Groundwater / Porous Flow Review

Aquifer – a geologic formation that (1) contains and (2) transmits water (2 gpm) under normal field conditions

Aquitard – same, but transmits at slow rate as compared to aquifer

Aquiclude – cannot transmit water

Aquifuge – cannot contain water

Porosity – ratio of void volume to aquifer volume $n = \frac{V_{void}}{V_{bulk}}$

Water Content – $\theta_w = \frac{V_{water}}{V_{bulk}}$ $\theta_w = n$ if completely saturated

Confined aquifer – bounded above & below by relatively impervious formations. Formations are upper/lower flow boundaries

Unconfined aquifer – Upper flow boundary is free surface

Leaky aquifer – gains/loses water to adjacent aquifers through vertical leakage

Hydraulic conductivity – measure of relative ease of flow through an aquifer. Combined property of porous medium and fluid, K

Transmissivity – measure of aquifers ability to transmit water through its entire thickness

Storativity – amount of water added to/released from storage per unit change in head per unit area of aquifer

$$S = \frac{V_w}{A \Delta h}$$

Mechanisms – confined: aquifer & water compression/decompression

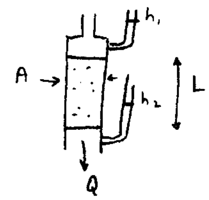
unconfined: same + drainage/filling pore space

Confined: S – storage coefficient

Unconfined: S_y – specific yield, S_r – specific retention $\Big\}$ $n = S_y + S_r$

Darcy's Law (Equation of Motion)

$$Q = -KA\left(\frac{h_2-h_1}{L}\right)$$



$$\lim_{L\to 0} \frac{h_2-h_1}{L} = \frac{dh}{dL}$$

$$\therefore Q = -KA\frac{dh}{dl}$$

Specific discharge is $U = \frac{Q}{A}$. Average linear velocity is $v = \frac{U}{n} = \frac{Q}{nA}$

In 3-D $\quad \underline{Q} = (Q_x, Q_y, Q_z) = -\underline{K}A \cdot \text{grad}(h) = -A\,\underline{K}\cdot\nabla h$

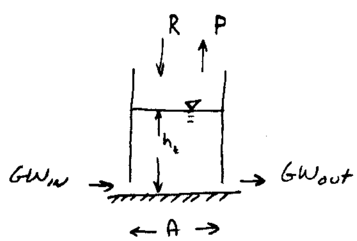
$$\underline{U} = (U, V, W) = -\underline{K}\cdot\nabla h$$

$$\underline{V} = (u, v, w) = \frac{1}{n}\underline{U} = \left(\frac{U}{n}, \frac{V}{n}, \frac{W}{n}\right) = -\frac{1}{n}\underline{K}\cdot\nabla h$$

Cell Balance Approach

Treats aquifer as linked single cell models.

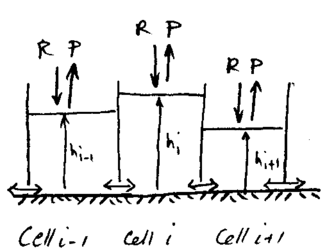
- Write cell balance for each cell
- Use Darcy's law to link the cells
- Generalize expressions
- Apply solution algorithm to resulting equations



Generic cell

$$SA\frac{dh}{dt} = AR - P + GW_{IN} - GW_{OUT}$$

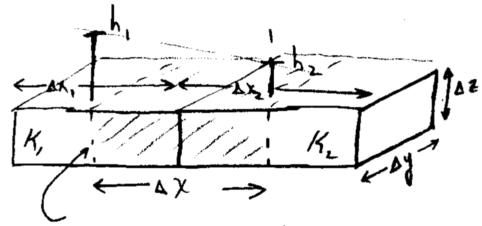
Linked cells



$$S_i A_i \frac{dh_i}{dt} = A_i R_i - P_i + GW_{i-1\to i} - GW_{i\to i+1}$$

Groundwater flow terms

$$Q_{1\to 2} = \bar{K}\,\Delta y\,\Delta z\,\frac{h_1 - h_2}{\Delta x}$$



Apply Darcy's law in shaded area

$$\bar{K} = \frac{K_1 + K_2}{2}$$ (Arithmetic mean, OK if $K_1 \simeq K_2$)

$$\bar{K} = \frac{\Delta x_1 + \Delta x_2}{\frac{\Delta x_1}{K_1} + \frac{\Delta x_2}{K_2}}$$ (Harmonic mean, Use if cells are big and $K_1 \neq K_2$)

$$\therefore\ Q_{1\to 2} = \underbrace{\bar{K}\,\Delta z}_{\bar{T}}\ \frac{\Delta y}{\Delta x}\ h_1 - h_2$$

but $\Delta x = \Delta x_1 + \Delta x_2$
if $\Delta x_1 = \Delta x_2$ then

$$Q_{1\to 2} = \bar{T}\frac{\Delta y}{\Delta x}(h_1 - h_2) = -\bar{T}\frac{\Delta y}{\Delta x}\Delta h \qquad \Delta h = h_2 - h_1$$

$$\therefore\ S_i A_i \frac{dh_i}{dt} = A_i R_i - P_i + \bar{T}_{i-1}\frac{\Delta y}{\Delta x}(h_{i-1} - h_i) - \bar{T}_{i+1}\frac{\Delta y}{\Delta x}(h_i - h_{i+1})$$

$A_i = \Delta x_i \Delta y_i$ divide by A_i, use arithmetic mean as example

$$S_i \frac{dh_i}{dt} = R_i - \frac{P_i}{\Delta x_i \Delta y_i} + \left(\frac{T_{i-1} + T_i}{2}\right)\left(\frac{h_{i-1} - h_i}{\Delta x^2}\right) - \left(\frac{T_i + T_{i+1}}{2}\right)\left(\frac{h_i - h_{i+1}}{\Delta x^2}\right)$$

In limit as $\Delta x, \Delta y \to 0$
we have

$$S\frac{\partial h}{\partial t} = r - p - \mathrm{div}(\underline{T}\cdot \mathrm{grad}(h)) = r - p - \nabla\cdot(\underline{T}\cdot\nabla h)$$

(Same result using fluid mechanics principles)

Return to the cell balance equation – replace $\frac{\partial h}{\partial t}$ by finite-difference approximation

time level, determines implicit/explicit scheme

$$S_i \frac{h_i^{t+\Delta t} - h_i^{t}}{\Delta t} = R_i - \frac{P_i}{\Delta x_i \Delta y_i} + \left(\frac{T_{i-1} + T_i}{2}\right)\left(\frac{h_{i-1} - h_i}{\Delta x^2}\right) - \left(\frac{T_i + T_{i+1}}{2}\right)\left(\frac{h_i - h_{i+1}}{\Delta x^2}\right)$$