

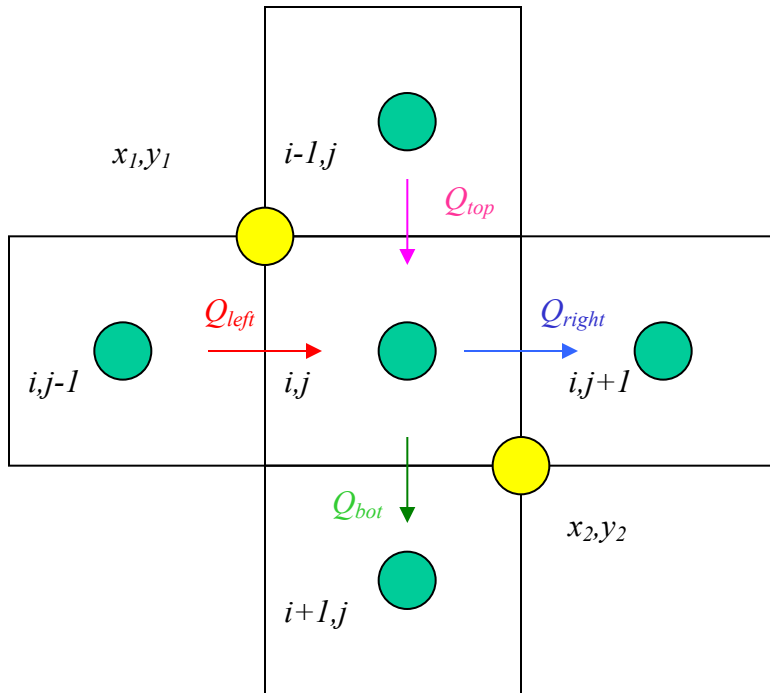
Particle Tracking on a Spreadsheet

- Particle or front tracking is typically performed using special software.
- It can be performed using a spreadsheet.
- The spreadsheet exercise is useful to illustrate the principles involved in particle tracking calculations.
- Particle tracking with reactions is very computationally intensive and is beyond practical application in a spreadsheet.

Velocity Field

- If analytical functions are available for the velocity field then tracking is relatively easy.
- Usually the velocity field is determined numerically at discrete points in space, and this is the situation of interest.
- The interpolation schemes in common use are simple; simple, simple-linear, and multi-linear schemes.
- Only the simple-linear scheme preserves cell-by-cell mass balances.

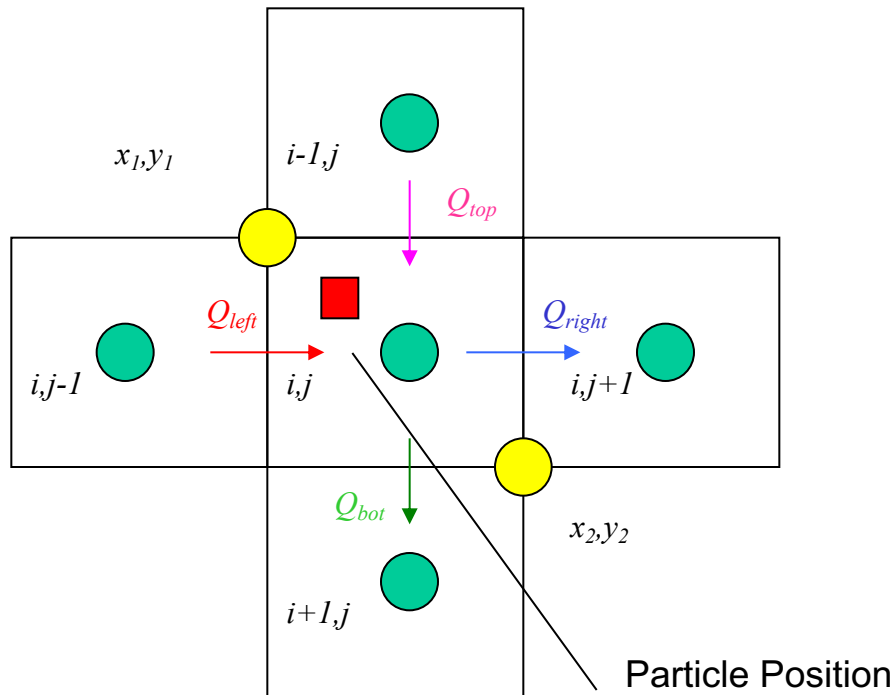
Simple Velocity Scheme



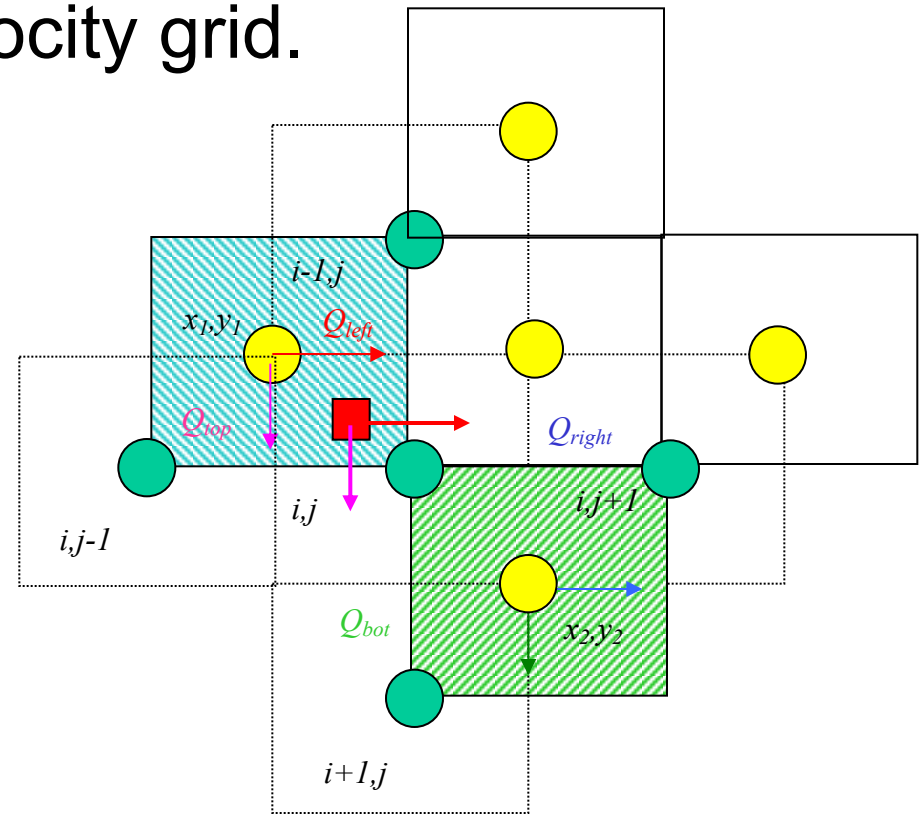
- Typical computational grid for heads.
- Arrows are the interfacial fluxes.
- The simple scheme assigns the top and left flux to (x_1, y_1)
- The simple scheme assigns the right and bottom flux to (x_2, y_2) .

Simple Velocity Grid

- The particle velocity is determined by position of the particle relative to the velocity grid.



Grid for Head Distribution



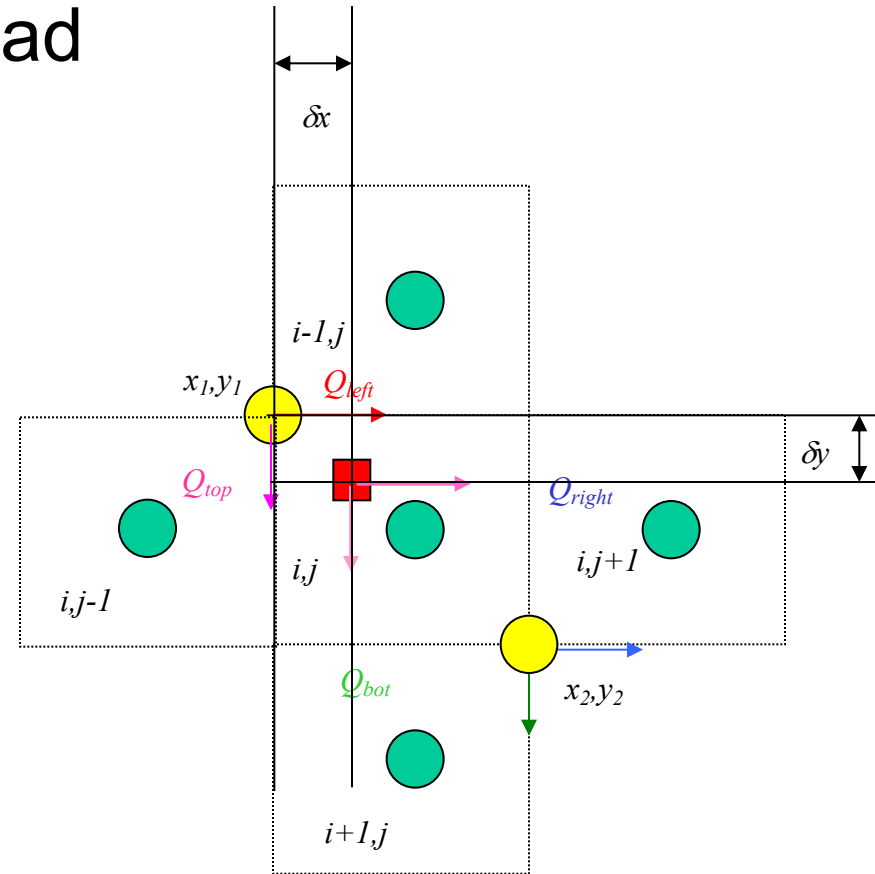
Grid for Velocity Distribution
(Head grid shown as dashed cells)

Linear Interpolation

- Use the same grid as the head scheme.
- Velocity is the distance weighted average of the cell that the particle occupies.

$$u_p = \frac{1 - \delta x}{\Delta x} u(x_1, y_1) + \frac{\delta x}{\Delta x} u(x_2, y_2)$$

$$v_p = \frac{1 - \delta y}{\Delta y} v(x_1, y_1) + \frac{\delta y}{\Delta y} v(x_2, y_2)$$



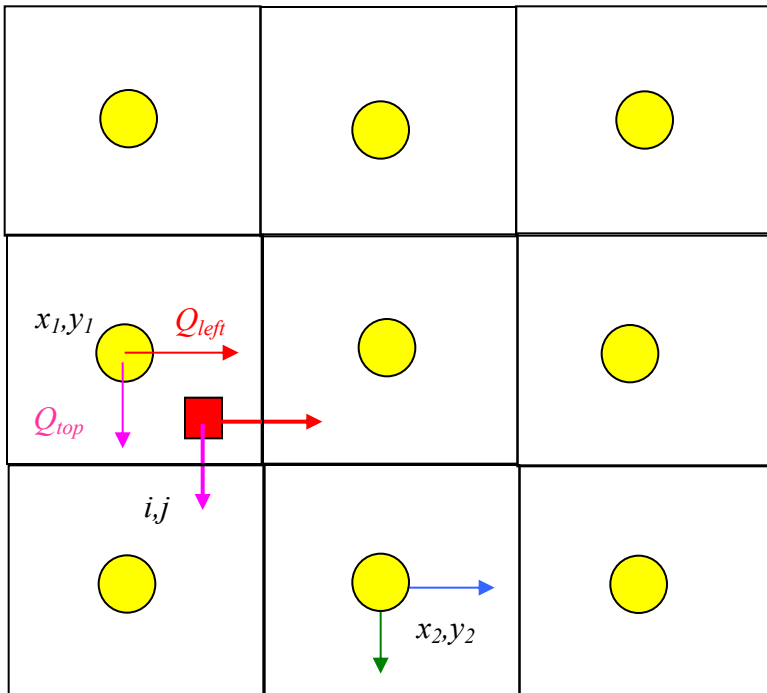
Multi-Linear Interpolation

- Higher order schemes produce smoother velocity fields at the expense of cell mass balances and computational ease.
- The USGS-MOC model uses a bi-linear scheme where the velocities at the four corners of the occupied cell are used.
- When transient flow fields occur, averaging in time is also used.
- The differences in the schemes are hard to detect when the grid spacing is small and the flow field is smoothly varying.

Spreadsheet Approach

- To illustrate particle tracking the simple velocity scheme is used.
- Extension to higher order schemes is straight forward.

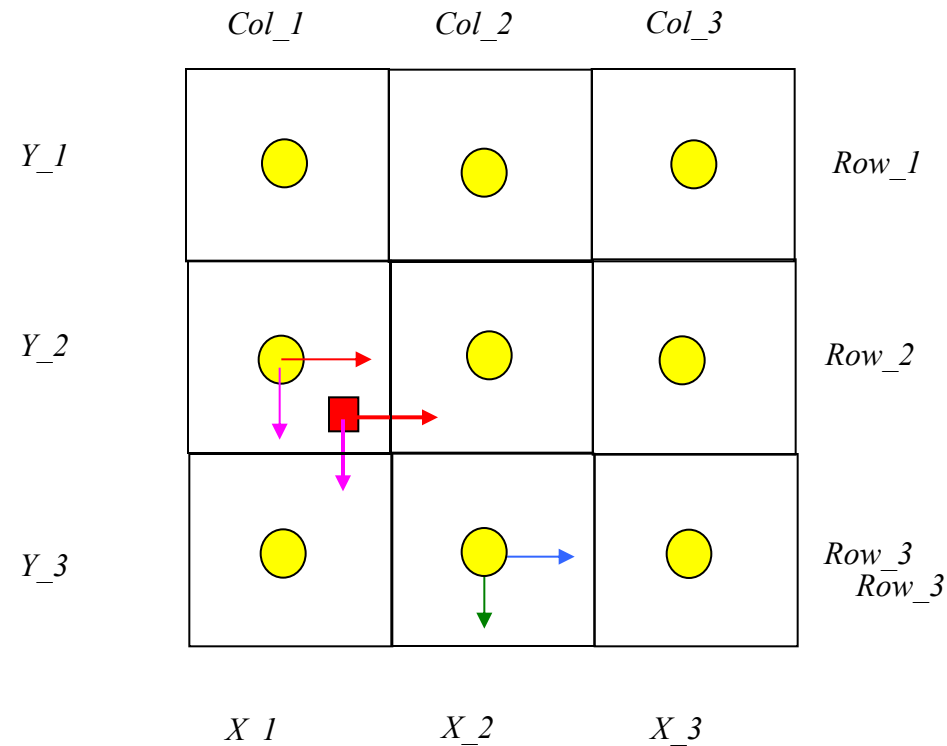
Spreadsheet Approach



- Illustrate with simple scheme.
 - Large rectangles represent the velocity grid.
 - Circles represent the geometric location where velocity is known.
 - Small rectangle represents the particle that we wish to track.

Cell Indexing

- Each cell represents a grid location in the velocity field. Thus each cell has a unique row and column index.



- Each cell centroid also has a unique geometric (x,y) location.
- The particle in the figure is located in cell named: Col_1, Row_2 .
- The cell is located at position: (X_1, Y_2) .
- The particle position is (X_P, Y_P) .

Locating the Particle

- At the start of a time-step
 - particle position is known.
 - cell positions are known.
 - cell that the particle occupies is unknown.
- Construct a distance table
 - The distance from each cell to the particle is calculated and stored in a table.
- Search the table, find the cell nearest the particle.
 - The cell coordinates of the smallest distance in the table is determined

Locating the Particle in EXCEL

- The spreadsheet function that finds the value in an array (rectangular area of cells), given the position in the array to search is the function

INDEX(array,row_index,column_index)

- The spreadsheet function that can find the position in an array where a particular value appears is the function

MATCH(value,array,type)

INDEX

- ***INDEX(array,row_index,column_index)***

array is the location of the rectangular area of cells to search (eg. A3:C6).

row_index is number of rows down from the starting row to search.

column_index is the number of columns across from the starting column to search.

MATCH

- ***MATCH(value,array,type)***

value is the numerical value to search for in the array.

array is the location of the rectangular area of cells to search (eg. A3:C6).

type is the type of match to use. type=0 means exact matching.

Using the functions

- The INDEX function allows us to select the correct values of velocity if we know which cell the particle resides in.
- The MATCH function allows us to compare values in an array and determine the position in the array that these values are found. Thus the MATCH function lets us search a distance table, find the cell center nearest the particle, and then use the index to find the correct velocity.

Moving the Particle

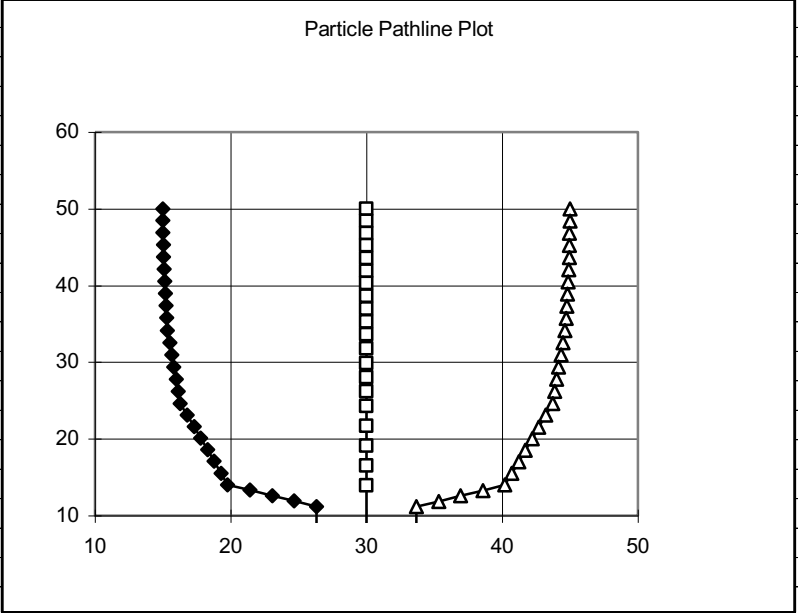
- Once the cell containing the particle is identified, the particle is assigned the velocity values for that cell.
- The particle is then “moved” by the simple kinematic calculation:

$$x_p(t + \Delta t) = x_p(t) + u_p(t)\Delta t$$

$$y_p(t + \Delta t) = y_p(t) + v_p(t)\Delta t$$

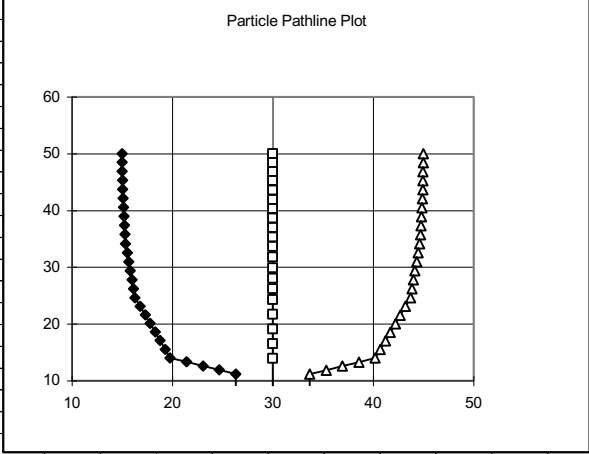
Illustrative Example Spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W			
1	Particle Tracking Spreadsheet																									
2																										
3				u(i,j)		10	20	30	40	50																
4																										
5				50		0.009	0.016	8E-14	-0.016	-0.009		1														
6				40		0.03	0.055	2E-13	-0.055	-0.03		2														
7				30		0.087	0.174	2E-13	-0.174	-0.087		3														
8				20		0.231	0.555	2E-13	-0.555	-0.231		4														
9				10		0.511	1.814	2E-13	-1.814	-0.511		5														
10																										
11						1	2	3	4	5																
12																										
13				v(i,j)		10	20	30	40	50																
14																										
15				50		-1.727	-1.758	-1.783	-1.758	-1.727		1														
16				40		-1.688	-1.766	-1.846	-1.766	-1.688		2														
17				30		-1.571	-1.771	-2.07	-1.771	-1.571		3														
18				20		-1.253	-1.677	-2.893	-1.677	-1.253		4														
19				10		-0.511	-0.792	-6.147	-0.792	-0.511		5														
20																										
21						1	2	3	4	5																
22																										
23																										
24					col		1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4		
25					row		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	2		
26					col_dist		10	10	10	10	10	20	20	20	20	20	30	30	30	30	30	40	40	40		
27					row_dist		50	40	30	20	10	50	40	30	20	10	50	40	30	20	10	50	40	40		
28	dt	0.9			u(col,row)		0.009	0.03	0.087	0.231	0.511	0.016	0.055	0.174	0.555	1.814	8E-14	2E-13	2E-13	2E-13	2E-13	-0.016	-0.055			
29	Particle # 1				v(col,row)		-1.727	-1.688	-1.571	-1.253	-0.511	-1.758	-1.766	-1.771	-1.677	-0.792	-1.783	-1.846	-2.07	-2.893	-6.147	-1.758	-1.766			
30	t	xp	yp	up	vp	Min Dis	Distance Table from Particle to Cell Centers																			
31	0	15	50	0.009	-1.727	5	5	11.18	20.62	30.41	40.31	5	11.18	20.62	30.41	40.31	15	18.03	25	33.54	42.72	25	26.93			
32	0.9	15.01	48.45	0.016	-1.758	5.228	5.244	9.819	=SQRT((B31-G\$26)^2+(C31-G\$27)^2)								17.21	23.77	32.15	41.26	25.04	26.38				
33	1.8	15.02	46.86	0.016	-1.758	5.883	5.922	8.505	=INDEX(\$G\$28:\$AE\$28,1,MATCH(\$F\$31,\$G\$31:\$AE\$31,0))								16.47	22.55	30.76	39.79	25.17	25.9				
34																										
35																										



Segment #1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AE
1	Particle Tracking Spreadsheet																														
2																															
3				u(i,j)		10	20	30	40	50																					
4																															
5			50		0.009	0.016	8E-14	-0.016	-0.009			1																			
6			40		0.03	0.055	2E-13	-0.055	-0.03			2																			
7			30		0.087	0.174	2E-13	-0.174	-0.087			3																			
8			20		0.231	0.555	2E-13	-0.555	-0.231			4																			
9			10		0.511	1.814	2E-13	-1.814	-0.511			5																			
10																															
11					1	2	3	4	5																						
12																															
13				v(i,j)		10	20	30	40	50																					
14																															
15			50		-1.727	-1.758	-1.783	-1.758	-1.727			1																			
16			40		-1.688	-1.766	-1.846	-1.766	-1.688			2																			
17			30		-1.571	-1.771	-2.07	-1.771	-1.571			3																			
18			20		-1.253	-1.677	-2.893	-1.677	-1.253			4																			
19			10		-0.511	-0.792	-6.147	-0.792	-0.511			5																			
20																															
21																															
22						1	2	3	4	5																					
23																															
24							1	1	1	1	1	1	2	2	2	2	3	3	3	3	3	4	4	4	4	4	5	5	5	5	5
25					col																										
26					row		1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
27					col_dist		10	10	10	10	10	20	20	20	20	20	30	30	30	30	30	40	40	40	40	40	50	50	50	50	50
28					row_dist		50	40	30	20	10	50	40	30	20	10	50	40	30	20	10	50	40	30	20	10	50	40	30	20	10
29					u(col,row)		0.009	0.03	0.087	0.231	0.511	0.016	0.055	0.174	0.555	1.814	8E-14	2E-13	2E-13	2E-13	2E-13	-0.016	-0.055	-0.174	-0.555	-1.814	-0.009	-0.03	-0.087	-0.231	-0.511
30					v(col,row)		-1.727	-1.688	-1.571	-1.253	-0.511	-1.758	-1.766	-1.771	-1.677	-0.792	-1.783	-1.846	-2.07	-2.893	-6.147	-1.758	-1.766	-1.771	-1.677	-0.792	-1.727	-1.688	-1.571	-1.253	-0.511
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=INDEX(\$F\$5:\$J\$9,G25,G24)

=SQRT((\$B31-G\$26)^2+(C31-G\$27)^2)

=INDEX(\$G\$28:\$AE\$28,1,MATCH(\$F31,\$G\$31:\$AE\$31,0))

Summary

- Particle tracking is a tool to determine the position of a fluid particle in a flow field.
- A two-step approach is required:
 - Determine particle velocity
 - Locate the particle relative to known velocity locations.
 - Assign the velocity to the particle based on an interpolation scheme.
 - Move the particle.
- All particle tracking programs use this type of two-step logic.