

**CE 5364 Groundwater Transport Phenomena**  
**Exercise Set 1**

**Exercises**

1. A sand column has the following characteristics<sup>1</sup>:

$$K = 10^{-4} \frac{cm}{s}; \quad A = 75 cm^2; \quad \frac{dh}{dl} = 0.01; \quad n = 0.20 \quad (1)$$

Determine:

- (a) Sketch the system.
- (b) The specific discharge.
- (c) The pore velocity.
- (d) The volumetric flow rate through the column.

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<sup>1</sup>Problem 2-3, pg. 578 in Bedient, et. al.

Sketch the system.

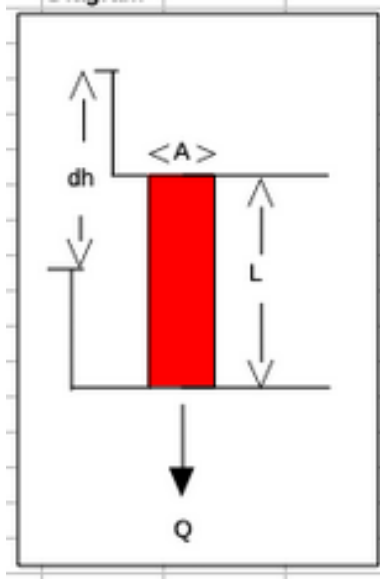


Figure 1: Sketch of Sand Column

Specific discharge

```
[61]: K = 1.0e-04  
      A = 75  
      dhdl = 0.01  
      n = 0.20  
  
[62]: q = K * dhdl  
      print("specific discharge = ", round(q, 6), " cm/sec")  
  
specific discharge = 1e-06 cm/sec
```

Figure 2: Specific discharge calculations

### Pore velocity

```
[63]: u=q/n  
      print("pore velocity = ",round(u,6)," cm/sec")  
      pore velocity = 5e-06 cm/sec
```

Figure 3: Specific discharge calculations

### Volumetric flow rate through the column

```
[60]: Q=q*A  
      print("Volumetric discharge = ",round(Q,6)," ml/sec")  
      Volumetric discharge = 7.5e-05 ml/sec
```

Figure 4: Volumetric discharge calculations

2. Three geologic formations overlies one another with the characteristics listed below.<sup>2</sup>

$$\begin{aligned}
 b_1 &= 50 \text{ ft} & K_1 &= 0.0002 \frac{\text{ft}}{\text{s}} \\
 b_2 &= 20 \text{ ft} & K_2 &= 0.000005 \frac{\text{ft}}{\text{s}} \\
 b_3 &= 210 \text{ ft} & K_3 &= 0.001 \frac{\text{ft}}{\text{s}}
 \end{aligned} \tag{2}$$

A constant velocity vertical flow field exists across the three formations. The hydraulic head at the top of the formations (top of formation 1) is 33 feet. The hydraulic head at the bottom of the formations (bottom of formation 3) is 21 feet.

Determine:

- (a) Sketch the system.
- (b) The hydraulic head at the internal boundary between formation 1 and 2.
- (c) The hydraulic head at the internal boundary between formation 2 and 3.
- (d) Approximate time for a tracer to flow (vertically) through the three layers if the porosities  $n_1$ ,  $n_2$ , and  $n_3$  are 0.30, 0.42, and 0.35, respectively

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<sup>2</sup>Problem 2-12, pg. 579 in Bedient, et. al.

Sketch the system.

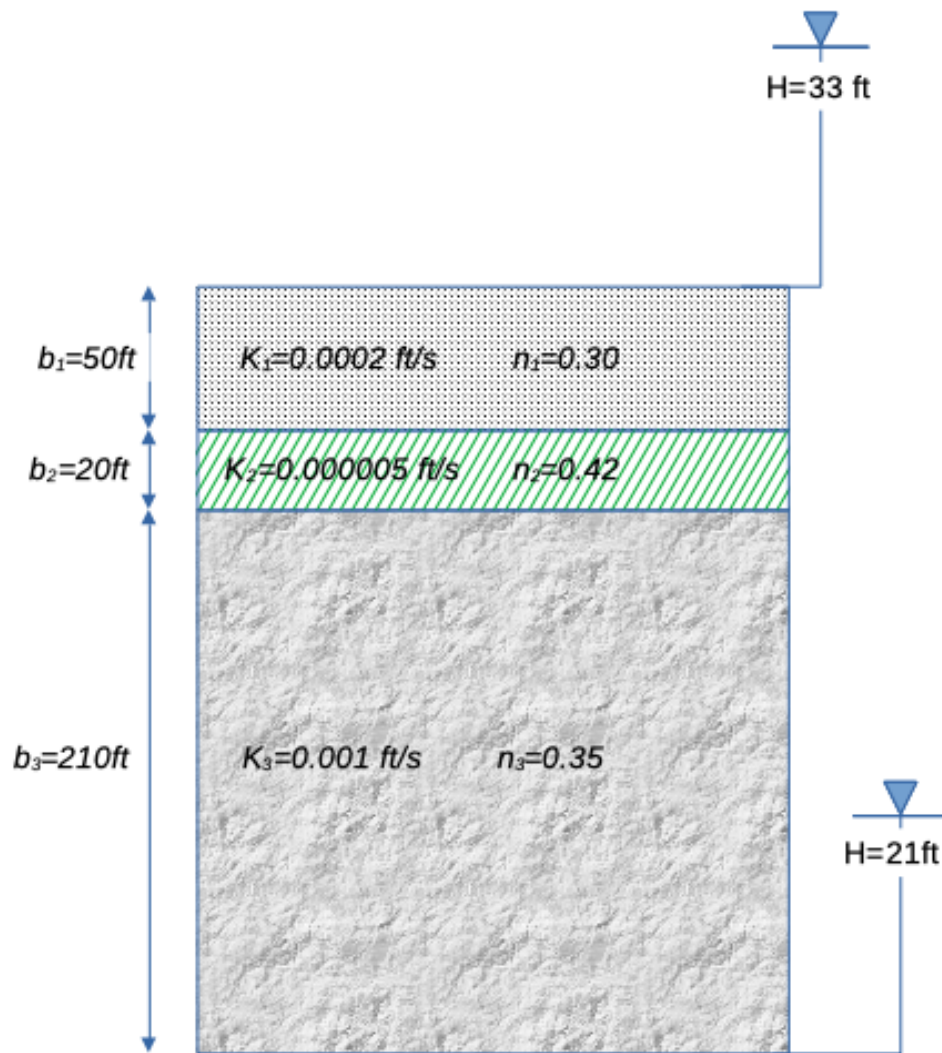


Figure 5: Sketch of Layered System

The hydraulic head at the internal boundary between formation 1 and 2.

```
[50]: # Find total discharge per unit area through entire formation
      htop=33
      hbot=21
      k1=0.0002
      k2=0.000005
      k3=0.001
      l1=50
      l2=20
      l3=210
      n1=0.30
      n2=0.42
      n3=0.35
      Kbar=(l1+l2+l3)/((l1/k1) + (l2/k2) + (l3/k3))
      print("vertical composite Kv: ",round(Kbar,6)," ft/sec")
      dH=htop - hbot
      dl = l1+l2+l3
      qt=Kbar*dH/dl
      print("vertical hydraulic gradient :",round(dH/dl,6)," ft/ft")
      print("vertical specific discharge q: ",round(qt,6)," ft/sec")

      vertical composite Kv:  6.3e-05  ft/sec
      vertical hydraulic gradient : 0.042857  ft/ft
      vertical specific discharge q:  3e-06  ft/sec

[51]: # find head loss in layer 1
      dh1 = l1*qt/k1
      h12 = htop -dh1
      print("head at layer 1-2 interface :",round(h12,2)," ft ")

      head at layer 1-2 interface : 32.33  ft
```

Figure 6: Head at layer 1-2 interface

**The hydraulic head at the internal boundary between formation 2 and 3.**

```
head at layer 1-2 interface : 32.33  ft

[52]: # find head loss in layer 2
      dh2 = l2*qt/k2
      h23 = htop - dh1 -dh2
      print("head at layer 2-3 interface :",round(h23,2)," ft ")

head at layer 2-3 interface : 21.57  ft
```

Figure 7: Head at layer 2-3 interface

Approximate time for a tracer to flow (vertically) through the three layers if the porosities  $n_1$ ,  $n_2$ , and  $n_3$  are 0.30, 0.42, and 0.35, respectively

head at layer 2-3 interface : 21.57 ft

```
[53]: # time in layer 1
time1 = l1*n1/qt
print("tracer time in layer 1 :",round(time1,6)," sec")

tracer time in layer 1 : 5575000.0 sec

[54]: # time in layer 2
time2 = l2*n2/qt
print("tracer time in layer 2 :",round(time2,6)," sec")

tracer time in layer 2 : 3122000.0 sec

[55]: # time in layer 3
time3 = l3*n3/qt
print("tracer time in layer 3 :",round(time3,6)," sec")

tracer time in layer 3 : 27317500.0 sec

[56]: totaltime=time1+time2+time3
totaltime=totaltime/86400
print("Tracer travel time :",round(totaltime,3)," days ")

Tracer travel time : 416.834 days
```

Figure 8: Travel times in layered system