 Hypothetical market demand and supply curves for Idaho potatoes, given for 100-lb sacks.

rium prices and quantities are also affected. Suppose higher labor costs were to increase the cost of growing potatoes. In Fig. 3-8, this would cause a shift from SS to S_1S_1. At the original price P, there would be a shortage and the price would be bid up to P_1. The opposite will happen if lower production costs should shift the supply curve to the right. There will be a surplus and a downward pressure on prices to a new equilibrium point.

3-8 SIGNIFICANCE OF MARKET EQUILIBRIUM Supply and demand curves provide additional background for understanding the automatic way the market system handles the allocation of goods or answers the basic economic questions of what, how, and for whom. *For whom* is partially determined by individual willingness to pay. If you have the money and wish to eat Idaho potatoes three times a day, you merely pay the market price. On the other hand, you may not like Idaho potatoes

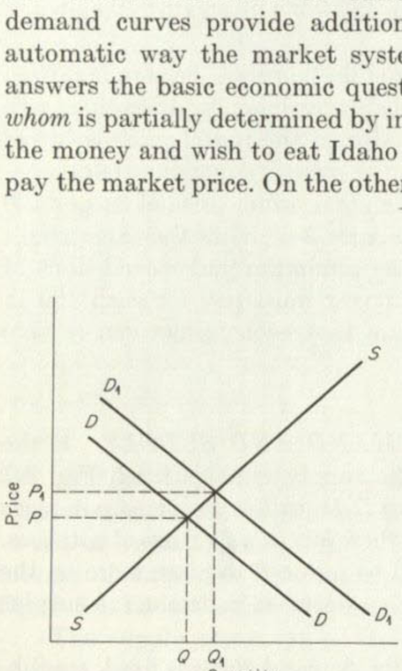

FIGURE 3-7 Shift in demand curve.

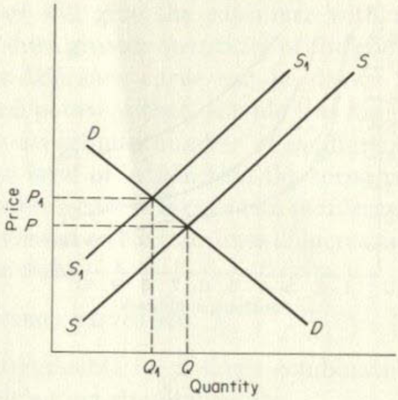

FIGURE 3-8 Shift in supply curve.

and spend your entire income on other goods. *What* is partially determined by the price for potatoes' determining the degree to which farmers shift productive resources from the production of other crops. *How* is also partially answered because the price for potatoes determines the money farmers have to invest in potato processing equipment, sprinkler systems, and fertilizer. A given pair of supply and demand curves only partially answers these questions because events in other food markets and resource markets also affect economic equilibrium. Therefore, the *partial equilibrium* solution for Idaho potatoes plays only a small part in determining the total price structure.

Consumer Demand

The above bird's-eye view of the interaction between supply and demand to achieve equilibrium prices provides the background for a more thorough discussion of the economic principles governing demand. Two approaches have been used to derive a theory of consumer demand. For years, classical economics has used the utility approach based on values assigned in absolute units. More recently, the indifference-curve approach has been developed because it avoids the problem of evaluation in absolute units by using relative values. For brevity, we shall only explain the more recent indifference-curve approach.[1]

[1] Leftwich, "The Price System and Resource Allocation," gives both approaches in chaps. 4 and 5.

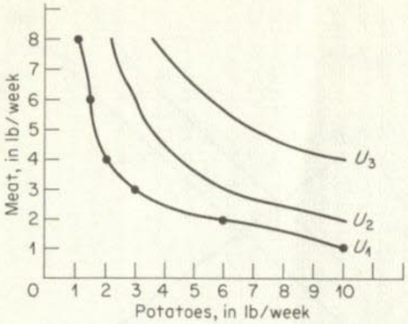

FIGURE 3-9 Indifference curves.

3-9 INDIFFERENCE CURVES An indifference curve (sometimes called the *equal-utility contour*) shows the consumption combinations which give a consumer equal satisfaction. It is called an indifference curve because the consumer is equally satisfied with any of the combinations depicted by the curve. An indifference curve is theoretically obtained by asking a consumer which combinations of goods yield equal satisfaction, and its development assumes he can order his preferences. The indifference curves described below will be two dimensional so they can be graphically presented. However, real consumers must choose in hundreds of dimensions, one for each good consumed. The true indifference curve is a multidimensional indifference surface.

If a consumer buys only two goods with his income, meat and potatoes, the combinations in Table 3-2 might give equal satisfaction. More meat compensates for giving up potatoes and vice versa. The plot of these combinations gives a single indifference curve (Fig. 3-9). Greater

TABLE 3-2 Combinations of Meat and Potatoes Giving Equal Satisfaction

Meat, lb wk^{-1}	Potatoes, lb wk^{-1}
1	10
2	6
3	3
4	2
6	1.5
8	1

quantities of both meat and potatoes will give the consumer with an inadequate diet a greater total value since greater quantities of the goods bring more satisfaction. A second indifference curve can be drawn to indicate the combinations of meat and potatoes that provide this higher level of satisfaction. Thus, there are an infinite number of indifference curves, each one indicating a separate level of satisfaction the consumer may experience. This system of indifference curves is called an *indifference map*. The indifference curves may be viewed as contour lines of increasing elevation as one moves upward to the right.

Some of the properties of indifference curves are:

1 They cannot intersect since it is impossible for a single combination of goods to yield two levels of satisfaction simultaneously.
2 They slope downward to the right. As the amount of one good is increased, the other must decrease if equal satisfaction is to be maintained. The slope of the indifference curve is called the *marginal rate of substitution*.
3 They tend to approach the axes asymptotically because as less and less of a good is consumed, the sacrifice of parting with an additional unit becomes greater. Many more units of the other good must be substituted to bring equal satisfaction.

3-10 MAXIMIZATION OF SATISFACTION A consumer maximizes his satisfaction by picking the highest indifference curve available to him as determined by his income and the prices of the two goods. These two consumer's opportunity factors determine what may be called a *line of attainable combinations*. It intercepts each axis at a value equal to the income divided by the price. For example, one intercept would be the amount of potatoes which could be purchased were the entire income spent for that purpose. The other intercept indicates the amount of meat which could be purchased by the entire income. Points on a straight line between these intercepts indicate the combinations of meat and potatoes open to the consumer. The highest attainable indifference curve is the one tangent to the line of attainable combinations (Fig. 3-10).

The total income I spent on two goods y_a and y_b with respective prices per unit of P_{ya} and P_{yb} equals

$$I = P_{ya}y_a + P_{yb}y_b \tag{3-2}$$

The number of units of y_a which can be purchased thus equals

$$X = \frac{-P_{yb}}{P_{ya}}\,y_b + \frac{I}{P_{ya}} \tag{3-3}$$

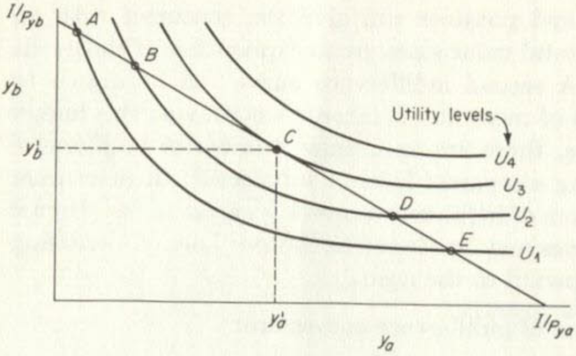

FIGURE 3-10 Income allocation.

This is the equation of a straight line (the line of attainable combinations) with a slope of $-P_{yb}/P_{ya}$.

The consumer is able to attain points A, B, C, D, and E in Fig. 3-10. If he chooses A, he is on U_1, which gives less satisfaction than U_2 or U_3. Point C gives maximum satisfaction. All the other points are on lower indifference curves. The income constraint prevents achievement of U_4. At point C, the slopes of the indifference curves and the line of attainable combinations are equal. The marginal rate of substitution (the slope of the indifference curve) equals the ratio of the prices (the slope of the line of attainable combinations),

$$MRS_{y_a y_b} = \frac{P_{yb}}{P_{ya}} \tag{3-4}$$

where $MRS_{y_a y_b}$ is read as the marginal rate of substitution of y_a for y_b.

3-11 CONSUMER-DEMAND CURVES Consumer-demand curves are derived by varying the price of one good, P_{ya}, while keeping constant the income, consumer preferences (position and shape of indifference curves), and the price of the other good, P_{yb}, or in the general case, of all other goods. By changing the price of y_a, the slope of the line of attainable combinations changes to become tangent to a different indifference curve. Suppose the initial line of attainable combinations (AB in Fig. 3-11) is tangent to U_1, which means that for an income of I, an amount of y_a' will be purchased at P_{y_a}', and an amount y_b at P_{y_b}'. If P_{y_a} increases to P_{y_a}'', the new line of attainable combinations AC will have a steeper slope than AB because fewer units of y_a can be purchased when all of I is spent

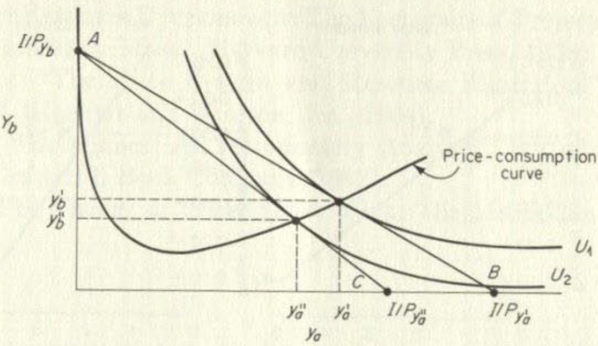

FIGURE 3-11 Derivation of price-consumption curve.

on y_a. The line of attainable combinations pivots about A because I/P_{yb} is constant. The price increase lowers the level of satisfaction the consumer on a fixed income can attain. He moves to a lower indifference curve and now would allocate I so as to purchase y_a'' units of y_a and y_b'' units of y_b. If P_{ya} is assigned a series of values, the line connecting the points of tangency is called the price-consumption curve. The demand curve of a specific individual for y_a can be found by plotting corresponding values of y_a and P_{ya} taken from the price-consumption curve (Fig. 3-12).

3-12 AGGREGATE-DEMAND CURVES Individual demand curves for Idaho potatoes may be used to develop a combined demand curve (Fig. 3-13). At each price, the demand by each individual may be added

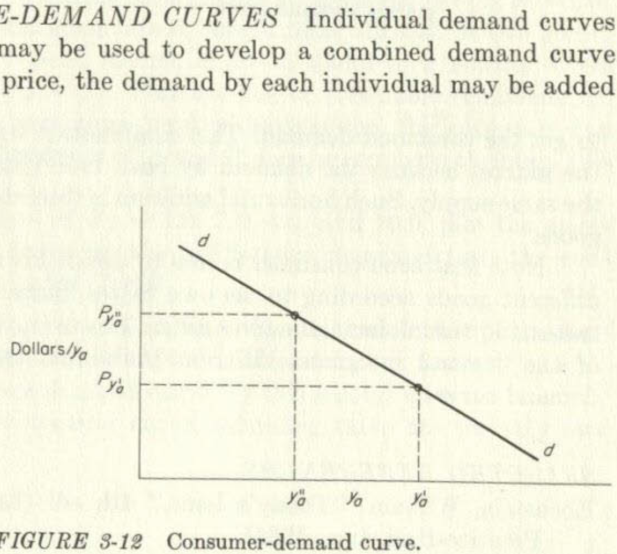

FIGURE 3-12 Consumer-demand curve.

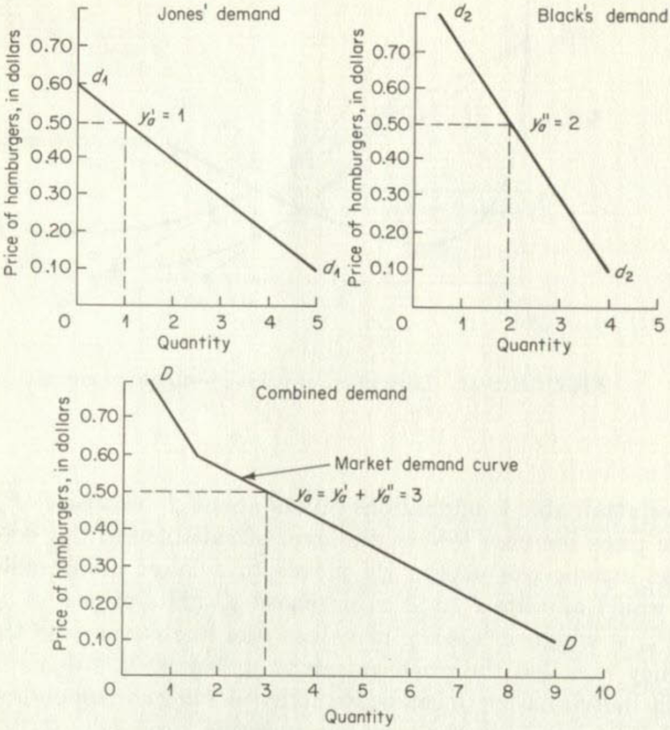

FIGURE 3-13 Summation of consumer demands to obtain market demand curve.

to get the combined demand. This combination occurs automatically in the market because the demand by both individuals must be met from the same supply. Such horizontal addition is thus characteristic of market goods.

Note that each consumer is free to decide his relative preference for different goods according to his own tastes. Since people have different tastes, individual demand curves differ. However, the horizontal addition of the demand integrates different individual tastes into the market demand curve.

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PROBLEMS

3-1 The supply and demand schedules for fidwots is:

Price, $:	1	2	3	4	5	6	7
Demand:	41	30	21	14	9	6	5
Supply:	12	14	18	24	32	42	55

a Plot the supply and demand curves.

b What is the equilibrium market price for fidwots?

c What is the price elasticity of demand at a price of $2? Of $6?

d At what price is there unitary elasticity?

e What equilibrium price would result from a doubling of demand?

3-2 Lines of consumer indifference between commodities A and B are represented by the indifference map represented by the equation $0.1A^2B = V$, where V is a scalar measure of satisfaction.

a Construct an indifference map covering the region $A < 10, B < 50$.

b What level of satisfaction is gained at the point $A = 5$, $B = 20$? What is the marginal rate of substitution, dA/dB, at this point?

c A consumer has an income of 20 to spend in a market where $P_A = 2$ and $P_B = 0.5$. Plot the line of attainable combinations. What is the maximum level of satisfaction the consumer can reach? What amounts of A and B does he purchase to obtain this satisfaction?

d Based on values of $P_B = 1.0$, 2.0, 4.0, and 10.0, plot the price-consumption curve on the indifference map and then the consumer-demand curve for B.

3-3 A second consumer is in a situation identical with that of the consumer in Prob. 3-2*c* except his available income is 5.

a Plot a consumer-demand curve for this second man.

b Plot a market-demand curve, assuming these are the only two consumers.