ANALYTICAL SOLUTIONS FOR ONE-, TWO-, AND THREE-DIMENSIONAL SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

U.S. GEOLOGICAL SURVEY

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CONVERSION FACTORS AND ABBREVIATIONS

For the convenience of readers who may prefer to use metric (International System) units, rather than the inch-pound units used in this report, values may be converted by using the following factors:

Multiply inch-pound unit	By	To obtain metric unit
inch (in.)	25.4	millimeter (mm)
inch per hour (in/h)	25.4	millimeter per hour (mm/h)
foot (ft)	0.3048	meter (m)
foot per day (ft/d)	0.3048	meter per day (m/d)
gallon (gal)	0.003785	cubic meter (m^3)
square inch per hour (in^2/h)	6.4516	square centimeter per hour (cm^2/h)
foot squared per day (ft^2/d)	0.09294	meter squared per day (m^2/d)
cubic foot (ft^3)	0.02832	cubic meter (m^3)
cubic foot per day (ft^3/d)	3,785	cubic meter per day (m^3/d)
pound per cubic foot (lb/ft^3)	0.01602	gram per cubic centimeter (g/cm^3)

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By Eliezer J. Wexler

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PREFACE

This report presents nine analytical solutions to the advective-dispersive solute-transport equation and describes computer programs written to evaluate these solutions. Users of these solutions and computer programs are cordially requested to notify me of any errors found and of any comments or suggestions regarding the report. This will allow for updates and corrections to be made prior to publication as a Techniques of Water-Resources Investigations Report. Copies of the computer programs are available at the U.S. Geological Survey, 9100 N.W. 36th Street, Suite 107, Miami, Florida 33178.

E.J.W.

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DEFINITION OF TERMS

Symbol	Dimensions	Definition
a_l	L	Longitudinal dispersivity
a_t	L	Transverse dispersivity
C	M/L^3	Volumetric concentration
C_o	M/L^3	Specified concentration value along a boundary
CEC	1/M	Cation exchange capacity
D	L^2/T	Coefficient of hydrodynamic dispersion for one-dimensional solute transport
$\bar{\bar{D}}$	L^2/T	Hydrodynamic dispersion tensor in an isotropic system
$\bar{\bar{D}}_d$	L^2/T	Molecular diffusion tensor
$\bar{\bar{D}}_m$	L^2/T	Mechanical dispersion tensor
D^*	L^2/T	Dispersion coefficient divided by the retardation factor, R
D_x, D_y, D_z	L^2/T	Magnitudes of the hydrodynamic dispersion tensor in a system with uniform flow
h	L	Head
$\vec{J}$	M/L^2T	Solute flux due to dispersion or diffusion
k	L^3/M	Slope of the linear equilibrium adsorption isotherm
$\bar{\bar{K}}$	L/T	Hydraulic conductivity tensor
k_d	L^3/M	Distribution coefficient
K_s	--	Selectivity coefficient for ion exchange reactions
n	--	Effective porosity
P	--	Column Peclet number
p	M/LT^2	Fluid pressure
Q_s	M/L^3T	General solute source term
R	--	Retardation factor for adsorbed solute
S	--	Mass concentration of adsorbed solute
T	--	Number of displaced pore volumes
$T_{1/2}$	T	Half-life of radioactive solute
V	L/T	Magnitude of the average interstitial fluid velocity
$\vec{V}$	L/T	Average interstitial fluid velocity
V^*	L/T	Average interstitial fluid velocity divided by the retardation factor, R
V_x, V_y, V_t	L/T	Magnitudes of the average interstitial velocity components
z	L	Elevation head
∞	--	Infinity
Σ	--	Summation
γ	M/L^2T^2	Specific weight of water
$\vec{\nabla}$	--	Gradient operator
∂	--	Partial derivative
θ	--	Soil moisture content (equal to porosity for saturated soils)
λ	1/T	First-order chemical transformation rate
ρ	M/L^3	Density of water
ρ_b	M/L^3	Bulk density of aquifer material

ANALYTICAL SOLUTIONS FOR ONE-, TWO-, AND THREE-DIMENSIONAL SOLUTE TRANSPORT IN GROUND-WATER SYSTEMS WITH UNIFORM FLOW

By Eliezer J. Wexler

ABSTRACT

Analytical solutions to the advective-dispersive solute-transport equation are useful in predicting the fate of solutes in ground water. Analytical solutions compiled from available literature or derived by the author are presented in this report for a variety of boundary condition types and solute-source configurations in one-, two-, and three-dimensional systems with uniform ground-water flow. A set of user-oriented computer programs was created to evaluate these solutions and to display the results in tabular and computer-graphics format. These programs incorporate many features that enhance their accuracy, ease of use, and versatility. Documentation for the programs describes their operation and required input data, and presents the results of sample problems. Derivations of select solutions, source codes for the computer programs, and samples of program input and output also are included.

INTRODUCTION

Contamination of ground water by inorganic and organic chemicals has become an increasing concern in recent years. These chemicals enter the ground-water system by a wide variety of mechanisms including accidental spills, land disposal of domestic and industrial waste, and through application of agricultural fertilizers and pesticides. Once introduced into an aquifer, these solutes will be transported by flowing ground water and may degrade water quality at nearby wells and streams.

To improve the management and protection of ground-water resources, it is important to first understand the physical, chemical, and biological processes that control the transport of solutes in ground water. Predictions of the fate of ground-water contaminants can then be made to assess the effect of these chemicals on local water resources and to evaluate the effectiveness of remedial actions.

Two physical processes that govern the movement of ground-water solutes are: (1) advection, which describes the transport of solutes by the bulk motion of flowing ground water (Freeze and Cherry, 1979); and (2) hydrodynamic dispersion, which describes the spread of solutes along and transverse to the direction of flow resulting from both mechanical mixing and molecular diffusion (Bear, 1979, p. 230). Chemical reactions, including those mediated by micro-organisms or caused by interaction with aquifer material or other solutes, may also affect the concentration of the solute under consideration.

These processes have been described quantitatively by a partial differential equation referred to as the "advective-dispersive solute-transport equation." Solution of this equation yields the solute concentration as a function of time and distance from the contaminant source. To apply this equation to a particular ground-water contamination problem, data must be provided on the ground-water velocity, coefficients of hydrodynamic dispersion, rates of chemical reactions, initial concentrations of solutes in the aquifer, configuration of the solute source, and boundary conditions to be specified along the physical boundaries of the ground-water flow system.

In ground-water systems with irregular geometry and nonuniform aquifer properties, numerical techniques are used to determine approximate solutions to the solute-transport equation. In aquifers with simple flow systems and relatively uniform hydrologic properties, analytical solutions, which represent exact mathematical solutions to the solute-transport equation, have been used to predict contaminant migration. These solutions are also used extensively in the analysis of data from soil-column experiments and field tracer tests in order to determine aquifer properties and have also been used to verify the soundness of numerical models. In complex hydrogeologic systems, analytical solutions can still be useful to the hydrologist because they can provide estimates of rates of solute spread and, thus, guide data collection and water-quality monitoring efforts.

While deriving an analytical solution for the solute-transport equation requires a knowledge of higher mathematics, analytical solutions have already been derived and published for many combinations of solute-source configurations and boundary-condition types. After the solutions have been derived, they can be easily evaluated using electronic calculators or digital computers.

Purpose and Scope

This report briefly describes the theoretical background of solute transport in a porous medium and then presents analytical solutions to the advective-dispersive solute-transport equation for a variety of aquifer and solute-source configurations and boundary conditions in systems with uniform (unidirectional) ground-water flow. Solutions for one-dimensional solute transport were compiled from various journals and reports, many of which are not readily available. Many of the solutions for two- and three-dimensional solute transport were modified from those presented in a report by Cleary and Ungs (1978), whereas others were derived by the author using integral transform techniques. (Detailed derivations of these solutions are provided in attachment 1.) All solutions are given in a simplified format together with information on important assumptions in their derivation and limitations on their use.

Simple computer programs, written in FORTRAN-77, have been provided for the evaluation of the analytical solutions presented. The programs were designed for ease of use and for enhanced accuracy. Documentation for these programs includes descriptions of program operation and the input data required. Source codes and samples of program output are provided at the end of the report. Subroutines that allow for graphical display of the program output, created using DISSPLA[1] software, are also described. Computer-generated plots are presented within the report.

Previous Studies

Analytical solutions for the one-dimensional form of the solute-transport equation have appeared in reports and journals concerning physical chemistry, soil science, and water resources. These solutions, generally determined through Laplace transform techniques, have been applied to studies of solute movement in laboratory columns, unsaturated soils, and natural-gradient tracer tests. Solutions to the one-dimensional solute-transport equation for most combinations of boundary and initial conditions are given in van Genuchten and Alves (1982); some of the more useful solutions appear in this report. Other sources that list several analytical solutions include Gershon and Nir (1969), Bear (1972), and Bear (1979).

Fewer analytical solutions have been published for the two- or three-dimensional form of the solute-transport equation. Cleary and Ungs (1978) give several solutions derived using integral transform techniques, and Yeh (1981) gives a computer program that evaluates Green's function to model one-, two-, and three-dimensional transport.

Acknowledgments

This report was prepared at the request of Thomas E. Reilly of the U.S. Geological Survey, Office of Ground Water, in Reston, Va., who recognized the need for a compilation of analytical solutions to the transport equation. Thanks are extended to Robert Cleary, formerly of Princeton University, and Edward Sudicky of the University of Waterloo, Ontario, Canada, for introducing the author to the application of analytical solutions to the analysis of water-quality problems.

THEORETICAL BACKGROUND

Most models which simulate migration of dissolved contaminants in ground-water solve the advective-dispersive solute-transport equation. This partial differential equation is derived from the conservation of mass principle (continuity equation), in which the net rate of change of solute mass within a volume of porous media is equal to the difference between the flux of solute into and out of the volume adjusted for the loss or gain of solute mass because of chemical reactions (Freeze and Cherry, 1979). The flux of solute into the volume is controlled by two physical processes--advection and hydrodynamic dispersion. Hydrodynamic dispersion, in turn, represents the combined effects of two other physical processes--molecular diffusion and mechanical dispersion.

[1]Use of brand, trade, or firm names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

Advection

Advective transport describes the bulk movement of solute particles along the mean direction of fluid flow at a rate equal to the average interstitial fluid velocity. In a saturated medium, this velocity can be calculated from Darcy's Law, such that

$$\vec{V} = - \frac{\bar{\bar{K}}}{n} \cdot \vec{\nabla}h, \tag{1}$$

where

$\vec{V}$ is the average interstitial fluid velocity [L/T],

$\bar{\bar{K}}$ is the hydraulic conductivity tensor for the medium [L/T],

$\vec{\nabla}h$ is the gradient in head [dimensionless] (equal to dh/dx for uniform flow along x-axis), and

n is the effective porosity [dimensionless].

Head, h in equation 1, is equal to the sum of the elevation head, Z, with respect to a datum level, and the pressure head, p/γ, where p is the fluid pressure (gage pressure) and γ is the specific weight of water (Bear, 1979, p. 62). Water flows from areas of higher head toward areas of lower head. Effective porosity, n, differs from the total porosity (volume of pore space per unit volume of aquifer material) in that it does not include pores that are too small to transmit water or "dead-end" pores, those that are not interconnected with other pores.

In unsaturated porous media, the average interstitial fluid velocity can be approximated (Bear, 1979, p. 209) as

$$\vec{V} = - \frac{\bar{\bar{K}}(\theta)}{\theta} \vec{\nabla}h, \tag{2}$$

where

$\vec{V}$ is the average interstitial fluid velocity [L/T],

θ is the moisture content of soil [dimensionless], and

$\bar{\bar{K}}(\theta)$ is the unsaturated hydraulic conductivity tensor for the medium, which is a function of moisture content [L/T].

This form of the equation assumes that the movement of air in the soil is negligible and that the density of water is constant.

Molecular Diffusion

In addition to advective transport, solutes spread through the porous medium by molecular diffusion. Diffusion results from the random collisions of solute molecules and produces a flux of solute particles from areas of higher to lower solute concentration (Bear, 1979). The solute flux, $\vec{J}$, can be given by Fick's first law as

$$\vec{J} = \bar{\bar{D}}_d \cdot \vec{\nabla}C, \qquad (3A)$$

where

C is the concentration of solute (mass of solute per unit volume of fluid) $[M/L^3]$,

$\vec{\nabla}C$ is the concentration gradient $[M/L^4]$, and

$\bar{\bar{D}}_d$ is the second-rank diagonal tensor of molecular diffusion $[L^2/T]$.

Bear and Bachmat (1967) state that the coefficients of molecular diffusion in an isotropic medium are dependent on the diffusion coefficient of the particular solute in water and the tortuosity of the medium. Rates of molecular diffusion are independent of the ground-water velocity, and diffusion occurs even in the absence of fluid movement.

Mechanical Dispersion

The average interstitial fluid velocity represents a mathematical approximation. True velocities at points in the aquifer will differ from this average value, both in magnitude and direction. Local variations in ground-water velocity may not greatly affect the bulk movement of ground water but do control the fate of solute particles.

Mechanical dispersion describes the mixing and spreading of solutes along and transverse to the direction of flow in response to local variations in interstitial fluid velocities. On a microscopic scale (the scale of individual pores), mechanical dispersion results from: (1) the distribution of velocities within an individual pore due to friction effects along the surface of soil grains; (2) differences in size of pores; (3) differences in path length for individual solute particles; and (4) the effect of converging and diverging flow paths (Freeze and Cherry, 1979, p. 75). On a larger (macroscopic) scale, mechanical dispersion results from local variations in hydraulic conductivity and, thus, fluid velocity due to the heterogeneity of aquifer material (Bear, 1979, p. 229).

Laboratory tests on soil columns have shown that the flux of solutes due to mechanical dispersion can also be described by using Fick's first law as

$$\vec{J} = - \bar{\bar{D}}_m \cdot \vec{\nabla}C, \qquad (3B)$$

where $\bar{\bar{D}}_m$ is the second-rank symmetric tensor of mechanical dispersion $[L^2/T]$.

Scheidegger (1961) stated that the coefficients of mechanical dispersion can be related to the average interstitial fluid velocity by means of the geometric dispersivity of the medium. For a saturated porous medium, the geometric dispersivity is dependent upon the hydraulic conductivity, length of a characteristic flow path, and tortuosity (Bear, 1972, p. 614). In a medium that is isotropic with respect to dispersion, the geometric dispersivity can be expressed in terms of just two coefficients--longitudinal dispersivity, a_l, and transverse dispersivity, a_t (Bear, 1979, p. 234).

The elements of the mechanical dispersion tensor can be expressed in terms of longitudinal and transverse dispersivities, the magnitude of the velocity vector, V, and magnitudes of its components, V_x, V_y, and V_z (Bear, 1979, p. 235) as

$$D_{m_{xx}} = [a_l V_x^2 + a_t (V_y^2 + V_z^2)]/V \qquad D_{m_{yy}} = [a_l V_y^2 + a_t (V_x^2 + V_z^2)]/V$$

$$D_{m_{xy}} = D_{m_{yx}} = (a_l - a_t) V_x V_y/V \qquad D_{m_{yz}} = D_{m_{zy}} = (a_l - a_t) V_y V_z/V$$

$$D_{m_{xz}} = D_{m_{zx}} = (a_l - a_t) V_x V_z/V \qquad D_{m_{zz}} = [a_l V_z^2 + a_t (V_x^2 + V_y^2)]/V. \tag{4}$$

If a coordinate system is chosen, such that the direction of the average ground-water velocity is aligned with the x-direction ($V=V_x$ and $V_y=V_z=0$), the off-diagonal terms in the dispersion tensor (eq. 4) will equal zero, and the mechanical dispersion tensor can be simplified to

$$D_{m_x} = D_{m_{xx}} = a_l V$$

$$D_{m_y} = D_{m_{yy}} = a_t V$$

$$D_{m_z} = D_{m_{zz}} = a_t V. \tag{5}$$

Hydrodynamic Dispersion

As stated earlier, hydrodynamic dispersion is the flux of solute due to the combined effect of molecular diffusion and mechanical dispersion. Solute flux, $\vec{J}$, is given by Fick's first law as

$$\vec{J} = - \bar{\bar{D}} \cdot \vec{\nabla} C, \tag{6}$$

where $\bar{\bar{D}}$ is the hydrodynamic dispersion tensor.

In a flow system with uniform flow aligned with the x-axis, the coefficients of the hydrodynamic dispersion tensor, D_x, D_y, and D_z, are given by

$$D_x = D_{m_x} + D_d = a_l V + D_d$$

$$D_y = D_{m_y} + D_d = a_t V + D_d$$

$$D_z = D_{m_z} + D_d = a_t V + D_d. \tag{7}$$

The effects of mechanical dispersion are generally much greater than those of molecular diffusion and, except at low ground-water velocities, the contribution of molecular diffusion is often negligible.

In laboratory experiments with homogeneous materials, values for longitudinal dispersivity, a_l, is typically between 0.004 to 0.4 inch; whereas in field studies, longitudinal dispersivities of as much as 328 feet have been determined (Freeze and Cherry, 1979). The larger field values can be attributed to increased mixing due to local variations in hydraulic conductivity (macrodispersion). A discussion of the apparent scale dependency of hydrodynamic dispersion is given in Anderson (1984). Transverse dispersivity is generally less than longitudinal dispersivity, by a factor of 5 to 20 (Freeze and Cherry, 1979, p. 400).

Advective-Dispersive Solute-Transport Equation

The advective-dispersive solute-transport equation for a single solute can be written as

$$\frac{\partial \theta C}{\partial t} = - \vec{\nabla} \cdot [\theta C\vec{V} - \theta \overline{\overline{D}} \cdot \vec{\nabla}C] + \theta Q_s, \tag{8}$$

where Q_s is the general source or sink term for production or loss of solute within the system.

Equation 8 (after Bear, 1979, p. 241) can be written in terms of volumetric rather than mass concentrations because the fluid density is assumed to be constant. This is usually valid for most ground-water flow systems where solutes are present in relatively low concentrations.

The analytical solutions presented in this report are derived for idealized systems in which the ground-water velocity is assumed to be uniform, aligned with the x-axis, and of constant magnitude. The moisture content (equal to porosity for saturated material) and the coefficients of hydrodynamic dispersion (see eq. 7) are also assumed to be constant. Given these assumptions, the three-dimensional form of the solute-transport equation in a uniform flow system can be expressed as

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} + Q_s. \tag{9}$$

In a thin aquifer, where the solute is uniformly mixed in the vertical (y-z) plane at the inflow boundary, the concentration gradient in the z-direction, $\partial C/\partial z$, equals zero. The two-dimensional solute-transport equation can be expressed as

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x} + Q_s. \tag{10}$$

Finally, if the solute concentration is uniform over the entire inflow boundary, such as in a soil column, the term $\partial C/\partial y$ would also equal zero, yielding the one-dimensional solute-transport equation that can be expressed as

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} + Q_s, \tag{11}$$

where D is identical to D_x.

Chemical Transformation

In addition to the physical mechanisms that govern the movement of solutes through the ground-water system, chemical transformations may alter the concentration of a contaminant species in solution. Possible chemical transformations include dissolution, precipitation, oxidation, reduction, biological degradation, radioactive decay, and adsorption and ion-exchange reactions between the solute and the solid matrix of the aquifer.

If the processes involved in chemical transformation can be described mathematically, they then can be incorporated within the source term, Q_s, in the solute-transport equation for each chemical species. The analytical solutions described herein have been derived for systems in which the chemical transformation terms are given by first-order (linear) relations. The relations and their incorporation into the solute-transport equation are described below.

Linear Equilibrium Adsorption

Many ionic inorganic solutes and nonpolar organic solutes can be removed from solution through adsorption onto the surface of soil particles. The solute may be attracted to soil surfaces by either electrical attraction, Van der Wals forces, or chemical bonding (chemisorption). A general expression for the change in solute concentration due to partitioning of solute particles on the solid matrix can be stated as

$$\theta \frac{\partial C}{\partial t} = -\rho_b \frac{\partial S}{\partial t}, \tag{12}$$

where

ρ_b is the bulk density of the solid matrix measured as mass per unit volume of aquifer material $[M/L^3]$, and

S is the mass of solute adsorbed on the solid matrix per unit mass of solid material [dimensionless].

The amount of solute remaining in solution depends upon the amount of solute in the adsorbed phase. The functional relation is usually determined experimentally through a series of batch tests in which solutions of known initial concentration are mixed with differing amounts of adsorbate. After equilibrium is achieved, the final solute concentration of each solution is measured, and the mass of solute adsorbed is calculated. An equilibrium adsorption curve can then be fitted to these data. Equilibrium concentrations are dependent upon temperature, and the adsorption curve at a particular temperature is termed an "equilibrium adsorption isotherm." A typical equilibrium adsorption isotherm is shown in figure 1.

A linear approximation to the equilibrium adsorption isotherm is generally applicable in systems where the solute concentration is low relative to the adsorbtive capacity of the porous medium. The adsorption of various nonionic organic solutes at trace concentrations onto sediments and soils has also been shown to be linear (Cherry and others, 1984). Many nonlinear forms for the adsorption isotherm, some empirical and some that account for the physical mechanisims of adsorption, are suggested in the literature (see Helfferich, 1962). However, the transport equation that incorporates these other forms must be solved by numerical methods.

Because the amount adsorbed depends solely on the solute concentration, equation 12 can be expressed as

$$\theta \frac{\partial C}{\partial t} = -\rho_b \frac{\partial S}{\partial C} \frac{\partial C}{\partial t}, \tag{13}$$

where $\partial S/\partial C$ is determined from the functional relation between C and S. For a linear equilibrium adsorption isotherm where S equals kC, $\partial S/\partial C$ is equal to k, the slope of the linearized adsorption isotherm, often termed the "partitioning coefficient." The source term can be incorporated into the general three-dimensional form of the solute-transport equation (eq. 9) to yield

$$R \frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x}, \tag{14}$$

where R is referred to as the "retardation factor" given by

$$R = 1 + \frac{k\rho_b}{\theta}. \tag{15}$$

Dividing through R yields

$$\frac{\partial C}{\partial t} = D_x^* \frac{\partial^2 C}{\partial x^2} + D_y^* \frac{\partial^2 C}{\partial y^2} + D_z^* \frac{\partial^2 C}{\partial z^2} - V^* \frac{\partial C}{\partial x}, \tag{16}$$

where V^* and D_x^*, D_y^*, and D_z^* are the scaled (or retarded) velocity and dispersion coeffecients, respectively. Equation 15 shows that transport of solutes subject to linear adsorption can be simulated in the same manner as a nonadsorbed solute. Because the apparent velocity of the adsorbed solute is reduced, the solute will arrive at a given point later than a nonadsorbed solute.

The use of equilibrium isotherms assumes that equilibrium between the porous medium and the solute in solution exists at all times. This assumption is generally valid when the adsorption process is fast in relation to the ground-water velocity (Cherry and others, 1984). If adsorption proceeds slowly, the kinetics of the reaction must be considered. Nonequilibrium adsorption relations can be incorporated into the transport equation, but numerical methods are needed for solution of the resulting equation.

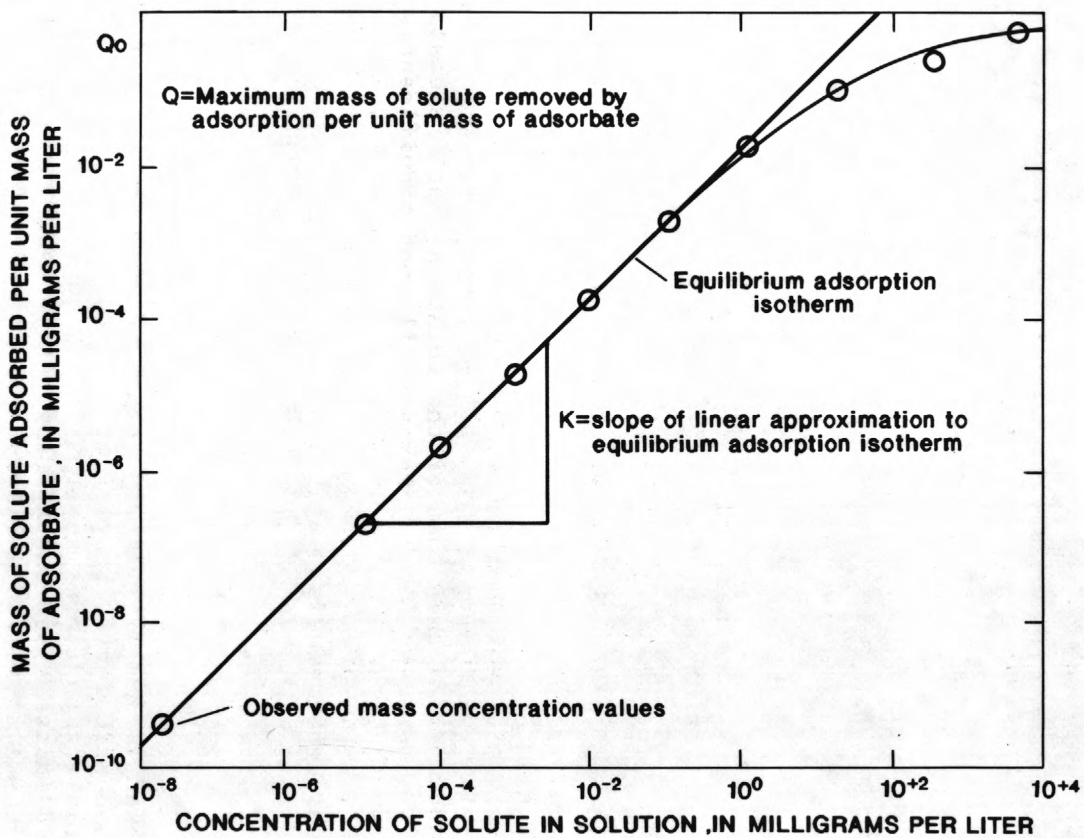


Figure 1.--Typical shape of equilibrium adsorption.

The process of adsorption is also assumed to be reversible. If hysteresis effects during desorption are significant, other forms of the adsorption isotherms must be considered, and numerical methods would be required.

Ion Exchange

Ion exchange is an adsorption process in which a cation in solution, such as potassium, calcium, or magnesium ion, replaces another cation that is electrically bound to collodial material in the soil. Under certain conditions, ion exchange can be modeled in a manner similar to linear adsorption (R.W. Cleary, Princeton University, written commun., 1977). The exchange reaction for monovalent ions can be expressed as

$$A^{+} + B^{+}R \rightleftharpoons B^{+} + A^{+}R, \tag{17}$$

where A^{+} is the cation in solution, R is the exchange medium, and B^{+} is the counter ion released from the exchanger. At equilibrium, a selectivity coefficient, K_s, can be defined, such that

$$K_s = \frac{[B^{+}]\ [A^{+}R]}{[A^{+}]\ [B^{+}R]}, \tag{18}$$

where the bracketed terms represent the activities of each constituent.

Measured values of K_s can be used in simulating transport by making the following assumptions: (1) If all exchange sites are assumed to be occupied initially, then $[B^{+}R]$ represents the total cation exchange capacity (CEC) of the medium, which can be determined experimentally and then treated as a constant; (2) the counter ion, B^{+}, is usually present in solution at much greater concentrations than that of the solute A^{+}, and releases of additional amounts of the counter ion by exchange will not significantly alter its concentration, thus, $[B^{+}]$ can also be treated as a constant; and (3) the relation between the amount of solute on the exchange sites and that remaining in solution can be defined as

$$k_d = \frac{[A^{+}]}{[A^{+}R]}, \tag{19}$$

where the distribution coefficient, k_d, is determined through laboratory batch tests.

Given these assumptions, the general expression for the change in solute concentration due to cation exchange can be expressed as

$$\theta \frac{\partial C}{\partial t} = -\rho_b\, k_d \frac{\partial C}{\partial t}, \tag{20}$$

where

$$k_d = \frac{K_s \cdot CEC}{[B^{+}]}. \tag{21}$$

This term would replace k in equation 15. For monovalent-divalent cation exchange, where

$$A^{++} + 2B^{+}R \rightleftharpoons 2B^{+} + A^{++}R, \tag{22}$$

the distribution coefficient can be given as

$$k_d = \frac{K_s \cdot CEC^2}{[B^+]^2}. \tag{23}$$

First-Order Chemical Reactions

Simple chemical reaction terms can be formulated to account for the kinetics of reactions under nonequilibrium conditions. A first-order chemical process, such as radioactive decay or biological degradation, involves the irreversible unimolecular conversion of a solute A to solute B (A → B). The rate of the reaction can be given by

$$\frac{d[A]}{dt} -\lambda\ [A], \tag{24}$$

where λ is defined as the rate coefficient [1/T]. The rate coefficient can be expressed in terms of the half-life of the solute, $T_{1/2}$ (the time required for the concentration of the solute species to be reduced to half the initial concentration) as

$$\lambda = \ln(2)/T_{1/2} = 0.693/T_{1/2}. \tag{25}$$

Equation 9 can be written to incorporate first-order reaction as

$$\theta \frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C. \tag{26}$$

If the solute is subject to linear adsorption and to first-order chemical transformation in both the solute and adsorbed phases, equation 9 can be expressed as

$$\left[1 + \frac{k\rho_b}{\theta}\right] \frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \left[1 + \frac{k\rho_b}{\theta}\right] \lambda C \tag{27}$$

or

$$\frac{\partial C}{\partial t} = D_x^* \frac{\partial^2 C}{\partial x^2} + D_y^* \frac{\partial^2 C}{\partial y^2} + D_z^* \frac{\partial^2 C}{\partial z^2} - V^* \frac{\partial C}{\partial x} - \lambda C, \tag{28}$$

where V^* and D^* represent the scaled velocity and dispersion coefficients. If the adsorbed phase is not subject to chemical transformation, λ in equation 28 should be replaced by λ^*, where

$$\lambda^* = \lambda / \left[1 + \frac{k\rho_b}{\theta}\right]. \tag{29}$$

Some multiple-ion reactions can be approximated as a first-order reaction if the concentrations of all ions, except the species being considered, are in excess (R.W. Cleary, Princeton University, written commun., 1977). For example, if the reaction involves the conversion of solutes A and B to form solute C ($A + B \rightarrow C$), the rate of reaction would be given as

$$\frac{d[A]}{dt} = -\lambda_{AB} [A] [B]. \tag{30}$$

If solute B is in excess, its concentration will remain relatively constant, and equations 26 or 28 can be used with a modified λ term, where $\lambda = \lambda_{AB} [B]$. General bimolecular or multiple-ion reactions result in nonlinear chemical source terms. Reversible reactions and multistep reactions require the simultaneous solution of the transport equation written for each species. Simulation of transport involving these chemical processes usually requires numerical methods.

Initial Conditions

To solve the solute-transport equation, a complete set of boundary and initial conditions must be specified. Initial conditions are used to define the solute concentration in the aquifer at the time inflow of solute begins. For the analytical solutions presented in this report, the initial conditions are specified such that all initial concentrations are equal to zero. If the solute is conservative, a constant initial background solute concentration can be added to the calculated concentrations. Analytical solutions for one-dimensional transport of nonconservative solute transport with nonzero initial concentrations are given in van Genuchten and Alves (1982).

Boundary Conditions

Three types of boundary conditions are generally associated with the solute-transport equation. The first-type (or Dirichlet) boundary condition specifies the value of the concentration along a section of the flow-system boundary. A second-type (or Neumann) boundary condition specifies the gradient in solute concentration across a section of the boundary. The third-type (or Cauchy) boundary condition is applied where the flux of solute across the boundary is dependent upon the difference between a specified concentration value on one side of the boundary and the solute concentration on the opposite side of the boundary. These different boundary condition types will be used to describe conditions on the inflow and outflow ends of the flow system and also along the lateral boundaries of two- and three-dimensional systems.

Inflow Boundary

A third-type boundary condition best describes solute concentrations at the inflow end in a uniform flow system (Bear, 1979 p. 268), where a well mixed solute enters the system by advection across the boundary and is transported away from the boundary by advection and dispersion. The boundary conditions can be given as

$$VC - D_x \frac{\partial C}{\partial x} = VC_o, \qquad x = 0, \tag{31}$$

where C_o is the known measured concentration in the influent water. The third-type boundary condition allows for solute concentration at the inflow boundary to be lower than C_o initially and then increase as more solute enters the system. Over time, the concentration gradient across the boundary, $\partial C/\partial x$, decreases as the concentration at the inflow boundary approaches C_o.

Alternatively, a first-type boundary condition can be specified at the inflow end, such that

$$C = C_o, \qquad x = 0. \tag{32}$$

Application of this simpler form of a boundary condition presumes that the concentration gradient across the boundary equals zero as soon as flow begins. However, this may lead to an overestimation of the mass of solute in the system at early times.

Equation 31 would indicate that the difference between concentrations predicted for a system with a first-type source boundary concentration and a system with a third-type boundary condition should decrease as the quantity D/V decreases. Additional discussions of the effect and relative merits of the different inflow boundary condition types are presented in Gershon and Nir (1969), van Genuchten and Alves (1982), and Parker and van Genuchten (1984).

Outflow Boundary

Often, the outflow boundary of the system being simulated is far enough away from the solute source such that the boundary will not affect solute concentrations within the area of interest. Such a system can be treated as being "semi-infinite," and either a first-type or second-type boundary condition can be specified as

$$C, \qquad \frac{\partial C}{\partial x} = 0, \qquad x = \infty. \tag{33}$$

When the system has a finite length, and solute concentrations near the outflow boundary are of interest, selection of an appropriate boundary condition becomes more difficult. In general, if the system discharges to a large, well-mixed reservoir and the additional solute will not significantly alter reservoir concentrations, then a third-type or first-type boundary condition (similar to the inflow boundary) can be used. If the res- ervoir is small or not well mixed, such as at the end of the soil column in figure 2A, concentrations in the reservoir would equal solute concentration at the discharge end of the system and, thus, no concentration gradient would exist across the boundary. This can be specified by a second-type boundary condition as

$$\frac{\partial C}{\partial x} = 0, \qquad x = L, \tag{34}$$

where L is the length of the finite system.

Van Genuchten and Alves (1982, p. 90-96) analyzed the difference between predicted concentrations obtained with analytical solutions for a semi-infinite system and a finite system with a second-type boundary condition in terms of two dimensionless numbers: (1) the column Peclet number (P), and (2) the number of displaced pore volumes (T), which are defined by

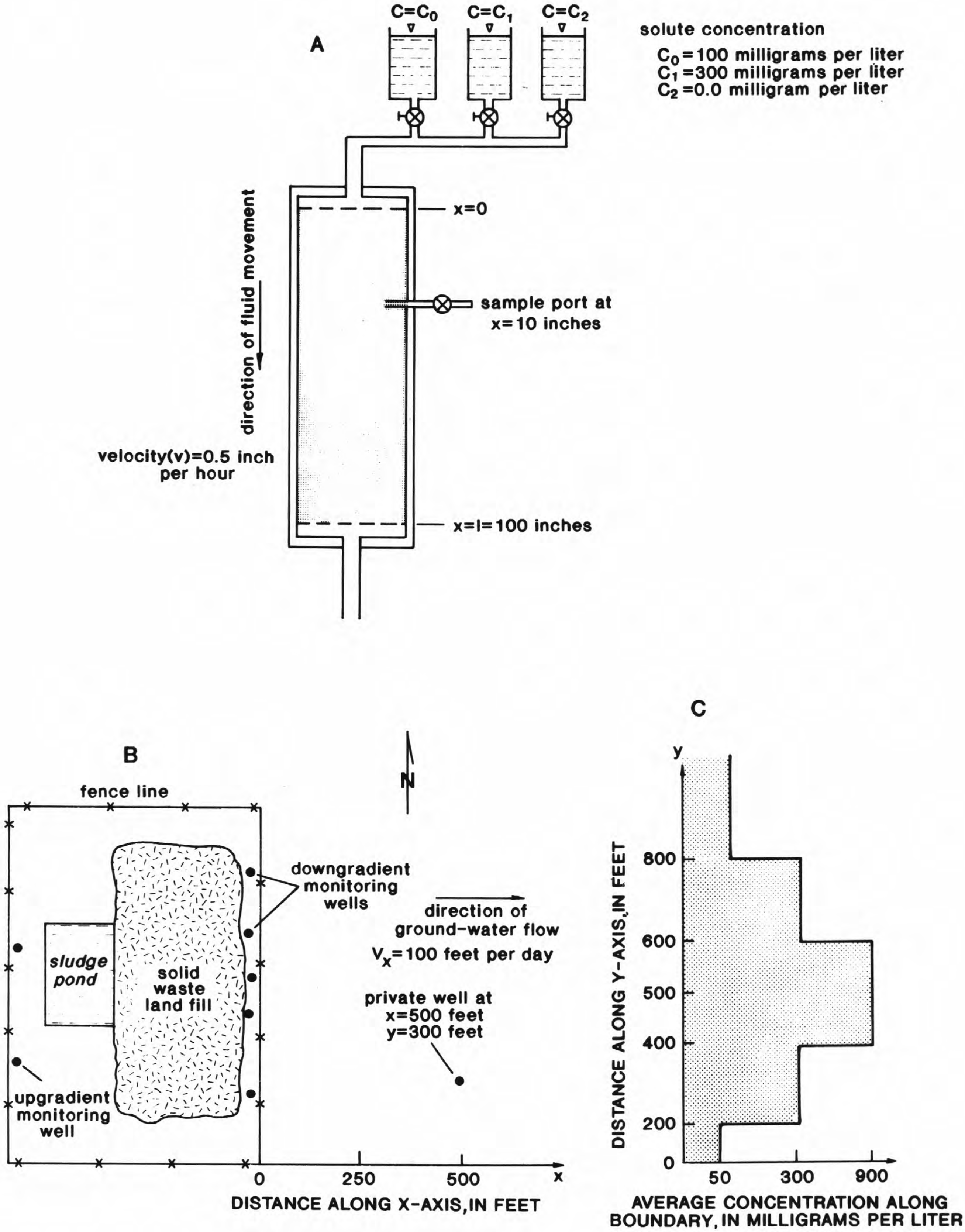


Figure 2.--Examples of situations where the principle of superposition can be applied: (A) soil column with time varying input concentration (cases A and B in text), (B) waste-disposal site with spatially varying input concentrations (case C in text), and (C) plot of average concentration measured along waste-disposal site boundary.

$$P = \frac{VL}{D} \tag{35}$$

and

$$T = \frac{Vt}{L}. \tag{36}$$

They found that predicted concentration at points near the outflow boundary begins to differ significantly for T greater than 0.25 and the differences increase as T approaches 1 (corresponding to movement of the solute front closer to the outflow boundary). The magnitude of the difference and distance inward from the outflow boundary at which the solutions diverge decreases as P values increase.

Lateral Boundaries

In two- and three-dimensional systems, impermeable or no-flow boundaries may be present at the base, top, or sides of the aquifer. Because there is no advective flux across the boundary, and molecular diffusion across the boundary is assumed to be negligible, the general third-type boundary condition simplifies to a second-type boundary condition, expressed as

$$\frac{\partial C}{\partial y} = 0, \qquad y = 0 \text{ and } y = W \tag{37A}$$

and

$$\frac{\partial C}{\partial z} = 0, \qquad z = 0 \text{ and } z = H \tag{37B}$$

where W and H are width and height of the aquifer, respectively.

In many cases, lateral boundaries of the system may be far enough away from the area of interest that the system can be treated as being infinite along the y- and z-axes. Boundary conditions can then be specified as

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = \pm\infty \tag{38A}$$

and

$$C, \frac{\partial C}{\partial z} = 0, \qquad z = \pm\infty. \tag{38B}$$

Superposition

Because the solute-transport equation is a linear partial-differential equation, the principle of superposition can be used to calculate concentrations in the system if solute concentrations at the inflow boundary vary over time. The general form of the solution can be expressed as

$$C = C_o \cdot A\ (x,y,z,t) + (C_1 - C_o) \cdot A\ (x,y,z\ [t-t_o]), \tag{39}$$

where

C_o is the initial solute concentration at boundary,

t_1 is the time at which solute concentration changes at boundary,

C_1 is the solute concentration at the boundary after $t = t_1$, and

A (x,y,z,t) is the general form of the analytical solution where concentration is function of space and time.

The principle of superposition should be familiar to most hydrologists who have used analytical solutions (such as the Theis equation) in the analysis of aquifer tests. Several examples are provided to illustrate its application to solute-transport simulation.

Case A:

A solution is passed through a 100-inch long soil column (fig. 2a) for a period of 10 hours with V = 0.5 in/h, D = 0.05 in^2/h, and C_o = 100 mg/L (milligrams per liter). At the end of the 10-hour period, the concentration of the influent is increased to C_1 = 300 mg/L. Of interest is the concentration at x = 10 inches at the end of a total elapsed time of 20 hours.

The analytical solution for transport of a conservative solute in a semi-infinite column (assuming that boundary effects at x = L are negligible) with a first-type inflow boundary condition was given by Ogata and Banks (1961) as

$$A\ (x,t) = \frac{1}{2}\left\{\text{erfc}\left[\frac{x-Vt}{2\sqrt{Dt}}\right] + \exp\left[\frac{Vx}{D}\right] \cdot \text{erfc}\left[\frac{x+Vt}{2\sqrt{Dt}}\right]\right\},$$

where erfc is the complementary error function. (The solution is described in more detail later.) For the values given, equation 39 becomes

$$\begin{aligned} C\ (10\text{ inches, }20\text{ hours}) &= 100\text{ mg/L} \cdot A\ (10\text{ inches, }20\text{ hours}) + (300\text{ mg/L} \\ &\quad - 100\text{ mg/L}) \cdot A\ (10\text{ inches, }[20\text{ hours} - 10\text{ hours}]) \\ &= 100\text{ mg/L} \cdot (0.984) + 200\text{ mg/L} \cdot (0.088) \\ &= 116.0\text{ mg/L.} \end{aligned}$$

Case B:

This case is similar to case A, except that at the end of 10 hours, solute-free water (C_1 = 0.0 mg/L) is passed through the soil column, thus, creating a solute pulse of finite duration. The concentration of solute at x = 10 inches and t = 20 hours can be given from equation 39 as

$$C\text{ (10 inches, 20 hours)} = 100\text{ mg/L} \cdot A\text{ (10 inches, 20 hours)} + (0 - 100\text{ mg/L}) \cdot A\text{ (10 inches, [20 - 10 hours])}$$

$$= 100\text{ mg/L} \cdot (0.984) - 100\text{ mg/L} \cdot (0.088)$$

$$= 89.6\text{ mg/L}.$$

The principle of superposition can also be used to simulate more complex solute configurations at the boundary of two- and three-dimensional systems as provided in the following example. Also, if solute sources are at two locations, the calculated concentration from the first source at a particular point of interst can simply be added to the calculated concentration from the second source at that point.

Case C:

A waste-disposal site, shown in plan view in figure 2b, has a solid-waste landfill and a smaller area for sludge disposal. Measured concentrations in fully screened wells along the eastern boundary downgradient of the landfill had chloride concentrations averaging 300 mg/L. Wells downgradient of both the sludge ponds and the landfill had concentrations averaging 900 mg/L. Background chloride concentrations are 50 mg/L. Given $V_x = 1$ ft/d, $D_x = 20$ ft²/d, and $D_y = 4$ ft²/d, calculate the concentration at a private well located at x = 500 feet and y = 300 feet at the end of 1 year.

The analytical solution for transport of a conservative solute in an infinitely wide aquifer with a finite-width or "strip" source along the inflow boundary is given by

$$A\ (x,y,Y_1,Y_2,t) = \frac{x}{4\sqrt{\pi D_x}} \exp\left(\frac{Vx}{2D_x}\right) \int_0^t \tau^{-3/2} \exp\left[\frac{-x^2}{4D_x\tau} - \frac{V^2\tau}{4D_x}\right] \cdot \left\{\operatorname{erfc}\left[\frac{Y_1 - y}{2\sqrt{D_y\tau}}\right] - \operatorname{erfc}\left[\frac{Y_2 - y}{2\sqrt{D_y\tau}}\right]\right\} d\tau,$$

where Y_1 and Y_2 are coordinates of the end points of the source on the y-axis and τ is a dummy variable of integration. The solute source can be represented by two strip sources: The first extending from $Y_1 = 200$ to $Y_2 = 800$ feet with an effective concentration of 250 mg/L (difference between measured and background concentration) and the second extending from $Y_1 = 400$ to $Y_2 = 600$ feet with a concentration of 600 mg/L (measured concentration minus first source effective concentration and background concentration). The concentration at the private well can be calculated as

$$C\text{ (500 feet, 300 feet)} = C_{\text{background}} + C_1 \cdot A\text{ (500 feet, 300 feet, 200 feet, 800 feet, 365 days)} + C_2 \cdot A\text{ (500 feet, 300 feet, 400 feet, 600 feet, 365 days)}$$

$$= 50\text{ mg/L} + 250\text{ mg/L} \cdot (0.1612) + 600\text{ mg/L} \cdot (0.1354)$$

$$= 171.5\text{ mg/L}.$$

ONE-DIMENSIONAL SOLUTE TRANSPORT

Many analytical solutions for the one-dimensional form of the solute-transport equation (eq. 11) were developed for study of dispersion phenomena in soil or adsorption columns. Some field situations can also be idealized as one-dimensional transport systems; two examples are shown in figure 3. Figure 3A represents steady vertical flow through the unsaturated zone beneath a septic tank drainfield. Transport at the center of the field is simulated, and the horizontal spread of solutes along edges of the field is neglected. Figure 3B represents a case of steady horizontal ground-water flow from river A which has been contaminated to river B.

One-dimensional systems can be finite, semi-infinite, or infinite in extent. In the finite or semi-infinite systems, water containing a known concentration of a contaminant species enters the system at the origin (at $x=0$). Water and solute exit at the opposite end of the system (at $x=L$) which could represent the water table, a stream, or the end of a soil column (fig. 3).

In the finite-length system, the outflow boundary is close enough that it will have an effect on the magnitude of concentrations within the area of interest. If the outflow boundary is far enough away that the effects will not be felt in the area of interest (equivalent to $T < 0.25$, where T is the number of displaced pore volumes), solutions for a semi-infinite system can be used and are generally easier to evaluate.

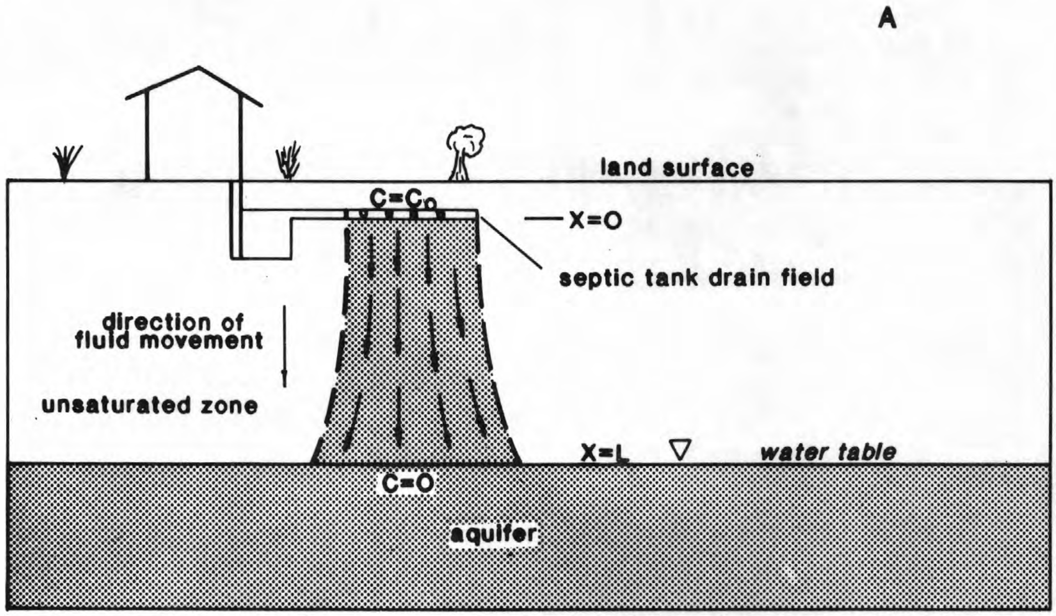
An example of transport in an infinite system might be the injection of a solute into the center of a long soil column. In this case, the spread of solute, both upgradient and downgradient of the source, is of interest. Solutions for an infinite system can be found in van Genuchten and Alves (1982) and Bear (1972; 1979).

For the four analytical solutions presented in this section, either a first- or third-type boundary condition is specified at the inflow end of a finite or semi-infinite system. Specifically, the solutions are for a:

- Finite system with a first-type boundary condition at the inflow end,
- Finite system with a third-type boundary condition at the inflow end,
- Semi-infinite system with a first-type boundary condition at the inflow end,
- Semi-infinite system with a third-type boundary condition at the inflow end.

Solutions for the finite systems assume a second-type boundary condition at the outflow end.

Two computer programs, FINITE and SEMINF, were developed to calculate concentrations in these four systems as a function of distance and elapsed time. These programs are also described in this section. The format used in presenting each of the solutions may seem repetitive, but it provides for easy reference.



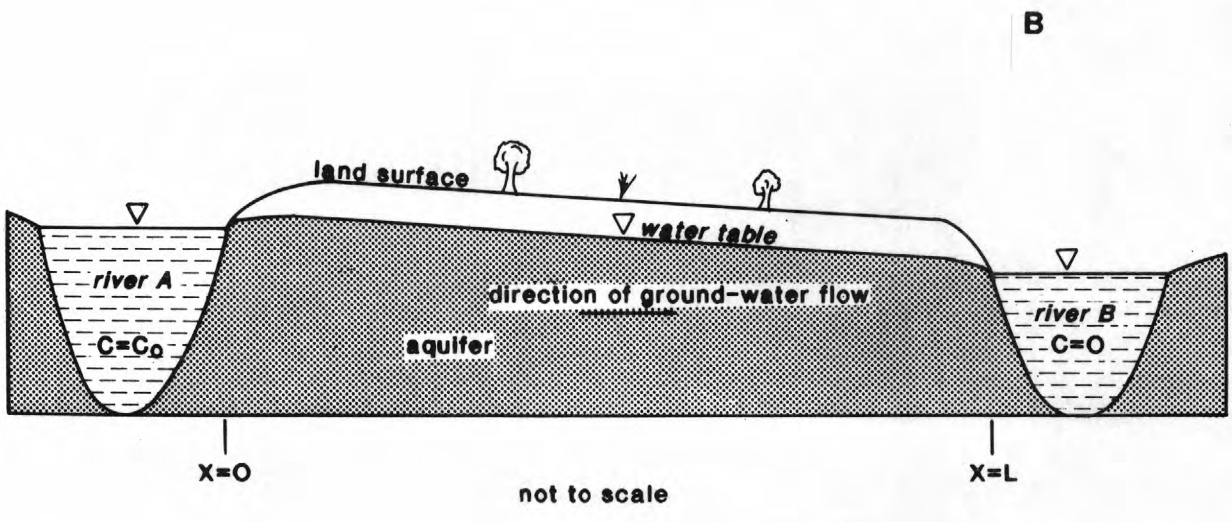


Figure 3.--Two examples (A and B) of contaminant movement in field settings that can be simulated as one-dimensional solute-transport systems.

Finite System with First-Type Source Boundary Condition

Governing Equation

One-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (40)$$

Boundary conditions:

$$C = C_o, \qquad x = 0 \qquad (41)$$

$$\frac{\partial C}{\partial x} = 0, \qquad x = L \qquad (42)$$

Initial condition:

$$C = 0, \qquad 0 < x < L \qquad \text{at } t = 0 \qquad (43)$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The longitudinal dispersion coefficient (D), which is equivalent to D_x (eq. 7), is constant.

Analytical Solution

The following equation is modified from van Genuchten and Alves (1982, p. 63-65) and can be expressed as

$$C(x,t) = C_o \left\{ \frac{\exp\left[\frac{(V-U)x}{2D}\right] + \frac{(U-V)}{(U+V)} \exp\left[\left(\frac{V+U}{2D}\right)x - \frac{UL}{D}\right]}{\left[1 + \frac{(U-V)}{(U+V)} \exp\left(\frac{-UL}{D}\right)\right]} \right.$$

$$\left. - 2 \exp\left[\frac{Vx}{2D} - \lambda t - \frac{V^2 t}{4D}\right] \sum_{i=1}^{\infty} \frac{\beta_i \sin\left(\frac{\beta_i x}{L}\right) \left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2\right] \exp\left[-\frac{\beta_i^2 Dt}{L^2}\right]}{\left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2 + \frac{VL}{2D}\right] \left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2 + \frac{\lambda L^2}{D}\right]} \right\}, \qquad (44)$$

where $U = \sqrt{V^2 + 4\lambda D}$ and β_i are the roots of the equation

$$\beta \cot \beta + \frac{VL}{2D} = 0. \qquad (45)$$

Comments:

Values of the first six roots of the equation a • cot(a) + c = 0 are tabulated in Carslaw and Jaeger (1959, p. 492) for various values of the constant c. Additional roots of equation 45 can be found through standard root-search techniques.

The maximum number of terms that should be computed in the infinite series summation depends upon how fast the series converges. Convergence is usually a problem at early times (T<<1) near the origin (x=o) especially when the column Peclet number (P in eq. 35) is relatively large. The program described below determines that the series has converged if the absolute value of the last term in the series is less than 1×10^{-12}. A good initial estimate for the maximum number of terms is 100, but more should be used if the program indicates that the series did not converge. A minimum of 25 roots is used by the program.

For a solute that is not subject to first-order chemical transformation (λ = 0), equation 44 can be replaced (Cleary and Adrian, 1973; Wexler and Cleary, 1979) by

$$C(x,t) = C_o \left\{ 1-2\exp\left[\frac{Vx}{2D} - \frac{V^2t}{4D}\right] \sum_{i=1}^{\infty} \frac{\beta_i \sin\left(\frac{\beta_i x}{L}\right) \exp\left(-\frac{\beta_i^2 Dt}{L^2}\right)}{\beta_i^2 + \left(\frac{VL}{2D}\right)^2 + \frac{VL}{2D}} \right\}. \qquad (46)$$

For large values of time (steady-state solution), equation 44 can be reduced (van Genuchten and Alves, 1982, p. 58) to

$$C(x) = C_o \exp \frac{\left[\frac{(V-U)x}{2D}\right] + \frac{(U-V)}{(U+V)} \exp\left[\frac{(V+U)x}{2D} - \frac{UL}{D}\right]}{\left[1 + \frac{(U-V)}{(U+V)} \exp\left(\frac{-UL}{D}\right)\right]}. \qquad (47)$$

Linear equilibrium adsorption and ion exchange can be simulated by first dividing the coefficients D and V by the retardation factor, R (eq. 15). (Note: U in equations 44 and 47 would be given by $U = \sqrt{V^{*}+4\lambda D^{*}}$). Temporal variations in source concentration can be simulated through the principle of superposition (eq. 39).

Description of Program FINITE

The program FINITE computes the analytical solution to the one-dimensional solute-transport equation for a finite system with a first-type (eq. 44) or third-type (eq. 52) source boundary condition at the inflow end. It consists of a main program and four subroutines--ROOT1, ROOT3, CNRML1, and CNRML3. The function of the main program and subroutines ROOT1 and CNRML1 are outlined below; the program code listing is presented in attachment 2. Subroutines ROOT3 and CNRML3 are called when a third-type boundary condition is specified and are described in a subsequent section.

The program also calls the output subroutines TITLE, OFILE, and PLOT1D which are common to most of the programs described in this report. These subroutines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 1.

The program calls subroutine ROOT1 to compute the positive roots of equation 45 when a first-type source boundary condition is specified and then executes a set of nested loops. The inner loop calls subroutine CNRML1 to calculate the concentration for a particular time value and distance; the outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time value. Graphs of concentration in relation to distance can also be plotted.

Subroutines ROOT1 and CNRML1

Subroutine ROOT1 calculates the roots of the equation

$$a \cdot \cot(a) + c = 0$$

by an iterative procedure. The first root is known to lie between $\pi/2$ and π and an initial estimate of 0.75 π is made. Newton's second-order method (Salvadori and Baron, 1961, p. 6) corrects and updates the estimate at each iteration. A maximum of 50 iterations and a convergence criterion of 1.0×10^{-10} are set in the subroutine. Each subsequent root of the equation is about π greater than the previous one. This value is used as an initial estimate in the search for the remaining roots.

Subroutine CNRML1 calculates the normalized concentration (C/C_o) for a particular time value and x-distance value using equation 44 for a solute subject to first-order chemical transformation and equation 46 if the solute is conservative (λ=0). The number of terms taken in the infinite series summation is specified in the input data.

Sample Problems 1A and 1B

Two sample problems are presented that use data similar to that given in Lai and Jurinak (1972). In sample problem 1a, a conservative solute is introduced into a saturated soil column under steady flow. Model variables are

Velocity (V)	= 0.6 in/h
Longitudinal dispersion (D)	= 0.6 in^2/h
System length (L)	= 12 inches
Solute concentration at inflow boundary (C_o)	= 1.0 mg/L.

Concentrations are calculated for points 0.5 inch apart at elapsed times of 2.5, 5, 10, 15, and 20 hours.

Table 1.--Input data format for the program FINITE

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NBC	Boundary condition type (NBC = 1 for a first-type boundary condition; NBC = 3 for a third-type boundary condition).
	5 - 8	I4	NX	Number of x-coordinates at which solution will be evaluated.
	9 - 12	I4	NT	Number of time values at which solution will be evaluated.
	13 - 16	I4	NROOT	Number of terms used in infinite series summation.
	17 - 20	I4	IPLT	Plot control variable. Concentration profiles will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.[1]
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.[1]
	31 - 40	F10.0	DK	First-order solute-decay coefficient.[1]
	41 - 50	F10.0	XL	Length of flow system.[1]
	51 - 60	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
5	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
6	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).

[1]All units must be consistent.

In sample problem 1B, a solute is removed by linear equilibrium adsorption. Additional model variables are

Soil bulk density (ρ_b) = 0.047 lb(mass)/in^3
Porosity (θ) = 0.45
Slope of adsorption isotherm (k) = 70 in^3/lb (mass).

From these values and equations 15 and 16 (substituting n for θ), the terms obtained are

Retardation factor (R) = 8.31
Scaled velocity (V^*) = 0.072 in/h
Scaled dispersion coefficient (D^*) = 0.072 in^2/h.

Concentrations are calculated for points 0.5 inch apart at elapsed times of 20, 50, 100, and 150 hours.

The input data sets for sample problems 1A and 1B are shown in figures 4A and 5A; computer plots of concentration profiles generated by the program FINITE are shown in figures 4B and 4C. Comparison of the concentration profiles at 20 hours shows the retarding effect of adsorption on solute movement.

Program output for the two sample problems are presented in attachment 4. Sample problems 1A and 1B each required 3.9 seconds of central processing unit (CPU) time on a Prime model 9955 Mod II.

Finite System with Third-Type Source Boundary Condition

Governing Equation

One-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - \lambda C \tag{48}$$

Boundary conditions:

$$VC_o = VC - D \frac{\partial C}{\partial x}, \qquad x = 0 \tag{49}$$

$$\frac{\partial C}{\partial x} = 0, \qquad x = L \tag{50}$$

Initial condition:

$$C = 0, \qquad 0 < x < L \qquad \text{at } t = 0 \tag{51}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The longitudinal dispersion coefficient (D), which is equivalent to D_x (eq. 7), is constant.

A

```
Sample Problem 1a -- Solute transport in a finite-length
soil column with a first-type boundary condition at x=0
Model Parameters:  L=12 in, V=0.6 in/h, D=0.6 in**2/h
                   K1=0.0 per h, C0=1.0 mg/L
====
  1  25  05  50   1
MG/L       IN/H       IN**2/H   PER HOUR  INCHES     HOURS
       1.0        0.6        0.6       0.0      12.0        1.2
       0.0        0.5        1.0       1.5       2.0        2.5        3.0        3.5
       4.0        4.5        5.0       5.5       6.0        6.5        7.0        7.5
       8.0        8.5        9.0       9.5      10.0       10.5       11.0       11.5
      12.0
       2.5        5.0       10.0      15.0      20.0
```

B

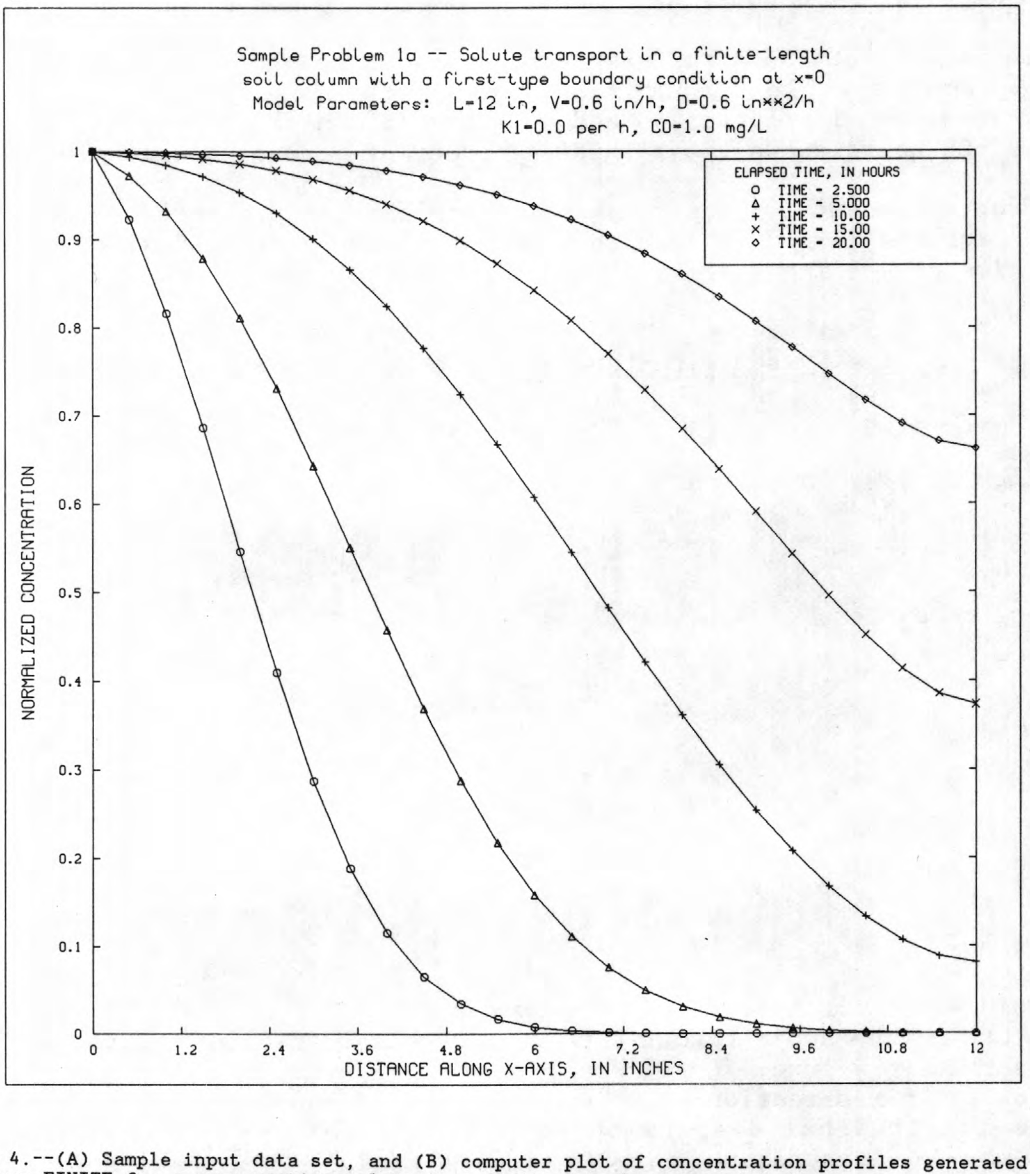


Figure 4.--(A) Sample input data set, and (B) computer plot of concentration profiles generated by the program FINITE for a conservative solute in a finite-length system with first-type source boundary condition after 2.5, 5, 10, 15, and 20 hours (sample problem 1A).

A

```
Sample Problem 1b -- Solute transport in a finite-length
soil column with a first-type boundary condition at x=0
Model Parameters:  L=12 in, V=0.072 in/h, D=0.072 in**2/h
                   K1=0.0 per h, C0=1.0 mg/L
Solute is subject to linear adsorption
====
   1  25  04  50   1
MG/L       IN/H       IN**2/H    PER HOUR  INCHES     HOURS
       1.0      0.072      0.072       0.0      12.0       1.2
       0.0        0.5        1.0       1.5       2.0       2.5       3.0       3.5
       4.0        4.5        5.0       5.5       6.0       6.5       7.0       7.5
       8.0        8.5        9.0       9.5      10.0      10.5      11.0      11.5
      12.0
      20.0       50.0      100.0     150.0
```

B

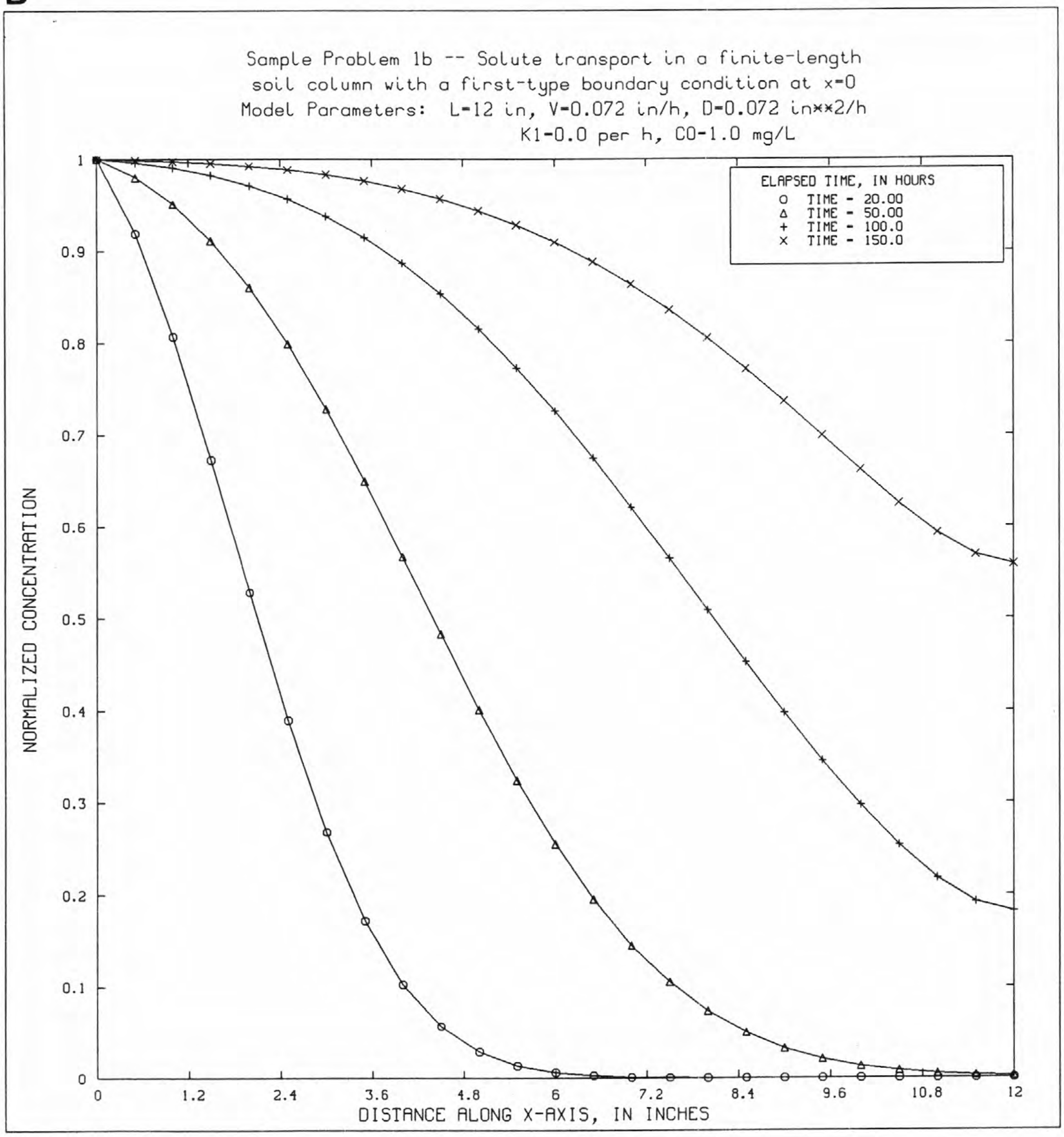


Figure 5.--(A) Sample input data set, and (B) computer plot of concentration profiles generated by the program FINITE for a solute subject to linear adsorption in a finite-length system with first-type source boundary condition after 20, 50, 100, and 150 hours (sample problem 1B).

Analytical Solution

The solution to equation 48 was first presented by Selim and Mansell (1976). The following equation is modified from a form presented in van Genuchten and Alves (1982, p. 66-67) and can be expressed as

$$C(x,t) = C_o \left\{ \frac{\exp\left[\frac{(V-U)x}{2D}\right] + \frac{(U-V)}{(U+V)} \exp\left[\frac{(V+U)x}{2D} - \frac{UL}{D}\right]}{\left[\frac{(U+V)}{2V} - \frac{(U-V)^2}{2V(U+V)} \exp\left(-\frac{UL}{D}\right)\right]} \right.$$

$$- 2 \frac{VL}{D} \exp\left[\frac{Vx}{2D} - \frac{V^2 t}{4D} - \lambda t\right] \sum_{i=1}^{\infty} \frac{\beta_i \left[\beta_i \cos\left(\frac{\beta_i x}{L}\right) + \left(\frac{VL}{2D}\right) \sin\left(\frac{\beta_i x}{L}\right)\right]}{\left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2 + \frac{VL}{D}\right]}$$

$$\left. \cdot \frac{\exp\left[-\frac{\beta_i^2 D t}{L^2}\right]}{\left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2 + \frac{\lambda L^2}{D}\right]} \right\}, \tag{52}$$

where $U = \sqrt{V^2 + 4\lambda D}$ and β_i are the roots of the equation

$$\beta \cot \beta - \frac{\beta^2 D}{VL} + \frac{VL}{4D} = 0. \tag{53}$$

For a solute that is not subject to first-order chemical transformation (λ=0), equation 52 can be simplified (Gershon and Nir, 1969, p. 837; van Genuchten and Alves, 1982, p. 13) as

$$C(x,t) = C_o \left\{ 1 - 2 \frac{VL}{D} \exp\left[\frac{Vx}{2D} - \frac{V^2 t}{4D}\right] \right.$$

$$\left. \cdot \sum_{i=1}^{\infty} \frac{\beta_i \left[\beta_i \cos\left(\frac{\beta_i x}{L}\right) + \left(\frac{VL}{2D}\right) \sin\left(\frac{\beta_i x}{L}\right)\right]}{\left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2 + \frac{VL}{D}\right]} \cdot \frac{\exp\left[-\frac{\beta_i^2 D t}{L^2}\right]}{\left[\beta_i^2 + \left(\frac{VL}{2D}\right)^2\right]} \right\}. \tag{54}$$

For large values of time (steady-state solution), equation 52 can be reduced (van Genuchten and Alves, 1982, p. 59) as

$$C(x) = C_o \left\{ \frac{\exp\left[\frac{(V-U)x}{2D}\right] + \frac{(U-V)}{(U+V)} \exp\left[\frac{(V+U)x}{2D} - \frac{UL}{D}\right]}{\left[\frac{(U+V)}{2V} - \frac{(U-V)^2}{2V(U+V)} \exp\left[-\frac{UL}{D}\right]\right]} \right\}. \tag{55}$$

Comments:

The roots of equation 53 can be found by standard root-search techniques. An iterative technique using Newton's second-order correction method was described in the preceding section.

Linear equilibrium adsorption and ion exchange can be simulated by dividing first the coefficients D and V by the retardation factor, R (eq. 15). (Note: U in eqs. 52 and 55 would be given by $U = \sqrt{V^* + 4\lambda D^*}$). Temporal variations in source concentration can be simulated through the principle of superposition (eq. 39).

Description of Program FINITE

The analytical solution to the one-dimensional solute-transport equation for a finite system with a third-type (or first-type) source boundary condition at the inflow end is computed by the program FINITE, described in detail in the preceding section. The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 1.

The main program then calls subroutine ROOT3 to compute the positive roots of equation 53 when a third-type source boundary condition is specified and then executes a set of nested loops. The inner loop calls subroutine CNRML3 to calculate the concentration for a particular time value and distance; the outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time value. Graphs of concentration in relation to distance can also be plotted.

Subroutines ROOT3 and CNRML3

Subroutine ROOT3 calculates the roots of the equation $a \cdot \cot(a) - b \cdot a^2 + c = 0$. The procedure followed is similar to that for subroutine ROOT1 described in the preceding section), with an initial estimate for the first root taken as $\pi/2$.

Subroutine CNRML3 calculates the normalized concentration (C/C_o) for a particular time value and distance value using equation 52 for a solute subject to first-order chemical transformation and equation 54 if the solute is conservative ($\lambda = 0$). The number of terms taken in the infinite series summation is specified in the input data.

Sample Problem 2

In sample problem 2, the solute introduced into the soil column is assumed to be conservative. Model variables are identical to those in sample problem 1A and are

Velocity (V)	= 0.6 in/h
Longitudinal dispersion (D)	= 0.6 in^2/h
System length (L)	= 12 inches
Solute concentration opposite inflow boundary (C_o)	= 1.0 mg/L.

Concentrations are calculated for points 0.5 inch apart at elapsed times of 2.5, 5, 10, 15, and 20 hours.

The input data set for sample problem 2 is shown in figure 6A; a computer plot of concentration profiles generated by the program FINITE is shown in figure 6B. Output for this sample problem is presented in attachment 4. Sample problem 2 required 4.3 seconds of CPU time on a Prime model 9955 Mod II.

Comparison of figures 4B and 6B shows that the principal difference between the solutions for a first-type and a third-type source boundary condition is reflected in the solute concentrations near the inflow boundary at early times. As mentioned previously, these differences decrease with decreasing values for the quantity D/V.

Semi-Infinite System with First-Type Source Boundary Condition

Governing Equation

One-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - \lambda C \tag{56}$$

Boundary conditions:

$$C = C_o, \qquad x = 0 \tag{57}$$

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \infty \tag{58}$$

Initial condition:

$$C = 0, \qquad 0 < x < \infty \qquad \text{at } t = 0 \tag{59}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The longitudinal dispersion coefficient (D), which is equivalent to D_x (eq. 7), is constant.

A

```
Sample Problem 2 -- Solute transport in a finite-length
soil column with a third-type boundary condition at x=0
Model Parameters:  L=12 in, V=0.6 in/h, D=0.6 in**2/h
                   K1=0.0 per h, C0=1.0 mg/L
====
   3  25  05  50   1
MG/L      IN/H       IN**2/H   PER HOUR  INCHES    HOURS
      1.0       0.6       0.6       0.0      12.0       1.2
      0.0       0.5       1.0       1.5       2.0       2.5       3.0       3.5
      4.0       4.5       5.0       5.5       6.0       6.5       7.0       7.5
      8.0       8.5       9.0       9.5      10.0      10.5      11.0      11.5
     12.0
      2.5       5.0      10.0      15.0      20.0
```

B

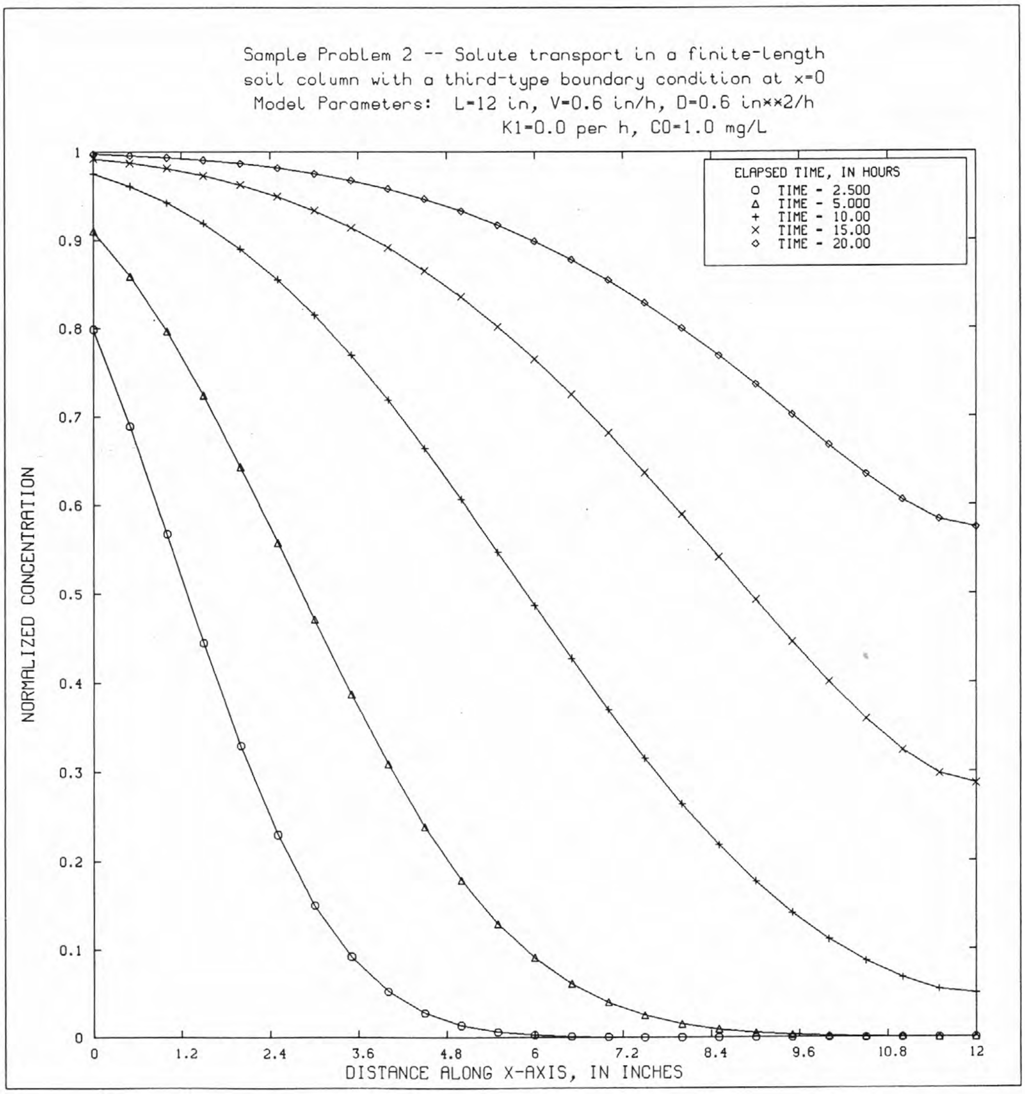


Figure 6.--(A) Sample input data set, and (B) computer plot of concentration profiles generated by the program FINITE for a conservative solute in a finite-length system with third-type source boundary condition after 2.5, 5, 10, 15, and 20 hours (sample problem 2).

Analytical Solution

The following equation, modified from Bear (1972, p. 630) and van Genuchten and Alves (1982, p. 60) can be expressed as

$$C(x,t) = \frac{C_o}{2}\left\{\exp\left[\frac{x}{2D}(V-U)\right] \cdot \operatorname{erfc}\left[\frac{x-Ut}{2\sqrt{Dt}}\right] + \exp\left[\frac{x}{2D}(V+U)\right] \cdot \operatorname{erfc}\left[\frac{x+Ut}{2\sqrt{Dt}}\right]\right\}, \qquad (60)$$

where $U = \sqrt{V^2 + 4\lambda D}$.

The analytical solution for a solute not subject to first-order chemical transformation ($\lambda = 0$) was derived by Ogata and Banks (1961) as

$$C(x,t) = \frac{C_o}{2}\left\{\operatorname{erfc}\left[\frac{x-Vt}{2\sqrt{Dt}}\right] + \exp\left[\frac{xV}{D}\right] \cdot \operatorname{erfc}\left[\frac{x+Vt}{2\sqrt{Dt}}\right]\right\}. \qquad (61)$$

For large values of time (steady-state solution), equation 60 reduces (modified from Bear, 1972, p. 631) to

$$C(x) = C_o \exp\left[\frac{x}{2D}(V-U)\right]. \qquad (62)$$

Comments:

Equations 60 and 61 are presented in this form to utilize computer routines that accurately compute the product of an exponential term (exp [x]) and the complementary error function (denoted as erfc [y]).

Linear equilibrium adsorption and ion exchange can be simulated by first dividing the coefficients D and V by the retardation factor, R (eq. 15). (Note: U in eqs. 60 and 62 would be given by $U = \sqrt{V^* + 4\lambda D^*}$). Temporal variations in source concentration can be simulated through the principle of superposition (eq. 39).

Description of Program SEMINF

The program SEMINF computes the analytical solution to the one-dimensional solute-transport equation for a semi-infinite system with a first-type or third-type source boundary condition at the inflow end. It consists of a main program and two subroutines--CNRML1 and CNRML3. The function of the main program and subroutine CNRML1 are outlined below; the program code listing is presented in attachment 2. Subroutine CNRML3 is called when a third-type boundary condition is specified and is described in a subsequent section.

The program also calls the subroutine EXERFC and the output subroutines TITLE, OFILE and PLOT1D, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 2.

The program next executes a set of nested loops. The inner loop calls subroutine CNRML1 to calculate the concentration for a particular time value and distance. The outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time value. Graphs of concentration in relation to distance can also be plotted.

Subroutine CNRML1

Subroutine CNRML1 calculates the normalized concentration (C/C_o) for a particular time value and distance using equation 60 for a solute subject to first-order chemical transformation and using equation 61 if the solute is conservative ($\lambda = 0$).

Sample Problems 3A and 3B

Two sample problems are presented. In sample problem 3A, a conservative solute is introduced into a long soil column. The system is idealized as being semi-infinite in length with model variables as

Velocity (V)	= 0.6 in/h
Longitudinal dispersion (D)	= 0.6 in^2/h
Solute concentration at inflow boundary (C_o)	= 1.0 mg/L.

Concentrations are calculated for points 0.5 inch apart at elapsed times of 2.5, 5, 10, 15, and 20 hours.

In sample problem 3B, solute is removed by both first-order solute decay and linear equilibrium adsorption. Additional model variables are

Solute half life ($T_{1/2}$)	= 7.6 days
Soil bulk density (ρ_b)	= 0.047 lb(mass)/in^3
Porosity (n)	= 0.45
Slope of adsorption isotherm (k)	= 70 in^3/lb(mass).

From these values, the following terms are obtained using equations 15 and 25:

Decay constant (λ)	= 0.0038 per hour
Retardation factor (R)	= 8.31
Scaled velocity (V^*)	= 0.072 in/h
Scaled dispersion coefficient (D^*)	= 0.072 in^2/h.

Table 2.--Input data format for the program SEMINF

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NBC	Boundary condition type (NBC = 1 for first-type boundary condition; NBC = 3 for third-type boundary condition).
	5 - 8	I4	NX	Number of x-coordinates at which solution will be evaluated.
	9 - 12	I4	NT	Number of time values at which solution will be evaluated.
	13 - 16	I4	IPLT	Plot control variable. Concentration profiles will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.[1]
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.[1]
	31 - 40	F10.0	DK	First-order solute decay coefficient.[1]
	41 - 50	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
5	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
6	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).

[1]All units must be consistent.

Concentrations are calculated for points 0.5 inch apart at elapsed times of 20, 50, 100, and 150 hours.

Input data sets for sample problems 3A and 3B are shown in figures 7A and 8A; computer plots of concentration profiles generated by the program SEMINF are also shown. Output for these sample problems is presented in attachment 4. Sample problems 3A and 3B each required 3 seconds of CPU time on a Prime model 9955 Mod II.

Comparison of the concentration profiles at 20 hours in each plot (figs. 7B and 8B) shows the effect of both solute decay and adsorption on solute movement. Comparison of figures 7B and 4B shows the difference in soil profiles that would result if the solution for a semi-infinite system was used to simulate transport in a finite system. The most significant difference is the steeper concentration gradients near x = 12.0 inches (fig. 7B). As mentioned previously, differences between the two solutions decrease with increased column Peclet number (P) and lower values for the number of displaced pore volumes (T).

Semi-Infinite System with Third-Type Source Boundary Condition

Governing Equation

One-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (63)$$

Boundary conditions:

$$VC_o = VC + D \frac{\partial C}{\partial x}, \qquad x = 0 \qquad (64)$$

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \infty \qquad (65)$$

Initial condition:

$$C = 0, \qquad 0 < x < \infty \qquad \text{at } t = 0 \qquad (66)$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The longitudinal dispersion coefficient (D), which is equivalent to D_x (eq. 7), is constant.

A

```
Sample Problem 3a -- Solute transport in a semi-infinite
soil column with a first-type boundary condition at x=0
Model Parameters:  V=0.6 in/h, D=0.6 in**2/h
                   K1=0.0 per h, C0=1.0 mg/L
====
   1  25  05   1
MG/L      IN/H       IN**2/H   PER HOUR  INCHES     HOURS
      1.0       0.6       0.6       0.0       1.2
      0.0       0.5       1.0       1.5       2.0       2.5       3.0       3.5
      4.0       4.5       5.0       5.5       6.0       6.5       7.0       7.5
      8.0       8.5       9.0       9.5      10.0      10.5      11.0      11.5
     12.0
      2.5       5.0      10.0      15.0      20.0
```

B

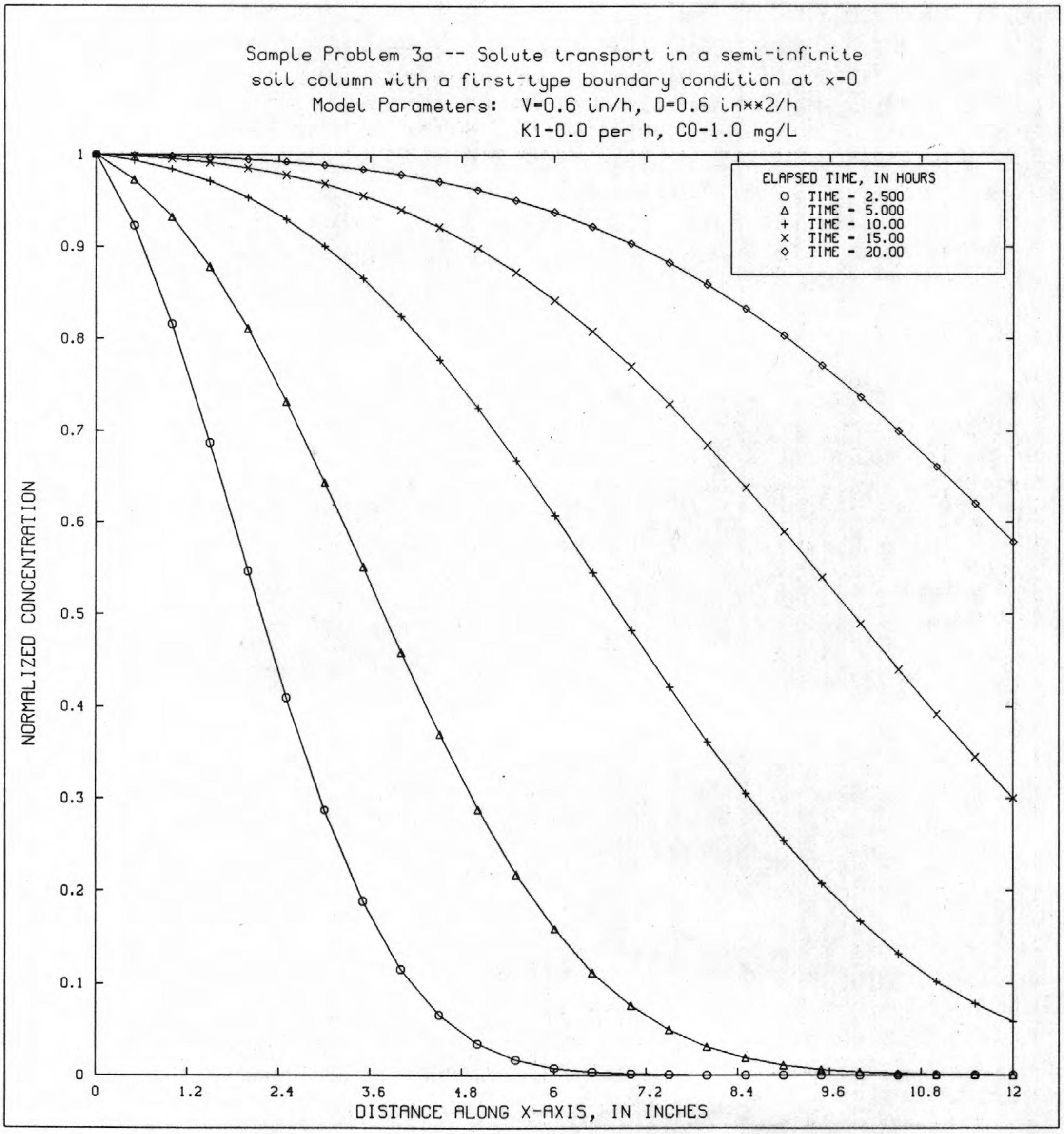


Figure 7.--(A) Sample input data set, and (B) computer plot of concentration profiles generated by the program SEMINF for a conservative solute in a semi-infinite system with first-type source boundary condition after 2.5, 5, 10, 15, and 20 hours (sample problem 3A).

A

```
Sample Problem 3b -- Solute transport in a semi-infinite
soil column with a first-type boundary condition at x=0
Model Parameters:  V=0.072 in/h, D=0.072 in**2/h
                   K1=0.0038 per h, C0=1.0 mg/L
Solute is subject to first-order decay and linear adsorption
====
   1  25  04   1
MG/L      IN/H      IN**2/H   PER HOUR  INCHES    HOURS
       1.0     0.072     0.072    0.0038       1.2
       0.0       0.5       1.0       1.5       2.0       2.5       3.0       3.5
       4.0       4.5       5.0       5.5       6.0       6.5       7.0       7.5
       8.0       8.5       9.0       9.5      10.0      10.5      11.0      11.5
      12.0
      20.0      50.0     100.0     150.0
```

B

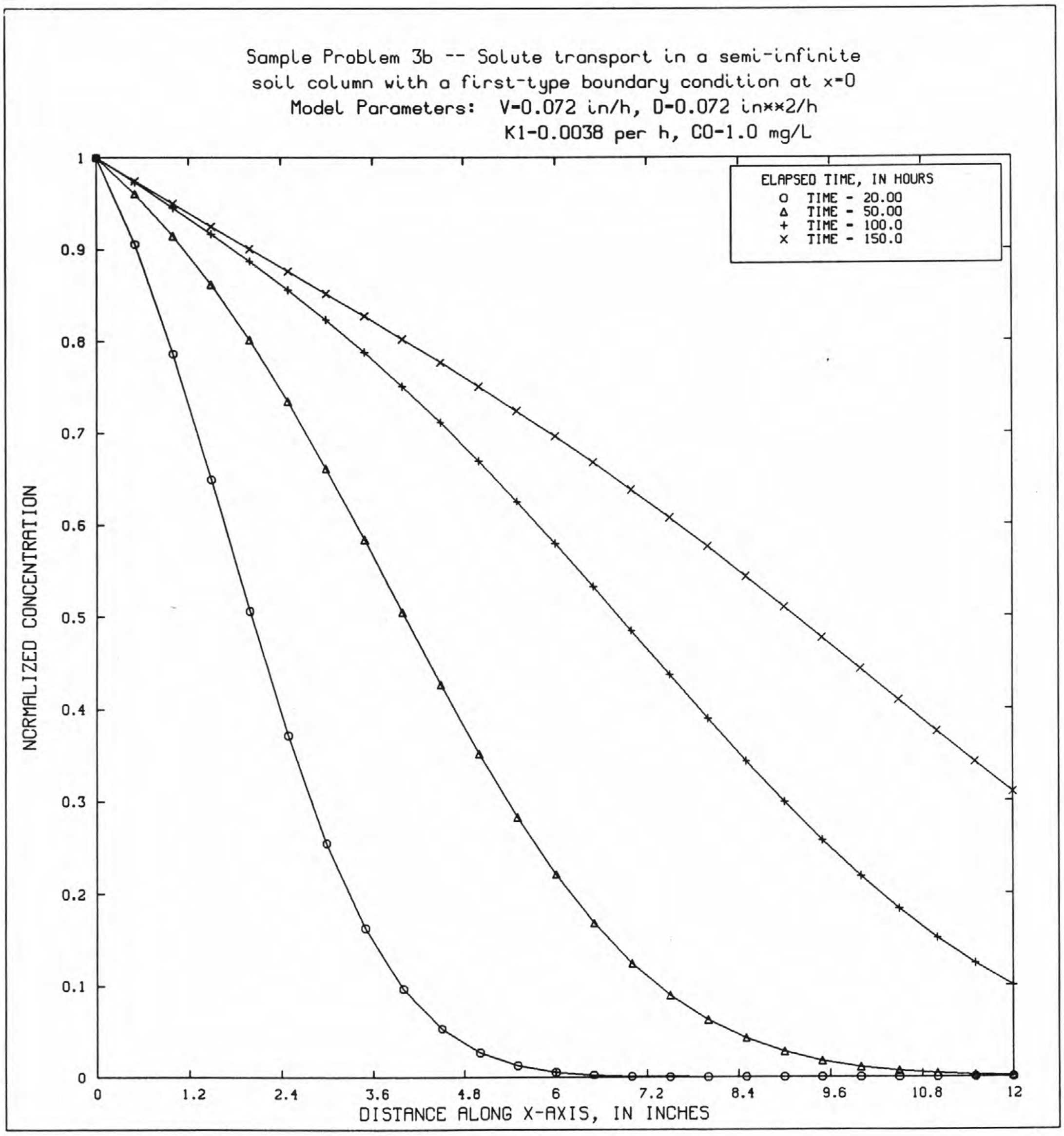


Figure 8.--(A) Sample input data set, and (B) computer plot of concentration profiles generated by the program SEMINF for a solute subject to first-order decay and linear equilibrium adsorption in a semi-infinite system with first-type source boundary condition after 20, 50, 100, and 150 hours (sample problem 3B).

Analytical Solution

The following equation is modified from Cleary and Ungs (1978, p. 10) and can be expressed as

$$C(x,t) = \frac{C_o V^2}{4D\lambda} \left\{ 2 \exp\left[\frac{xV}{D} - \lambda t\right] \cdot \operatorname{erfc}\left[\frac{x+Vt}{2\sqrt{Dt}}\right] + \left(\frac{U}{V} - 1\right) \exp\left[\frac{x}{2D}(V-U)\right] \cdot \operatorname{erfc}\left[\frac{x-Ut}{2\sqrt{Dt}}\right] - \left(\frac{U}{V} + 1\right) \exp\left[\frac{x}{2D}(V+U)\right] \cdot \operatorname{erfc}\left[\frac{x+Ut}{2\sqrt{Dt}}\right] \right\}, \quad (67)$$

where $U = \sqrt{V^2 + 4\lambda D}$.

For a conservative solute ($\lambda = 0$), the solution to equation 63 is given by Lindstrom and others (1967) and van Genuchten and Alves (1982, p. 10) as

$$C(x,t) = C_o \left\{ \frac{1}{2} \operatorname{erfc}\left[\frac{x-Vt}{2\sqrt{Dt}}\right] + \sqrt{\frac{V^2 t}{\pi D}} \exp\left[-\frac{(x-Vt)^2}{4Dt}\right] - \frac{1}{2}\left[1 + \frac{Vx}{D} + \frac{V^2 t}{D}\right] \exp\left(\frac{Vx}{D}\right) \cdot \operatorname{erfc}\left[\frac{x+Vt}{2\sqrt{Dt}}\right] \right\}. \quad (68)$$

For large values of time (steady-state solution), equation 67 can be reduced (Gershon and Nir, 1969, p. 837) to

$$C(x) = C_o \frac{2V}{(V+U)} \cdot \exp\left[\frac{x}{2D}(V-U)\right]. \quad (69)$$

Comments:

Equations 67 and 68 are presented in this form to utilize computer routines that compute the product of an exponential term and the complementary error function. For extremely small values of λ, calculations of concentration values using equation 67 may be subject to round-off errors as the denominator in the first term and terms within the bracket both approach zero.

Linear equilibrium adsorption can be simulated by dividing the coefficients D and V by the retardation factor, R (eq. 15). Temporal variations in source concentration can be simulated through the principle of superposition (eq. 39).

Description of Program SEMINIF

The analytical solution to the one-dimensional solute-transport equation for a semi-infinite system with a third-type (or first-type) source boundary condition is computed by the program SEMINF, described in detail in the preceding section. The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 2.

The program next executes a set of nested loops. The inner loop calls subroutine CNRML3 to calculate the concentration for a particular time value and distance. The outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time value. Graphs of concentration in relation to distance can also be plotted.

Subroutine CNRML3

Subroutine CNRML3 calculates the normalized concentration (C/C_o) for a particular time value and distance using equation 67 for a solute subject to first-order chemical transformation and using equation 68 if the solute is conservative ($\lambda = 0$).

Sample Problem 4

In sample problem 4, a conservative solute is introduced into a long soil column. The system is idealized as being semi-infinite in length with model variables as

Velocity (V)	= 0.6 in/h
Longitudinal dispersion (D)	= 0.6 in^2/h
Solute concentration opposite inflow boundary (C_o)	= 1.0 mg/L.

Concentrations are calculated for points spaced 0.5 inch apart at elapsed times of 2.5, 5, 10, 15, and 20 hours.

The input data set for sample problem 4 is shown in figure 9A; a computer plot of concentration profiles generated by the program SEMINF is shown in figure 9B. Because of the third-type boundary condition, solute concentration computed near x = 0 at early times differs from C_o.

Program output for this sample problem is presented in attachment 4. Sample problem 4 required 3.6 seconds of CPU time on a Prime model 9955 Mod II.

A

```
Sample Problem 4 -- Solute transport in a semi-infinite
soil column with a third-type boundary condition at x=0
Model Parameters:  V=0.6 in/h, D=0.6 in**2/h
                   K1=0.0 per h, C0=1.0 mg/L
====
   3  25  05   1
MG/L      IN/H      IN**2/H   PER HOUR  INCHES    HOURS
         1.0       0.6       0.6       0.0       1.2
         0.0       0.5       1.0       1.5       2.0       2.5       3.0       3.5
         4.0       4.5       5.0       5.5       6.0       6.5       7.0       7.5
         8.0       8.5       9.0       9.5      10.0      10.5      11.0      11.5
        12.0
         2.5       5.0      10.0      15.0      20.0
```

B

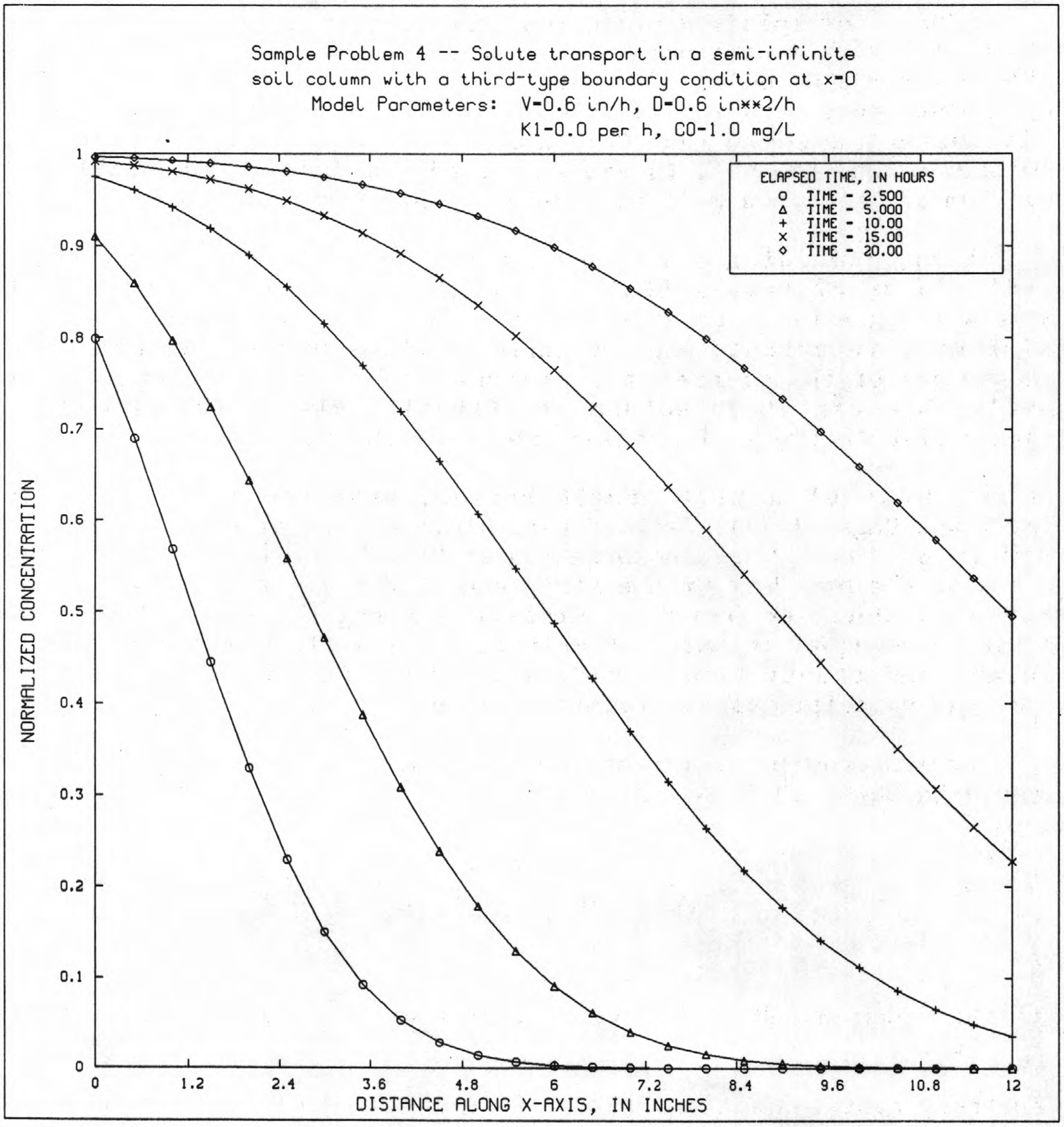


Figure 9.--(A) Sample input data set, and (B) computer plot of concentration profiles generated by the program SEMINF for a conservative solute in a semi-infinite system with third-type source boundary condition after 2.5, 5, 10, 15, and 20 hours (sample problem 4).

TWO-DIMENSIONAL SOLUTE TRANSPORT

Several analytical solutions are available for the two-dimensional form of the solute-transport equation (eq. 10). These solutions can be used to simulate transport of contaminants from sources within relatively thin aquifers, provided that the solute is generally well mixed throughout the thickness of the aquifer and vertical concentration gradients are negligible. Transport of contaminants within a vertical section along the centerline of a contaminant plume in a thick aquifer can be simulated with these solutions if the solute source is wide enough such that horizontal concentration gradients, which cause solute movement perpendicular to the centerline, are negligible.

In the first solution presented, the aquifer is assumed to be of infinite areal extent with a continuous point source in the x,y plane (equivalent to a line source extending the entire thickness of the aquifer). Fluid with a known solute concentration is injected into the aquifer at a constant rate. It is further assumed that the injection rate is small, and that the uniform flowfield around the well is not disturbed. Solutions in which radial flow away from an injection well is considered are discussed by Hseih (1986).

For the remaining solutions presented in this section, aquifers are assumed to be of semi-infinite length with a solute source at the inflow boundary (at x = 0). The width of the aquifer can be treated as being finite or infinite in extent. In the infinite-width system, impermeable boundaries at the edges of the aquifer are presumed to be far enough away as to have a negligible effect on solute distribution within the aquifer. Idealized diagrams of both types of systems are shown in figure 10.

One type of source configuration, referred to as a "strip" source (Cleary and Ungs, 1978), has a finite width extending from $y = Y_1$ to $y = Y_2$ at x = 0 (fig. 10). The concentration within the strip is uniform and equal to C_o, and at the boundary of the strip source (at $y = Y_1$ or $y = Y_2$), the concentration is equal to $0.5\ C_o$. Elsewhere along the inflow boundary, the concentration is equal to zero. Combinations of strip sources could be used to simulate odd-shaped concentration distributions or multiple sources through use of the principle of superposition as previously described.

A solute source can also have a "gaussian" concentration distribution (Cleary and Ungs, 1978, p. 80) given by

$$C = C_m \exp\left[\frac{-(y-Y_c)^2}{2\sigma^2}\right], \qquad x = 0, \qquad (70)$$

where

C_m is the maximum concentration at center of gaussian concentration distribution,

Y_c is the y-coordinate of center of solute source ($X_c = 0$), and

σ is the standard deviation of gaussian distribution.

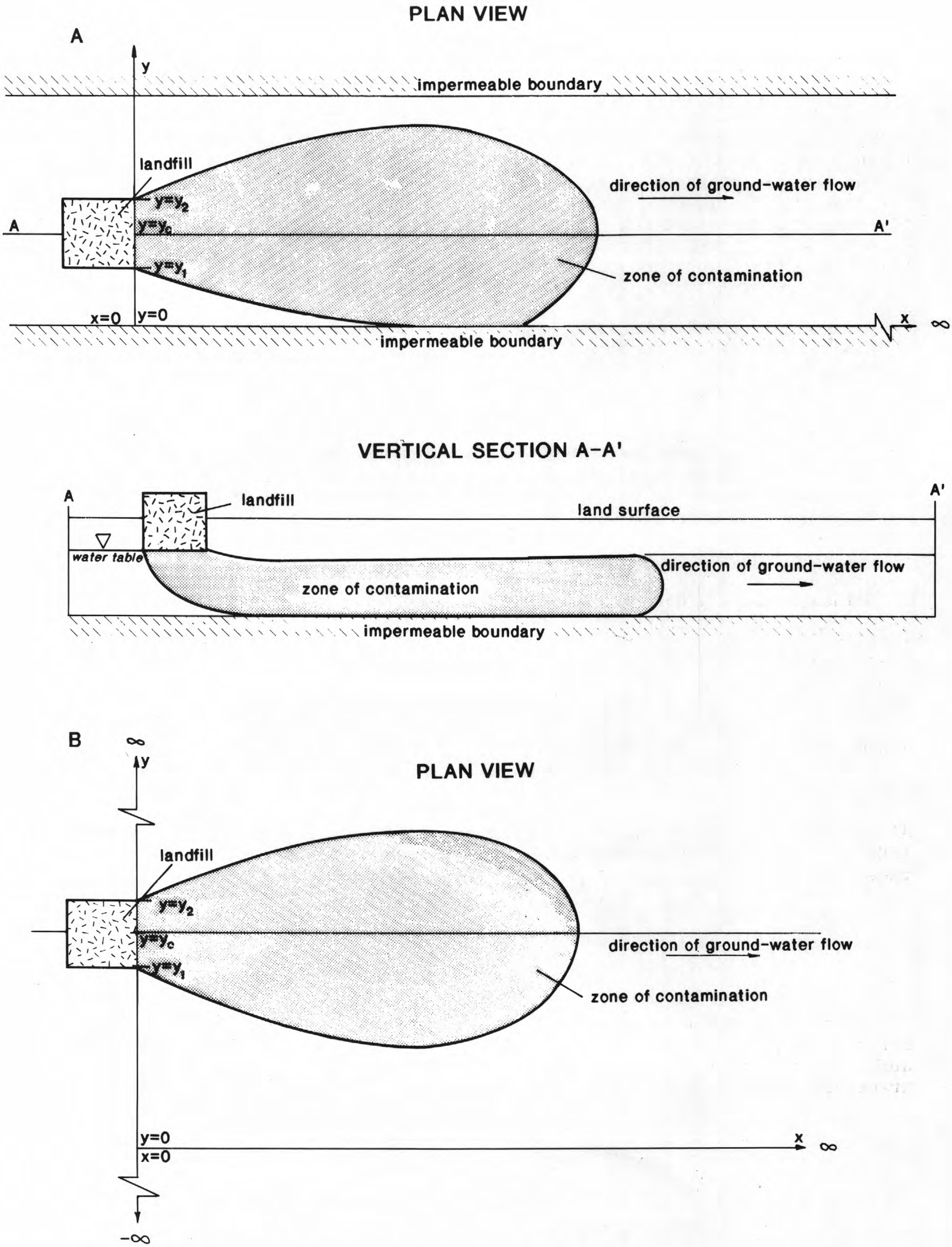


Figure 10.--(A) Plan view and vertical section of idealized two-dimensional solute transport in an aquifer of semi-infinite length and finite width, and (B) plan view of idealized two-dimensional solute transport in an aquifer of semi-infinite length and infinite width.

A field situation in which a gaussian distribution may be found is shown in figure 11, in which the solute concentration at the waste-disposal pond is unknown, but a line of monitoring wells downgradient from the site and normal to the direction of flow shows a concentration distribution that approximates a gaussian curve. (This is expected as the concentration distribution along a cross section normal to the direction of flow taken at any point downgradient of an ideal point source would be gaussian.) The standard deviation of the distribution can be determined from the data as

$$\sigma = \frac{(y-Y_c)}{\sqrt{-2 \ln (C/C_m)}}, \tag{71}$$

where C is the concentration observed at a well a distance $(y-Y_c)$ away from the point of maximum concentration.

Solving equation 71 may lead to differing values of σ if the observed data are not perfectly gaussian. An alternative procedure (R.M. Cleary, Princeton University, written commun., 1978) is to: (1) Normalize the data by dividing the observed concentrations by C_m, (2) plot a histogram of the normalized concentration with respect to y, and (3) calculate the area under the curve. The standard deviation can be approximated by $\sigma = \text{area}/\sqrt{2\pi}$. A sample problem illustrating the use of both methods is presented later.

This section presents analytical solutions for a/an:

- Aquifer of infinite areal extent with a continuous point source in which fluid is injected at a constant rate and concentration,
- Semi-infinite aquifer of finite width with a strip source,
- Semi-infinite aquifer of infinite width with a strip source,
- Semi-infinite aquifer of infinite width with a gaussian source.

All solutions can account for first-order solute decay. Four computer programs (POINT2, STRIPF, STRIPI and GAUSS) were written to calculate concentrations in these systems as a function of distance and elapsed time.

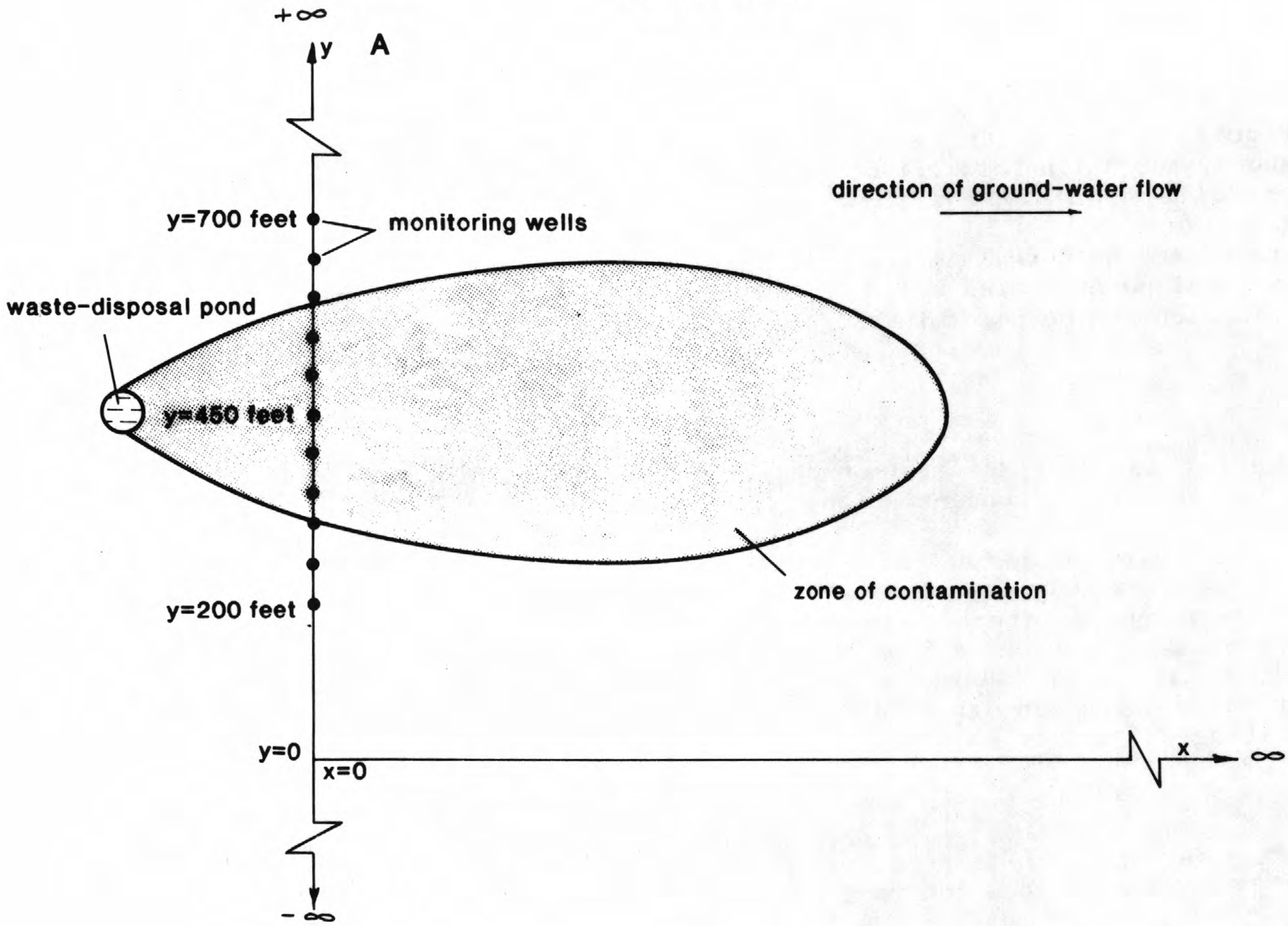
Aquifer of Infinite Areal Extent with Continuous Point Source

Governing Equation

The analytical solution for a continuous point source has been derived by first solving the solute transport equation for an instantaneous point source and then integrating the solution over time. The two-dimensional solute-transport equation for an instantaneous point source is given by

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x} - \lambda C + Q' \, dt \, C_o \, \delta(x-X_c) \, \delta(y-Y_c) \, \delta(t-t') \tag{72}$$

PLAN VIEW



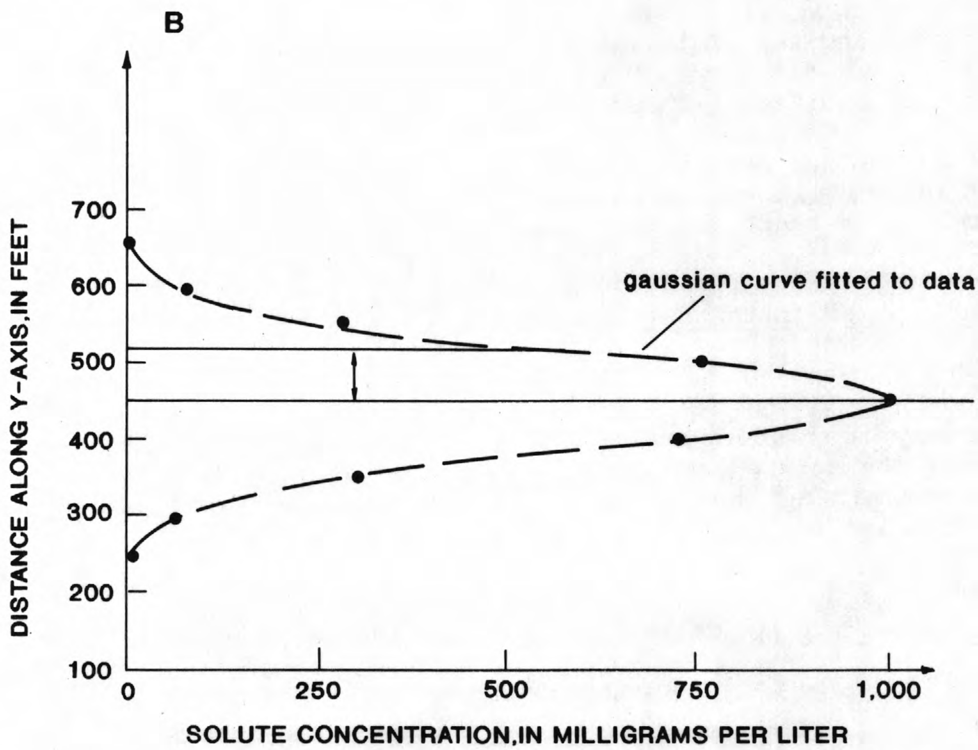


Figure 11.--(A) Plan view of a semi-infinite aquifer of infinite width showing location of waste-disposal pond and monitoring wells, and (B) observed solute concentration values and gaussian curve used to approximate concentration distribution at x = 0.

Boundary conditions:

$$C,\ \frac{\partial C}{\partial x} = 0, \qquad x = \pm\infty \tag{73}$$

$$C,\ \frac{\partial C}{\partial y} = 0, \qquad y = \pm\infty, \tag{74}$$

where

V is V_x, the velocity in x-direction,

X_c, Y_c are the x- and y-coordinates of the point source,

Q' is the fluid injection rate per unit thickness of aquifer,

dt is an infinitesimal time interval,

$\delta(\quad)$ is the Dirac delta (impulse) function, and

t' is the instant at which the point source occurs (assumed to equal 0).

Initial condition:

$$C = 0, \qquad -\infty < y < +\infty \text{ and } -\infty < x < +\infty \qquad \text{at } t = 0 \tag{75}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant (no radial flow).
4. The longitudinal and transverse dispersion coefficients (D_x and D_y) are constant.

Analytical Solution

The following equation, modified from Bear (1979, p. 274), represents the analytical solution for an instantaneous point source integrated with respect to time, such that

$$C(x,y,t) = \frac{C_o\, Q'}{4\pi\sqrt{D_x D_y}} \exp\left[\frac{V(x-X_c)}{2D_x}\right] \int_0^t \frac{1}{\tau}$$

$$\cdot \exp\left[-\left(\frac{V^2}{4D_x} + \lambda\right)\tau - \frac{(x-X_c)^2}{4D_x\tau} - \frac{(y-Y_c)^2}{4D_y\tau}\right] d\tau, \tag{76}$$

where τ is the dummy variable of integration for the time integral.

The steady-state solution is given (modified from Bear, 1979, p. 274) as

$$C(x,y) = \frac{Q' \; C_o \; \exp\left[\frac{V(x-X_c)}{2Dx}\right]}{2\pi\sqrt{D_x D_y}} K_o \left\{\sqrt{\left(\frac{V^2}{4D_x} + \lambda\right) \left[\frac{(x-X_c)^2}{D_x} + \frac{(y-Y_c)^2}{D_y}\right]}\right\}, \quad (77)$$

where K_o is the modified Bessel function of second kind and zero order. Tables of values and polynomial approximations for $K_o(x)$ are given by Abramowitz and Stegun (1964, p. 37 and p. 417-422).

Comments:

The integral in equation 76 could not be simplified further and must, therefore, be evaluated numerically. A Gauss-Legendre numerical integration technique, used in the computer program written to evaluate the analytical solution (eq. 76), is described later.

The integral in equation 76 is difficult to evaluate correctly at x and y values near the point source. (Mathematically, when $(x-X_c)$ and $(y-Y_c)$ approach zero, the integral in equation 76 becomes a form of the exponential integral, $E_1(t)$, which becomes infinite at $t = 0$; see Abramowitz and Stegun, 1964, p. 228.) Farther away from the point source, generally when $(x-X_c)^2$ is larger than V^2, a meaningful solution can be obtained.

Linear equilibrium adsorption and ion exchange can be simulated by first dividing the coefficients D_x, D_y, and V by the retardation factor, R (eq. 15). Temporal variations in source concentration or multiple sources can be simulated through the principle of superposition.

Description of Program POINT2

The program POINT2 computes the analytical solution to the two-dimensional solute-transport equation for an aquifer of infinite areal extent with a continuous point source. It consists of a main program and the subroutine CNRML2. The function of the main program and subroutine are outlined below; the program code listing is presented in attachment 2.

The program also calls subroutine GLQPTS and the output subroutines TITLE, OFILE, PLOT2D, and CNTOUR, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 3. The routine then calls the subroutine GLQPTS which reads the data file GLQ.PTS containing values of the positive roots and weighting functions used in the Gauss-Legendre numerical integration technique.

Table 3.--Input data format for the program POINT2

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NT	Number of time values at which solution will be evaluated.
	13 - 16	I4	NMAX	Number of terms used in the numerical integration technique (must be equal to 4, 20, 60, 104, or 256).
	17 - 20	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration in injected fluid.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient.
	41 - 50	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	XC	X-coordinate of point source.
	11 - 20	F10.0	YC	Y-coordinate of point source.
	21 - 30	F10.0	QM	Fluid injection rate per unit thickness of aquifer[1].
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[2]9	1 - 10	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]For the solution to be consistent, units of QM must be identical to those of the dispersion coefficients

[2]Data line is needed only if IPLT (in data set 2) is greater than 0.

The program next executes a set of three nested loops. The inner loop calls subroutine CNRML2 to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value and time. The middle loop cycles through all x-coordinate values. The outer loop cycles through all specified time values and prints a table of concentrations in relation to distance for each time value. Model output can also be plotted as a map showing lines of equal solute concentration.

Subroutine CNRML2

Subroutine CNRML2 calculates the normalized concentration (C/C_o) for a particular time value and distance. The integral in equation 76 is evaluated through a Guass-Legendre numerical integration technique. The Gauss integration formula used is given by Abramowitz and Stegun (1964) as

$$\int_{-1}^{1} f(x)dx = \sum_{i=1}^{n} w_i \cdot f(z_i), \tag{78}$$

where

n is the order of the Legendre polynomial,

z_i are the roots of the n^{th} order polynomial,

w_i are the weighting functions, and

$f(z_i)$ is the value of the integrand calculated with the variable of integration equal to z_i.

The normalized roots of the Legendre polynomial and the corresponding weighting functions are passed by subroutine GLQPTS and scaled in the subroutine to account for the non-normalized limits of integration (from 0 to t rather than -1 to +1).

The number of terms summed in the numerical integration (equivalent to the order of the polynomial) is specified by the user. Roots of the Legendre polynomial of order 4, 20, 60, 104, and 256 (from data in Cleary and Ungs, 1978) are provided in data file GLQ.PTS. In general, the more terms used in the integration, the more accurate the approximation; however, this must be weighed against the corresponding increase in computational effort and time. Additional discussions on the numerical integration technique are presented in a later section describing subroutine GLQPTS.

Sample Problem 5

In sample problem 5, an abandoned borehole that penetrates a brackish artesian formation is discharging into an overlying freshwater aquifer. Model variables are

Aquifer thickness	= 100 feet
Discharge rate	= 5,000 ft^3/d
Ground-water velocity (V_x)	= 2 ft/d
Longitudinal dispersivity (a_l)	= 30 feet
Transverse dispersivity (a_t)	= 6 feet
Source concentration (C_o)	= 1,000 mg/L
Point-source location (X_c,Y_c)	= 0, 500 feet.

From these values, the terms obtained are

Discharge rate per unit thickness of aquifer (Q')	= 50 ft^2/d
Coefficient of longitudinal dispersion (D_x)	= 60 ft^2/d
Coefficient of transverse dispersion (D_y)	= 12 ft^2/d.

Concentrations are calculated at 10-foot intervals along the x-axis from x = -60 feet to x = 200 feet and at 5-foot intervals along the y-axis from y = 450 feet to y = 550 feet. Chloride concentration distribution after 25 days and 100 days is simulated.

The input data set for sample problem 5 is shown in figure 12A. A computer-generated contour plot of normalized concentrations (C/C_o) at both time values is shown in figure 12B. Program output for this sample problem is presented in attachment 4. Sample problem 5 required 9 seconds of CPU time on a Prime model 9955 Mod II.

Aquifer of Finite Width with Finite-Width Solute Source

Governing Equation

Two-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (79)$$

Boundary conditions:

$$C = C_o, \qquad x = 0 \text{ and } Y_c - W_s/2 < y < Y_c + W_s/2 \qquad (80)$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = 0 \qquad (81)$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = W \qquad (82)$$

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \infty \qquad (83)$$

A

```
Sample Problem 5 -- Solute transport in an aquifer of
infinite areal extent with a continuous point source
Model Data:   V=2.0 ft/d, DX=60.0 ft**2/d, DY=12.0 ft**2/d
              QM=50.0 ft**2/d, C0=1000.0 mg/L
====
  26  21  02 104    3
MG/L        FT/D       FT**2/D   PER DAY   FEET       DAYS
    1000.0       2.0      60.0      12.0
       0.0     500.0      50.0
     -60.0     -50.0    -40.00     -30.0     -20.0     -10.0      10.0      20.0
      30.0      40.0      50.0      60.0      70.0      80.0      90.0     100.0
     110.0     120.0     130.0     140.0     150.0     160.0     170.0     180.0
     190.0     200.0
     450.0     455.0     460.0     465.0     470.0     475.0     480.0     485.0
     490.0     495.0     500.0     505.0     510.0     515.0     520.0     525.0
     530.0     535.0     540.0     545.0     550.0
      25.0     100.0
      30.0      30.0       0.1
```

B

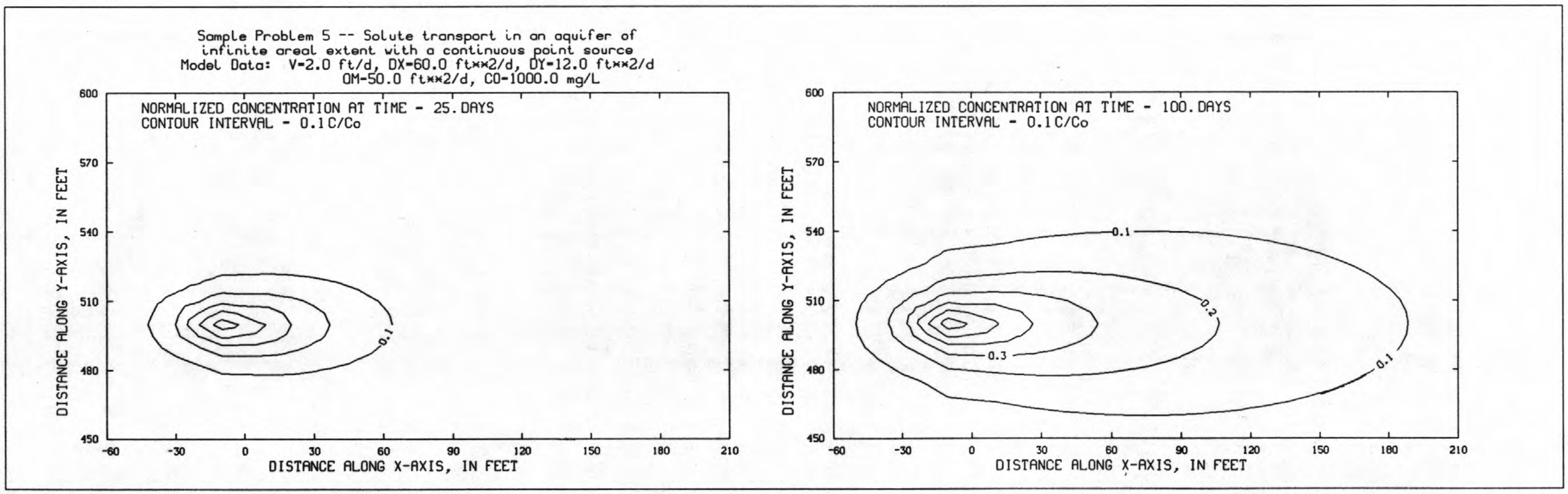


Figure 12.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program POINT2 for a conservative solute injected continuously into an aquifer of infinite areal extent after 25 and 100 days (sample problem 5).

where

V is V_x, the velocity in x-direction,

W is the aquifer width,

W_s is solute-source width, and

Y_c is y-coordinate of center of solute source ($X_c = 0$).

Initial condition:

$$C = 0, \qquad 0 < x < \infty \text{ and } 0 < y < W \qquad \text{at } t = 0 \tag{84}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The longitudinal and transverse dispersion coefficients (D_x, D_y) are constant.

Analytical Solution

The following equation is modified from Hewson (1976) and can be expressed as

$$C(x,y,t) = C_o \sum_{n=0}^{\infty} L_n P_n \cos(\eta y)$$

$$\cdot \left\{ \exp\left[\frac{x(v-\beta)}{2D_x}\right] \operatorname{erfc}\left[\frac{x-\beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\beta)}{2D_x}\right] \operatorname{erfc}\left[\frac{x+\beta t}{2\sqrt{D_x t}}\right] \right\}, \tag{85}$$

where

$$L_n = \begin{cases} 1/2, & n = 0 \\ 1, & n > 0 \end{cases}$$

$$P_n = \begin{cases} \dfrac{Y_2 - Y_1}{W} & n = 0 \\ \dfrac{[\sin(\eta Y_2) - \sin(\eta Y_1)]}{n\pi}, & n > 0 \end{cases}$$

$$Y_1 = Y_c - W_s/2$$

$$Y_2 = Y_c + W_s/2$$

$$\eta = n\pi/W, \qquad n = 0,1,2,3. \ . \ .$$

$$\beta = \sqrt{V^2 + 4D_x (\eta^2 D_y + \lambda)}$$

Comments:

Terms in the infinite series in equation 85 tend to oscillate, and the series converges slowly for small values of x; thus, a large number of terms may be needed to assure convergence. A good initial estimate is 100 terms. For larger values of x, the series converges more quickly.

The solution can yield results with either D_y or $\lambda = 0$. Linear equilibrium adsorption and ion exchange can be simulated by first dividing the coefficients D_x, D_y, and V by the retardation factor, R (eq. 15). Temporal variations in solute concentration and odd-shaped source configurations can be simulated through the principle of superposition.

Description of Program STRIPF

The program STRIPF computes the analytical solution to the two-dimensional solute-transport equation for an aquifer of finite width with a finite-width or "strip" solute source at the inflow boundary. It consists of a main program and subroutine CNRMLF. The function of the main program and subroutine are outlined below; the program code listing is presented in attachment 2.

The program also calls the subroutine EXERFC and the output subroutines TITLE, OFILE, PLOT2D, and CNTOUR, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 4.

The program next executes a set of three nested loops. The inner loop calls subroutine CNRLMF to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value and time. The middle loop cycles through all x-coordinate values. The outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time. Model output can also be plotted as a map showing lines of equal solute concentration.

Subroutine CNRMLF

Subroutine CNRMLF calculates the normalized concentration (C/C_o) for a particular time value and distance using equation 85. The maximum number of terms in the infinite series summation is specified by the user. Because terms in the series tend to oscillate, a subtotal of the last 10 terms is kept, and when the subtotal is less than a convergence criterion set at 1×10^{-12}, the series summation is halted. If the series does not converge after the specified maximum number of terms are taken, a warning message is printed on the program output.

Table 4.--Input data format for the program STRIPF

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NT	Number of time values at which solution will be evaluated.
	13 - 16	I4	NMAX	Maximum number of terms used in the infinite series summation.
	17 - 20	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	C0	Solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient.
	41 - 50	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	W	Aquifer width (aquifer extends from y = 0 to y = W).
	11 - 20	F10.0	YC	Y-coordinate of center of finite-width solute source.
	21 - 30	F10.0	WS	Full width of finite-width (strip) solute source.
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[1]9	1 - 10	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]Data line is needed only if IPLT (in data set 2) is greater than 0.

Sample Problem 6

In sample problem 6, migration of chloride ion in landfill leachate through a narrow, relatively thin, valley-fill aquifer is simulated. Model variables are

Aquifer width (W)	= 3,000 feet
Source width (W_s)	= 1,600 feet
Location of source center (Y_c)	= 1,200 feet
Ground-water velocity (V_x)	= 1 ft/d
Longitudinal dispersivity (a_l)	= 200 feet
Transverse dispersivity (a_t)	= 60 feet
Source concentration (C_o)	= 1,000 mg/L.

From these values, the terms obtained are

Dispersion in x-direction (D_x)	= 200 ft^2/d
Dispersion in y-direction (D_y)	= 60 ft^2/d.

Concentrations are calculated at 150-foot intervals along the x-axis for 4,500 feet and at 100-foot intervals along the y-axis for 3,000 feet. Chloride concentration distribution after 1,500 and 3,000 days is simulated.

The input data set for sample problem 6 is shown in figure 13A. A computer-generated contour plot of normalized concentration (C/C_o) at both time values is shown in figure 13B. The lack of symmetry about the center line of the chloride plume is due to the effect of the closer lateral boundary (at y = 0). Lines of equal concentration are perpendicular to the lateral boundary, indicating that concentration gradients in the y-direction equal zero and, thus, no solute flux occurs across the boundary. Program output for this sample problem is presented in attachment 4. Sample problem 6 required 52 seconds of CPU time on a Prime model 9955 Mod II.

Aquifer of Infinite Width with Finite-Width Solute Source

Governing Equation

Two-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (86)$$

Boundary conditions:

$$C = C_o, \qquad x = 0 \text{ and } Y_c - W_s/2 < y < Y_c + W_s/2 \qquad (87)$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = \pm\infty \qquad (88)$$

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \infty, \qquad (89)$$

A

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Sample Problem 6 -- Solute transport in a semi-infinite
aquifer of finite width with a continuous 'strip' source
Model Data:  V=1.0 ft/d, DX=200.0 ft**2/d, DY=60.0 ft**2/d
     W=3000 ft, WS=1600 ft, YC=1200 ft, C0=1000.0 mg/L
====
  31  27  02 300    1
MG/L        FT/D        FT**2/D   PER DAY   FEET       DAYS
    1000.0        1.0      200.0      60.0       0.0
    3000.0     1200.0     1600.0
       0.0      150.0      300.0     450.0     600.0     750.0     900.0    1050.0
    1200.0     1350.0     1500.0    1650.0    1800.0    1950.0    2100.0    2250.0
    2400.0     2550.0     2700.0    2850.0    3000.0    3150.0    3300.0    3450.0
    3600.0     3750.0     3900.0    4050.0    4200.0    4350.0    4500.0
       0.0      100.0      200.0     300.0     400.0     500.0     600.0     700.0
     800.0      900.0     1000.0    1100.0    1200.0    1300.0    1400.0    1500.0
    1600.0     1700.0     1800.0    1900.0    2000.0    2100.0    2200.0    2300.0
    2400.0     2500.0     2600.0    2700.0    2800.0    2900.0    3000.0
    1500.0     3000.0
      750.       750.        0.1
```

B

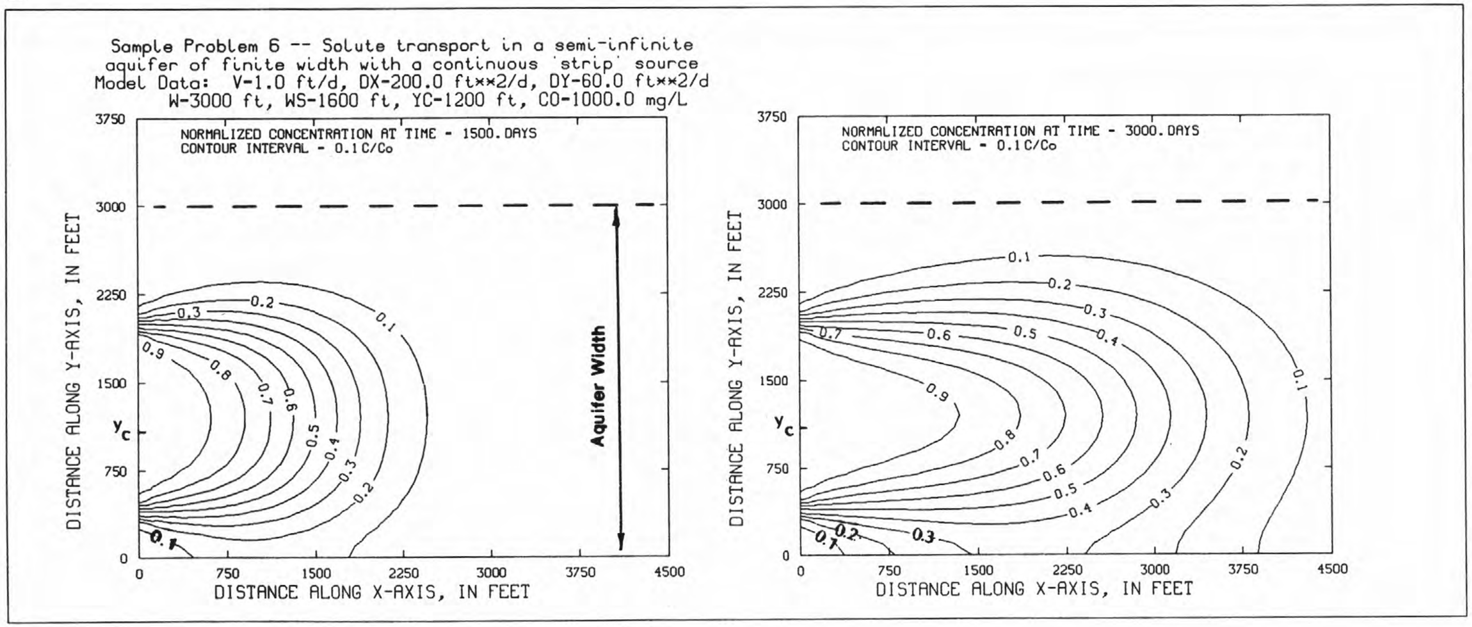

Figure 13.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program STRIPF for a conservative solute in an aquifer of finite width with finite-width solute source after 1,500 and 3,000 days (sample problem 6).

where

V is V_x, the velocity in x-direction,

W_s is the width of solute source, and

Y_c is the y-coordinate of center of solute source ($X_c = 0$).

Initial condition:

$$C = 0, \quad 0 < x < \infty \text{ and } -\infty < y < +\infty \quad \text{at } t = 0 \tag{90}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The longitudinal and transverse dispersion coefficients (D_x, D_y) are constant.

Analytical Solution

The following equation is modified from Cleary and Ungs (1978, p. 17) and can be expressed as

$$C(x,y,t) = \frac{C_o x}{4\sqrt{\pi D_x}} \exp\left(\frac{Vx}{2D_x}\right)$$
$$\cdot \int_{\tau=0}^{\tau=t} \tau^{-3/2} \exp\left[-\left(\frac{V^2}{4D_x} + \lambda\right)\tau - \frac{x^2}{4D_x\tau}\right]$$
$$\cdot \left\{\text{erfc}\left[\frac{(Y_1 - y)}{2\sqrt{D_y\tau}}\right] - \text{erfc}\left[\frac{(Y_2 - y)}{2\sqrt{D_y\tau}}\right]\right\} d\tau, \tag{91}$$

where

$$Y_1 = Y_c - W_s/2$$

and

$$Y_2 = Y_c - W_s/2.$$

Comments:

The integral in equation 91 could not be simplified further and must be evaluated numerically. A Gauss-Legendre numerical integration technique was used in the computer program written to evaluate the analytical solution and is described later.

Linear equilibrium adsorption and ion exchange can be simulated by dividing the coefficients D_x, D_y, and V by the retardation factor, R (eq. 15). Temporal variations in solute concentration and odd-shaped source configurations can be simulated through the principle of superposition.

Description of Program STRIPI

The program STRIPI computes the analytical solution to the two-dimensional solute-transport equation for an aquifer of infinite width with a finite-width or "strip" solute source at the inflow boundary. It consists of a main program and the subroutine CNRMLI. The function of the main program and subroutine are outlined below; the program code listing is presented in attachment 2.

The program also calls subroutines EXERFC and GLQPTS and the output subroutines TITLE, OFILE, PLOT2D, and CNTOUR, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 5. The routine then calls the subroutine GLQPTS which reads the data file GLQ.PTS containing values of the positive roots and weighting functions used in the Gauss-Legendre numerical integration technique.

The program next executes a set of three nested loops. The inner loop calls subroutine CNRMLI to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value and time. The middle loop cycles through all x-coordinate values. The outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time. Model output can also be plotted as a map showing lines of equal solute concentration.

Subroutine CNRMLI

Subroutine CNRMLI calculates the normalized concentrations (C/C_o) for a particular time value and distance. The integral in equation 91 is evaluated through a Guass-Legendre numerical integration technique. The normalized roots of the Legendre polynomial and the corresponding weighting functions are passed by subroutine GLQPTS and scaled in the subroutine to account for the non-normalized limits of integration (from 0 to t rather than -1 to +1).

The number of terms summed in the numerical integration (equivalent to the order of the polynomial) is specified by the user. Roots of the Legendre polynomial of order 4, 20, 60, 104, and 256 are provided in data file GLQ.PTS. In general, the more terms used in the integration, the more accurate the approximation; however, this must be weighed against the corresponding

Table 5.--Input data format for the program STRIPI

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NT	Number of time values at which solution will be evaluated.
	13 - 16	I4	NMAX	Number of terms used in the numerical integration techniques (must be equal to 4, 20, 60, 104, or 256).
	17 - 20	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient.
	41 - 50	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	YC	Y-coordinate of center of finite-width solute source.
	11 - 20	F10.0	WS	Full width of finite-width (strip) solute source.
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[1]9	1 - 10	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]Data line is needed only if IPLT (in data set 2) is greater than 0.

increase in computational effort and time. Additional discussions on the numerical integration technique are presented in a later section describing subroutine GLQPTS.

Sample Problem 7

In sample problem 7, contaminant migration from a waste-disposal pond through the upper glacial aquifer of Long Island, N.Y., is simulated. Data are taken from a numerical modeling study by Pinder (1973). Model variables are

Source width (W_s)	= 230 feet
Location of source center (Y_c)	= 750 feet
Ground-water velocity (V_x)	= 1.43 ft/d
Longitudinal dispersivity (a_l)	= 70 feet
Transverse dispersivity (a_t)	= 14 feet
Source concentration (C_o)	= 40 mg/L.

Lateral boundaries are far enough away from the area of interest so that the aquifer can be treated as being infinite in width. From these values, the terms obtained are

Dispersion in x-direction (D_x)	= 100 ft^2/d
Dispersion in y-direction (D_y)	= 20 ft^2/d.

Concentrations are calculated at 100-foot intervals along the x-axis for 3,000 feet and at 50-foot intervals on the y-axis for 1,500 feet. Concentration distributions after 5 years (1,826 days) are simulated.

The input data set for sample problem 7 is shown in figure 14A. A computer-generated contour plot of normalized concentration (C/C_o) at both time values is shown in figure 14B. Program output for this sample problem is presented in attachment 4. Sample problem 7 required 1 minute and 25 seconds of CPU time on a Prime model 9955 Mod II.

Aquifer of Infinite Width with Solute Source Having Gaussian Concentration Distribution

Governing Equation

Two-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (92)$$

Boundary conditions:

$$C = C_m \exp\left[\frac{-(y-Y_c)^2}{2\sigma^2}\right], \qquad x = 0 \qquad (93)$$

$$C,\ \frac{\partial C}{\partial y} = 0, \qquad y = \pm\infty \qquad (94)$$

$$C,\ \frac{\partial C}{\partial x} = 0, \qquad x = \infty, \qquad (95)$$

A

```
Sample Problem 7 -- Solute transport in a semi-infinite
aquifer of infinite width with a continuous 'strip' source
Model Data:  V=1.42 ft/d, DX=100.0 ft**2/d, DY=20.0 ft**2/d
             WS=230 ft, YC=750 ft, C0=40.0 mg/L
====
  31  31   1 104   1
MG/L       FT/D       FT**2/D   PER DAY   FEET       DAYS
     40.0      1.42     100.0      20.0        0.0
    750.0     230.0
      0.0     100.0     200.0     300.0     400.0     500.0     600.0     700.0
    800.0     900.0    1000.0    1100.0    1200.0    1300.0    1400.0    1500.0
   1600.0    1700.0    1800.0    1900.0    2000.0    2100.0    2200.0    2300.0
   2400.0    2500.0    2600.0    2700.0    2800.0    2900.0    3000.0
      0.0      50.0     100.0     150.0     200.0     250.0     300.0     350.0
    400.0     450.0     500.0     550.0     600.0     650.0     700.0     750.0
    800.0     850.0     900.0     950.0    1000.0    1050.0    1100.0    1150.0
   1200.0    1250.0    1300.0    1350.0    1400.0    1450.0    1500.0
   1826.0
     500.      500.       0.1
```

B

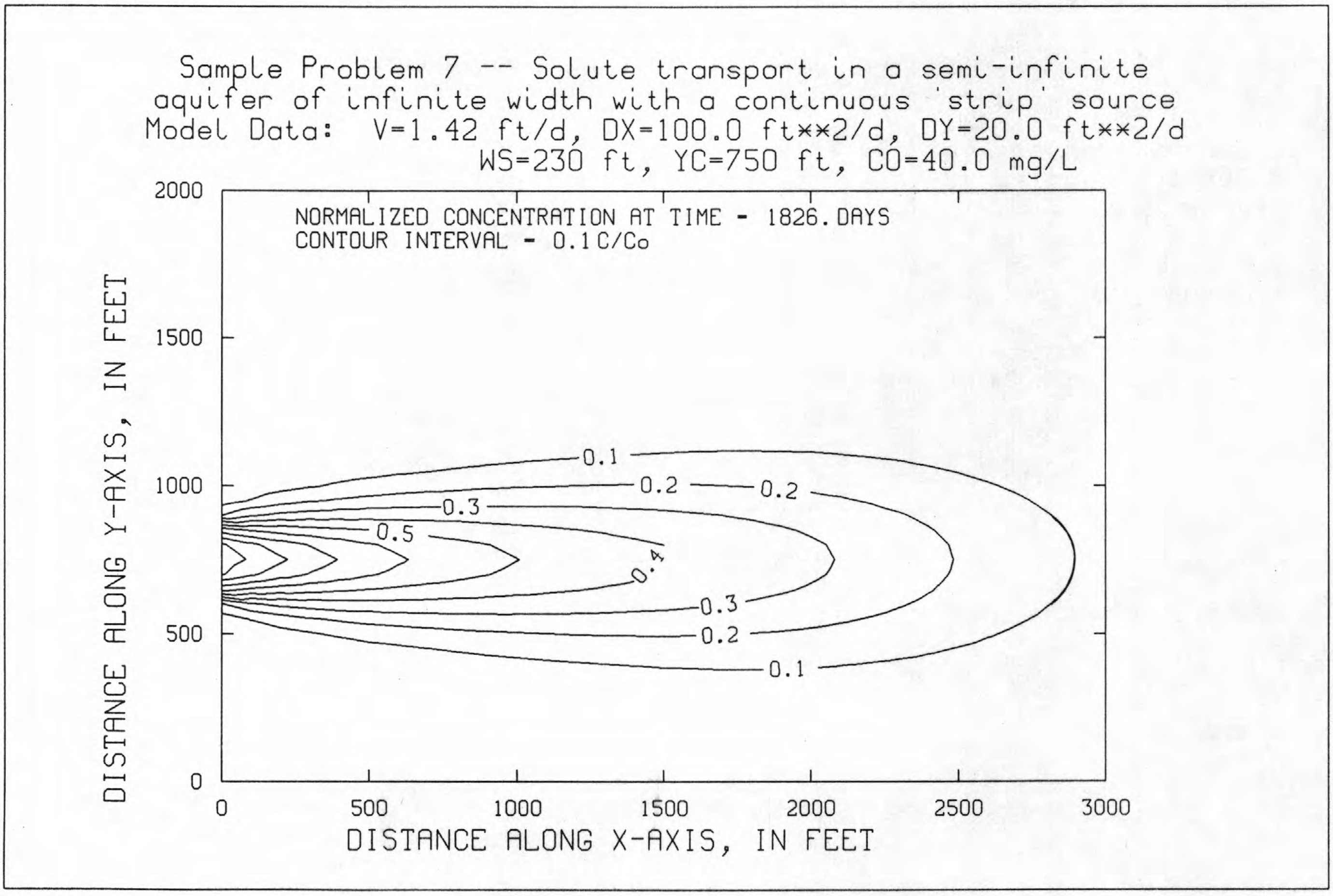

Figure 14.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program STRIPI for a conservative solute in an aquifer of infinite width with finite-width solute source after 1,826 days (sample problem 7).

where

C_m is the maximum concentration at center of gaussian solute source,

Y_c is the y-coordinate of center of solute source ($X_c = 0$), and

σ is the standard deviation of gaussian distribution.

Initial conditions:

$$C = 0, \qquad 0 < x < \infty \text{ and } -\infty < y < +\infty \qquad \text{at } t = 0 \tag{96}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only ($V_y = 0$), and velocity is constant.
4. The longitudinal and transverse dispersion coefficients (D_x, D_y) are constant.

Analytical Solution

The following equation is modified from Gureghian and others (1980, p. 905) and can be expressed as

$$C(x,y,t) = \frac{C_m x\sigma}{\sqrt{8\pi D_x}} \exp\left[\frac{V_x}{2D_x}\right] \cdot \int_0^t \frac{\exp\left[-\beta\tau - \frac{x^2}{4D_x\tau} - \frac{(y-Y_c)^2}{4(D_y\tau+\frac{\sigma^2}{2})}\right] d\tau}{\tau^{3/2}\sqrt{D_y\tau + \frac{\sigma^2}{2}}}, \tag{97}$$

where

$$\beta = \frac{V_x^2}{4D_x} + \lambda$$

and τ is dummy variable of integration for the time integral.

To improve the accuracy of the numerical integration, a variable substitution (modified from Cleary and Ungs, 1978, p. 20) can be made where $\tau = Z^4$, yielding

$$C(x,y,t) = \frac{2C_m x\sigma\sqrt{\beta}}{\sqrt{2\pi D_x}} \cdot \exp\left[\frac{V_x x}{2D_x}\right] \cdot \int_0^{(t\beta)^{1/4}} \frac{\exp\left[-Z^4 - \frac{x^2\beta}{4D_x Z^4} - \frac{(y-Y_c)^2}{4\gamma}\right] dZ}{Z^3\sqrt{\gamma}}, \tag{98}$$

where

$$\gamma = \left(\frac{D_y Z^4}{\beta} + \frac{\sigma^2}{2}\right).$$

Comments:

The integral in equation 98 could not be simplified further and must be evaluated numerically. A Gauss-Legendre numerical integration technique was used in the computer program written to evaluate the analytical solution and is described later.

Linear equilibrium adsorption and ion exchange can be simulated by first dividing the coefficients D_x, D_y, and V by the retardation factor, R (eq. 15). Temporal variations in solute concentration can be simulated through the principle of superposition.

Description of Program GAUSS

The program GAUSS computes the analytical solution to the two-dimensional solute-transport equation for an aquifer of infinite width with a solute source having a gaussian concentration distribution along the inflow boundary. It consists of a main program and the subroutine CNRMLG. The function of the main program and subroutine are outlined below; the program code listing is presented in attachment 2.

The program also calls the subroutine GLQPTS and the output subroutines TITLE, OFILE, and PLOT2D, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 6. The routine then calls the subroutine GLQPTS which reads the data file GLQ.PTS containing values of the positive roots and weighting functions used in the Gauss-Legendre numerical integration technique.

The program next executes a set of three nested loops. The inner loop calls subroutine CNRMLG to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value and time. The middle loop cycles through all x-coordinate values. The outer loop cycles through all specified time values and prints a table of concentration in relation to distance for each time value. Model output can also be plotted as a map showing lines of equal solute concentration.

Subroutine CNRMLG

Subroutine CNRMLG calculates the normalized concentration (C/C_m) for a particular time value and distance. The integral in equation 98 is evaluated through a Guass-Legendre numerical integration technique. The normalized roots of the Legendre polynomial and the corresponding weighting functions are passed by subroutine GLQPTS and scaled in the subroutine to account for the non-normalized limits of integration, from

$$0 \text{ to } (t\beta)^{1/4},$$

rather than -1 to +1.

Table 6.--Input data format for the program GAUSS

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NT	Number of time values at which solution will be evaluated.
	13 - 16	I4	NMAX	Number of terms used in the numerical integration technique (must be equal to 4, 20, 60, 104, or 256).
	17 - 20	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CM	Maximum solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient.
	41 - 50	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	YC	Y-coordinate of center of gaussian-distributed solute source.
	11 - 20	F10.0	WS	Standard deviation of gaussian distribution describing solute source.
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[1]9	1 - 10	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]Data line is needed only if IPLT (in data set 2) is greater than 0.

The number of terms summed in the the numerical integration (equivalent to the order of the polynomial) is specified by the user. Roots of the Legendre polynomial of order 4, 20, 60, 104, and 256 are provided in data file GLQ.PTS. In general, the more terms used in the integration, the more accurate the approximation; however, this must be weighed against the corresponding increase in computational effort and time. Additional discussions on the numerical integration technique are presented in a later section describing subroutine GLQPTS.

Sample Problems 8A and 8B

Two sample problems are presented. Sample problem 8A is modified from an example presented in Gureghian and others (1980) for a conservative solute uniformly mixed in a thin aquifer of infinite width. Model variables are

Maximum concentration (C_m)	= 1,000 mg/L
Standard deviation of gaussian distribution (σ)	= 130 feet
Center of solute source (Y_c)	= 450 feet
Ground-water velocity (V_x)	= 4 ft/d
Coefficient of longitudinal dispersion (D_x)	= 150 ft^2/d
Coefficient of transverse dispersion (D_y)	= 30 ft^2/d.

Concentrations are calculated at 50-foot intervals along the x-axis for 1,700 feet and at 25-foot intervals on the y-axis for 900 feet. The chloride concentration distribution after 300 days is simulated.

Sample problem 8B demonstrates two methods of calculating a value for σ. Aquifer dimensions, ground-water velocity, and dispersion coefficients are the same as in problem 8A. Concentrations measured in monitoring wells 500 feet downgradient of a waste-disposal site are presented in table 7; figure 15 presents a plot of the normalized concentration (C/C_m) in relation to distance along the y-axis (normal to the direction of flow). An average value of σ, calculated from the observed concentrations (table 7) using equation 70, equals 66.1 feet. The area under the curve in figure 15 can also be approximated and yields a σ value of 65.0 feet. A value of 65 feet was used in the input data for sample problem 8B.

Input data sets for sample problems 8A and 8B are shown in figures 16A and 17A. Computer-generated contour plots of normalized concentration (C/C_m) at both time values are shown in figure 16B and 17B. Comparison of figures 16B and 17B shows the effect of varying σ on the concentration distribution. Program outputs for the sample problem are presented in attachment 4. Sample problems 8A and 8B required 24 seconds of CPU time on a Prime model 9955 Mod II.

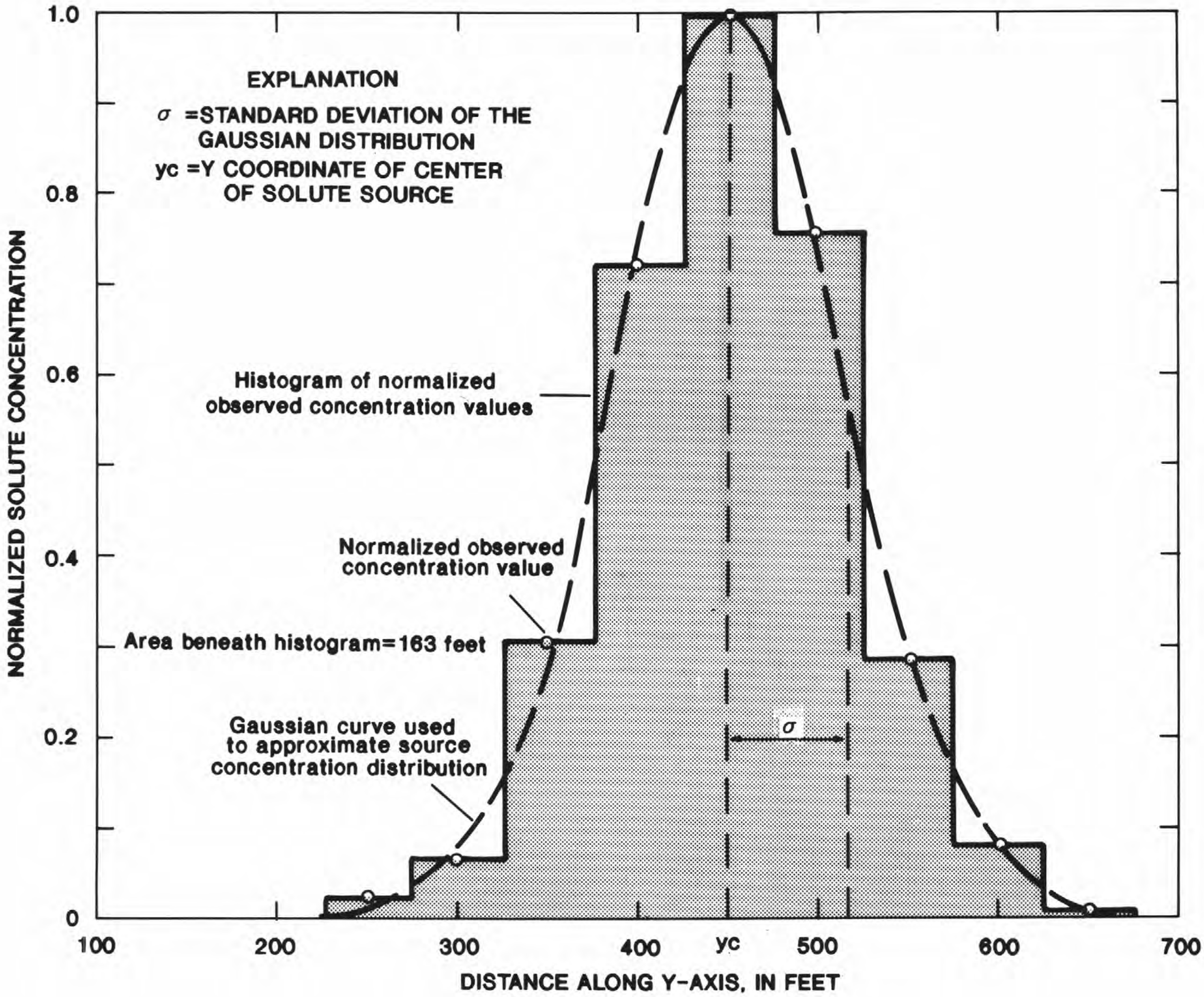

Figure 15.--Normalized concentrations in relation to distance for the waste-disposal site in sample problem 8A and fitted gaussian distribution.

A

```
Sample Problem 8a -- Solute transport in a semi-infinite
aquifer of infinite width with a continuous gaussian source
Model Data:  V=4.0 ft/d, DX=150.0 ft**2/d, DY=30.0 ft**2/d
             WS=130 ft, YC=450 ft, C0=1000.0 mg/L
====
  33  37   1 104   1
MG/L       FT/D        FT**2/D   PER DAY   FEET      DAYS
    1000.0       4.00      150.0      30.0       0.0
     450.0     130.0
       0.0      50.0      100.0     150.0     200.0     250.0     300.0     350.0
     400.0     450.0      500.0     550.0     600.0     650.0     700.0     750.0
     800.0     850.0      900.0     950.0    1000.0    1050.0    1100.0    1150.0
    1200.0    1250.0     1300.0    1350.0    1400.0    1450.0    1500.0    1550.0
    1600.0
       0.0      25.0       50.0      75.0     100.0     125.0     150.0     175.0
     200.0     225.0      250.0     275.0     300.0     325.0     350.0     375.0
     400.0     425.0      450.0     475.0     500.0     525.0     550.0     575.0
     600.0     625.0      650.0     675.0     700.0     725.0     750.0     775.0
     800.0     825.0      850.0     875.0     900.0
     300.0
     250.0     250.0        0.1
```

B

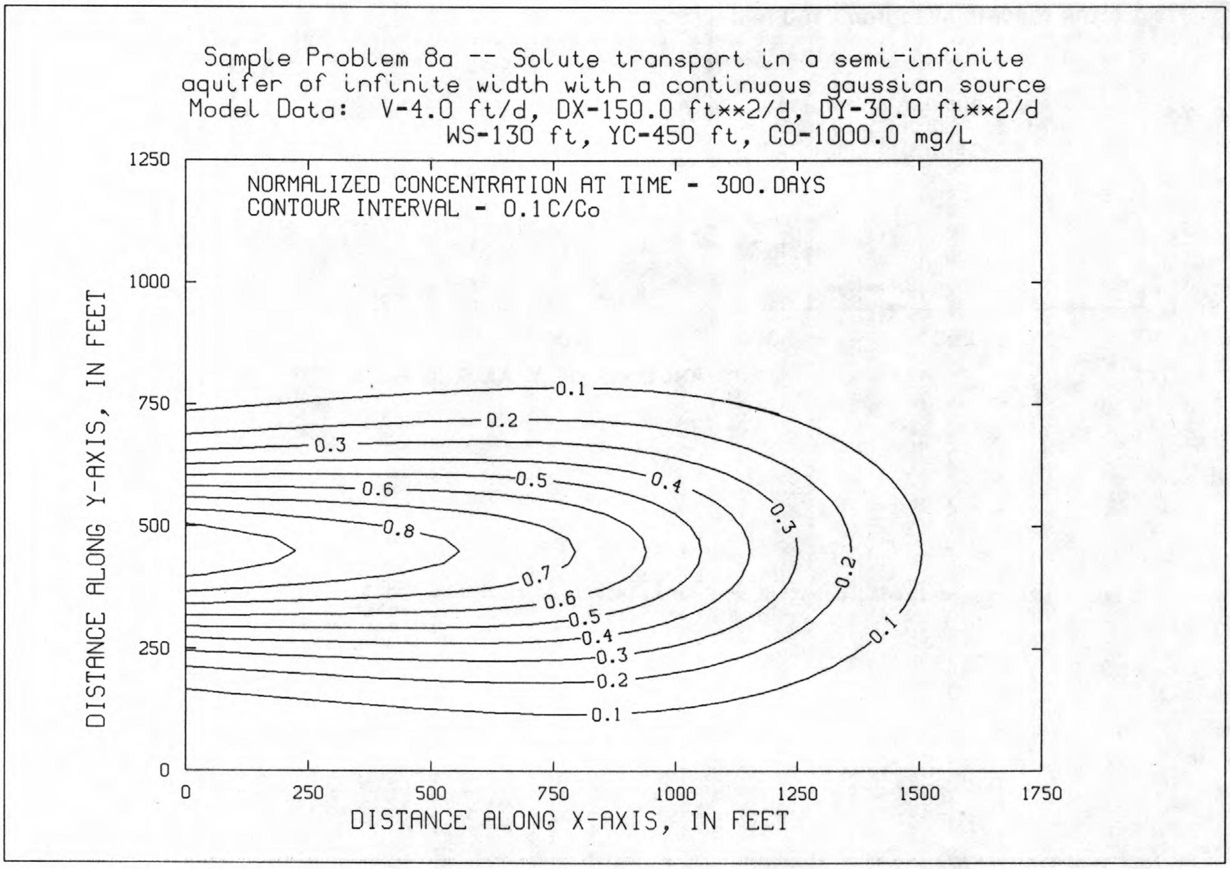

Figure 16.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program GAUSS for a conservative solute in an aquifer of infinite width having a gaussian concentration distribution (σ = 150 feet) at the inflow boundary at 300 days (sample problem 8A).

A

```
Sample Problem 8b -- Solute transport in a semi-infinite
aquifer of infinite width with a continuous gaussian source
Model Data:  V=4.0 ft/d, DX=150.0 ft**2/d, DY=30.0 ft**2/d
             WS=65 ft, YC=450 ft, C0=1000.0 mg/L
====
 33  37   1 104   1
MG/L       FT/D       FT**2/D    PER DAY    FEET       DAYS
    1000.0       4.00      150.0       30.0        0.0
    450.0       65.0
      0.0       50.0      100.0      150.0      200.0      250.0      300.0      350.0
    400.0      450.0      500.0      550.0      600.0      650.0      700.0      750.0
    800.0      850.0      900.0      950.0     1000.0     1050.0     1100.0     1150.0
   1200.0     1250.0     1300.0     1350.0     1400.0     1450.0     1500.0     1550.0
   1600.0
      0.0       25.0       50.0       75.0      100.0      125.0      150.0      175.0
    200.0      225.0      250.0      275.0      300.0      325.0      350.0      375.0
    400.0      425.0      450.0      475.0      500.0      525.0      550.0      575.0
    600.0      625.0      650.0      675.0      700.0      725.0      750.0      775.0
    800.0      825.0      850.0      875.0      900.0
    300.0
    250.0      250.0        0.1
```

B

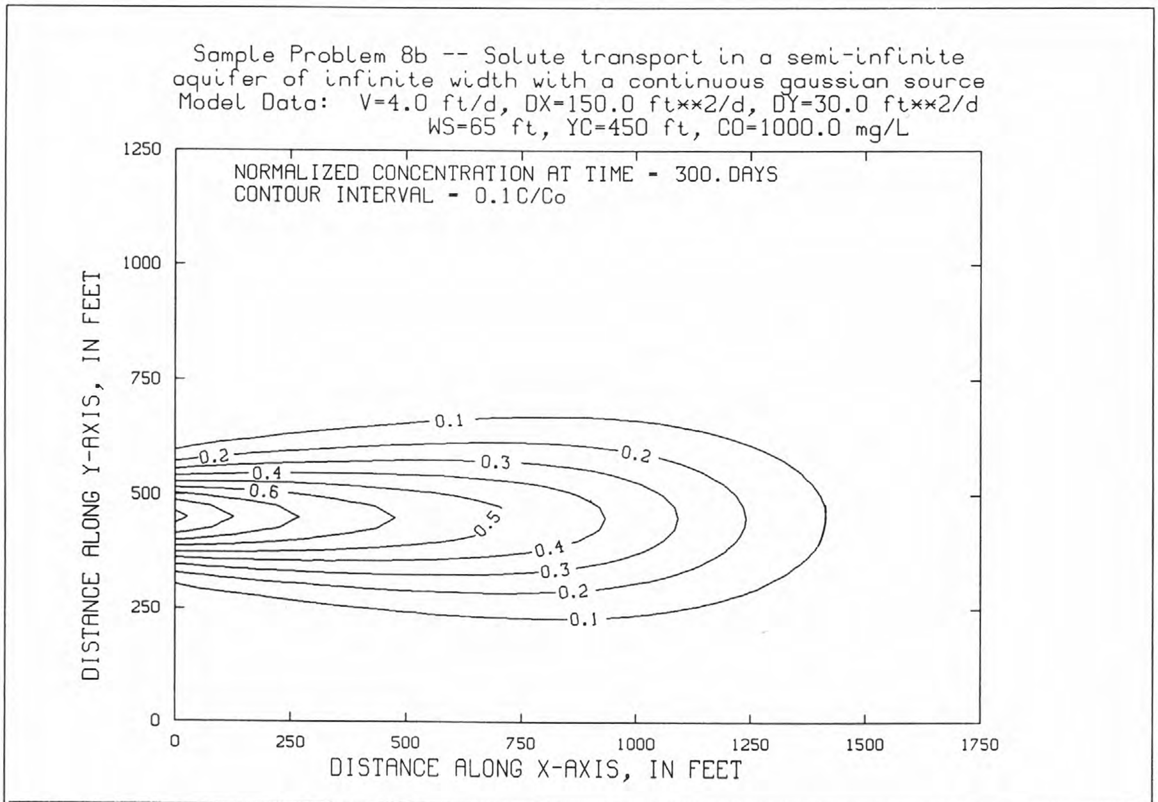

Figure 17.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program GAUSS for a conservative solute in an aquifer of infinite width having a gaussian concentration distribution (σ = 65 feet) at the inflow boundary at 300 days (sample problem 8B).

Table 7.--Measured solute concentrations in monitoring wells downgradient of the waste-disposal site in sample problem 8B

[Well locations shown in figure 12]

Well location (x and y coordinates), in feet	Measured solute concentration, in milligrams per liter	Calculated value of σ, in feet (from eq. 71)
0, 200	2	70.9
0, 250	12	67.2
0, 300	65	64.2
0, 350	310	65.3
0, 400	725	62.3
0, 450	1,000	--
0, 500	760	67.5
0, 550	290	63.6
0, 600	82	67.1
0, 650	9	65.2
0, 700	1	67.3

THREE-DIMENSIONAL SOLUTE TRANSPORT

Relatively few analytical solutions are available for the three-dimensional form of the solute-transport equation (eq. 9). These solutions are particularly useful as they can simulate transport of contaminants from sources in relatively thick aquifers where both vertical and horizontal spread of the solute is of interest. In addition to a solution modified from Cleary and Ungs (1978, p. 24-25), two solutions were derived by the author for this report. Detailed derivations of these solutions are presented in attachment 1.

In the first solution presented, the aquifer is assumed to be of infinite extent along all three coordinate axes. Fluid is injected into the aquifer through a point source at a constant rate and solute concentration (C_o). In the remaining solutions presented in this section, the aquifer is assumed to be semi-infinite in length with a solute source located along the inflow boundary. The semi-infinite aquifer can be either finite in both width and height, extending from y = 0 to y = W and from z = 0 (the base of the aquifer) to z = H, or infinite in width and height. A diagram of an idealized three-dimensional aquifer of semi-infinite length and finite width and height is presented in figure 18.

The solute source is referred to as a "patch" source (Cleary and Ungs, 1978) with a finite width and height extending from $y = Y_1$ to $y = Y_2$ and from $z = Z_1$ to $z = Z_2$ at x = 0 (fig. 18). The concentration within the patch is uniform and equal to C_o, except along the boundary of the patch source where it is equal to 0.5 C_o. Elsewhere along the inflow boundary, the concentration is equal to 0. Combinations of patch sources could be used to simulate odd-shaped concentration distributions or multiple sources through the principle of superposition. First-order solute decay, adsorption, and ion exchange can also be simulated.

Three computer programs, POINT3, PATCHF, and PATCHI, were developed to calculate concentrations in these systems as a function of distance and elapsed time. They are described in this section.

Aquifer of Infinite Extent with Continuous Point Source

Governing Equation

The analytical solution for a continuous point source has been derived by first solving the solute-transport equation for an instantaneous point source and then integrating the solution over time. The three-dimensional solute-transport equation for an instantaneous point source is given by

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C + Q dt\, C_o \cdot \delta(x-X_c)\, \delta(y-Y_c)\, \delta(z-Z_c)\, \delta(t-t'). \quad (99)$$

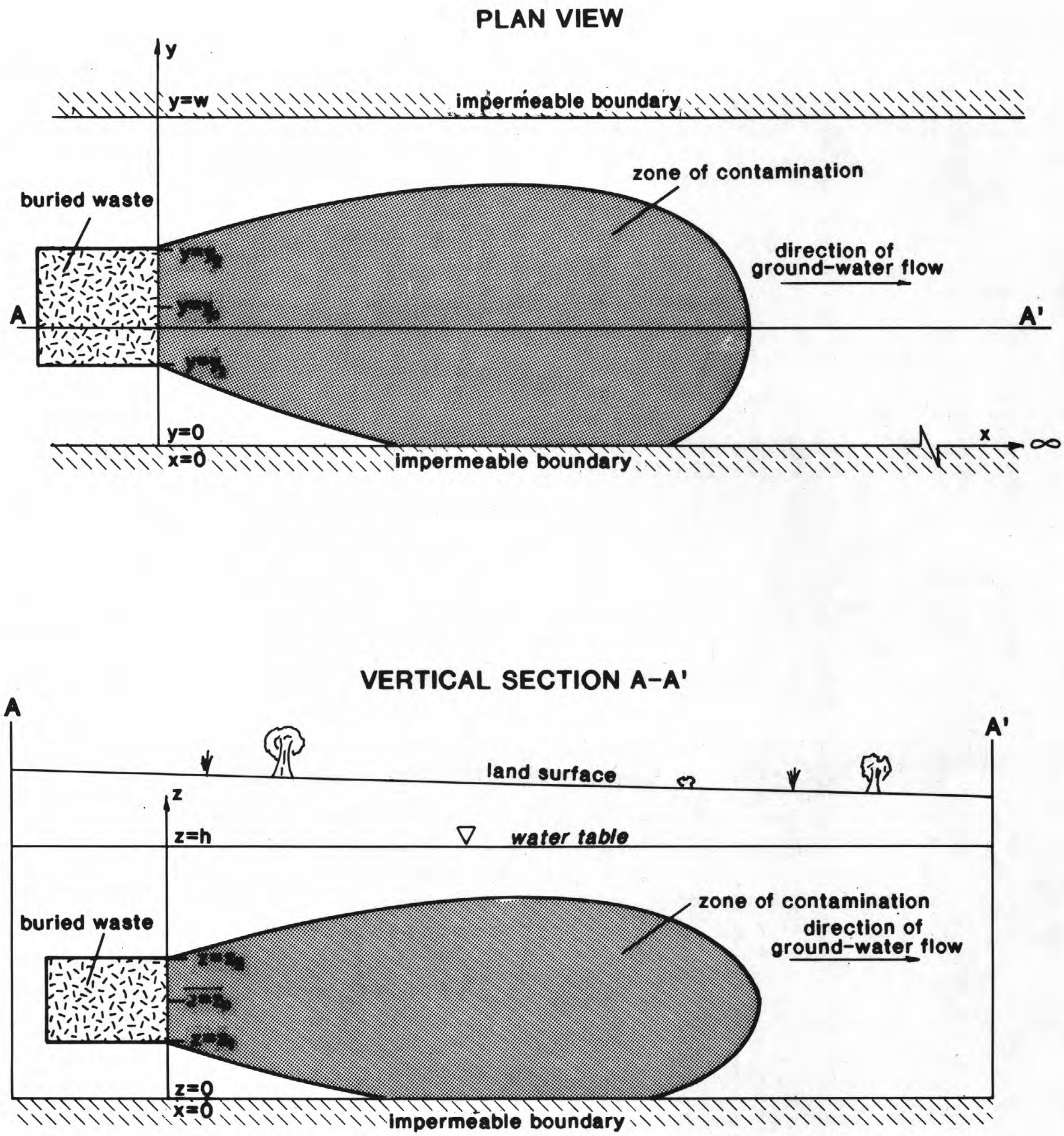

Figure 18.--Plan view and vertical section of idealized three-dimensional transport in an aquifer of semi-infinite length and finite width and height.

Boundary conditions:

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \pm\infty \tag{100}$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = \pm\infty \tag{101}$$

$$C, \frac{\partial C}{\partial z} = 0, \qquad z = \pm\infty, \tag{102}$$

where

V is V_x, the velocity in x-direction ($V_y = V_z = 0$),

Q is the fluid injection rate,

dt is the infinitesimal time interval,

X_c, Y_c, Z_c are the coordinates of point source,

$\delta(\ )$ is the Dirac delta function, and

t' is the time at which the instantaneous point source occurs (assumed to equal 0).

Initial Condition:

$$C = 0, \qquad -\infty < x < \infty,\ -\infty < y < \infty,\ -\infty < z < \infty \qquad \text{at } t' = 0 \tag{103}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant. This presumes that the fluid injection rate is small and that the spread of solute due to radially diverging flow paths is negligible.
4. The coefficients of longitudinal dispersion (D_x) and transverse dispersion (D_y, D_z), from equation 7, are constant.

Analytical Solution

Solution for the instantaneous point source was derived by the author using exponential Fourier transforms (detailed derivation in attachment 1) and can be expressed as

$$C(x,y,z,t) = \frac{C_o\, Q dt \exp\left[\frac{V(x-X_c)}{2D_x} - \left(\frac{V^2}{4D_x} + \lambda\right)(t-t')\right]}{8\pi^{3/2}\,(t-t')^{3/2}\,\sqrt{D_x D_y D_z}}$$

$$\cdot \exp\left[-\frac{(x-X_c)^2}{4D_x\,(t-t')} - \frac{(y-Y_c)^2}{4D_y\,(t-t')} - \frac{(z-Z_c)^2}{4D_z\,(t-t')}\right]. \tag{104}$$

Equation 104 can be integrated with respect to time to yield a closed form solution for the continuous solute source as

$$C(x,y,z,t) = \frac{C_o\, Q \exp\left[\frac{V(x-X_c)}{2D_x}\right]}{8\pi\gamma\sqrt{D_y D_z}} \cdot \left\{\exp\left[\frac{\gamma\beta}{2D_x}\right] \operatorname{erfc}\left[\frac{(\gamma+\beta t)}{2\sqrt{D_x t}}\right] + \exp\left[\frac{-\gamma\beta}{2D_x}\right] \operatorname{erfc}\left[\frac{(\gamma-\beta t)}{2\sqrt{D_x t}}\right]\right\}, \qquad (105)$$

where

$$\gamma = \left[(x-Xc)^2 + \frac{D_x(y-Y_c)^2}{D_y} + \frac{D_x(z-Z_c)^2}{D_z}\right]^{1/2}$$

and

$$\beta = \left[V^2 + 4D_x\lambda\right]^{1/2}.$$

Comments:

Equation 105 is valid only when γ does not equal zero. Also, concentrations determined at locations close to the point source may exceed C_o for certain combinations of values for Q, V, D_x, D_y, and D_z. In general, this can occur when Q is large relative to

$$\gamma \cdot \sqrt{D_y D_z}.$$

A solution which accounts for radial flow away from the well would be more accurate than equation 105 at large injection rates.

Linear equilibrium adsorption and ion exchange can be simulated by dividing the coefficients V, D_x, D_y, and D_z by the retardation factor, R (eq. 15). Temporal variations in solute concentration can be simulated through the principle of superposition.

Description of Program POINT3

The program POINT3 computes the analytical solution to the three-dimensional solute-transport equation for an aquifer of infinite extent with a continuous point source. It consists of a main program and the subroutine CNRML3. The function of the main program and subroutine are outlined below; the program code listing is presented in attachment 2.

The program also calls the subroutine EXERFC and the output subroutines TITLE, OFILE, PLOT3, and CNTOUR, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 8.

The program next executes a set of four nested loops. The innermost loop calls subroutine CNRML3 to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value, z-coordinate value, and time. The second loop cycles through all x-coordinate values. The third loop cycles through all z-coordinate values and prints a table of concentration in relation to x and y for each z value. The outer loop cycles through all specified time values. Model output can be plotted as a series of maps showing lines of equal solute concentration in a horizontal (x-y plane) cross section at each point along the z-axis.

Subroutine CNRML3

Subroutine CNRML3 calculates the normalized concentration (C/C_o) for a particular time value and distance using equation 105. A warning message is printed on the program output if the values of ($x-X_c$), ($y-Y_c$), and ($z-Z_c$) all equal to zero are passed to the subroutine.

Sample Problem 9

In sample problem 9, a natural gradient tracer test was conducted by injecting a chloride solution into an aquifer. The solution was injected through three wells spaced 2 feet apart, laterally, each having a small screened interval centered about Z = 10 feet. A total of 90 gallons (12 ft^3) of solution was injected during a 24-hour period. Other model variables are

Ground-water velocity (V_x)	= 0.1 ft/d
Longitudinal dispersivity (a_l)	= 0.60 feet
Horizontal transverse dispersivity (a_{th})	= 0.03 feet
Vertical transverse dispersivity (a_{tv})	= 0.006 feet
Chloride concentration in injected solution	= 1,000 mg/L
Injection well coordinates (X_c, Y_c, Z_c)	
Well 1	= (0, 98, 10)
Well 2	= (0, 100, 10)
Well 3	= (0, 102, 10).

Table 8.--Input data format for the program POINT3

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NZ	Number of z-coordinates at which solution will be evaluated.
	13 - 16	I4	NT	Number of time values at which solution will be evaluated.
	17 - 20	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	QUNITS	Units of solution injection rate.
	61 - 70	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration in injected fluid.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient in y-direction.
	41 - 50	F10.0	DZ	Transverse dispersion coefficient in z-direction.
	51 - 60	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	XC	X-coordinate of continuous point source.
	11 - 20	F10.0	YC	Y-coordinate of continuous point source.
	21 - 30	F10.0	ZC	Z-coordinate of continuous point source.
	31 - 40	F10.0	QM	Solution injection rate.
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	Z(I)	Z-coordinates at which solution will be evaluated (eight values per line).
9	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[1]10	1 - 10	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]Data line is needed only if IPLT (in data set 2) is greater than 0.

From these values, the terms obtained are

Coefficient of longitudinal dispersion (D_x)	= 0.06 ft^2/d
Coefficient of horizontal transverse dispersion (D_y)	= 0.003 ft^2/d
Coefficient of vertical transverse dispersion (D_z)	= 0.0006 ft^2/d
Injection rate per well (Q_m)	= 4.0 ft^3/d.

Chloride concentrations are computed in the z = 10-foot plane at x = 0 and at 2-foot intervals along the x-axis from x = 20 feet to x = 60 feet and at 1-foot intervals along the y-axis from y = 90 feet to y = 110 feet after an elapsed time of 400 days. The injection period was simulated using the principle of superposition by first calculating the concentrations resulting from a continuous point source after 400 days and subtracting the concentrations resulting from a continuous point source after 399 days. The effect of the multiple injection wells was simulated by summing the calculated concentrations for each individual well.

Rather than running the program POINT3 six times and then summing all the concentration values manually, it was easier to temporarily modify the main program by adding nine lines within the innermost loop as

```
      DO 50 IY=1,NY
      YY=Y(IY)-YC
      CALL CNRML3(QM,DK,T(IT),XX,YY,ZZ,DX,DY,DZ,VX,CN,NMAX)
      CXY(IX,IY)=CO*CN
        YY1=YY+2.0
        YY2=YY-2.0
        CALL CNRML3(QM,DK,T(IT),XX,YY1,ZZ,DX,DY,DZ,VX,CN1,NMAX)
        CALL CNRML3(QM,DK,T(IT),XX,YY2,ZZ,DX,DY,DZ,VX,CN2,NMAX)
        T1=T(IT)-1.0
        CALL CNRML3(QM,DK,T1,XX,YY,ZZ,DX,DY.DZ,VX,CN3,NMAX)
        CALL CNRML3(QM,DX,T1,XX,YY1,ZZ,DX,DY,DZ,VX,CN4,NMAX)
        CALL CNRML3(QM,DK,T1,XX,YY2,ZZ,DX,DY,DZ,VX,CN5,NMAX)
        CXY(IX,IY)=CXY(IX,IY)+CO*(CN1+CN2-CN3-CN4-CN5)
   50 CONTINUE
```

The input data set for sample problem 9 is shown in figure 19A; computer-generated contour plots of normalized concentrations (C/C_o) in the x-y plane at z = 10 feet are shown in figure 19B. Program output for this sample problem is presented in attachment 4. Sample problem 9 required 5 seconds of CPU time on a Prime model 9955 Mod II.

Aquifer of Finite Width and Height with Finite-Width and Finite-Height Solute Source

Governing Equation

Three-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (106)$$

A

```
Sample Problem 9 -- Solute transport in an infinite aquifer
with multiple point sources of finite duration
Model Data:  V=0.1 ft/d, DX=0.06 ft**2/d, DY=0.003 ft**2/d
          DZ=0.0006 ft**2/d QM=4.0 ft**3/d, C0=1000.0 mg/L
====
  21  21   1  01 104   1
MG/L       FT/D      FT**2/D   PER DAY   FEET       FT**3/D   DAYS
    1000.0       0.1      0.06     0.003    0.0006
       0.0     100.0      10.0      4.00
      20.0      22.0      24.0      26.0      28.0      30.0      32.0      34.0
      36.0      38.0      40.0      42.0      44.0      46.0      48.0      50.0
      52.0      54.0      56.0      58.0      60.0
      90.0      91.0      92.0      93.0      94.0      95.0      96.0      97.0
      98.0      99.0     100.0     101.0     102.0     103.0     104.0     105.0
     106.0     107.0     108.0     109.0     110.0
      10.0
     400.0
       5.0       5.0      0.01
```

B

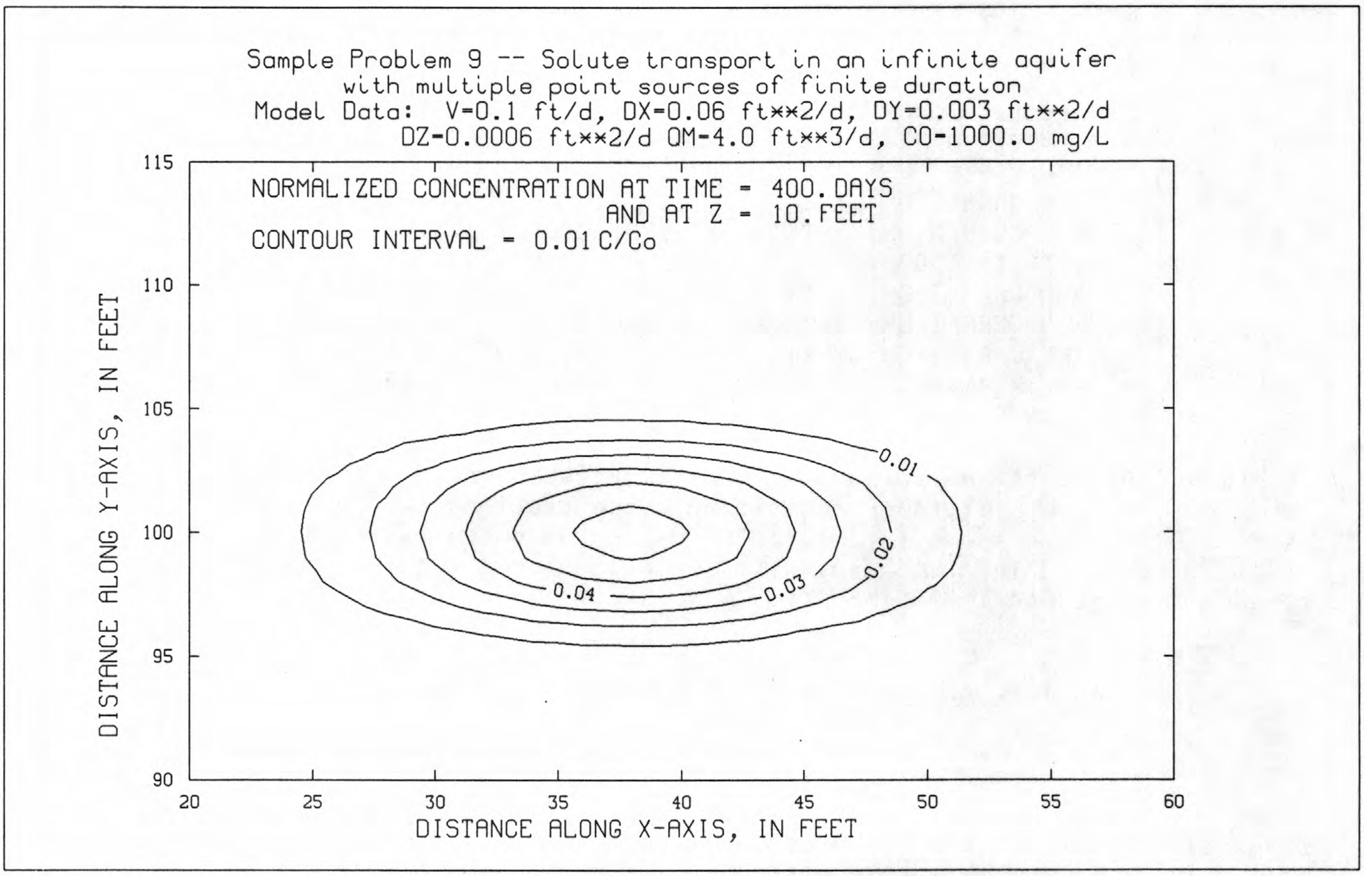

Figure 19.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program POINT3 for a natural gradient tracer test in an aquifer of infinite extent after 400 days in the z = 10-foot plane (sample problem 9).

Boundary conditions:

$$C = C_o, \qquad x = 0 \text{ and } Y_c - W_s/2 < y < Y_c + W_s/2 \text{ and } Z_c - H_s/2 < z < Z_c + H_s/2 \tag{107}$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = 0 \tag{108}$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = W \tag{109}$$

$$C, \frac{\partial C}{\partial z} = 0, \qquad z = 0 \tag{110}$$

$$C, \frac{\partial C}{\partial z} = 0 \qquad z = H \tag{111}$$

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \infty \tag{112}$$

where

V is V_x, the velocity in x-direction (V_y and $V_z = 0$),

W is the aquifer width,

H is the aquifer height,

W_s is the width of solute source,

H_s is the height of solute source,

Y_c is the y-coordinate of center of solute source, and

Z_c is the z-coordinate of center of solute source ($X_c = 0$).

Initial condition:

$$C = 0, \qquad 0 < x < \infty,\ 0 < y < W, \text{ and } 0 < z < H \qquad \text{at } t = 0 \tag{113}$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The coefficients of longitudinal dispersion (D_x) and transverse dispersion (D_y, D_z), from equation 7, are constant.

Analytical Solution

The solution to equation 106 was first derived by Cleary and Ungs (1978, p. 24-25). A modified form of the equation (derived in detail by the author in attachment 1) can be given as

$$C(x,y,z,t) = C_o \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} L_{mn} O_m P_n \cos(\zeta z) \cos(\eta y) \cdot \left\{ \exp\left[\frac{x(v-\beta)}{2D_x}\right] \cdot \operatorname{erfc}\left[\frac{x-\beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\beta)}{2D_x}\right] \cdot \operatorname{erfc}\left[\frac{x+\beta t}{2\sqrt{D_x t}}\right]\right\}, \quad (114)$$

where

$$L_{mn} = \begin{cases} 1/2 & m = 0, \text{ and } n = 0 \\ 1 & m = 0, \text{ and } n > 0 \\ 1 & m > 0, \text{ and } n = 0 \\ 2 & m > 0, \text{ and } n > 0 \end{cases}$$

$$O_m = \begin{cases} \dfrac{Z_2 - Z_1}{H} & m = 0 \\ \dfrac{[\sin(\zeta Z_2) - \sin(\zeta Z_1)]}{m\pi} & m > 0 \end{cases}$$

$$P_n = \begin{cases} \dfrac{Y_2 - Y_1}{W} & n = 0 \\ \dfrac{[\sin(\eta Y_2) - \sin(\eta Y_1)]}{n\pi} & n > 0 \end{cases}$$

$$Z_1 = Z_c - H_s/2$$

$$Z_2 = Z_c + H_s/2$$

$$Y_1 = Y_c - W_s/2$$

$$Y_2 = Y_c + W_s/2$$

$$\zeta = m\pi/H \qquad m = 0,1,2,3\ldots$$

$$\eta = n\pi/W \qquad n = 0,1,2,3\ldots$$

$$\beta = \sqrt{V^2 + 4D_x(\eta^2 D_y + \zeta^2 D_z + \lambda)}.$$

Comments:

The terms in the infinite series in equation 114 tend to oscillate and the double series converges slowly for small values of x and time. Therefore, many terms may be needed to assure convergence. A good initial estimate is 200 terms for each series.

The solution can yield results with either D_y, D_z, or $\lambda = 0$. Linear equilibrium adsorption and ion exchange can be simulated by dividing the coefffficients D_x, D_y, D_z, and V by the retardation factor, R (eq. 15). Temporal variations in solute concentration and odd-shaped source configurations can be simulated through the principle of superposition.

Description of Program PATCHF

The program PATCHF computes the analytical solution to the three-dimensional solute-transport equation for an aquifer of finite width and height with a finite-width and finite-height solute source at the inflow boundary. It consists of a main program and subroutine CNRMLF. The function of the main program and subroutine are outlined below; the program code listed is presented in attachment 2.

The program also calls the subroutine EXERFC and output subroutines TITLE, OFILE, PLOT3D, and CNTOUR, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 9.

The program next executes a set of four nested loops. The innermost loop calls subroutine CNRMLF to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value, z-coordinate value, and time. The second loop cycles through all x-coordinate values. The third loop cycles through all z-coordinate values and prints a table of concentration in relation to x and y for each z value. The outer loop cycles through all specified time values. Model output can be plotted as a series of maps showing lines of equal solute concentration in the horizontal (x-y) plane at each point along the z-axis.

Subroutine CNRMLF

Subroutine CNRMLF calculates the normalized concentration (C/C_o) for a particular time value and distance using equation 114. The maximum number of terms in the infinite series summation is specified by the user. Because terms in the series tend to oscillate, a subtotal of the last 10 terms is kept, and when the subtotal is less than a convergence criterion set at 1×10^{-12}, the series summation is halted. If the series does not converge after the specified maximum number of terms are taken, a warning message is printed on the program output.

Table 9.--Input data format for the program PATCHF

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as title for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NZ	Number of z-coordinates at which solution will be evaluated.
	13 - 16	I4	NT	Number of time values at which solution will be evaluated.
	17 - 20	I4	NMAX	Maximum number of terms to be used in inner loop of the infinite series summation.
	21 - 24	I4	MMAX	Maximum number of terms to be used in outer loop of the infinite series summation.
	25 - 28	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient in y-direction.
	41 - 50	F10.0	DZ	Transverse dispersion coefficient in z-direction.
	51 - 60	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	W	Aquifer width (aquifer extends from y = 0 to y = W).
	11 - 20	F10.0	H	Aquifer thickness (aquifer extends from z = 0 to z = H).
	21 - 30	F10.0	YC	Y-coordinate of center of patch solute source.
	31 - 40	F10.0	ZC	Z-coordinate of center of patch solute source.
	41 - 50	F10.0	WS	Full width of patch solute source.
	51 - 60	F10.0	HS	Full height of patch solute source.
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	Z(I)	Z-coordinates at which solution will be evaluated (eight values per line).
9	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[1]10	1 - 10	F10.0	XSCLP	Scaling factor by which x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]Data line is needed only if IPLT (in data set 2) is greater than 0.

Sample Problem 10

In sample problem 10, migration of chloride ion from a landfill, created by filling in a gravel pit excavated in a valley-fill aquifer, is simulated. Model variables are

Aquifer width (W)	= 3,000 feet
Aquifer height (H)	= 100 feet
Y-coordinate of source center (Y_c)	= 1,200 feet
Z-coordinate of source center (Z_c)	= 75 feet
Source width (W_s)	= 1,600 feet
Source height (H_s)	= 50 feet
Source concentration (C_o)	= 1,000 mg/L
Ground-water velocity (V_x)	= 1 ft/d
Dispersion in x-direction (D_x)	= 200 ft^2/d
Dispersion in y-direction (D_y)	= 60 ft^2/d
Dispersion in z-direction (D_z)	= 10 ft^2/d.

Concentrations are calculated at 150-foot intervals along the x-axis for 3,900 feet and at 100-foot intervals along the y-axis for 3,000 feet. Chloride concentration distributions after 3,000 days for z-coordinates of 50 and 75 feet (z = 0 is at the base of the aquifer) are simulated.

The input data set for sample problem 10 is shown in figure 20A; computer-generated contour plots of normalized concentration (C/C_o) in x-y planes defined by the two z-coordinates are shown in figure 20B. The plot of concentrations along the center line of the plume (at z = 75 feet) can be compared with the second plot in figure 13 to show the effect of vertical dispersion on both the shape of the chloride plume and simulated concentrations. This demonstrates the type of errors that can be introduced by using a two-dimensional solution when a three-dimensional solution is required.

Program output for sample problem 10 is presented in attachment 4. The sample problem required 7 minutes and 50 seconds of CPU time on a Prime model 9955 Mod II.

Aquifer of Infinite Width and Height with Finite-Width and Finite-Height Solute Source

Governing Equation

Three-dimensional solute-transport equation:

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C \qquad (115)$$

A

```
Sample Problem 10 -- Solute transport in a semi-infinite
aquifer of finite width and height with a 'patch' source
Model Data:  V=1.0 ft/d, DX=200.0, DY=60.0, DZ=10.0 ft**2/d
 W=3000 ft, H=100 ft, WS=1600 ft, HS=50 ft, C0=1000.0 mg/L
====
  29  27   2   1 350 350   1
MG/L        FT/D       FT**2/D   PER DAY   FEET       DAYS
    1000.0       1.0     200.0      60.0     10.0       0.0
    3000.0     100.0    1200.0      75.0   1600.0      50.0
       0.0     150.0     300.0     450.0    600.0     750.0     900.0    1050.0
    1200.0    1350.0    1500.0    1650.0   1800.0    1950.0    2100.0    2250.0
    2400.0    2550.0    2700.0    2850.0   3000.0    3150.0    3300.0    3450.0
    3600.0    3750.0    3900.0    4050.0   4200.0
       0.0     100.0     200.0     300.0    400.0     500.0     600.0     700.0
     800.0     900.0    1000.0    1100.0   1200.0    1300.0    1400.0    1500.0
    1600.0    1700.0    1800.0    1900.0   2000.0    2100.0    2200.0    2300.0
    2400.0    2500.0    2600.0    2700.0   2800.0    2900.0    3000.0
      75.0      50.0
    3000.0
      750.      750.       0.1
```

B

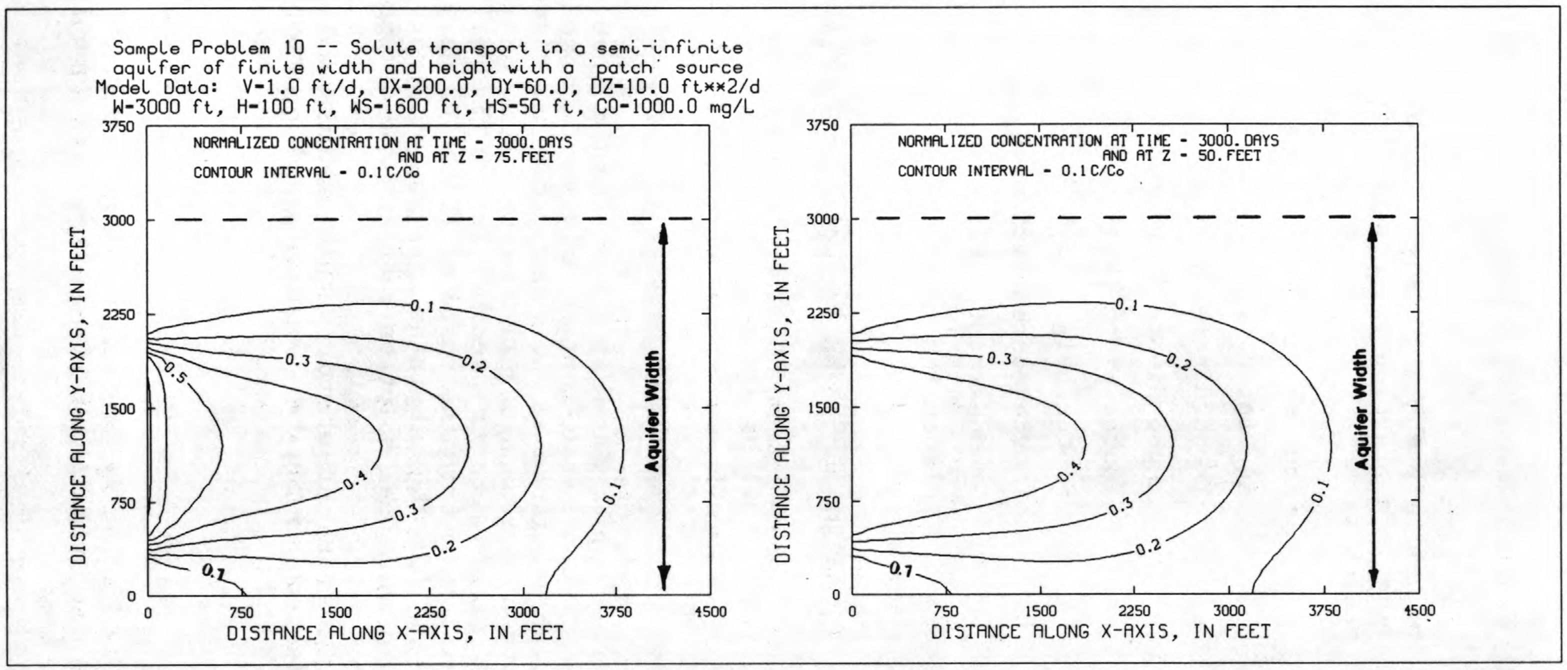

Figure 20.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program PATCHF for a conservative solute in an aquifer of finite height and width after 3,000 days at heights of 75, 50, and 25 feet above base of the aquifer (sample problem 10).

Boundary conditions:

$$C = C_o, \qquad x = 0 \text{ and } Y_c - W_s/2 < y < Y_c + W_s/2 \quad (116)$$

$$\text{and } Z_c - H_s/2 < z < Z_c + H_s/2$$

$$C, \frac{\partial C}{\partial y} = 0, \qquad y = \pm\infty \quad (117)$$

$$C, \frac{\partial C}{\partial z} = 0, \qquad z = \pm\infty \quad (118)$$

$$C, \frac{\partial C}{\partial x} = 0, \qquad x = \infty \quad (119)$$

where

V is V_x, the velocity in x-direction (V_y and $V_z = 0$),

W_s is the width of solute source,

H_s is the height of solute source,

Y_c is the y-coordinate of center of solute source, and

Z_c is the z-coordinate of center of solute source ($X_c = 0$).

Initial condition:

$$C = 0, \qquad 0 < x < \infty, -\infty < y < +\infty, \text{ and } -\infty < z < +\infty \qquad \text{at } t = 0 \quad (120)$$

Assumptions:

1. Fluid is of constant density and viscosity.
2. Solute may be subject to first-order chemical transformation (for a conservative solute, $\lambda = 0$).
3. Flow is in x-direction only, and velocity is constant.
4. The coefficients of longitudinal dispersion (D_x) and transverse dispersion (D_y, D_z), from equation 7, are constant.

Analytical Solution

The following analytical solution was derived by the author using Fourier transforms (detailed derivation presented in attachment 1) and can be expressed as

$$C(x,y,z,t) = \frac{C_o\, x \exp\left[\frac{Vx}{2D_x}\right]}{8\sqrt{\pi D_x}} \cdot \int_o^t \tau^{-3/2} \exp\left[-\left(\frac{V^2}{4D_x} + \lambda\right)\tau - \frac{x^2}{4D_x\tau}\right]$$

$$\cdot \left\{\mathrm{erfc}\left[\frac{(Y_1-y)}{2\sqrt{D_y\tau}}\right] - \mathrm{erfc}\left[\frac{(Y_2-y)}{2\sqrt{D_y\tau}}\right]\right\}$$

$$\cdot \left\{\mathrm{erfc}\left[\frac{(Z_1-z)}{2\sqrt{D_z\tau}}\right] - \mathrm{erfc}\left[\frac{(Z_2-z)}{2\sqrt{D_z\tau}}\right]\right\} d\tau, \qquad (121)$$

where

$$Y_1 = Y_c - \frac{W_s}{2}$$

$$Y_2 = Y_c + \frac{W_s}{2}$$

$$Z_1 = Z_c - \frac{H_s}{2}$$

$$Z_2 = Z_c + \frac{H_s}{2}$$

and τ is dummy variable of integration for the time integral.

Comments:

The integral in equation 121 could not be simplified further and must be evaluated numerically. A Gauss-Legendre numerical integration technique was used in the computer program written to evaluate the analytical solution and is described later.

Linear equilibrium adsorption and ion exchange can be simulated by dividing the coefficients D_x, D_y, D_z, and V by the retardation factor, R (eq. 15). Temporal variations in solute concentration and odd-shaped source configurations can be simulated through the principle of superposition.

Description of Program PATCHI

The program PATCHI computes the analytical solution to the three-dimensional solute-transport equation for an aquifer of infinite width and height with finite-width and finite-height solute source at the inflow boundary. It consists of a main program and the subroutine CNRMLI. The functions of the main program and the subroutine are outlined below; the program code listing is presented in attachment 2.

The program also calls subroutines EXERFC and GLQPTS and the output subroutines TITLE, OFILE, and PLOT3D, which are common to most programs described in this report. These routines are described in detail later.

Main program

The main program reads and prints all input data needed to specify model variables. The required input data and the format used in preparing a data file are shown in table 10.

The program next executes a set of four nested loops. The innermost loop calls subroutine CNRMLI to calculate the concentration at all specified y-coordinate values for a particular x-coordinate value, z-coordinate value, and time. The second loop cycles through all x-coordinate values. The third loop cycles through all z-coordinate values and prints a table of concentration in relation to x and y for each z value. The outer loop cycles through all specified time values. Model output can also be plotted as a series of maps showing lines of equal solute concentration in the horizontal (x-y) plane at each point along the z-axis.

Subroutine CNRMLI

Subroutine CNRMLI calculates the normalized concentration (C/C_o) for a particular time value and distance. The integral in equation 121 is evaluated through a Guass-Legendre numerical integration technique. The normalized roots of the Legendre polynomial and the corresponding weighting coefficients are passed by subroutine GLQPTS and scaled in the subroutine to account for the non-normalized limits of integration (from 0 to t rather than -1 to +1).

The number of terms summed in the numerical integration (equivalent to the order of the polynomial) is specified by the user. Roots of the Legendre polynomial of order 4, 20, 60, 104, and 256 are provided in data file GLQ.PTS. In general, the more terms used in the integration, the more accurate the approximation; however, this must be weighed against the corresponding increase in computational effort and time. Additional discussions on the numerical integration technique are presented in a later section describing subroutine GLQPTS.

Sample Problem 11

In sample problem 11, a contaminant plume containing ^{90}Sr (strontium-90) from a deep radioactive waste storage facility migrates through a thick, confined aquifer. Model variables are

Source width (W_s)	= 1,200 feet
Source height (H_s)	= 300 feet
Y-coordinate of source center (Y_c)	= 1,500 feet
Z-coordinate of source center (H_c)	= 1,500 feet
Ground-water velocity (V_x)	= 1 ft/d
Longitudinal dispersivity (a_l)	= 100 feet
Transverse dispersivity (a_t)	= 20 feet
Source concentration (C_o)	= 100 mg/L
Half-life of ^{90}Sr	= 28 years.

Table 10.--Input data format for the program PATCHI

Data set	Columns	Format	Variable name	Description
1	1 - 60	A60	TITLE	Data to be printed in a title box on the first page of program output. Last line in data set must have an "=" in column 1. First four lines are also used as titles for plot.
2	1 - 4	I4	NX	Number of x-coordinates at which solution will be evaluated.
	5 - 8	I4	NY	Number of y-coordinates at which solution will be evaluated.
	9 - 12	I4	NZ	Number of z-coordinates at which solution will be evaluated.
	13 - 16	I4	NT	Number of time values at which solution will be evaluated.
	17 - 20	I4	NMAX	Number of terms to be used in numerical integration technique (must be equal to 4, 20, 60, 104, or 256).
	21 - 24	I4	IPLT	Plot control variable. Contours of normalized concentration will be plotted if IPLT is greater than 0.
3	1 - 10	A10	CUNITS	Character variable used as label for units of concentration in program output.
	11 - 20	A10	VUNITS	Units of ground-water velocity.
	21 - 30	A10	DUNITS	Units of dispersion coefficient.
	31 - 40	A10	KUNITS	Units of solute-decay coefficient.
	41 - 50	A10	LUNITS	Units of length.
	51 - 60	A10	TUNITS	Units of time.
4	1 - 10	F10.0	CO	Solute concentration at inflow boundary.
	11 - 20	F10.0	VX	Ground-water velocity in x-direction.
	21 - 30	F10.0	DX	Longitudinal dispersion coefficient.
	31 - 40	F10.0	DY	Transverse dispersion coefficient in y-direction.
	41 - 50	F10.0	DZ	Transverse dispersion coefficient in z-direction.
	51 - 60	F10.0	DK	First-order solute-decay coefficient.
5	1 - 10	F10.0	YC	Y-coordinate of center of finite width and height solute source.
	11 - 20	F10.0	ZC	Z-coordinate of center of finite width and height solute source.
	21 - 30	F10.0	WS	Full width of finite width and height solute source.
	31 - 40	F10.0	HS	Full height of finite width and height solute source.
6	1 - 80	8F10.0	X(I)	X-coordinates at which solution will be evaluated (eight values per line).
7	1 - 80	8F10.0	Y(I)	Y-coordinates at which solution will be evaluated (eight values per line).
8	1 - 80	8F10.0	Z(I)	Z-coordinates at which solution will be evaluated (eight values per line).
9	1 - 80	8F10.0	T(I)	Time values at which solution will be evaluated (eight values per line).
[1]10	1 - 10	F10.0	XSCLP	Scaling factor by which to divide x-coordinate values are divided to convert them to plotter inches.
	11 - 20	F10.0	YSCLP	Scaling factor used to convert y-coordinates into plotter inches.
	21 - 30	F10.0	DELTA	Contour increment for plot of normalized concentration (must be between 0.0 and 1.0).

[1]Data line is needed only if IPLT (in data set 2) is greater than 0.

From these values, the terms obtained are

Coefficient of longitudinal dispersion (D_x) = 100 ft^2/d
Coefficients of transverse dispersion (D_y and D_z) = 20 ft^2/d
First-order solute-decay coefficient (λ) = 6.78×10^{-5} per day.

Concentrations are calculated at 150-foot intervals along the x-axis for 3,900 feet and at 100-foot intervals along the y-axis for 2,600 feet. The ^{90}Sr concentration distribution after 10 years (3,652 days) for z-coordinates of 1,650, 1,700, and 1,750 feet (z = 0 at the base of the aquifer and z = 1,650 at the top of the storage facility) is simulated.

The input data set for sample problem 11 is shown in figure 21A; computer-generated contour plots of the normalized concentration (C/C_o) in x-y planes defined by the three z-coordinates are shown in figure 21B. Program output for this sample problem is presented in attachment 4. Sample problem 11 required 3 minutes and 20 seconds of CPU time on a Prime model 9955 Mod II.

DESCRIPTION OF SUBROUTINES

The subroutines described below are common to most of the programs developed to evaluate the analytical solutions. The subroutines EXERFC and GLQPTS are used in evaluating terms in the analytical solutions, OFILE and TITLE are used in program input and output, and PLOT1D, PLOT2D, PLOT3D, and CNTOUR are used to graphically display program results. Descriptions of these subroutines are provided below. Subroutine listings are presented in attachment 3.

Mathematical Subroutines

Subroutines EXERFC and GLQPTS

Subroutine EXERFC is called to evaluate the product of an exponential and complementary error function (exp[x] • erfc[y]) where the error function, erf(y), is defined as

$$\mathrm{erf}(y) = \frac{2}{\sqrt{\pi}} \int_0^y \exp[-\epsilon^2]\,d\epsilon, \qquad (122)$$

and the complementary error function, erfc(y), is defined as

$$\mathrm{erfc}(y) = 1.0 - \mathrm{erf}(y). \qquad (123)$$

A

```
Sample Problem 11 -- Solute transport in a semi-infinite
aquifer of infinite width and height with a 'patch' source
Model Data:  V=1.0 ft/d, DX=100.0, DY=20.0, DZ=20.0 ft**2/d
  WS=1200 ft, HS=300 ft, DK=6.78E-05 per day, C0=100.0 mg/L
====
  27  27   3   1 104   1
MG/L        FT/D        FT**2/D    PER DAY   FEET        DAY
     100.0         1.0      100.0       20.0       20.0  6.78E-05
    1500.0      1500.0     1200.0      300.0
       0.0       150.0      300.0      450.0      600.0      750.0      900.0     1050.0
    1200.0      1350.0     1500.0     1650.0     1800.0     1950.0     2100.0     2250.0
    2400.0      2550.0     2700.0     2850.0     3000.0     3150.0     3300.0     3450.0
    3600.0      3750.0     3900.0
       0.0       100.0      200.0      300.0      400.0      500.0      600.0      700.0
     800.0       900.0     1000.0     1100.0     1200.0     1300.0     1400.0     1500.0
    1600.0      1700.0     1800.0     1900.0     2000.0     2100.0     2200.0     2300.0
    2400.0      2500.0     2600.0     2700.0     2800.0     2900.0     3000.0
    1650.0      1700.0     1750.0
    3652.5
 1000.     1000.      0.1
```

B

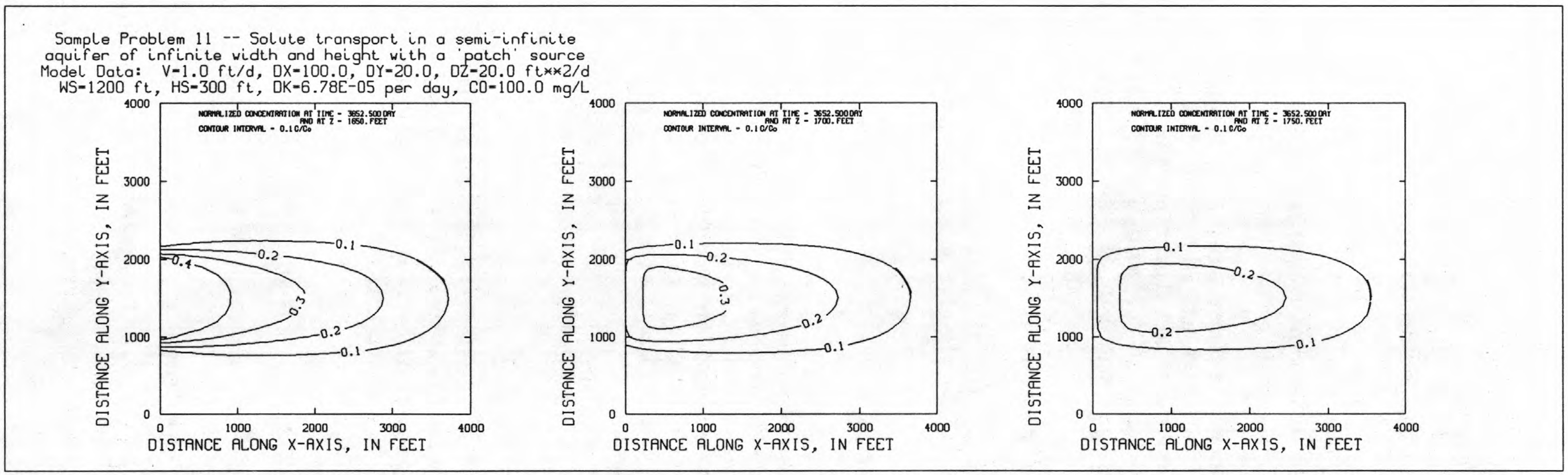

Figure 21.--(A) Sample input data set, and (B) computer plot of normalized concentration contours generated by the program PATCHI for a solute subject to first-order chemical transformation in an aquifer of infinite height and width with solute source of finite height and width after 3,000 days at heights of 1,500, 1,250, and 1,050 feet above base of the aquifer (sample problem 11).

Often, the values of x and y are such that erfc(y) is very small (less than 1×10^{-12} for y = 5), whereas exp(x) is very large. To accurately calculate the product of the two functions, a high degree of accuracy is needed in the calculation of erfc(y). Subroutine EXERFC uses a rational Chebyshev approximation (Cody, 1969), accurate to between 10 and 13 significant figures, to calculate erf(y) or erfc(y). The two variables x and y are passed to the subroutine. To calculate only erfc(y), the routine EXERFC can be called with the value of x set to zero.

For absolute values of y less than 0.469, the rational Chebyshev approximation is given by

$$\mathrm{erf}(y) = y \sum_{i=0}^{n} P1_i \, y^{2i} \Big/ \sum_{i=0}^{n} Q1_i \, y^{2i}, \tag{124}$$

where P1 and Q1 are the coefficients of the rational approximation given by Cody (1969) for n = 5. For negative values of y, the symmetry condition that erf(-y) = -erf(y) (Abramowitz and Stegun, 1964) is used. Erfc(y) is then given by equation 123.

For absolute values of y between 0.469 and 4.0, a rational approximation for erfc(y) is used, which is given by

$$\mathrm{erfc}(y) = \exp[-y^2] \sum_{i=0}^{n} P2_i \, y^{i} \Big/ \sum_{i=0}^{n} Q2_i \, y^{i}, \tag{125}$$

where P2 and Q2 are the coefficients given by Cody (1969) for n = 8. For negative values of y, the identity that erfc(-y) = 2 - erfc(y) is used.

For absolute values of y greater than 4.0, a second rational approximation for erfc(y) is given by

$$\mathrm{erfc}(y) = \frac{\exp[-y^2]}{y} \left\{ \frac{1}{\sqrt{\pi}} + \frac{1}{y^2} \cdot \sum_{i=0}^{n} P3_i \, y^{-2i} \Big/ \sum_{i=0}^{n} Q3_i \, y^{-2i} \right\}, \tag{126}$$

where P3 and Q3 are coefficients given by Cody (1969) for n = 5. When a product of the exp(x) and erfc(y) is calculated, the argument for the exponential in equations 125 and 126 are changed to $(x-y^2)$.

Subroutine GLQPTS is called to numerically evaluate the time integral found in several of the analytical solutions. The Gauss integration formula used is given by Abramowitz and Stegun (1964) as

$$\int_{-1}^{1} f(x)\,dx = \sum_{i=1}^{n} w_i \, f(z_i), \tag{127}$$

where z_i are the roots of the Legendre polynomial for a particular value of n, and

w_i are the corresponding weighting functions.

Positive roots of the Legendre polynomials for n = 4, 20, 60, 104, and 256 and their weighting functions, as given in Cleary and Ungs (1978), have been tabulated and are read from a data file called GLQ.PTS. Subroutine GLQPTS fills in the negative roots and their weighting coefficients. These values are passed to the other subroutines through an array in common. A listing of file GLQ.PTS is presented in attachment 3.

As stated earlier, accuracy of the numerical integration is increased if the user selects a larger value for n. However, computational effort is also increased. Checks can be made to determine whether a smaller value for n produces reasonable results by comparing the solution for a particular n with that obtained using the next higher value. Roots and weighting coefficients for additional values of n can be found in Abramowitz and Stegun (1964, p. 916-919).

The subroutine is set up to read data file GLQ.PTS on logical unit 77 on the Prime system. For systems other than Prime, this routine should be modified to include the correct system-dependent file opening statements. Also, file-naming conventions for the particular system must be observed, and the data file renamed appropriately.

Input/Output Routines

Subroutines OFILE and TITLE

Subroutine OFILE is used to open disk files for program input and output on the Prime computer system. It assigns logical unit 15 to the input data file and logical unit 16 to the file for program output. The user is queried at the terminal (logical unit 1) for the name of the appropriate disk files and any file name up to 50 characters in length can be entered. For output to be sent directly to the terminal, the user should type an asterisk (*) in column 1 when asked for the output file name.

For systems other than the Prime, this routine should be modified to include the correct system-dependent file opening statements. Also, the logical units (1, 15 and 16) should be changed if they are not appropriate for the particular system.

Subroutine TITLE is called by all programs to print a title box on the first page of model output. Titles are supplied as the first cards of the input data set. Titles are automatically centered, and the routine closes the title box when it encounters an equal sign (=) in column 1 of a data card. The routine also prints the date and time the program began execution. The first four title lines are used as titles for plots.

Subroutine TITLE calls the Prime-supplied functions TIME$A and DATE$A found in the library VAPPLB. For non-Prime systems, these calls should be modified or, if similar functions are not available, they should be deleted.

Graphics Subroutines

Four subroutines, PLOT1D, PLOT2D, PLOT3D, and CNTOUR, were developed to graphically display select output from the programs described in this report. These subroutines contain calls for DISSPLA graphics software (Integrated Software Systems Corporation, 1981), and the DISSPLA library must be loaded when compiling the programs. Users who do not have access to DISSPLA software can easily modify the DISSPLA software calls to those appropriate to their own graphics software.

Subroutines PLOT1D, PLOT2D, and PLOT3 contain a call to COMPRS, which creates a META file that can be output, at a later time, to a wide variety of plotter devices through the DISSPLA post-processor. This call can be replaced with a call to directly nominate a plotter device (such as a graphics terminal) so that plots can be drawn as the programs execute. The user should consult the DISSPLA Users Manual (Integrated Software Systems Corporation, 1981) for more information.

Subroutines PLOT1D, PLOT2D, PLOT3D, and CNTOUR

Subroutine PLOT1D is called by the programs FINITE and SEMINF to create plots of the normalized concentration C/C_o in relation to distance for each of the time values specified in the input data. An example of typical plotter output is shown in figure 3B. DISSPLA software calls are used to draw the axes and plot the data points. The height of the plot is 12.5 inches. The width is controlled by the difference between the minimum and maximum x-coordinate value and by the scale factor XSCLP specified in data set 4 (tables 1 and 2). If no plotter is available, the user can either specify a value of 0 for IPLT in data set 2 (tables 1 and 2) or delete the call to PLOT1D in the main programs of FINITE and SEMINF.

Subroutine PLOT2D is called by the programs POINT2, STRIPF, STRIPI, and GAUSS to initialize a plot of lines of equal normalized concentration (C/C_o) in the x-y plane for each of the specified time values. A typical example is shown in figure 13B.

The size of each subplot depends upon the difference between the maximum and minimum x- and y-coordinate values and the plot scaling factors, XSCLP and YSCLP specified by the user in data set 9 (tables 3-5). The overall length of the plot is determined by the number of time values specified. The contour increment DELTA (a value between 0.0 and 1.0) is specified by the user in data set 9.

Subroutine PLOT2D defines the plot and subplot sizes, draws and labels the axes, and then calls subroutine CNTOUR which draws and labels the contours. If no plotter is available, IPLT in data set 2 (tables 3-6) can be set to 0, or the call to PLOT2D in the main programs STRIPF, STRIPI, and GAUSS can be deleted.

Subroutine PLOT3D is called by the programs FOINT3, PATCHF, and PATCHI to initialize a plot of lines of equal normalized concentration (C/C_o) in the x-y plane for each of the z-coordinates and time values specified in the input data. An example of plotter output from this subroutine is shown in figure 20B.

The size of each subplot depends upon the difference between the maximum and minimum x- and y-coordinates and the plot scaling factors XSCLP and YSCLP specified by the user in data set 10 (tables 7 and 8). The overall length of the plot is determined by the number of z-values specified. Separate plots are drawn for each specified time value. The contour increment DELTA (a value between 0.0 and 1.0) can also be specified by the user in data set 10.

Subroutine PLOT3D defines the plot and subplot sizes, draws and labels the axes, and then calls subroutine CNTOUR which draws and labels the contours. If no plotter is available, IPLT in data set 2 (tables 8-10) can be set to zero, or the call to PLOT3D in the main programs of POINT3, PATCHF, and PATCHI can be deleted.

Subroutine CNTOUR is called to produce simplified plots of lines of equal normalized concentration (C/C_o) in the x-y plane for each of the time values or z-coordinates specified in the input data. Although there are many software packages that contour gridded data, such as concentration in relation to x and y, some of these require the grid to be equally spaced and others, such as that contained in DISSPLA, can interpolate scattered data onto regular grids but at the cost of considerable computational effort and time.

The subroutine first creates a rectangular grid based upon the x- and y-coordinates supplied in the input data. Each rectangular block in the grid is then subdivided into two triangles defined by a diagonal drawn across the block. Next, contour segments are drawn by connecting points of equal concentration determined by linear interpolation along the axes of each triangular element.

The number of contours drawn is determined by the difference between the maximum and minimum normalized concentration values and the contour increment, DELTA. The subroutine uses a relatively complex algorithm to connect the contour segments defining a contour line and determine whether a contour line has exited the grid or formed a closed loop. Contour lines are labeled after all NUM contour segments are drawn. NUM is set to 40 in the code, but this can be changed by the user. The routine requires three work arrays--XPC, YPC, and IFLAG--to store contouring data. IFLAG must be dimensioned to twice the number of rectangular blocks. XPC and YPC are dimensioned by 50 in the subroutine and in common block PDAT in the main programs. This number must be changed if the user increases the value of NUM to greater than 50.

Running the Programs

Array Dimensions

Dimensions of arrays used by the programs are set by a PARAMETER statement in which the following array sizes are set as

```
PARAMETER MAXX=100, MAXY=50, MAXZ=30, MAXT=20, MAXXY=5000, MAXXY2=10000, MAXRT=1000,
```

where

MAXX is maximum number of x-coordinates,
MAXY is maximum number of y-coordinates,
MAXZ is maximum number of z-coordinates,
MAXT is maximum number of time values,
MAXXY is product of MAXX AND MAXY,
MAXXY2 is twice MAXXY, and
MAXRT is maximum number of roots used in series summation in program FINITE.

The user can modify the PARAMETER statment to increase or decrease these limits.

Compiling and Loading

The following describes the procedure for compiling and running the programs on the Prime system. For convenience, the user should first create a single file called SUBS.F77 that contains the following subroutines: EXERFC, GLQPTS, OFILE, TITLE, PLOT1D, PLOT2D, PLOT3D, AND CNTOUR. The user should then type

```
F77 PROGRAM.F77 -BIG -SILENT
F77 SUBS.F77 -BIG -SILENT
SEG -LOAD
LOAD PROGRAM
LOAD SUBS
LIBRARY DISSPA
LIBRARY VAPPLB
LI
SAVE
QUIT
```

where PROGRAM indicates the name of the main program (for example, STRIPF or GAUSS). After the message "LOAD COMPLETE" is received at the terminal, the user can run the program by typing

```
SEG PROGRAM
```

The following message will appear

"TYPE IN INPUT FILE NAME"

The user can respond with the name of the file containing the data set (see description of subroutine OFILE). The following will then appear:

"TYPE IN OUTPUT FILE NAME"

The user can respond with the name of the output file name or an asterisk (*) to cause output to come to the terminal.

These programs can be run on other computer systems although some device-dependent subroutine calls may have to be modified. These statements are identified in the previous section.

SUMMARY

The physical, chemical, and biological processes that govern transport of solutes in ground water can be described quantitatively by the advective-dispersive solute-transport equation. Analytical solutions, which are exact mathematical solutions for this partial differential equation, have been derived for many combinations of aquifer geometry, solute-source configurations, and boundary and initial conditions. These solutions can be used to mathematically model the movement of solutes in homogeneous aquifers with simple flow systems in which the chemical and biological processes can be described by linear relations.

This report presents analytical solutions for solute transport in one-, two-, and three-dimensional systems with uniform flow, which were compiled from those published in various journals and reports or derived by the author. The solutions for one-dimensional solute transport are for: (1) a finite-length system with a first-type boundary condition at the inflow end, (2) a finite-length system with a third-type boundary condition at the inflow end, (3) a semi-infinite system with a first-type boundary condition at the inflow end, and (4) a semi-infinite system with a third-type boundary condition at the inflow end. Solutions for the finite-length system assume a second-type boundary condition at the outflow end.

Solutions for two-dimensional solute transport were presented for: (1) an aquifer of infinite areal extent with a continuous point source at which fluid is injected at a constant rate and concentration, (2) a semi-infinite aquifer of finite width with a strip source along the inflow boundary, (3) a semi-infinite aquifer of infinite width with a strip source along the inflow boundary, and (4) a semi-infinite aquifer of infinite width with a solute source having a gaussian concentration distribution. Solution for three-dimensional solute transport were presented for: (1) an aquifer of infinite extent with a continuous point source, (2) a semi-infinite aquifer of finite height and width with a patch source along the inflow boundary, and (3) a semi-infinite aquifer of infinite width and height with a patch source along the inflow boundary. All the solutions presented can account for first-order solute decay due to chemical or biological processes and linear equilibrium adsorption.

A set of computer programs were written to evaluate these solutions and produce tables and graphs of solute concentration as a function of time and distance from the solute source. Documentation of these programs includes instruction on their use, description of input data format, sample problems, and sample data sets. Source codes for the programs and output for the sample problems are presented in the attachments to the report.

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ATTACHMENT 1.--DERIVATION OF SELECT ANALYTICAL SOLUTIONS

Aquifer of infinite width and height with finite width and finite height solute source

Aquifer of finite width and height with finite width and finite height solute source

Aquifer of infinite width and height with continuous point source

AQUIFER OF INFINITE WIDTH AND HEIGHT WITH FINITE WIDTH AND FINITE HEIGHT SOLUTE SOURCE

The following is a step-by-step derivation of the analytical solution for solute transport in an aquifer of infinite length, width, and height containing a solute source of finite width and finite height (patch source) in a steady flow field (eq. 121 in the text).

The governing three-dimensional solute-transport equation is

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C. \qquad \text{(A1.1)}$$

Boundary and initial conditions are

$$C = C_o, \qquad x = 0 \text{ and } Y_c - W_s/2 < y < Y_c + W_s/2 \qquad \text{(A1.2)}$$

$$\text{and } Z_c - H_s/2 < z < Z_c + H_s/2$$

$$C(\infty, y, z, t) = 0 \qquad \text{(A1.3)}$$

$$C(x, \pm\infty, z, t) = 0 \qquad \text{(A1.4)}$$

$$C(x, y, \pm\infty, t) = 0 \qquad \text{(A1.5)}$$

$$C(x, y, z, o) = 0, \qquad \text{(A1.6)}$$

where

V is V_x, the velocity in x-direction (V_y and $V_z = 0$),

W_s is the width of solute source,

H_s is the height of solute source,

Y_c is the y-coordinate of center of solute source, and

Z_c is the z-coordinate of center of solute source ($X_c = 0$).

STEP 1:

To solve equation A1.1 for the patch source, first solve the partial differential equation for solute transport in an aquifer with an instantaneous point source at the inflow end (at $x = 0$). The governing equations are identical, but the boundary condition at $x = 0$ (eq. A1.2) is rewritten as

$$C\ (o,\ y,\ z,\ t) = C_o\ \delta(y-y')\ \delta(z-z')\ \delta(t-t') \text{ at } x = 0,$$

where

δ (   ) is the dirac delta function,

y' and z' are the coordinates of the point source, and

t' is time at which the instantaneous point source starts and ends.

STEP 2:

A variable transformation is applied to remove the advective and solute-decay terms, where

$$c = C \exp\left[- \frac{Vx}{2D_x} + \frac{V^2 t}{4D_x} + \lambda t\right]. \tag{A1.7}$$

The resulting transformed solute-transport equation and boundary and initial conditions are

$$\frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} + D_z \frac{\partial^2 c}{\partial z^2}$$

$$c\ (o,\ y,\ z,\ t) = C_o \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \delta(y-y')\ \delta(z-z')\ \delta(t-t') \tag{A1.8}$$

$$c\ (\infty,\ y,\ z,\ t) = 0 \tag{A1.9}$$

$$c\ (x,\ \pm\infty,\ z,\ t) = 0 \tag{A1.10}$$

$$c\ (x,\ y,\ \pm\infty,\ t) = 0 \tag{A1.11}$$

$$c\ (x,\ y,\ z,\ o) = 0. \tag{A1.12}$$

STEP 3:

The x-derivative term is removed by applying the Fourier sine transform, defined by Churchill (1972, p. 401-402) as

$$S\,[F(x)] = \overline{F}(\alpha) = \int_{o}^{\infty} F(x)\,\sin(\alpha x)\,dx \tag{A1.13}$$

with inverse

$$S^{-1}\,[\overline{F}(\alpha)] = F(x) = \frac{2}{\pi}\int_{o}^{\infty} \overline{F}(\alpha)\,\sin(\alpha x)\,d\alpha \tag{A1.14}$$

and with an operational property

$$S\left[\frac{d^2F(x)}{dx^2}\right] = -\alpha^2\,\overline{F} + \alpha\,F(o), \tag{A1.15}$$

where F(o) is the function evaluated at x = 0. The transformed equation and boundary and initial conditions are

$$\frac{\partial\overline{c}}{\partial t} + \alpha^2 D_x\overline{c} - D_y\frac{\partial^2\overline{c}}{\partial y^2} - D_z\frac{\partial^2\overline{c}}{\partial z^2}$$

$$- \alpha\,D_x\,C_o\,\exp\left(\frac{V^2t}{4D_x} + \lambda t\right)\,\delta(y-y')\,\delta(z-z')\,\delta(t-t') = 0 \tag{A1.16}$$

$$\overline{c}\,(\alpha,\ \pm\infty,\ z,\ t) = 0 \tag{A1.17}$$

$$\overline{c}\,(\alpha,\ y,\ \pm\infty,\ t) = 0 \tag{A1.18}$$

$$\overline{c}\,(\alpha,\ y,\ z,\ o) = 0. \tag{A1.19}$$

STEP 4:

The y-derivative is removed by applying the exponential Fourier transform, defined by Churchill (1972, p. 384-385) as

$$E\,[G(y)] = \overline{G}(\beta) = \int_{-\infty}^{+\infty} G(y)\,\exp\,[-i\beta y]\,dy \qquad \text{(A1.20)}$$

with inverse

$$E^{-1}\,[\overline{G}(\beta)] = G(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{G}(\beta)\,\exp\,[i\beta y]\,d\beta \qquad \text{(A1.21)}$$

and with an operational property

$$E\left[\frac{d^2G(y)}{dy^2}\right] = -\beta^2\,\overline{G}(\beta), \qquad \text{(A1.22)}$$

where $i = \sqrt{-1}$. The transformed equation and boundary and initial conditions are

$$\frac{\partial\overline{\overline{c}}}{\partial t} + \alpha^2 D_x\overline{\overline{c}} + \beta^2 D_y\overline{\overline{c}} - D_z\frac{\partial^2\overline{\overline{c}}}{\partial z^2} - \alpha\,D_x\,C_o\,\exp\left[\frac{V^2t}{4D_x} + \lambda t\right] \cdot \int_{-\infty}^{+\infty} e^{-i\beta y}\,\delta(y-y')\,\delta(z-z')\,\delta(t-t')\,dy = 0 \qquad \text{(A1.23)}$$

$$\overline{\overline{c}}\,(\alpha,\,\beta,\,\pm\infty,\,t) = 0 \qquad \text{(A1.24)}$$

$$\overline{\overline{c}}\,(\alpha,\,\beta,\,z,\,o) = 0. \qquad \text{(A1.25)}$$

STEP 5:

The exponential Fourier transform is applied again to remove the z-derivative. Also, by definition, the integral of a function multiplied by the dirac delta function (last term in eq. A1.23) is equal to the function evaluated at the coordinate of the point source; that is

$$\int F(x)\ \delta(x-x')\ dx = F(x'). \qquad \text{(A1.26)}$$

Thus, the transformed equation and initial condition are given by

$$\frac{d\bar{\bar{c}}}{dt} + \left(\alpha^2 D_x + \beta^2 D_y + \gamma^2 D_z\right) \bar{\bar{c}}$$

$$- \alpha D_x C_o \exp\left[\frac{V^2 t}{4D_x} + \lambda t - i\beta y' - i\gamma z'\right] \cdot \delta(t-t') = 0 \qquad \text{(A1.27)}$$

$$\bar{\bar{c}}\ (\alpha,\ \beta,\ \gamma,\ o) = 0. \qquad \text{(A1.28)}$$

STEP 6:

The transformed ordinary differential equation is solved for $\bar{\bar{\bar{c}}}$ using an integrating factor; that is, given a differential equation of the form

$$\frac{dw}{dt} + gw = h(t), \tag{A1.29}$$

the solution is given by

$$w = \frac{1}{p(t)} \int_{t_o}^{t} p(\tau)\, h(\tau)\, d\tau + w_o \frac{p(t)}{p(t_o)}, \tag{A1.30}$$

where the integrating factor p(t) is given by

$$p(t) = \exp\left[\int g(\tau) d\tau\right]. \tag{A1.31}$$

Applied to equation A1.27, this yields

$$\bar{\bar{\bar{c}}} = \frac{\alpha\, D_x\, C_o \exp\,[-i\beta y' - i\gamma z']}{\exp\,[\alpha^2 D_x t + \beta^2 D_y t + \gamma^2 D_z t]} \int_{o}^{t} \exp\left[\alpha^2 D_x + \beta^2 D_y + \gamma^2 D_z + \frac{V^2}{4D_x} + \lambda\right] \cdot \tau\, \delta(\tau - t')\, d\tau. \tag{A1.32}$$

Integrating equation A1.32 and grouping like terms gives

$$\bar{\bar{\bar{c}}} = \alpha\, D_x\, C_o \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \alpha^2 D_x\,(t-t') - i\beta y' - \beta^2 D_y\,(t-t') - i\gamma z' - \gamma^2 D_z\,(t-t')\right]. \tag{A1.33}$$

STEP 7:

The inverse Fourier sine transform (eq. A1.14) is applied to remove the α term; that is

$$\bar{\bar{c}} = D_x\ C_o \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - i\beta y' - \beta^2 D_y\ (t-t') - i\gamma z' - \gamma^2 D_z\ (t-t')\right] \cdot S^{-1}\left\{\alpha \exp\left[-\alpha^2\ D_x\ (t-t')\right]\right\}. \quad \text{(A1.34)}$$

From a table of inverse Fourier sine transforms given in Churchill (1972, p. 424, eq. D.1.26)

$$S^{-1}\left[\alpha \exp\left(-a\alpha^2\right)\right] = \frac{x}{2a\sqrt{\pi a}} \exp\left(\frac{-x^2}{4a}\right). \quad \text{(A1.35)}$$

Applied to equation A1.35, this yields

$$\bar{\bar{c}} = C_o \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - i\beta y' - \beta^2 D_y\ (t-t') - i\gamma z' - \gamma^2 D_z\ (t-t')\right] \cdot \frac{x}{2\ (t-t')\ \sqrt{\pi D_x\ (t-t')}} \exp\left[\frac{-x^2}{4D_x\ (t-t')}\right]. \quad \text{(A1.36)}$$

STEP 8:

The inverse exponential Fourier transform (eq. A1.21) is applied to remove the β terms; that is

$$\bar{c} = \frac{C_o\ x}{2\ (t-t')\ \sqrt{\pi D_x\ (t-t')}} \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \frac{x^2}{4D_x\ (t-t')} - i\gamma z' - \gamma^2 D_z\ (t-t')\right] \cdot E^{-1}\left\{\exp\left[-\ i\beta y' - \beta^2\ D_y\ (t-t')\right]\right\}. \quad (A1.37)$$

Multiplying through by $\dfrac{2\sqrt{\pi D_y\ (t-t')}}{2\sqrt{\pi D_y\ (t-t')}}$

and using the shift theorem (Churchill, 1972, p. 471, eq. C.1.5) given by

$$E^{-1}\left\{\exp\left[ia\beta\right]\ \bar{G}(\beta)\right\} = F\ (y + a) \quad (A1.38)$$

and equation C.1.20 from the table of inverse exponential Fourier transforms (Churchill, 1972, p. 472) given by

$$E^{-1}\left\{2\sqrt{\pi a}\ \exp\left[-a(\beta)^2\right]\right\} = \exp\left[-\ \frac{y^2}{4a}\right], \quad (A1.39)$$

yields

$$\bar{c} = \frac{C_o\ x}{4\pi\ (t-t')^2\ \sqrt{D_x D_y}} \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \frac{x^2}{4D_x\ (t-t')} - i\gamma z' - \gamma^2 D_z\ (t-t')\right] \cdot \exp\left[-\ \frac{(y-y')^2}{4D_y\ (t-t')}\right]. \quad (A1.40)$$

STEP 9:

Next multiply through by $\frac{2\sqrt{\pi D_z (t-t')}}{2\sqrt{\pi D_z (t-t')}}$ and apply the inverse exponential Fourier transform (eq. A1.21) to remove the γ terms; that is

$$c = \frac{C_o\, x}{8\pi^{3/2}\ (t-t')^{5/2}\ \sqrt{D_x D_y D_z}}\ \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \frac{x^2}{4D_x\ (t-t')} - \frac{(y-y')^2}{4D_y\ (t-t')}\right]$$
$$\cdot\ E^{-1}\left\{2\sqrt{\pi D_z\ (t-t')}\ \exp\left[-i\gamma z' - \gamma^2 D_z\ (t-t')\right]\right\}. \quad \text{(A1.41)}$$

Applying the shift theorem and inverse transform (eqs. A1.38 and A1.39) yields

$$c = \frac{C_o\, x}{8\pi^{3/2}\ (t-t')^{5/2}\ \sqrt{D_x D_y D_z}}\ \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \frac{x^2}{4D_x\ (t-t')} - \frac{(y-y')^2}{4D_y\ (t-t')} - \frac{(z-z')^2}{4D_z\ (t-t')}\right]. \quad \text{(A1.42)}$$

STEP 10:

The transformed variable is converted back from c to C by multiplying both sides of equation A1.42 by

$$\exp\left[\frac{Vx}{2D_x} - \frac{V^2 t}{4D_x} - \lambda t\right]$$

(see eq. A1.7) to yield the analytical solution to the solute-transport equation for an <u>instantaneous point</u> source

$$C = \frac{C_o\ x}{8\pi^{3/2}\ (t-t')^{5/2}\ \sqrt{D_x D_y D_z}} \exp\left[- \frac{V^2\ (t-t')}{4D_x} - \lambda\ (t-t') + \frac{Vx}{2D_x} - \frac{x^2}{4D_x\ (t-t')} - \frac{(y-y')^2}{4D_y\ (t-t')} - \frac{(z-z')^2}{4D_z\ (t-t')}\right]. \qquad \text{(A1.43)}$$

STEP 11:

The equation for an instantaneous line source of finite length along the Y-axis is derived by integrating equation A1.43 from $y' = Y_1$ to $y' = Y_2$; that is

$$C = \frac{C_o\, x}{8\pi^{3/2}\, (t-t')^{5/2}\, \sqrt{D_x D_y D_z}} \exp\left[- \frac{V^2\, (t-t')}{4D_x} - \lambda\, (t-t') + \frac{Vx}{2D_x} - \frac{x^2}{4D_x\, (t-t')} - \frac{(z-z')^2}{4D_z\, (t-t')}\right] \cdot \int_{Y_1}^{Y_2} \exp\left[- \frac{(y-y')^2}{4D_y\, (t-t')}\right] dy' . \qquad \text{(A1.44)}$$

The integral in equation A1.44 can be found in a table of integrals by Abramowitz and Stegun (1964, p. 303, eq. 7.4.32) given as

$$\int \exp\left[- \left(ax^2 + 2bx + c\right)\right] dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \exp\left[\frac{b^2 - ac}{a}\right] \cdot \operatorname{erf}\left(\sqrt{a}\, x + \frac{b}{\sqrt{a}}\right) + C, \qquad \text{(A1.45)}$$

where erf(x) is the error function, and C is an arbitrary constant. Letting

$$x = y', \ \eta = 4D_y\, (t-t'), \ a = \frac{1}{\eta}, \ b = \frac{-y}{\eta}, \text{ and } c = \frac{y^2}{\eta},$$

the integral in equation A1.44 can be simplified to

$$I = \frac{\sqrt{\pi\eta}}{2} \left\{\frac{\operatorname{erf}(Y_2-y)}{\sqrt{\eta}} - \frac{\operatorname{erf}(Y_1-y)}{\sqrt{\eta}}\right\} \qquad \text{(A1.46)}$$

or

$$I = \sqrt{\pi D_y\, (t-t')} \left\{\operatorname{erfc}\left[\frac{Y_1-y}{2\sqrt{D_y\, (t-t')}}\right] - \operatorname{erfc}\left[\frac{Y_2-y}{2\sqrt{D_y\, (t-t')}}\right]\right\}, \qquad \text{(A1.47)}$$

where erfc is the complementary error function, 1-erf(x); thus, the analytical solution for an <u>instantaneous line</u> source is given by

$$C = \frac{C_o x}{8\pi\, (t-t')^2\, \sqrt{D_x D_z}} \exp\left[- \frac{V^2\, (t-t')}{4D_x} - \lambda\, (t-t') + \frac{Vx}{2D_x} - \frac{x^2}{4D_x\, (t-t')} - \frac{(z-z')^2}{4D_z\, (t-t')}\right] \cdot \left\{\operatorname{erfc}\left[\frac{Y_1-y}{2\sqrt{D_y\, (t-t')}}\right] - \operatorname{erfc}\left[\frac{Y_2-y}{2\sqrt{D_y\, (t-t')}}\right]\right\}. \qquad \text{(A1.48)}$$

STEP 12

The z' terms in equation A1.44 are integrated similarly from $z' = Z_1$ to $z' = Z_2$ to obtain the solution for an <u>instantaneous patch</u> source using equation A1.47; that is

$$C = \frac{C_o x}{8\sqrt{\pi D_x}\ (t-t')^{3/2}} \exp\left[-\frac{V^2\ (t-t')}{4D_x} - \lambda\ (t-t') + \frac{Vx}{2D_x} - \frac{x^2}{4D_x\ (t-t')}\right]$$

$$\cdot \left\{\operatorname{erfc}\left[\frac{Y_1-y}{2\sqrt{D_y\ (t-t')}}\right] - \operatorname{erfc}\left[\frac{Y_2-y}{2\sqrt{D_y\ (t-t')}}\right]\right\}$$

$$\cdot \left\{\operatorname{erfc}\left[\frac{Z_1-z}{2\sqrt{D_z\ (t-t')}}\right] - \operatorname{erfc}\left[\frac{Z_2-z}{2\sqrt{D_z\ (t-t')}}\right]\right\}. \qquad \text{(A1.49)}$$

STEP 13:

To derive a solution for a continuous patch source, integrate equation A1.49 from $t' = 0$ to $t' = t$. To simplify the integration, let $\tau = (t-t')$ and $d\tau = -dt'$; that is

$$C = \frac{C_o \ x \exp\left(\frac{Vx}{2D_x}\right)}{8\sqrt{\pi D_x}} \int_o^t \tau^{-3/2} \exp\left[-\frac{V^2\tau}{4D_x} - \lambda\tau - \frac{x^2}{4D_x\tau}\right]$$

$$\cdot \left\{\mathrm{erfc}\left[\frac{Y_1-y}{2\sqrt{D_y\tau}}\right] - \mathrm{erfc}\left[\frac{Y_2-y}{2\sqrt{D_y\tau}}\right]\right\}$$

$$\cdot \left\{\mathrm{erfc}\left[\frac{Z_1-z}{2\sqrt{D_z\tau}}\right] - \mathrm{erfc}\left[\frac{Z_2-z}{2\sqrt{D_z\tau}}\right]\right\} d\tau. \qquad \text{(A1.50)}$$

Equation A1.50 is identical to equation 121 in the text. The integral in the solution could not easily be simplified further and must be evaluated numerically.

AQUIFER OF FINITE WIDTH AND HEIGHT WITH FINITE WIDTH AND FINITE HEIGHT SOLUTE SOURCE

The following is a step-by-step derivation of the analytical solution for solute transport in an aquifer of infinite length and finite width and height containing a solute source of finite width and finite height (patch source) in a steady flow field (eq. 102 in the text).

The governing three-dimensional solute-transport equation is

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C. \qquad \text{(A1.51)}$$

Boundary and initial conditions are

$$C\ (o,\ y,\ z,\ t) = C_o, \qquad \text{for } Y_c - W_s/2 < y < Y_c + W_s/2 \quad Z_c - H_s/2 < z < Z_c + H_s/2 \qquad \text{(A1.52)}$$

$$C,\ \frac{\partial C}{\partial y} = 0, \qquad y = 0,\ y = W \qquad \text{(A1.53)}$$

$$C,\ \frac{\partial C}{\partial z} = 0, \qquad z = 0,\ z = H \qquad \text{(A1.54)}$$

$$C,\ \frac{\partial C}{\partial x} = 0, \qquad x = \infty \qquad \text{(A1.55)}$$

$$C\ (x,\ y,\ z,\ o) = 0, \qquad \text{(A1.56)}$$

where

V is V_x, the velocity in x-direction (V_y and $V_z = 0$),
W_s is the width of solute source,
H_s is the height of solute source,
Y_c is the y-coordinate of center of solute source,
Z_c is the z-coordinate of center of solute source ($X_c = 0$),
W is the aquifer width, and
H is the aquifer height.

STEP 1:

To solve equation A1.51 for the patch source, a variable transformation is applied to remove the advective and solute-decay terms, where

$$c = C \exp\left[-\frac{Vx}{2D_x} + \frac{V^2 t}{4D_x} + \lambda t\right]. \tag{A1.57}$$

The resulting transformed solute-transport equation and boundary and initial conditions are

$$\frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} + D_z \frac{\partial^2 c}{\partial z^2}$$

$$c\,(o, y, z, t) = C_o \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right], \qquad \text{for } Y_c - W_s/2 < y < Y_c + W_s/2 \quad Z_c - H_s/2 < z < Z_c + H_s/2 \tag{A1.58}$$

$$c,\ \frac{\partial c}{\partial y} = 0, \qquad y = 0,\ y = W \tag{A1.59}$$

$$c,\ \frac{\partial c}{\partial z} = 0, \qquad z = 0,\ z = H \tag{A1.60}$$

$$c,\ \frac{\partial c}{\partial x} = 0, \qquad x = \infty \tag{A1.61}$$

$$c\,(x, y, z, o) = 0. \tag{A1.62}$$

STEP 2:

The x-derivative term is removed by applying the Fourier sine transform as defined by Churchill (1972, p. 401-402); that is

$$S\ [F(x)] = \overline{F}(\alpha) = \int_{o}^{\infty} F(x)\ \sin\ (\alpha x)\ dx \qquad (A1.63)$$

with inverse

$$S^{-1}\ [\overline{F}(\alpha)] = F(x) = \frac{2}{\pi} \int_{o}^{\infty} \overline{F}(\alpha)\ \sin\ (\alpha x)\ d\alpha \qquad (A1.64)$$

and with an operational property

$$S \left[\frac{d^2F(x)}{dx^2}\right] = -\alpha^2\ \overline{F} + \alpha\ F(o), \qquad (A1.65)$$

where F(o) is the function evaluated at x = 0. The transformed equation and boundary and initial conditions are

$$\frac{\partial \overline{c}}{\partial t} + \alpha^2\ D_x \overline{c} - D_y \frac{\partial^2 \overline{c}}{\partial y^2} - D_z \frac{\partial^2 \overline{c}}{\partial z^2} - \alpha\ D_x\ c\ (o,\ y,\ z,\ t) = 0 \qquad (A1.66)$$

$$\overline{c},\ \frac{\partial \overline{c}}{\partial y} = 0 \qquad y = 0,\ y = W \qquad (A1.67)$$

$$c,\ \frac{\partial \overline{c}}{\partial z} = 0 \qquad z = 0,\ z = H \qquad (A1.68)$$

$$\overline{c}\ (\alpha,\ y,\ z,\ o) = 0, \qquad (A1.69)$$

where c (o, y, z, t) is the patch source boundary condition specified in equation A1.58.

STEP 3:

The y-derivative is removed by applying the finite Fourier cosine transform as defined by Churchill (1972, p. 354-356); that is

$$F_c\ [G(y)] = \bar{G}(n) = \int_o^W G(y)\ \cos\left(\frac{n\pi y}{W}\right)\ dy \qquad (A1.70)$$

with inverse

$$F_c^{-1}\ [\bar{G}(n)] = G(y) = \frac{\bar{G}(o)}{W} + \frac{2}{W} \sum_{n=1}^{\infty} \bar{G}(n)\ \cos\left(\frac{n\pi y}{W}\right) \qquad (A1.71)$$

and with an operational property

$$F_c \left[\frac{d^2G(y)}{dy^2}\right] = (-1)^n \left.\frac{dG}{dy}\right|_{y=W} - \left.\frac{dG}{dy}\right|_{y=0} - \frac{n^2\pi^2}{W^2}\ \bar{G}. \qquad (A1.72)$$

The transformed equation and boundary and initial conditions are

$$\frac{\partial \bar{\bar{c}}}{\partial t} + \alpha^2 D_x \bar{\bar{c}} + \eta^2 D_y \bar{\bar{c}} - D_z \frac{\partial^2 \bar{\bar{c}}}{\partial z^2} - \alpha\ D_x \int_o^W c\ (o,\ y,\ z,\ t)\ \cos\ (\eta y)\ dy = 0 \qquad (A1.73)$$

$$\bar{\bar{c}},\ \frac{\partial \bar{\bar{c}}}{\partial z} = 0, \qquad z = 0,\ z = H \qquad (A1.74)$$

$$\bar{\bar{c}}\ (\alpha,\ n,\ z,\ o) = 0. \qquad (A1.75)$$

where $\eta = \frac{n\pi}{W}$.

STEP 4:

The finite Fourier cosine transform is applied again to remove the z-derivative. Note that when equation A1.58 is used to define the patch source boundary term, the integral in equation A1.73 has a nonzero value only over the interval from Y_1 to Y_2 and from Z_1 to Z_2. Thus, the transformed equation and initial condition are given by

$$\frac{d\bar{\bar{\bar{c}}}}{dt} + \left(\alpha^2 D_x + \eta^2 D_y + \zeta^2 D_z\right) \bar{\bar{\bar{c}}} - \alpha\, D_x\, C_o \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \cdot \int_{Z_1}^{Z_2} \int_{Y_1}^{Y_2} \cos(\eta y)\, \cos(\zeta z)\, dy\, dz = 0 \qquad \text{(A1.76)}$$

$$\bar{\bar{\bar{c}}}\ (\alpha,\ n,\ m,\ o) = 0, \qquad \text{(A1.77)}$$

where $\zeta = \frac{m\pi}{H}$.

STEP 5:

The transformed ordinary differential equation is solved for $\bar{\bar{c}}$ using an integrating factor (see eqs. A1.29 to A1.31); that is

$$\bar{\bar{c}} = \frac{\alpha\, D_x\, C_o\, I_{zy}}{\exp\,[\alpha^2 D_x t + \eta^2 D_y t + \zeta^2 D_z t]} \int_o^t \exp\left[\alpha^2 D_x + \eta^2 D_y + \zeta^2 D_z + \frac{V^2}{4D_x} + \lambda\right] \cdot \tau\, d\tau, \qquad \text{(A1.78)}$$

where

$$I_{zy} = \int_{Z_1}^{Z_2} \int_{Y_1}^{Y_2} \cos(\eta y)\, \cos(\zeta z)\, dy\, dz.$$

Integrating equation A1.78 over time gives

$$\bar{\bar{c}} = \frac{\alpha\, D_x\, C_o\, I_{zy}}{\left(\alpha^2 D_x + \eta^2 D_y + \zeta^2 D_z + \frac{V^2}{4D_x} + \lambda\right)} \left\{\exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] - \exp\left[-\alpha^2 D_x t - \eta^2 D_y t - \zeta^2 D_z t\right]\right\}. \qquad \text{(A1.79)}$$

STEP 6:

The inverse Fourier sine transform (eq. A1.64) is applied to remove the α term; that is

$$\bar{\bar{c}} = C_o \, I_{zy} \left\{ \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] S^{-1}\left[\frac{\alpha}{\alpha^2 + \frac{\eta^2 D_y}{D_x} + \frac{\zeta^2 D_z}{D_x} + \frac{V^2}{4D_x^{\,2}} + \frac{\lambda}{D_x}}\right]\right.$$

$$\left. - \exp\left[-\eta^2 D_y t - \zeta^2 D_z t\right] S^{-1}\left[\frac{\alpha \exp\left[-\alpha^2 D_x t\right]}{\alpha^2 + \frac{\eta^2 D_y}{D_x} + \frac{\zeta^2 D_z}{D_x} + \frac{V^2}{4D_x^{\,2}} + \frac{\lambda}{D_x}}\right]\right\}. \qquad \text{(A1.80)}$$

The first inverse transform can be evaluated using equation D.1.16 in the table of inverse Fourier sine transforms in Churchill (1972, p. 474), where

$$S^{-1}\left[\frac{\alpha}{\alpha^2+b^2}\right] = \exp\left[-bx\right]. \qquad \text{(A1.81)}$$

Unfortunately, the second inverse transform cannot be found in the tables. Instead, it can be determined by performing the integration as defined in equation A1.64, where

$$S^{-1}\left[\frac{\alpha \exp\left[-a\alpha^2\right]}{a^2+b^2}\right] = \frac{2}{\pi}\int_o^\infty \frac{\alpha \exp\left[-a\alpha^2\right]}{\alpha^2 + b^2} \sin \alpha x d\alpha. \qquad \text{(A1.82)}$$

The integral in equation A1.82 is given in Gradshteyn and Ryzhik (1980, p. 497, eq. 3.954); that is

$$I = -\frac{\pi}{4}\exp\left[ab^2\right]\left\{2\sinh(xb) + \exp\left[-xb\right]\operatorname{erf}\left[b\sqrt{a} - \frac{x}{2\sqrt{a}}\right] - \exp\left[xb\right]\operatorname{erf}\left[b\sqrt{a} + \frac{x}{2\sqrt{a}}\right]\right\}, \qquad \text{(A1.83)}$$

where sinh(xb) is the hyperbolic sine. When written in terms of the complementary error function, erfc, the inverse Fourier sine transform can be written as

$$S^{-1}\left[\frac{\alpha \exp\left[-a\alpha^2\right]}{\alpha^2 + b^2}\right] = \frac{1}{2}\exp\left[ab^2\right]\left\{\exp\left[-xb\right]\operatorname{erfc}\left[b\sqrt{a} - \frac{x}{2\sqrt{a}}\right] - \exp\left[xb\right]\operatorname{erfc}\left[b\sqrt{a} + \frac{x}{2\sqrt{a}}\right]\right\}. \tag{A1.84}$$

Letting $a = D_x t$ and $b = \left(\frac{\eta^2 D_y}{D_x} + \frac{\zeta^2 D_z}{D_x} + \frac{V^2}{4D_x^2} + \frac{\lambda}{D_x}\right)^{1/2}$, equation A1.80 can be evaluated as

$$\bar{\bar{c}} = C_o\, I_{zy}\left\{\exp\left[\frac{V^2 t}{4D_x} + \lambda t - \frac{\beta x}{2D_x}\right] - \frac{1}{2}\exp\left[\frac{V^2 t}{4D_x} + \lambda t - \frac{\beta x}{2D_x}\right]\operatorname{erfc}\left[\frac{\beta t - x}{2\sqrt{D_x t}}\right] + \frac{1}{2}\exp\left[\frac{V^2 t}{4D_x} + \lambda t + \frac{\beta x}{2D_x}\right]\operatorname{erfc}\left[\frac{\beta t + x}{2\sqrt{D_x t}}\right]\right\}, \tag{A1.85}$$

where $\beta = [V^2 + 4D_x(\lambda + \eta^2 D_y + \zeta^2 D_z)]^{1/2}$. The second term in equation A1.85 can be rewritten using the identity erfc (-x) = 2 - erfc (x) to cancel the first term, yielding

$$\bar{\bar{c}} = C_o \frac{I_{zy}}{2}\exp\left[\frac{V^2 t}{4D_x} + \lambda t\right]\left\{\exp\left[-\frac{\beta x}{2D_x}\right]\operatorname{erfc}\left[\frac{x - \beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{\beta x}{2D_x}\right]\operatorname{erfc}\left[\frac{x + \beta t}{2\sqrt{D_x t}}\right]\right\}. \tag{A1.86}$$

STEP 7:

The inverse finite Fourier cosine transform (eq. A1.71) is applied to remove the n terms; that is

$$\bar{c} = \left.\frac{\bar{\bar{c}}}{W}\right|_{n=0} + \frac{2}{W} \sum_{n=1}^{\infty} \bar{\bar{c}}\ (n) \cos\ (\eta y). \tag{A1.87}$$

Integrals involving n in the term I_{zy} are also evaluated at this point to give

$$\bar{c} = C_o \frac{(Y_2 - Y_1)}{2W} \int_{Z_1}^{Z_2} \cos\ (\zeta z)\ dz \cdot \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right]$$

$$\left\{\exp\left[\frac{-\gamma x}{2D_x}\right] \operatorname{erfc}\left[\frac{x - \gamma t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{\gamma x}{2D_x}\right] \operatorname{erfc}\left[\frac{x + \gamma t}{2\sqrt{D_x t}}\right]\right\}$$

$$+ \frac{C_o}{W} \int_{Z_1}^{Z_2} \cos\ (\zeta z)\ dz \cdot \sum_{n=1}^{\infty} \left[\frac{\sin\ (\eta Y_2) - \sin\ (\eta Y_1)}{\eta}\right]$$

$$\cdot \cos\ (\eta y) \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \left\{\exp\left[\frac{-\beta x}{2D_x}\right] \operatorname{erfc}\left[\frac{x - \beta t}{2\sqrt{D_x t}}\right]\right.$$

$$\left. + \exp\left[\frac{\beta x}{2D_x}\right] \operatorname{erfc}\left[\frac{x + \beta t}{2\sqrt{D_x t}}\right]\right\}, \tag{A1.88}$$

where

$$\gamma = \left[V^2 + 4D_x\ (\lambda + \zeta^2 D_z)\right]^{1/2}.$$

STEP 8:

Apply the inverse finite Fourier cosine transform to remove the m terms; that is

$$c = \frac{C_o}{2} \frac{(Y_2-Y_1)}{W} \frac{(Z_2-Z_1)}{H} \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \left\{\exp\left[\frac{-\omega x}{2D_x}\right] \operatorname{erfc}\left[\frac{x-\omega t}{2\sqrt{D_x t}}\right]\right.$$

$$\left. + \exp\left[\frac{\omega x}{2D_x}\right] \operatorname{erfc}\left[\frac{x+\omega t}{2\sqrt{D_x t}}\right]\right\} + C_o \left(\frac{Z_2-Z_1}{H}\right) \sum_{n=1}^{\infty} \left[\frac{\sin(\eta Y_2) - \sin(\eta Y_1)}{n\pi}\right] \cos \eta y$$

$$\cdot \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \left\{\exp\left[\frac{-ex}{2D_x}\right] \operatorname{erfc}\left[\frac{x-\epsilon t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{\epsilon x}{2D_x}\right] \operatorname{erfc}\left[\frac{x+\epsilon t}{2\sqrt{D_x t}}\right]\right\}$$

$$+ C_o \left(\frac{Y_2-Y_1}{W}\right) \sum_{m=1}^{\infty} \left[\frac{\sin(\zeta Z_2) - \sin(\zeta Z_1)}{m\pi}\right] \cos(\zeta z)$$

$$\cdot \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \left\{\exp\left[\frac{-\gamma x}{2D_x}\right] \operatorname{erfc}\left[\frac{x-\gamma t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{\gamma x}{2D_x}\right] \operatorname{erfc}\left[\frac{x+\gamma t}{2\sqrt{D_x t}}\right]\right\}$$

$$+ 2C_o \sum_{m=1}^{\infty} \left[\frac{\sin(\zeta Z_2) - \sin(\zeta Z_1)}{m\pi}\right] \cos \zeta z \cdot \sum_{n=1}^{\infty} \left[\frac{\sin(\eta Y_2) - \sin(\eta Y_1)}{n\pi}\right] \cos(\eta y)$$

$$\cdot \exp\left[\frac{V^2 t}{4D_x} + \lambda t\right] \left\{\exp\left[\frac{-\beta x}{2D_x}\right] \operatorname{erfc}\left[\frac{x-\beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{\beta x}{2D_x}\right] \operatorname{erfc}\left[\frac{x+\beta t}{2\sqrt{D_x t}}\right]\right\}, \quad \text{(A1.89)}$$

where
$$\omega = (V^2 + 4\lambda D_x)^{1/2}$$

and
$$\epsilon = [V^2 + 4D_x (\lambda + \eta^2 D_y)]^{1/2}.$$

STEP 9:

Multiply both sides of equation A1.90 by

$$\exp\left[\frac{Vx}{2D_x} - \frac{V^2t}{4D_x} - \lambda t\right]$$

to convert the transformed variable c back to C (see eq. A1.57) which yields

$$C = \frac{C_o}{2}\left[\frac{Y_2-Y_1}{W}\right]\left[\frac{Z_2-Z_1}{H}\right]\left\{\exp\left[\frac{x(v-\omega)}{2D_x}\right]\text{erfc}\left[\frac{x-\omega t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\omega)}{2D_x}\right]\text{erfc}\left[\frac{x+\omega t}{2\sqrt{D_x t}}\right]\right\}$$

$$+ C_o\frac{(Z_2-Z_1)}{H}\sum_{n=1}^{\infty}\left[\frac{\sin(\eta Y_2) - \sin(\eta Y_1)}{n\pi}\right]\cos(\eta y)$$

$$\cdot\left\{\exp\left[\frac{x(v-\epsilon)}{2D_x}\right]\text{erfc}\left[\frac{x-\epsilon t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\epsilon)}{2D_x}\right]\text{erfc}\left[\frac{x+\epsilon t}{2\sqrt{D_x t}}\right]\right\}$$

$$+ C_o\frac{(Y_2-Y_1)}{W}\sum_{m=1}^{\infty}\left[\frac{\sin(\zeta Z_2) - \sin(\zeta Z_1)}{m\pi}\right]\cos(\zeta z)$$

$$\cdot\left\{\exp\left[\frac{x(v-\gamma)}{2D_x}\right]\text{erfc}\left[\frac{x-\gamma t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\gamma)}{2D_x}\right]\text{erfc}\left[\frac{x+\gamma t}{2\sqrt{D_x t}}\right]\right\}$$

$$+ 2C_o\sum_{m=1}^{\infty}\left[\frac{\sin(\zeta Z_2) - \sin(\zeta Z_1)}{m\pi}\right]\cos(\zeta z)\cdot\sum_{n=1}^{\infty}\left[\frac{\sin(\eta Y_2) - \sin(\eta Y_1)}{n\pi}\right]\cos\eta y$$

$$\cdot\left\{\exp\left[\frac{x(v-\beta)}{2D_x}\right]\text{erfc}\left[\frac{x-\beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\beta)}{2D_x}\right]\text{erfc}\left[\frac{x+\beta t}{2\sqrt{D_x t}}\right]\right\}. \qquad \text{(A1.90)}$$

Equation A1.90 represents a final form of the analytical solution for the patch source. It can also be written in a form similar to that of Cleary (1978, p. 24-25) and equation 114 in the text; that is

$$C = C_o \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} L_{mn}\, O_m\, P_n \cos(\zeta z) \cos(\eta y)$$

$$\cdot \left\{ \exp\left[\frac{x(v-\beta)}{2D_x}\right] \operatorname{erfc}\left[\frac{x-\beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{x(v+\beta)}{2D_x}\right] \operatorname{erfc}\left[\frac{x+\beta t}{2\sqrt{D_x t}}\right] \right\}, \quad \text{(A1.91)}$$

where

$$L_{mn} = \begin{cases} 1/2 & m = 0, \text{ and } n = 0 \\ 1 & m = 0, \text{ and } n > 0 \\ 1 & m > 0, \text{ and } n = 0 \\ 2 & m > 0, \text{ and } n > 0 \end{cases}$$

$$O_m = \begin{cases} \dfrac{Z_2 - Z_1}{H} & m = 0 \\ \left[\dfrac{\sin(\zeta Z_2) - \sin(\zeta Z_1)}{m\pi}\right] & m > 0 \end{cases}$$

$$P_n = \begin{cases} \dfrac{Y_2 - Y_1}{W} & n = 0 \\ \left[\dfrac{\sin(\eta Y_2) - \sin(\eta Y_1)}{n\pi}\right] & n > 0. \end{cases}$$

AQUIFER OF INFINITE WIDTH AND HEIGHT WITH CONTINUOUS POINT SOURCE

The following is a step-by-step derivation of the analytical solution for solute transport in an aquifer of infinite length, width, and height containing a continuous point solute source injecting solute with a concentration C_o at a rate Q in a steady flow field (eq. 105 in the text).

The governing three-dimensional solute-transport equation is

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda C$$

$$+ Qt\, C_o\, \delta(\quad)\, \delta(x-X_c)\, \delta(y-Y_c)\, \delta(z-Z_c). \quad (A1.92)$$

Boundary and initial conditions are

$$C,\ \frac{\partial C}{\partial x} = 0 \qquad x = \pm\infty \qquad (A1.93)$$

$$C,\ \frac{\partial C}{\partial y} = 0 \qquad y = \pm\infty \qquad (A1.94)$$

$$C,\ \frac{\partial C}{\partial z} = 0 \qquad z = \pm\infty \qquad (A1.95)$$

$$C\ (x,\ y,\ z,\ o) = 0 \qquad (A1.96)$$

where

V is V_x, the velocity in x-direction (V_y and $V_z = 0$),

$Qt\, C_o$ is the mass of solute injected into the aquifer over time,

X_c, Y_c, Z_c are the coordinates of the point source, and

$\delta(\quad)$ is the dirac delta function.

STEP 1:

To solve equation A1.91 for the continuous point source, first solve the partial differential equation for solute transport in an aquifer with an instantaneous point source. The governing equation is rewritten as

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - V \frac{\partial C}{\partial x} - \lambda c$$

$$+ Qdt\ C_o\ \delta(x-X_c) \cdot \delta(y-Y_c)\ \delta(z-Z_c)\ \delta(t-t'), \quad \text{(A1.97)}$$

where t' is time at which the instantaneous point source starts and ends, and $Qdt\ C_o$ represents the mass of solute injected in that time period. Boundary conditions remain the same.

STEP 2:

A variable transformation is applied to remove the advective and solute-decay terms, where

$$c = C \exp\left[- \frac{Vx}{2D_x} + \frac{V^2 t}{4D_x} + \lambda t\right]. \quad \text{(A1.98)}$$

The resulting transformed solute-transport equation and boundary and initial conditions are

$$\frac{\partial c}{\partial t} = D_x \frac{\partial^2 c}{\partial x^2} + D_y \frac{\partial^2 c}{\partial y^2} + D_z \frac{\partial^2 c}{\partial z^2} + Qdt\ C_o \exp\left[- \frac{Vx}{2D_x} + \frac{V^2 t}{4D_x} + \lambda t\right]$$

$$\cdot\ \delta(x-X_c) \cdot \delta(y-Y_c)\ \delta(z-Z_c)\ \delta(t-t') \quad \text{(A1.99)}$$

$$c,\ \frac{\partial c}{\partial x} = 0 \qquad x = \pm\infty \quad \text{(A1.100)}$$

$$c,\ \frac{\partial c}{\partial y} = 0 \qquad y = \pm\infty \quad \text{(A1.101)}$$

$$c,\ \frac{\partial c}{\partial z} = 0 \qquad z = \pm\infty \quad \text{(A1.102)}$$

$$c\ (x,\ y,\ z,\ o) = 0. \quad \text{(A1.103)}$$

STEP 3:

The x-derivative term is removed by applying the exponential Fourier transform as defined by Churchill (1972, p. 384-385); that is

$$E\,[F(x)] = \bar{F}(\alpha) = \int_{-\infty}^{+\infty} F(x)\,\exp\,[-i\alpha x]\,dx \tag{A1.104}$$

with inverse

$$E^{-1}\,[\bar{F}(\alpha)] = F(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \bar{F}(\alpha)\,\exp\,[i\alpha x]\,d\alpha \tag{A1.105}$$

and with an operational property

$$E\left[\frac{d^2F(x)}{dx^2}\right] = -\alpha^2\,\bar{F}(\alpha), \tag{A1.106}$$

where $i = \sqrt{-1}$. The y- and z-derivatives can be removed similarly yielding the transformed equation and initial condition

$$\frac{\partial\bar{\bar{c}}}{\partial t} + \left[\alpha^2 D_x + \beta^2 D_y + \gamma^2 D_z\right]\bar{\bar{\bar{c}}} - Qdt\;C_o\,\exp\left[\frac{V^2t}{4D_x} + \lambda t\right]$$

$$\cdot\int_{\infty}^{\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp\left[-i\alpha x - \frac{Vx}{2D_x} - i\beta y - i\gamma z\right]$$

$$\delta(x-X_c)\;\delta(y-Y_c)\;\delta(z-Z_c)\;\delta(t-t')\;dx\;dy\;dz = 0 \tag{A1.107}$$

$$\bar{\bar{\bar{c}}}\;(\alpha,\;\beta,\;\gamma,\;o) = 0. \tag{A1.108}$$

By definition, the integral of a function multiplied by the dirac delta function (last term in eq. A1.107) is equal to the function evaluated at the coordinate of the point source. Thus, the transformed equation is given by

$$\frac{d\bar{\bar{\bar{c}}}}{dt} + (\alpha^2 D_x + \beta^2 D_y + \gamma^2 D_z)\;\bar{\bar{\bar{c}}} - Qdt\;C_o$$

$$\exp\left[\frac{V^2t}{4D_x} + \lambda t - i\alpha X_c - \frac{VX_c}{2D_x} - i\beta Y_c - i\gamma Z_c\right]\delta(t-t') = 0. \tag{A1.109}$$

STEP 4:

The transformed ordinary differential equation is solved for $\bar{\bar{c}}$ using an integrating factor (see eqs. A1.29 to A1.31); that is

$$\bar{\bar{c}} = \frac{Qdt\ C_o \exp\left[-\ i\alpha X_c - \frac{VX_c}{2D_x} - i\beta Y_c - i\gamma Z_c\right]}{\exp\ [\alpha^2 D_x t + \beta^2 D_y t + \gamma^2 D_z t]} \int_o^t \exp\left[\alpha^2 D_x + \beta^2 D_y\right.$$

$$\left. + \gamma^2 D_z + \frac{V^2}{4D_x} + \lambda\right] \bullet \tau\ \delta(\tau - t')\ d\tau. \qquad \text{(A1.110)}$$

Integrating equation A1.110 and grouping like terms gives

$$\bar{\bar{c}} = Qdt\ C_o \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - i\alpha X_c - \frac{VX_c}{2D_x} - \alpha^2 D_x\ (t-t')\ - i\beta Y_c\right.$$

$$\left. -\ \beta^2 D_y\ (t-t') - i\gamma Z_c - \gamma^2 D_z\ (t-t')\right]. \qquad \text{(A1.111)}$$

STEP 5:

The inverse exponential Fourier transform (eq. A1.105) is applied three times to remove the α, β, and γ terms; that is

$$\bar{c} = Qdt\ C_o \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \frac{VX_c}{2D_x}\right] \cdot E^{-1}\left\{\exp\left[-\ i\alpha X_c - \alpha^2 D_x\ (t-t')\right]\right\}$$

$$\cdot\ E^{-1}\left\{\exp\left[-i\beta Y_c - \beta^2 D_y\ (t-t')\right]\right\} \cdot E^{-1}\left\{\exp\left[-i\gamma Z_c - \gamma^2 D_z\ (t-t')\right]\right\}. \quad \text{(A1.112)}$$

Multiplying through by

$$\frac{2\sqrt{\pi D_x\ (t-t')} \cdot 2\sqrt{\pi D_y\ (t-t')} \cdot 2\sqrt{\pi D_z\ (t-t')}}{8\pi^{3/2}\ (t-t')^{3/2}\ \sqrt{D_x D_y D_z}}$$

and using the shift theorem (Churchill, 1972, p. 471, eq. C.1.5) given by

$$E^{-1}\left\{\exp\left[i a \alpha\right]\ \bar{F}(\alpha)\right\} = F\ (x+a) \quad \text{(A1.113)}$$

and equation C.1.20 from the table of inverse exponential Fourier transforms (Churchill, 1972, p. 472) given by

$$E^{-1}\left\{2\sqrt{\pi a}\ \exp\left[-a(\alpha)^2\right]\right\} = \exp\left[-\ \frac{x^2}{4a}\right], \quad \text{(A1.114)}$$

yields

$$c = \frac{Qdt\ C_o}{8\pi^{3/2}\ (t-t')^{3/2}\ \sqrt{D_x D_y D_z}} \exp\left[\frac{V^2 t'}{4D_x} + \lambda t' - \frac{VX_c}{2D_x}\right.$$

$$\left. -\ \frac{(x-X_c)^2}{4D_x\ (t-t')} - \frac{(y-Y_c)^2}{4D_y\ (t-t')} - \frac{(z-Z_c)^2}{4D_z\ (t-t')}\right]. \quad \text{(A1.115)}$$

STEP 6:

The transformed variable is converted back from c to C by multiplying both sides of equation A1.115 by

$$\exp\left[\frac{Vx}{2D_x} - \frac{V^2 t}{4D_x} - \lambda t\right]$$

(see eq. A1.98) to yield the analytical solution to the solute-transport equation for an instantaneous point source (identical to eq. 104 in the text); that is

$$C = \frac{C_o\ Qdt}{8\pi^{3/2}\ (t-t')^{3/2}\ \sqrt{D_x D_y D_z}} \exp\left[- \frac{V^2\ (t-t')}{4D_x} - \lambda\ (t-t')\right.$$

$$\left. + \frac{V(x-X_c)}{2D_x} - \frac{(x-X_c)^2}{4D_x\ (t-t')} - \frac{(y-Y_c)^2}{4D_y\ (t-t')} - \frac{(z-Z_c)^2}{4D_z\ (t-t')}\right]. \quad \text{(A1.116)}$$

STEP 7:

To derive a solution for a continuous point source, integrate equation A1.116 from $t' = 0$ to $t' = t$. To simplify the integration, let $\tau = (t-t')$ and $d\tau = -dt'$:

$$C = \frac{C_o\, Q \exp\left[\frac{V(x-X_c)}{2D_x}\right]}{8\pi^{3/2}\sqrt{D_x D_y D_z}} \cdot \int_t^o - \tau^{-3/2}$$

$$\exp\left[-\frac{(x-X_c)^2}{4D_x\tau} - \frac{(y-Y_c)^2}{4D_y\tau} - \frac{(z-Z_c)^2}{4D_z\tau} - \left(\frac{V^2}{4D_x} + \lambda\right)\tau\right] d\tau. \quad \text{(A1.117)}$$

The integral in equation A1.117 can be evaluated by first reversing the limits of integration and then using an indefinite integral given in a table by Cho (1971, eq. 2.9.5), where

$$\int_o^t \tau^{-3/2} \exp\left[-\frac{a^2}{\tau} - b^2\tau\right] d\tau = \frac{\sqrt{\pi}}{2a}\left\{\exp\left[-2ab\right] \operatorname{erfc}\left[\frac{a}{\sqrt{t}} - b\sqrt{t}\right]\right.$$

$$\left. + \exp\left[2ab\right] \operatorname{erfc}\left[\frac{a}{\sqrt{t}} + b\sqrt{t}\right]\right\}, \quad \text{(A1.118)}$$

let

$$\gamma = \left[(x-X_c)^2 + \frac{D_x}{D_y}(y-Y_c)^2 + \frac{D_x}{D_z}(z-Z_c)^2\right]^{1/2}$$

and

$$\beta = (V^2 + 4D_x\lambda)^{1/2},$$

the integral can be rewritten as

$$I = \frac{\sqrt{\pi D_x}}{\gamma}\left\{\exp\left[-\frac{\gamma\beta}{2D_x}\right] \operatorname{erfc}\left[\frac{\gamma-\beta t}{2\sqrt{D_x t}}\right] + \exp\left[\frac{\gamma\beta}{2D_x}\right] \operatorname{erfc}\left[\frac{\gamma+\beta t}{2\sqrt{D_x t}}\right]\right\}. \quad \text{(A1.119)}$$

Substituting in equation A1.117 yields the final closed form of the analytical solution for a continuous point source (identical to eq. 105 in the text); that is

$$C = \frac{C_o\, Q \exp\left[\frac{V(x-X_c)}{2D_x}\right]}{8\pi\gamma\sqrt{D_y D_z}} \cdot \left\{\exp\left[-\frac{\gamma\beta}{2D_x}\right] \operatorname{erfc}\left[\frac{\gamma-\beta t}{2\sqrt{D_x t}}\right]\right.$$

$$\left. + \exp\left[\frac{\gamma\beta}{2D_x}\right] \operatorname{erfc}\left[\frac{\gamma+\beta t}{2\sqrt{D_x t}}\right]\right\}. \quad \text{(A1.120)}$$

At steady state, the solution is given by

$$C = \frac{C_o\, Q}{4\pi\gamma\sqrt{D_y D_z}} \exp\left[\frac{V(x-X_c) - \gamma\beta}{2D_x}\right]. \quad \text{(A1.121)}$$

ATTACHMENT 2.--PROGRAM SOURCE-CODE LISTINGS

FINITE
SEMINF
POINT2
STRIPF
STRIPI
GAUSS
POINT3
PATCHF
PATCHI

```
C
C        ********************************************************
C        *                                                      *
C        *                  **** FINITE ****                    *
C        *                                                      *
C        * ONE-DIMENSIONAL GROUND-WATER SOLUTE-TRANSPORT MODEL  *
C        *                                                      *
C        *   FOR A FINITE SYSTEM WITH A FIRST- OR THIRD-TYPE    *
C        *                                                      *
C        *             BOUNDARY CONDITION AT X=0                *
C        *                                                      *
C        *           VERSION CURRENT AS OF 11/30/88             *
C        *                                                      *
C        ********************************************************
C
C       THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C       GREATER THAN THOSE GIVEN HERE.
C         MAXX = MAXIMUM NUMBER OF X-VALUES
C         MAXT = MAXIMUM NUMBER OF TIME VALUES
C         MAXRT = MAXIMUM NUMBER OF ROOTS USED IN THE SERIES SUMMATION
      PARAMETER MAXX=100,MAXT=20,MAXRT=1000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL XP(MAXX),CP(MAXX),TP,XSCLP
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      CHARACTER*1 IERR(MAXX,MAXT)
      DIMENSION CXT(MAXX,MAXT),X(MAXX),T(MAXT)
      DIMENSION ROOT(MAXRT)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C          NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C          MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C          LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CO        SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C     DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T         TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C     CN        NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C     CXT       SOLUTE CONCENTRATION C(X,T) [M/L**3]
C     XL        LENGTH OF THE FLOW SYSTEM [L]
C     ROOT(N)   ROOTS OF EQ. USED IN INFINITE SERIES SUMMATION
C
C     NBC       SOURCE BOUNDARY CONDITION TYPE (1 OR 3)
C     NX        NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NT        NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C     NROOT     NUMBER OF ROOTS USED IN INFINITE SERIES SUMMATION
C     IPLT      PLOT CONTROL. IF IPLT>0, CONCENTRATION PROFILES ARE PLOTT
C
```

```
C        CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C      CUNITS   UNITS OF CONCENTRATION (M/L**3)
C      VUNITS   UNITS OF GROUND-WATER VELOCITY (L/T)
C      DUNITS   UNITS OF DISPERSION COEFFICIENT (L**2/T)
C      KUNITS   UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C      LUNITS   UNITS OF LENGTH (L)
C      TUNITS   UNITS OF TIME (T)
C
C        DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
      CALL OFILE
      CALL TITLE
      WRITE(IO,201)
C
C        READ IN MODEL PARAMETERS
      READ(IN,101) NBC,NX,NT,NROOT,IPLT
      IF(NBC.EQ.1) WRITE(IO,202)
      IF(NBC.EQ.3) WRITE(IO,203)
      WRITE(IO,205) NX,NT,NROOT
      READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      READ(IN,110) C0,VX,DX,DK,XL,XSCLP
      WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DK,KUNITS,XL,LUNITS,
     1 XSCLP
      READ(IN,110) (X(I),I=1,NX)
      WRITE(IO,215) LUNITS
      WRITE(IO,220) (X(I),I=1,NX)
      READ(IN,110) (T(I),I=1,NT)
      WRITE(IO,225) TUNITS
      WRITE(IO,220) (T(I),I=1,NT)
C
C        GET EIGENVALUES (BETA) USED IN SERIES SUMMATION BY SOLVING FOR
C        THE POSITIVE ROOTS OF: BETA*COTAN(BETA)+VX*XL/(2*DX)=0.0
C        FOR A FIRST-TYPE SOURCE BOUNDARY CONDITION,
C        OR: BETA*COTAN(BETA)-BETA**2*DX/(VX*XL)+VX*XL/(4*DX)=0.0
C        FOR A THIRD-TYPE SOURCE BOUNDARY CONDITION.
C
      IF (NBC.EQ.1) THEN
        C=VX*XL/(2.0D0*DX)
        CALL ROOT1(C,ROOT,NROOT)
      ELSE
        A=0.250D0*VX*XL/DX
        C=DX/(XL*VX)
        CALL ROOT3(A,C,ROOT,NROOT)
      END IF
C
C        BEGIN TIME LOOP
      DO 40 IT=1,NT
C
C        BEGIN X-COORDINATE LOOP
      DO 50 IX=1,NX
C
C        CALL ROUTINE TO CALCULATE NORMALIZED CONCENTRATION
C        BASED ON TYPE OF BOUNDARY CONDITION SPECIFIED
      IF(NBC.EQ.1) CALL CNRML1(XL,T(IT),X(IX),DX,VX,DK,ROOT,CN,NROOT,
```

```
     1 IERR(IX,IT))
      IF(NBC.EQ.3) CALL CNRML3(XL,T(IT),X(IX),DX,VX,DK,ROOT,CN,NROOT,
     1 IERR(IX,IT))
      CXT(IX,IT)=CN*C0
50    CONTINUE
C
C       CONVERT X AND C TO SINGLE PRECISION AND DIVIDE BY C0 TO
C       PLOT NORMALIZED CONCENTRATION PROFILE FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 40
      DO 60 I=1,NX
      XP(I)=SNGL(X(I))
60    CP(I)=SNGL(CXT(I,IT)/C0)
      TP=SNGL(T(IT))
      CALL PLOT1D(XP,CP,NX,TP,IT,NT,TUNITS,LUNITS,XSCLP)
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NT-1)/9
      DO 80 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) TUNITS
      IF(NP.NE.1) WRITE(IO,231) TUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NT) NP2=NT-NP1
      WRITE(IO,235) (T(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXT(IX,NP1+J),IERR(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) LUNITS
      WRITE(IO,235) (T(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
80    CONTINUE
C
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C        FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
201   FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL'/
     1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE-TRANSPORT EQUATION'/
     2 1H ,36X,'FOR A SYSTEM OF FINITE LENGTH'///1H0,40X,'INPUT DATA'/
     3 1H ,40X,10(1H-))
202   FORMAT(1H0,25X,'FIRST-TYPE BOUNDARY CONDITION AT X = 0.0')
203   FORMAT(1H0,25X,'THIRD-TYPE BOUNDARY CONDITION AT X = 0.0')
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
     1 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,'NUMBER OF ROOTS ',
     2 'USED IN INFINITE SERIES SUMMATION (NROOT) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =',
```

```
     1 1P1E13.6,1X,A10/1H ,25X,
     2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
     3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
     4 1H ,25X,'FIRST-ORDER SOLUTE-DECAY RATE (DK) =',1P1E13.6,1X,A10/
     5 1H ,25X,'LENGTH OF FINITE FLOW SYSTEM (XL) =',1P1E13.6,1X,A10/
     6 1H ,25X,'PLOT SCALING FACTOR (XSCLP) =',1P1E13.6)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
230   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME',
     1 15X,'*  INDICATES SOLUTION DID NOT CONVERGE'/
     2 1H0,25X,'TIME VALUES, IN ',A10)
231   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME =',
     1 5X,'(CONTINUED)'/
     2 1H0,25X,'TIME VALUES, IN ',A10)
235   FORMAT(1H ,20X,9F12.4)
236   FORMAT(1H ,19X,'*',108(1H-)/
     1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
     2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240   FORMAT(1H ,5X,F12.4,2X,'! ',9(F11.5,A1))
241   FORMAT(1H ,19X,'!')
      END
      SUBROUTINE CNRML1(XL,T,X,D,V,DK,ROOT,CN,NROOT,IERR)
      IMPLICIT DOUBLE PRECISION  (A-H,O-Z)
      CHARACTER*1 IERR
      DIMENSION ROOT(NROOT)
C
C       SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE-TRANSPORT EQUATION
C       FOR A SYSTEM OF FINITE LENGTH WITH A FIRST-TYPE SOURCE
C       BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
C       CONCENTRATION AT A GIVEN X-COORDINATE AND TIME VALUE.
C       FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.
C
      IERR=' '
      XL2=XL*XL
      V2D=V/(2.0D0*D)
      VX2D=V2D*X
      VL2D=V2D*XL
      VL2D2=VL2D*VL2D
      DKL2D=DK*XL*XL/D
      VSQT4D=V*V*T/(4.0D0*D)
      IF(DK.EQ.0.0D0) GO TO 20
C
C       BEGIN SERIES SUMMATION FOR SOLUTE WITH DECAY
      SIGMA=0.0
      DO 10 N=1,NROOT
      BETA=ROOT(N)
      BETA2=BETA*BETA
C
C       TERM 1
      X1=(BETA2+VL2D2)*DEXP(-BETA2*D*T/XL2)
```

```
C
C       TERM 2
      DENOM=(BETA2+VL2D2+VL2D)*(BETA2+VL2D2+DKL2D)
      X2=BETA*DSIN(BETA*X/XL)/DENOM
      SIGMA=SIGMA+X1*X2
C
C     CHECK FOR CONVERGENCE OF SERIES
      IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 15
10    CONTINUE
      IERR='*'
15    CONTINUE
C
C       TERM 3
      U=DSQRT(V*V+4.0D0*DK*D)
      VMU=V-U
      VPU=V+U
      VUPM=(U-V)/VPU
      D2=D*2.0D0
      X3=DEXP(VMU*X/D2)+VUPM*DEXP((VPU*X-2.0D0*U*XL)/D2)
      X3=X3/(1.0D0+VUPM*DEXP(-U*XL/D))
      CN=X3-2.0D0*DEXP(VX2D-VSQT4D-DK*T)*SIGMA
      RETURN
C
C       BEGIN SERIES SUMMATION FOR SOLUTE WITH NO DECAY
20    SIGMA=0.0
      DO 30 N=1,NROOT
      BETA=ROOT(N)
      BETA2=BETA*BETA
C
C       TERM 1
      DENOM=BETA2+VL2D2+VL2D
      X1=BETA*DSIN(BETA*X/XL)*DEXP(-BETA2*D*T/XL2)
      X1=X1/DENOM
      TERM=X1
      SIGMA=SIGMA+X1
      IF(N.GT.25 .AND. DABS(X1).LT.1.0D-14) GO TO 35
30    CONTINUE
      IERR='*'
35    CONTINUE
      CN=1.0D0-2.0D0*DEXP(VX2D-VSQT4D)*SIGMA
      RETURN
      END
      SUBROUTINE CNRML3(XL,T,X,D,V,DK,ROOT,CN,NROOT,IERR)
      IMPLICIT DOUBLE PRECISION  (A-H,O-Z)
      CHARACTER*1 IERR
      DIMENSION ROOT(NROOT)
C
C       SOLUTION FOR THE ONE DIMENSIONAL SOLUTE-TRANSPORT EQUATION
C       FOR A SYSTEM OF FINITE LENGTH WITH A THIRD-TYPE SOURCE
C       BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
C       CONCENTRATION AT A GIVEN X-COORDINATE AND TIME VALUE.
C       FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.
C
```

```
      IERR=' '
      XL2=XL*XL
      V2D=V/(2.0D0*D)
      VLD=V*XL/D
      VX2D=V2D*X
      VL2D=V2D*XL
      VL2D2=VL2D*VL2D
      DKL2D=DK*XL*XL/D
      VSQT4D=V*V*T/(4.0D0*D)
      IF(DK.EQ.0.0D0) GO TO 20
C
C       BEGIN SERIES SUMMATION FOR SOLUTE WITH DECAY
      SIGMA=0.0
      DO 10 N=1,NROOT
      BETA=ROOT(N)
      BETA2=BETA*BETA
C
C       TERM 1
      BETAXL=BETA*X/XL
      X1=BETA*(BETA*DCOS(BETAXL)+VL2D*DSIN(BETAXL))
C
C       TERM 2
      DENOM=(BETA2+VL2D2+VLD)*(BETA2+VL2D2+DKL2D)
      X2=DEXP(-BETA2*D*T/XL2)/DENOM
      SIGMA=SIGMA+X1*X2
C
C     CHECK FOR CONVERGENCE OF SERIES
      IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 15
10    CONTINUE
      IERR='*'
15    CONTINUE
C
C       TERM 3
      U=DSQRT(V*V+4.0D0*DK*D)
      VMU=V-U
      VPU=V+U
      VUPM=(U-V)/VPU
      D2=D*2.0D0
      X3=DEXP(VMU*X/D2)+VUPM*DEXP((VPU*X-2.0D0*U*XL)/D2)
      X3=2.0D0*V*X3/(VPU+VMU*VUPM*DEXP(-U*XL/D))
      CN=X3-2.0D0*VLD*DEXP(VX2D-VSQT4D-DK*T)*SIGMA
      RETURN
C
C       BEGIN SERIES SUMMATION FOR SOLUTE WITH NO DECAY
20    SIGMA=0.0
      DO 30 N=1,NROOT
      BETA=ROOT(N)
      BETA2=BETA*BETA
C
C       TERM 1
      BETAXL=BETA*X/XL
      X1=BETA*(BETA*DCOS(BETAXL)+VL2D*DSIN(BETAXL))
C
```

```
C         TERM 2
       DENOM=(BETA2+VL2D2+VLD)*(BETA2+VL2D2)
       X2=DEXP(-BETA2*D*T/XL2)/DENOM
C
       SIGMA=SIGMA+X1*X2
       IF(N.GT.25 .AND. DABS(X1*X2).LT.1.0D-14) GO TO 35
30     CONTINUE
       IERR='*'
35     CONTINUE
       CN=1.0D0-2.0D0*SIGMA
       CN=1.0D0-2.0D0*VLD*DEXP(VX2D-VSQT4D)*SIGMA
       RETURN
       END
       SUBROUTINE ROOT1 (C,ROOT,NROOT)
       IMPLICIT DOUBLE PRECISION (A-H,O-Z)
       DIMENSION ROOT(NROOT)
       COMMON /IOUNIT/ IN,IO
       DATA MAXIT,EPS/50,1.0D-10/
C
C         THIS ROUTINE CALCULATES ROOTS OF THE EQUATION: B*COTAN(B)+C=0
C         USING NEWTON'S SECOND-ORDER METHOD.
C
C                             PROGRAM VARIABLES
C         MAXIT     MAXIMUM NUMBER OF ITERATIONS ALLOWED IN ROOT SEARCH
C         EPS       CONVERGENCE CRITERION
C         F1,F2     1ST AND 2ND DERIVATIVES OF THE EQUATION
C         H         SECOND-ORDER CORRECTION FACTOR
C
C         FIRST ROOT LIES BETWEEN PI/2 AND PI. START WITH .75*PI
       PI=3.14159265359D0
       ROOT(1)=0.750D0*PI
C
C         START LOOP FOR EACH ROOT SEARCH
       DO 10 N=1,NROOT
C
C            BEGIN ITERATIVE LOOP

       DO 20 I=1,MAXIT
       X=ROOT(N)
       SINX2=DSIN(X)*DSIN(X)
       COTX=1.0D0/DTAN(X)
       F=X*COTX+C
C         IF F IS 0.0, EXACT ROOT HAS BEEN FOUND
       IF(F.EQ.0.0) GO TO 30
       F1=COTX-X/SINX2
       F2=-1.0D0/SINX2-(SINX2-X*DSIN(X*2.0D0))/(SINX2*SINX2)
       H=(F2/2.0D0)/F1-F1/F
       H=1.0D0/H
       ROOT(N)=X+H
C
C      CHECK FOR CONVERGENCE. IF NOT ACHIEVED, RE-ITERATE
       IF(DABS(H).LT.EPS) GO TO 30
   20  CONTINUE
```

```
      WRITE(IO,201) MAXIT,N
      STOP
C
C        NEXT ROOT IS ABOUT PI GREATER THAN LAST ROOT
   30 IF(N.NE.NROOT) ROOT(N+1)=ROOT(N)+PI
   10 CONTINUE
      RETURN
C
C        FORMAT STATEMENTS
201   FORMAT(1H ,5X,'**** WARNING ****  ROOT SEARCH ROUTINE DID NOT',
     1 'CONVERGE AFTER ',I4,'ITERATIONS WHILE SEARCHING FOR ROOT',I5)
      END
      SUBROUTINE ROOT3 (A,C,ROOT,NROOT)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION ROOT(NROOT)
      COMMON /IOUNIT/ IN,IO
      DATA MAXIT,EPS/50,1.0D-10/
C
C        THIS ROUTINE CALCULATES ROOTS OF THE EQ: B*COTAN(B)-C*B**2+A=0
C        USING NEWTON'S SECOND-ORDER METHOD.
C
C                              PROGRAM VARIABLES
C        MAXIT    MAXIMUM NUMBER OF ITERATIONS ALLOWED IN ROOT SEARCH
C        EPS      CONVERGENCE CRITERION
C        F1,F2    1ST AND 2ND DERIVATIVES OF THE EQUATION
C        H        SECOND-ORDER CORRECTION FACTOR
C
C        FIRST ROOT LIES BETWEEN 0.0 AND PI. START WITH 0.5*PI
      PI=3.14159265359D0
      ROOT(1)=0.50D0*PI
C
C        START LOOP FOR EACH ROOT SEARCH
      DO 10 N=1,NROOT
C
C        BEGIN ITERATIVE LOOP
      DO 20 I=1,MAXIT
      X=ROOT(N)
      SINX2=DSIN(X)*DSIN(X)
      COTX=1.0D0/DTAN(X)
      F=X*COTX-C*X*X+A
C        IF F IS 0.0, EXACT ROOT HAS BEEN FOUND
      IF(F.EQ.0.0) GO TO 30
      F1=COTX-X/SINX2-(2.0D0*C*X)
      F2=-1.0D0/SINX2-(SINX2-X*DSIN(X*2.0D0))/(SINX2*SINX2)-2.0D0*C
      H=(F2/2.0D0)/F1-F1/F
      H=1.0D0/H
      ROOT(N)=X+H
C
C     CHECK FOR CONVERGENCE. IF NOT ACHIEVED, RE-ITERATE
      IF(DABS(H).LT.EPS) GO TO 30
   20 CONTINUE
      WRITE(IO,201) MAXIT,N
      STOP
```

```
C
C        NEXT ROOT IS ABOUT PI GREATER THAN LAST ROOT
   30 IF(N.NE.NROOT) ROOT(N+1)=ROOT(N)+PI
   10 CONTINUE
      RETURN
C
C        FORMAT STATEMENTS
201   FORMAT(1H ,5X,'**** WARNING ****  ROOT SEARCH ROUTINE DID NOT',
     1 'CONVERGE AFTER ',I4,'ITERATIONS WHILE SEARCHING FOR ROOT',I5)
      END
```

```
C
C        ********************************************************
C        *                                                      *
C        *                    **** SEMINF ****                  *
C        *                                                      *
C        * ONE-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT MODEL  *
C        *                                                      *
C        *   FOR A SEMI-INFINITE SYSTEM WITH A FIRST-TYPE OR    *
C        *                                                      *
C        *         THIRD-TYPE BOUNDARY CONDITION AT X=0         *
C        *                                                      *
C        *            VERSION CURRENT AS OF 11/30/88            *
C        *                                                      *
C        ********************************************************
C
C   THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C   GREATER THAN THOSE GIVEN HERE.
C     MAXX = MAXIMUM NUMBER OF X-VALUES
C     MAXT = MAXIMUM NUMBER OF TIME VALUES
      PARAMETER MAXX=100,MAXT=20
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      REAL XP(MAXX),CP(MAXX),TP,XSCLP
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      DIMENSION CXT(MAXX,MAXT),X(MAXX),T(MAXT)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C          NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C          MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C          LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CO        SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C     DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T         TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C     CN        NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C     CXT       SOLUTE CONCENTRATION C(X,T) [M/L**3]
C
C     NBC       SOURCE BOUNDARY CONDITION TYPE (1 OR 3)
C     NX        NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NT        NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C     IPLT      PLOT CONTROL. IF IPLT>0, CONCENTRATION PROFILES ARE PLOTT
C
C       CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C     CUNITS   UNITS OF CONCENTRATION (M/L**3)
C     VUNITS   UNITS OF GROUND-WATER VELOCITY (L/T)
C     DUNITS   UNITS OF DISPERSION COEFFICIENT (L**2/T)
C     KUNITS   UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C     LUNITS   UNITS OF LENGTH (L)
```

```
C     TUNITS   UNITS OF TIME (T)
C
C       DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
      CALL OFILE
      CALL TITLE
      WRITE(IO,201)
C
C       READ IN MODEL PARAMETERS
      READ(IN,101) NBC,NX,NT,IPLT
      IF(NBC.EQ.1) WRITE(IO,202)
      IF(NBC.EQ.3) WRITE(IO,203)
      WRITE(IO,205) NX,NT
      READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      READ(IN,110) CO,VX,DX,DK,XSCLP
      WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DK,KUNITS,XSCLP
      READ(IN,110) (X(I),I=1,NX)
      WRITE(IO,215) LUNITS
      WRITE(IO,220) (X(I),I=1,NX)
      READ(IN,110) (T(I),I=1,NT)
      WRITE(IO,225) TUNITS
      WRITE(IO,220) (T(I),I=1,NT)
C
C       BEGIN TIME LOOP
      DO 40 IT=1,NT
C
C       BEGIN X-COORDINATE LOOP
      DO 50 IX=1,NX
C
C       CALL ROUTINE TO CALCULATE NORMALIZED CONCENTRATION
C       BASED ON TYPE OF BOUNDARY CONDITION SPECIFIED
      IF(NBC.EQ.1) CALL CNRML1(DK,T(IT),X(IX),DX,VX,CN)
      IF(NBC.EQ.3) CALL CNRML3(DK,T(IT),X(IX),DX,VX,CN)
      CXT(IX,IT)=CN*CO
50    CONTINUE
C
C       CONVERT X AND C TO SINGLE PRECISION AND DIVIDE BY CO TO
C       PLOT NORMALIZED CONCENTRATION PROFILE FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 40
      DO 60 I=1,NX
      XP(I)=SNGL(X(I))
60    CP(I)=SNGL(CXT(I,IT)/CO)
      TP=SNGL(T(IT))
      CALL PLOT1D(XP,CP,NX,TP,IT,NT,TUNITS,LUNITS,XSCLP)
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NT-1)/9
      DO 80 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) TUNITS
      IF(NP.NE.1) WRITE(IO,231) TUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NT) NP2=NT-NP1
```

```
      WRITE(IO,235) (T(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXT(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) LUNITS
      WRITE(IO,235) (T(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
80    CONTINUE
C
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C         FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
201   FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL'/
     1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
     2 1H ,38X,'FOR A SEMI-INFINITE SYSTEM'///1H0,40X,'INPUT DATA'/
     3 1H ,40X,10(1H-))
202   FORMAT(1H0,25X,'FIRST-TYPE BOUNDARY CONDITION AT X = 0.0')
203   FORMAT(1H0,25X,'THIRD-TYPE BOUNDARY CONDITION AT X = 0.0')
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
     1 'NUMBER OF TIME VALUES (NT) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =',
     1 1P1E13.6,1X,A10/1H ,25X,
     2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
     3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
     4 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10/
     5 1H ,25X,'PLOT SCALING FACTOR (XSCLP) =',1P1E13.6)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
230   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME'/
     2 1H0,25X,'TIME VALUES, IN ',A10)
231   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AS A FUNCTION OF TIME =',
     1 5X,'(CONTINUED)'/
     2 1H0,25X,'TIME VALUES, IN ',A10)
235   FORMAT(1H ,20X,9F12.4)
236   FORMAT(1H ,19X,'*',108(1H-)/
     1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
     2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240   FORMAT(1H ,5X,F12.4,2X,'! ',9F12.5)
241   FORMAT(1H ,19X,'!')
      END
      SUBROUTINE CNRML1(DK,T,X,D,V,CN)
      IMPLICIT DOUBLE PRECISION  (A-H,O-Z)
C
```

```
C        SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE TRANSPORT EQUATION
C        FOR A SEMI-INFINITE SYSTEM WITH A FIRST-TYPE SOURCE
C        BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
C        CONCENTRATION AT A GIVEN X-COORDINATE AND TIME.
C        FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.
C
      ALPHA=2.0D0*DSQRT(D*T)
      U=DSQRT(V*V+4.0D0*D*DK)
      X2D=X/(2.0D0*D)
C
C        SOLUTION WITH SOLUTE DECAY
      IF(DK.EQ.0.0) GO TO 10
C
C        TERM 1
      X1=X2D*(V-U)
      Y1=(X-U*T)/ALPHA
      CALL EXERFC(X1,Y1,Z1)
C
C        TERM 2
      X2=X2D*(V+U)
      Y2=(X+U*T)/ALPHA
      CALL EXERFC(X2,Y2,Z2)
      CN=(Z1+Z2)/(2.0D0)
      RETURN
C
C        SOLUTION WITH NO SOLUTE DECAY
C        TERM 1
10    Y1=(X-V*T)/ALPHA
      CALL EXERFC(0.0D0,Y1,Z1)
      X2=X*V/D
C
C        TERM 2
      Y2=(X+V*T)/ALPHA
      CALL EXERFC(X2,Y2,Z2)
      CN=(Z1+Z2)/(2.0D0)
      RETURN
      END
      SUBROUTINE CNRML3(DK,T,X,D,V,CN)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
C        SOLUTION FOR THE ONE-DIMENSIONAL SOLUTE TRANSPORT EQUATION
C        FOR A SEMI-INFINITE SYSTEM WITH A THIRD-TYPE SOURCE
C        BOUNDARY CONDITION. VALUE RETURNED IS THE NORMALIZED SOLUTE
C        CONCENTRATION AT A GIVEN X-COORDINATE AND TIME.
C        FOR NO SOLUTE DECAY, A SIMPLIFIED SOLUTION IS USED.
C
      ALPHA=2.0D0*DSQRT(D*T)
      U=DSQRT(V*V+4.0D0*D*DK)
      X2D=X/(2.0D0*D)
      VXD=V*X/D
C
C        SOLUTION WITH SOLUTE DECAY
      IF(DK.EQ.0.0) GO TO 10
```

```
C
C       TERM 1
      X1=VXD-DK*T
      Y1=(X+V*T)/ALPHA
      CALL EXERFC(X1,Y1,Z1)
      Z1=Z1*2.0D0
C
C       TERM 2
      X2=X2D*(V-U)
      Y2=(X-U*T)/ALPHA
      CALL EXERFC(X2,Y2,Z2)
      Z2=Z2*(U/V-1.0D0)
C
C       TERM 3
      X3=X2D*(V+U)
      Y3=(X+U*T)/ALPHA
      CALL EXERFC(X3,Y3,Z3)
      Z3=Z3*(U/V+1.0D0)
      CN=V*V*(Z1+Z2-Z3)/(4.0D0*D*DK)
      RETURN
C
C       SOLUTION FOR NO SOLUTE DECAY
10    PI=3.14159265358979D0
C
C       TERM 1
      Y1=(X-V*T)/ALPHA
      CALL EXERFC(0.0D0,Y1,Z1)
      Z1=0.50D0*Z1
C
C       TERM 2
      X2=X*V/D
      Y2=(X+V*T)/ALPHA
      CALL EXERFC(X2,Y2,Z2)
      Z2=Z2*0.50D0*(1.0D0+V*(X+V*T)/D)
C
C       TERM 3
      Z3=DEXP(-1.0D0*(X-V*T)*(X-V*T)/(4.0D0*D*T))
      Z3=Z3*V*DSQRT(T/(PI*D))
      CN=Z1-Z2+Z3
      RETURN
      END
```

```
C
C      ********************************************************
C      *                                                      *
C      *                  **** POINT2 ****                    *
C      *                                                      *
C      * TWO-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT MODEL  *
C      *                                                      *
C      *    FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A    *
C      *                                                      *
C      *   CONTINUOUS POINT SOURCE LOCATED AT X=XC AND Y=YC   *
C      *                                                      *
C      *        GROUND-WATER FLOW IN X-DIRECTION ONLY         *
C      *                                                      *
C      *            VERSION CURRENT AS OF 8/13/87             *
C      *                                                      *
C      ********************************************************
C
C    THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C    GREATER THAN THOSE GIVEN HERE.
C      MAXX = MAXIMUM NUMBER OF X-VALUES
C      MAXY = MAXIMUM NUMBER OF Y-VALUES
C      MAXT = MAXIMUM NUMBER OF TIME VALUES
C      MAXXY = MAXX * MAXY
C      MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C           NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C           MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C           LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C      CO        SOLUTE CONCENTRATION IN INJECTED FLUID [M/L**3]
C      DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C      DY        TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
C      VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C      DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C      X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C      Y         Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C      T         TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C      CN        NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C      CXY       SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
C      XC        X-COORDINATE OF POINT SOURCE [L]
C      YC        Y-COORDINATE OF POINT SOURCE [L]
C      QM        FLUID INJECTION RATE PER UNIT THICKNESS OF AQUIFER [L**2/
```

```
C                (UNITS MUST BE SAME AS DISPERSION COEFFICIENT)
C
C       NX       NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C       NY       NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C       NT       NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C       NMAX     NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL
C                INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)
C
C       IPLT     PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C       XSCLP    SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C       YSCLP    SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C       DELTA    CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C         CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C       CUNITS   UNITS OF CONCENTRATION (M/L**3)
C       VUNITS   UNITS OF GROUND-WATER VELOCITY (L/T)
C       DUNITS   UNITS OF DISPERSION COEFFICIENT (L**2/T)
C       KUNITS   UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C       LUNITS   UNITS OF LENGTH (L)
C       TUNITS   UNITS OF TIME (T)
C
C         DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
        CALL OFILE
        CALL TITLE
        WRITE(IO,201)
C
C         READ IN MODEL PARAMETERS
        READ(IN,101) NX,NY,NT,NMAX,IPLT
        WRITE(IO,205) NX,NY,NT,NMAX
        READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
        READ(IN,110) CO,VX,DX,DY,DK
        WRITE(IO,210) CO,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS
        READ(IN,110) XC,YC,QM
        WRITE(IO,212) XC,LUNITS,YC,LUNITS,QM,DUNITS
        READ(IN,110) (X(I),I=1,NX)
        WRITE(IO,215) LUNITS
        WRITE(IO,220) (X(I),I=1,NX)
        READ(IN,110) (Y(I),I=1,NY)
        WRITE(IO,216) LUNITS
        WRITE(IO,220) (Y(I),I=1,NY)
        READ(IN,110) (T(I),I=1,NT)
        WRITE(IO,225) TUNITS
        WRITE(IO,220) (T(I),I=1,NT)
        IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
        IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C         READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS
        CALL GLQPTS (NMAX)
C
C         BEGIN TIME LOOP
        DO 20 IT=1,NT
C
C         BEGIN X LOOP
```

```
      DO 40 IX=1,NX
      XX=X(IX)-XC
C
C     CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)
      DO 50 IY=1,NY
      YY=Y(IY)-YC
      CALL CNRML2(QM,DK,T(IT),XX,YY,DX,DY,VX,CN,NMAX)
      CXY(IX,IY)=C0*CN
50    CONTINUE
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NY-1)/9
      DO 60 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
      IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NY) NP2=NY-NP1
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) T(IT),TUNITS,LUNITS
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60    CONTINUE
C
C       CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C       PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C       CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 20
      NXY=NX*NY
      DO 80 I=1,NX
      IP=(I-1)*NY
      XP(I)=SNGL(X(I))
      DO 80 J=1,NY
      IF(I.EQ.1) YP(J)=SNGL(Y(J))
      CP(IP+J)=SNGL(CXY(I,J)/C0)
80    CONTINUE
      TP=SNGL(T(IT))
      NXY2=NXY*2
      CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
     1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
20    CONTINUE
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C        FORMAT STATEMENTS
101   FORMAT(20I4)
```

```
105     FORMAT(8A10)
110     FORMAT(8F10.0)
201     FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
       1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
       2 1H ,28X,'FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A'/
       3 1H ,31X,'CONTINUOUS POINT SOURCE AT X=0 AND Y=YC'
       4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205     FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
       1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
       2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
       3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)
210     FORMAT(1H0,25X,'SOLUTE CONCENTRATION IN INJECTED FLUID (C0) =',
       1 1P1E13.6,1X,A10/1H ,25X,
       2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
       3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
       4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
       5 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212     FORMAT(1H0,25X,'AQUIFER IS OF INFINITE AREAL EXTENT'/1H ,25X,
       1 'CONTINUOUS POINT SOURCE IS LOCATED AT X =',1P1E13.6,1X,A10/1H ,
       1 63X,'Y =',1P1E13.6,1X,A10/1H ,25X,
       2 'FLUID INJECTION RATE PER UNIT THICHKNESS OF AQUIFER (QM) =',
       3 1P1E13.6,1X,A10)
215     FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
       1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216     FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
       1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220     FORMAT(1H ,5X,8F12.4)
225     FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
       1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227     FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
       1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
       2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230     FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
       1 F12.4,1X,A10/
       2 1H0,25X,'Y-COORDINATE, IN ',A10)
231     FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
       1 F12.4,1X,A10,5X,'(CONTINUED)'/
       2 1H0,25X,'Y-COORDINATE, IN ',A10)
235     FORMAT(1H ,20X,9F12.4)
236     FORMAT(1H ,19X,'*',108(1H-)/
       1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
       2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240     FORMAT(1H ,5X,F12.4,2X,'!',9F12.5)
241     FORMAT(1H ,19X,'!')
        END
        SUBROUTINE CNRML2(QM,DK,T,X,Y,DX,DY,VX,CN,NMAX)
        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
        COMMON /IOUNIT/ IN,IO
        COMMON /GLPTS/ WN(256),ZN(256)
C
C          THIS ROUTINE CALCULATES SOLUTE CONCENTRATION AT X,Y BASED ON
C          THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-
C          DISPERSIVE SOLUTE TRANSPORT EQUATION FOR AN AQUIFER OF
```

```
C          INFINITE AREAL EXTENT WITH A CONTINUOUS POINT SOURCE LOCATED
C          AT X=XC AND Y=YC.  THE INTEGRAL FROM 0 TO T IS EVALUATED
C          USING A GAUSS-LEGENDRE QUADRATURE INTEGRATION TECHNIQUE.
C
        PI=3.14159265358979D0
        CN=0.0D0
C
C          FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
        IF(T.LE.0.0D0) RETURN
C
C          START NUMERICAL INTEGRATION LOOP
        ALPHA=X*X/(4.0D0*DX)+Y*Y/(4.0D0*DY)
        BETA=VX*VX/(4.0D0*DX)+DK
        VX2D=VX*X/(2.0D0*DX)
        SUM=0.0D0
        DO 20 I=1,NMAX
C
C          SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE
C          NON-NORMALIZED LIMITS OF INTEGRATION
        WI=WN(I)*T/2.0D0
        ZI=T*(ZN(I)+1.0D0)/2.0D0
C
C          TERM 1
        X1=-ALPHA/ZI-BETA*ZI
        X1=DEXP(X1)/ZI
        SUM=SUM+X1*WI
   20   CONTINUE
        CN=QM*SUM*DEXP(VX2D)/(4.0D0*PI*DSQRT(DX*DY))
        RETURN
        END
```

```
C
C       ********************************************************
C       *                                                      *
C       *                  **** STRIPF ****                    *
C       *                                                      *
C       * TWO-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT MODEL  *
C       *                                                      *
C       *   FOR A SEMI-INFINITE AQUIFER WITH A FINITE WIDTH    *
C       *                                                      *
C       *     A STRIP SOURCE EXTENDS FROM Y1 TO Y2 AT X=0      *
C       *                                                      *
C       *         GROUND-WATER FLOW IN X-DIRECTION ONLY        *
C       *                                                      *
C       *            VERSION CURRENT AS OF 11/30/88            *
C       *                                                      *
C       ********************************************************
C
C    THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C    GREATER THAN THOSE GIVEN HERE.
C      MAXX = MAXIMUM NUMBER OF X-VALUES
C      MAXY = MAXIMUM NUMBER OF Y-VALUES
C      MAXT = MAXIMUM NUMBER OF TIME VALUES
C      MAXXY = MAXX * MAXY
C      MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      CHARACTER*1 IERR(MAXX,MAXY)
      REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C          NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C          MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C          LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     C0        SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C     DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     DY        TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
C     VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Y         Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T         TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C     CN        NORMALIZED CONCENTRATION C/C0 [DIMENSIONLESS]
C     CXY       SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
C     W         AQUIFER WIDTH (AQUIFER EXTENDS FROM Y=0 TO Y=W) [L]
C     WS        WIDTH OF FINITE-WIDTH (STRIP) SOLUTE SOURCE [L]
```

```
C      YC        Y-COORDINATE OF CENTER OF STRIP SOLUTE SOURCE [L]
C
C      NX        NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C      NY        NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C      NT        NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C      NMAX      NUMBER OF TERMS USED IN INFINITE SERIES SUMMATION
C
C      IPLT      PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C      XSCLP     SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C      YSCLP     SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C      DELTA     CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C        CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C      CUNITS   UNITS OF CONCENTRATION (M/L**3)
C      VUNITS   UNITS OF GROUND-WATER VELOCITY (L/T)
C      DUNITS   UNITS OF DISPERSION COEFFICIENT (L**2/T)
C      KUNITS   UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C      LUNITS   UNITS OF LENGTH (L)
C      TUNITS   UNITS OF TIME (T)
C
C        DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
       CALL OFILE
       CALL TITLE
       WRITE(IO,201)
C
C        READ IN MODEL PARAMETERS
       READ(IN,101) NX,NY,NT,NMAX,IPLT
       WRITE(IO,205) NX,NY,NT,NMAX
       READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
       READ(IN,110) C0,VX,DX,DY,DK
       WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS
       READ(IN,110) W,YC,WS
       WRITE(IO,212) W,LUNITS,YC,LUNITS,WS,LUNITS
       Y1=YC-WS/2.0D0
       Y2=YC+WS/2.0D0
       READ(IN,110) (X(I),I=1,NX)
       WRITE(IO,215) LUNITS
       WRITE(IO,220) (X(I),I=1,NX)
       READ(IN,110) (Y(I),I=1,NY)
       WRITE(IO,216) LUNITS
       WRITE(IO,220) (Y(I),I=1,NY)
       READ(IN,110) (T(I),I=1,NT)
       WRITE(IO,225) TUNITS
       WRITE(IO,220) (T(I),I=1,NT)
       IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
       IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C        BEGIN TIME LOOP
       DO 20 IT=1,NT
C
C        BEGIN X LOOP
       DO 40 IX=1,NX
C
```

```
C      CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)
       DO 50 IY=1,NY
       CALL CNRMLF(DK,T(IT),X(IX),Y(IY),W,Y1,Y2,DX,DY,
      1 VX,CN,NMAX,IERR(IX,IY))
       CXY(IX,IY)=C0*CN
50     CONTINUE
40     CONTINUE
C
C        PRINT OUT TABLES OF CONCENTRATION VALUES
       NPAGE=1+(NY-1)/9
       DO 60 NP=1,NPAGE
       IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
       IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
       NP1=(NP-1)*9
       NP2=9
       IF((NP1+NP2).GT.NY) NP2=NY-NP1
       WRITE(IO,235) (Y(NP1+J),J=1,NP2)
       WRITE(IO,236) CUNITS,LUNITS
       DO 70 IX=1,NX
       WRITE(IO,240) X(IX),(CXY(IX,NP1+J),IERR(IX,NP1+J),J=1,NP2)
       IF(MOD(IX,45).NE.0) GO TO 70
       WRITE(IO,231) T(IT),TUNITS,LUNITS
       WRITE(IO,235) (Y(NP1+J),J=1,NP2)
       WRITE(IO,236) CUNITS,LUNITS
70     IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60     CONTINUE
C
C        CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C        PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C        CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
       IF(IPLT.LT.1) GO TO 20
       NXY=NX*NY
       DO 80 I=1,NX
       IP=(I-1)*NY
       XP(I)=SNGL(X(I))
       DO 80 J=1,NY
       IF(I.EQ.1) YP(J)=SNGL(Y(J))
       CP(IP+J)=SNGL(CXY(I,J)/C0)
80     CONTINUE
       TP=SNGL(T(IT))
       NXY2=NXY*2
       CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
      1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
20     CONTINUE
       CLOSE (IN)
       CLOSE (IO)
       STOP
C
C         FORMAT STATEMENTS
101    FORMAT(20I4)
105    FORMAT(8A10)
110    FORMAT(8F10.0)
201    FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
```

```
     1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
     2 1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF FINITE WIDTH'/
     3 1H ,26X,'WITH A FINITE-WIDTH (STRIP) SOLUTE SOURCE AT X=0.0'
     4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
     1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
     2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
     3 'NUMBER OF TERMS IN INFINTE SERIES SUMMATION (NMAX) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =',
     1 1P1E13.6,1X,A10/1H ,25X,
     2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
     3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
     4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
     5 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212   FORMAT(1H0,25X,'AQUIFER WIDTH (W) =',1P1E13.6,1X,A10/1H ,25X,
     1 'SOLUTE SOURCE IS CENTERED AT Y =',1P1E13.6,1X,A10/1H ,25X,
     2 'FINITE-WIDTH OF SOLUTE SOURCE (WS) =',1P1E13.6,1X,A10)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216   FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227   FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
     1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
     2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10,15X,'*  INDICATES SOLUTION DID NOT CONVERGE'/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
231   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10,5X,'(CONTINUED)'/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
235   FORMAT(1H ,20X,9F12.4)
236   FORMAT(1H ,19X,'*',108(1H-)/
     1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
     2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240   FORMAT(1H ,5X,F12.4,2X,'! ',9(F11.5,A1))
241   FORMAT(1H ,19X,'!')
      END
      SUBROUTINE CNRMLF(DK,T,X,Y,W,Y1,Y2,DX,DY,VX,CN,NMAX,IERR)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      CHARACTER*1 IERR
      COMMON /IOUNIT/ IN,IO
C
C        THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y
C        BASED ON THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL
C        ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
C        INFINITE AQUIFER WITH A FINITE WIDTH.  A FINITE-WIDTH (STRIP)
C        SOLUTE SOURCE EXTENDS FROM Y=Y1 TO Y=Y2. THE SOLUTION
C        CONTAINS AN INFINITE SERIES SUMMATION WHICH MAY TAKE A LARGE
C        NUMBER OF TERMS TO CONVERGE FOR SMALL VALUES OF X.
C
```

```
      PI=3.14159265358979D0
      CN=0.0D0
      IERR=' '
C
C       FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
      IF(T.LE.0.0D0) RETURN
C
C       FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
      IF(X.GT.0.0D0) GO TO 10
      IF(Y.GT.Y1 .AND. Y.LT.Y2) CN=1.0D0
      IF(Y.EQ.Y1) CN=0.50D0
      IF(Y.EQ.Y2) CN=0.50D0
      RETURN
C
C       BEGIN SUMMATION OF TERMS IN INFINITE SERIES
10    RTDXT=2.0D0*DSQRT(DX*T)
      SIGMA=0.0D0
      SUBTOT=0.0D0
      NMAX1=NMAX+1
      DO 20 NN=1,NMAX1
      N=NN-1
      ETA=N*PI/W
      PN=(Y2-Y1)/(2.0D0*W)
      IF(N.NE.0) PN=(DSIN(ETA*Y2)-DSIN(ETA*Y1))/(N*PI)
      COSRY=DCOS(ETA*Y)
      ALPHA=4.0D0*DX*(ETA*ETA*DY+DK)
      BETA=DSQRT(VX*VX+ALPHA)
      BETAT=BETA*T
C
C       CALCULATE TERM 1
      A1=X*(VX-BETA)/(2.0D0*DX)
      B1=(X-BETAT)/RTDXT
      CALL EXERFC(A1,B1,C1)
C
C       CALCULATE TERM 2
      A2=X*(VX+BETA)/(2.0D0*DX)
      B2=(X+BETAT)/RTDXT
      CALL EXERFC(A2,B2,C2)
C
C       ADD TERMS TO SUMMATION
      TERM=PN*COSRY*(C1+C2)
      SIGMA=SIGMA+TERM
C
C       CHECK FOR CONVERGENCE. BECAUSE SERIES OSCILLATES, CHECK
C       SUBTOTAL OF LAST 10 TERMS.
      SUBTOT=SUBTOT+TERM
      IF(MOD(NN,10).NE.0) GO TO 20
      IF(DABS(SUBTOT).LT.1.0D-12) GO TO 30
      SUBTOT=0.0D0
20    CONTINUE
      IERR='*'
30    CN=SIGMA
      RETURN
```

```
C
C        ***********************************************************
C        *                                                         *
C        *                    **** STRIPI ****                     *
C        *                                                         *
C        * TWO-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT MODEL     *
C        *                                                         *
C        *    FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH        *
C        *                                                         *
C        *     A STRIP SOURCE EXTENDS FROM Y1 TO Y2 AT X=0         *
C        *                                                         *
C        *         GROUND-WATER FLOW IN X-DIRECTION ONLY           *
C        *                                                         *
C        *            VERSION CURRENT AS OF 8/13/87                *
C        *                                                         *
C        ***********************************************************
C
C     THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C     GREATER THAN THOSE GIVEN HERE.
C       MAXX = MAXIMUM NUMBER OF X-VALUES
C       MAXY = MAXIMUM NUMBER OF Y-VALUES
C       MAXT = MAXIMUM NUMBER OF TIME VALUES
C       MAXXY = MAXX * MAXY
C       MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C           NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C           MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C           LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CO       SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C     DX       LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     DY       TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
C     VX       GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK       FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X        X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Y        Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T        TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C     CN       NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C     CXY      SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
C     WS       WIDTH OF FINITE-WIDTH (STRIP) SOLUTE SOURCE [L]
C     YC       Y-COORDINATE OF CENTER OF STRIP SOLUTE SOURCE [L]
C
```

```
C       NX        NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C       NY        NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C       NT        NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C       NMAX      NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL
C                 INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)
C
C       IPLT      PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C       XSCLP     SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C       YSCLP     SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C       DELTA     CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C         CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C       CUNITS    UNITS OF CONCENTRATION (M/L**3)
C       VUNITS    UNITS OF GROUND-WATER VELOCITY (L/T)
C       DUNITS    UNITS OF DISPERSION COEFFICIENT (L**2/T)
C       KUNITS    UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C       LUNITS    UNITS OF LENGTH (L)
C       TUNITS    UNITS OF TIME (T)
C
C         DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
        CALL OFILE
        CALL TITLE
        WRITE(IO,201)
C
C         READ IN MODEL PARAMETERS
        READ(IN,101) NX,NY,NT,NMAX,IPLT
        WRITE(IO,205) NX,NY,NT,NMAX
        READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
        READ(IN,110) C0,VX,DX,DY,DK
        WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS
        READ(IN,110) YC,WS
        WRITE(IO,212) YC,LUNITS,WS,LUNITS
        Y1=YC-WS/2.0D0
        Y2=YC+WS/2.0D0
        READ(IN,110) (X(I),I=1,NX)
        WRITE(IO,215) LUNITS
        WRITE(IO,220) (X(I),I=1,NX)
        READ(IN,110) (Y(I),I=1,NY)
        WRITE(IO,216) LUNITS
        WRITE(IO,220) (Y(I),I=1,NY)
        READ(IN,110) (T(I),I=1,NT)
        WRITE(IO,225) TUNITS
        WRITE(IO,220) (T(I),I=1,NT)
        IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
        IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C         READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS
        CALL GLQPTS (NMAX)
C
C         BEGIN TIME LOOP
        DO 20 IT=1,NT
C
C         BEGIN X LOOP
```

```
      DO 40 IX=1,NX
C
C     CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)
      DO 50 IY=1,NY
      CALL CNRMLI(DK,T(IT),X(IX),Y(IY),Y1,Y2,DX,DY,VX,CN,NMAX)
      CXY(IX,IY)=C0*CN
50    CONTINUE
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NY-1)/9
      DO 60 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
      IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NY) NP2=NY-NP1
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) T(IT),TUNITS,LUNITS
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60    CONTINUE
C
C       CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C       PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C       CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 20
      NXY=NX*NY
      DO 80 I=1,NX
      IP=(I-1)*NY
      XP(I)=SNGL(X(I))
      DO 80 J=1,NY
      IF(I.EQ.1) YP(J)=SNGL(Y(J))
      CP(IP+J)=SNGL(CXY(I,J)/C0)
80    CONTINUE
      TP=SNGL(T(IT))
      NXY2=NXY*2
      CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
     1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
20    CONTINUE
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C        FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
```

```
201    FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
      1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
      2 1H ,29X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/
      3 1H ,26X,'WITH A FINITE-WIDTH (STRIP) SOLUTE SOURCE AT X=0.0'
      4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205    FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
      1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
      2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
      3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)
210    FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =',
      1 1P1E13.6,1X,A10/1H ,25X,
      2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
      3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
      4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
      5 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212    FORMAT(1H0,25X,'AQUIFER WIDTH (W) IS INFINITE'/1H ,25X,
      1 'SOLUTE SOURCE IS CENTERED AT Y =',1P1E13.6,1X,A10/1H ,25X,
      2 'FINITE-WIDTH OF SOLUTE SOURCE (WS) =',1P1E13.6,1X,A10)
215    FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
      1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216    FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
      1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220    FORMAT(1H ,5X,8F12.4)
225    FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
      1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227    FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
      1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
      2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230    FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
      1 F12.4,1X,A10/
      2 1H0,25X,'Y-COORDINATE, IN ',A10)
231    FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
      1 F12.4,1X,A10,5X,'(CONTINUED)'/
      2 1H0,25X,'Y-COORDINATE, IN ',A10)
235    FORMAT(1H ,20X,9F12.4)
236    FORMAT(1H ,19X,'*',108(1H-)/
      1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
      2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240    FORMAT(1H ,5X,F12.4,2X,'!',9F12.5)
241    FORMAT(1H ,19X,'!')
       END
       SUBROUTINE CNRMLI(DK,T,X,Y,Y1,Y2,DX,DY,VX,CN,NMAX)
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       COMMON /IOUNIT/ IN,IO
       COMMON /GLPTS/ WN(256),ZN(256)
C
C        THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y
C        BASED ON THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL
C        ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
C        INFINITE AQUIFER OF INFINITE WIDTH.  A FINITE-WIDTH (STRIP)
C        SOLUTE SOURCE EXTENDS FROM Y=Y1 TO Y=Y2. THE SOLUTION CONTAINS
C        AN INTEGRAL FROM 0 TO T WHICH IS EVALUATED USING A GAUSS-
C        LEGENDRE QUADRATURE NUMERICAL INTEGRATION TECHNIQUE.
```

```
C
      PI=3.14159265358979D0
      CN=0.0D0
C
C       FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
      IF(T.LE.0.0D0) RETURN
C
C       FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
      IF(X.GT.0.0D0) GO TO 10
      IF(Y.GT.Y1 .AND. Y.LT.Y2) CN=1.0D0
      IF(Y.EQ.Y1) CN=0.50D0
      IF(Y.EQ.Y2) CN=0.50D0
      RETURN
C
C       START NUMERICAL INTEGRATION LOOP
10    SUM=0.0D0
      DO 20 I=1,NMAX
C
C       SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE
C       NON-NORMALIZED LIMITS OF INTEGRATION
      WI=WN(I)*T/2.0D0
      ZI=T*(ZN(I)+1.0D0)/2.0D0
C
C       TERM 1
      XVT=X-VX*ZI
      EXP1=-XVT*XVT/(4.0D0*DX*ZI)-DK*ZI
      ERFC1=(Y1-Y)/(2.0D0*DSQRT(DY*ZI))
      CALL EXERFC(EXP1,ERFC1,Z1)
C
C       TERM 2
      ERFC2=(Y2-Y)/(2.0D0*DSQRT(DY*ZI))
      CALL EXERFC(EXP1,ERFC2,Z2)
      TERM=(Z1-Z2)*WI/(ZI**1.5)
      SUM=SUM+TERM
20    CONTINUE
      CN=SUM*X/(4.0D0*DSQRT(PI*DX))
      RETURN
      END
```

```
C
C        ********************************************************
C        *                                                      *
C        *                   **** GAUSS ****                    *
C        *                                                      *
C        * TWO-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT MODEL  *
C        *                                                      *
C        *   FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH. A   *
C        *                                                      *
C        *  SOURCE HAVING A GAUSSIAN-SHAPED CONCENTRATION DIS-  *
C        *                                                      *
C        * TRIBUTION IS LOCATED AT X=0 AND CENTERED ABOUT Y=YC  *
C        *                                                      *
C        *        GROUND-WATER FLOW IN X-DIRECTION ONLY         *
C        *                                                      *
C        *            VERSION CURRENT AS OF 8/13/87             *
C        *                                                      *
C        ********************************************************
C
C       THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C       GREATER THAN THOSE GIVEN HERE.
C         MAXX = MAXIMUM NUMBER OF X-VALUES
C         MAXY = MAXIMUM NUMBER OF Y-VALUES
C         MAXT = MAXIMUM NUMBER OF TIME VALUES
C         MAXXY = MAXX * MAXY
C         MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      REAL XP,YP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C           NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C           MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C           LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CM        MAXIMUM SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L*
C     DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     DY        TRANSVERSE DISPERSION COEFFICIENT [L**2/T]
C     VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Y         Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T         TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C     CN        NORMALIZED CONCENTRATION C/CM [DIMENSIONLESS]
C     CXY       SOLUTE CONCENTRATION C(X,Y,T) [M/L**3]
C     YC        Y-COORDINATE OF THE CENTER OF SOLUTE SOURCE AT X=0 [L]
```

```
C       SIGMA     STANDARD DEVIATION OF GAUSSIAN CONCENTRATION DISTRIBUTION
C                 FOR THE SOLUTE SOURCE [L]
C
C       NX        NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C       NY        NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C       NT        NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C       NMAX      NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL
C                 INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)
C
C       IPLT      PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C       XSCLP     SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C       YSCLP     SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C       DELTA     CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C         CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C       CUNITS    UNITS OF CONCENTRATION (M/L**3)
C       VUNITS    UNITS OF GROUND-WATER VELOCITY (L/T)
C       DUNITS    UNITS OF DISPERSION COEFFICIENT (L**2/T)
C       KUNITS    UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C       LUNITS    UNITS OF LENGTH (L)
C       TUNITS    UNITS OF TIME (T)
C
C         DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
        CALL OFILE
        CALL TITLE
        WRITE(IO,201)
C
C         READ IN MODEL PARAMETERS
        READ(IN,101) NX,NY,NT,NMAX,IPLT
        WRITE(IO,205) NX,NY,NT,NMAX
        READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
        READ(IN,110) CM,VX,DX,DY,DK
        WRITE(IO,210) CM,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DK,KUNITS
        READ(IN,110) YC,SIGMA
        WRITE(IO,212) YC,LUNITS,SIGMA,LUNITS
        READ(IN,110) (X(I),I=1,NX)
        WRITE(IO,215) LUNITS
        WRITE(IO,220) (X(I),I=1,NX)
        READ(IN,110) (Y(I),I=1,NY)
        WRITE(IO,216) LUNITS
        WRITE(IO,220) (Y(I),I=1,NY)
        READ(IN,110) (T(I),I=1,NT)
        WRITE(IO,225) TUNITS
        WRITE(IO,220) (T(I),I=1,NT)
        IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
        IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C         READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS
        CALL GLQPTS (NMAX)
C
C         BEGIN TIME LOOP
        DO 20 IT=1,NT
C
```

```
C         BEGIN X LOOP
       DO 40 IX=1,NX
C
C      CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX)
       DO 50 IY=1,NY
       CALL CNRMLG(DK,T(IT),X(IX),Y(IY),YC,SIGMA,DX,DY,VX,CN,NMAX)
       CXY(IX,IY)=CM*CN
50     CONTINUE
40     CONTINUE
C
C         PRINT OUT TABLES OF CONCENTRATION VALUES
       NPAGE=1+(NY-1)/9
       DO 60 NP=1,NPAGE
       IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,LUNITS
       IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,LUNITS
       NP1=(NP-1)*9
       NP2=9
       IF((NP1+NP2).GT.NY) NP2=NY-NP1
       WRITE(IO,235) (Y(NP1+J),J=1,NP2)
       WRITE(IO,236) CUNITS,LUNITS
       DO 70 IX=1,NX
       WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
       IF(MOD(IX,45).NE.0) GO TO 70
       WRITE(IO,231) T(IT),TUNITS,LUNITS
       WRITE(IO,235) (Y(NP1+J),J=1,NP2)
       WRITE(IO,236) CUNITS,LUNITS
70     IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60     CONTINUE
C
C         CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C         PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY CM TO PLOT
C         CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
       IF(IPLT.LT.1) GO TO 20
       NXY=NX*NY
       DO 80 I=1,NX
       IP=(I-1)*NY
       XP(I)=SNGL(X(I))
       DO 80 J=1,NY
       IF(I.EQ.1) YP(J)=SNGL(Y(J))
       CP(IP+J)=SNGL(CXY(I,J)/CM)
80     CONTINUE
       TP=SNGL(T(IT))
       NXY2=NXY*2
       CALL PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,TUNITS,
     1 LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
20     CONTINUE
       CLOSE (IN)
       CLOSE (IO)
       STOP
C
C          FORMAT STATEMENTS
101    FORMAT(20I4)
105    FORMAT(8A10)
```

```
110     FORMAT(8F10.0)
201     FORMAT(/////1H ,30X,'ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL'/
       1 1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
       2 1H ,29X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/
       3 1H ,25X,'WITH A SOLUTE SOURCE HAVING A GAUSSIAN CONCENTRATION'/
       4 1H ,24X,'DISTRIBUTION LOCATED AT X=0.0 AND CENTERED ABOUT Y=YC'
       5 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205     FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
       1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
       2 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
       3 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)
210     FORMAT(1H0,25X,'MAXIMUM SOLUTE CONCENTRATION AT THE BOUNDARY',
       1 ' CM) =',1P1E13.6,1X,A10/1H ,25X,
       2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
       3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
       4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
       5 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212     FORMAT(1H0,25X,'AQUIFER WIDTH (W) IS INFINITE'/1H ,25X,
       1 'SOLUTE SOURCE IS CENTERED AT Y =',1P1E13.6,1X,A10/1H ,25X,
       2 'STANDARD DEVIATION OF GAUSSIAN DISTRIBUTION (SIGMA) =',
       3 1P1E13.6,1X,A10)
215     FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
       1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216     FORMAT(1H0,25X,'Y COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
       1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220     FORMAT(1H ,5X,8F12.4)
225     FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
       1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227     FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
       1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
       2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230     FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
       1 F12.4,1X,A10/
       2 1H0,25X,'Y-COORDINATE, IN ',A10)
231     FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
       1 F12.4,1X,A10,5X,'(CONTINUED)'/
       2 1H0,25X,'Y-COORDINATE, IN ',A10)
235     FORMAT(1H ,20X,9F12.4)
236     FORMAT(1H ,19X,'*',108(1H-)/
       1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
       2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240     FORMAT(1H ,5X,F12.4,2X,'!',9F12.6)
241     FORMAT(1H ,19X,'!')
        END
        SUBROUTINE CNRMLG(DK,T,X,Y,YC,SIGMA,DX,DY,VX,CN,NMAX)
        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
        COMMON /IOUNIT/ IN,IO
        COMMON /GLPTS/ WN(256),ZN(256)
C
C         THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y
C         BASED ON THE ANALYTIC SOLUTION TO THE TWO-DIMENSIONAL
C         ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
C         INFINITE AQUIFER OF INFINITE WIDTH. THE SOLUTE SOURCE, LOCATED
```

```
C        AT X=0.0 AND CENTERED ABOUT Y=YC, HAS A GAUSSIAN CONCENTRATION
C        DISTRIBUTION WITH A STANDARD DEVIATION OF SIGMA. THE SOLUTION
C        CONTAINS AN INTEGRAL FROM 0 TO (T*BETA)**.25 WHICH IS EVALUATED
C        USING A GAUSS-LEGENDRE QUADRATURE INTEGRATION TECHNIQUE.
C
      PI=3.14159265358979D0
      Y1=Y-YC
      SIGSQ=SIGMA*SIGMA
      BETA=DK+VX*VX/(4.0D0*DX)
      VX2D=VX*X/(2.0D0*DX)
      CN=0.0D0
C
C        FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
      IF(T.LE.0.0D0) RETURN
C
C        FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
      IF(X.GT.0.0D0) GOTO 10
      CN=DEXP(-Y1*Y1/(2.0D0*SIGSQ))
      RETURN
C
C        START NUMERICAL INTEGRATION LOOP
10    SUM=0.0D0
      DO 20 I=1,NMAX
C
C        SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE
C        NON-NORMALIZED LIMITS OF INTEGRATION
C        LIMITS OF INTEGRATION ARE FROM 0 TO (T*BETA)**0.25
      TT=(T*BETA)**0.250D0
      WI=WN(I)*TT/2.0D0
      ZI=TT*(ZN(I)+1.0D0)/2.0D0
C
C        TERM 1
      Z4=ZI**4
      ALPHA=Z4*DY/BETA + SIGSQ/2.0D0
      X1=DEXP(-Z4 -X*X*BETA/(4.0D0*DX*Z4) -Y1*Y1/(4.0D0*ALPHA))
      X1=X1/((ZI**3)*DSQRT(ALPHA))
      SUM=SUM+X1*WI
20    CONTINUE
C
C        TERM 2
      X2=SUM*4.0D0*DSQRT(BETA)*DEXP(VX2D)
      CN=X*SIGMA*X2/(DSQRT(8.0D0*PI*DX))
      RETURN
      END
```

```
C
C        *********************************************************
C        *                                                       *
C        *                    **** POINT3 ****                   *
C        *                                                       *
C        *    THREE-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT    *
C        *                                                       *
C        *     MODEL FOR AN AQUIFER OF INFINITE EXTENT WITH A    *
C        *                                                       *
C        *    CONTINUOUS POINT SOURCE AT X=XC, Y=YC, AND Z=ZC    *
C        *                                                       *
C        *         GROUND-WATER FLOW IN X-DIRECTION ONLY         *
C        *                                                       *
C        *              VERSION CURRENT AS OF 11/30/88           *
C        *                                                       *
C        *********************************************************
C
C     THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C     GREATER THAN THOSE GIVEN HERE.
C       MAXX = MAXIMUM NUMBER OF X-VALUES
C       MAXY = MAXIMUM NUMBER OF Y-VALUES
C       MAXZ = MAXIMUM NUMBER OF Z-VALUES
C       MAXT = MAXIMUM NUMBER OF TIME VALUES
C       MAXXY = MAXX * MAXY
C       MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,QUNITS,TUNITS
      REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C          NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C          MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C          LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CO       SOLUTE CONCENTRATION IN INJECTED FLUID [M/L**3]
C     DX       LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     DY       TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C     DZ       TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C     VX       GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK       FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X        X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Y        Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Z        Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T        TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C     CN       NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C     CXY      SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]
```

```
C     XC       X-COORDINATE OF CONTINUOUS POINT SOURCE [L]
C     YC       Y-COORDINATE OF CONTINUOUS POINT SOURCE [L]
C     ZC       Z-COORDINATE OF CONTINUOUS POINT SOURCE [L]
C     QM       FLUID INJECTION RATE [L**3/T]
C
C     NX       NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NY       NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NZ       NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NT       NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C     NMAX     NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL
C              INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)
C
C     IPLT     PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C     XSCLP    SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C     YSCLP    SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C     DELTA    CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C       CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C     CUNITS   UNITS OF CONCENTRATION (M/L**3)
C     VUNITS   UNITS OF GROUND-WATER VELOCITY (L/T)
C     DUNITS   UNITS OF DISPERSION COEFFICIENT (L**2/T)
C     KUNITS   UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C     LUNITS   UNITS OF LENGTH (L)
C     QUNITS   UNITS OF FLUID INJECTION RATE (L**3/T)
C     TUNITS   UNITS OF TIME (T)
C
C       DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
      CALL OFILE
      CALL TITLE
      WRITE(IO,201)
C
C       READ IN MODEL PARAMETERS
      READ(IN,101) NX,NY,NZ,NT,NMAX,IPLT
      WRITE(IO,205) NX,NY,NZ,NT,NMAX
      READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,QUNITS,TUNITS
      READ(IN,110) C0,VX,DX,DY,DZ,DK
      WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,
     1 DK,KUNITS
      READ(IN,110) XC,YC,ZC,QM
      WRITE(IO,212) XC,LUNITS,YC,LUNITS,ZC,LUNITS,QM,QUNITS
      READ(IN,110) (X(I),I=1,NX)
      WRITE(IO,215) LUNITS
      WRITE(IO,220) (X(I),I=1,NX)
      READ(IN,110) (Y(I),I=1,NY)
      WRITE(IO,216) LUNITS
      WRITE(IO,220) (Y(I),I=1,NY)
      READ(IN,110) (Z(I),I=1,NZ)
      WRITE(IO,217) LUNITS
      WRITE(IO,220) (Z(I),I=1,NZ)
      READ(IN,110) (T(I),I=1,NT)
      WRITE(IO,225) TUNITS
      WRITE(IO,220) (T(I),I=1,NT)
      IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
```

```
      IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C       READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS
      CALL GLQPTS (NMAX)
C
C       BEGIN TIME LOOP
      DO 20 IT=1,NT
C
C       BEGIN Z LOOP
      DO 30 IZ=1,NZ
      ZZ=Z(IZ)-ZC
C
C       BEGIN X LOOP
      DO 40 IX=1,NX
      XX=X(IX)-XC
C
C     CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ
      DO 50 IY=1,NY
      YY=Y(IY)-YC
      CALL CNRML3(QM,DK,T(IT),XX,YY,ZZ,DX,DY,DZ,VX,CN,NMAX)
      CXY(IX,IY)=C0*CN
50    CONTINUE
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NY-1)/9
      DO 60 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NY) NP2=NY-NP1
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60    CONTINUE
C
C       CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C       PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C       CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 30
      NXY=NX*NY
      DO 80 I=1,NX
      IP=(I-1)*NY
      XP(I)=SNGL(X(I))
      DO 80 J=1,NY
      IF(I.EQ.1) YP(J)=SNGL(Y(J))
```

```
      CP(IP+J)=SNGL(CXY(I,J)/C0)
80    CONTINUE
      TP=SNGL(T(IT))
      ZP=SNGL(Z(IZ))
      NXY2=NXY*2
      CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT,
     1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
30    CONTINUE
20    CONTINUE
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C         FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
201   FORMAT(/////1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL'
     1 /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
     2 1H ,34X,'FOR AN AQUIFER OF INFINITE EXTENT'/
     3 1H ,30X,'WITH A CONTINUOUS POINT SOURCE AT XC,YC,ZC'
     4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
     1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
     2 'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X,
     3 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
     4 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION IN INJECTED FLUID (C0) =',
     1 1P1E13.6,1X,A10/1H ,25X,
     2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
     3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
     4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
     5 1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) =',1P1E13.6,1X,A10/
     6 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212   FORMAT(1H0,25X,'AQUIFER IS OF INFINITE EXTENT'
     2 /1H0,25X,'CONTINUOUS POINT SOURCE IS AT X =',1P1E13.6,1X,A10/
     3 1H ,55X,'Y =',1P1E13.6,1X,A10/1H ,55X,'Z =',1P1E13.6,1X,A10/
     5 1H ,25X,'FLUID INJECTION RATE (QM) =',1P1E13.6,1X,A10)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216   FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
217   FORMAT(1H0,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227   FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
     1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
     2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10/1H ,35X,'AND AT Z =',F12.4,1X,A10/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
```

```
231   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z =',F12.4,1X,A10/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
235   FORMAT(1H ,20X,9F12.4)
236   FORMAT(1H ,19X,'*',108(1H-)/
     1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
     2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240   FORMAT(1H ,5X,F12.4,2X,'!',9F12.6)
241   FORMAT(1H ,19X,'!')
      END
      SUBROUTINE CNRML3(QM,DK,T,X,Y,Z,DX,DY,DZ,VX,CN,NMAX)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /IOUNIT/ IN,IO
C
C       THIS ROUTINE CALCULATES SOLUTE CONCENTRATION AT X,Y,Z BASED ON
C       THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL ADVECTIVE-
C       DISPERSIVE SOLUTE TRANSPORT EQUATION FOR AN AQUIFER OF
C       INFINITE EXTENT WITH A CONTINUOUS POINT SOURCE LOCATED AT
C       X=XC, Y=YC, AND Z=ZC.  A CLOSED FORM SOLUTION WAS OBTAINED.
C
      PI=3.14159265358979D0
      CN=0.0D0
C
C       FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
      IF(T.LE.0.0D0) RETURN
C
C        CHECK FOR X=Y=Z=0
      IF(X.EQ.0.0D0 .AND. Y.EQ.0.0D0 .AND. Z.EQ.0.0D0) THEN
        WRITE(IO,200)
        RETURN
      END IF
C
      BETA=DSQRT(VX*VX+4.0D0*DX*DK)
      GAMMA=DSQRT(X*X+Y*Y*DX/DY+Z*Z*DX/DZ)
      RTDXT=2.0D0*DSQRT(DX*T)
C
C       TERM 1
      X1=(VX*X-GAMMA*BETA)/(2.0D0*DX)
      Y1=(GAMMA-BETA*T)/RTDXT
      CALL EXERFC(X1,Y1,Z1)
C
C       TERM 2
      X2=(VX*X+GAMMA*BETA)/(2.0D0*DX)
      Y2=(GAMMA+BETA*T)/RTDXT
      CALL EXERFC(X2,Y2,Z2)
C
C       TERM 3
      Z3=Z1+Z2
      Z4=DSQRT(DY*DZ)
      CN=QM*Z3/(8.0D0*PI*GAMMA*Z4)
      RETURN
C
C       FORMAT STATEMENTS
```

```
200   FORMAT (1H0,5X,'**** WARNING ****  A SOLUTION CAN NOT BE COMPUTED'
     1 ' FOR X=XC,Y=YC,Z=ZC'/)
      END
```

```
C
C        ********************************************************
C        *                                                      *
C        *                  **** PATCHF ****                    *
C        *                                                      *
C        *   THREE-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT    *
C        *                                                      *
C        *   MODEL FOR A SEMI-INFINITE AQUIFER WITH A FINITE    *
C        *                                                      *
C        *    WIDTH AND HEIGHT. A PATCH SOURCE EXTENDS FROM     *
C        *                                                      *
C        *             Y1 TO Y2 AND Z1 TO Z2 AT X=0             *
C        *                                                      *
C        *         GROUND-WATER FLOW IN X-DIRECTION ONLY        *
C        *                                                      *
C        *            VERSION CURRENT AS OF 11/30/88            *
C        *                                                      *
C        ********************************************************
C
C       THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C       GREATER THAN THOSE GIVEN HERE.
C         MAXX = MAXIMUM NUMBER OF X-VALUES
C         MAXY = MAXIMUM NUMBER OF Y-VALUES
C         MAXZ = MAXIMUM NUMBER OF Z-VALUES
C         MAXT = MAXIMUM NUMBER OF TIME VALUES
C         MAXXY = MAXX * MAXY
C         MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      CHARACTER*1 IERR(MAXX,MAXY)
      REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C           NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C           MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C           LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CO        SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C     DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     DY        TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C     DZ        TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C     VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Y         Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Z         Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
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C      T          TIME AT WHICH CONCENTRATION IS EVALUATED [T]
C      CN         NORMALIZED CONCENTRATION C/CO [DIMENSIONLESS]
C      CXY        SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]
C      W          AQUIFER WIDTH (AQUIFER EXTENDS FROM Y=0 TO Y=W) [L]
C      H          AQUIFER HEIGHT (AQUIFER EXTENDS FROM Z=0 TO Z=H) [L]
C      WS         WIDTH OF PATCH SOLUTE SOURCE [L]
C      HS         HEIGHT OF PATCH SOLUTE SOURCE [L]
C      YC         Y-COORDINATE OF CENTER OF PATCH SOLUTE SOURCE [L]
C      ZC         Z-COORDINATE OF CENTER OF PATCH SOLUTE SOURCE [L]
C
C      NX         NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C      NY         NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C      NZ         NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED
C      NT         NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C      NMAX       NUMBER OF TERMS USED IN INNER INFINITE SERIES SUMMATION
C      MMAX       NUMBER OF TERMS USED IN OUTER INFINITE SERIES SUMMATION
C
C      IPLT       PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C      XSCLP      SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C      YSCLP      SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C      DELTA      CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C        CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C      CUNITS   UNITS OF CONCENTRATION (M/L**3)
C      VUNITS   UNITS OF GROUND-WATER VELOCITY (L/T)
C      DUNITS   UNITS OF DISPERSION COEFFICIENT (L**2/T)
C      KUNITS   UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C      LUNITS   UNITS OF LENGTH (L)
C      TUNITS   UNITS OF TIME (T)
C
C        DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
       CALL OFILE
       CALL TITLE
       WRITE(IO,201)
C
C        READ IN MODEL PARAMETERS
       READ(IN,101) NX,NY,NZ,NT,NMAX,MMAX,IPLT
       WRITE(IO,205) NX,NY,NZ,NT,NMAX,MMAX
       READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
       READ(IN,110) C0,VX,DX,DY,DZ,DK
       WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,
      1 DK,KUNITS
       READ(IN,110) W,H,YC,ZC,WS,HS
       WRITE(IO,212) W,LUNITS,H,LUNITS,YC,LUNITS,ZC,LUNITS,WS,LUNITS,
      1 HS,LUNITS
       Y1=YC-WS/2.0D0
       Y2=YC+WS/2.0D0
       Z1=ZC-HS/2.0D0
       Z2=ZC+HS/2.0D0
       READ(IN,110) (X(I),I=1,NX)
       WRITE(IO,215) LUNITS
       WRITE(IO,220) (X(I),I=1,NX)
       READ(IN,110) (Y(I),I=1,NY)
```

```
      WRITE(IO,216) LUNITS
      WRITE(IO,220) (Y(I),I=1,NY)
      READ(IN,110) (Z(I),I=1,NZ)
      WRITE(IO,217) LUNITS
      WRITE(IO,220) (Z(I),I=1,NZ)
      READ(IN,110) (T(I),I=1,NT)
      WRITE(IO,225) TUNITS
      WRITE(IO,220) (T(I),I=1,NT)
      IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
      IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C       BEGIN TIME LOOP
      DO 20 IT=1,NT
C
C       BEGIN Z LOOP
      DO 30 IZ=1,NZ
C
C       BEGIN X LOOP
      DO 40 IX=1,NX
C
C     CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ
      DO 50 IY=1,NY
      CALL CNRMLP(DK,T(IT),X(IX),Y(IY),Z(IZ),W,H,Y1,Y2,Z1,Z2,DX,
     1 DY,DZ,VX,CN,NMAX,MMAX,IERR(IX,IY))
      CXY(IX,IY)=C0*CN
50    CONTINUE
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NY-1)/9
      DO 60 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NY) NP2=NY-NP1
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXY(IX,NP1+J),IERR(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60    CONTINUE
C
C       CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C       PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C       CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 30
      NXY=NX*NY
      DO 80 I=1,NX
```

```
      IP=(I-1)*NY
      XP(I)=SNGL(X(I))
      DO 80 J=1,NY
      IF(I.EQ.1) YP(J)=SNGL(Y(J))
      CP(IP+J)=SNGL(CXY(I,J)/C0)
80    CONTINUE
      TP=SNGL(T(IT))
      ZP=SNGL(Z(IZ))
      NXY2=NXY*2
      CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT,
     1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
30    CONTINUE
20    CONTINUE
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C        FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
201   FORMAT(/////1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL'
     1 /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
     2 1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF FINITE WIDTH'/
     3 1H ,28X,'AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0'
     4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
     1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
     2 'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X,
     3 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
     4 'NUMBER OF TERMS IN INNER INFINTE SERIES SUMMATION (NMAX) = ',
     5 I4/1H ,25X,
     6 'NUMBER OF TERMS IN OUTER INFINTE SERIES SUMMATION (MMAX) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =',
     1 1P1E13.6,1X,A10/1H ,25X,
     2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
     3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
     4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
     5 1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) =',1P1E13.6,1X,A10/
     6 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212   FORMAT(1H0,25X,'AQUIFER WIDTH (W) =',1P1E13.6,1X,A10/1H ,25X,
     1 'AQUIFER HEIGHT (H) =',1P1E13.6,1X,A10/1H ,25X,
     2 'SOLUTE SOURCE IS CENTERED AT Y =',1P1E13.6,1X,A10/1H ,50X,
     3 'AND Z =',1P1E13.6,1X,A10/1H ,25X,
     4 'FINITE-WIDTH OF SOLUTE SOURCE (WS) =',1P1E13.6,1X,A10/1H ,25X,
     5 'FINITE-HEIGHT OF SOLUTE SOURCE (HS) =',1P1E13.6,1X,A10)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216   FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
217   FORMAT(1H0,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
```

```
225    FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227    FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
     1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
     2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230    FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10/1H ,35X,'AND AT Z =',F12.4,1X,A10,
     1 15X,'*  INDICATES SOLUTION DID NOT CONVERGE'/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
231    FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z =',F12.4,1X,
     2 A10/1H0,25X,'Y-COORDINATE, IN ',A10)
235    FORMAT(1H ,20X,9F12.4)
236    FORMAT(1H ,19X,'*',108(1H-)/
     1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
     2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240    FORMAT(1H ,5X,F12.4,2X,'! ',9(F11.5,A1))
241    FORMAT(1H ,19X,'!')
       END
       SUBROUTINE CNRMLP(DK,T,X,Y,Z,W,H,Y1,Y2,Z1,Z2,DX,DY,DZ,VX,CN,NMAX,
     1 MMAX,IERR)
       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
       CHARACTER*1 IERR
       COMMON /IOUNIT/ IN,IO
C
C        THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y,Z
C        BASED ON THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL
C        ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
C        INFINITE AQUIFER WITH A FINITE WIDTH AND HEIGHT. THE SOLUTE
C        SOURCE HAS A FINITE WIDTH AND HEIGHT, EXTENDING FROM Y=Y1 TO
C        Y=Y2 AND Z=Z1 TO Z=Z2. SOLUTE MAY BE SUBJECT TO FIRST-ORDER
C        CHEMICAL TRANSFORMATION. THE SOLUTION CONTAINS TWO INFINITE
C        SERIES SUMMATIONS WHICH MAY CONVERGE SLOWLY.
C
       PI=3.14159265358979D0
       CN=0.0D0
       IERR=' '
C
C        FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
       IF(T.LE.0.0D0) RETURN
C
C        FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
       IF(X.GT.0.0D0) GO TO 10
       IF(Y.EQ.Y1.OR.Y.EQ.Y2) THEN
         IF(Z.GT.Z1.AND.Z.LT.Z2) CN=0.50D0
         IF(Z.EQ.Z1.OR.Z.EQ.Z2) CN=0.25D0
       END IF
       IF(Z.EQ.Z1.OR.Z.EQ.Z2) THEN
         IF(Y.GT.Y1.AND.Y.LT.Y2) CN=0.50D0
       END IF
       IF(Y.GT.Y1.AND.Y.LT.Y2.AND.Z.GT.Z1.AND.Z.LT.Z2) CN=1.0D0
       RETURN
10     RTDXT=2.0D0*DSQRT(DX*T)
```

```
C
C        BEGIN SUMMATION OF TERMS IN INFINITE SERIES (OUTER SERIES)
      NMAX1=NMAX+1
      MMAX1=MMAX+1
      SIGMAM=0.0D0
      SUBTM=0.0D0
      DO 20 MM=1,MMAX1
      M=MM-1
      ZETA=M*PI/H
      OM=(Z2-Z1)/H
      IF(M.NE.0) OM=(DSIN(ZETA*Z2)-DSIN(ZETA*Z1))/(M*PI)
      COSSZ=DCOS(ZETA*Z)
C
C        BEGIN SUMMATION OF TERMS IN INFINITE SERIES (INNER SERIES)
      SIGMAN=0.0D0
      SUBTN=0.0D0
      DO 30 NN=1,NMAX1
      N=NN-1
      ETA=N*PI/W
      PN=(Y2-Y1)/W
      IF(N.NE.0) PN=(DSIN(ETA*Y2)-DSIN(ETA*Y1))/(N*PI)
      COSRY=DCOS(ETA*Y)
      ALPHA=4.0D0*DX*(ETA*ETA*DY+ZETA*ZETA*DZ+DK)
      BETA=DSQRT(VX*VX+ALPHA)
      BETAT=BETA*T
C
C     IF M>0 AND N>0, USE GENERAL FORM
C       TERM 1
      A1=X*(VX-BETA)/(2.0D0*DX)
      B1=(X-BETAT)/RTDXT
      CALL EXERFC(A1,B1,C1)
      A2=X*(VX+BETA)/(2.0D0*DX)
      B2=(X+BETAT)/RTDXT
      CALL EXERFC(A2,B2,C2)
      TERM1=COSRY*PN*(C1+C2)
C
C       MULTIPLY TERM BY L(MN)
      IF(M.EQ.0 .AND. N.EQ.0) TERM1=TERM1*0.50D0
      IF(M.GT.0 .AND. N.GT.0) TERM1=TERM1*2.0D0
C
C       ADD TERM TO SUMMATION
      SIGMAN=SIGMAN+TERM1
C
C       CHECK FOR CONVERGENCE OF INNER SERIES.  BECAUSE SERIES
C       OSCILLATES, CHECK SUBTOTAL OF LAST 10 TERMS.
      SUBTN=SUBTN+TERM1
      IF(MOD(NN,10).NE.0) GO TO 30
      IF(DABS(SUBTN).LT.1.0D-12) GO TO 25
      SUBTN=0.0D0
30    CONTINUE
      IERR='*'
25    SIGMAM=SIGMAM+SIGMAN*COSSZ*OM
C
```

```
C         CHECK FOR CONVERGENCE OF OUTER SERIES.  BECAUSE SERIES
C         OSCILLATES, CHECK SUBTOTAL OF LAST 10 TERMS.
      SUBTM=SUBTM+SIGMAN*OM*COSSZ
      IF(MOD(MM,10).NE.0) GO TO 20
      IF(DABS(SUBTM).LT.1.0D-12) GO TO 35
      SUBTM=0.0D0
20    CONTINUE
      IERR='*'
35    CN=SIGMAM
      RETURN
      END
```

```
C
C       *******************************************************
C       *                                                     *
C       *                   **** PATCHI ****                  *
C       *                                                     *
C       *   THREE-DIMENSIONAL GROUND-WATER SOLUTE TRANSPORT   *
C       *                                                     *
C       *    MODEL FOR A SEMI-INFINITE AQUIFER OF INFINITE    *
C       *                                                     *
C       *    WIDTH AND HEIGHT. PATCH SOURCE EXTENDING FROM    *
C       *                                                     *
C       *         Y1 TO Y2 AND Z1 TO Z2 LOCATED AT X=0        *
C       *                                                     *
C       *        GROUND-WATER FLOW IN X-DIRECTION ONLY        *
C       *                                                     *
C       *            VERSION CURRENT AS OF 11/30/88           *
C       *                                                     *
C       *******************************************************
C
C     THE FOLLOWING CARD MUST BE CHANGED IF PROBLEM DIMENSIONS ARE
C     GREATER THAN THOSE GIVEN HERE.
C       MAXX = MAXIMUM NUMBER OF X-VALUES
C       MAXY = MAXIMUM NUMBER OF Y-VALUES
C       MAXZ = MAXIMUM NUMBER OF Z-VALUES
C       MAXT = MAXIMUM NUMBER OF TIME VALUES
C       MAXXY = MAXX * MAXY
C       MAXXY2 = 2 * MAXX * MAXY
      PARAMETER MAXX=100,MAXY=50,MAXZ=30,MAXT=20,MAXXY=5000,MAXXY2=10000
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      CHARACTER*10 CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      REAL XP,YP,ZP,CP,TP,DELTA,XPC,YPC,XSCLP,YSCLP
      DIMENSION CXY(MAXX,MAXY),X(MAXX),Y(MAXY),Z(MAXZ),T(MAXT)
      COMMON /PDAT/ XP(MAXX),YP(MAXY),CP(MAXXY),XPC(50),YPC(50),
     1 IFLAG(MAXXY2)
      COMMON /IOUNIT/ IN,IO
C
C     PROGRAM VARIABLES
C
C          NOTE:  ANY CONSISTANT SET OF UNITS MAY BE USED IN THE
C          MODEL.  NO FORMAT STATEMENTS NEED TO BE CHANGED AS
C          LABELS FOR ALL VARIABLES ARE SPECIFIED IN MODEL INPUT.
C
C     CO        SOLUTE CONCENTRATION AT THE INFLOW BOUNDARY [M/L**3]
C     DX        LONGITUDINAL DISPERSION COEFFICIENT [L**2/T]
C     DY        TRANSVERSE (Y-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C     DZ        TRANSVERSE (Z-DIRECTION) DISPERSION COEFFICIENT [L**2/T]
C     VX        GROUND-WATER VELOCITY IN X-DIRECTION [L/T]
C     DK        FIRST-ORDER SOLUTE DECAY CONSTANT [1/T]
C     X         X-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Y         Y-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     Z         Z-POSITION AT WHICH CONCENTRATION IS EVALUATED [L]
C     T         TIME AT WHICH CONCENTRATION IS EVALUATED [T]
```

```
C     CN        NORMALIZED CONCENTRATION C/C0 [DIMENSIONLESS]
C     CXY       SOLUTE CONCENTRATION C(X,Y,Z,T) [M/L**3]
C     WS        WIDTH OF PATCH SOLUTE SOURCE [L]
C     HS        HEIGHT OF PATCH SOLUTE SOURCE [L]
C     YC        Y-COORDINATE OF CENTER OF PATCH SOLUTE SOURCE [L]
C     ZC        Z-COORDINATE OF CENTER OF PATCH SOLUTE SOURCE [L]
C
C     NX        NUMBER OF X-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NY        NUMBER OF Y-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NZ        NUMBER OF Z-POSITIONS AT WHICH SOLUTION IS EVALUATED
C     NT        NUMBER OF TIME VALUES AT WHICH SOLUTION IS EVALUATED
C     NMAX      NUMBER OF TERMS USED IN GAUSS-LEGENDRE NUMERICAL
C               INTEGRATION TECHNIQUE (MUST EQUAL 4, 20, 60, 104 OR 256)
C
C     IPLT      PLOT CONTROL. IF IPLT>0, CONTOUR MAPS ARE PLOTTED
C     XSCLP     SCALING FACTOR TO CONVERT X TO PLOTTER INCHES
C     YSCLP     SCALING FACTOR TO CONVERT Y TO PLOTTER INCHES
C     DELTA     CONTOUR INCREMENT FOR PLOT. (VALUE BETWEEN 0 AND 1.0)
C
C       CHARACTER VARIABLES USED TO SPECIFY UNITS FOR MODEL PARAMETERS
C     CUNITS    UNITS OF CONCENTRATION (M/L**3)
C     VUNITS    UNITS OF GROUND-WATER VELOCITY (L/T)
C     DUNITS    UNITS OF DISPERSION COEFFICIENT (L**2/T)
C     KUNITS    UNITS OF SOLUTE DECAY CONSTANT  (1/T)
C     LUNITS    UNITS OF LENGTH (L)
C     TUNITS    UNITS OF TIME (T)
C
C       DEFINE INPUT/OUTPUT FILES AND PRINT TITLE PAGE
      CALL OFILE
      CALL TITLE
      WRITE(IO,201)
C
C       READ IN MODEL PARAMETERS
      READ(IN,101) NX,NY,NZ,NT,NMAX,IPLT
      WRITE(IO,205) NX,NY,NZ,NT,NMAX
      READ(IN,105) CUNITS,VUNITS,DUNITS,KUNITS,LUNITS,TUNITS
      READ(IN,110) C0,VX,DX,DY,DZ,DK
      WRITE(IO,210) C0,CUNITS,VX,VUNITS,DX,DUNITS,DY,DUNITS,DZ,DUNITS,
     1 DK,KUNITS
      READ(IN,110) YC,ZC,WS,HS
      WRITE(IO,212) YC,LUNITS,ZC,LUNITS,WS,LUNITS,HS,LUNITS
      Y1=YC-WS/2.0D0
      Y2=YC+WS/2.0D0
      Z1=ZC-HS/2.0D0
      Z2=ZC+HS/2.0D0
      READ(IN,110) (X(I),I=1,NX)
      WRITE(IO,215) LUNITS
      WRITE(IO,220) (X(I),I=1,NX)
      READ(IN,110) (Y(I),I=1,NY)
      WRITE(IO,216) LUNITS
      WRITE(IO,220) (Y(I),I=1,NY)
      READ(IN,110) (Z(I),I=1,NZ)
      WRITE(IO,217) LUNITS
```

```
      WRITE(IO,220) (Z(I),I=1,NZ)
      READ(IN,110) (T(I),I=1,NT)
      WRITE(IO,225) TUNITS
      WRITE(IO,220) (T(I),I=1,NT)
      IF(IPLT.GT.0) READ(IN,110) XSCLP,YSCLP,DELTA
      IF(IPLT.GT.0) WRITE(IO,227) XSCLP,YSCLP,DELTA,CUNITS
C
C       READ IN GAUSS-LEGENDRE POINTS AND WEIGHTING FACTORS
      CALL GLQPTS (NMAX)
C
C       BEGIN TIME LOOP
      DO 20 IT=1,NT
C
C       BEGIN Z LOOP
      DO 30 IZ=1,NZ
C
C       BEGIN X LOOP
      DO 40 IX=1,NX
C
C     CALCULATE NORMALIZED CONCENTRATION FOR ALL Y AT X=X(IX) AND Z=Z(IZ
      DO 50 IY=1,NY
      CALL CNRMLP(DK,T(IT),X(IX),Y(IY),Z(IZ),Y1,Y2,Z1,Z2,DX,
     1 DY,DZ,VX,CN,NMAX)
      CXY(IX,IY)=C0*CN
50    CONTINUE
40    CONTINUE
C
C       PRINT OUT TABLES OF CONCENTRATION VALUES
      NPAGE=1+(NY-1)/9
      DO 60 NP=1,NPAGE
      IF(NP.EQ.1) WRITE(IO,230) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      IF(NP.NE.1) WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      NP1=(NP-1)*9
      NP2=9
      IF((NP1+NP2).GT.NY) NP2=NY-NP1
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
      DO 70 IX=1,NX
      WRITE(IO,240) X(IX),(CXY(IX,NP1+J),J=1,NP2)
      IF(MOD(IX,45).NE.0) GO TO 70
      WRITE(IO,231) T(IT),TUNITS,Z(IZ),LUNITS,LUNITS
      WRITE(IO,235) (Y(NP1+J),J=1,NP2)
      WRITE(IO,236) CUNITS,LUNITS
70    IF(MOD(IX,5).EQ.0 .AND. MOD(IX,45).NE.0) WRITE(IO,241)
60    CONTINUE
C
C       CONVERT X AND Y TO SINGLE PRECISION AND DIVIDE BY THE
C       PLOT SCALING FACTORS. CONVERT C(X,Y) AND DIVIDE BY C0 TO PLOT
C       CONTOUR MAPS OF NORMALIZED CONCENTRATION FOR EACH TIME VALUE.
      IF(IPLT.LT.1) GO TO 30
      NXY=NX*NY
      DO 80 I=1,NX
      IP=(I-1)*NY
```

```
      XP(I)=SNGL(X(I))
      DO 80 J=1,NY
      IF(I.EQ.1) YP(J)=SNGL(Y(J))
      CP(IP+J)=SNGL(CXY(I,J)/C0)
80    CONTINUE
      TP=SNGL(T(IT))
      ZP=SNGL(Z(IZ))
      NXY2=NXY*2
      CALL PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT,
     1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
30    CONTINUE
20    CONTINUE
      CLOSE (IN)
      CLOSE (IO)
      STOP
C
C        FORMAT STATEMENTS
101   FORMAT(20I4)
105   FORMAT(8A10)
110   FORMAT(8F10.0)
201   FORMAT(/////1H ,29X,'ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL'
     1 /1H ,28X,'ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION'/
     2 1H ,30X,'FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH'/
     3 1H ,28X,'AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0'
     4 ///1H0,40X,'INPUT DATA'/1H ,40X,10(1H-))
205   FORMAT(1H0,25X,'NUMBER OF X-COORDINATES (NX) = ',I4/1H ,25X,
     1 'NUMBER OF Y-COORDINATES (NY) = ',I4/1H ,25X,
     2 'NUMBER OF Z-COORDINATES (NZ) = ',I4/1H ,25X,
     3 'NUMBER OF TIME VALUES (NT) = ',I4/1H ,25X,
     4 'NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = ',I4)
210   FORMAT(1H0,25X,'SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) =',
     1 1P1E13.6,1X,A10/1H ,25X,
     2 'GROUND-WATER VELOCITY IN X-DIRECTION (VX) =',1P1E13.6,1X,A10/
     3 1H ,25X,'DISPERSION IN THE X-DIRECTION (DX) =',1P1E13.6,1X,A10/
     4 1H ,25X,'DISPERSION IN THE Y-DIRECTION (DY) =',1P1E13.6,1X,A10/
     5 1H ,25X,'DISPERSION IN THE Z-DIRECTION (DZ) =',1P1E13.6,1X,A10/
     6 1H ,25X,'FIRST-ORDER SOLUTE DECAY RATE (DK) =',1P1E13.6,1X,A10)
212   FORMAT(1H0,25X,'AQUIFER WIDTH (W) AND HEIGHT (H) ARE INFINITE'
     2 /1H ,25X,'SOLUTE SOURCE IS CENTERED AT Y =',1P1E13.6,1X,A10/
     3 1H ,50X,'AND Z =',1P1E13.6,1X,A10/1H ,25X,
     4 'FINITE-WIDTH OF SOLUTE SOURCE (WS) =',1P1E13.6,1X,A10/1H ,25X,
     5 'FINITE-HEIGHT OF SOLUTE SOURCE (HS) =',1P1E13.6,1X,A10)
215   FORMAT(1H0,25X,'X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
216   FORMAT(1H0,25X,'Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
217   FORMAT(1H0,25X,'Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS ',
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,78(1H-)/)
220   FORMAT(1H ,5X,8F12.4)
225   FORMAT(1H0,25X,'TIMES AT WHICH SOLUTE CONCENTRATIONS '
     1 'WILL BE CALCULATED, IN ',A10/1H ,25X,70(1H-)/)
227   FORMAT(1H0,25X,'PLOT SCALING FACTOR FOR X (XSCLP) =',1P1E13.6/
     1 1H ,25X,'PLOT SCALING FACTOR FOR Y (YSCLP) =',1P1E13.6/
```

```
     2 1H ,25X,'CONTOUR INCREMENT (DELTA) =',1P1E13.6,1X,A10)
230   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10/1H ,35X,'AND AT Z =',F12.4,1X,A10/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
231   FORMAT(1H1/1H0,15X,'SOLUTE CONCENTRATION AT TIME =',
     1 F12.4,1X,A10,5X,'(CONTINUED)'/1H ,35X,'AND AT Z =',F12.4,1X,A10/
     2 1H0,25X,'Y-COORDINATE, IN ',A10)
235   FORMAT(1H ,20X,9F12.4)
236   FORMAT(1H ,19X,'*',108(1H-)/
     1 1H ,4X,'X-COORDINATE,',2X,'!',44X,'SOLUTE CONCENTRATION, IN '
     2 A10/1H ,4X,'IN ',A10,2X,1H!/1H ,19X,'!')
240   FORMAT(1H ,5X,F12.4,2X,'!',9F12.6)
241   FORMAT(1H ,19X,'!')
      END
      SUBROUTINE CNRMLP(DK,T,X,Y,Z,Y1,Y2,Z1,Z2,DX,DY,DZ,VX,CN,NMAX)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON /IOUNIT/ IN,IO
      COMMON /GLPTS/ WN(256),ZN(256)
C
C        THIS ROUTINE CALCULATES THE NORMALIZED CONCENTRATION AT X,Y,Z
C        BASED ON THE ANALYTIC SOLUTION TO THE THREE-DIMENSIONAL
C        ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-
C        INFINITE AQUIFER WITH INFINITE WIDTH AND HEIGHT. THE SOLUTE
C        SOURCE HAS A FINITE WIDTH AND HEIGHT, EXTENDING FROM Y=Y1 TO
C        Y=Y2 AND Z=Z1 TO Z=Z2. THE SOLUTE MAY BE SUBJECT TO FIRST-ORDER
C        CHEMICAL TRANSFORMATION. THE SOLUTION CONTAINS AN INTEGRAL
C        FROM 0 TO T WHICH IS EVALUATED NUMERICALLY USING A GAUSS-
C        LEGENDRE QUADRATURE TECHNIQUE.
C
      PI=3.14159265358979D0
      CN=0.0D0
C
C        FOR T=0, ALL CONCENTRATIONS EQUAL 0.0
      IF(T.LE.0.0D0) RETURN
C
C        FOR X=0.0, CONCENTRATIONS ARE SPECIFIED BY BOUNDARY CONDITIONS
      IF(X.GT.0.0D0) GO TO 10
      IF(Y.EQ.Y1.OR.Y.EQ.Y2) THEN
        IF(Z.GT.Z1.AND.Z.LT.Z2) CN=0.50D0
        IF(Z.EQ.Z1.OR.Z.EQ.Z2) CN=0.25D0
      END IF
      IF(Z.EQ.Z1.OR.Z.EQ.Z2) THEN
        IF(Y.GT.Y1.AND.Y.LT.Y2) CN=0.50D0
      END IF
      IF(Y.GT.Y1.AND.Y.LT.Y2.AND.Z.GT.Z1.AND.Z.LT.Z2) CN=1.0D0
      RETURN
C
C        START NUMERICAL INTEGRATION LOOP
10    SUM=0.0D0
      DO 20 I=1,NMAX
C
C        SCALE THE GAUSS-LEGENDRE COEFFICIENTS TO ACCOUNT FOR THE
C        NON-NORMALIZED LIMITS OF INTEGRATION
```

```
      WI=WN(I)*T/2.0D0
      ZI=T*(ZN(I)+1.0D0)/2.0D0
C
C       TERM 1
      XVT=X-VX*ZI
      EXP1=-XVT*XVT/(4.0D0*DX*ZI)-DK*ZI
      ERFC1=(Y1-Y)/(2.0D0*DSQRT(DY*ZI))
      CALL EXERFC(EXP1,ERFC1,Q1)
C
C       TERM 2
      ERFC2=(Y2-Y)/(2.0D0*DSQRT(DY*ZI))
      CALL EXERFC(EXP1,ERFC2,Q2)
C
C       TERM 3
      EXP2=0.0D0
      ERFC1=(Z1-Z)/(2.0D0*DSQRT(DZ*ZI))
      CALL EXERFC(EXP2,ERFC1,Q3)
      ERFC2=(Z2-Z)/(2.0D0*DSQRT(DZ*ZI))
      CALL EXERFC(EXP2,ERFC2,Q4)
      TERM=(Q1-Q2)*(Q3-Q4)*WI/(ZI**1.5)
      SUM=SUM+TERM
20    CONTINUE
      CN=SUM*X/(8.0D0*DSQRT(PI*DX))
      RETURN
      END
```

ATTACHMENT 3.--SUBROUTINE LISTING AND DATA FILE GLQ.PTS

Subroutine EXERFC
Subroutine GLQPTS
Subroutine OFILE
Subroutine TITLE
Subroutine PLOT1D
Subroutine PLOT2D
Subroutine PLOT3D
Subroutine CNTOUR
Data file GLQ.PTS

```
C
C     **********************************************************
C     *                                                        *
C     *                 SUBROUTINE EXERFC                      *
C     *                                                        *
C     *           VERSION CURRENT AS OF 10/01/87               *
C     *                                                        *
C     **********************************************************
C
      SUBROUTINE EXERFC (X,YY,Z)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION P1(5),Q1(5),P2(9),Q2(9),P3(6),Q3(6)
C
C       THIS ROUTINE USES RATIONAL CHEBYSHEV APPROXIMATIONS
C       FOR EVALUATING THE ERROR FUNCTION AND COMPLEMENTARY
C       ERROR FUNCTION IN ORDER TO EVALUATE THE PRODUCT OF
C       EXP(X) AND ERFC(Y)
C
      DATA P1/3.209377589138469472562D03,3.774852376853020208137D02,
     1        1.138641541510501556495D02,3.161123743870565596947D0,
     2        1.857777061846031526730D-01/
      DATA Q1/2.844236833439170622273D03,1.282616526077372275645D03,
     1        2.440246379344441733056D02,2.360129095234412093499D01,
     2        1.0D0 /
      DATA P2/1.23033935479799725272D03,2.05107837782607146532D03,
     1        1.71204761263407058314D03,8.81952221241769090411D02,
     2        2.98635138197400131132D02,6.61191906371416294775D01,
     3       8.88314979438837594118D00,5.64188496988670089180D-01,
     4       2.15311535474403846343D-08/
      DATA Q2/1.23033935480374942043D03,3.43936767414372163696D03,
     1        4.36261909014324715820D03,3.29079923573345962678D03,
     2        1.62138957456669018874D03,5.37181101862009857509D02,
     3        1.17693950891312499305D02,1.57449261107098347253D01,
     4        1.0D0 /
      DATA P3/-6.58749161529837803157D-04,-1.60837851487422766278D-02,
     1        -1.25781726111229246204D-01,-3.60344899949804439429D-01,
     2        -3.05326634961232344035D-01,-1.63153871373020978498D-02/
      DATA Q3/2.33520497626869185443D-03,6.05183413124413191178D-02,
     1        5.27905102951428412248D-01,1.87295284992346047209D00,
     2        2.56852019228982242072D00,1.0D0/
C
      IF(YY.EQ.0.0D0) Z=DEXP(X)
      IF(YY.EQ.0.0D0) RETURN
      Y=DABS(YY)
C
C        FOR 0.0 < Y < .46875
      IF (Y.GT.0.46875D0) GO TO 20
      SUMP=0.0D0
      SUMQ=0.0D0
      DO 10 I=1,5
      Y2I=Y**(2*(I-1))
      SUMP=SUMP+P1(I)*Y2I
      SUMQ=SUMQ+Q1(I)*Y2I
```

```
10      CONTINUE
        ERF=Y*SUMP/SUMQ
        IF(YY.LT.0.0) ERF=-ERF
        ERFCY=1.0D0-ERF
        Z=DEXP(X)*ERFCY
        RETURN
C
C          FOR 0.0 < Y < .46875
20      IF (Y.GT.4.0D0) GO TO 40
        SUMP=0.0D0
        SUMQ=0.0D0
        DO 30 I=1,9
        YI=Y**(I-1)
        SUMP=SUMP+P2(I)*YI
        SUMQ=SUMQ+Q2(I)*YI
30      CONTINUE
        Z=DEXP(X-Y*Y)*SUMP/SUMQ
        IF(YY.LT.0.0D0) Z=2.0D0*DEXP(X)-Z
        RETURN
40      SUMP=0.0D0
        SUMQ=0.0D0
        DO 50 I=1,6
        Y2I=Y**(-2*(I-1))
        SUMP=SUMP+P3(I)*Y2I
        SUMQ=SUMQ+Q3(I)*Y2I
50      CONTINUE
        SQRTPI=0.56418958354775628694810D0
        Z=SQRTPI+SUMP/(Y*Y*SUMQ)
        Z=DEXP(X-Y*Y)*Z/Y
        IF(YY.LT.0.0D0) Z=2.0D0*DEXP(X)-Z
        RETURN
        END
```

```
C
C         ********************************************************
C         *                                                      *
C         *                 SUBROUTINE GLQPTS                    *
C         *                                                      *
C         *           VERSION CURRENT AS OF 10/01/87             *
C         *                                                      *
C         ********************************************************
C
      SUBROUTINE GLQPTS (N)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      CHARACTER*1 SKIP
      COMMON /GLPTS/ WN(256),ZN(256)
      COMMON /IOUNIT/ IN,IO
C
C       THIS ROUTINE READS THE NORMALIZED ROOTS ZN(I) AND WEIGHTS WN(I)
C       OF THE LEGENDRE POLYNOMIALS FROM THE DATA FILE 'GLQ.PTS'
C
C       N IS THE NUMBER OF INTEGRATION POINTS AND CAN ONLY HAVE A
C       VALUE OF EITHER 4,20,60,104,OR 256
C
      IN2=77
      OPEN(IN2,FILE='GLQ.PTS',STATUS='OLD')
C
C       SKIP LINES IN FILE UNTIL CORRECT COEFFICIENTS ARE REACHED
      ISKIP=-1
      IF(N.EQ.4) ISKIP=7
      IF(N.EQ.20) ISKIP=9
      IF(N.EQ.60) ISKIP=15
      IF(N.EQ.104) ISKIP=31
      IF(N.EQ.256) ISKIP=57
      IF (ISKIP.EQ.-1) WRITE(IO,201)
      IF (ISKIP.EQ.-1) STOP
      DO 60 I=1,ISKIP
60    READ(IN2,101) SKIP
C
C       READ IN ZN(I) AND WN(I), FOUR VALUES PER LINE
      NC=N/8
      IF (MOD(N,8).NE.0) NC=NC+1
      DO 80 I=1,NC
      K=(I-1)*8-1
      READ(IN2,102) (ZN(K+J*2),J=1,4)
80    CONTINUE
      DO 100 I=1,NC
      K=(I-1)*8-1
      READ(IN2,102) (WN(K+J*2),J=1,4)
100   CONTINUE
C
C       FILL IN THE SYMMETRIC TERMS
      DO 120 J=2,N,2
      J1=J-1
      ZN(J)=-ZN(J1)
120   WN(J)=WN(J1)
```

```
      CLOSE(IN2)
      RETURN
C
C        FORMAT STATEMENTS
101   FORMAT(A1)
102   FORMAT(4D20.0)
201   FORMAT(1H0,20X,'*****  ERROR IN ROUTINE GLQPTS  *****'/
     1 1H ,20X,'NO. OF ROOTS SPECIFIED MUST EQUAL 4,20,60,104 OR 256')
      END
```

```
C
C          ********************************************************
C          *                                                      *
C          *                  SUBROUTINE OFILE                    *
C          *                                                      *
C          *            VERSION CURRENT AS OF 10/01/87            *
C          *                                                      *
C          ********************************************************
C
      SUBROUTINE OFILE
      CHARACTER*50 IFNAME,OFNAME
      CHARACTER*1 STAR
      COMMON /IOUNIT/ IN,IO
      DATA STAR/'*'/
      IN=15
      IO=16
      WRITE(1,5)
      READ(1,7) IFNAME
      WRITE(1,6)
      READ(1,7) OFNAME
      OPEN (IN,FILE=IFNAME,STATUS='OLD')
      IF(OFNAME(1:1).EQ.STAR) IO=1
      IF(OFNAME(1:1).NE.STAR) OPEN (IO,FILE=OFNAME)
      RETURN
C
C        FORMAT STATEMENTS
5     FORMAT(5X,'TYPE IN INPUT FILE NAME')
6     FORMAT(5X,'TYPE IN OUTPUT FILE NAME')
7     FORMAT(A50)
      END
```

```
C
C         ********************************************************
C         *                                                      *
C         *                  SUBROUTINE TITLE                    *
C         *                                                      *
C         *            VERSION CURRENT AS OF 10/01/87            *
C         *                                                      *
C         ********************************************************
C
      SUBROUTINE TITLE
      CHARACTER*1 LINE1(60),EQUAL,BLANK
      CHARACTER DATE*16,TIME*8
      CHARACTER*60 LINE
      CHARACTER*61 TITLE,T1
      COMMON /IOUNIT/ IN,IO
      COMMON /TITLES/ TITLE(4)
      DATA EQUAL/'='/,BLANK/' '/
      DATA TITLE/4*'
     1          '/
C
C        THIS ROUTINE CREATES A TITLE BOX ON THE FIRST PAGE OF
C        PROGRAM OUTPUT. THE ROUTINE READS AND PRINTS ALL DATA
C        CARDS UNTIL IT ENCOUNTERS AN '=' IN COLUMN 1. THE FIRST 4
C        LINES READ IN ARE ALSO USED AS TITLES ON PLOTS.
C
      CALL TIME$A (TIME)
      CALL DATE$A(DATE)
      WRITE(IO,201)
      DO 10 L=1,60
      READ(IN,101,END=20) LINE
      IF (LINE(1:1).EQ.EQUAL) GOTO 60
      T1=LINE
C       STRIP OFF TRAILING BLANKS AND CENTER LINE
      DO 15 N=1,60
      NN=61-N
15    IF(LINE(NN:NN).NE.BLANK) GOTO 20
20    NN1=NN+1
      T1(NN1:NN1)='$'
      IF(L.LT.5) TITLE(L)=T1
      NS=(60-NN)/2
      IF(NS.EQ.0) GO TO 35
      DO 30 I=1,60
30    LINE1(I)=BLANK
35    NS1=NS+1
      DO 40 I=1,NN
40    LINE1(NS+I)=LINE(I:I)
10    WRITE(IO,202) (LINE1(I),I=1,60)
60    WRITE(IO,203) DATE,TIME
      RETURN
C
C       FORMAT STATEMENTS
101   FORMAT (A60)
201   FORMAT(1H1//////////1H ,16X,68(1H*))
```

```
202     FORMAT(1H ,16X,1H*,66X,1H*/1H ,16X,1H*,3X,60A1,3X,1H*)
203     FORMAT(1H ,16X,1H*,66X,1H*/1H ,16X,1H*,12X,'PROGRAM RUN ON ',
       1 A16,' AT ',A8,11X,1H*/1H ,16X,1H*,66X,1H*/1H ,16X,68(1H*)
       2 /1H1)
        END
```

```
C
C         *******************************************************
C         *                                                     *
C         *                  SUBROUTINE PLOT1D                  *
C         *                                                     *
C         *           VERSION CURRENT AS OF 10/01/87            *
C         *                                                     *
C         *******************************************************
C
      SUBROUTINE PLOT1D (X,C,NX,T,IT,NT,TUNITS,LUNITS,XSCLP)
      COMMON /XP/ XPOS,YPOS,XPOS1,YPOS1,XAXIS,YAXIS
      COMMON /TITLES/ TITLE(4)
      DIMENSION X(NX),C(NX)
      COMMON ID
      CHARACTER*10 LUNITS,TUNITS
      CHARACTER*17 LAB
      CHARACTER*27 LAB1
      CHARACTER*26 LABX
      CHARACTER*36 LABX1
      CHARACTER*24 LABY
      CHARACTER*61 TITLE
C
C       THIS ROUTINE PLOTS CONCENTRATION VS. DISTANCE AT EACH OF THE
C       TIMES SPECIFIED IN THE INPUT DATA.  THE ROUTINE USES DISSPLA
C       SOFTWARE PLOT CALLS.
C
C       INITIALIZE PLOT - SCALE BASED ON MAXIMUM X-DISTANCE
      HITE=0.1
      IF(IT.EQ.1) THEN
        CALL COMPRS
        CALL SETCLR ('BLUE')
        X1=X(NX)-X(1)
        XAXIS=INT(X1/XSCLP)
        X11=X1/XSCLP
        IF((X11-XAXIS).GT.0.0) XAXIS=XAXIS+1.0
        YAXIS=10.0
        XPM=XAXIS+1.5
        YPM=12.2
        CALL PAGE(XPM,YPM)
        CALL AREA2D (XAXIS,YAXIS)
        CALL HEADIN (TITLE(1),100,1.,4)
        CALL HEADIN (TITLE(2),100,1.,4)
        CALL HEADIN (TITLE(3),100,1.,4)
        CALL HEADIN (TITLE(4),100,1.,4)
C         LABEL AXES
        LABX='DISTANCE ALONG X-AXIS, IN '
        LABX1=LABX//LUNITS
        LABY='NORMALIZED CONCENTRATION'
        CALL XNAME (LABX1,36)
        CALL YNAME (LABY,24)
C         DRAW AND NUMBER AXES
        CALL INTAXS
        CALL YAXANG (0.)
```

```
          CALL XREVTK
          CALL YREVTK
          XMIN=X(1)
          XMAX=XSCLP*XAXIS
          YMAX=1.0
          CALL GRAF(XMIN,XSCLP,XMAX,0.0,0.1,YMAX)
          CALL RESET('XREVTK')
          CALL RESET('YREVTK')
C              DRAW EXTRA AXIS TO CLOSE BOX
          CALL XNONUM
          CALL XGRAXS(XMIN,XSCLP,XMAX,XAXIS,' ',1,0.0,YAXIS)
          CALL YNONUM
          CALL YGRAXS(0.0,0.1,YMAX,YAXIS,' ',1,XAXIS,0.0)
          CALL RESET('XNONUM')
          CALL RESET('YNONUM')
C           BEGIN LEGEND
          XPOS=XAXIS-.85*HITE*(27+4)-.1
          YPOS=YAXIS-.1-2.0*HITE
          CALL HEIGHT (0.1)
          LAB='ELAPSED TIME, IN '
          LAB1=LAB//TUNITS
          CALL MESSAG (LAB1,27,XPOS,YPOS)
          YPOS=YPOS-.5*HITE
C           BLANK OUT AREA FOR MESSAGE
          WIDE=HITE*0.85*35.
          HIGH=HITE*1.5*(NT+3)
          XPOS=XAXIS-WIDE-0.1
          YPOS1=YAXIS-HIGH-0.1
          CALL BLREC(XPOS,YPOS1,WIDE,HIGH,1.0)
          CALL BLKEY(ID)
          XPOS=XAXIS-2.75
        END IF
C
C         DRAW PLOT OF C VS X
        CALL MARKER(IT)
        CALL CURVE (X,C,NX,1)
        CALL MARKER(IT)
C         PLACE LABEL IN BOX
        CALL BLOFF(ID)
        YPOS=YPOS-1.5*HITE
        XPOS1=XPOS+3.*.85*HITE
        XP=XPOS1*XSCLP
        YP=(YPOS+0.05)/10.
        CALL CURVE (XP,YP,1,-1)
        XPOS1=XPOS+6*.85*HITE
        CALL MESSAG ('TIME =',6,XPOS1,YPOS)
        IPL=104
        CALL REALNO (T,IPL,'ABUT','ABUT')
        CALL BLON(ID)
C
C         CLOSE PLOT FILE
        IF(IT.EQ.NT) THEN
          CALL ENDPL (0)
```

```
  CALL DONEPL
END IF
RETURN
END
```

```
C
C        *******************************************************
C        *                                                     *
C        *                SUBROUTINE PLOT2D                    *
C        *                                                     *
C        *          VERSION CURRENT AS OF 11/30/88             *
C        *                                                     *
C        *******************************************************
C
      SUBROUTINE PLOT2D (XP,YP,CP,TP,DELTA,NX,NY,NXY,NXY2,IT,NT,IPLT,
     1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
      CHARACTER*10 TUNITS,LUNITS
      CHARACTER*26 LABX,LABY
      CHARACTER*36 LABX1,LABY1
      CHARACTER*61 TITLE
      DIMENSION XP(NX),YP(NY),CP(NXY),XPC(50),YPC(50),IFLAG(NXY2)
      COMMON /IOUNIT/ IN,IO
      COMMON /TITLES/ TITLE(4)
C
C       THIS ROUTINE INITIALIZES A CONTOUR PLOT ON THE RECTANGULAR GRID
C       DEFINED IN THE X-Y PLANE BY THE X AND Y VALUES READ IN. ONE
C       SUBPLOT IS GENERATED FOR EACH TIME VALUE.  THE ROUTINE USES
C       DISSPLA (ISCO) SOFTWARE SUBROUTINE CALLS.
C
C       CALCULATE PLOT SIZE AND DRAW BORDER
      XSPC=1.5
      YSPC=2.0
      X1=XP(NX)-XP(1)
      XAXIS=INT(X1/XSCLP)
      IF(AMOD(X1,XSCLP).GT.0.0) XAXIS=XAXIS+1.0
      Y1=YP(NY)-YP(1)
      YAXIS=INT(Y1/YSCLP)+1.0
      IF(AMOD(Y1,YSCLP).GT.0.0) YAXIS=YAXIS+1.0
      IF(IT.EQ.1) THEN
        CALL COMPRS
        XPM=(XAXIS+XSPC)*NT+XSPC
        YPM=YAXIS+YSPC
        CALL PAGE(XPM,YPM)
      END IF
C       CHOOSE PLOT SIZE BASED ON MAXIMUM COORDINATE VALUES
      XORIG=(IT-1)*(XAXIS+XSPC)+XSPC
      YORIG=0.75
      CALL SETCLR ('BLUE')
      CALL PHYSOR(XORIG,YORIG)
      CALL AREA2D (XAXIS,YAXIS)
      IF(IT.EQ.1) THEN
        CALL HEADIN (TITLE(1),100,1.,4)
        CALL HEADIN (TITLE(2),100,1.,4)
        CALL HEADIN (TITLE(3),100,1.,4)
        CALL HEADIN (TITLE(4),100,1.,4)
      END IF
C       ROTATE Y VALUES, PUT TICK MARKS ON INSIDE, AND DEFINE AXES LABEL
      CALL INTAXS
```

```
      CALL YAXANG (0.)
      CALL XREVTK
      CALL YREVTK
      LABX='DISTANCE ALONG X-AXIS, IN '
      LABY='DISTANCE ALONG Y-AXIS, IN '
      LABX1=LABX//LUNITS
      LABY1=LABY//LUNITS
C       DRAW AND LABEL AXES
      CALL XNAME(LABX1,36)
      CALL YNAME(LABY1,36)
      XMIN=XP(1)
      YMIN=YP(1)
      XMAX=XSCLP*XAXIS + XMIN
      YMAX=YSCLP*YAXIS + YMIN
      CALL GRAF(XMIN,XSCLP,XMAX,YMIN,YSCLP,YMAX)
C       DRAW EXTRA AXIS TO CLOSE BOX
      CALL RESET('XREVTK')
      CALL RESET('YREVTK')
      CALL XNONUM
      CALL YNONUM
      CALL XGRAXS(XMIN,XSCLP,XMAX,XAXIS,' ',1,0.0,YAXIS)
      CALL YGRAXS(YMIN,YSCLP,YMAX,YAXIS,' ',1,XAXIS,0.0)
      CALL RESET('XNONUM')
      CALL RESET('YNONUM')
C       PRINT TITLE
      HITE=(XAXIS-1.0)/(55.*.86)
      IF(HITE.GT.0.14) HITE=0.14
      CALL HEIGHT (HITE)
      YP3=YAXIS-0.07-1.5*HITE
      CALL MESSAG('NORMALIZED CONCENTRATION AT TIME =$',100,0.5,YP3)
      IPL=3
      IF(AMOD(TP,0.01).EQ.0.0) IPL=2
      IF(AMOD(TP,0.1).EQ.0.0) IPL=1
      IF((TP-INT(TP)).EQ.0.0) IPL=0
      CALL REALNO(TP,IPL,'ABUT','ABUT')
      CALL MESSAG(TUNITS,10,'ABUT','ABUT')
C       COUNT NUMBER OF DIGITS IN CONTOUR LABEL
      YP3=YP3-1.5*HITE
      CALL MESSAG('CONTOUR INTERVAL =$',100,0.5,YP3)
      IPL=3
      IF(AMOD(DELTA,0.01).EQ.0.0) IPL=2
      IF(AMOD(DELTA,0.1).EQ.0.0) IPL=1
      CALL REALNO(DELTA,IPL,'ABUT','ABUT')
      CALL MESSAG('C/Co$',100,'ABUT','ABUT')
C
C       CALL ROUTINE THAT ACTUALLY DOES THE CONTOURING
      CALL CNTOUR(XP,YP,CP,DELTA,NX,NY,NXY,NXY2,XSCLP,YSCLP,XPC,YPC,
     1 IFLAG,IPL)
C
C       SUBPLOT IS FINISHED
      CALL ENDGR(0)
      IF (IT.EQ.NT) THEN
        CALL ENDPL (0)
```

```
  CALL DONEPL
END IF
RETURN
END
```

```
C
C        *******************************************************
C        *                                                     *
C        *                 SUBROUTINE PLOT3D                   *
C        *                                                     *
C        *          VERSION CURRENT AS OF 11/30/88             *
C        *                                                     *
C        *******************************************************
C
      SUBROUTINE PLOT3D (XP,YP,ZP,CP,TP,DELTA,NX,NY,NXY,NXY2,IZ,NZ,IPLT,
     1 TUNITS,LUNITS,XSCLP,YSCLP,XPC,YPC,IFLAG)
      CHARACTER*10 TUNITS,LUNITS
      CHARACTER*26 LABX,LABY
      CHARACTER*36 LABX1,LABY1
      CHARACTER*61 TITLE
      DIMENSION XP(NX),YP(NY),CP(NXY),XPC(50),YPC(50),IFLAG(NXY2)
      COMMON /IOUNIT/ IN,IO
      COMMON /TITLES/ TITLE(4)
C
C       THIS ROUTINE INITIALIZES A CONTOUR PLOT ON THE RECTANGULAR GRID
C       DEFINED IN THE X-Y PLANE BY THE X AND Y VALUES READ IN. ONE
C       SUBPLOT IS GENERATED FOR EACH Z VALUE AND A NEW PLOT IS
C       GENERATED FOR EACH TIME VALUE.  THE ROUTINE USES DISSPLA (ISCO)
C       SOFTWARE SUBROUTINE CALLS.
C       PLOT SCALING FACTORS (XSCLP,YSCLP) AND CONTOUR INTERVAL (DELTA)
C       ARE SPECIFIED IN THE MAIN PROGRAM.
C
C       CALCULATE PLOT SIZE AND DRAW BORDER
      XSPC=1.5
      YSPC=2.0
      X1=XP(NX)-XP(1)
      XAXIS=INT(X1/XSCLP)
      IF(AMOD(X1,XSCLP).GT.0.0) XAXIS=XAXIS+1.0
      Y1=YP(NY)-YP(1)
      YAXIS=INT(Y1/YSCLP)+1.0
      IF(AMOD(Y1,YSCLP).GT.0.0) YAXIS=YAXIS+1.0
      IF(IZ.EQ.1) THEN
        CALL COMPRS
        XPM=(XAXIS+XSPC)*NZ+XSPC
        YPM=YAXIS+YSPC
        CALL PAGE(XPM,YPM)
      END IF
C       CHOOSE PLOT SIZE BASED ON MAXIMUM COORDINATE VALUES
      XORIG=(IZ-1)*(XAXIS+XSPC)+XSPC
      YORIG=0.75
      CALL SETCLR ('BLUE')
      CALL PHYSOR(XORIG,YORIG)
      CALL AREA2D (XAXIS,YAXIS)
      IF(IZ.EQ.1) THEN
        CALL HEADIN (TITLE(1),100,1.,4)
        CALL HEADIN (TITLE(2),100,1.,4)
        CALL HEADIN (TITLE(3),100,1.,4)
        CALL HEADIN (TITLE(4),100,1.,4)
```

```
      END IF
C       ROTATE Y VALUES, PUT TICK MARKS ON INSIDE, AND DEFINE AXES LABEL
      CALL INTAXS
      CALL YAXANG (0.)
      CALL XREVTK
      CALL YREVTK
      LABX='DISTANCE ALONG X-AXIS, IN '
      LABY='DISTANCE ALONG Y-AXIS, IN '
      LABX1=LABX//LUNITS
      LABY1=LABY//LUNITS
C       DRAW AND LABEL AXES
      CALL XNAME(LABX1,36)
      CALL YNAME(LABY1,36)
      XMIN=XP(1)
      YMIN=YP(1)
      XMAX=XSCLP*XAXIS + XMIN
      YMAX=YSCLP*YAXIS + YMIN
      CALL GRAF(XMIN,XSCLP,XMAX,YMIN,YSCLP,YMAX)
C       DRAW EXTRA AXIS TO CLOSE BOX
      CALL RESET('XREVTK')
      CALL RESET('YREVTK')
      CALL XNONUM
      CALL YNONUM
      CALL XGRAXS(XMIN,XSCLP,XMAX,XAXIS,' ',1,0.0,YAXIS)
      CALL YGRAXS(YMIN,YSCLP,YMAX,YAXIS,' ',1,XAXIS,0.0)
      CALL RESET('XNONUM')
      CALL RESET('YNONUM')
C       PRINT TITLE
      HITE=(XAXIS-1.0)/(55.*.86)
      IF(HITE.GT.0.14) HITE=0.14
      CALL HEIGHT (HITE)
      YP3=YAXIS-0.07-1.5*HITE
      CALL MESSAG('NORMALIZED CONCENTRATION AT TIME =$',100,0.5,YP3)
      IPL=3
      IF(AMOD(TP,0.01).EQ.0.0) IPL=2
      IF(AMOD(TP,0.1).EQ.0.0) IPL=1
      IF((TP-INT(TP)).EQ.0.0) IPL=0
      CALL REALNO(TP,IPL,'ABUT','ABUT')
      CALL MESSAG(TUNITS,10,'ABUT','ABUT')
      YP3=YP3-1.5*HITE
      CALL MESSAG('                        AND AT Z =$',100,0.5,YP3)
      IPL=3
      IF(AMOD(ZP,0.01).EQ.0.0) IPL=2
      IF(AMOD(ZP,0.1).EQ.0.0) IPL=1
      IF((ZP-INT(ZP)).EQ.0.0) IPL=0
      CALL REALNO(ZP,IPL,'ABUT','ABUT')
      CALL MESSAG(LUNITS,10,'ABUT','ABUT')
C       COUNT NUMBER OF DIGITS IN CONTOUR LABEL
      YP3=YP3-1.5*HITE
      CALL MESSAG('CONTOUR INTERVAL =$',100,0.5,YP3)
      IPL=3
      IF(AMOD(DELTA,0.01).EQ.0.0) IPL=2
      IF(AMOD(DELTA,0.1).EQ.0.0) IPL=1
```

```
      CALL REALNO(DELTA,IPL,'ABUT','ABUT')
      CALL MESSAG('C/Co$',100,'ABUT','ABUT')
C
C       CALL ROUTINE THAT ACTUALLY DOES THE CONTOURING
      CALL CNTOUR(XP,YP,CP,DELTA,NX,NY,NXY,NXY2,XSCLP,YSCLP,XPC,YPC,
     1 IFLAG,IPL)
C
C       SUBPLOT IS FINISHED
      CALL ENDGR(0)
      IF (IZ.EQ.NZ) THEN
        CALL ENDPL (0)
        CALL DONEPL
      END IF
      RETURN
      END
```

```
C
C        ******************************************************
C        *                                                    *
C        *                SUBROUTINE CNTOUR                   *
C        *                                                    *
C        *          VERSION CURRENT AS OF 10/01/87            *
C        *                                                    *
C        ******************************************************
C
      SUBROUTINE CNTOUR (XP,YP,CP,DELTA,NX,NY,NXY,NXY2,XSCLP,YSCLP,
     1 XPC,YPC,IFLAG,IPL)
      DIMENSION XP(NX),YP(NY),CP(NXY),XPC(50),YPC(50),IFLAG(NXY2)
C
C       THIS ROUTINE IS CALLED BY PLOT2D AND PLOT3D TO CONTOUR VALUES
C       OF NORMALIZED CONCENTRATION VALUES ON THE RECTANGULAR GRID.
C       NUMBER OF SEGMENTS DRAWN BEFORE THE CONTOUR LINE IS LABELED
C       (NUM), AND CHARACTER HEIGHT ARE SET HERE, BUT CAN BE
C       EASILY MODIFIED.
C       XPC, YPC, AND IFLAG ARE WORK ARRAYS USED BY THIS ROUTINE.
C       IFLAG MUST BE DIMENSIONED TO TWICE THE NUMBER OF RECTANGULAR
C       BLOCKS SINCE EACH BLOCK IS DIVIDED INTO TWO TRIANGLES.
C
      NUM=40
      HITE=0.10
      RAD=57.2957795
C       COMPUTE SPACE NEEDED FOR CONTOUR LABEL
      CALL HEIGHT (HITE)
      CALL NUMODE('NOLEADSPACE')
      SPC1=(IPL+2)*HITE
      CALL SETCLR ('RED')
C
C        FIND MIN AND MAX VALUES AND NUMBER OF CONTOURS
      VMIN=1.0E36
      VMAX=-1.0E36
      DO 10 N=1,NXY
      VAL=CP(N)
      IF(VAL.GT.VMAX) VMAX=VAL
      IF(VAL.LT.VMIN) VMIN=VAL
10    CONTINUE
      GDEL=VMAX-VMIN
      MAXCNT=GDEL/DELTA
      MAXCNT=MAXCNT+1
C
C        FIND FIRST CONTOUR VALUE
      INC=VMIN/DELTA
      VALINC=INC*DELTA
C
C        SET UP MASTER LOOP FOR ALL CONTOURS
C          EACH RECTANGULAR BLOCK IS DIVIDED INTO TWO TRIANGLES.
C          CONTOURS ARE DRAWN BY LINEARLY INTERPOLATING ACROSS EACH
C          TRIANGLE.
      NTR=(NX-1)*(NY-1)*2
      NY2=(NY-1)*2
```

```
      DO 20 M=1,MAXCNT
      VALINC=VALINC+DELTA
      IF(VALINC.GT.VMAX) GOTO 20
C
C       INITIALIZE FLAGS ON TRIANGLES WITH CONTOURS PASSING THROUGH
      IFIRST=0
      DO 30 N=1,NTR
        N1=(N-1)/NY2
        N2=(N-(N1*NY2)+1)/2
        NG1=N1*NY+N2
        NG2=NG1+NY
        NG3=NG1+1
        IF(MOD(N,2).EQ.0) THEN
          NG1=NG1+1
          NG2=NG1+NY-1
          NG3=NG1+NY
        END IF
      IFLAG(N)=0
      CP1=CP(NG1)
      CP2=CP(NG2)
      CP3=CP(NG3)
      CPMAX=AMAX1(CP1,CP2,CP3)
      CPMIN=AMIN1(CP1,CP2,CP3)
      IF(CPMAX.LT.VALINC .OR. CPMIN.GT.VALINC) GOTO 30
      IFLAG(N)=1
      IF(IFIRST.EQ.0) IFIRST=N
      ILAST=N
30    CONTINUE
C
C       LOOP THROUGH ALL FLAGGED TRIANGLES
      DO 40 N=IFIRST,ILAST
      IF(IFLAG(N).EQ.0) GO TO 40
C
C       START UP A NEW CONTOUR SEGMENT
      ISTART=0
      ICHK=0
      IPT=1
      NEXT=N
C
C       CONTROL LOOP FOR FOLLOWING CONTOUR SEGMENT THROUGH ELEMENTS
50      N1=(NEXT-1)/NY2
        N2=(NEXT-(N1*NY2)+1)/2
      IEVEN=0
      IF(MOD(NEXT,2).EQ.0) IEVEN=1
        NG1=N1*NY+N2
        NG2=NG1+NY
        NG3=NG1+1
        IF(IEVEN.EQ.1) THEN
          NG1=NG1+1
          NG2=NG1+NY-1
          NG3=NG1+NY
        END IF
      CP1=CP(NG1)
```

```
      CP2=CP(NG2)
      CP3=CP(NG3)
      DELV21=CP2-CP1
      DELV31=CP3-CP1
      DELV32=CP3-CP2
      X1=XP(N1)
      X2=XP(N1+1)
      X3=XP(N1)
      Y1=YP(N2)
      Y2=YP(N2)
      Y3=YP(N2+1)
      IF(IEVEN.EQ.1) THEN
        X3=XP(N1+1)
        Y1=YP(N2+1)
      END IF
      DELX21=X2-X1
      DELX31=X3-X1
      DELX32=X3-X2
      DELY21=Y2-Y1
      DELY31=Y3-Y1
      DELY32=Y3-Y2
C       RESET FLAG, INCREMENT COUNTER, AND FIND NEIGHBORING ELEMENTS
      IFLAG(NEXT)=0
      IPT=IPT+1
      IUP=NEXT+1
      IDN=NEXT-1
      ISIDE=NEXT-NY2+1
      IF(IEVEN.EQ.1) ISIDE=NEXT+NY2-1
C
C       SPECIAL CASE 1.  CONTOURS ALONG ELEMENT SIDES
      IF(CP1.EQ.CP2 .AND. CP1.EQ.VALINC) THEN
        NEXT=-1
        XPC(1)=X1
        YPC(1)=Y1
        XPC(2)=X2
        YPC(2)=Y2
        IF(CP3.NE.CP1) GO TO 60
        IPT=4
        XPC(3)=X3
        YPC(3)=Y3
        XPC(4)=X1
        YPC(4)=Y1
      ELSE IF(CP1.EQ.CP3 .AND. CP1.EQ.VALINC) THEN
        NEXT=-1
        XPC(1)=X3
        YPC(1)=Y3
        XPC(2)=X1
        YPC(2)=Y1
      ELSE IF(CP2.EQ.CP3 .AND. CP2.EQ.VALINC) THEN
        NEXT=-1
        XPC(1)=X2
        YPC(1)=Y2
        XPC(2)=X3
```

```
        YPC(2)=Y3
      END IF
      IF(NEXT.EQ.-1) GO TO 60
C
C       SPECIAL CASE 2.  SINGLE POINTS EQUAL TO CONTOUR VALUE
C         CHECK NODE 1 FIRST
      JUMP=0
C
C       CHECK IF SEGMENT DEAD-ENDS AT NODE 1
      IF(CP1.EQ.VALINC .AND. ISTART.EQ.2) THEN
        NEXT=-1
        XPC(IPT)=X1
        YPC(IPT)=Y1
C
C       OTHERWISE, START NEW SEGMENT AT NODE 1
      ELSE IF(CP1.EQ.VALINC .AND. ISTART.EQ.0) THEN
        IF((CP2.GT.VALINC .AND. CP3.GT.VALINC) .OR. (CP2.LT.VALINC
     1  .AND. CP3.LT.VALINC)) GO TO 40
        JUMP=1
        XPC(1)=X1
        YPC(1)=Y1
        NEXT=IUP
        ISTART=1
        IF(IEVEN.EQ.1) THEN
          NEXT=ISIDE
          ISTART=3
        END IF
        RATIO=(VALINC-CP2)/DELV32
        XPC(2)=X2+RATIO*DELX32
        YPC(2)=Y2+RATIO*DELY32
C
C         NEXT CHECK NODE 2
      ELSE IF(CP2.EQ.VALINC .AND. ISTART.EQ.3) THEN
        NEXT=-1
        XPC(IPT)=X2
        YPC(IPT)=Y2
      ELSE IF(CP2.EQ.VALINC .AND. ISTART.EQ.0) THEN
        IF((CP1.GT.VALINC .AND. CP3.GT.VALINC) .OR. (CP1.LT.VALINC
     1  .AND. CP3.LT.VALINC)) GO TO 40
        JUMP=1
        XPC(1)=X2
        YPC(1)=Y2
        NEXT=ISIDE
        ISTART=2
        IF(IEVEN.EQ.1) THEN
          NEXT=IUP
          ISTART=2
        END IF
        RATIO=(VALINC-CP1)/DELV31
        XPC(2)=X1+RATIO*DELX31
        YPC(2)=Y1+RATIO*DELY31
C
C         NEXT CHECK NODE 3
```

```
      ELSE IF(CP3.EQ.VALINC .AND. ISTART.EQ.1) THEN
        NEXT=-1
        XPC(IPT)=X3
        YPC(IPT)=Y3
      ELSE IF(CP3.EQ.VALINC .AND. ISTART.EQ.0) THEN
        IF((CP1.GT.VALINC .AND. CP2.GT.VALINC) .OR. (CP1.LT.VALINC
     1  .AND. CP2.LT.VALINC)) GO TO 40
        JUMP=1
        XPC(1)=X3
        YPC(1)=Y3
        NEXT=IDN
        ISTART=3
        IF(IEVEN.EQ.1) ISTART=2
        RATIO=(VALINC-CP1)/DELV21
        XPC(2)=X1+RATIO*DELX21
        YPC(2)=Y1+RATIO*DELY21
      END IF
      IF(JUMP.EQ.1 .OR. NEXT.EQ.-1) GO TO 60
C
C       ROUTINE FOR DRAWING CONTOUR SEGMENT THROUGH TYPICAL ELEMENTS
C         START SEGMENT, IF NECCESSARY
      IF(ISTART.EQ.0) THEN
C       CHECK FOR CONTOUR ENTERING ON BOTTOM OF TRIANGLE (SIDE 1-2)
        IF((CP1.GT.VALINC .AND. CP2.LT.VALINC) .OR. (CP1.LT.VALINC
     1  .AND. CP2.GT.VALINC)) THEN
          ISTART=1
          RATIO=(VALINC-CP1)/DELV21
          XPC(1)=X1+RATIO*DELX21
          YPC(1)=Y1+RATIO*DELY21
C
C         CONTOUR MUST START ON SIDE 2 OR 3. PICK STARTING POINT
C         BASED ON WHETHER ELEMENT IS ODD OR EVEN
        ELSE
C           FOR ODD ELEMENT, START ON SIDE 1-3
          IF(MOD(NEXT,2).NE.0) THEN
            ISTART=3
            RATIO=(VALINC-CP1)/DELV31
            XPC(1)=X1+RATIO*DELX31
            YPC(1)=Y1+RATIO*DELY31
C           IF EVEN,START CONTOUR ON SIDE 2-3
          ELSE
            ISTART=2
            RATIO=(VALINC-CP2)/DELV32
            XPC(1)=X2+RATIO*DELX32
            YPC(1)=Y2+RATIO*DELY32
          END IF
        END IF
      END IF
C
C       CHECK FOR CONTOUR ENTERING ON BOTTOM OF TRIANGLE (SIDE 1-2)
      IF(ISTART.EQ.1) THEN
C
C       CHECK WHETHER CONTOUR EXITS SIDE OR TOP
```

```
      IF((CP3.GT.VALINC .AND. CP1.LT.VALINC) .OR. (CP3.LT.VALINC
     1 .AND. CP1.GT.VALINC)) THEN
C
C       CONTOUR MUST EXIT BETWEEN NODES 1 AND 3
        NEXT=ISIDE
        ISTART=2
        IF(IEVEN.EQ.1) THEN
          NEXT=IUP
          ISTART=1
        END IF
        RATIO=(VALINC-CP1)/DELV31
        XPC(IPT)=X1+RATIO*DELX31
        YPC(IPT)=Y1+RATIO*DELY31
C
C       CONTOUR MUST EXIT BETWEEN NODES 2 AND 3
      ELSE
        NEXT=IUP
        ISTART=1
        IF(IEVEN.EQ.1) THEN
          NEXT=ISIDE
          ISTART=3
        END IF
        RATIO=(VALINC-CP2)/DELV32
        XPC(IPT)=X2+RATIO*DELX32
        YPC(IPT)=Y2+RATIO*DELY32
      END IF
C
C       CHECK FOR CONTOUR ENTERING ON SIDE 2-3
     ELSE IF(ISTART.EQ.2) THEN
C
C       CHECK WHETHER CONTOUR EXITS BOTTOM OR SIDE 1-3
      IF((CP3.GT.VALINC .AND. CP1.LT.VALINC) .OR. (CP3.LT.VALINC
     1 .AND. CP1.GT.VALINC)) THEN
C
C       CONTOUR MUST EXIT BETWEEN NODES 1 AND 3
        NEXT=ISIDE
        ISTART=2
        IF(IEVEN.EQ.1) THEN
          NEXT=IUP
          ISTART=1
        END IF
        RATIO=(VALINC-CP1)/DELV31
        XPC(IPT)=X1+RATIO*DELX31
        YPC(IPT)=Y1+RATIO*DELY31
C
C       CONTOUR MUST EXIT BETWEEN NODES 1 AND 2
      ELSE
        NEXT=IDN
        ISTART=3
        IF(IEVEN.EQ.1) ISTART=2
        RATIO=(VALINC-CP1)/DELV21
        XPC(IPT)=X1+RATIO*DELX21
        YPC(IPT)=Y1+RATIO*DELY21
```

```
          END IF
C
C         CHECK FOR START OF CONTOUR SEGMENT ALONG SIDE 3-1
        ELSE IF(ISTART.EQ.3) THEN
C
C         CHECK WHETHER CONTOUR EXITS BOTTOM OR SIDE 2-3
          IF((CP2.GT.VALINC .AND. CP1.LT.VALINC) .OR. (CP2.LT.VALINC
     1    .AND. CP1.GT.VALINC)) THEN
C
C         CONTOUR MUST EXIT BETWEEN NODES 1 AND 2
            NEXT=IDN
            ISTART=3
            IF(IEVEN.EQ.1) ISTART=2
            RATIO=(VALINC-CP1)/DELV21
            XPC(IPT)=X1+RATIO*DELX21
            YPC(IPT)=Y1+RATIO*DELY21
C
C           CONTOUR MUST EXIT BETWEEN NODES 2 AND 3
          ELSE
            NEXT=IUP
            ISTART=1
            IF(IEVEN.EQ.1) THEN
              NEXT=ISIDE
              ISTART=3
            END IF
            RATIO=(VALINC-CP2)/DELV32
            XPC(IPT)=X2+RATIO*DELX32
            YPC(IPT)=Y2+RATIO*DELY32
          END IF
        END IF
C         CHECK IF CONTOUR LINE SEGMENT HAS ENDED
60      IF(NEXT.EQ.-1) GO TO 70
C         CHECK IF CONTOUR LINE SEGMENT HAS LEFT BOUNDARY
        IF(NEXT.LT.1 .OR. NEXT.GT.NTR) GO TO 80
        IF(MOD(NEXT,NY2).EQ.0 .AND. ISTART.EQ.3) GO TO 80
        IF(MOD((NEXT-1),NY2).EQ.0 .AND. ISTART.EQ.1) GO TO 80
C         CHECK FOR END OF CLOSED CONTOUR LOOP
        IF(IFLAG(NEXT).EQ.0) GO TO 70
C
C         OTERWISE, CONTINUE SEGMENT, OR BREAK AFTER 'NUM' SEGMENTS
        IF(IPT.NE.NUM) GO TO 50
        ICHK=1
C
C         BLANK OUT SPACE AT END OF SEGMENT TO WRITE LABEL
80      IF(IPT.LT.NUM) GOTO 70
        XPT=XPC(IPT)
        YPT=YPC(IPT)
        IP1=IPT
C         CHECK IF ENOUGH SPACE IS CREATED BY BLANKING OUT ONE POINT
90      IP1=IP1-1
        IF(IP1.LE.1) GO TO 100
        XP1=XPC(IP1)
        YP1=YPC(IP1)
```

```
      DELX=(XPT-XP1)/XSCLP
      DELY=(YPT-YP1)/YSCLP
      XLEN=SQRT(DELX*DELX+DELY*DELY)
C       IF NOT, DROP ANOTHER POINT ON CURVE
      IF(XLEN.LT.SPC1) GO TO 90
C       MAKE SURE LABELS ARE RIGHT-SIDE UP
100     OFSET=(XLEN-SPC1)/2.0
      ANG=90.
      IF(DELY.LT.0.0) ANG=270.
      IF(DELX.NE.0.0) ANG=ATAN2(DELY,DELX)*RAD
      IF (ABS(ANG).LE.90) THEN
        CALL ANGLE(ANG)
        XP1=XP1+(OFSET*COS(ANG/RAD)+HITE*SIN(ANG/RAD)/2.0)*XSCLP
        YP1=YP1+(OFSET*SIN(ANG/RAD)-HITE*COS(ANG/RAD)/2.0)*YSCLP
        CALL RLREAL(VALINC,IPL,XP1,YP1)
      ELSE
        ANG=ANG-180.
        CALL ANGLE(ANG)
        XPT=XPT+(OFSET*COS(ANG/RAD)+HITE*SIN(ANG/RAD)/2.0)*XSCLP
        YPT=YPT+(OFSET*SIN(ANG/RAD)-HITE*COS(ANG/RAD)/2.0)*YSCLP
        CALL RLREAL(VALINC,IPL,XPT,YPT)
      END IF
      CALL RESET ('ANGLE')
      IPT=IP1
C
C       DRAW CONTOUR SEGMENT
70    CALL CURVE(XPC,YPC,IPT,0)
C       EITHER CONTINUE CONTOUR SEGMENT WHERE IT LEFT OFF
      IF (ICHK.EQ.1) THEN
        ICHK=0
        ISTART=0
        IPT=1
        GO TO 50
      END IF
C       OR START SEARCH FOR NEXT SEGMENT
40    CONTINUE
20    CONTINUE
      CALL RESET('HEIGHT')
      CALL RESET('NUMODE')
      RETURN
      END
```

```
          ************************************************************
          *                                                          *
          *                   DATA FILE GLQ.PTS                      *
          *                                                          *
          ************************************************************

0.33998104358485D+000.86113631159405D+00
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0.99549645448109D+000.99658260202338D+000.99751925275672D+000.99830626647300D+00
```

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0.99894352584341D+000.99943093746626D+000.99976843740926D+000.99995605001899D+00   89
0.12247671640290D-010.12245834369748D-010.12242160104273D-010.12236649395040D-01   90
0.12229303068710D-010.12220122227304D-010.12209108248037D-010.12196262783115D-01   91
0.12181587759482D-010.12165085378535D-010.12146758115794D-010.12126608720527D-01   92
0.12104640215340D-010.12080855895724D-010.12055259329560D-010.12027854356582D-01   93
0.11998645087806D-010.11967635904906D-010.11934831459564D-010.11900236672766D-01   94
0.11863856734071D-010.11825697100824D-010.11785763497343D-010.11744061914061D-01   95
0.11700598606621D-010.11655380094945D-010.11608413162253D-010.11559704854044D-01   96
0.11509262477039D-010.11457093598091D-010.11403206043039D-010.11347607895545D-01   97
0.11290307495875D-010.11231313439650D-010.11170634576553D-010.11108280009010D-01   98
0.11044259090814D-010.10978581425729D-010.10911256866049D-010.10842295511115D-01   99
0.10771707705805D-010.10699504038980D-010.10625695341897D-010.10550292686581D-01  100
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0.72536389258339D-020.71322239610754D-020.70097390929698D-020.68862026954463D-02  109
0.67616333001738D-020.66360495937811D-020.65094704150536D-020.63819147521079D-02  110
0.62534017395424D-020.61239506555679D-020.59935809191153D-020.58623120869226D-02  111
0.57301638506014D-020.55971560336829D-020.54633085886443D-020.53286415939159D-02  112
0.51931752508693D-020.50569298807868D-020.49199259218138D-020.47821839258926D-02  113
0.46437245556800D-020.45045685814479D-020.43647368779680D-020.42242504213815D-02  114
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0.11608435575677D-020.10114243932084D-020.86185370142008D-030.71215416347332D-03  120
0.56234895403141D-030.41246325442617D-030.26253494429644D-030.11278901782227D-03  121
0.33998104358485D+000.86113631159405D+00                                           122
0.65214515486254D+000.34785484513745D+00                                           123
0.76526521133497D-010.22778585114164D+000.37370608871542D+000.51086700195082D+00  124
0.63605368072651D+000.74633190646015D+000.83911697182221D+000.91223442825132D+00  125
0.96397192727791D+000.99312859918509D+00                                           126
0.15275338713072D+000.14917298647260D+000.14209610931838D+000.13168863844917D+00  127
0.11819453196152D+000.10193011981724D+000.83276741576705D-010.62672048334108D-01  128
0.40601429800387D-010.17614007139152D-01                                           129
0.25959772301248D-010.77809333949536D-010.12944913539694D+000.18073996487342D+00  130
0.23154355137603D+000.28172293742326D+000.33114284826845D+000.37967005657680D+00  131
0.42717374158308D+000.47352584176171D+000.51860140005856D+000.56227890075394D+00  132
0.60444059704851D+000.64497282848947D+000.68376632738135D+000.72071651335573D+00  133
0.75572377530658D+000.78869373993226D+000.81953752616214D+000.84817198478593D+00  134
0.87451992264690D+000.89851031081004D+000.92007847617762D+000.93916627611642D+00  135
0.95572225583999D+000.96970178876505D+000.98106720175259D+000.98978789522222D+00  136
0.99584052511884D+000.99921012322744D+00                                           137
0.51907877631221D-010.51767943174910D-010.51488451500981D-010.51070156069855D-01  138
0.50514184532509D-010.49822035690550D-010.48995575455757D-010.48037031819971D-01  139
0.46948988848912D-010.45734379716114D-010.44396478795787D-010.42938892835935D-01  140
0.41365551235585D-010.39680695452381D-010.37888867569243D-010.35994898051084D-01  141
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0.13487007296848D+000.16459270634967D+000.19416659381859D+000.22356500882721D+00  147
0.25276138340572D+000.28172933217251D+000.31044267617922D+000.33887546656923D+00  148
0.36700200802816D+000.39479688200531D+000.42223496968490D+000.44929147468653D+00  149
0.47594194547414D+000.50216229745345D+000.52792883473767D+000.55321827156203D+00  150
0.57800775332758D+000.60227487725540D+000.62599771263251D+000.64915482063112D+00  151
0.67172527368334D+000.69368867439393D+000.71502517397392D+000.73571549017836D+00  152
0.75574092473217D+000.77508338022830D+000.79372537648276D+000.81165006633204D+00  153
0.82884125085838D+000.84528339402936D+000.86096163673850D+000.87586181023427D+00  154
0.88997044892541D+000.90327480255122D+000.91576284770591D+000.92742329870708D+00  155
0.93824561779910D+000.94822002468348D+000.95733750537007D+000.96558982034615D+00  156
0.97296991206736D+000.97946991179059D+000.98508514581446D+000.98981014133672D+00  157
0.99364063268970D+000.99657317140603D+000.99860516265198D+000.99973522187609D+00  158
0.30059347260915D-010.30032181992593D-010.29977876005781D-010.29896478377947D-01  159
0.29788062669856D-010.29652726859082D-010.29490593251467D-010.29301808370591D-01  160
0.29086542825356D-010.28844991155801D-010.28577371657294D-010.28283926183256D-01  161
0.27964919926590D-010.27620641180020D-010.27251401075562D-010.26857533303340D-01  162
0.26439393810028D-010.25997360477175D-010.25531832779705D-010.25043231424904D-01  163
0.24531997972222D-010.23998594434228D-010.23443502859080D-010.22867224894895D-01  164
0.22270281336402D-010.21653211654298D-010.21016573507721D-010.20360942240297D-01  165
0.19686910360204D-010.18995087004733D-010.18286097389822D-010.17560582245083D-01  166
0.16819197234815D-010.16062612365553D-010.15291511380693D-010.14506591142768D-01  167
0.13708561003966D-010.12898142165527D-010.12076067026716D-010.11243078524165D-01  168
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0.33820208186870D-020.24826218021768D-020.15810952911948D-020.67947618248455D-03  171
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0.55087655694634D-010.67312521165716D-010.79527289100232D-010.91730127163519D-01  173
0.10391920481051D+000.11609269356033D+000.12824876727061D+000.14038560241137D+00  174
0.15250137833866D+000.16459427756755D+000.17666248604490D+000.18870419342139D+00  175
0.20071759332312D+000.21270088362263D+000.22465226670913D+000.23656994975828D+00  176
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0.24845214500106D+000.26029706999194D+000.27210294787633D+000.28386800765708 0+00   177
0.29559048446013D+000.30726861979932D+000.31890066184010D+000.33048486566242D+00   178
0.34201949352237D+000.35350281511297D+000.36493310782365D+000.37630865699871D+00   179
0.38762775619452D+000.39888870743546D+000.41008982146872D+000.42122941801762D+00   180
0.43230582603374D+000.44331738394753D+000.45426243991759D+000.46513935207848D+00   181
0.47594648878698D+000.48668222886689D+000.49734496185218D+000.50793308822861D+00   182
0.51844501967367D+000.52887917929482D+000.53923400186606D+000.54950793406272D+00   183
0.55969943469448D+000.56980697493657D+000.57982903855908D+000.58976412215445D+00   184
0.59961073536297D+000.60936740109633D+000.61903265575926D+000.62860504946901D+00   185
0.63808314627290D+000.64746552436372D+000.65675077629297D+000.66593750918204D+00   186
0.67502434493116D+000.68400992042607D+000.69289288774257D+000.70167191434868D+00   187
0.71034568330454D+000.71891289345997D+000.72737225964965D+000.73572251288591D+00   188
0.74396240054911D+000.75209068657549D+000.76010615164265D+000.76800759335244D+00   189
0.77579382641132D+000.78346368280818D+000.79101601198954D+000.79844968103217D+00   190
0.80576357481299D+000.81295659617643D+000.82002766609892D+000.82697572385081 0+00   191
0.83379972715550D+000.84049865234576D+000.84707149451729D+000.85351726767950D+00   192
0.85983500490337D+000.86602375846655D+000.87208259999549D+000.87801062060471D+00   193
0.88380693103316D+000.88947066177761D+000.89500096322308D+000.90039700577030 0+00   194
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0.95798769241117D+000.96142848853073D+000.96472506097570D+000.96787691522849D+00   198
0.97088357848074D+000.97374459970437D+000.97645954971923D+000.97902802125762D+00   199
0.98144962902546D+000.98372400976031D+000.98585082228612D+000.98782974756486D+00   200
0.98966048874506D+000.99134277120758D+000.99287634260882D+000.99426097292240 0+00   201
0.99549645448109D+000.99658260202338D+000.99751925275672D+000.99830626647300D+00   202
0.99894352584341D+000.99943093746626D+000.99976843740926D+000.99995605001899D+00   203
0.12247671640290D-010.12245834369748D-010.12242160104273D-010.12236649395040D-01   204
0.12229303068710D-010.12220122227304D-010.12209108248037D-010.12196262783115D-01   205
0.12181587759482D-010.12165085378535D-010.12146758115794D-010.12126608720527D-01   206
0.12104640215340D-010.12080855895724D-010.12055259329560D-010.12027854356582D-01   207
0.11998645087806D-010.11967635904906D-010.11934831459564D-010.11900236672766D-01   208
0.11863856734071D-010.11825697100824D-010.11785763497343D-010.11744061914051D-01   209
0.11700598606621D-010.11655380094945D-010.11608413162253D-010.11559704854044D-01   210
0.11509262477039D-010.11457093598091D-010.11403206043039D-010.11347607895545D-01   211
0.11290307495875D-010.11231313439650D-010.11170634576553D-010.11108280009010D-01   212
0.11044259090814D-010.10978581425729D-010.10911256866049D-010.10842295511115D-01   213
0.10771707705805D-010.10699504038980D-010.10625695341897D-010.10550292686581D-01   214
0.10473307384170D-010.10394750983212D-010.10314635267934D-010.10232972256478D-01   215
0.10149774199095D-010.10065053576306D-010.99788230970349D-020.98910956966958D-02   216
0.98018845352573D-020.97112029952662D-020.96190646798406D-020.95254834106292D-02   217
0.94304732257376D-020.93340483776232D-020.92362233309562D-020.91370127604508D-02   218
0.90364315486628D-020.89344947837582D-020.88312177572487D-020.87266159616988D-02   219
0.86207050884010D-020.85135010250225D-020.84050198532215D-020.82952778462352D-02   220
0.81842914664382D-020.80720773628734D-020.79586523687543D-020.78440334989397D-02   221
0.77282379473815D-020.76112830845456D-020.74931864548058D-020.73739657738123D-02   222
0.72536389258339D-020.71322239610754D-020.70097390929698D-020.68862026954463D-02   223
0.67616333001738D-020.66360495937811D-020.65094704150536D-020.63819147521079D-02   224
0.62534017395424D-020.61239506555679D-020.59935809191153D-020.58623120869226D-02   225
0.57301638506014D-020.55971560336829D-020.54633085886443D-020.53286415939159D-02   226
0.51931752508693D-020.50569298807868D-020.49199259218138D-020.47821839258926D-02   227
0.46437245556800D-020.45045685814479D-020.43647368779680D-020.42242504213815D-02   228
0.40831302860526D-020.39413976414088D-020.37990737487662D-020.36561799581425D-02   229
0.35127377050563D-020.33687685073155D-020.32242939617942D-020.30793357411993D-02   230
0.29339155908297D-020.27880553253277D-020.26417768254275D-020.24951020347037D-02   231
0.23480529563273D-020.22006516498399D-020.20529202279661D-020.19048808534997D-02   232
0.17565557363307D-020.16079671307493D-020.14591373333107D-020.13100886819025D-02   233
0.11608435575677D-020.10114243932084D-020.86185370142008D-030.71215416347332D-03   234
0.56234895403141D-030.41246325442617D-030.26253494429644D-030.11278901782227D-03   235
```

ATTACHMENT 4.--PROGRAM OUTPUT FOR SAMPLE PROBLEMS

Sample problem 1A
Sample problem 1B
Sample problem 2
Sample problem 3A
Sample problem 3B
Sample problem 4
Sample problem 5
Sample problem 6
Sample problem 7
Sample problem 8A
Sample problem 8B
Sample problem 9
Sample problem 10
Sample problem 11

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*****************************************************************************************************
*                 SAMPLE PROBLEM 1A.--SOLUTE TRANSPORT IN A FINITE-LENGTH SOIL                      *
*                       COLUMN WITH A FIRST-TYPE BOUNDARY CONDITION AT X=0                          *
*                                                                                                   *
*          MODEL PARAMETERS:  L=12 INCHES, V=0.6 INCH PER HOUR, D=0.6 IN**2 PER HOUR                *
*                          K1=0.0 PER HOUR, C0=1.0 MILLIGRAM PER LITER                              *
*                                                                                                   *
*                    PROGRAM RUN ON TUESDAY, OCTOBER 20, 1987, AT 13:54:41                          *
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ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL ADVECTIVE-DISPERSIVE
SOLUTE-TRANSPORT EQUATION FOR A SYSTEM OF FINITE LENGTH

INPUT DATA

FIRST-TYPE BOUNDARY CONDITION AT X = 0.0

NUMBER OF X-COORDINATES (NX) = 25
NUMBER OF TIME VALUES (NT) = 5
NUMBER OF ROOTS USED IN INFINITE SERIES SUMMATION (NROOT) = 50

SOLUTE CONCENTRATION ON MODEL BOUNDARY) (C0) = 1.000000E+00 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 6.000000E-01 INCHES PER HOUR
DISPERSION IN THE X-DIRECTION (DX) = 6.000000E-01 IN**2 PER HOUR
FIRST-ORDER SOLUTE-DECAY RATE (DK) = 0.000000E-01 PER HOUR
LENGTH OF FINITE FLOW SYSTEM (XL) = 1.200000E+01 INCHES
PLOT SCALING FACTOR (XSCLP) = 1.200000E+00

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN INCHES

0.0000	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000
8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000	11.5000
12.0000							

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN HOURS

2.5000	5.0000	10.0000	15.0000	20.0000

SOLUTE CONCENTRATION AS A FUNCTION OF TIME

X-COORDINATE, IN INCHES	TIME VALUES, IN HOURS 2.5000	5.0000	10.0000	15.0000	20.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER				
0.0000	1.00000	1.00000	1.00000	1.00000	1.00000
0.5000	0.92277	0.97244	0.99378	0.99816	0.99939
1.0000	0.81598	0.93216	0.98440	0.99537	0.99845
1.5000	0.68658	0.87818	0.97112	0.99132	0.99708
2.0000	0.54642	0.81077	0.95319	0.98570	0.99515
2.5000	0.40929	0.73160	0.92996	0.97816	0.99251
3.0000	0.28739	0.64367	0.90091	0.96833	0.98900
3.5000	0.18856	0.55099	0.86574	0.95583	0.98441
4.0000	0.11530	0.45802	0.82441	0.94030	0.97855
4.5000	0.06558	0.36915	0.77717	0.92141	0.97119
5.0000	0.03463	0.28806	0.72461	0.89890	0.96211
5.5000	0.01696	0.21737	0.66761	0.87258	0.95108
6.0000	0.00769	0.15846	0.60731	0.84236	0.93788
6.5000	0.00323	0.11150	0.54507	0.80827	0.92232
7.0000	0.00125	0.07567	0.48231	0.77049	0.90426
7.5000	0.00045	0.04950	0.42052	0.72933	0.88361
8.0000	0.00015	0.03119	0.36105	0.68526	0.86036
8.5000	0.00005	0.01893	0.30514	0.63891	0.83464
9.0000	0.00001	0.01106	0.25377	0.59112	0.80673
9.5000	0.00000	0.00621	0.20771	0.54294	0.77717
10.0000	0.00000	0.00336	0.16752	0.49577	0.74689
10.5000	0.00000	0.00175	0.13367	0.45152	0.71732
11.0000	0.00000	0.00088	0.10681	0.41301	0.69072
11.5000	0.00000	0.00045	0.08826	0.38453	0.67059
12.0000	0.00000	0.00031	0.08096	0.37289	0.66227

```
*********************************************************************************************
*              SAMPLE PROBLEM 1B.--SOLUTE TRANSPORT IN A FINITE-LENGTH SOIL                 *
*                 COLUMN WITH A FIRST-TYPE BOUNDARY CONDITION AT X=0                        *
*                                                                                           *
*    MODEL PARAMETERS:  L=12 INCHES, V=0.072 INCH PER HOUR, D=0.072 IN**2 PER HOUR          *
*                   K1=0.0 PER HOUR, C0=1.0 MILLIGRAM PER LITER                             *
*                                                                                           *
*                      SOLUTE IS SUBJECT TO LINEAR ADSORPTION                               *
*                                                                                           *
*             PROGRAM RUN ON TUESDAY, OCTOBER 20, 1987, AT 13:58:11                         *
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ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL ADVECTIVE-DISPERSIVE
SOLUTE-TRANSPORT EQUATION FOR A SYSTEM OF FINITE LENGTH

INPUT DATA

FIRST-TYPE BOUNDARY CONDITION AT X = 0.0

NUMBER OF X-COORDINATES (NX) = 25
NUMBER OF TIME VALUES (NT) = 4
NUMBER OF ROOTS USED IN INFINITE SERIES SUMMATION (NROOT) = 50

SOLUTE CONCENTRATION ON MODEL BOUNDARY) (C0) = 1.000000E+00 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 7.200000E-01 INCHES PER HOUR
DISPERSION IN THE X-DIRECTION (DX) = 7.200000E-02 IN**2 PER HOUR
FIRST-ORDER SOLUTE-DECAY RATE (DK) = 0.000000E-01 PER HOUR
LENGTH OF FINITE FLOW SYSTEM (XL) = 1.200000E+01 INCHES
PLOT SCALING FACTOR (XSCLP) = 1.200000E+00

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN INCHES

0.0000	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000
8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000	11.5000
12.0000							

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN HOURS

20.0000 50.0000 100.0000 150.0000

SOLUTE CONCENTRATION AS A FUNCTION OF TIME

X-COORDINATE, IN INCHES	TIME VALUES, IN HOURS 20.0000	50.0000	100.0000	150.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER			
0.0000	1.00000	1.00000	1.00000	1.00000
0.5000	0.91872	0.98031	0.99626	0.99906
1.0000	0.80683	0.95124	0.99059	0.99762
1.5000	0.67234	0.91158	0.98247	0.99553
2.0000	0.52831	0.86079	0.97136	0.99259
2.5000	0.38947	0.79921	0.95671	0.98861
3.0000	0.26826	0.72813	0.93801	0.98333
3.5000	0.17207	0.64972	0.91482	0.97651
4.0000	0.10251	0.56689	0.88680	0.96788
4.5000	0.05661	0.48292	0.85378	0.95715
5.0000	0.02893	0.40114	0.81576	0.94407
5.5000	0.01366	0.32453	0.77296	0.92837
6.0000	0.00595	0.25546	0.72580	0.90985
6.5000	0.00239	0.19550	0.67491	0.88833
7.0000	0.00089	0.14534	0.62113	0.86372
7.5000	0.00030	0.10490	0.56541	0.83602
8.0000	0.00010	0.07346	0.50885	0.80533
8.5000	0.00003	0.04989	0.45256	0.77192
9.0000	0.00001	0.03285	0.39771	0.73624
9.5000	0.00000	0.02096	0.34546	0.69904
10.0000	0.00000	0.01296	0.29702	0.66144
10.5000	0.00000	0.00778	0.25383	0.62517
11.0000	0.00000	0.00457	0.21779	0.59286
11.5000	0.00000	0.00277	0.19194	0.56857
12.0000	0.00000	0.00215	0.18156	0.55857

```
************************************************************************************************
*                SAMPLE PROBLEM 2.--SOLUTE TRANSPORT IN A FINITE-LENGTH SOIL                   *
*                    COLUMN WITH A THIRD-TYPE BOUNDARY CONDITION AT X=0                        *
*                                                                                              *
*        MODEL PARAMETERS:  L=12 INCHES, V=0.6 INCH PER HOUR, D=0.6 IN**2 PER HOUR             *
*                       K1=0.0 PER HOUR, C0=1.0 MILLIGRAM PER LITER                            *
*                                                                                              *
*                  PROGRAM RUN ON TUESDAY, OCTOBER 20, 1987, AT 14:00:17                       *
************************************************************************************************
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ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL ADVECTIVE-DISPERSIVE
SOLUTE-TRANSPORT EQUATION FOR A SYSTEM OF FINITE LENGTH

INPUT DATA

THIRD-TYPE BOUNDARY CONDITION AT X = 0.0

NUMBER OF X-COORDINATES (NX) = 25
NUMBER OF TIME VALUES (NT) = 5
NUMBER OF ROOTS USED IN INFINITE SERIES SUMMATION (NROOT) = 50

SOLUTE CONCENTRATION ON MODEL BOUNDARY) (C0) = 1.000000E+00 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 6.000000E-01 INCHES PER HOUR
DISPERSION IN THE X-DIRECTION (DX) = 6.000000E-01 IN**2 PER HOUR
FIRST-ORDER SOLUTE-DECAY RATE (DK) = 0.000000E-01 PER HOUR
LENGTH OF FINITE FLOW SYSTEM (XL) = 1.200000E+01 INCHES
PLOT SCALING FACTOR (XSCLP) = 1.200000E+00

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN INCHES

0.0000	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000
8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000	11.5000
12.0000							

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN HOURS

2.5000	5.0000	10.0000	15.0000	20.0000

SOLUTE CONCENTRATION AS A FUNCTION OF TIME

X-COORDINATE, IN INCHES	TIME VALUES, IN HOURS 2.5000	5.0000	10.0000	15.0000	20.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER				
0.0000	0.79858	0.90992	0.97530	0.99197	0.99716
0.5000	0.68921	0.85904	0.96098	0.98727	0.99549
1.0000	0.56799	0.79673	0.94230	0.98097	0.99322
1.5000	0.44466	0.72419	0.91871	0.97276	0.99021
2.0000	0.32919	0.64364	0.88977	0.96231	0.98629
2.5000	0.22958	0.55821	0.85524	0.94926	0.98128
3.0000	0.15033	0.47151	0.81509	0.93331	0.97499
3.5000	0.09217	0.38726	0.76955	0.91415	0.96720
4.0000	0.05280	0.30880	0.71911	0.89156	0.95771
4.5000	0.02820	0.23875	0.66455	0.86537	0.94630
5.0000	0.01402	0.17878	0.60686	0.83551	0.93276
5.5000	0.00648	0.12953	0.54722	0.80201	0.91692
6.0000	0.00278	0.09072	0.48691	0.76503	0.89862
6.5000	0.00111	0.06137	0.42724	0.72482	0.87775
7.0000	0.00041	0.04008	0.36949	0.68179	0.85425
7.5000	0.00014	0.02525	0.31477	0.63644	0.82814
8.0000	0.00004	0.01534	0.26404	0.58940	0.79952
8.5000	0.00001	0.00898	0.21800	0.54138	0.76864
9.0000	0.00000	0.00507	0.17710	0.49322	0.73590
9.5000	0.00000	0.00275	0.14160	0.44591	0.70194
10.0000	0.00000	0.00144	0.11154	0.40065	0.66775
10.5000	0.00000	0.00072	0.08691	0.35904	0.63487
11.0000	0.00000	0.00035	0.06782	0.32340	0.60563
11.5000	0.00000	0.00018	0.05487	0.29733	0.58368
12.0000	0.00000	0.00012	0.04982	0.28674	0.57463

```
*****************************************************************************************************
*              SAMPLE PROBLEM 3A.--SOLUTE TRANSPORT IN A SEMI-INFINITE LENGTH SOIL                  *
*                   COLUMN WITH A FIRST-TYPE BOUNDARY CONDITION AT X=0                              *
*                                                                                                   *
*                MODEL PARAMETERS:  V=0.6 INCH PER HOUR, D=0.6 IN**2 PER HOUR,                      *
*                        K1=0.0 PER HOUR, C0=1.0 MILLIGRAM PER LITER                                *
*                                                                                                   *
*                    PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 14:43:22                           *
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ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL ADVECTIVE-DISPERSIVE
SOLUTE-TRANSPORT EQUATION FOR A SYSTEM OF SEMI-INFINITE LENGTH

INPUT DATA

FIRST-TYPE BOUNDARY CONDITION AT X = 0.0

NUMBER OF X-COORDINATES (NX) = 25
NUMBER OF TIME VALUES (NT) = 5

SOLUTE CONCENTRATION ON MODEL BOUNDARY) (C0) = 1.000000E+00 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 6.000000E-01 INCHES PER HOUR
DISPERSION IN THE X-DIRECTION (DX) = 6.000000E-01 IN**2 PER HOUR
FIRST-ORDER SOLUTE-DECAY RATE (DK) = 0.000000E-01 PER HOUR
PLOT SCALING FACTOR (XSCLP) = 1.200000E+00

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN INCHES

0.0000	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000
8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000	11.5000
12.0000							

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN HOURS

2.5000 5.0000 10.0000 15.0000 20.0000

SOLUTE CONCENTRATION AS A FUNCTION OF TIME

X-COORDINATE, IN INCHES	TIME VALUES, IN HOURS 2.5000	5.0000	10.0000	15.0000	20.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER				
0.0000	1.00000	1.00000	1.00000	1.00000	1.00000
0.5000	0.92277	0.97244	0.99378	0.99816	0.99939
1.0000	0.81598	0.93216	0.98440	0.99537	0.99845
1.5000	0.68658	0.87818	0.97112	0.99132	0.99708
2.0000	0.54642	0.81077	0.95319	0.98570	0.99515
2.5000	0.40929	0.73160	0.92996	0.97816	0.99251
3.0000	0.28739	0.64367	0.90091	0.96833	0.98899
3.5000	0.18856	0.55099	0.86574	0.95583	0.98441
4.0000	0.11530	0.45802	0.82441	0.94030	0.97854
4.5000	0.06558	0.36915	0.77717	0.92141	0.97118
5.0000	0.03463	0.28806	0.72461	0.89890	0.96208
5.5000	0.01696	0.21737	0.66761	0.87257	0.95103
6.0000	0.00769	0.15846	0.60731	0.84234	0.93779
6.5000	0.00323	0.11150	0.54506	0.80823	0.92215
7.0000	0.00125	0.07567	0.48231	0.77039	0.90395
7.5000	0.00045	0.04950	0.42051	0.72913	0.88303
8.0000	0.00015	0.03119	0.36103	0.68485	0.85930
8.5000	0.00005	0.01893	0.30508	0.63809	0.83274
9.0000	0.00001	0.01106	0.25362	0.58950	0.80336
9.5000	0.00000	0.00621	0.20733	0.53978	0.77126
10.0000	0.00000	0.00336	0.16661	0.48968	0.73663
10.5000	0.00000	0.00175	0.13157	0.43997	0.69968
11.0000	0.00000	0.00087	0.10208	0.39138	0.66074
11.5000	0.00000	0.00042	0.07778	0.34460	0.62017
12.0000	0.00000	0.00019	0.05819	0.30022	0.57840

```
*******************************************************************************************
*          SAMPLE PROBLEM 3B:--SOLUTE TRANSPORT IN A SEMI-INFINITE LENGTH SOIL           *
*                    COLUMN WITH A FIRST-TYPE BOUNDARY CONDITION AT X=0                   *
*                                                                                         *
*          MODEL PARAMETERS:  V=0.072 INCH PER HOUR, D=0.072 IN**2 PER HOUR,             *
*                      K1=0.0038 PER HOUR, C0=1.0 MILLIGRAM PER LITER                     *
*                                                                                         *
*              SOLUTE IS SUBJECT TO FIRST-ORDER DECAY AND LINEAR ADSORPTION               *
*                                                                                         *
*                  PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 14:43:37                   *
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ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL ADVECTIVE-DISPERSIVE
SOLUTE-TRANSPORT EQUATION FOR A SYSTEM OF SEMI-INFINITE LENGTH

INPUT DATA

FIRST-TYPE BOUNDARY CONDITION AT X = 0.0

NUMBER OF X-COORDINATES (NX) = 25
NUMBER OF TIME VALUES (NT) = 4

SOLUTE CONCENTRATION ON MODEL BOUNDARY) (C0) = 1.000000E+00 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 7.200000E-02 INCHES PER HOUR
DISPERSION IN THE X-DIRECTION (DX) = 7.200000E-02 IN**2 PER HOUR
FIRST-ORDER SOLUTE-DECAY RATE (DK) = 3.800000E-03 PER HOUR
PLOT SCALING FACTOR (XSCLP) = 1.200000E+00

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN INCHES

0.0000	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000
8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000	11.5000
12.0000							

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN HOURS

20.0000 50.0000 100.0000 150.0000

SOLUTE CONCENTRATION AS A FUNCTION OF TIME

X-COORDINATE, IN INCHES	TIME VALUES, IN HOURS 20.0000	50.0000	100.0000	150.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER			
0.0000	1.00000	1.00000	1.00000	1.00000
0.5000	0.90569	0.96058	0.97294	0.97473
1.0000	0.78624	0.91485	0.94534	0.94982
1.5000	0.64915	0.86197	0.91688	0.92520
2.0000	0.50636	0.80166	0.88723	0.90075
2.5000	0.37113	0.73431	0.85606	0.87636
3.0000	0.25445	0.66104	0.82307	0.85191
3.5000	0.16261	0.58369	0.78802	0.82725
4.0000	0.09660	0.50462	0.75072	0.80225
4.5000	0.05322	0.42643	0.71112	0.77677
5.0000	0.02714	0.35174	0.66926	0.75068
5.5000	0.01280	0.28284	0.62532	0.72384
6.0000	0.00557	0.22146	0.57962	0.69616
6.5000	0.00224	0.16869	0.53261	0.66757
7.0000	0.00083	0.12490	0.48483	0.63801
7.5000	0.00028	0.08982	0.43692	0.60748
8.0000	0.00009	0.06271	0.38958	0.57601
8.5000	0.00003	0.04247	0.34348	0.54369
9.0000	0.00001	0.02789	0.29931	0.51065
9.5000	0.00000	0.01776	0.25764	0.47705
10.0000	0.00000	0.01095	0.21898	0.44311
10.5000	0.00000	0.00655	0.18371	0.40908
11.0000	0.00000	0.00379	0.15206	0.37522
11.5000	0.00000	0.00212	0.12414	0.34184
12.0000	0.00000	0.00115	0.09993	0.30920

```
*****************************************************************************************
*        SAMPLE PROBLEM 4.--SOLUTE TRANSPORT IN A SEMI-INFINITE LENGTH SOIL             *
*              COLUMN WITH A THIRD-TYPE BOUNDARY CONDITION AT X=0                        *
*                                                                                        *
*         MODEL PARAMETERS:  V=0.6 INCH PER HOUR, D=0.6 IN**2 PER HOUR,                  *
*                K1=0.0 PER HOUR, C0=1.0 MILLIGRAM PER LITER                             *
*                                                                                        *
*              PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 14:43:49                      *
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ANALYTICAL SOLUTION TO THE ONE-DIMENSIONAL ADVECTIVE-DISPERSIVE
SOLUTE-TRANSPORT EQUATION FOR A SYSTEM OF SEMI-INFINITE LENGTH

INPUT DATA

FIRST-TYPE BOUNDARY CONDITION AT X = 0.0

NUMBER OF X-COORDINATES (NX) = 25
NUMBER OF TIME VALUES (NT) = 5

SOLUTE CONCENTRATION ON MODEL BOUNDARY) (C0) = 1.000000E+00 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 6.000000E-01 INCHES PER HOUR
DISPERSION IN THE X-DIRECTION (DX) = 6.000000E-01 IN**2 PER HOUR
FIRST-ORDER SOLUTE-DECAY RATE (DK) = 0.000000E-01 PER HOUR
PLOT SCALING FACTOR (XSCLP) = 1.200000E+00

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN INCHES

0.0000	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000
4.0000	4.5000	5.0000	5.5000	6.0000	6.5000	7.0000	7.5000
8.0000	8.5000	9.0000	9.5000	10.0000	10.5000	11.0000	11.5000
12.0000							

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN HOURS

2.5000	5.0000	10.0000	15.0000	20.0000

SOLUTE CONCENTRATION AS A FUNCTION OF TIME

X-COORDINATE, IN INCHES	TIME VALUES, IN HOURS 2.5000	5.0000	10.0000	15.0000	20.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER				
0.0000	0.79858	0.90992	0.97530	0.99197	0.99716
0.5000	0.68921	0.85904	0.96098	0.98727	0.99549
1.0000	0.56799	0.79673	0.94230	0.98097	0.99322
1.5000	0.44466	0.72419	0.91871	0.97276	0.99021
2.0000	0.32919	0.64364	0.88977	0.96231	0.98629
2.5000	0.22958	0.55821	0.85524	0.94926	0.98128
3.0000	0.15033	0.47151	0.81509	0.93331	0.97498
3.5000	0.09217	0.38726	0.76955	0.91415	0.96720
4.0000	0.05280	0.30880	0.71911	0.89156	0.95770
4.5000	0.02820	0.23875	0.66455	0.86537	0.94629
5.0000	0.01402	0.17878	0.60686	0.83551	0.93274
5.5000	0.00648	0.12953	0.54722	0.80201	0.91688
6.0000	0.00278	0.09072	0.48691	0.76501	0.89855
6.5000	0.00111	0.06137	0.42724	0.72479	0.87761
7.0000	0.00041	0.04008	0.36949	0.68173	0.85399
7.5000	0.00014	0.02525	0.31477	0.63631	0.82766
8.0000	0.00004	0.01534	0.26403	0.58912	0.79865
8.5000	0.00001	0.00898	0.21796	0.54080	0.76705
9.0000	0.00000	0.00507	0.17702	0.49206	0.73302
9.5000	0.00000	0.00275	0.14138	0.44359	0.69680
10.0000	0.00000	0.00144	0.11102	0.39610	0.65867
10.5000	0.00000	0.00072	0.08568	0.35022	0.61897
11.0000	0.00000	0.00035	0.06498	0.30653	0.57809
11.5000	0.00000	0.00016	0.04840	0.26551	0.53646
12.0000	0.00000	0.00007	0.03542	0.22755	0.49452

```
***************************************************************************
*   SAMPLE PROBLEM 5.--SOLUTE TRANSPORT IN AN AQUIFER OF INFINITE        *
*            AREAL EXTENT WITH A CONTINUOUS POINT SOURCE                  *
*                                                                         *
*   MODEL PARAMETERS: V=2.0 FEET PER DAY, DX=60.0 FT**2 PER DAY,          *
*      QM=50.0 FT**2 PER DAY, CO=1000.0 MILLIGRAMS PER LITER              *
*                                                                         *
*        PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 13:06:17             *
***************************************************************************
```

ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE-TRANSPORT EQUATION FOR AN AQUIFER OF INFINITE AREAL EXTENT WITH A CONTINUOUS POINT SOURCE AT X = 0 AND Y = YC

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 26
NUMBER OF Y-COORDINATES (NY) = 21
NUMBER OF TIME VALUES (NT) = 2
NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = 104

SOLUTE CONCENTRATION IN INJECTED FLUID (C0) = 1.000000E+03 MILLIGRAMS PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 2.000000E+00 FEET PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 6.000000E+01 FT**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 1.200000E+01 FT**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER IS OF INFINITE AREAL EXTENT
CONTINUOUS POINT SOURCE IS LOCATED AT X = 0.000000E-01 FEET AND Y = 5.000000E+02 FEET
FLUID INJECTION RATE PER UNIT THICHKNESS OF AQUIFER (QM) = 5.000000E+01 FT**2 PER DAY

PLOT SCALING FACTOR FOR X (XSCLP) = 3.000000E+01
PLOT SCALING FACTOR FOR Y (YSCLP) = 3.000000E+01
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

-60.0000	-50.0000	-40.0000	-30.0000	-20.0000	-10.0000	10.0000	20.0000
30.0000	40.0000	50.0000	60.0000	70.0000	80.0000	90.0000	100.0000
110.0000	120.0000	130.0000	140.0000	150.0000	160.0000	170.0000	180.0000
190.0000	200.0000						

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

450.0000	455.0000	460.0000	465.0000	470.0000	475.0000	480.0000	485.0000
490.0000	495.0000	500.0000	505.0000	510.0000	515.0000	520.0000	525.0000
530.0000	535.0000	540.0000	545.0000	550.0000			

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS

25.0000 100.0000

SOLUTE CONCENTRATION AT TIME = 25.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET						
	450.0000	455.0000	460.0000	465.0000	470.0000	475.0000	480.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
-60.0000	0.76669	1.30334	2.13436	3.36337	5.09043	7.37647	10.18468
-50.0000	1.15536	1.98256	3.28385	5.24710	8.07655	11.94306	16.88386
-40.0000	1.67061	2.89186	4.84237	7.84393	12.28439	18.56731	26.97509
-30.0000	2.31428	4.03659	6.82489	11.19467	17.82358	27.54022	41.21752
-20.0000	3.06704	5.38132	9.16873	15.19368	24.53064	38.65681	59.53951
-10.0000	3.88398	6.84047	11.71295	19.54101	31.84967	50.89469	80.11494
10.0000	5.42054	9.54665	16.34674	27.27168	44.44979	71.02926	111.80940
20.0000	5.97378	10.48137	17.85824	29.59324	47.77916	75.29319	115.96713
30.0000	6.29088	10.97258	18.55198	30.43026	48.44952	74.86207	112.04084
40.0000	6.33774	10.97074	18.37034	29.75725	46.60288	70.43819	102.33453
50.0000	6.11705	10.49665	17.38630	27.78073	42.76119	63.23244	89.39142
60.0000	5.66509	9.63046	15.77088	24.85211	37.61345	54.50515	75.25518
70.0000	5.04202	8.48957	13.74552	21.37167	31.84182	45.31882	61.34321
80.0000	4.31885	7.20353	11.53637	17.71292	26.01858	36.45706	48.54373
90.0000	3.56506	5.89247	9.34016	14.17691	20.56372	28.43043	37.34576
100.0000	2.83919	4.65264	7.30515	10.97368	15.74224	21.51732	27.94931
110.0000	2.18354	3.54977	5.52540	8.22361	11.68373	15.81534	20.35221
120.0000	1.62296	2.61912	4.04489	5.97078	8.41198	11.29257	14.41923
130.0000	1.16656	1.86996	2.86757	4.20217	5.87708	7.83386	9.93758
140.0000	0.81129	1.29253	1.96954	2.86764	3.98515	5.27977	6.66057
150.0000	0.54610	0.86522	1.31094	1.89787	2.62282	3.45668	4.34016
160.0000	0.35590	0.56104	0.84575	1.21825	1.67542	2.19807	2.74872
170.0000	0.22461	0.35248	0.52892	0.75848	1.03866	1.35730	1.69142
180.0000	0.13730	0.21457	0.32067	0.45802	0.62485	0.81373	1.01097
190.0000	0.08129	0.12657	0.18847	0.26824	0.36473	0.47355	0.58678
200.0000	0.04663	0.07235	0.10738	0.15235	0.20654	0.26745	0.33064

X-COORDINATE, IN FEET	SOLUTE CONCENTRATION AT TIME = 25.0000 DAYS						
	Y-COORDINATE, IN FEET						
	485.0000	490.0000	495.0000	500.0000	505.0000	510.0000	515.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
-60.0000	13.30406	16.29009	18.50348	19.32850	18.50348	16.29009	13.30406
-50.0000	22.63445	28.42428	32.91679	34.63912	32.91679	28.42428	22.63445
-40.0000	37.36163	48.59217	57.95451	61.72005	57.95451	48.59217	37.36163
-30.0000	59.38493	81.08913	101.44444	110.40503	101.44444	81.08913	59.38493
-20.0000	89.56223	130.42813	177.15302	202.14826	177.15302	130.42813	89.56223
-10.0000	125.04770	195.15640	303.97931	400.01490	303.97931	195.15640	125.04770
10.0000	174.51813	272.36270	424.23730	558.26576	424.23730	272.36270	174.51813
20.0000	174.44341	254.03932	345.04697	393.73105	345.04697	254.03932	174.44341
30.0000	161.42497	220.42311	275.75457	300.11199	275.75457	220.42311	161.42497
40.0000	141.73761	184.34254	219.86016	234.14537	219.86016	184.34254	141.73761
50.0000	119.83788	150.49204	174.27760	183.39646	174.27760	150.49204	119.83788
60.0000	98.30441	120.36838	136.72322	142.81935	136.72322	120.36838	98.30441
70.0000	78.51359	94.36677	105.76504	109.93869	105.76504	94.36677	78.51359
80.0000	61.14038	72.46844	80.44277	83.32810	80.44277	72.46844	61.14038
90.0000	46.43950	54.45882	60.01899	62.01409	60.01899	54.45882	46.43950
100.0000	34.40041	40.00546	43.84841	45.21902	43.84841	40.00546	34.40041
110.0000	24.84227	28.69898	31.32090	32.25181	31.32090	28.69898	24.84227
120.0000	17.48075	20.08685	21.84700	22.46977	21.84700	20.08685	17.48075
130.0000	11.97971	13.70563	14.86531	15.27452	14.86531	13.70563	11.97971
140.0000	7.99150	9.10984	9.85818	10.12167	9.85818	9.10984	7.99150
150.0000	5.18676	5.89475	6.36693	6.53290	6.36693	5.89475	5.18676
160.0000	3.27380	3.71121	4.00212	4.10423	4.00212	3.71121	3.27380
170.0000	2.00871	2.27215	2.44697	2.50825	2.44697	2.27215	2.00871
180.0000	1.19763	1.35219	1.45455	1.49040	1.45455	1.35219	1.19763
190.0000	0.69362	0.78188	0.84023	0.86066	0.84023	0.78188	0.69362
200.0000	0.39010	0.43912	0.47149	0.48281	0.47149	0.43912	0.39010

X-COORDINATE, IN FEET	SOLUTE CONCENTRATION AT TIME = 25.0000 DAYS						
	Y-COORDINATE, IN FEET						
	520.0000	525.0000	530.0000	535.0000	540.0000	545.0000	550.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
-60.0000	10.18468	7.37647	5.09043	3.36337	2.13436	1.30334	0.76669
-50.0000	16.88386	11.94306	8.07655	5.24710	3.28385	1.98256	1.15536
-40.0000	26.97509	18.56731	12.28439	7.84393	4.84237	2.89186	1.67061
-30.0000	41.21752	27.54022	17.82358	11.19467	6.82489	4.03659	2.31428
-20.0000	59.53951	38.65681	24.53064	15.19368	9.16873	5.38132	3.06704
-10.0000	80.11494	50.89469	31.84967	19.54101	11.71295	6.84047	3.88398
10.0000	111.80940	71.02926	44.44979	27.27168	16.34674	9.54665	5.42054
20.0000	115.96713	75.29319	47.77916	29.59324	17.85824	10.48137	5.97378
30.0000	112.04084	74.86207	48.44952	30.43026	18.55198	10.97258	6.29088
40.0000	102.33453	70.43819	46.60288	29.75725	18.37034	10.97074	6.33774
50.0000	89.39142	63.23244	42.76119	27.78073	17.38630	10.49665	6.11705
60.0000	75.25518	54.50515	37.61345	24.85211	15.77088	9.63046	5.66509
70.0000	61.34321	45.31882	31.84182	21.37167	13.74552	8.48957	5.04202
80.0000	48.54373	36.45706	26.01858	17.71292	11.53637	7.20353	4.31885
90.0000	37.34576	28.43043	20.56372	14.17691	9.34016	5.89247	3.56506
100.0000	27.94931	21.51732	15.74224	10.97368	7.30515	4.65264	2.83919
110.0000	20.35221	15.81534	11.68373	8.22361	5.52540	3.54977	2.18354
120.0000	14.41923	11.29257	8.41198	5.97078	4.04489	2.61912	1.62296
130.0000	9.93758	7.83386	5.87708	4.20217	2.86757	1.86996	1.16656
140.0000	6.66057	5.27977	3.98515	2.86764	1.96954	1.29253	0.81129
150.0000	4.34016	3.45668	2.62282	1.89787	1.31094	0.86522	0.54610
160.0000	2.74872	2.19807	1.67542	1.21825	0.84575	0.56104	0.35590
170.0000	1.69142	1.35730	1.03866	0.75848	0.52892	0.35248	0.22461
180.0000	1.01097	0.81373	0.62485	0.45802	0.32067	0.21457	0.13730
190.0000	0.58678	0.47355	0.36473	0.26824	0.18847	0.12657	0.08129
200.0000	0.33064	0.26745	0.20654	0.15235	0.10738	0.07235	0.04663

SOLUTE CONCENTRATION AT TIME = 100.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 450.0000	455.0000	460.0000	465.0000	470.0000	475.0000	480.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
-60.0000	8.18811	10.36703	13.02748	16.22753	19.99683	24.30255	28.99636
-50.0000	10.76639	13.75207	17.46474	22.03273	27.56714	34.11397	41.56120
-40.0000	13.92227	17.92889	22.99846	29.37634	37.32958	47.09881	58.77108
-30.0000	17.67917	22.92825	29.67313	38.33290	49.43167	63.58999	81.43064
-20.0000	22.01602	28.70976	37.41686	48.78238	63.68958	83.35578	109.43514
-10.0000	26.85753	35.14568	46.01223	60.35179	79.45619	105.26367	140.83004
10.0000	37.48270	49.04974	64.21524	84.22770	110.89004	146.90729	196.54416
20.0000	42.88135	55.91898	72.87809	95.01510	124.05036	162.35488	213.15055
30.0000	48.05695	62.32543	80.65992	104.19962	134.36921	172.85551	221.35142
40.0000	52.81647	68.01624	87.24852	111.44406	141.61602	178.67724	222.95795
50.0000	57.00253	72.81021	92.46687	116.65205	145.95395	180.61606	220.04534
60.0000	60.50239	76.60259	96.26081	119.90615	147.75773	179.57292	214.25570
70.0000	63.24891	79.35689	98.66894	121.39337	147.46542	176.33642	206.70523
80.0000	65.21591	81.09004	99.78935	121.34430	145.49266	171.52139	198.10319
90.0000	66.41058	81.85632	99.75158	119.99388	142.19692	165.58154	188.88874
100.0000	66.86523	81.73331	98.69684	117.56087	137.87113	158.84469	179.33337
110.0000	66.62975	80.81118	96.76589	114.23970	132.75043	151.54787	169.60797
120.0000	65.76544	79.18528	94.09253	110.19956	127.02324	143.86535	159.82434
130.0000	64.34027	76.95136	90.80085	105.58669	120.84202	135.92913	150.06023
140.0000	62.42548	74.20281	87.00459	100.52788	114.33242	127.84302	140.37430
150.0000	60.09316	71.02904	82.80749	95.13395	107.60025	119.69217	130.81484
160.0000	57.41456	67.51476	78.30418	89.50267	100.73661	111.54913	121.42468
170.0000	54.45887	63.73950	73.58095	83.72115	93.82139	103.47773	112.24375
180.0000	51.29235	59.77744	68.71647	77.86743	86.92562	95.53531	103.31020
190.0000	47.97764	55.69727	63.78230	72.01163	80.11292	87.77390	94.66060
200.0000	44.57321	51.56197	58.84319	66.21662	73.44022	80.24074	86.32969

SOLUTE CONCENTRATION AT TIME = 100.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 485.0000	490.0000	495.0000	500.0000	505.0000	510.0000	515.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
-60.0000	33.74777	37.99784	41.01199	42.11132	41.01199	37.99784	33.74777
-50.0000	49.48401	56.95787	62.51791	64.60604	62.51791	56.95787	49.48401
-40.0000	71.98979	85.41774	96.17377	100.41700	96.17377	85.41774	71.98979
-30.0000	103.21300	127.72368	149.85968	159.43074	149.85968	127.72368	103.21300
-20.0000	143.97267	188.34516	237.29557	263.05412	237.29557	188.34516	143.97267
-10.0000	191.27817	265.67166	377.21240	474.16187	377.21240	265.67166	191.27817
10.0000	266.95019	370.77467	526.44232	661.74620	526.44232	370.77467	266.95019
20.0000	280.42047	366.84628	462.18866	512.35946	462.18866	366.84628	280.42047
30.0000	280.56203	347.18895	407.36085	433.37767	407.36085	347.18895	280.56203
40.0000	273.10536	324.04653	364.85134	380.94873	364.85134	324.04653	273.10536
50.0000	261.99259	301.56287	331.00043	342.05601	331.00043	301.56287	261.99259
60.0000	249.36418	280.76816	303.03991	311.16288	303.03991	280.76816	249.36418
70.0000	236.27211	261.71549	279.18450	285.43689	279.18450	261.71549	236.27211
80.0000	223.20440	244.19337	258.27781	263.25442	258.27781	244.19337	223.20440
90.0000	210.37306	227.95083	239.55022	243.61130	239.55022	227.95083	210.37306
100.0000	197.85954	212.76405	222.47601	225.85324	222.47601	212.76405	197.85954
110.0000	185.68664	198.45179	206.68906	209.53868	206.68906	198.45179	185.68664
120.0000	173.85371	184.87441	191.93159	194.36312	191.93159	184.87441	173.85371
130.0000	162.35387	171.92836	178.02181	180.11454	178.02181	171.92836	162.35387
140.0000	151.18202	159.53998	164.83260	166.64557	164.83260	159.53998	151.18202
150.0000	140.33818	147.65978	152.27703	153.85528	152.27703	147.65978	140.33818
160.0000	129.82839	136.25749	140.29796	141.67662	140.29796	136.25749	129.82839
170.0000	119.66433	125.31773	128.86042	130.06744	128.86042	125.31773	119.66433
180.0000	109.86229	114.83635	117.94568	119.00372	117.94568	114.83635	109.86229
190.0000	100.44177	104.81723	107.54664	108.47440	107.54664	104.81723	100.44177
200.0000	91.42403	95.26955	97.66405	98.47722	97.66405	95.26955	91.42403

SAMPLE PROBLEM 5.--SOLUTE TRANSPORT IN AN AQUIFER OF INFINITE AREAL EXTENT WITH A CONTINUOUS POINT SOURCE--CONTINUED

	SOLUTE CONCENTRATION AT TIME = 100.0000 DAYS						
X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET						
	520.0000	525.0000	530.0000	535.0000	540.0000	545.0000	550.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
-60.0000	28.99636	24.30255	19.99683	16.22753	13.02748	10.36703	8.18811
-50.0000	41.56120	34.11397	27.56714	22.03273	17.46474	13.75207	10.76639
-40.0000	58.77108	47.09881	37.32958	29.37634	22.99846	17.92889	13.92227
-30.0000	81.43064	63.58999	49.43167	38.33290	29.67313	22.92825	17.67917
-20.0000	109.43514	83.35578	63.68958	48.78238	37.41686	28.70976	22.01602
-10.0000	140.83004	105.26367	79.45619	60.35179	46.01223	35.14568	26.85753
10.0000	196.54416	146.90729	110.89004	84.22770	64.21524	49.04974	37.48270
20.0000	213.15055	162.35488	124.05036	95.01510	72.87809	55.91898	42.88135
30.0000	221.35142	172.85551	134.36921	104.19962	80.65992	62.32543	48.05695
40.0000	222.95795	178.67724	141.61602	111.44406	87.24852	68.01624	52.81647
50.0000	220.04534	180.61606	145.95395	116.65205	92.46687	72.81021	57.00253
60.0000	214.25570	179.57292	147.75773	119.90615	96.26081	76.60259	60.50239
70.0000	206.70523	176.33642	147.46542	121.39337	98.66894	79.35689	63.24891
80.0000	198.10319	171.52139	145.49266	121.34430	99.78935	81.09004	65.21591
90.0000	188.88874	165.58154	142.19692	119.99388	99.75158	81.85632	66.41058
100.0000	179.33337	158.84469	137.87113	117.56087	98.69684	81.73331	66.86523
110.0000	169.60797	151.54787	132.75043	114.23970	96.76589	80.81118	66.62975
120.0000	159.82434	143.86535	127.02324	110.19956	94.09253	79.18528	65.76544
130.0000	150.06023	135.92913	120.84202	105.58669	90.80085	76.95136	64.34027
140.0000	140.37430	127.84302	114.33242	100.52788	87.00459	74.20281	62.42548
150.0000	130.81484	119.69217	107.60025	95.13395	82.80749	71.02904	60.09316
160.0000	121.42468	111.54913	100.73661	89.50267	78.30418	67.51476	57.41456
170.0000	112.24375	103.47773	93.82139	83.72115	73.58095	63.73950	54.45887
180.0000	103.31020	95.53531	86.92562	77.86743	68.71647	59.77744	51.29235
190.0000	94.66060	87.77390	80.11292	72.01163	63.78230	55.69727	47.97764
200.0000	86.32969	80.24074	73.44022	66.21662	58.84319	51.56197	44.57321

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*  SAMPLE PROBLEM 6.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER   *
*          OF FINITE WIDTH WITH A CONTINUOUS "STRIP" SOURCE          *
*                                                                    *
* MODEL PARAMETERS: V=1.0 FEET PER DAY, DX=200.0 FT**2 PER DAY,     *
*        DY=60.0 FT**2 PER DAY, W=3000 FEET, WS=1600 FEET,          *
*           YC=1200 FEET, CO=1000.0 MILLIGRAMS PER LITER             *
*                                                                    *
*        PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 13:03:02        *
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ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE-TRANSPORT EQUATION
FOR A SEMI-INFINITE AQUIFER OF FINITE-WIDTH (STRIP) SOLUTE SOURCE AT X = 0.0

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 31
NUMBER OF Y-COORDINATES (NY) = 27
NUMBER OF TIME VALUES (NT) = 2
NUMBER OF TERMS IN INFINITE SERIES SUMMATION (NMAX) = 300

SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) = 1.000000E+03 MILLIGRAMS PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 1.000000E+00 FEET PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 2.000000E+02 FT**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 6.000000E+01 FT**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER WIDTH (W) = 3.000000E+03 FEET
SOLUTE SOURCE IS CENTERED AT Y = 1.200000E+03 FEET
FINITE-WIDTH OF SOLUTE SOURCE (WS) = 1.600000E+03 FEET

PLOT SCALING FACTOR FOR X (XSCLP) = 7.500000E+02
PLOT SCALING FACTOR FOR Y (YSCLP) = 7.500000E+02
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	150.0000	300.0000	450.0000	600.0000	750.0000	900.0000	1050.0000
1200.0000	1350.0000	1500.0000	1650.0000	1800.0000	1950.0000	2100.0000	2250.0000
2400.0000	2550.0000	2700.0000	2850.0000	3000.0000	3150.0000	3300.0000	3450.0000
3600.0000	3750.0000	3900.0000	4050.0000	4200.0000	4350.0000	4500.0000	

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000
1600.0000	1700.0000	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS

1500.0000 3000.0000

SOLUTE CONCENTRATION AT TIME = 1500.0000 DAYS

SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
0.0000	0.00000	0.00000	0.00000	0.00000	500.00000	1000.00000	1000.00000	1000.00000	1000.00000
150.0000	16.56160	23.02965	50.80833	147.86124	497.55471	847.47601	945.35723	975.11265	986.13848
300.0000	41.00855	54.74523	107.40032	239.90280	493.14659	746.98145	881.61007	939.22558	963.95142
450.0000	69.69482	88.67497	154.16407	286.72329	485.96041	686.26791	822.61775	896.66821	933.60837
600.0000	98.19577	119.58950	188.12702	310.36580	475.10498	641.45765	769.30685	850.05657	895.61261
750.0000	122.90887	144.48110	210.43963	319.93844	459.72835	601.66826	718.50343	799.85118	850.23024
900.0000	141.53960	161.92122	222.52938	319.16892	439.16727	561.77654	667.16695	745.51371	797.51763
1050.0000	152.97020	171.45262	225.53432	309.85242	413.10179	519.29501	613.32205	686.50047	737.65024
1200.0000	156.99776	173.28129	220.49518	293.20957	381.67847	473.26774	556.13047	622.79991	671.25067
1350.0000	154.11246	168.13025	208.56970	270.45262	345.56873	423.82109	495.78415	555.13932	599.57652
1500.0000	145.31984	157.13587	191.13206	242.99473	305.94180	371.89592	433.32889	484.97024	524.53239
1650.0000	131.97589	141.73033	169.75792	212.46336	264.35061	318.99985	370.42468	414.29814	448.52111
1800.0000	115.61714	123.49695	146.12567	180.59866	222.54993	266.93856	309.05493	345.41120	374.17913
1950.0000	97.78952	104.01040	121.87304	149.09524	182.28171	217.53866	251.21840	280.56745	304.05868
2100.0000	79.89259	84.68532	98.44873	119.43711	145.06953	172.39728	198.64942	221.70202	240.32579
2250.0000	63.05973	66.65787	76.99319	92.76565	112.06056	132.69458	152.60944	170.20755	184.53320
2400.0000	48.08900	50.71776	58.27079	69.80607	83.93942	99.09368	113.77735	126.82067	137.50530
2550.0000	35.42920	37.29587	42.66083	50.86048	60.92100	71.73293	82.24395	91.62152	99.34348
2700.0000	25.21438	26.50129	30.20110	35.85967	42.81114	50.29661	57.59431	64.12900	69.53356
2850.0000	17.33175	18.19232	20.66710	24.45450	29.11243	34.13681	39.04698	43.45745	47.11867
3000.0000	11.50466	12.06238	13.66666	16.12325	19.14746	22.41447	25.61385	28.49527	30.89472
3150.0000	7.37346	7.72351	8.73065	10.27367	12.17487	14.23139	16.24895	18.07015	19.59080
3300.0000	4.56210	4.77474	5.38668	6.32464	7.48123	8.73374	9.96443	11.07753	12.00910
3450.0000	2.72451	2.84946	3.20912	3.76062	4.44114	5.17884	5.90467	6.56227	7.11373
3600.0000	1.57028	1.64127	1.84565	2.15917	2.54627	2.96626	3.37999	3.75539	4.07074
3750.0000	0.87332	0.91230	1.02455	1.19680	1.40959	1.64064	1.86849	2.07549	2.24966
3900.0000	0.46862	0.48930	0.54887	0.64030	0.75330	0.87608	0.99727	1.10751	1.20038
4050.0000	0.24259	0.25319	0.28372	0.33059	0.38854	0.45156	0.51380	0.57048	0.61829
4200.0000	0.12114	0.12638	0.14149	0.16469	0.19339	0.22461	0.25548	0.28361	0.30736
4350.0000	0.05834	0.06085	0.06807	0.07916	0.09287	0.10781	0.12258	0.13605	0.14744
4500.0000	0.02710	0.02826	0.03159	0.03670	0.04303	0.04992	0.05674	0.06297	0.06824

SOLUTE CONCENTRATION AT TIME = 1500.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000
150.0000	990.69626	992.67298	993.50125	993.72894	993.50071	992.67133	990.69182	986.12717	975.08483
300.0000	974.94058	979.90096	982.02697	982.61765	982.02550	979.89646	974.92853	963.92076	939.15041
450.0000	951.56838	960.12965	963.92001	964.98964	963.91714	960.12086	951.54493	933.54891	896.52309
600.0000	919.77489	931.98696	937.59695	939.20918	937.59218	931.97237	919.73613	895.51476	849.81920
750.0000	879.03702	894.42603	901.76174	903.90986	901.75461	894.40431	878.97956	850.08601	799.50377
900.0000	829.13988	846.87819	855.62816	858.23649	855.61837	846.84844	829.06162	797.32246	745.04731
1050.0000	770.31751	789.41386	799.12164	802.06216	799.10910	789.37592	770.21825	737.40442	685.91802
1200.0000	703.41780	722.87290	733.01984	736.13619	733.00475	722.82744	703.29958	670.96002	622.11730
1350.0000	629.99582	648.91103	658.98957	662.12121	658.97241	648.85959	629.86284	599.25205	554.38397
1500.0000	552.27837	569.92298	579.49154	582.49361	579.47304	569.86777	552.13655	524.18889	484.17755
1650.0000	472.98951	488.83480	497.55239	500.30927	497.53342	488.77845	472.84565	448.17521	413.50658
1800.0000	395.06711	408.79408	416.43571	418.86802	416.41717	408.73926	394.92799	373.84695	344.65714
1950.0000	321.32889	332.81476	339.27064	341.33641	339.25334	332.76382	321.20037	303.75388	279.88077
2100.0000	254.15672	263.44541	268.70767	270.39878	268.69223	263.40014	254.04314	240.05815	221.10332
2250.0000	195.26062	202.52298	206.66409	207.99959	206.65090	202.48446	195.16447	184.30802	169.70715
2400.0000	145.56110	151.05104	154.19836	155.21629	154.18756	151.01962	145.48307	137.32357	126.41931
2550.0000	105.19871	109.21104	111.52157	112.27064	111.51309	109.18646	105.13794	99.20269	91.31234
2700.0000	73.65117	76.48583	78.12432	78.65656	78.11793	76.46737	73.60572	69.42879	63.90011
2850.0000	49.91928	51.85482	52.97712	53.34229	52.97250	51.84150	49.88663	47.04373	43.29452
3000.0000	32.73642	34.01346	34.75592	34.99782	34.75270	34.00422	32.71387	30.84318	28.38370
3150.0000	20.76139	21.57537	22.04969	22.20441	22.04754	21.56921	20.74641	19.55670	17.99663
3300.0000	12.72801	13.22913	13.52171	13.61724	13.52033	13.22518	12.71844	11.98739	11.03091
3450.0000	7.54024	7.83815	8.01239	8.06932	8.01153	7.83572	7.53435	7.10044	6.53380
3600.0000	4.31510	4.48610	4.58626	4.61901	4.58575	4.48466	4.31162	4.06290	3.73865
3750.0000	2.38484	2.47958	2.53515	2.55333	2.53486	2.47876	2.38286	2.24521	2.06602
3900.0000	1.27257	1.32324	1.35299	1.36272	1.35283	1.32279	1.27149	1.19795	1.10234
4050.0000	0.65549	0.68164	0.69701	0.70204	0.69692	0.68140	0.65492	0.61700	0.56776
4200.0000	0.32586	0.33888	0.34654	0.34905	0.34650	0.33876	0.32557	0.30671	0.28223
4350.0000	0.15632	0.16257	0.16626	0.16746	0.16623	0.16251	0.15618	0.14712	0.13538
4500.0000	0.07235	0.07525	0.07696	0.07752	0.07695	0.07522	0.07228	0.06809	0.06266

SOLUTE CONCENTRATION AT TIME = 1500.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	1000.00000	1000.00000	500.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150.0000	945.29105	847.32303	497.20943	147.09558	49.12656	19.33007	8.28101	3.70026	1.68348
300.0000	881.43188	746.57150	492.22728	237.88237	103.01926	45.29340	20.50485	9.45368	4.38568
450.0000	822.27574	685.48702	484.22678	282.96519	146.17114	71.90957	34.84854	16.76902	8.00195
600.0000	768.75163	640.20225	472.35339	304.50230	175.94410	94.85307	49.09977	24.74246	12.19791
750.0000	717.69809	599.86789	455.83954	311.80788	193.96459	112.12424	61.45727	32.36589	16.49737
900.0000	666.09638	559.41261	434.13986	308.86258	202.16093	123.16296	70.77371	38.77067	20.39904
1050.0000	611.99894	516.41045	407.06271	297.70918	202.09797	128.11948	76.49014	43.34908	23.47538
1200.0000	554.59612	469.96494	374.86877	279.76567	195.10738	127.51576	78.50498	45.78475	25.43459
1350.0000	494.10394	420.24872	338.30951	256.36247	182.47668	122.11780	77.06320	46.03435	26.14599
1500.0000	431.58339	368.22804	298.58871	228.94097	165.55296	112.87510	72.66748	44.28443	25.63598
1650.0000	368.69841	315.41204	257.24685	199.07308	145.75353	100.86086	65.99574	40.89379	24.06248
1800.0000	307.42530	263.58588	215.98630	168.37837	124.50681	87.19364	57.81624	36.32713	21.67542
1950.0000	249.74691	214.53947	176.46963	138.39204	103.15486	72.94621	48.90196	31.08647	18.77076
2100.0000	197.37647	169.82475	140.12968	110.42477	82.84969	59.05705	39.95276	25.64820	15.64579
2250.0000	151.55310	130.57625	108.02607	85.47003	64.47088	46.26139	31.53542	20.41355	12.56212
2400.0000	112.93564	97.41755	80.77041	64.11849	48.58227	35.05380	24.04907	15.67797	9.72101
2550.0000	81.59946	70.45760	58.52549	46.58965	35.43366	25.68602	17.71821	11.62086	7.25260
2700.0000	57.11980	49.36299	41.06769	32.76959	25.00220	18.19600	12.60992	8.31362	5.21801
2850.0000	38.71089	33.47893	27.89032	22.29968	17.06012	12.45796	8.66786	5.74040	3.62078
3000.0000	25.38476	21.96812	18.32216	14.67480	11.25290	8.24120	5.75372	3.82538	2.42332
3150.0000	16.09863	13.93973	11.63785	9.33503	7.17265	5.26619	3.68766	2.46013	1.56439
3300.0000	9.86945	8.55016	7.14448	5.73821	4.41669	3.24985	2.28165	1.52671	0.97408
3450.0000	5.84687	5.06751	4.23763	3.40737	2.62664	1.93641	1.36263	0.91417	0.58500
3600.0000	3.34611	2.90122	2.42772	1.95401	1.50830	1.11382	0.78536	0.52812	0.33885
3750.0000	1.84936	1.60402	1.34303	1.08192	0.83612	0.61836	0.43679	0.29433	0.18929
3900.0000	0.98687	0.85621	0.71728	0.57827	0.44736	0.33129	0.23438	0.15823	0.10198
4050.0000	0.50835	0.44117	0.36975	0.29830	0.23099	0.17125	0.12133	0.08205	0.05298
4200.0000	0.25272	0.21938	0.18394	0.14849	0.11508	0.08541	0.06059	0.04103	0.02654
4350.0000	0.12124	0.10527	0.08830	0.07132	0.05531	0.04109	0.02918	0.01979	0.01282
4500.0000	0.05612	0.04873	0.04089	0.03304	0.02564	0.01907	0.01356	0.00920	0.00597

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.00000	0.00000	0.00000	0.00000	500.00000	1000.00000	1000.00000	1000.00000	1000.00000
150.0000	18.65022	25.17248	53.10822	150.40522	500.40619	850.67045	948.90158	978.98812	990.30556
300.0000	46.82229	60.70951	113.80082	246.98128	501.07924	755.86685	891.46764	950.00324	975.53948
450.0000	81.47913	100.76332	167.13338	301.06194	502.02445	704.25677	842.57102	918.48105	957.05951
600.0000	118.81534	140.73828	210.80931	335.43198	503.17542	672.88032	804.15163	888.14182	936.55366
750.0000	155.76568	178.17550	246.56127	359.83386	504.38064	651.62990	773.88698	860.37089	915.27836
900.0000	190.38215	211.99844	276.18489	378.38881	505.40294	635.84559	749.23891	835.17054	893.86534
1050.0000	221.59573	241.79552	300.85499	392.91537	505.92963	623.03033	728.20666	811.95821	872.44076
1200.0000	248.87275	267.42850	321.22922	404.19114	505.58952	611.62831	709.26984	789.96403	850.80338
1350.0000	271.95873	288.85173	337.62718	412.48269	503.97364	600.53638	691.24012	768.39392	828.56656
1500.0000	290.72841	306.03782	350.16392	417.79609	500.65828	588.89601	673.15431	746.49311	805.25587
1650.0000	305.11272	318.95537	358.83993	420.01029	495.22963	576.00492	654.21468	723.57519	780.37632
1800.0000	315.07315	327.57311	363.60438	418.95788	487.30991	561.28472	633.76333	699.04286	753.46313
1950.0000	320.60286	331.87734	364.40325	414.48058	476.58417	544.27632	611.27806	672.40870	724.12362
2100.0000	321.74115	331.89504	361.21811	406.47093	462.82644	524.64869	586.38017	643.31687	692.07333
2250.0000	318.59282	327.71776	354.09778	394.90463	445.92307	502.21233	558.84686	611.56360	657.16626
2400.0000	311.34653	319.52204	343.18236	379.86431	425.89091	476.93141	528.62249	577.11312	619.41717
2550.0000	300.28788	307.58352	328.71869	361.55387	402.88781	448.93025	495.82401	540.10579	579.01385
2700.0000	285.80465	292.28250	311.06592	340.30262	377.21362	418.49114	460.73752	500.85561	536.31705
2850.0000	268.38260	274.09954	290.69049	316.55838	349.30038	386.04162	423.80382	459.83588	491.84694
3000.0000	248.59151	253.60126	268.15081	290.87009	319.69200	352.13124	385.59310	417.65273	446.25669
3150.0000	227.06259	231.41730	244.07290	263.86111	289.01461	317.39883	346.76963	375.00824	400.29452
3300.0000	204.45932	208.21066	219.11917	236.19570	257.94009	282.53296	308.04944	332.65594	354.75783
3450.0000	181.44456	184.64430	193.95368	208.54192	227.14614	248.22892	270.15460	291.35279	310.44334
3600.0000	158.64732	161.34752	169.20708	181.53454	197.27655	215.14635	233.76814	251.81193	268.09821
3750.0000	136.63248	138.88516	145.44477	155.74147	168.90551	183.87118	199.49393	214.66066	228.37645
3900.0000	115.87617	117.73284	123.14122	131.63671	142.50903	154.88539	167.82480	180.40723	191.80476
4050.0000	96.74903	98.25992	102.66242	109.58210	118.44563	128.54677	139.12141	149.41903	158.76020
4200.0000	79.50819	80.72144	84.25762	89.81864	96.94742	105.07966	113.60293	121.91324	129.46097
4350.0000	64.29800	65.25887	68.06015	72.46755	78.12136	84.57663	91.34914	97.95959	103.96986
4500.0000	51.15852	51.90874	54.09635	57.53967	61.95944	67.00961	72.31267	77.49373	82.20877

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000
150.0000	995.10063	997.25103	998.18430	998.44588	998.17929	997.23888	995.07624	990.25884	978.89986
300.0000	987.18818	992.63136	995.04926	995.73412	995.03527	992.59748	987.12034	975.40987	949.75925
450.0000	976.35313	985.89081	990.27156	991.53159	990.24305	985.82194	976.21569	956.79812	917.99183
600.0000	963.04138	976.95647	983.59634	985.54052	983.54609	976.83545	962.80095	936.09913	887.29758
750.0000	947.77408	965.86560	974.83586	977.51046	974.75501	965.67156	947.39070	914.55883	859.04662
900.0000	930.94092	952.67601	963.84382	967.23020	963.72211	952.38510	930.36973	892.80220	833.23391
1050.0000	912.71929	937.39678	950.48161	954.50764	950.30790	936.98342	911.91326	870.95392	809.27960
1200.0000	893.08108	919.95436	934.59106	939.14868	934.35404	919.39300	891.99437	848.81757	786.42746
1350.0000	871.83832	900.18978	915.98112	920.94376	915.67018	899.45694	870.43019	826.01802	763.90765
1500.0000	848.69906	877.87716	894.43042	899.66706	894.03663	876.95358	846.93771	802.09856	740.99882
1650.0000	823.32211	852.75494	869.70395	875.08870	869.22098	851.62771	821.18828	776.58725	717.05493
1800.0000	795.36805	824.56414	841.58041	846.99836	841.00535	823.22839	792.85770	749.04603	691.52307
1950.0000	764.54591	793.08926	809.88621	815.23636	809.22018	791.54934	761.67195	719.11074	663.96109
2100.0000	730.65506	758.19836	774.53163	779.72809	773.78009	756.46836	727.44771	686.52510	634.05611
2250.0000	693.62063	719.87876	735.54471	740.51665	734.71737	717.98222	690.12650	651.16881	601.64201
2400.0000	653.52018	678.26478	693.09828	697.78944	692.20873	676.23366	649.80006	613.07808	566.71279
2550.0000	610.59902	633.65429	647.52650	651.89439	646.59151	631.52725	606.72456	572.45624	529.42878
2700.0000	565.27217	586.51171	599.32800	603.34290	598.36652	584.33191	561.32183	529.67265	490.11303
2850.0000	518.11201	537.45616	549.15412	552.79811	548.18619	535.26871	514.16649	485.24880	449.23658
3000.0000	469.82185	487.23521	497.78263	501.04848	496.82818	485.08456	465.95958	439.83191	407.39259
3150.0000	421.19768	436.68621	446.07879	448.96881	445.15650	434.61366	417.49059	394.15783	365.26082
3300.0000	373.08114	386.68871	394.94742	397.47219	394.07375	384.73033	369.59118	349.00629	323.56510
3450.0000	326.30856	338.11260	345.28058	347.45762	344.46900	336.29760	323.08508	305.15264	283.02779
3600.0000	281.66073	291.76703	297.90602	299.75839	297.16656	290.11680	278.73899	263.32066	244.32512
3750.0000	239.81829	248.35518	253.54158	255.09642	252.88058	246.88290	237.21905	224.14079	208.04780
3900.0000	201.32680	208.43879	212.75955	214.04666	212.17976	207.14970	199.05695	188.11746	174.67010
4050.0000	166.57415	172.41546	175.96391	177.01441	175.46483	171.30764	164.62816	155.60808	144.52986
4200.0000	135.78155	140.50991	143.38176	144.22685	142.96011	139.57536	134.14355	126.81472	117.82040
4350.0000	109.00764	112.77862	115.06845	115.73835	114.71877	112.00467	107.65388	101.78810	94.59395
4500.0000	86.16408	89.12626	90.92447	91.44759	90.63980	88.49701	85.06547	80.44214	74.77499

SAMPLE PROBLEM 6.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER OF FINITE WIDTH
WITH A CONTINUOUS "STRIP" SOURCE--CONTINUED

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	1000.00000	1000.00000	500.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150.0000	948.73546	850.35729	499.81217	149.26559	50.88247	20.70766	9.32861	4.47340	2.23969
300.0000	891.01051	755.01009	499.46645	243.91876	107.90460	49.12699	23.42094	11.60651	5.93512
450.0000	841.66103	702.56697	498.88141	295.18767	156.06583	79.67689	40.75962	21.13541	11.14674
600.0000	802.59623	670.02662	497.94856	325.85617	193.23812	108.43589	59.44321	32.38905	17.71060
750.0000	771.47475	647.26627	496.52863	345.76814	221.48272	133.75160	77.94045	44.56398	25.30261
900.0000	745.75554	629.64123	494.44943	359.22363	242.99519	155.28238	95.27853	56.92808	33.52655
1050.0000	723.45297	614.69935	491.50601	368.26474	259.35094	173.19786	110.92388	68.90211	41.98418
1200.0000	703.07932	600.95506	487.46396	373.90837	271.56948	187.78760	124.61011	80.05911	50.31405
1350.0000	683.49433	587.39420	482.06661	376.65564	280.27931	199.31263	136.20979	90.09169	58.20528
1500.0000	663.79457	573.25874	475.04629	376.72986	285.85115	207.96588	145.65938	98.77706	65.39762
1650.0000	643.24944	557.95118	466.13985	374.20299	288.49271	213.87581	152.92262	105.95110	71.67676
1800.0000	621.27078	540.99562	455.10749	369.07232	288.31657	217.12566	157.97782	111.49337	76.87001
1950.0000	597.40411	522.02722	441.75392	361.31306	285.39087	217.77730	160.81847	115.32103	80.84428
2100.0000	571.33206	500.79599	425.94981	350.91759	279.77852	215.89486	161.46075	117.38884	83.50652
2250.0000	542.88302	477.17601	407.65152	337.92547	271.56764	211.56593	159.95347	117.69248	84.80581
2400.0000	512.03923	451.17442	386.91659	322.44539	260.89413	204.91845	156.38749	116.27270	84.73575
2550.0000	478.94020	422.93601	363.91286	304.66884	247.95635	196.13244	150.90260	113.21847	83.33613
2700.0000	443.87811	392.74029	338.91958	284.87505	233.02167	185.44537	143.69039	108.66760	80.69260
2850.0000	407.28358	360.98974	312.31939	263.42688	216.42479	173.15099	134.99234	102.80399	76.93381
3000.0000	369.70144	328.18872	284.58156	240.75827	198.55830	159.59166	125.09292	95.85127	72.22571
3150.0000	331.75789	294.91438	256.23738	217.35422	179.85667	145.14518	114.30833	88.06293	66.76305
3300.0000	294.12116	261.78160	227.85002	193.72552	160.77526	130.20735	102.97180	79.70986	60.75884
3450.0000	257.45948	229.40519	199.98152	170.38071	141.76677	115.17221	91.41703	71.06633	54.43250
3600.0000	222.39985	198.36281	173.16022	147.79800	123.25751	100.41210	79.96132	62.39581	47.99779
3750.0000	189.49158	169.16199	147.85168	126.40014	105.62588	86.25940	68.89017	53.93782	41.65175
3900.0000	159.17762	142.21416	124.43558	106.53433	89.18520	72.99190	58.44475	45.89730	35.56540
4050.0000	131.77596	117.81764	103.19062	88.45897	74.17214	60.82280	48.81330	38.43700	29.87711
4200.0000	107.47189	96.15051	84.28800	72.33785	60.74166	49.89615	40.12692	31.67364	24.68887
4350.0000	86.32090	77.27306	67.79348	58.24181	48.96816	40.28760	32.45983	25.67762	20.06569
4500.0000	68.26081	61.13852	53.67678	46.15680	38.85226	32.00982	25.83357	20.47610	16.03755

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*   SAMPLE PROBLEM 7.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER          *
*        OF INFINITE WIDTH WITH A CONTINUOUS "STRIP" SOURCE                 *
*                                                                           *
*  MODEL PARAMETERS:  V=1.42 FEET PER DAY, DX=100.0 FT**2 PER DAY,          *
*          DY=20.0 FT**2 PER DAY, WS=230 FEET, VC=750 FEET,                 *
*                   CO=40.0 MILLIGRAMS PER LITER                            *
*                                                                           *
*        PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 13:02:04               *
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ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH WITH A FINITE-WIDTH (STRIP) SOLUTE SOURCE AT X = 0.0

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 31
NUMBER OF Y-COORDINATES (NY) = 31
NUMBER OF TIME VALUES (NT) = 1
NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = 104

SOLUTE CONCENTRATION ON MODEL BOUNDARY (CO) = 4.000000E+01 MILLIGRAMS PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 1.420000E+00 FEET PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 1.000000E+02 FT**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 2.000000E+01 FT**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER WIDTH (W) IS INFINITE
SOLUTE SOURCE IS CENTERED AT Y = 7.500000E+02 FEET
FINITE-WIDTH OF SOLUTE SOURCE (WS) = 2.300000E+02 FEET

PLOT SCALING FACTOR FOR X (XSCLP) = 5.000000E+02
PLOT SCALING FACTOR FOR Y (YSCLP) = 5.000000E+02
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000
1600.0000	1700.0000	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000
2400.0000	2500.0000	2600.0000	2700.0000	2800.0000	2900.0000	3000.0000	

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	50.0000	100.0000	150.0000	200.0000	250.0000	300.0000	350.0000
400.0000	450.0000	500.0000	550.0000	600.0000	650.0000	700.0000	750.0000
800.0000	850.0000	900.0000	950.0000	1000.0000	1050.0000	1100.0000	1150.0000
1200.0000	1250.0000	1300.0000	1350.0000	1400.0000	1450.0000	1500.0000	

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS
1826.0000

SOLUTE CONCENTRATION AT TIME = 1826.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET							
	0.0000	50.0000	100.0000	150.0000	200.0000	250.0000	300.0000	350.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER							
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100.0000	0.00003	0.00006	0.00016	0.00041	0.00105	0.00275	0.00731	0.01998
200.0000	0.00010	0.00024	0.00060	0.00150	0.00381	0.00978	0.02548	0.06763
300.0000	0.00026	0.00063	0.00156	0.00385	0.00958	0.02400	0.06058	0.15403
400.0000	0.00058	0.00141	0.00341	0.00826	0.02002	0.04858	0.11779	0.28462
500.0000	0.00118	0.00279	0.00663	0.01565	0.03684	0.08621	0.20006	0.45793
600.0000	0.00218	0.00508	0.01176	0.02702	0.06158	0.13870	0.30759	0.66727
700.0000	0.00376	0.00858	0.01938	0.04328	0.09536	0.20664	0.43823	0.90335
800.0000	0.00611	0.01364	0.03003	0.06511	0.13874	0.28944	0.58818	1.15645
900.0000	0.00943	0.02057	0.04413	0.09295	0.19167	0.38557	0.75282	1.41763
1000.0000	0.01390	0.02963	0.06196	0.12685	0.25350	0.49276	0.92722	1.67921
1100.0000	0.01965	0.04097	0.08358	0.16651	0.32305	0.60823	1.10646	1.93476
1200.0000	0.02675	0.05460	0.10880	0.21126	0.39868	0.72887	1.28580	2.17882
1300.0000	0.03519	0.07039	0.13718	0.26006	0.47837	0.85133	1.46067	2.40661
1400.0000	0.04486	0.08802	0.16803	0.31153	0.55974	0.97206	1.62659	2.61369
1500.0000	0.05553	0.10702	0.20040	0.36402	0.64017	1.08736	1.77911	2.79571
1600.0000	0.06684	0.12670	0.23310	0.41559	0.71678	1.19344	1.91380	2.94835
1700.0000	0.07835	0.14628	0.26480	0.46416	0.78660	1.28645	2.02630	3.06727
1800.0000	0.08952	0.16483	0.29403	0.50755	0.84666	1.36269	2.11244	3.14829
1900.0000	0.09978	0.18141	0.31933	0.54366	0.89416	1.41875	2.16846	3.18769
2000.0000	0.10855	0.19509	0.33932	0.57058	0.92666	1.45178	2.19137	3.18257
2100.0000	0.11529	0.20507	0.35283	0.58676	0.94228	1.45976	2.17919	3.13134
2200.0000	0.11957	0.21070	0.35902	0.59118	0.93993	1.44171	2.13137	3.03407
2300.0000	0.12111	0.21161	0.35746	0.58344	0.91943	1.39792	2.04897	2.89288
2400.0000	0.11979	0.20772	0.34819	0.56385	0.88160	1.33002	1.93478	2.71202
2500.0000	0.11570	0.19927	0.33172	0.53343	0.82824	1.24097	1.79327	2.49780
2600.0000	0.10912	0.18679	0.30901	0.49383	0.76203	1.13486	1.63036	2.25829
2700.0000	0.10046	0.17104	0.28139	0.44720	0.68629	1.01661	1.45296	2.00275
2800.0000	0.09029	0.15296	0.25040	0.39599	0.60475	0.89158	1.26847	1.74096
2900.0000	0.07920	0.13357	0.21769	0.34275	0.52117	0.76513	1.08420	1.48242
3000.0000	0.06778	0.11387	0.18484	0.28988	0.43908	0.64221	0.90678	1.23569

SOLUTE CONCENTRATION AT TIME = 1826.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 400.0000	450.0000	500.0000	550.0000	600.0000	650.0000	700.0000	750.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER							
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000	40.00000	40.00000	40.00000
100.0000	0.05651	0.16767	0.53412	1.90013	7.86633	25.98603	36.42156	38.24154
200.0000	0.18345	0.51060	1.45943	4.22045	11.42789	23.57398	32.76983	35.49065
300.0000	0.39397	1.00810	2.54357	6.11176	13.02727	22.40683	29.94800	32.60906
400.0000	0.68123	1.59727	3.59536	7.50817	13.85347	21.53500	27.72478	30.03286
500.0000	1.02530	2.21592	4.53098	8.52497	14.28926	20.77006	25.90567	27.84477
600.0000	1.40286	2.82091	5.33061	9.26630	14.49599	20.06362	24.37914	26.00439
700.0000	1.79335	3.38768	6.00064	9.80615	14.55741	19.40258	23.07531	24.44807
800.0000	2.18102	3.90442	6.55563	10.19584	14.52278	18.78257	21.94577	23.11782
900.0000	2.55477	4.36686	7.01117	10.47132	14.42298	18.20055	20.95468	21.96653
1000.0000	2.90706	4.77466	7.38120	10.65793	14.27763	17.65272	20.07411	20.95685
1100.0000	3.23286	5.12920	7.67710	10.77337	14.09878	17.13409	19.28133	20.05874
1200.0000	3.52855	5.43220	7.90748	10.82949	13.89303	16.63842	18.55689	19.24725
1300.0000	3.79123	5.68491	8.07822	10.83352	13.66276	16.15810	17.88327	18.50075
1400.0000	4.01810	5.88766	8.19255	10.78883	13.40696	15.68425	17.24398	17.79962
1500.0000	4.20618	6.03961	8.25134	10.69565	13.12185	15.20671	16.62294	17.12538
1600.0000	4.35206	6.13882	8.25342	10.55167	12.80150	14.71430	16.00431	16.46030
1700.0000	4.45206	6.18240	8.19609	10.35277	12.43850	14.19526	15.37254	15.78736
1800.0000	4.50241	6.16702	8.07577	10.09388	12.02485	13.63796	14.71296	15.09067
1900.0000	4.49967	6.08944	7.88883	9.76989	11.55296	13.03180	14.01252	14.35625
2000.0000	4.44125	5.94727	7.63251	9.37685	11.01679	12.36836	13.26084	13.57297
2100.0000	4.32604	5.73972	7.30590	8.91297	10.41305	11.64253	12.45135	12.73369
2200.0000	4.15490	5.46833	6.91076	8.37966	9.74217	10.85354	11.58227	11.83623
2300.0000	3.93111	5.13744	6.45216	7.78217	9.00912	10.00571	10.65732	10.88408
2400.0000	3.66054	4.75448	5.93878	7.12996	8.22361	9.10868	9.68595	9.88658
2500.0000	3.35154	4.32979	5.38272	6.43643	7.39988	8.17711	8.68294	8.85856
2600.0000	3.01453	3.87617	4.79888	5.71826	6.55586	7.22969	7.66741	7.81924
2700.0000	2.66133	3.40798	4.20403	4.99419	5.71181	6.28774	6.66125	6.79070
2800.0000	2.30425	2.94013	3.61547	4.28358	4.88871	5.37335	5.68721	5.79591
2900.0000	1.95521	2.48690	3.04969	3.60486	4.10648	4.50750	4.76689	4.85666
3000.0000	1.62478	2.06088	2.52114	2.97400	3.38235	3.70828	3.91888	3.99173

SOLUTE CONCENTRATION AT TIME = 1826.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 800.0000	850.0000	900.0000	950.0000	1000.0000	1050.0000	1100.0000	1150.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER							
0.0000	40.00000	40.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100.0000	36.42156	25.98603	7.86633	1.90013	0.53412	0.16767	0.05651	0.01998
200.0000	32.76983	23.57398	11.42789	4.22045	1.45943	0.51060	0.18345	0.06763
300.0000	29.94800	22.40683	13.02727	6.11176	2.54357	1.00810	0.39397	0.15403
400.0000	27.72478	21.53500	13.85347	7.50817	3.59536	1.59727	0.68123	0.28462
500.0000	25.90567	20.77006	14.28926	8.52497	4.53098	2.21592	1.02530	0.45793
600.0000	24.37914	20.06362	14.49599	9.26630	5.33061	2.82091	1.40286	0.66727
700.0000	23.07531	19.40258	14.55741	9.80615	6.00064	3.38768	1.79335	0.90335
800.0000	21.94577	18.78257	14.52278	10.19584	6.55563	3.90442	2.18102	1.15645
900.0000	20.95468	18.20055	14.42298	10.47132	7.01117	4.36686	2.55477	1.41763
1000.0000	20.07411	17.65272	14.27763	10.65793	7.38120	4.77466	2.90706	1.67921
1100.0000	19.28133	17.13409	14.09878	10.77337	7.67710	5.12920	3.23286	1.93476
1200.0000	18.55689	16.63842	13.89303	10.82949	7.90748	5.43220	3.52855	2.17882
1300.0000	17.88327	16.15810	13.66276	10.83352	8.07822	5.68491	3.79123	2.40661
1400.0000	17.24398	15.68425	13.40696	10.78883	8.19255	5.88766	4.01810	2.61369
1500.0000	16.62294	15.20671	13.12185	10.69565	8.25134	6.03961	4.20618	2.79571
1600.0000	16.00431	14.71430	12.80150	10.55167	8.25342	6.13882	4.35206	2.94835
1700.0000	15.37254	14.19526	12.43850	10.35277	8.19609	6.18240	4.45206	3.06727
1800.0000	14.71296	13.63796	12.02485	10.09388	8.07577	6.16702	4.50241	3.14829
1900.0000	14.01252	13.03180	11.55296	9.76989	7.88883	6.08944	4.49967	3.18769
2000.0000	13.26084	12.36836	11.01679	9.37685	7.63251	5.94727	4.44125	3.18257
2100.0000	12.45135	11.64253	10.41305	8.91297	7.30590	5.73972	4.32604	3.13134
2200.0000	11.58227	10.85354	9.74217	8.37966	6.91076	5.46833	4.15490	3.03407
2300.0000	10.65732	10.00571	9.00912	7.78217	6.45216	5.13744	3.93111	2.89288
2400.0000	9.68595	9.10868	8.22361	7.12996	5.93878	4.75448	3.66054	2.71202
2500.0000	8.68294	8.17711	7.39988	6.43643	5.38272	4.32979	3.35154	2.49780
2600.0000	7.66741	7.22969	6.55586	5.71826	4.79888	3.87617	3.01453	2.25829
2700.0000	6.66125	6.28774	5.71181	4.99419	4.20403	3.40798	2.66133	2.00275
2800.0000	5.68721	5.37335	4.88871	4.28358	3.61547	2.94013	2.30425	1.74096
2900.0000	4.76689	4.50750	4.10648	3.60486	3.04969	2.48690	1.95521	1.48242
3000.0000	3.91888	3.70828	3.38235	2.97400	2.52114	2.06088	1.62478	1.23569

SAMPLE PROBLEM 7.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER OF INFINITE WIDTH WITH A CONTINUOUS "STRIP" SOURCE--CONTINUED

X-COORDINATE, IN FEET	SOLUTE CONCENTRATION AT TIME = 1826.0000 DAYS Y-COORDINATE, IN FEET						
	1200.0000	1250.0000	1300.0000	1350.0000	1400.0000	1450.0000	1500.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
100.0000	0.00731	0.00275	0.00105	0.00041	0.00016	0.00006	0.00003
200.0000	0.02548	0.00978	0.00381	0.00150	0.00060	0.00024	0.00010
300.0000	0.06058	0.02400	0.00958	0.00385	0.00156	0.00063	0.00026
400.0000	0.11779	0.04858	0.02002	0.00826	0.00341	0.00141	0.00058
500.0000	0.20006	0.08621	0.03684	0.01565	0.00663	0.00279	0.00118
600.0000	0.30759	0.13870	0.06158	0.02702	0.01176	0.00508	0.00218
700.0000	0.43823	0.20664	0.09536	0.04328	0.01938	0.00858	0.00376
800.0000	0.58818	0.28944	0.13874	0.06511	0.03003	0.01364	0.00611
900.0000	0.75282	0.38557	0.19167	0.09295	0.04413	0.02057	0.00943
1000.0000	0.92722	0.49276	0.25350	0.12685	0.06196	0.02963	0.01390
1100.0000	1.10646	0.60823	0.32305	0.16651	0.08358	0.04097	0.01965
1200.0000	1.28580	0.72887	0.39868	0.21126	0.10880	0.05460	0.02675
1300.0000	1.46067	0.85133	0.47837	0.26006	0.13718	0.07039	0.03519
1400.0000	1.62659	0.97206	0.55974	0.31153	0.16803	0.08802	0.04486
1500.0000	1.77911	1.08736	0.64017	0.36402	0.20040	0.10702	0.05553
1600.0000	1.91380	1.19344	0.71678	0.41559	0.23310	0.12670	0.06684
1700.0000	2.02630	1.28645	0.78660	0.46416	0.26480	0.14628	0.07835
1800.0000	2.11244	1.36269	0.84666	0.50755	0.29403	0.16483	0.08952
1900.0000	2.16846	1.41875	0.89416	0.54366	0.31933	0.18141	0.09978
2000.0000	2.19137	1.45178	0.92666	0.57058	0.33932	0.19509	0.10855
2100.0000	2.17919	1.45976	0.94228	0.58676	0.35283	0.20507	0.11529
2200.0000	2.13137	1.44171	0.93993	0.59118	0.35902	0.21070	0.11957
2300.0000	2.04897	1.39792	0.91943	0.58344	0.35746	0.21161	0.12111
2400.0000	1.93478	1.33002	0.88160	0.56385	0.34819	0.20772	0.11979
2500.0000	1.79327	1.24097	0.82824	0.53343	0.33172	0.19927	0.11570
2600.0000	1.63036	1.13486	0.76203	0.49383	0.30901	0.18679	0.10912
2700.0000	1.45296	1.01661	0.68629	0.44720	0.28139	0.17104	0.10046
2800.0000	1.26847	0.89158	0.60475	0.39599	0.25040	0.15296	0.09029
2900.0000	1.08420	0.76513	0.52117	0.34275	0.21769	0.13357	0.07920
3000.0000	0.90678	0.64221	0.43908	0.28988	0.18484	0.11387	0.06778

```
******************************************************************************
*   SAMPLE PROBLEM 8A.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER          *
*        OF INFINITE WIDTH WITH A CONTINUOUS GAUSSIAN SOURCE                 *
*                                                                            *
*    MODEL PARAMETERS:  V=4.0 FEET PER DAY, DX=150.0 FT**2 PER DAY,          *
*           DY=30.0 FT**2 PER DAY, WS=130 FEET, YC=450 FEET,                 *
*                   CO=1000.0 MILLIGRAMS PER LITER                           *
*                                                                            *
*         PROGRAM RUN ON TUESDAY, OCTOBER 20, 1987, AT 15:38:47              *
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ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH WITH A SOLUTE SOURCE HAVING A GAUSSIAN CONCENTRATION DISTRIBUTION LOCATED AT X=0.0 AND CENTERED ABOUT Y=YC

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 33
NUMBER OF Y-COORDINATES (NY) = 37
NUMBER OF TIME VALUES (NT) = 1
NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = 104

MAXIMUM SOLUTE CONCENTRATION AT THE BOUNDARY CM) = 1.000000E+03 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 4.000000E+00 FEET PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 1.500000E+02 FT**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 3.000000E+01 FT**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER WIDTH (W) IS INFINITE
SOLUTE SOURCE IS CENTERED AT Y = 4.500000E+02 FEET
STANDARD DEVIATION OF GAUSSIAN DISTRIBUTION (SIGMA) = 1.300000E+02 FEET

PLOT SCALING FACTOR FOR X (XSCLP) = 2.500000E+02
PLOT SCALING FACTOR FOR Y (YSCLP) = 2.500000E+02
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	50.0000	100.0000	150.0000	200.0000	250.0000	300.0000	350.0000
400.0000	450.0000	500.0000	550.0000	600.0000	650.0000	700.0000	750.0000
800.0000	850.0000	900.0000	950.0000	1000.0000	1050.0000	1100.0000	1150.0000
1200.0000	1250.0000	1300.0000	1350.0000	1400.0000	1450.0000	1500.0000	1550.0000
1600.0000							

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	25.0000	50.0000	75.0000	100.0000	125.0000	150.0000	175.0000
200.0000	225.0000	250.0000	275.0000	300.0000	325.0000	350.0000	375.0000
400.0000	425.0000	450.0000	475.0000	500.0000	525.0000	550.0000	575.0000
600.0000	625.0000	650.0000	675.0000	700.0000	725.0000	750.0000	775.0000
800.0000	825.0000	850.0000	875.0000	900.0000			

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS
300.0000

SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET							
	0.0000	25.0000	50.0000	75.0000	100.0000	125.0000	150.0000	175.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER							
0.0000	2.500851	4.777036	8.793629	15.599710	26.668815	43.936934	69.758089	106.732878
50.0000	3.225607	5.952161	10.619335	18.312183	30.513181	49.118280	76.370655	114.675294
100.0000	4.041862	7.241570	12.574552	21.152175	34.454442	54.326777	82.896076	122.376117
150.0000	4.947792	8.639387	14.647958	24.102292	38.470017	59.537713	89.312902	129.824215
200.0000	5.940450	10.138642	16.827462	27.145020	42.538105	64.728015	95.601712	137.009935
250.0000	7.015645	11.731108	19.099936	30.262276	46.636912	69.875071	101.743493	143.923096
300.0000	8.167680	13.406946	21.450700	33.434617	50.743503	74.955111	107.717523	150.550415
350.0000	9.388944	15.154134	23.862707	36.640091	54.832188	79.941057	113.498630	156.872206
400.0000	10.669308	16.957647	26.315412	39.852673	58.872425	84.799784	119.053769	162.858266
450.0000	11.995366	18.798414	28.783342	43.040342	62.826274	89.488855	124.337962	168.463020
500.0000	13.349543	20.652107	31.234465	46.162905	66.645562	93.952922	129.289877	173.620226
550.0000	14.709198	22.487926	33.628556	49.169838	70.269090	98.120214	133.827516	178.237859
600.0000	16.045867	24.267581	35.915849	51.998522	73.620383	101.899725	137.844828	182.194122
650.0000	17.324879	25.944794	38.036381	54.573404	76.606636	105.179956	141.210245	185.335822
700.0000	18.505596	27.465644	39.920475	56.806656	79.119609	107.830119	143.768302	187.480520
750.0000	19.542511	28.770072	41.490771	58.600865	81.039140	109.704651	145.345359	188.423672
800.0000	20.387368	29.794773	42.666088	59.854108	82.239708	110.651581	145.760094	187.951569
850.0000	20.992324	30.477481	43.367132	60.467431	82.600065	110.524732	144.838708	185.859981
900.0000	21.313991	30.762418	43.523722	60.354263	82.015320	109.198981	142.433885	181.977311
950.0000	21.317960	30.606371	43.082814	59.450851	80.410271	106.587017	138.445527	176.189834
1000.0000	20.983246	29.984621	42.016278	57.726300	77.752178	102.655343	132.840457	168.465572
1050.0000	20.305956	28.895784	40.327199	55.190638	74.060922	97.436869	125.667822	158.872826
1100.0000	19.301519	27.364677	38.053491	51.899314	69.414530	91.037619	117.067094	147.589604
1150.0000	18.004958	25.442494	35.267917	47.952979	63.948607	83.635678	107.266403	134.901186
1200.0000	16.468948	23.204010	32.074146	43.492056	57.849057	75.471660	96.570348	121.184854
1250.0000	14.759805	20.741960	28.599073	38.686474	51.338615	66.831411	85.338178	106.882920
1300.0000	12.951864	18.159298	24.982348	33.721780	44.658798	58.022970	73.954955	92.467277
1350.0000	11.121010	15.560342	21.364500	28.783502	38.049691	49.350920	62.799590	78.400273
1400.0000	9.338266	13.042070	17.875320	24.041936	31.730380	41.091675	52.214174	65.097315
1450.0000	7.664310	10.686710	14.624057	19.639395	25.882661	33.473028	42.478748	52.896279
1500.0000	6.145572	8.556556	11.692625	15.681478	20.640016	26.660462	33.794582	42.037424
1550.0000	4.812303	6.691462	9.132445	12.233150	16.082831	20.751421	26.277441	32.655586
1600.0000	3.678601	5.109050	6.964905	9.319587	12.239784	15.777423	19.960602	24.784323

	SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS							
X-COORDINATE,	Y-COORDINATE, IN FEET							
IN FEET	200.0000	225.0000	250.0000	275.0000	300.0000	325.0000	350.0000	375.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER							
0.0000	157.376788	223.625821	306.225980	404.111241	513.923697	629.847151	743.893062	846.690449
50.0000	166.270886	232.764935	314.581732	410.421545	516.867536	628.286172	737.133197	834.698648
100.0000	174.746801	241.319546	322.239745	416.017885	519.210231	626.372943	730.379771	823.122868
150.0000	182.810790	249.317946	329.250018	420.965031	521.017999	624.158040	723.651435	811.939595
200.0000	190.468762	256.784998	335.655314	425.316946	522.344107	621.679299	716.956939	801.120248
250.0000	197.724048	263.739907	341.489117	429.115017	523.227235	618.959994	710.292719	790.627837
300.0000	204.574493	270.193149	346.772551	432.384990	523.688309	616.005321	703.638704	780.411864
350.0000	211.008687	276.142352	351.510006	435.132357	523.725559	612.796899	696.952043	770.401111
400.0000	217.001215	281.567003	355.683361	437.336095	523.307668	609.285193	690.158626	760.494188
450.0000	222.507036	286.422089	359.244933	438.940905	522.365237	605.380077	683.142651	750.548074
500.0000	227.455323	290.631120	362.109674	439.848573	520.781243	600.940311	675.735056	740.365530
550.0000	231.743536	294.079388	364.147643	439.909617	518.381818	595.763389	667.702401	729.683048
600.0000	235.232849	296.608831	365.178309	438.917026	514.929362	589.577972	658.738582	718.161861
650.0000	237.746420	298.016255	364.968739	436.604395	510.120593	582.041768	648.462465	705.385288
700.0000	239.072179	298.056845	363.237903	432.651024	503.592371	572.747973	636.424803	690.865981
750.0000	238.971592	296.454719	359.669110	426.696255	494.937851	561.243074	622.127437	674.066232
800.0000	237.195343	292.921589	353.931790	418.364396	483.734459	547.057628	605.056533	654.433207
850.0000	233.505802	287.183353	345.712408	407.300005	469.583394	529.749690	584.729447	631.448635
900.0000	227.704824	279.012902	334.752483	393.211153	452.158003	508.957921	560.752020	604.689554
950.0000	219.663952	268.265640	320.889650	375.916075	431.255831	484.458672	532.880097	573.893485
1000.0000	209.352877	254.912844	304.096099	355.386697	406.847092	456.219032	501.076611	539.018849
1050.0000	196.861382	239.067234	284.507878	331.781671	379.111274	424.436732	465.554386	500.290142
1100.0000	182.410280	220.995466	262.438951	305.461944	348.454140	389.558421	426.795497	458.218166
1150.0000	166.348075	201.113755	238.375663	276.983984	315.499656	352.270326	385.540748	413.588485
1200.0000	149.132219	179.965331	212.950152	247.069021	281.055104	313.459429	342.747294	367.416052
1250.0000	131.296391	158.181472	186.894797	216.550774	246.052170	274.148276	299.517811	320.869652
1300.0000	113.407714	136.430798	160.983183	186.307933	211.471093	235.411276	257.009762	275.175285
1350.0000	96.019700	115.363673	135.965566	157.190545	178.258157	198.283816	216.337033	231.511559
1400.0000	79.627453	95.559458	112.507818	129.950553	147.248007	163.676866	178.477646	190.911658
1450.0000	64.631191	77.483741	91.142119	105.185896	119.101313	132.308625	144.200050	154.185169
1500.0000	51.312508	61.460747	72.235386	83.304943	94.265357	104.661506	114.016922	121.869249
1550.0000	39.825479	47.663322	55.978173	64.514360	72.960919	80.968156	88.170434	94.213277
1600.0000	30.202128	36.119922	42.393305	48.829476	55.194397	61.225263	66.647586	71.195487

	SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS						
X-COORDINATE,	Y-COORDINATE, IN FEET						
IN FEET	400.0000	425.0000	450.0000	475.0000	500.0000	525.0000	550.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	928.704665	981.678788	1000.000000	981.678788	928.704665	846.690449	743.893062
50.0000	912.215420	962.146379	979.391312	962.146379	912.215420	834.698648	737.133197
100.0000	896.522475	943.679504	959.945412	943.679504	896.522475	823.122868	730.379771
150.0000	881.559980	926.181011	941.553616	926.181011	881.559980	811.939595	723.651435
200.0000	867.262418	909.558289	924.113369	909.558289	867.262418	801.120248	716.956939
250.0000	853.560239	893.718110	907.522784	893.718110	853.560239	790.627837	710.292719
300.0000	840.373820	878.559871	891.673616	878.559871	840.373820	780.411864	703.638704
350.0000	827.605342	863.966802	876.442234	863.966802	827.605342	770.401111	696.952043
400.0000	815.128433	849.794969	861.678376	849.794969	815.128433	760.494188	690.158626
450.0000	802.775808	835.860286	847.191948	835.860286	802.775808	750.548074	683.142651
500.0000	790.325804	821.924449	832.738740	821.924449	790.325804	740.365530	675.735056
550.0000	777.489533	807.681559	818.006857	807.681559	777.489533	729.683048	667.702401
600.0000	763.901297	792.748125	802.606587	792.748125	763.901297	718.161861	658.738582
650.0000	749.115656	776.659934	786.067187	776.659934	749.115656	705.385288	648.462465
700.0000	732.614866	758.879589	767.844456	758.879589	732.614866	690.865981	636.424803
750.0000	713.829992	738.818098	747.342483	738.818098	713.829992	674.066232	622.127437
800.0000	692.177597	715.872452	723.951522	715.872452	692.177597	654.433207	605.056533
850.0000	667.111537	689.478718	697.101522	689.478718	667.111537	631.448635	584.729447
900.0000	638.186270	659.176922	666.327543	659.176922	638.186270	604.689554	560.752020
950.0000	605.124761	624.680648	631.339924	624.680648	605.124761	573.893485	532.880097
1000.0000	567.881379	585.941451	592.089219	585.941451	567.881379	539.018849	501.076611
1050.0000	526.688833	543.196895	548.814589	543.196895	526.688833	500.290142	465.554386
1100.0000	482.079040	496.991787	502.065177	496.991787	482.079040	458.218166	426.795497
1150.0000	434.870786	448.165353	452.687105	448.165353	434.870786	413.588485	385.540748
1200.0000	386.122086	397.802165	401.773919	397.802165	386.122086	367.416052	342.747294
1250.0000	337.051047	347.150789	350.584484	347.150789	337.051047	320.869652	299.517811
1300.0000	288.934823	297.519972	300.438231	297.519972	288.934823	275.175285	257.009762
1350.0000	243.000292	250.166394	252.601918	250.166394	243.000292	231.511559	216.337033
1400.0000	200.321709	206.189641	208.183688	206.189641	200.321709	190.911658	178.477646
1450.0000	161.739182	166.448590	168.048754	166.448590	161.739182	154.185169	144.200050
1500.0000	127.807850	131.509370	132.766938	131.509370	127.807850	121.869249	114.016922
1550.0000	98.782095	101.629291	102.596516	101.629291	98.782095	94.213277	88.170434
1600.0000	74.633153	76.775075	77.502649	76.775075	74.633153	71.195487	66.647586

SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET						
	575.0000	600.0000	625.0000	650.0000	675.0000	700.0000	725.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	629.847151	513.923697	404.111241	306.225980	223.625821	157.376788	106.732878
50.0000	628.286172	516.867536	410.421545	314.581732	232.764935	166.270886	114.675294
100.0000	626.372943	519.210231	416.017885	322.239745	241.319546	174.746801	122.376117
150.0000	624.158040	521.017999	420.965031	329.250018	249.317946	182.810790	129.824215
200.0000	621.679299	522.344107	425.316946	335.655314	256.784998	190.468762	137.009935
250.0000	618.959994	523.227235	429.115017	341.489117	263.739907	197.724048	143.923096
300.0000	616.005321	523.688309	432.384990	346.772551	270.193149	204.574493	150.550415
350.0000	612.796899	523.725559	435.132357	351.510006	276.142352	211.008687	156.872206
400.0000	609.285193	523.307668	437.336095	355.683361	281.567003	217.001215	162.858266
450.0000	605.380077	522.365237	438.940905	359.244933	286.422089	222.507036	168.463020
500.0000	600.940311	520.781243	439.848573	362.109674	290.631120	227.455323	173.620226
550.0000	595.763389	518.381818	439.909617	364.147643	294.079388	231.743536	178.237859
600.0000	589.577972	514.929362	438.917026	365.178309	296.608831	235.232849	182.194122
650.0000	582.041768	510.120593	436.604395	364.968739	298.016255	237.746420	185.335822
700.0000	572.747973	503.592371	432.651024	363.237903	298.056845	239.072179	187.480520
750.0000	561.243074	494.937851	426.696255	359.669110	296.454719	238.971592	188.423672
800.0000	547.057628	483.734459	418.364396	353.931790	292.921589	237.195343	187.951569
850.0000	529.749690	469.583394	407.300005	345.712408	287.183353	233.505802	185.859981
900.0000	508.957921	452.158003	393.211153	334.752483	279.012902	227.704824	181.977311
950.0000	484.458672	431.255831	375.916075	320.889650	268.265640	219.663952	176.189834
1000.0000	456.219032	406.847092	355.386697	304.096099	254.912844	209.352877	168.465572
1050.0000	424.436732	379.111274	331.781671	284.507878	239.067234	196.861382	158.872826
1100.0000	389.558421	348.454140	305.461944	262.438951	220.995466	182.410280	147.589604
1150.0000	352.270326	315.499656	276.983984	238.375663	201.113755	166.348075	134.901186
1200.0000	313.459429	281.055104	247.069021	212.950152	179.965331	149.132219	121.184854
1250.0000	274.148276	246.052170	216.550774	186.894797	158.181472	131.296391	106.882920
1300.0000	235.411276	211.471093	186.307933	160.983183	136.430798	113.407714	92.467277
1350.0000	198.283816	178.258157	157.190545	135.965566	115.363673	96.019700	78.400273
1400.0000	163.676866	147.248007	129.950553	112.507818	95.559458	79.627453	65.097315
1450.0000	132.308625	119.101313	105.185896	91.142119	77.483741	64.631191	52.896279
1500.0000	104.661506	94.265357	83.304943	72.235386	61.460747	51.312508	42.037424
1550.0000	80.968156	72.960919	64.514360	55.978173	47.663322	39.825479	32.655586
1600.0000	61.225263	55.194397	48.829476	42.393305	36.119922	30.202128	24.784323

SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET						
	750.0000	775.0000	800.0000	825.0000	850.0000	875.0000	900.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	69.758089	43.936934	26.668815	15.599710	8.793629	4.777036	2.500851
50.0000	76.370655	49.118280	30.513181	18.312183	10.619335	5.952161	3.225607
100.0000	82.896076	54.326777	34.454442	21.152175	12.574552	7.241570	4.041862
150.0000	89.312902	59.537713	38.470017	24.102292	14.647958	8.639387	4.947792
200.0000	95.601712	64.728015	42.538105	27.145020	16.827462	10.138642	5.940450
250.0000	101.743493	69.875071	46.636912	30.262276	19.099936	11.731108	7.015645
300.0000	107.717523	74.955111	50.743503	33.434617	21.450700	13.406946	8.167680
350.0000	113.498630	79.941057	54.832188	36.640091	23.862707	15.154134	9.388944
400.0000	119.053769	84.799784	58.872425	39.852673	26.315412	16.957647	10.669308
450.0000	124.337962	89.488855	62.826274	43.040342	28.783342	18.798414	11.995366
500.0000	129.289877	93.952922	66.645562	46.162905	31.234465	20.652107	13.349543
550.0000	133.827516	98.120214	70.269090	49.169838	33.628556	22.487926	14.709198
600.0000	137.844828	101.899725	73.620383	51.998522	35.915849	24.267581	16.045867
650.0000	141.210245	105.179956	76.606636	54.573404	38.036381	25.944794	17.324879
700.0000	143.768302	107.830119	79.119609	56.806656	39.920475	27.465644	18.505596
750.0000	145.345359	109.704651	81.039140	58.600865	41.490771	28.770072	19.542511
800.0000	145.760094	110.651581	82.239708	59.854108	42.666088	29.794773	20.387368
850.0000	144.838708	110.524732	82.600065	60.467431	43.367132	30.477481	20.992324
900.0000	142.433885	109.198981	82.015320	60.354263	43.523722	30.762418	21.313991
950.0000	138.445527	106.587017	80.410271	59.450851	43.082814	30.606371	21.317960
1000.0000	132.840457	102.655343	77.752178	57.726300	42.016278	29.984621	20.983246
1050.0000	125.667822	97.436869	74.060922	55.190638	40.327199	28.895784	20.305956
1100.0000	117.067094	91.037619	69.414530	51.899314	38.053491	27.364677	19.301519
1150.0000	107.266403	83.635678	63.948607	47.952979	35.267917	25.442494	18.004958
1200.0000	96.570348	75.471660	57.849057	43.492056	32.074146	23.204010	16.468948
1250.0000	85.338178	66.831411	51.338615	38.686474	28.599073	20.741960	14.759805
1300.0000	73.954955	58.022970	44.658798	33.721780	24.982348	18.159298	12.951864
1350.0000	62.799590	49.350920	38.049691	28.783502	21.364500	15.560342	11.121010
1400.0000	52.214174	41.091675	31.730380	24.041936	17.875320	13.042070	9.338266
1450.0000	42.478748	33.473028	25.882661	19.639395	14.624057	10.686710	7.664310
1500.0000	33.794582	26.660462	20.640016	15.681478	11.692625	8.556556	6.145572
1550.0000	26.277441	20.751421	16.082831	12.233150	9.132445	6.691462	4.812303
1600.0000	19.960602	15.777423	12.239784	9.319587	6.964905	5.109050	3.678601

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********************************************************************************
*    SAMPLE PROBLEM 8B.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER           *
*          OF INFINITE WIDTH WITH A CONTINUOUS GAUSSIAN SOURCE                 *
*                                                                              *
*    MODEL PARAMETERS:  V=4.0 FEET PER DAY, DX=150.0 FEET**2 PER DAY          *
*          DY=30.0 FEET**2 PER DAY, WS=65 FEET, YC=450 FEET,                   *
*                    C0=1000.0 MILLIGRAMS PER LITER                            *
*                                                                              *
*          PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 11:59:08                *
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ANALYTICAL SOLUTION TO THE TWO-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH WITH A SOLUTE SOURCE HAVING A GAUSSIAN CONCENTRATION DISTRIBUTION LOCATED AT X=0.0 AND CENTERED ABOUT Y=YC

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 33
NUMBER OF Y-COORDINATES (NY) = 37
NUMBER OF TIME VALUES (NT) = 1
NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = 104

MAXIMUM SOLUTE CONCENTRATION AT THE BOUNDARY CM) = 1.000000E+03 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 4.000000E+00 FEET PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 1.500000E+02 FEET**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 3.000000E+01 FEET**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER WIDTH (W) IS INFINITE
SOLUTE SOURCE IS CENTERED AT Y = 4.500000E+02 FEET
STANDARD DEVIATION OF GAUSSIAN DISTRIBUTION (SIGMA) = 6.500000E+01 FEET

PLOT SCALING FACTOR FOR X (XSCLP) = 2.500000E+02
PLOT SCALING FACTOR FOR Y (YSCLP) = 2.500000E+02
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	50.0000	100.0000	150.0000	200.0000	250.0000	300.0000	350.0000
400.0000	450.0000	500.0000	550.0000	600.0000	650.0000	700.0000	750.0000
800.0000	850.0000	900.0000	950.0000	1000.0000	1050.0000	1100.0000	1150.0000
1200.0000	1250.0000	1300.0000	1350.0000	1400.0000	1450.0000	1500.0000	1550.0000
1600.0000							

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	25.0000	50.0000	75.0000	100.0000	125.0000	150.0000	175.0000
200.0000	225.0000	250.0000	275.0000	300.0000	325.0000	350.0000	375.0000
400.0000	425.0000	450.0000	475.0000	500.0000	525.0000	550.0000	575.0000
600.0000	625.0000	650.0000	675.0000	700.0000	725.0000	750.0000	775.0000
800.0000	825.0000	850.0000	875.0000	900.0000			

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS
300.0000

SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS

SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER

X-COORIDNATE, IN FEET	Y-COORDINATE, IN FEET: 0.0000	25.0000	50.0000	75.0000	100.0000	125.0000	150.0000	175.0000
0.0000	0.000000	0.000001	0.000006	0.000059	0.000506	0.003727	0.023680	0.129776
50.0000	0.000872	0.002138	0.005318	0.013522	0.035438	0.096477	0.272970	0.792857
100.0000	0.003089	0.007478	0.018259	0.045100	0.112929	0.286625	0.734335	1.881000
150.0000	0.007743	0.018413	0.043911	0.105100	0.252330	0.606141	1.449290	3.420997
200.0000	0.016356	0.038081	0.088466	0.204947	0.472660	1.081569	2.442921	5.408293
250.0000	0.030860	0.070234	0.158777	0.356120	0.790687	1.731798	3.723855	7.813942
300.0000	0.053513	0.118981	0.261777	0.569072	1.219403	2.566952	5.285720	10.591414
350.0000	0.086744	0.188451	0.403832	0.852206	1.766901	3.588045	7.109155	13.682120
400.0000	0.132938	0.282379	0.590059	1.210986	2.435585	4.787058	9.163713	17.019231
450.0000	0.194164	0.403653	0.823671	1.647159	3.221606	6.147155	11.409301	20.529794
500.0000	0.271887	0.553860	1.105359	2.158110	4.114467	7.642883	13.797042	24.135433
550.0000	0.366658	0.732845	1.432760	2.736352	5.096750	9.240313	16.269670	27.752083
600.0000	0.477840	0.938350	1.800056	3.369201	6.144015	10.897225	18.761721	31.289336
650.0000	0.603393	1.165758	2.197748	4.038699	7.224986	12.563539	21.199895	34.650065
700.0000	0.739761	1.408025	2.612712	4.721906	8.302168	14.182239	23.504057	37.730989
750.0000	0.881899	1.655819	3.028570	5.391631	9.333057	15.691079	25.589273	40.424779
800.0000	1.023474	1.897943	3.426458	6.017716	10.272081	17.025248	27.369199	42.624107
850.0000	1.157243	2.122014	3.786183	6.568857	11.073269	18.121059	28.760902	44.227732
900.0000	1.275604	2.315403	4.087729	7.014892	11.693567	18.920504	29.690875	45.148263
950.0000	1.371263	2.466332	4.312984	7.329382	12.096506	19.376273	30.101702	45.320838
1000.0000	1.437949	2.565024	4.447491	7.492178	12.255813	19.456662	29.958538	44.711526
1050.0000	1.471093	2.604750	4.482009	7.491646	12.158464	19.149615	29.254355	43.324059
1100.0000	1.468346	2.582608	4.413628	7.326178	11.806655	18.465213	28.012998	41.203542
1150.0000	1.429876	2.499925	4.246255	7.004730	11.218301	17.436020	26.289246	38.436108
1200.0000	1.358377	2.362171	3.990346	6.546210	10.425862	16.115041	24.165554	35.144073
1250.0000	1.258784	2.178414	3.661911	5.977766	9.473544	14.571396	21.745668	31.476905
1300.0000	1.137727	1.960365	3.280896	5.332164	8.413229	12.884220	19.145836	27.599020
1350.0000	1.002822	1.721153	2.869189	4.644638	7.299665	11.135605	16.484788	23.676042
1400.0000	0.861881	1.474034	2.448524	3.949637	6.185599	9.403537	13.873847	19.861370
1450.0000	0.722183	1.231179	2.038586	3.277928	5.117493	7.755790	11.408477	16.284851
1500.0000	0.589880	1.002728	1.655535	2.654394	4.132353	6.245491	9.162293	13.044919
1550.0000	0.469610	0.796196	1.311124	2.096768	3.255970	4.908784	7.184061	10.204888
1600.0000	0.364344	0.616258	1.012423	1.615313	2.502616	3.764619	5.497726	7.793406

SAMPLE PROBLEM 8B.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER OF INFINITE WIDTH
WITH A CONTINUOUS GAUSSIAN SOURCE--CONTINUED

	SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS							
X-COORDINATE,	Y-COORDINATE, IN FEET							
IN FEET	200.0000	225.0000	250.0000	275.0000	300.0000	325.0000	350.0000	375.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER							
0.0000	0.613427	2.500851	8.793629	26.668815	69.758089	157.376788	306.225980	513.923697
50.0000	2.305483	6.506740	17.313880	42.448335	94.309941	187.729866	332.242804	520.121341
100.0000	4.746848	11.592871	26.903731	58.356764	116.699210	212.850911	351.238450	521.301444
150.0000	7.885965	17.531593	37.104888	73.853468	136.759161	233.493402	364.918208	519.175747
200.0000	11.635253	24.093471	47.550247	88.597647	154.515015	250.359179	374.557223	514.877709
250.0000	15.888255	31.066334	57.958139	102.390243	170.092150	264.061993	381.105961	509.161434
300.0000	20.531518	38.263254	68.118557	115.127389	183.658962	275.117786	385.269526	502.525953
350.0000	25.451655	45.523381	77.876810	126.764568	195.391428	283.946426	387.565038	495.292817
400.0000	30.538510	52.708459	87.117252	137.289349	205.450827	290.877217	388.361722	487.653870
450.0000	35.685434	59.696961	95.748256	146.700584	213.969220	296.154542	387.907960	479.699748
500.0000	40.787620	66.377190	103.688940	154.992455	221.039590	299.942404	386.348916	471.436073
550.0000	45.739429	72.640388	110.858001	162.142432	226.709192	302.327961	383.738046	462.792353
600.0000	50.431603	78.374779	117.165100	168.102921	230.975819	303.325124	380.045597	453.627707
650.0000	54.749244	83.461443	122.505428	172.796907	233.787581	302.879849	375.167143	443.736985
700.0000	58.571428	87.772917	126.758204	176.118302	235.047180	300.879007	368.934980	432.860507
750.0000	61.773184	91.175265	129.789795	177.937703	234.621772	297.164526	361.134787	420.700008
800.0000	64.230272	93.534046	131.461850	178.113987	232.359050	291.553903	351.528998	406.942335
850.0000	65.826823	94.724102	131.644201	176.511436	228.109316	283.867025	339.887018	391.290956
900.0000	66.465301	94.642414	130.231525	173.021127	221.752133	273.957804	326.020641	373.503423
950.0000	66.077692	93.222511	127.161814	167.584234	213.224830	261.747575	309.821308	353.431000
1000.0000	64.636334	90.448344	122.434002	160.213997	202.549054	247.255923	291.294326	331.055069
1050.0000	62.162486	86.365178	116.121674	151.012652	189.851037	230.623975	270.584479	306.514173
1100.0000	58.730872	81.085191	108.379990	140.179860	175.371507	212.125518	247.987861	280.115978
1150.0000	54.468831	74.786079	99.443724	128.010131	159.462365	192.162689	223.946275	252.330188
1200.0000	49.549526	67.702003	89.615652	114.878395	142.569203	171.245224	199.023137	223.761255
1250.0000	44.179672	60.107497	79.246147	101.214852	125.201075	149.955019	173.862918	195.103149
1300.0000	38.583177	52.296236	68.706405	87.472129	107.891248	128.900352	149.139082	167.081677
1350.0000	32.982876	44.557474	58.358914	74.089228	91.154276	108.666046	125.497648	140.392201
1400.0000	27.582841	37.153412	48.529257	61.457286	75.445448	89.766573	103.504287	115.641433
1450.0000	22.553639	30.300520	39.483059	49.891812	61.128117	72.608513	83.602154	93.301231
1500.0000	18.022301	24.157079	31.410885	39.614788	48.452938	57.466944	66.085628	73.679989
1550.0000	14.067892	18.818039	24.422424	30.748214	37.550818	44.477837	51.092180	56.914054
1600.0000	10.722625	14.317059	18.549700	23.318720	28.439058	33.645773	38.611574	42.978207

	SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS						
X-COORDINATE,	Y-COORDINATE, IN FEET						
IN FEET	400.0000	425.0000	450.0000	475.0000	500.0000	525.0000	550.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	743.893062	928.704665	1000.000000	928.704665	743.893062	513.923697	306.225980
50.0000	717.838661	871.589208	929.950741	871.589208	717.838661	520.121341	332.242804
100.0000	693.012200	822.956642	871.621384	822.956642	693.012200	521.301444	351.238450
150.0000	669.683281	781.022783	822.250323	781.022783	669.683281	519.175747	364.918208
200.0000	647.911956	744.456411	779.864235	744.456411	647.911956	514.877709	374.557223
250.0000	627.648205	712.243052	743.015305	712.243052	627.648205	509.161434	381.105961
300.0000	608.783283	683.590967	710.612061	683.590967	608.783283	502.525953	385.269526
350.0000	591.174499	657.864012	681.806284	657.864012	591.174499	495.292817	387.565038
400.0000	574.654811	634.531757	655.914263	634.531757	574.654811	487.653870	388.361722
450.0000	559.033638	613.130930	632.359650	613.130930	559.033638	479.699748	387.907960
500.0000	544.092809	593.234777	610.630560	593.234777	544.092809	471.436073	386.348916
550.0000	529.580612	574.428826	590.247090	574.428826	529.580612	462.792353	383.738046
600.0000	515.206593	556.292963	570.737751	556.292963	515.206593	453.627707	380.045597
650.0000	500.639775	538.390752	551.624912	538.390752	500.639775	443.736985	375.167143
700.0000	485.512879	520.267531	532.420231	520.267531	485.512879	432.860507	368.934980
750.0000	469.434715	501.458748	512.631227	501.458748	469.434715	420.700008	361.134787
800.0000	452.011998	481.509302	491.779510	481.509302	452.011998	406.942335	351.528998
850.0000	432.880298	460.003256	469.429884	460.003256	432.880298	391.290956	339.887018
900.0000	411.741939	436.601360	445.227619	436.601360	411.741939	373.503423	326.020641
950.0000	388.406619	411.081834	418.939184	411.081834	388.406619	353.431000	309.821308
1000.0000	362.828874	383.378142	390.490072	383.378142	362.828874	331.055069	291.294326
1050.0000	335.135726	353.606780	359.992573	353.606780	335.135726	306.514173	270.584479
1100.0000	305.638383	322.078600	327.756940	322.078600	305.638383	280.115978	247.987861
1150.0000	274.823704	289.289211	294.281388	289.289211	274.823704	252.330188	223.946275
1200.0000	243.324208	255.887158	260.219632	255.887158	243.324208	223.761255	199.023137
1250.0000	211.869049	222.622410	226.328507	222.622410	211.869049	195.103149	173.862918
1300.0000	181.221855	190.281297	193.401889	190.281297	181.221855	167.081677	149.139082
1350.0000	152.113820	159.616620	162.199785	159.616620	152.113820	140.392201	125.497648
1400.0000	125.181336	131.282612	133.382368	131.282612	125.181336	115.641433	103.504287
1450.0000	100.916604	105.783517	107.457857	105.783517	100.916604	93.301231	83.602154
1500.0000	79.637192	83.441969	84.750489	83.441969	79.637192	73.679989	66.085628
1550.0000	61.477088	64.389802	65.391248	64.389802	61.477088	56.914054	51.092180
1600.0000	46.398157	48.580134	49.330153	48.580134	46.398157	42.978207	38.611574

	SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS						
X-COORDINATE,	Y-COORDINATE, IN FEET						
IN FEET	575.0000	600.0000	625.0000	650.0000	675.0000	700.0000	725.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	157.376788	69.758089	26.668815	8.793629	2.500851	0.613427	0.129776
50.0000	187.729866	94.309941	42.448335	17.313880	6.506740	2.305483	0.792857
100.0000	212.850911	116.699210	58.356764	26.903731	11.592871	4.746848	1.881000
150.0000	233.493402	136.759161	73.853468	37.104888	17.531593	7.885965	3.420997
200.0000	250.359179	154.515015	88.597647	47.550247	24.093471	11.635253	5.408293
250.0000	264.061993	170.092150	102.390243	57.958139	31.066334	15.888255	7.813942
300.0000	275.117786	183.658962	115.127389	68.118557	38.263254	20.531518	10.591414
350.0000	283.946426	195.391428	126.764568	77.876810	45.523381	25.451655	13.682120
400.0000	290.877217	205.450827	137.289349	87.117252	52.708459	30.538510	17.019231
450.0000	296.154542	213.969220	146.700584	95.748256	59.696961	35.685434	20.529794
500.0000	299.942404	221.039590	154.992455	103.688940	66.377190	40.787620	24.135433
550.0000	302.327961	226.709192	162.142432	110.858001	72.640388	45.739429	27.752083
600.0000	303.325124	230.975819	168.102921	117.165100	78.374779	50.431603	31.289336
650.0000	302.879849	233.787581	172.796907	122.505428	83.461443	54.749244	34.650065
700.0000	300.879007	235.047180	176.118302	126.758204	87.772917	58.571428	37.730989
750.0000	297.164526	234.621772	177.937703	129.789795	91.175265	61.773184	40.424779
800.0000	291.553903	232.359050	178.113987	131.461850	93.534046	64.230272	42.624107
850.0000	283.867025	228.109316	176.511436	131.644201	94.724102	65.826823	44.227732
900.0000	273.957804	221.752133	173.021127	130.231525	94.642414	66.465301	45.148263
950.0000	261.747575	213.224830	167.584234	127.161814	93.222511	66.077692	45.320838
1000.0000	247.255923	202.549054	160.213997	122.434002	90.448344	64.636334	44.711526
1050.0000	230.623975	189.851037	151.012652	116.121674	86.365178	62.162486	43.324059
1100.0000	212.125518	175.371507	140.179860	108.379990	81.085191	58.730872	41.203542
1150.0000	192.162689	159.462365	128.010131	99.443724	74.786079	54.468831	38.436108
1200.0000	171.245224	142.569203	114.878395	89.615652	67.702003	49.549526	35.144073
1250.0000	149.955019	125.201075	101.214852	79.246147	60.107497	44.179672	31.476905
1300.0000	128.900352	107.891248	87.472129	68.706405	52.296236	38.583177	27.599020
1350.0000	108.666046	91.154276	74.089228	58.358914	44.557474	32.982876	23.676042
1400.0000	89.766573	75.445448	61.457286	48.529257	37.153412	27.582841	19.861370
1450.0000	72.608513	61.128117	49.891812	39.483059	30.300520	22.553639	16.284851
1500.0000	57.466944	48.452938	39.614788	31.410885	24.157079	18.022301	13.044919
1550.0000	44.477837	37.550818	30.748214	24.422424	18.818039	14.067892	10.204888
1600.0000	33.645773	28.439058	23.318720	18.549700	14.317059	10.722625	7.793406

	SOLUTE CONCENTRATION AT TIME = 300.0000 DAYS						
X-COORDINATE,	Y-COORDINATE, IN FEET						
IN FEET	750.0000	775.0000	800.0000	825.0000	850.0000	875.0000	900.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
0.0000	0.023680	0.003727	0.000506	0.000059	0.000006	0.000001	0.000000
50.0000	0.272970	0.096477	0.035438	0.013522	0.005318	0.002138	0.000872
100.0000	0.734335	0.286625	0.112929	0.045100	0.018259	0.007478	0.003089
150.0000	1.449290	0.606141	0.252330	0.105100	0.043911	0.018413	0.007743
200.0000	2.442921	1.081569	0.472660	0.204947	0.088466	0.038081	0.016356
250.0000	3.723855	1.731798	0.790687	0.356120	0.158777	0.070234	0.030860
300.0000	5.285720	2.566952	1.219403	0.569072	0.261777	0.118981	0.053513
350.0000	7.109155	3.588045	1.766901	0.852206	0.403832	0.188451	0.086744
400.0000	9.163713	4.787058	2.435585	1.210986	0.590059	0.282379	0.132938
450.0000	11.409301	6.147155	3.221606	1.647159	0.823671	0.403653	0.194164
500.0000	13.797042	7.642883	4.114467	2.158110	1.105359	0.553860	0.271887
550.0000	16.269670	9.240313	5.096750	2.736352	1.432760	0.732845	0.366658
600.0000	18.761721	10.897225	6.144015	3.369201	1.800056	0.938350	0.477840
650.0000	21.199895	12.563539	7.224986	4.038699	2.197748	1.165758	0.603393
700.0000	23.504057	14.182239	8.302168	4.721906	2.612712	1.408025	0.739761
750.0000	25.589273	15.691079	9.333057	5.391631	3.028570	1.655819	0.881899
800.0000	27.369199	17.025248	10.272081	6.017716	3.426458	1.897943	1.023474
850.0000	28.760902	18.121059	11.073269	6.568857	3.786183	2.122014	1.157243
900.0000	29.690875	18.920504	11.693567	7.014892	4.087729	2.315403	1.275604
950.0000	30.101702	19.376273	12.096506	7.329382	4.312984	2.466332	1.371263
1000.0000	29.958538	19.456662	12.255813	7.492178	4.447491	2.565024	1.437949
1050.0000	29.254355	19.149615	12.158464	7.491646	4.482009	2.604750	1.471093
1100.0000	28.012998	18.465213	11.806655	7.326178	4.413628	2.582608	1.468346
1150.0000	26.289246	17.436020	11.218301	7.004730	4.246255	2.499925	1.429876
1200.0000	24.165554	16.115041	10.425862	6.546210	3.990346	2.362171	1.358377
1250.0000	21.745668	14.571396	9.473544	5.977766	3.661911	2.178414	1.258784
1300.0000	19.145836	12.884220	8.413229	5.332164	3.280896	1.960365	1.137727
1350.0000	16.484788	11.135605	7.299665	4.644638	2.869189	1.721153	1.002822
1400.0000	13.873847	9.403537	6.185599	3.949637	2.448524	1.474034	0.861881
1450.0000	11.408477	7.755790	5.117493	3.277928	2.038586	1.231179	0.722183
1500.0000	9.162293	6.245491	4.132353	2.654394	1.655535	1.002728	0.589880
1550.0000	7.184061	4.908784	3.255970	2.096768	1.311124	0.796196	0.469610
1600.0000	5.497726	3.764619	2.502616	1.615313	1.012423	0.616258	0.364344

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*************************************************************************
*      SAMPLE PROBLEM 9.--SOLUTE TRANSPORT IN AN INFINITE AQUIFER       *
*             WITH MULTIPLE POINT SOURCES OF FINITE DURATION            *
*                                                                       *
*   MODEL PARAMETERS:  V=0.1 FOOT PER DAY, DX=0.06 FEET**2 PER DAY,     *
*          DY=0.003 FEET**2 PER DAY, DZ=0.0006 FEET**2 PER DAY,         *
*         QM=4.0 FEET**3 PER DAY, C0=1000.0 MILLIGRAMS PER LITER        *
*                                                                       *
*        PROGRAM RUN ON MONDAY, OCTOBER 12, 1987, AT 13:39:07           *
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ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR AN AQUIFER OF INFINITE EXTENT WITH A CONTINUOUS POINT SOURCE AT XC,YC,ZC

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 21
NUMBER OF Y-COORDINATES (NY) = 21
NUMBER OF Z-COORDINATES (NZ) = 1
NUMBER OF TIME VALUES (NT) = 1
NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = 104

SOLUTE CONCENTRATION IN INJECTED FLUID (C0) = 1.000000E+03 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 1.000000E-01 FOOT PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 6.000000E-02 FEET**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 3.000000E-03 FEET**2 PER DAY
DISPERSION IN THE Z-DIRECTION (DZ) = 6.000000E-04 FEET**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER IS OF INFINITE EXTENT
CONTINUOUS POINT SOURCE IS AT X = 0.000000E-01 FEET, Y = 1.000000E+02 FEET, AND Z = 1.000000E+01 FEET
FLUID INJECTION RATE (QM) = 4.000000E+00 FEET**3 PER DAY

PLOT SCALING FACTOR FOR X (XSCLP) = 5.000000E+00
PLOT SCALING FACTOR FOR Y (YSCLP) = 5.000000E+00
CONTOUR INCREMENT (DELTA) = 1.000000E-02 MILLIGRAM PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

20.0000	22.0000	24.0000	26.0000	28.0000	30.0000	32.0000	34.0000
36.0000	38.0000	40.0000	42.0000	44.0000	46.0000	48.0000	50.0000
52.0000	54.0000	56.0000	58.0000	60.0000			

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

90.0000	91.0000	92.0000	93.0000	94.0000	95.0000	96.0000	97.0000
98.0000	99.0000	100.0000	101.0000	102.0000	103.0000	104.0000	105.0000
106.0000	107.0000	108.0000	109.0000	110.0000			

Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

10.0000

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS

400.0000

SOLUTE CONCENTRATION AT TIME = 400.0000 DAYS AND AT Z = 10.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET						
	90.0000	91.0000	92.0000	93.0000	94.0000	95.0000	96.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
20.0000	0.000001	0.000020	0.000296	0.002949	0.019440	0.085396	0.253395
22.0000	0.000002	0.000043	0.000653	0.006501	0.042858	0.188267	0.558646
24.0000	0.000004	0.000088	0.001324	0.013184	0.086924	0.381835	1.133022
26.0000	0.000007	0.000164	0.002470	0.024599	0.162183	0.712428	2.113993
28.0000	0.000012	0.000281	0.004240	0.042223	0.278377	1.222840	3.628543
30.0000	0.000019	0.000444	0.006696	0.066671	0.439569	1.930911	5.729603
32.0000	0.000028	0.000645	0.009726	0.096849	0.638533	2.804907	8.323011
34.0000	0.000038	0.000862	0.012998	0.129425	0.853304	3.748336	11.122444
36.0000	0.000046	0.001060	0.015979	0.159112	1.049031	4.608105	13.673629
38.0000	0.000052	0.001198	0.018072	0.179950	1.186415	5.211589	15.464337
40.0000	0.000055	0.001247	0.018803	0.187225	1.234381	5.422282	16.089510
42.0000	0.000052	0.001193	0.017997	0.179201	1.181478	5.189889	15.399919
44.0000	0.000046	0.001051	0.015847	0.157791	1.040319	4.569811	13.559955
46.0000	0.000037	0.000851	0.012837	0.127817	0.842697	3.701713	10.984046
48.0000	0.000028	0.000634	0.009566	0.095249	0.627973	2.758492	8.185230
50.0000	0.000019	0.000435	0.006558	0.065297	0.430502	1.891060	5.611306
52.0000	0.000012	0.000274	0.004136	0.041181	0.271503	1.192623	3.538846
54.0000	0.000007	0.000159	0.002400	0.023892	0.157521	0.691937	2.053166
56.0000	0.000004	0.000085	0.001281	0.012752	0.084075	0.369313	1.095851
58.0000	0.000002	0.000042	0.000629	0.006262	0.041282	0.181337	0.538076
60.0000	0.000001	0.000019	0.000284	0.002828	0.018647	0.081912	0.243053

SAMPLE PROBLEM 9.--SOLUTE TRANSPORT IN AN INFINITE AQUIFER WITH MULTIPLE POINT SOURCES OF FINITE DURATION--CONTINUED

SOLUTE CONCENTRATION AT TIME = 400.0000 DAYS AND AT Z = 10.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 97.0000	98.0000	99.0000	100.0000	101.0000	102.0000	103.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
20.0000	0.522808	0.791993	0.957310	1.006805	0.957310	0.791993	0.522808
22.0000	1.152604	1.746060	2.110526	2.219644	2.110526	1.746060	1.152604
24.0000	2.337661	3.541283	4.280478	4.501786	4.280478	3.541283	2.337661
26.0000	4.361606	6.607324	7.986513	8.399429	7.986513	6.607324	4.361606
28.0000	7.486434	11.341072	13.708367	14.417112	13.708367	11.341072	7.486434
30.0000	11.821352	17.907963	21.646009	22.765143	21.646009	17.907963	11.821352
32.0000	17.172082	26.013690	31.443697	33.069390	31.443697	26.013690	17.172082
34.0000	22.947878	34.763339	42.019718	44.192210	42.019718	34.763339	22.947878
36.0000	28.211484	42.737078	51.657869	54.328671	51.657869	42.737078	28.211484
38.0000	31.906065	48.333927	58.422987	61.443559	58.422987	48.333927	31.906065
40.0000	33.195909	50.287881	60.784803	63.927487	60.784803	50.287881	33.195909
42.0000	31.773128	48.132529	58.179551	61.187541	58.179551	48.132529	31.773128
44.0000	27.976899	42.381684	51.228295	53.876894	51.228295	42.381684	27.976899
46.0000	22.662273	34.330650	41.496716	43.642176	41.496716	34.330650	22.662273
48.0000	16.887748	25.582927	30.923024	32.521804	30.923024	25.582927	16.887748
50.0000	11.577227	17.538116	21.198966	22.294995	21.198966	17.538116	11.577227
52.0000	7.301330	11.060641	13.369403	14.060629	13.369403	11.060641	7.301330
54.0000	4.236080	6.417153	7.756649	8.157684	7.756649	6.417153	4.236080
56.0000	2.260951	3.425069	4.140007	4.354054	4.140007	3.425069	2.260951
58.0000	1.110154	1.681749	2.032792	2.137892	2.032792	1.681749	1.110154
60.0000	0.501465	0.759658	0.918227	0.965702	0.918227	0.759658	0.501465

SOLUTE CONCENTRATION AT TIME = 400.0000 DAYS AND AT Z = 10.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 104.0000	105.0000	106.0000	107.0000	108.0000	109.0000	110.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER						
20.0000	0.253395	0.085396	0.019440	0.002949	0.000296	0.000020	0.000001
22.0000	0.558646	0.188267	0.042858	0.006501	0.000653	0.000043	0.000002
24.0000	1.133022	0.381835	0.086924	0.013184	0.001324	0.000088	0.000004
26.0000	2.113993	0.712428	0.162183	0.024599	0.002470	0.000164	0.000007
28.0000	3.628543	1.222840	0.278377	0.042223	0.004240	0.000281	0.000012
30.0000	5.729603	1.930911	0.439569	0.066671	0.006696	0.000444	0.000019
32.0000	8.323011	2.804907	0.638533	0.096849	0.009726	0.000645	0.000028
34.0000	11.122444	3.748336	0.853304	0.129425	0.012998	0.000862	0.000038
36.0000	13.673629	4.608105	1.049031	0.159112	0.015979	0.001060	0.000046
38.0000	15.464337	5.211589	1.186415	0.179950	0.018072	0.001198	0.000052
40.0000	16.089510	5.422282	1.234381	0.187225	0.018803	0.001247	0.000055
42.0000	15.399919	5.189889	1.181478	0.179201	0.017997	0.001193	0.000052
44.0000	13.559955	4.569811	1.040319	0.157791	0.015847	0.001051	0.000046
46.0000	10.984046	3.701713	0.842697	0.127817	0.012837	0.000851	0.000037
48.0000	8.185230	2.758492	0.627973	0.095249	0.009566	0.000634	0.000028
50.0000	5.611306	1.891060	0.430502	0.065297	0.006558	0.000435	0.000019
52.0000	3.538846	1.192623	0.271503	0.041181	0.004136	0.000274	0.000012
54.0000	2.053166	0.691937	0.157521	0.023892	0.002400	0.000159	0.000007
56.0000	1.095851	0.369313	0.084075	0.012752	0.001281	0.000085	0.000004
58.0000	0.538076	0.181337	0.041282	0.006262	0.000629	0.000042	0.000002
60.0000	0.243053	0.081912	0.018647	0.002828	0.000284	0.000019	0.000001

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*     SAMPLE PROBLEM 10.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER     *
*            OF FINITE WIDTH AND HEIGHT WITH A "PATCH" SOURCE             *
*                                                                         *
*   MODEL PARAMETERS:  V=1.0 FOOT PER DAY, DX=200.0 FEET**2 PER DAY,      *
*    DY=60.0 FEET**2 PER DAY, DZ=10.0 FEET**2 PER DAY, W=3000 FEET,       *
* H=100 FEET, WS=1600 FEET, HS=50 FEET, CO=1000.0 MILLIGRAMS PER LITER    *
*                                                                         *
*        PROGRAM RUN ON WEDNESDAY, OCTOBER 21, 1987, AT 16:31:04          *
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ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-INFINITE AQUIFER OF FINITE WIDTH AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 29
NUMBER OF Y-COORDINATES (NY) = 27
NUMBER OF Z-COORDINATES (NZ) = 2
NUMBER OF TIME VALUES (NT) = 1
NUMBER OF TERMS IN INNER INFINTE SERIES SUMMATION (NMAX) = 350
NUMBER OF TERMS IN OUTER INFINTE SERIES SUMMATION (MMAX) = 350

SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) = 1.000000E+03 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 1.000000E+00 FEET PER DAY
DISPERSION IN THE X-DIRECTION (DX) = 2.000000E+02 FEET**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 6.000000E+01 FEET**2 PER DAY
DISPERSION IN THE Z-DIRECTION (DZ) = 1.000000E+01 FEET**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 0.000000E-01 PER DAY

AQUIFER WIDTH (W) = 3.000000E+03 FEET
AQUIFER HEIGHT (H) = 1.000000E+02 FEET
SOLUTE SOURCE IS CENTERED AT Y = 1.200000E+03 FEET AND Z = 7.500000E+01 FEET
FINITE-WIDTH OF SOLUTE SOURCE (WS) = 1.600000E+03 FEET
FINITE-HEIGHT OF SOLUTE SOURCE (HS) = 5.000000E+01 FEET

PLOT SCALING FACTOR FOR X (XSCLP) = 7.500000E+02
PLOT SCALING FACTOR FOR Y (YSCLP) = 7.500000E+02
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	150.0000	300.0000	450.0000	600.0000	750.0000	900.0000	1050.0000
1200.0000	1350.0000	1500.0000	1650.0000	1800.0000	1950.0000	2100.0000	2250.0000
2400.0000	2550.0000	2700.0000	2850.0000	3000.0000	3150.0000	3300.0000	3450.0000
3600.0000	3750.0000	3900.0000	4050.0000	4200.0000			

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000
800.0000	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000
1600.0000	1700.0000	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000
2400.0000	2500.0000	2600.0000					

Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

75.0000 50.0000

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS

3000.0000

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS AND AT Z = 75.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.00000	0.00000	0.00000	0.00000	500.00000	1000.00000	1000.00000	1000.00000	1000.00000
150.0000	9.47415	13.01090	29.04485	92.65859	361.38612	630.24567	694.32876	711.44962	717.44405
300.0000	23.71327	31.12532	60.52232	139.73960	301.69333	463.99276	544.42538	576.56928	589.92546
450.0000	41.11684	51.22067	86.68839	160.75926	275.22229	390.32158	466.59285	506.86407	526.76023
600.0000	59.76766	71.07035	107.53819	173.38836	263.09237	353.77872	422.96179	466.42402	491.10479
750.0000	78.17116	89.58856	124.57449	182.89236	257.66041	333.78132	396.59959	440.66475	468.43368
900.0000	95.39584	106.32301	138.82555	190.71028	255.30266	321.61074	379.09696	422.49498	452.03119
1050.0000	110.93146	121.09365	150.82591	197.21687	254.20190	313.23128	366.18530	408.27935	438.62644
1200.0000	124.51832	133.82740	160.82532	202.47169	253.38317	306.61569	355.60537	396.06010	426.53647
1350.0000	136.02739	144.48906	168.92297	206.42634	252.26675	300.64360	346.07337	384.70253	414.81830
1500.0000	145.39139	153.05333	175.13798	208.98858	250.46233	294.62422	336.78928	373.48380	402.88014
1650.0000	152.57134	159.49607	179.44837	210.04929	247.67820	288.08532	327.20676	361.89901	390.30705
1800.0000	157.54466	163.79623	181.81649	209.50041	243.68513	280.68138	316.92832	349.57378	376.78761
1950.0000	160.30572	165.94369	182.20877	207.25071	238.30645	272.15655	305.66095	336.22897	362.08823
2100.0000	160.87282	165.95011	180.61261	203.24051	231.42007	262.33303	293.20040	321.67002	346.04913
2250.0000	159.29757	163.86020	177.05065	197.45476	222.96480	251.11027	279.42829	305.78725	328.58901
2400.0000	155.67386	159.76169	171.59205	189.93334	212.94701	238.46764	264.31353	288.55913	309.71136
2550.0000	150.14424	153.79210	164.35977	180.77751	201.44465	224.46604	247.91309	270.05410	289.50824
2700.0000	142.90248	146.14142	155.53317	170.15158	188.60717	209.24601	230.36927	250.42838	268.15914
2850.0000	134.19138	137.04985	145.34535	158.27932	174.65036	193.02102	211.90215	229.91821	245.92376
3000.0000	124.29579	126.80067	134.07546	145.43511	159.84608	176.06572	192.79667	208.82649	223.12848
3150.0000	113.53131	115.70867	122.03647	131.93059	144.50734	158.69946	173.38487	187.50418	200.14733
3300.0000	102.22967	104.10534	109.55960	118.09786	128.97006	141.26650	154.02475	166.32800	177.37895
3450.0000	90.72228	92.32216	96.97685	104.27096	113.57308	124.11447	135.07731	145.67641	155.22169
3600.0000	79.32366	80.67376	84.60354	90.76727	98.63828	107.57318	116.88408	125.90597	134.04911
3750.0000	68.31624	69.44258	72.72239	77.87073	84.45276	91.93559	99.74697	107.33033	114.18823
3900.0000	57.93809	58.86642	61.57061	65.81836	71.25452	77.44270	83.91240	90.20362	95.90238
4050.0000	48.37452	49.12996	51.33121	54.79105	59.22282	64.27339	69.56071	74.70952	79.38010
4200.0000	39.75410	40.36072	42.12881	44.90932	48.47371	52.53983	56.80147	60.95662	64.73049

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS AND AT Z = 75.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000	1000.00000
150.0000	719.90169	720.98839	721.45731	721.58847	721.45480	720.98231	719.88949	717.42069	711.40548
300.0000	595.86942	598.61576	599.82990	600.17321	599.82290	598.59882	595.83550	589.86065	576.44729
450.0000	536.55222	541.35457	543.55250	544.18386	543.53825	541.32014	536.48350	526.62953	506.61945
600.0000	504.48229	511.47465	514.80316	515.77687	514.77804	511.41414	504.36208	490.87753	466.00189
750.0000	484.78426	493.86017	498.35346	499.69239	498.31304	493.76315	484.59257	468.07392	440.00260
900.0000	470.63877	481.52921	487.11990	488.81453	487.05904	481.38375	470.35318	451.49962	421.52665
1050.0000	458.80920	471.16370	477.71120	479.72535	477.62435	470.95702	458.40618	437.88301	406.94004
1200.0000	447.70081	461.14752	468.46938	470.74902	468.35087	460.86684	447.15745	425.54356	394.29181
1350.0000	436.46845	450.65027	458.54821	461.03009	458.39274	450.28384	435.76438	413.54402	382.45938
1500.0000	424.60946	439.20203	447.48006	450.09874	447.28317	438.74024	423.72879	401.30148	370.73665
1650.0000	411.78402	426.50240	434.97773	437.67033	434.73624	425.93879	410.71710	388.41251	358.63887
1800.0000	397.74218	412.34130	420.84990	423.55900	420.56237	411.67342	396.48701	374.57906	345.81388
1950.0000	382.30046	396.57270	404.97143	407.64658	404.63842	395.80274	380.86347	359.58179	332.00516
2100.0000	365.34053	379.11248	387.27926	389.87752	386.90348	378.24748	363.73685	343.27501	317.03964
2250.0000	346.81646	359.94568	367.77873	370.26472	367.36506	358.99741	345.06940	325.59028	300.82645
2400.0000	326.76300	339.13537	346.55216	348.89776	346.10739	338.11981	324.90294	306.54181	283.35896
2550.0000	305.30088	316.82856	323.76468	325.94863	323.29719	315.76504	303.36366	286.22943	264.71560
2700.0000	282.63674	293.25652	299.66468	301.67213	299.18394	292.16662	280.66156	264.83694	245.05709
2850.0000	259.05631	268.72839	274.57738	276.39938	274.09342	267.63467	257.08355	242.62469	224.61856
3000.0000	234.91107	243.61775	248.89147	250.52439	248.41424	242.54243	232.97994	219.91609	203.69642
3150.0000	210.59891	218.34318	223.03947	224.48448	222.57832	217.30690	208.74537	197.07898	182.63047
3300.0000	186.54060	193.34439	197.47375	198.73613	197.03691	192.36520	184.79562	174.50317	161.78258
3450.0000	163.15430	169.05632	172.64030	173.72882	172.23452	168.14882	161.54256	152.57633	141.51391
3600.0000	140.83037	145.88352	148.95302	149.87920	148.58329	145.05841	139.36950	131.66034	122.16257
3750.0000	119.90915	124.17759	126.77080	127.54822	126.44029	123.44145	118.60953	112.07040	104.02390
3900.0000	100.66340	104.21940	106.37978	107.02333	106.08988	103.57485	99.52848	94.05873	87.33505
4050.0000	83.28708	86.20773	87.98196	88.50720	87.73241	85.65382	82.31408	77.80404	72.26493
4200.0000	67.89078	70.25496	71.69088	72.11342	71.48005	69.78768	67.07178	63.40736	58.91020

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS AND AT Z = 75.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	1000.00000	1000.00000	500.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150.0000	694.24570	630.08906	361.08898	92.08817	27.92906	10.76407	4.73883	2.25112	1.12276
300.0000	544.19680	463.56431	300.88661	138.20689	57.56754	25.30261	11.86154	5.83470	2.97424
450.0000	466.13783	389.47656	273.65026	157.81986	81.14470	40.63401	20.56844	10.61115	5.58328
600.0000	422.18406	352.35171	260.47826	168.59765	98.74108	54.87281	29.90160	16.24088	8.86681
750.0000	395.39343	331.59932	253.73365	175.85656	112.02397	67.33517	39.11438	22.32343	12.66256
900.0000	377.35523	318.50837	249.82517	181.12497	122.22104	77.93240	47.74165	28.49662	16.77293
1050.0000	363.80841	309.06561	246.98943	184.88929	130.06640	86.77154	55.52874	34.47434	20.99958
1200.0000	352.51007	301.27890	244.31986	187.32856	135.99008	93.99148	62.34603	40.04503	25.16240
1350.0000	342.20043	294.07238	241.31282	188.51157	140.24542	99.70978	68.12891	45.05557	29.10626
1500.0000	332.10938	286.80548	237.65605	188.45462	142.97928	104.01150	72.84328	49.39439	32.70112
1650.0000	321.72412	279.05838	233.13311	187.14509	144.27334	106.95288	76.46880	52.97896	35.83980
1800.0000	310.68203	270.53677	227.58379	184.55728	144.17173	108.57057	78.99295	55.74861	38.43585
1950.0000	298.72397	261.03197	220.89125	180.66674	142.70209	108.89260	80.41138	57.66158	40.42263
2100.0000	285.67634	250.40666	212.98170	175.46372	139.89254	107.94944	80.73150	58.69500	41.75354
2250.0000	271.44636	238.59209	203.82899	168.96511	135.78543	105.78397	79.97732	58.84655	42.40306
2400.0000	256.02190	225.58914	193.45983	161.22384	130.44785	102.45973	78.19404	58.13651	42.36796
2550.0000	239.47118	211.46892	181.95716	152.33497	123.97856	98.06647	75.45145	56.60932	41.66811
2700.0000	221.93956	196.37058	169.46014	142.43779	116.51102	92.72281	71.84527	54.33384	40.34632
2850.0000	203.64203	180.49507	156.15986	131.71357	108.21248	86.57555	67.49621	51.40202	38.46692
3000.0000	184.85083	164.09446	142.29086	120.37919	99.27919	79.79586	62.54648	47.92565	36.11286
3150.0000	165.87900	147.45724	128.11873	108.67714	89.92836	72.57261	57.15417	44.03147	33.38153
3300.0000	147.06060	130.89082	113.92503	96.86277	80.38764	65.10368	51.48590	39.85493	30.37942
3450.0000	128.72975	114.70260	99.99077	85.19036	70.88339	57.58611	45.70852	35.53317	27.21625
3600.0000	111.19993	99.18141	86.58011	73.89900	61.62876	50.20605	39.98066	31.19790	23.99889
3750.0000	94.74579	84.58100	73.92584	63.20007	52.81294	43.12970	34.44509	26.96891	20.82587
3900.0000	79.58881	71.10708	62.21779	53.26717	44.59260	36.49595	29.22238	22.94865	17.78270
4050.0000	65.88798	58.90882	51.59531	44.22948	37.08607	30.41140	24.40665	19.21850	14.93855
4200.0000	53.73594	48.07526	42.14400	36.16892	30.37083	24.94807	20.06346	15.83682	12.34444

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS AND AT Z = 50.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET: 0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.00000	0.00000	0.00000	0.00000	500.00000	500.00000	500.00000	500.00000	500.00000
150.0000	9.32511	12.58624	26.55411	75.20261	250.20309	425.33523	474.45079	489.49406	495.15278
300.0000	23.41114	30.35476	56.90041	123.49064	250.53962	377.93343	445.73382	475.00162	487.76974
450.0000	40.73957	50.38166	83.56669	150.53097	251.01222	352.12838	421.28551	459.24053	478.52976
600.0000	59.40767	70.36914	105.40465	167.71599	251.58771	336.44016	402.07582	444.07091	468.27683
750.0000	77.88284	89.08775	123.28063	179.91693	252.19032	325.81495	386.94349	430.18544	457.63918
900.0000	95.19107	105.99922	138.09245	189.19440	252.70147	317.92279	374.61945	417.58527	446.93267
1050.0000	110.79787	120.89776	150.42750	196.45768	252.96482	311.51516	364.10333	405.97910	436.22038
1200.0000	124.43637	133.71425	160.61461	202.09557	252.79476	305.81416	354.63492	394.98201	425.40169
1350.0000	135.97936	144.42587	168.81359	206.24135	251.98682	300.26819	345.62006	384.19696	414.28328
1500.0000	145.36420	153.01891	175.08196	208.89804	250.32914	294.44800	336.57716	373.24656	402.62793
1650.0000	152.55636	159.47769	179.41996	210.00514	247.61481	288.00246	327.10734	361.78760	390.18816
1800.0000	157.53658	163.78656	181.80219	209.47894	243.65495	280.64236	316.88167	349.52143	376.73156
1950.0000	160.30143	165.93867	182.20162	207.24029	238.29209	272.13816	305.63903	336.20435	362.06181
2100.0000	160.87057	165.94752	180.60905	203.23546	231.41322	262.32435	293.19008	321.65843	346.03667
2250.0000	159.29641	163.85888	177.04889	197.45232	222.96154	251.10617	279.42343	305.78180	328.58313
2400.0000	155.67327	159.76102	171.59118	189.93216	212.94545	238.46570	264.31124	288.55656	309.70858
2550.0000	150.14394	153.79176	164.35934	180.77694	201.44391	224.46513	247.91201	270.05289	289.50693
2700.0000	142.90232	146.14125	155.53296	170.15131	188.60681	209.24557	230.36876	250.42781	268.15852
2850.0000	134.19130	137.04977	145.34524	158.27919	174.65019	193.02081	211.90191	229.91794	245.92347
3000.0000	124.29575	126.80063	134.07541	145.43505	159.84600	176.06562	192.79655	208.82637	223.12835
3150.0000	113.53129	115.70865	122.03645	131.93056	144.50730	158.69942	173.38481	187.50412	200.14726
3300.0000	102.22966	104.10533	109.55958	118.09785	128.97005	141.26648	154.02472	166.32797	177.37891
3450.0000	90.72228	92.32215	96.97684	104.27096	113.57307	124.11446	135.07730	145.67639	155.22167
3600.0000	79.32366	80.67376	84.60354	90.76727	98.63827	107.57317	116.88407	125.90597	134.04910
3750.0000	68.31624	69.44258	72.72239	77.87073	84.45276	91.93559	99.74696	107.33033	114.18822
3900.0000	57.93809	58.86642	61.57061	65.81836	71.25452	77.44270	83.91240	90.20361	95.90238
4050.0000	48.37452	49.12996	51.33121	54.79105	59.22282	64.27338	69.56071	74.70952	79.38010
4200.0000	39.75410	40.36072	42.12881	44.90932	48.47371	52.53983	56.80147	60.95662	64.73049

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS AND AT Z = 50.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET: 900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000	500.00000
150.0000	497.55031	498.62552	499.09215	499.22294	499.08964	498.61944	497.53812	495.12942	489.44993
300.0000	493.59409	496.31568	497.52463	497.86706	497.51763	496.29874	493.56017	487.70493	474.87963
450.0000	488.17657	492.94541	495.13578	495.76579	495.12152	492.91097	488.10785	478.39906	458.99591
600.0000	481.52069	488.47824	491.79817	492.77026	491.77304	488.41772	481.40048	468.04957	443.64879
750.0000	473.88704	482.93280	487.41793	488.75523	487.37750	482.83578	473.69535	457.27942	429.52331
900.0000	465.47046	476.33801	481.92191	483.61510	481.86105	476.19255	465.18487	446.40110	416.61695
1050.0000	456.35964	468.69839	475.24081	477.25382	475.15395	468.49171	455.95663	435.47696	404.63980
1200.0000	446.54054	459.97718	467.29553	469.57434	467.17702	459.69650	445.99719	424.40879	393.21373
1350.0000	435.91916	450.09489	457.99056	460.47188	457.83509	449.72847	435.21509	413.00901	381.95382
1500.0000	424.34953	438.93858	447.21521	449.83353	447.01831	438.47679	423.46885	401.04928	370.49941
1650.0000	411.66106	426.37747	434.85198	437.54435	434.61049	425.81386	410.59414	388.29362	358.52746
1800.0000	397.68403	412.28207	420.79021	423.49918	420.50267	411.61419	396.42885	374.52302	345.76154
1950.0000	382.27296	396.54463	404.94310	407.61818	404.61009	395.77467	380.83597	359.55537	331.98055
2100.0000	365.32753	379.09918	387.26582	389.86404	386.89004	378.23418	363.72385	343.26255	317.02806
2250.0000	346.81032	359.93938	367.77236	370.25832	367.35868	358.99111	345.06325	325.58441	300.82100
2400.0000	326.76009	339.13239	346.54914	348.89472	346.10436	338.11683	324.90003	306.53904	283.35639
2550.0000	305.29951	316.82714	323.76325	325.94719	323.29575	315.76363	303.36228	286.22812	264.71439
2700.0000	282.63609	293.25585	299.66400	301.67145	299.18326	292.16596	280.66091	264.83632	245.05652
2850.0000	259.05600	268.72808	274.57706	276.39906	274.09310	267.63436	257.08324	242.62440	224.61829
3000.0000	234.91093	243.61760	248.89132	250.52424	248.41409	242.54228	232.97979	219.91596	203.69630
3150.0000	210.59884	218.34311	223.03940	224.48440	222.57825	217.30683	208.74530	197.07892	182.63041
3300.0000	186.54057	193.34436	197.47371	198.73609	197.03688	192.36517	184.79559	174.50314	161.78255
3450.0000	163.15428	169.05630	172.64029	173.72881	172.23450	168.14880	161.54254	152.57632	141.51389
3600.0000	140.83037	145.88352	148.95301	149.87919	148.58328	145.05840	139.36949	131.66033	122.16256
3750.0000	119.90915	124.17759	126.77079	127.54821	126.44029	123.44145	118.60953	112.07040	104.02390
3900.0000	100.66340	104.21939	106.37978	107.02333	106.08988	103.57485	99.52848	94.05873	87.33505
4050.0000	83.28708	86.20773	87.98195	88.50720	87.73241	85.65382	82.31408	77.80404	72.26493
4200.0000	67.89077	70.25496	71.69088	72.11342	71.48005	69.78768	67.07178	63.40736	58.91020

SOLUTE CONCENTRATION AT TIME = 3000.0000 DAYS AND AT Z = 50.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	500.00000	500.00000	500.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
150.0000	474.36773	425.17865	249.90608	74.63279	25.44124	10.35383	4.66431	2.23670	1.11984
300.0000	445.50526	377.50504	249.73322	121.95938	53.95230	24.56349	11.71047	5.80326	2.96756
450.0000	420.83052	351.28348	249.44071	147.59383	78.03291	39.83845	20.37981	10.56771	5.57337
600.0000	401.29812	335.01331	248.97428	162.92809	96.61906	54.21794	29.72161	16.19453	8.85530
750.0000	385.73737	323.63314	248.26431	172.88407	110.74136	66.87580	38.97022	22.28199	12.65130
900.0000	372.87777	314.82062	247.22471	179.61181	121.49759	77.64119	47.63927	28.46404	16.76328
1050.0000	361.72648	307.34968	245.75300	184.13237	129.67547	86.59893	55.46194	34.45106	20.99209
1200.0000	351.53966	300.47753	243.73198	186.95418	135.78474	93.89380	62.30505	40.02956	25.15702
1350.0000	341.74717	293.69710	241.03330	188.32782	140.13965	99.65631	68.10490	45.04585	29.10264
1500.0000	331.89729	286.62937	237.52315	188.36493	142.92558	103.98294	72.82969	49.38853	32.69881
1650.0000	321.62472	278.97559	233.06992	187.10150	144.24636	106.93791	76.46131	52.97555	35.83838
1800.0000	310.63539	270.49781	227.55375	184.53616	144.15829	108.56283	78.98891	55.74668	38.43501
1950.0000	298.70205	261.01361	220.87696	180.65653	142.69543	108.88865	80.40923	57.66051	40.42214
2100.0000	285.66603	250.39800	212.97491	175.45879	139.88926	107.94743	80.73038	58.69442	41.75326
2250.0000	271.44151	238.58800	203.82576	168.96274	135.78382	105.78296	79.97674	58.84624	42.40290
2400.0000	256.01961	225.58721	193.45829	161.22270	130.44707	102.45922	78.19374	58.13635	42.36788
2550.0000	239.47010	211.46800	181.95643	152.33442	123.97818	98.06622	75.45130	56.60924	41.66807
2700.0000	221.93906	196.37014	169.45979	142.43752	116.51084	92.72269	71.84520	54.33380	40.34630
2850.0000	203.64179	180.49487	156.15969	131.71344	108.21239	86.57549	67.49617	51.40200	38.46691
3000.0000	184.85072	164.09436	142.29078	120.37913	99.27915	79.79583	62.54646	47.92564	36.11285
3150.0000	165.87894	147.45719	128.11869	108.67711	89.92834	72.57259	57.15416	44.03146	33.38152
3300.0000	147.06058	130.89080	113.92501	96.86276	80.38763	65.10367	51.48590	39.85493	30.37942
3450.0000	128.72974	114.70259	99.99076	85.19035	70.88339	57.58610	45.70852	35.53317	27.21625
3600.0000	111.19992	99.18140	86.58011	73.89900	61.62875	50.20605	39.98066	31.19790	23.99889
3750.0000	94.74579	84.58100	73.92584	63.20007	52.81294	43.12970	34.44509	26.96891	20.82587
3900.0000	79.58881	71.10708	62.21779	53.26717	44.59260	36.49595	29.22238	22.94865	17.78270
4050.0000	65.88798	58.90882	51.59531	44.22948	37.08607	30.41140	24.40665	19.21850	14.93855
4200.0000	53.73594	48.07526	42.14400	36.16892	30.37083	24.94807	20.06346	15.83682	12.34444

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*     SAMPLE PROBLEM 11.--SOLUTE TRANSPORT IN A SEMI-INFINITE AQUIFER       *
*           OF INFINITE WIDTH AND HEIGHT WITH A "PATCH" SOURCE              *
*                                                                           *
*    MODEL PARAMETERS:  V=1.0 FEET PER DAY, DX=100.0 FEET**2 PER DAY,       *
*     DY=20.0 FEET**2 PER DAY, DZ=20.0 FEET**2 PER DAY, WS=1200 FEET,       *
*     HS=300 FEET, DK=6.78E-05 PER DAY, C0=100.0 MILLIGRAMS PER LITER       *
*                                                                           *
*         PROGRAM RUN ON WEDNESDAY, OCTOBER 21, 1987, AT 11:46:57           *
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ANALYTICAL SOLUTION TO THE THREE-DIMENSIONAL ADVECTIVE-DISPERSIVE SOLUTE TRANSPORT EQUATION FOR A SEMI-INFINITE AQUIFER OF INFINITE WIDTH AND HEIGHT WITH A PATCH SOLUTE SOURCE AT X=0.0

INPUT DATA

NUMBER OF X-COORDINATES (NX) = 27
NUMBER OF Y-COORDINATES (NY) = 27
NUMBER OF Z-COORDINATES (NZ) = 3
NUMBER OF TIME VALUES (NT) = 1
NUMBER OF POINTS FOR NUMERICAL INTEGRATION (NMAX) = 104

SOLUTE CONCENTRATION ON MODEL BOUNDARY (C0) = 1.000000E+02 MILLIGRAM PER LITER
GROUND-WATER VELOCITY IN X-DIRECTION (VX) = 1.000000E+00 FT/D
DISPERSION IN THE X-DIRECTION (DX) = 1.000000E+02 FEET**2 PER DAY
DISPERSION IN THE Y-DIRECTION (DY) = 2.000000E+01 FEET**2 PER DAY
DISPERSION IN THE Z-DIRECTION (DZ) = 2.000000E+01 FEET**2 PER DAY
FIRST-ORDER SOLUTE DECAY RATE (DK) = 6.780000E-05 PER DAY

AQUIFER WIDTH (W) AND HEIGHT (H) ARE INFINITE
SOLUTE SOURCE IS CENTERED AT Y = 1.500000E+03 FEET AND Z = 1.500000E+03 FEET
FINITE-WIDTH OF SOLUTE SOURCE (WS) = 1.200000E+03 FEET
FINITE-HEIGHT OF SOLUTE SOURCE (HS) = 3.000000E+02 FEET

PLOT SCALING FACTOR FOR X (XSCLP) = 1.000000E+03
PLOT SCALING FACTOR FOR Y (YSCLP) = 1.000000E+03
CONTOUR INCREMENT (DELTA) = 9.999999E-02 MILLIGRAMS PER LITER

X-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

```
   0.0000   150.0000   300.0000   450.0000   600.0000   750.0000   900.0000  1050.0000
1200.0000  1350.0000  1500.0000  1650.0000  1800.0000  1950.0000  2100.0000  2250.0000
2400.0000  2550.0000  2700.0000  2850.0000  3000.0000  3150.0000  3300.0000  3450.0000
3600.0000  3750.0000  3900.0000
```

Y-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

```
   0.0000   100.0000   200.0000   300.0000   400.0000   500.0000   600.0000   700.0000
 800.0000   900.0000  1000.0000  1100.0000  1200.0000  1300.0000  1400.0000  1500.0000
1600.0000  1700.0000  1800.0000  1900.0000  2000.0000  2100.0000  2200.0000  2300.0000
2400.0000  2500.0000  2600.0000
```

Z-COORDINATES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN FEET

1650.0000 1700.0000 1750.0000

TIMES AT WHICH SOLUTE CONCENTRATIONS WILL BE CALCULATED, IN DAYS

3652.5000

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1650.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	0.000023	0.000087	0.000339	0.001354	0.005599	0.024288	0.112806	0.583416	3.641427
300.0000	0.000088	0.000332	0.001272	0.004978	0.019989	0.082898	0.357355	1.601835	7.136665
450.0000	0.000239	0.000887	0.003320	0.012574	0.048245	0.187307	0.729389	2.766086	9.367797
600.0000	0.000547	0.001979	0.007174	0.026050	0.094480	0.339389	1.183911	3.843439	10.688715
750.0000	0.001110	0.003901	0.013616	0.047117	0.160560	0.531578	1.667355	4.742020	11.443290
900.0000	0.002057	0.006988	0.023403	0.076910	0.245740	0.751070	2.138462	5.452420	11.843671
1050.0000	0.003531	0.011580	0.037135	0.115779	0.347171	0.984221	2.572522	5.996512	12.017822
1200.0000	0.005686	0.017973	0.055164	0.163271	0.460749	1.219208	2.957802	6.403117	12.044507
1350.0000	0.008659	0.026378	0.077531	0.218256	0.581918	1.446942	3.290682	6.698809	11.973021
1500.0000	0.012551	0.036878	0.103956	0.279101	0.706202	1.660910	3.571819	6.905205	11.834286
1650.0000	0.017412	0.049407	0.133853	0.343841	0.829462	1.856571	3.803628	7.038657	11.647256
1800.0000	0.023219	0.063734	0.166363	0.410298	0.947949	2.030676	3.988763	7.110734	11.422707
1950.0000	0.029865	0.079465	0.200393	0.476173	1.058250	2.180671	4.129276	7.128873	11.165554
2100.0000	0.037156	0.096046	0.234658	0.539095	1.157187	2.304275	4.226219	7.097059	10.876387
2250.0000	0.044812	0.112792	0.267732	0.596666	1.241755	2.399226	4.279569	7.016505	10.552630
2400.0000	0.052478	0.128912	0.298112	0.646526	1.309108	2.463222	4.288390	6.886399	10.189595
2550.0000	0.059747	0.143564	0.324293	0.686441	1.356633	2.494032	4.251212	6.704745	9.781579
2700.0000	0.066193	0.155913	0.344864	0.714427	1.382110	2.489754	4.166579	6.469317	9.323071
2850.0000	0.071403	0.165198	0.358623	0.728908	1.383921	2.449180	4.033694	6.178682	8.810021
3000.0000	0.075021	0.170809	0.364685	0.728878	1.361306	2.372200	3.853098	5.833186	8.241077
3150.0000	0.076789	0.172350	0.362589	0.714050	1.314580	2.260167	3.627249	5.435793	7.618606
3300.0000	0.076574	0.169689	0.352366	0.684959	1.245287	2.116136	3.360909	4.992622	6.949332
3450.0000	0.074387	0.162984	0.334567	0.642981	1.156223	1.944920	3.061235	4.513063	6.244431
3600.0000	0.070388	0.152674	0.310230	0.590268	1.051320	1.752905	2.737529	4.009424	5.519003
3750.0000	0.064864	0.139435	0.280794	0.529582	0.935378	1.547651	2.400654	3.496107	4.790956
3900.0000	0.058199	0.124114	0.247961	0.464058	0.813673	1.337298	2.062191	2.988442	4.079435

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1650.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000	50.000000
150.0000	24.626014	45.610599	48.668606	49.139199	49.227653	49.246090	49.249321	49.246090	49.227653
300.0000	24.107242	41.077815	46.612629	47.857044	48.131258	48.193226	48.204531	48.193226	48.131258
450.0000	23.440716	37.513626	44.115290	46.151813	46.693247	46.829876	46.856294	46.829876	46.693247
600.0000	22.665942	34.643147	41.488316	44.147449	44.990539	45.230252	45.279807	45.230252	44.990539
750.0000	21.834798	32.226259	38.927308	42.001178	43.134165	43.495468	43.575410	43.495468	43.134165
900.0000	20.990075	30.136390	36.527225	39.839726	41.222188	41.711103	41.826425	41.711103	41.222188
1050.0000	20.159862	28.301746	34.322327	37.743847	39.324102	39.935596	40.088345	39.935596	39.324102
1200.0000	19.359626	26.674483	32.314679	35.756069	37.482379	38.203648	38.393020	38.203648	37.482379
1350.0000	18.595877	25.218318	30.490679	33.892916	35.718941	36.532813	36.755751	36.532813	35.718941
1500.0000	17.869150	23.903390	28.829742	32.154718	34.041306	34.928938	35.180894	34.928938	34.041306
1650.0000	17.175944	22.703736	27.308490	30.532029	32.447102	33.389768	33.665402	33.389768	32.447102
1800.0000	16.509854	21.595765	25.902536	29.009422	30.927009	31.907111	32.200829	31.907111	30.927009
1950.0000	15.862211	20.557229	24.587133	27.567624	29.466652	30.468151	30.774448	30.468151	29.466652
2100.0000	15.222536	19.566588	23.337388	26.184815	28.047900	29.056386	29.370043	29.056386	28.047900
2250.0000	14.579000	18.602777	22.128516	24.837628	26.650033	27.652575	27.968732	27.652575	26.650033
2400.0000	13.919070	17.645438	20.936368	23.502250	25.251037	26.235967	26.550137	26.235967	25.251037
2550.0000	13.230397	16.675607	19.738377	22.155819	23.829249	24.785939	25.093986	24.785939	23.829249
2700.0000	12.501986	15.676832	18.514911	20.778168	22.365354	23.284070	23.582194	23.284070	22.365354
2850.0000	11.725524	14.636570	17.250917	19.353818	20.844630	21.716492	22.001225	21.716492	20.844630
3000.0000	10.896732	13.547641	15.937608	17.873959	19.259174	20.076223	20.344466	20.076223	19.259174
3150.0000	10.016490	12.409459	14.573865	16.338094	17.609731	18.365112	18.614189	18.365112	17.609731
3300.0000	9.091507	11.228732	13.167039	14.754973	15.906753	16.594961	16.822704	16.594961	15.906753
3450.0000	8.134340	10.019398	11.732852	13.142514	14.170358	14.787509	14.992345	14.787509	14.170358
3600.0000	7.162648	8.801666	10.294267	11.526575	12.429038	12.973104	13.154126	12.973104	12.429038
3750.0000	6.197726	7.600204	8.879388	9.938621	10.717173	11.188123	11.345138	11.188123	10.717173
3900.0000	5.262496	6.441686	7.518611	8.412575	9.071664	9.471475	9.605000	9.471475	9.071664

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1650.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	50.000000	50.000000	50.000000	50.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	49.139199	48.668606	45.610599	24.626014	3.641427	0.583416	0.112806	0.024288	0.005599
300.0000	47.857044	46.612629	41.077815	24.107242	7.136665	1.601835	0.357355	0.082898	0.019989
450.0000	46.151813	44.115290	37.513626	23.440716	9.367797	2.766086	0.729389	0.187307	0.048245
600.0000	44.147449	41.488316	34.643147	22.665942	10.688715	3.843439	1.183911	0.339389	0.094480
750.0000	42.001178	38.927308	32.226259	21.834798	11.443290	4.742020	1.667355	0.531578	0.160560
900.0000	39.839726	36.527225	30.136390	20.990075	11.843671	5.452420	2.138462	0.751070	0.245740
1050.0000	37.743847	34.322327	28.301746	20.159862	12.017822	5.996512	2.572522	0.984221	0.347171
1200.0000	35.756069	32.314679	26.674483	19.359626	12.044507	6.403117	2.957802	1.219208	0.460749
1350.0000	33.892916	30.490679	25.218318	18.595877	11.973021	6.698809	3.290682	1.446942	0.581918
1500.0000	32.154718	28.829742	23.903390	17.869150	11.834286	6.905205	3.571819	1.660910	0.706202
1650.0000	30.532029	27.308490	22.703736	17.175944	11.647256	7.038657	3.803628	1.856571	0.829462
1800.0000	29.009422	25.902536	21.595765	16.509854	11.422707	7.110734	3.988763	2.030676	0.947949
1950.0000	27.567624	24.587133	20.557229	15.862211	11.165554	7.128873	4.129276	2.180671	1.058250
2100.0000	26.184815	23.337388	19.566588	15.222536	10.876387	7.097059	4.226219	2.304275	1.157187
2250.0000	24.837628	22.128516	18.602777	14.579000	10.552630	7.016505	4.279569	2.399226	1.241755
2400.0000	23.502250	20.936368	17.645438	13.919070	10.189595	6.886399	4.288390	2.463222	1.309108
2550.0000	22.155819	19.738377	16.675607	13.230397	9.781579	6.704745	4.251212	2.494032	1.356633
2700.0000	20.778168	18.514911	15.676832	12.501986	9.323071	6.469317	4.166579	2.489754	1.382110
2850.0000	19.353818	17.250917	14.636570	11.725524	8.810021	6.178682	4.033694	2.449180	1.383921
3000.0000	17.873959	15.937608	13.547641	10.896732	8.241077	5.833186	3.853098	2.372200	1.361306
3150.0000	16.338094	14.573865	12.409459	10.016490	7.618606	5.435793	3.627249	2.260167	1.314580
3300.0000	14.754973	13.167039	11.228732	9.091507	6.949332	4.992622	3.360909	2.116136	1.245287
3450.0000	13.142514	11.732852	10.019398	8.134340	6.244431	4.513063	3.061235	1.944920	1.156223
3600.0000	11.526575	10.294267	8.801666	7.162648	5.519003	4.009424	2.737529	1.752905	1.051320
3750.0000	9.938621	8.879388	7.600204	6.197726	4.790956	3.496107	2.400654	1.547651	0.935378
3900.0000	8.412575	7.518611	6.441686	5.262496	4.079435	2.988442	2.062191	1.337298	0.813673

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1700.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	0.000020	0.000077	0.000296	0.001161	0.004680	0.019592	0.086366	0.410002	2.146034
300.0000	0.000079	0.000295	0.001114	0.004280	0.016771	0.067311	0.277151	1.161076	4.671373
450.0000	0.000215	0.000790	0.002915	0.010850	0.040716	0.153576	0.575721	2.077468	6.631630
600.0000	0.000493	0.001766	0.006320	0.022586	0.080317	0.281486	0.952499	2.983429	7.999031
750.0000	0.001003	0.003490	0.012041	0.041078	0.137594	0.446261	1.366635	3.787250	8.921862
900.0000	0.001862	0.006271	0.020783	0.067450	0.212358	0.638109	1.783083	4.460404	9.528720
1050.0000	0.003206	0.010427	0.033126	0.102153	0.302500	0.845718	2.178195	5.005318	9.911730
1200.0000	0.005175	0.016240	0.049431	0.144921	0.404663	1.058657	2.538596	5.436071	10.134777
1350.0000	0.007901	0.023916	0.069784	0.194850	0.514910	1.268420	2.858079	5.769242	10.241855
1500.0000	0.011482	0.033548	0.093972	0.250541	0.629215	1.468533	3.134707	6.020038	10.262996
1650.0000	0.015970	0.045092	0.121496	0.310242	0.743731	1.654191	3.368676	6.200871	10.218232
1800.0000	0.021349	0.058349	0.151588	0.371965	0.854876	1.821756	3.560903	6.321002	10.120213
1950.0000	0.027524	0.072963	0.183249	0.433565	0.959317	1.968273	3.712160	6.386622	9.975984
2100.0000	0.034319	0.088426	0.215290	0.492800	1.053909	2.091100	3.822607	6.401152	9.788257
2250.0000	0.041475	0.104102	0.246374	0.547377	1.135652	2.187687	3.891635	6.365678	9.556463
2400.0000	0.048661	0.119252	0.275079	0.595021	1.201696	2.255516	3.917955	6.279524	9.277746
2550.0000	0.055498	0.133082	0.299971	0.633566	1.249419	2.292197	3.899902	6.140975	8.948036
2700.0000	0.061582	0.144800	0.319701	0.661076	1.276576	2.295703	3.835912	5.948129	8.563219
2850.0000	0.066523	0.153681	0.333107	0.675996	1.281513	2.264708	3.725109	5.699840	8.120373
3000.0000	0.069984	0.159140	0.339329	0.677313	1.263393	2.198964	3.567928	5.396637	7.618974
3150.0000	0.071716	0.160791	0.337899	0.664696	1.222419	2.099635	3.366651	5.041520	7.061894
3300.0000	0.071589	0.158498	0.328820	0.638597	1.159966	1.969524	3.125767	4.640461	6.456030
3450.0000	0.069610	0.152397	0.312587	0.600270	1.078614	1.813123	2.852054	4.202525	5.812422
3600.0000	0.065922	0.142891	0.290157	0.551712	0.982033	1.636437	2.554342	3.739532	5.145772
3750.0000	0.060793	0.130610	0.262872	0.495505	0.874728	1.446602	2.242958	3.265291	4.473402
3900.0000	0.054583	0.116345	0.232327	0.434594	0.761671	1.251324	1.928927	2.794490	3.813786

SOLUTE CONCENTRATION AT TIME = 3652.5000 DAYS AND AT Z = 1700.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	9.984673	17.823311	19.559339	19.882960	19.949678	19.964370	19.967025	19.964370	19.949678
300.0000	14.147548	23.623720	27.134002	28.017869	28.227493	28.277214	28.286540	28.277214	28.227493
450.0000	15.820151	25.008664	29.562784	31.064375	31.485945	31.596680	31.618612	31.596680	31.485945
600.0000	16.523838	25.048624	30.064129	32.094704	32.764445	32.961060	33.002523	32.961060	32.764445
750.0000	16.747277	24.572650	29.707062	32.126960	33.044848	33.344963	33.412442	33.344963	33.044848
900.0000	16.701137	23.873473	28.941410	31.617417	32.757982	33.169221	33.267462	33.169221	32.757982
1050.0000	16.495646	23.079418	27.985167	30.810053	32.135311	32.655830	32.787149	32.655830	32.135311
1200.0000	16.194454	22.253890	26.951506	29.845419	31.314296	31.935100	32.099351	31.935100	31.314296
1350.0000	15.835728	21.429219	25.900138	28.805941	30.379588	31.087232	31.282226	31.087232	30.379588
1500.0000	15.442187	20.620804	24.861259	27.738915	29.383029	30.161925	30.384031	30.161925	29.383029
1650.0000	15.026485	19.833912	23.847738	26.669419	28.354791	29.188851	29.433594	29.188851	28.354791
1800.0000	14.594294	19.067234	22.861654	25.607912	27.310073	28.183719	28.446253	28.183719	27.310073
1950.0000	14.146180	18.314861	21.897968	24.554856	26.253325	27.152000	27.427437	27.152000	26.253325
2100.0000	13.678861	17.567522	20.946741	23.503700	25.181128	26.091463	26.375064	26.091463	25.181128
2250.0000	13.186219	16.813568	19.994731	22.443063	24.084422	24.994196	25.281470	24.994196	24.084422
2400.0000	12.660310	16.039988	19.026827	21.358648	22.950546	23.848554	24.135289	23.848554	22.950546
2550.0000	12.092503	15.233613	18.027602	20.235193	21.765378	22.641285	22.923545	22.641285	21.765378
2700.0000	11.474783	14.382556	16.983055	19.058584	20.515651	21.359897	21.634027	21.359897	20.515651
2850.0000	10.801173	13.477816	15.882513	17.818071	19.191402	19.995197	20.257830	19.995197	19.191402
3000.0000	10.069129	12.514852	14.720463	16.508408	17.788316	18.543727	18.791830	18.543727	17.788316
3150.0000	9.280696	11.494904	13.498083	15.131597	16.309647	17.009787	17.240719	17.009787	16.309647
3300.0000	8.443214	10.425766	12.224127	13.697921	14.767370	15.406639	15.618238	15.406639	14.767370
3450.0000	7.569392	9.321820	10.914952	12.225989	13.182250	13.756604	13.947272	13.756604	13.182250
3600.0000	6.676649	8.203192	9.593529	10.741664	11.582716	12.089890	12.258663	12.089890	11.582716
3750.0000	5.785758	7.094092	8.287520	9.275922	10.002574	10.442220	10.588816	10.442220	10.002574
3900.0000	4.918974	6.020530	7.026630	7.861924	8.477870	8.851572	8.976389	8.851572	8.477870

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1700.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	19.882960	19.559339	17.823311	9.984673	2.146034	0.410002	0.086366	0.019592	0.004680
300.0000	28.017869	27.134002	23.623720	14.147548	4.671373	1.161076	0.277151	0.067311	0.016771
450.0000	31.064375	29.562784	25.008664	15.820151	6.631630	2.077468	0.575721	0.153576	0.040716
600.0000	32.094704	30.064129	25.048624	16.523838	7.999031	2.983429	0.952499	0.281486	0.080317
750.0000	32.126960	29.707062	24.572650	16.747277	8.921862	3.787250	1.366635	0.446261	0.137594
900.0000	31.617417	28.941410	23.873473	16.701137	9.528720	4.460404	1.783083	0.638109	0.212358
1050.0000	30.810053	27.985167	23.079418	16.495646	9.911730	5.005318	2.178195	0.845718	0.302500
1200.0000	29.845419	26.951506	22.253890	16.194454	10.134777	5.436071	2.538596	1.058657	0.404663
1350.0000	28.805941	25.900138	21.429219	15.835728	10.241855	5.769242	2.858079	1.268420	0.514910
1500.0000	27.738915	24.861259	20.620804	15.442187	10.262996	6.020038	3.134707	1.468533	0.629215
1650.0000	26.669419	23.847738	19.833912	15.026485	10.218232	6.200871	3.368676	1.654191	0.743731
1800.0000	25.607912	22.861654	19.067234	14.594294	10.120213	6.321002	3.560903	1.821756	0.854876
1950.0000	24.554856	21.897968	18.314861	14.146180	9.975984	6.386622	3.712160	1.968273	0.959317
2100.0000	23.503700	20.946741	17.567522	13.678861	9.788257	6.401152	3.822607	2.091100	1.053909
2250.0000	22.443063	19.994731	16.813568	13.186219	9.556463	6.365678	3.891635	2.187687	1.135652
2400.0000	21.358648	19.026827	16.039988	12.660310	9.277746	6.279524	3.917955	2.255516	1.201696
2550.0000	20.235193	18.027602	15.233613	12.092503	8.948036	6.140975	3.899902	2.292197	1.249419
2700.0000	19.058584	16.983055	14.382556	11.474783	8.563219	5.948129	3.835912	2.295703	1.276576
2850.0000	17.818071	15.882513	13.477816	10.801173	8.120373	5.699840	3.725109	2.264708	1.281513
3000.0000	16.508408	14.720463	12.514852	10.069129	7.618974	5.396637	3.567928	2.198964	1.263393
3150.0000	15.131597	13.498083	11.494904	9.280696	7.061894	5.041520	3.366651	2.099635	1.222419
3300.0000	13.697921	12.224127	10.425766	8.443214	6.456030	4.640461	3.125767	1.969524	1.159966
3450.0000	12.225989	10.914952	9.321820	7.569392	5.812422	4.202525	2.852054	1.813123	1.078614
3600.0000	10.741664	9.593529	8.203192	6.676649	5.145772	3.739532	2.554342	1.636437	0.982033
3750.0000	9.275922	8.287520	7.094092	5.785758	4.473402	3.265291	2.242958	1.446602	0.874728
3900.0000	7.861924	7.026630	6.020530	4.918974	3.813786	2.794490	1.928927	1.251324	0.761671

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1750.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET 0.0000	100.0000	200.0000	300.0000	400.0000	500.0000	600.0000	700.0000	800.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	0.000018	0.000066	0.000250	0.000955	0.003729	0.014934	0.061649	0.262577	1.113243
300.0000	0.000069	0.000254	0.000941	0.003532	0.013429	0.051735	0.201078	0.773173	2.736257
450.0000	0.000188	0.000681	0.002471	0.008997	0.032840	0.119481	0.427042	1.447072	4.283524
600.0000	0.000432	0.001527	0.005378	0.018843	0.065371	0.222159	0.723651	2.166644	5.544902
750.0000	0.000881	0.003029	0.010296	0.034506	0.113122	0.357575	1.062746	2.850752	6.514288
900.0000	0.001640	0.005463	0.017866	0.057079	0.176419	0.518984	1.416613	3.460005	7.238186
1050.0000	0.002832	0.009121	0.028637	0.087104	0.253917	0.697633	1.764017	3.981739	7.766991
1200.0000	0.004587	0.014267	0.042975	0.124501	0.343063	0.884783	2.090993	4.416954	8.143314
1350.0000	0.007027	0.021101	0.061010	0.168613	0.440628	1.072798	2.389400	4.772430	8.400483
1500.0000	0.010246	0.029726	0.082604	0.218304	0.543131	1.255470	2.655021	5.056527	8.563518
1650.0000	0.014297	0.040119	0.107351	0.272072	0.647104	1.427887	2.885931	5.277076	8.650497
1800.0000	0.019172	0.052117	0.134591	0.328153	0.749208	1.586120	3.081275	5.440387	8.673778
1950.0000	0.024791	0.065409	0.163438	0.384598	0.846253	1.726863	3.240454	5.550848	8.641007
2100.0000	0.030997	0.079543	0.192814	0.439325	0.935174	1.847139	3.362630	5.610875	8.555965
2250.0000	0.037557	0.093940	0.221491	0.490181	1.013017	1.944119	3.446508	5.621086	8.419368
2400.0000	0.044170	0.107921	0.248149	0.535006	1.076955	2.015066	3.490356	5.580698	8.229717
2550.0000	0.050487	0.120753	0.271444	0.571726	1.124376	2.057425	3.492242	5.488095	7.984257
2700.0000	0.056133	0.131696	0.290102	0.598471	1.153030	2.069034	3.450434	5.341569	7.680056
2850.0000	0.060747	0.140072	0.303017	0.613723	1.161225	2.048433	3.363923	5.140155	7.315170
3000.0000	0.064011	0.145324	0.309357	0.616467	1.148050	1.995205	3.232975	4.884473	6.889777
3150.0000	0.065691	0.147081	0.308656	0.606326	1.113573	1.910285	3.059625	4.577450	6.407123
3300.0000	0.065661	0.145203	0.300883	0.583655	1.058973	1.796168	2.847997	4.224793	5.874133
3450.0000	0.063921	0.139803	0.286466	0.549566	0.986570	1.656952	2.604384	3.835094	5.301534
3600.0000	0.060598	0.131240	0.266269	0.505869	0.899718	1.498174	2.337017	3.419523	4.703428
3750.0000	0.055936	0.120088	0.241518	0.454933	0.802567	1.326449	2.055553	2.991118	4.096336
3900.0000	0.050264	0.107073	0.213680	0.399472	0.699721	1.148955	1.770323	2.563747	3.497840

SOLUTE CONCENTRATION AT TIME = 3652.5000 DAYS AND AT Z = 1750.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	900.0000	1000.0000	1100.0000	1200.0000	1300.0000	1400.0000	1500.0000	1600.0000	1700.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	3.678787	6.244332	7.094994	7.295909	7.342576	7.353596	7.355665	7.353596	7.342576
300.0000	7.254845	11.773431	13.736502	14.308546	14.457705	14.495324	14.502629	14.495324	14.457705
450.0000	9.607529	14.931526	17.767941	18.787835	19.094904	19.179754	19.197071	19.179754	19.094904
600.0000	11.072971	16.601023	19.979195	21.421878	21.922275	22.075213	22.108276	22.075213	21.922275
750.0000	11.974444	17.434564	21.097922	22.885302	23.588326	23.825510	23.879917	23.825510	23.588326
900.0000	12.509188	17.780118	21.557963	23.600202	24.494009	24.824170	24.904297	24.824170	24.494009
1050.0000	12.797570	17.828021	21.612683	23.828437	24.888533	25.312734	25.421080	25.312734	24.888533
1200.0000	12.916209	17.688889	21.414278	23.737093	24.933625	25.446638	25.583674	25.446638	24.933625
1350.0000	12.915343	17.429860	21.056402	23.434680	24.737210	25.329473	25.493884	25.329473	24.737210
1500.0000	12.828186	17.092337	20.597088	22.991765	24.371843	25.031306	25.220433	25.031306	24.371843
1650.0000	12.676301	16.701360	20.071608	22.453367	23.885598	24.599153	24.809464	24.599153	23.885598
1800.0000	12.472856	16.270903	19.499982	21.846698	23.308921	24.063363	24.290856	24.063363	23.308921
1950.0000	12.224719	15.807059	18.891573	21.186177	22.659170	23.441756	23.682253	23.441756	22.659170
2100.0000	11.933925	15.310124	18.248080	20.476871	21.943843	22.742544	22.991885	22.742544	21.943843
2250.0000	11.598869	14.776183	17.565740	19.717083	21.163125	21.966686	22.220834	21.966686	21.163125
2400.0000	11.215441	14.198537	16.837213	18.900602	20.312187	21.110084	21.365178	21.110084	20.312187
2550.0000	10.778251	13.569183	16.053443	18.018889	19.383497	20.165871	20.418243	20.165871	19.383497
2700.0000	10.281971	12.880425	15.205604	17.063344	18.369251	19.126869	19.373064	19.126869	18.369251
2850.0000	9.722767	12.126562	14.287109	16.027616	17.263861	17.988147	18.224948	17.988147	17.263861
3000.0000	9.099677	11.305518	13.295518	14.909773	16.066322	16.749470	16.973949	16.749470	16.066322
3150.0000	8.415764	10.420192	12.234111	13.714094	14.782144	15.417310	15.626893	15.417310	14.782144
3300.0000	7.678832	9.479279	11.112833	12.452148	13.424540	14.006087	14.198639	14.006087	13.424540
3450.0000	6.901542	8.497375	9.948418	11.142944	12.014602	12.538353	12.712265	12.538353	12.014602
3600.0000	6.100836	7.494259	8.763543	9.812009	10.580319	11.043777	11.198034	11.043777	10.580319
3750.0000	5.296714	6.493389	7.585096	8.489459	9.154515	9.556998	9.691225	9.556998	9.154515
3900.0000	4.510500	5.519814	6.441754	7.207319	7.771975	8.114632	8.229094	8.114632	7.771975

SOLUTE CONCENTRATION AT TIME = 3652.0000 DAYS AND AT Z = 1750.0000 FEET

X-COORDINATE, IN FEET	Y-COORDINATE, IN FEET								
	1800.0000	1900.0000	2000.0000	2100.0000	2200.0000	2300.0000	2400.0000	2500.0000	2600.0000
	SOLUTE CONCENTRATION, IN MILLIGRAMS PER LITER								
0.0000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
150.0000	7.295909	7.094994	6.244332	3.678787	1.113243	0.262577	0.061649	0.014934	0.003729
300.0000	14.308546	13.736502	11.773431	7.254845	2.736257	0.773173	0.201078	0.051735	0.013429
450.0000	18.787835	17.767941	14.931526	9.607529	4.283524	1.447072	0.427042	0.119481	0.032840
600.0000	21.421878	19.979195	16.601023	11.072971	5.544902	2.166644	0.723651	0.222159	0.065371
750.0000	22.885302	21.097922	17.434564	11.974444	6.514288	2.850752	1.062746	0.357575	0.113122
900.0000	23.600202	21.557963	17.780118	12.509188	7.238186	3.460005	1.416613	0.518984	0.176419
1050.0000	23.828437	21.612683	17.828021	12.797570	7.766991	3.981739	1.764017	0.697633	0.253917
1200.0000	23.737093	21.414278	17.688889	12.916209	8.143314	4.416954	2.090993	0.884783	0.343063
1350.0000	23.434680	21.056402	17.429860	12.915343	8.400483	4.772430	2.389400	1.072798	0.440628
1500.0000	22.991765	20.597088	17.092337	12.828186	8.563518	5.056527	2.655021	1.255470	0.543131
1650.0000	22.453367	20.071608	16.701360	12.676301	8.650497	5.277076	2.885931	1.427887	0.647104
1800.0000	21.846698	19.499982	16.270903	12.472856	8.673778	5.440387	3.081275	1.586120	0.749208
1950.0000	21.186177	18.891573	15.807059	12.224719	8.641007	5.550848	3.240454	1.726863	0.846253
2100.0000	20.476871	18.248080	15.310124	11.933925	8.555965	5.610875	3.362630	1.847139	0.935174
2250.0000	19.717083	17.565740	14.776183	11.598869	8.419368	5.621086	3.446508	1.944119	1.013017
2400.0000	18.900602	16.837213	14.198537	11.215441	8.229717	5.580698	3.490356	2.015066	1.076955
2550.0000	18.018889	16.053443	13.569183	10.778251	7.984257	5.488095	3.492242	2.057425	1.124376
2700.0000	17.063344	15.205604	12.880425	10.281971	7.680056	5.341569	3.450434	2.069034	1.153030
2850.0000	16.027616	14.287109	12.126562	9.722767	7.315170	5.140155	3.363923	2.048433	1.161225
3000.0000	14.909773	13.295518	11.305518	9.099677	6.889777	4.884473	3.232975	1.995205	1.148050
3150.0000	13.714094	12.234111	10.420192	8.415764	6.407123	4.577450	3.059625	1.910285	1.113573
3300.0000	12.452148	11.112833	9.479279	7.678832	5.874133	4.224793	2.847997	1.796168	1.058973
3450.0000	11.142944	9.948418	8.497375	6.901542	5.301534	3.835094	2.604384	1.656952	0.986570
3600.0000	9.812009	8.763543	7.494259	6.100836	4.703428	3.419523	2.337017	1.498174	0.899718
3750.0000	8.489459	7.585096	6.493389	5.296714	4.096336	2.991118	2.055553	1.326449	0.802567
3900.0000	7.207319	6.441754	5.519814	4.510500	3.497840	2.563747	1.770323	1.148955	0.699721

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