FLUIDMECHANICS . PDF

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1 Fluid properties

1.1 Introduction

The engineer involved in water supply and sanitation deals with a variety of fluids including clean water, sewage and industrial wastewaters, sludges, gases (including air), biogas, chlorine, oxygen, and so on. Although the physical properties of these fluids and their flow characteristics vary widely, they are all classified as fluids in so far as they flow or continuously deform under the action of any unbalanced external force, no matter how small that force may be. Properties which influence the flow behaviour of all fluids include density, viscosity, and surface tension. Where compressibility effects are significant, as is the case in the flow of gases under certain conditions, thermodynamic properties such as specific heat at constant volume or constant temperature must also be known.

1.2 Viscosity

The viscosity of a fluid is a measure of its resistance to flow under conditions where turbulence is suppressed. A commonly used flow environment for the definition of fluid viscosity is illustrated in Fig. 1.1. Consider the deformation of the fluid layer contained between the moving upper plate and the stationary lower plate. Assuming that there is no relative movement between the fluid and the plate surfaces, the movement of the upper plate at a uniform velocity v_p (m s^{-1}) results in a linear velocity gradient across the fluid. The force required to sustain the movement of the top plate can be expressed as a function of the velocity v_p, the plate area A, and the distance Y between the plates:

$$F \propto \frac{A v_p}{Y}. \qquad (1.1)$$

This proportional relationship may be written as an equation by introducing the correlating coefficient μ:

$$\frac{F}{A} = \mu \frac{v_p}{Y} \qquad (1.2)$$

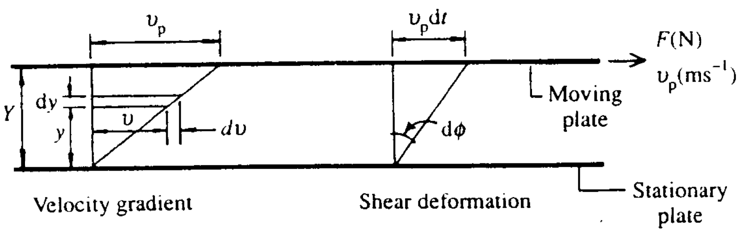

Fig. 1.1 Viscosity definition diagram.

where μ is the coefficient of **dynamic viscosity**. Equation (1.2) may be written in differential form as follows:

$$\tau = \mu \frac{dv}{dy}. \tag{1.3}$$

The units of μ are N s m^{-2}, that is, stress/velocity gradient. Thus, in fluid flow, the maintenance of a velocity gradient requires the application of a shear force.

While the concept of velocity gradient is a very useful one in the general description of fluid flow, particularly in contexts such as mixing, flow can also be represented as a rate of shear deformation, as illustrated on the right-hand side of Fig. 1.1:

$$\frac{dv}{dy} = \frac{d\phi}{dt} \tag{1.4}$$

where ϕ is the angular measure of shear deformation. Thus, the application of a shear stress to a fluid results in a **rate** of shear strain while its application to a solid causes a finite magnitude of strain or deformation.

Those fluids which exhibit the foregoing deformation behaviour are known as Newtonian fluids. They include waters, wastewaters, and gases.

The magnitude of the coefficient of dynamic viscosity μ for liquids decreases with increase in temperature. Its value for water in the temperature range 0 to 100°C is presented in Table 1.1.

The prevailing pressure has only a very minor influence on the dynamic viscosity value for water. At temperatures below 30°C the dynamic viscosity of water slightly decreases with increase in pressure, reaching a minimum value and thereafter increasing with further increase in applied pressure. This minimum disappears at temperatures above 30°C.

The dynamic viscosity of gases increases with increase in temperature. Maitland and Smith (1972) recommended the following empirical correlation

Table 1.1 Physical properties of water. (Source: CRC Handbook of Chemistry and Physics, 67th edn, 1987.)

Temperature (°C)	Density (kg m^{-3})	Saturation vapour pressure (N m^{-2} × 10^{-3})	Dynamic viscosity (N s m^{-2} × 10^{3})	Surface tension (N m^{-1} × 10^{3})
0	999.87	0.6107	1.787	75.64
5	999.99	0.8721	1.519	74.92
10	999.73	1.2277	1.307	74.22
15	999.13	1.7049	1.139	73.49
20	998.23	2.3378	1.002	72.75
25	997.07	3.1676	0.890	71.97
30	995.68	4.2433	0.798	71.18
35	994.06	5.6237	0.719	70.37
40	992.25	7.3774	0.653	69.56
45	990.24	9.5848	0.596	68.74
50	988.07	12.3380	0.547	67.91
55	985.73	15.7450	0.504	67.05
60	983.24	19.9240	0.467	66.18
65	980.59	25.0130	0.434	65.29
70	977.81	31.1660	0.404	64.40
75	974.89	38.5530	0.378	63.50
80	971.83	47.3640	0.355	62.60
85	968.65	57.8080	0.334	61.68
90	965.34	70.1120	0.315	60.76
95	961.92	84.5280	0.298	59.84
100	958.38	101.3250	0.282	58.90

of gas viscosity with temperature for eleven common gases at low pressure (<2 atm):

$$\ln\left(\frac{\mu}{S}\right) = A \ln \Theta + \frac{B}{\Theta} + \frac{C}{\Theta^2} + D \tag{1.5}$$

where μ is the dynamic viscosity (N s m^{-2}) at temperature Θ (K); S is the dynamic viscosity (N s m^{-2}) at a standard temperature of 293.2 K; and A, B, C, and D are coefficients determined from a least-squares regression analysis. Note that Kelvin temperature K = °C + 273.1.

Recommended values for the foregoing coefficients for air, oxygen, nitrogen, methane, and carbon dioxide are presented in Table 1.2.

The linear correlation of shear stress and velocity gradient, characteristic of Newtonian fluids, prevails only in the absence of turbulence in the flow field. This type of flow environment is described as **laminar** flow and, for Newtonian fluids, is confined to situations where random bulk fluid movement is suppressed as, for example, flow in small-bore pipes or through porous media or very close to solid boundaries. Where turbulence exists in

Table 1.2 Dynamic viscosity coefficients for gases.

Gas	A	B	C	D	S ($\mathrm{N\,s\,m^{-2}} \times 10^7$)
Air	0.63404	−45.6380	380.87	−3.4500	182.0
Oxygen	0.52662	−97.5893	2650.70	−2.6892	203.2
Methane	0.54188	−127.5700	4700.80	−2.6952	109.3
Carbon dioxide	0.44037	−288.4000	19 312.00	−1.7418	146.7

the flow, however, the shear resistance is greatly increased and the associated shear stress can, for convenience, be correlated to the velocity gradient by an expression of the same form as that used to define dynamic viscosity:

$$\tau = \varepsilon \frac{dv}{dy} \qquad (1.6)$$

where ε is the coefficient of **eddy viscosity** and is a characteristic of the flow, as distinct from μ which is a property of the fluid. The coefficient of eddy viscosity may be regarded as a coefficient of momentum transfer along the velocity gradient; its magnitude is dependent on the velocity gradient, shear stress, and other factors and is invariably much greater than the dynamic viscosity, μ.

Unlike water and gases, sludges typically exhibit non-Newtonian behaviour, particularly at high concentration. Such behaviour, as illustrated in Fig. 1.2, is characterized by a non-linear relation of shear stress and velocity gradient or rate of shear strain, and, in some fluids, by the existence of a yield stress

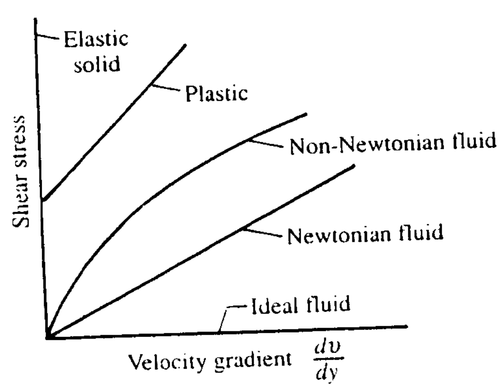

Fig. 1.2 Fluid flow classification.

which must be exceeded for flow to take place. The shear stress/rate of shear strain relation may be expressed in the form

$$\tau = \tau_y + K\left(\frac{dv}{dy}\right)^n \qquad (1.7)$$

where τ_y is the yield stress ($\mathrm{N\,m^{-2}}$), K is a consistency coefficient, and n is a consistency index. These flow parameters are further discussed in Chapter 2. Newtonian and non-Newtonian flow behaviours are illustrated in Fig. 1.2.

The ratio of fluid viscosity to fluid density, generally known as the **kinematic viscosity**, is a frequently encountered flow parameter in hydraulic computations:

$$\nu = \frac{\mu}{\rho} \qquad (1.8)$$

where ν is the kinematic viscosity ($\mathrm{m^2\,s^{-1}}$).

1.3 Surface tension

The interfacial liquid at the boundary between a liquid and a gas behaves rather like a membrane which possesses tensile strength. This membrane-like behaviour can be quantified as a strain energy per unit area, that is, N m/m^2 or force per unit length ($\mathrm{N\,m^{-1}}$), denoted by the symbol σ. The surface tension influence is generally very small in most fluid flow problems encountered by civil engineers. However, in certain applications such as hydraulic modelling, where the model flow depth may be very small, the surface tension influence may be of much greater relative significance in the model than in the prototype and thus distort model flow behaviour. Surface tension is also responsible for the capillary rise above the phreatic surface in fine-grained saturated soils and porous construction materials. When a liquid surface is penetrated by a solid object, surface tension causes the liquid surface in contact with the solid to be raised above the general liquid surface level in the case where the liquid 'wets' the solid surface. On the other hand where the liquid does not wet the solid surface, the liquid surface in contact with the solid is depressed.

The surface tension of water decreases with increase in temperature, as the data presented in Table 1.1 indicate.

1.4 Vapour pressure

When evaporation takes place from the surface of a liquid within an enclosed space or vessel, the partial pressure created by the vapour molecules is called vapour pressure. A liquid may, at any temperature, be considered to be in equilibrium with its own vapour when the rate of molecular transport through the separating gas–liquid interface is the same in both directions. The absolute pressure corresponding to this concentration of gas molecules is defined as the saturation vapour pressure of the liquid. The saturation vapour pressure of every liquid increases with increase in temperature. The temperature at which it reaches a value of 1 atm absolute is the boiling point, which for water is 100°C. Data on the saturation vapour pressure of water in the temperature range 0 to 100°C are presented in Table 1.1.

1.5 Thermodynamic properties

Thermodynamic properties are of particular relevance to gases. The equation of state for the so-called perfect gas is usually written in its general form as follows:

$$PV = mR_u\Theta \tag{1.9}$$

where P is the absolute pressure (N m^{-2}), V is the gas volume (m^3), m is the mass of gas (mole), R_u is the universal gas constant (J mole^{-1} K^{-1}), and Θ is the absolute temperature (K).

The perfect gas has an R_u-value of 8.3144 J mole^{-1} K^{-1}. The variation from this value for real gases is found to be less than 3 per cent (Daugherty and Ingersoll 1954).

Changing from mole to kg, eqn (1.9) may be written for individual gases in the form

$$\frac{P}{\rho} = R\Theta \tag{1.10}$$

where ρ is the gas density (kg m^{-3}) and R is the specific gas constant (J kg^{-1} K^{-1}), related to R_u as follows:

$$R = \frac{1000R_u}{w} \tag{1.11}$$

where w is the molecular weight.

The constant R can be shown to be the difference between the specific heat capacity of a gas at constant pressure (C_p) and its specific heat capacity at constant volume (C_v). Values for these thermodynamic properties for a number of gases are given in Table 1.3.

The relationship embodied in eqns (1.9) and (1.10) may also be expressed

Table 1.3 Thermodynamic properties of gases. (Source: CRC Handbook of Tables for Applied Engineering Science, 2nd edn, 1976.)

	(25°C and 1 atm)		
Gas	C_p (J kg^{-1} K^{-1})	C_p/C_v	R (J kg^{-1} K^{-1})
Air	1005.0	1.40	287.1
Oxygen	920.0	1.40	262.9
Nitrogen	1040.0	1.40	297.1
Methane	2260.0	1.31	534.8
Carbon dioxide	876.0	1.30	202.2

(K = °C + 273.15)

in the forms

$$PV^{\gamma} = \text{constant} \tag{1.12}$$

or

$$\frac{P}{\rho^{\gamma}} = \text{constant} \tag{1.13}$$

where V is the gas volume (m^3) and γ is the so-called polytropic exponent. The value of γ depends on the process by which the gas undergoes volume change. For adiabatic processes (zero internal energy loss), γ is equal to the specific heat ratio C_p/C_v, whereas for isothermal processes (zero temperature change), γ is equal to unity. Thus, in real situations, the value of γ lies within the range 1.0 to C_p/C_v.

1.6 Compressibility

Compressibility may be defined as the susceptibility of a material to volumetric change on the application of pressure. The coefficient of compressibility K is defined as follows:

$$K = \frac{-\Delta P}{\Delta V/V} \tag{1.14}$$

where K is the bulk modulus or coefficient of compressibility (N m^{-2}), ΔP is the change in pressure (N m^{-2}), and ΔV is the change in volume (m^3) of the original volume V (m^3).

Liquids are highly incompressible; for example, the K-value for water at

10°C is about 21.1×10^8 N m^{-2}. Its value increases marginally with pressure and temperature up to a temperature of about 50°C. Above 50°C, there is a slight decrease with increase in temperature.

Gases are relatively highly compressible, their compressibility depending on temperature and pressure. The coefficient of compressibility K for a gas is given by the relation

$$K = P\gamma \tag{1.15}$$

where γ is the polytropic gas volume exponent, as previously defined, and P is the absolute gas pressure in N m^{-2}. Thus, gas compressibility decreases linearly with increase in pressure. Fluid compressibility has a key influence on the speed of transmission of elastic waves through a fluid and is therefore an important fluid property in the analysis of unsteady flow phenomena such as waterhammer. Compressibility effects may also have to be considered in the steady flow of gases at high velocity.

1.7 Density

The density of a substance is defined as its mass per unit volume (kg m^{-3}). Density is influenced by temperature and pressure. As may be deduced from the preceding data on fluid compressibility, liquids are highly incompressible and thus exhibit negligible change in density with change in pressure. Change in liquid density with variation in temperature is also slight. Density data for water in the temperature range 0 to 100°C are presented in Table 1.1. Gases, on the other hand, are highly compressible and hence subject to significant density change with changing temperature and pressure. Equation (1.10) may be used to compute gas density as a function of temperature and pressure.

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2 Fluid flow

2.1 Introduction

From the macroscopic viewpoint of the engineer concerned with fluid transport it is a convenient idealization to treat fluid flow as that of a continuum, thereby neglecting the complex random motions at molecular level. Flow analysis is concerned with quantifying the flow variables throughout the flow field, as functions of time; these variables are velocity, pressure, and density. This approach may be contrasted with that adopted in solid particle mechanics, where the focus of kinematic analysis is on the motions of individual particles.

Velocity is, of course, a vector, that is, it has magnitude and direction. In a three-dimensional flow field, the components u, v, and w of the velocity vector **V**, in the x-, y-, and z-directions, respectively, can be written in functional form as follows:

$$u = f_1(x, y, z, t)$$

$$v = f_2(x, y, z, t)$$

$$w = f_3(x, y, z, t).$$

These components define the value of **V** in space and time.

A **streamline** is defined as a continuous curve in the flow field that is everywhere tangential to the local velocity vector. It is thus a flow path.

2.2 Flow classification

Flow is described as **steady** at a particular location if the velocity vector at the location does not change with time; it is described as **unsteady** if the velocity vector changes with time. In mathematical terms these definitions are written as follows overleaf.

(1) steady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} = 0;$$

2) unsteady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0,y_0,z_0} \neq 0.$$

Flow is said to be **uniform** if the velocity vector is constant along the flow path or streamline. Conversely, flow is described as **non-uniform** if the elocity vector varies along the flow path. These definitions are expressed in ıathematical terms as follows:

(1) uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t_0} = 0;$$

2) non-uniform flow

$$\left(\frac{\partial v}{\partial s}\right)_{t_0} \neq 0.$$

The most regulated of the foregoing flow types is steady uniform flow such as that which occurs in pipes of fixed diameter having a constant discharge ıte.

An example of steady non-uniform flow is that which occurs upstream of a weir in a river having a steady discharge rate. Examples of unsteady on-uniform flow include estuarine flows (due to variation in channel section ıd time-variation in flow associated with tides) and flood flows in rivers.

Flow is described as **rotational** if fluid elements undergo a rotation about their centres of mass, and **irrotational** if no such rotation exists. Where there a spatial velocity gradient in the flow field, as in very many real flow ..tuations, such as in boundary layer flow, there is inevitably some degree of rotation. Flow is obviously rotational where the streamlines are curved.

Flow is described as **compressible** if the fluid undergoes a significant change density along the flow path, and **incompressible** if there is no significant change in density. Flow of liquid is clearly incompressible flow. Flow of gases the velocities normally encountered in sanitary engineering practice may so be regarded as incompressible. There may be significant density changes along the flow path in high-velocity gas flows; hence thermodynamic behaviour must be taken into account in analysing such flows.

When fluid flow is confined by solid boundaries, such that random lateral

mixing in a direction perpendicular to that flow is suppressed, flow is described as **laminar**, that is, flowing, as it were, in separate layers with minimal lateral momentum transfer between layers. Where there is significant lateral mixing and momentum transfer in a direction normal to the flow direction, flow is classified as **turbulent**. The criteria used to define laminar and turbulent flow conditions are discussed further in Chapter 3.

2.3 Fluid acceleration

The acceleration at any point on a streamline can be expressed in terms of its tangential and normal components. The tangential component dv_s/dt may be derived as follows:

$$dv_s = \frac{\partial v_s}{\partial s}ds + \frac{\partial v_s}{\partial t}dt. \qquad (2.1a)$$

Hence

$$\frac{dv_s}{dt} = \frac{\partial v_s}{\partial s}\frac{ds}{dt} + \frac{\partial v_s}{\partial t} \qquad (2.1b)$$

or

$$\frac{dv_s}{dt} = v_s\frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t}. \qquad (2.1c)$$

Equations (2.1) show the tangential acceleration to be the sum of the spatial or convective acceleration $v_s(\partial v_s/\partial s)$ and the local or temporal acceleration $\partial v_s/\partial t$.

In steady flow $\partial v_s/\partial t$ is zero and hence the steady flow acceleration is

$$\frac{dv_s}{dt} = v_s\frac{\delta v_s}{\delta s}. \qquad (2.2)$$

The normal or centripetal acceleration dv_n/dt of a fluid element moving along a curved path or streamline can similarly be written as the sum of convective and temporal components:

$$\frac{dv_n}{dt} = \frac{v_s^2}{R} + \frac{\partial v_n}{\partial t} \qquad (2.3)$$

where R is the local radius of curvature of the streamline and v_n is the normal velocity.

2.4 Streamtube and control volume

The concepts of streamtube and control volume are widely used in fluid flow analysis. A streamtube is an elemental flow volume, the end areas of which are normal to the local flow directions and the peripheral surface of which is generated by streamlines. Flow into and out of the streamtube is through its end areas only; there is no flow normal to the peripheral surface since it is generated by streamlines and thus acts as a virtual boundary. The end areas are sufficiently small in extent that any variation in velocity over the cross-section may be neglected.

A bundle of adjacent streamtubes constitutes a control volume. A control volume has the same general characteristics as a streamtube except that there may be a variation in velocity over its end areas. These concepts are illustrated in Fig. 2.1.

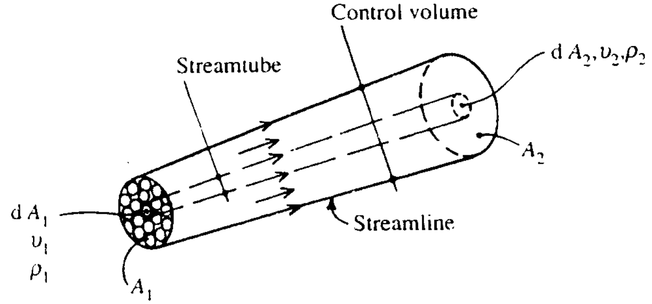

Fig. 2.1 Streamtube and control volume.

2.5 The continuity principle

The concepts of streamtube and control volume facilitate the application of the principle of mass conservation, or the continuity principle as it is known in fluid flow analysis. For example, under steady flow conditions, the mass of fluid contained within a streamtube or control volume does not change with time, hence the rate of mass flow out of such defined zones must equal its rate of inflow:

(1) streamtube

$$\rho_1\,dA_1\,v_1 = \rho_2\,dA_2\,v_2;$$

(2) control volume

$$\bar{\rho}_1 A_1 \bar{v}_1 = \bar{\rho}_2 A_2 \bar{v}_2;$$

where $\bar{\rho}$ and $\bar{v}$ represent the averaged values of these parameters. The mass conservation principle will be applied repeatedly in later chapters to develop the appropriate form of the continuity equation for the problem in hand. In unsteady compressible flow, for example, the fluid mass within the streamtube or control volume varies with time.

2.6 The momentum principle

Newton's second law relates force to the rate of change of momentum:

$$F = \frac{d}{dt}(mv).$$

Consider the application of this principle to steady flow through the streamtube illustrated in Fig. 2.2.

At time zero the streamtube contains a mass of fluid between end areas AA and BB. In the following time interval dt this mass moves to the space between A'A' and B'B'.

$$\text{Initial momentum} = \sum_{BB}^{AA} dm\,v$$

$$\text{Final momentum} = \sum_{B'B'}^{A'A'} dm\,v.$$

Since the flow is steady there is no change in momentum at any point within the streamtubes, that is, the momentum of the fluid in the space A'A'BB, which is common to both streamtubes, remains unaltered. Thus the change

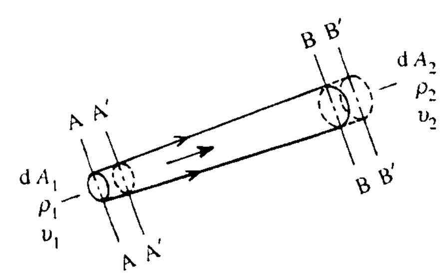

Fig. 2.2 Streamtube flow.

in momentum in the time dt can be written

$$\text{Momentum change} = \sum_{BB}^{B'B'} dm\, v - \sum_{A'A'}^{AA} dm\, v.$$

Written in terms of ρ, dA, and v, this becomes

$$\text{Momentum change} = (\rho_2\, dA_2\, v_2\, dt)v_2 - (\rho_1\, dA_2\, v_1\, dt)v_1.$$

The corresponding rate of change of momentum yields the magnitude of the applied force F:

$$F = \rho_2\, dA_2\, v_2^2 - \rho_1\, dA_1\, v_1^2 \qquad (2.4)$$

The term $\rho_2\, dA_2\, v_2^2$ represents the rate of outflow of momentum from the streamtube, while $\rho_1\, dA_1\, v_1^2$ is its rate of inflow. Thus, the applied force corresponds to the difference in momentum flux across the streamtube end areas. It should be noted that this force is the net force applied to the fluid mass within the streamtube by the surrounding bulk fluid.

In unsteady flow, as discussed in Chapters 6 and 10, the change in momentum of the fluid mass throughout the streamtube volume must also be taken into account.

Equation (2.4) may also be applied to a control volume:

$$F = \sum_{A_2} \rho_2\, dA_2\, v_2^2 - \sum_{A_1} \rho_1\, dA_1\, v_1^2.$$

Written in terms of the mean velocity $\bar{v}$:

$$F = \beta_2\rho_2 A_2\bar{v}_2^2 - \beta_1\rho_1 A_1\bar{v}_1^2 \qquad (2.5)$$

where β is the momentum correction factor (sometimes called the Boussinesq coefficient), which allows for the use of the mean velocity in the application of the momentum principle to control volumes. Its value is obtained as follows:

$$\beta\rho A\bar{v}^2 = \sum_{A} \rho\, dA\, v^2;$$

hence

$$\beta = \frac{1}{A}\sum\left(\frac{v}{\bar{v}}\right)^2 dA. \qquad (2.6)$$

In turbulent pipe flow, β is generally less than 1.1; in laminar pipe flow, β is 1.33.

2.7 The energy principle

Consider the idealized flow of an elemental fluid mass along a streamline as depicted in Fig. 2.3.

Applying Newton's second law to this elemental mass:

$$p\, dA - (p + dp)\, dA - \rho g\, dA\, ds \cos\theta = \rho\, dA\, ds \frac{dv}{dt}. \qquad (2.7)$$

Since $ds \cos\theta = dz$ and $dv/dt = v(dv/ds)$ in steady flow, eqn (2.7) may be written as follows:

$$\frac{dp}{\rho} + g\, dz + v\, dv = 0. \qquad (2.8)$$

This is the **Euler equation**; it relates to steady irrotational flow of a frictionless fluid along a streamline.

Integration of the Euler equation along a streamline yields

$$\int \frac{dp}{\rho} + gz + \frac{v^2}{2} = \text{constant}. \qquad (2.9)$$

If the flow is incompressible, that is, ρ is constant and independent of p, eqn (2.9) becomes

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}, \qquad (2.10)$$

which is the **Bernoulli equation**; it relates to steady, irrotational, incompressible flow of a frictionless fluid along a streamline.

When related to liquid flows, the Bernoulli equation is usually written in the form

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}. \qquad (2.11)$$

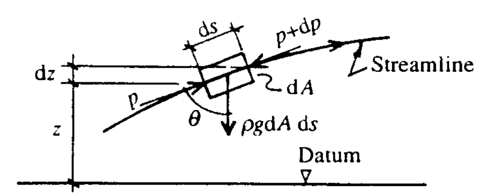

Fig. 2.3 Forces acting on an elemental fluid mass.

Each term in eqn (2.11) has units of length (m) or 'head'. Their sum represents the total head relative to a datum level defined by z. In dealing with incompressible flow the pressure term is conveniently taken as the gauge pressure.

When flow is compressible, integration of the pressure/density term in eqn (2.9) requires reference to the equation of state for gases:

$$\frac{p}{\rho^{\gamma}} = \text{constant}; \tag{1.13}$$

hence

$$\int \frac{\mathrm{d}p}{\rho} = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$$

or

$$\int \frac{\mathrm{d}p}{\rho} = \frac{\gamma}{\gamma - 1} R\Theta \tag{2.12}$$

which, from the definitions of R and γ (given in Section 1.5), can be written as follows:

$$\int \frac{\mathrm{d}p}{\rho} = C_p\Theta. \tag{2.13}$$

Thus, the integrated form of the Euler equation for steady compressible flow along a streamline becomes

$$C_p\Theta + gz + \frac{v^2}{2} = \text{constant}. \tag{2.14}$$

This is known as the **energy equation** and can also be written in the form

$$C_v\Theta + \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant} \tag{2.15}$$

since $R = C_p - C_v$ and $p/\rho = R\Theta$, where P is the absolute pressure. Each term in the energy equations has units of energy per unit mass ($\mathrm{J\,kg^{-1}}$).

The foregoing Euler, Bernoulli, and energy equations do not take into account the energy loss associated with the flow of all real fluids. Thus, in practice, the sum of the terms on the left-hand side of the Bernoulli and energy equations is not constant but decreases in the downstream direction along a streamline.

When dealing with practical flow situations it is convenient to use the mean velocity $\bar{v}$ and a kinetic energy factor α, where

$$\int \rho\, \mathrm{d}A\, v \cdot v^2 = \alpha\rho A\bar{v}\cdot\bar{v}^2;$$

hence

$$\alpha = \frac{1}{A}\int\left(\frac{v}{\bar{v}}\right)^3 \mathrm{d}A. \tag{2.16}$$

The value of α lies between 1.03 and 1.3 in turbulent flow and has the value of 2.0 in laminar flow.

2.8 Applications of continuity, energy, and momentum principle

2.8.1 Incompressible flow

Many examples of the application of the continuity, energy, and momentum principles are presented in later sections of this book. In this section, the illustrative examples are confined to steady flow problems.

The discharge rate through differential head flow devices such as the Venturi meter and orifice plate meters is derive from application of the Bernoulli and continuity equations. Assuming incompressible flow and neglecting losses, the Bernoulli equation may be applied to the central streamline flow of the Venturi meter illustrated in Fig. 2.4:

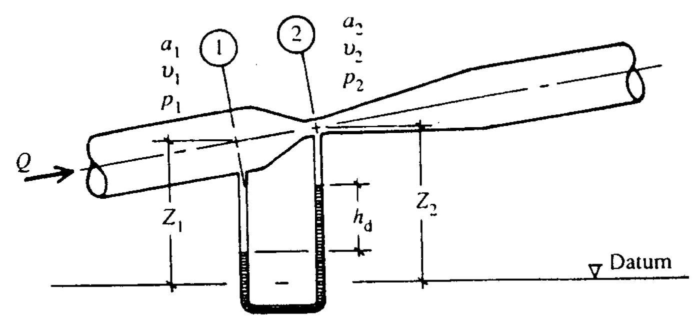

Fig. 2.4 Venturi meter.

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2. \tag{2.17}$$

Similarly, the continuity equation may be applied to the flow througl sections 1 and 2:

$$Q = a_1 v_1 = a_2 v_2. \tag{2.18}$$

Combining eqns (2.17) and (2.18), the discharge Q can be expressed as a function of the upstream and throat cross-sections and the differential head:

$$Q = a_2\left[\left(\frac{2g}{1-(a_2/a_1)^2}\right)\left(\frac{p_1 - p_2}{\rho g} + (z_1 - z_2)\right)\right]^{1/2}. \tag{2.19}$$

Also

$$\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = h_d\left(\frac{\rho_m}{\rho} - 1\right) \tag{2.20}$$

where h_d is the differential head of manometer fluid, ρ_m is the manometer fluid density, and ρ is the flowing fluid density. Because of flow losses between the upstream and throat pressure tappings, the measured differential head will exceed the theoretical value. This is taken into account in practical applications by introducing a discharge coefficient C_d. The practical discharge equation thus becomes:

$$Q = C_d a_2\left[\frac{2gh_d\left(\frac{\rho_m}{\rho} - 1\right)}{(1 - a_2/a_1)^2}\right]^{1/2} \tag{2.21}$$

The value of C_d lies between 0.96 and 0.99 for Venturi meters and in the range 0.60–0.63 for orifice plates. The diameter ratio d_2/d_1 is typically in the range 0.3 to 0.7.

The Pitot tube, illustrated in Fig. 2.5, is a flow measuring device which senses the kinetic streamline head at a point and hence can be used for flow velocity traversing. Writing the Bernoulli terms for the upstream section (1) and the stagnation point section (2):

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + 0.$$

Hence

$$v_1 = \sqrt{2g\left(\frac{p_2 - p_1}{\rho g}\right)} \tag{2.22}$$

or

$$v_1 = \sqrt{2gh}. \tag{2.23}$$

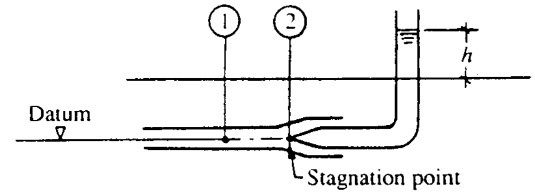

Fig. 2.5 Pitot tube.

The velocity given by the foregoing theoretical derivations is slightly too large and needs to be modified by a correction factor C_v which has a value typically in the range 0.95–1.0. The Bernoulli and continuity equations can also be applied to evaluate the velocity of discharge through a submerged orifice, as in Fig. 2.6:

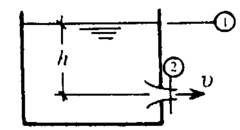

Fig. 2.6 Submerged orifice.

(1) Bernoulli

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + h = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + 0; \tag{2.24}$$

(2) continuity

$$a_1 v_1 = a_2 v_2. \tag{2.25}$$

Combining eqns (2.24) and (2.25) and assuming that p_1 and p_2 are both equal to atmospheric pressure:

$$v_2 = \left[\frac{2gh}{1-(a_2/a_1)^2}\right]^{1/2}. \tag{2.26}$$

The actual velocity is found to be somewhat less than the theoretical value. The cross-sectional area of the discharge jet may be effectively equal to the orifice area for a bellmouth orifice, reducing to about 0.6 of the orifice area for a sharp-edged orifice. Assuming that the velocity of approach is negligible, the discharge through a submerged orifice may be written in practical form as follows:

$$Q_0 = C_d a_0 \sqrt{2gh} \tag{2.27}$$

where Q_0 is the **orifice discharge**, a_0 is the orifice area, and C_d is a discharge coefficient, the value of which generally lies in the range 0.50–0.98, depending on orifice geometry, as shown in Fig. 2.7 (Featherstone and Nalluri 1988).

Computation of the forces exerted on pipe fittings such as the taper, illustrated in Fig. 2.8, or the bend, illustrated in Fig. 2.9, requires simultaneous application of the continuity, energy, and momentum principles. Applied to the vertical taper:

(1) continuity

$$Q = v_1 a_1 = v_2 a_2; \qquad (2.28)$$

(2) energy

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + 0 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + h; \qquad (2.29)$$

(3) momentum

$$p_1 a_1 - p_2 a_2 - W + F_T = \rho Q(v_2 - v_1); \qquad (2.30)$$

where F_T is the force exerted on the water mass in the taper (positive upward) and W is the weight of water within the taper. Thus, the force applied to the

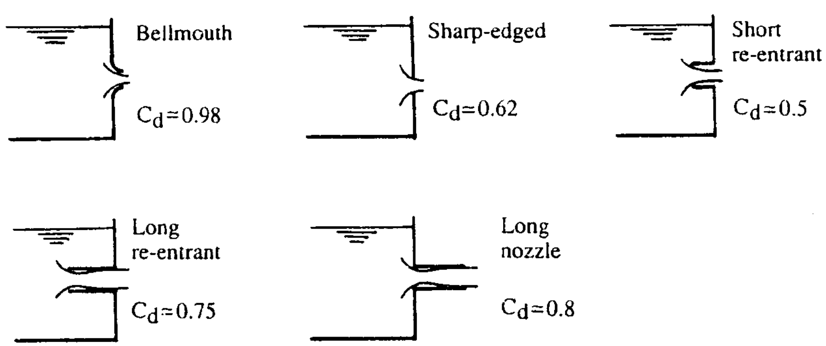

Fig. 2.7 Orifice discharge coefficients.

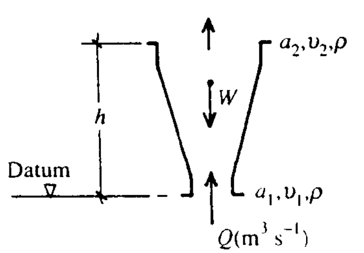

Fig. 2.8 Vertical pipe taper.

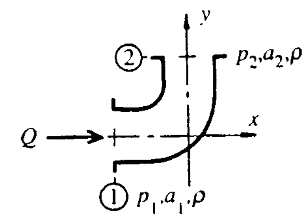

Fig. 2.9 Horizontal pipe bend.

taper, which is the unknown quantity generally sought by the designer, is $-F_T$. When Q and the dimensions of the taper are specified, the magnitude of F can be computed from the foregoing continuity, energy, and momentum equations.

The force exerted by steady flow through a horizontal pipe bend, as illustrated in Fig. 2.9, is also found from application of the continuity, energy, and momentum equations:

(1) continuity

$$Q = v_1 a_1 = v_2 a_2; \qquad (2.31)$$

(2) energy

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}; \qquad (2.32)$$

(3) momentum, x-direction

$$p_1 a_1 + F_x = -\rho Q v_1; \qquad (2.33)$$

(4) momentum, y-direction

$$-p_2 a_2 + F_y = \rho Q v_2 \qquad (2.34)$$

where F_x and F_y are the forces exerted on the fluid in the x- and y-directions, respectively, and $-F_x$ and $-F_y$ are the corresponding forces exerted by the flow on the bend.

2.8.2 Compressible flow

Compressible flow is marked by variations in fluid density along the flow path. The circumstances in which the related thermodynamic consequences must be taken into account can be examined as follows. The maximum temperature rise along a streamline occurs when the velocity is reduced to zero, that is, at a stagnation point. Applying the energy eqn (2.15) to this flow situation:

$$C_p\Theta + \frac{v^2}{2} = C_p(\Theta + \Delta\Theta)$$

upstream section — stagnation point

where $\Delta\Theta$ is the temperature rise. This relationship can be written in the following form:

$$\frac{\Delta\Theta}{\Theta} = \frac{v^2}{2C_p\Theta}. \qquad (2.35)$$

For practical design purposes it may be assumed that, where the potential incremental temperature change along the flow path is less than 1 per cent, flow may be regarded as incompressible. Applying this criterion to eqn (2.35), the **limiting value** of v is found to be:

$$v = \sqrt{0.02C_p\Theta} \qquad (2.36)$$

giving a limit value for air at 10°C of 75.4 m s^{-1}.

Where, however, thermodynamic changes are significant the flow must be treated as compressible flow. Consider, for example, the steady discharge of a gas from a pipe or reservoir through an orifice or nozzle, as illustrated in Fig. 2.10. The flow is defined by the following correlations:

(1) continuity

$$m = \rho_1 a_1 v_1 = \rho_2 a_2 v_2; \qquad (2.37)$$

(2) energy

$$\frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} + \frac{v_1^2}{2} = \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} + \frac{v_2^2}{2}; \qquad (2.38)$$

(3) pressure/density

$$\frac{P_1}{\rho_1^\gamma} = \frac{P_2}{\rho_2^\gamma}. \qquad (2.39)$$

These correlations assume idealized flow with zero energy loss. Using these equations, the mass discharge rate m (kg s^{-1}) can be expressed in terms of known parameter values, P_1, ρ_1, and P_2, and the sectional areas a_1 and a_2:

$$m = a_2\left[\left(\frac{P_1\rho_1}{(P_2/P_1)^{-2/\gamma} - (a_2/a_1)^2}\right)\left(\frac{2\gamma}{\gamma - 1}\right)\left(1 - (P_2/P_1)^{(\gamma-1)/\gamma}\right)\right]^{1/2}. \qquad (2.40)$$

The foregoing expression is valid for subsonic flow conditions, that is, up to the pressure ratio at which the velocity at the orifice or nozzle throat reaches

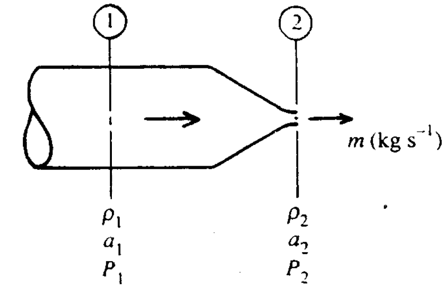

Fig. 2.10 Gas discharge through an orifice.

the sonic value. The sonic velocity α is the velocity of transmission of a weak pressure wave through the gas. Its magnitude (Benedict 1983) is

$$\alpha = \sqrt{\gamma P_2/\rho_2}. \qquad (2.41)$$

When the critical pressure ratio is exceeded, flow control changes from a dependency on the pressure ratio P_2/P_1 to a dependency on the upstream pressure P_1, the upstream flow remains subsonic, and the velocity through the throat remains at sonic value. The critical pressure ratio may be found by applying the energy equation to the upstream and throat sections at the onset of sonic flow at the throat section:

$$\left(\frac{\gamma}{\gamma - 1}\right)\frac{P_1}{\rho_1} + \frac{v_1^2}{2} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{P_2}{\rho_2} + \frac{\alpha^2}{2}; \qquad (2.42)$$

Combining eqns (2.41), (2.42), and (2.39) and neglecting v_1 (this is equivalent to assuming p_1 as the upstream stagnation pressure), the **critical pressure ratio** is found to be

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma-1)}. \qquad (2.43)$$

The critical pressure ratio for air ($\gamma = 1.4$) is found from eqn (2.43) to be 0.528.

When the pressure ratio exceeds the above limiting value, the rate of discharge through the orifice or nozzle is found by inserting the limiting value of P_2/P_1 in the discharge eqn (2.40). By omitting the area ratio term, a_2/a_1, the resulting discharge expression is found to be

$$m = a_2\left[P_1\rho_1\gamma\left(\frac{2}{\gamma + 1}\right)^{(\gamma+1)/(\gamma-1)}\right]^{1/2}. \qquad (2.44)$$

Equations (2.40) and (2.44) are theoretical expressions for the mass discharge rate of a compressible fluid through an orifice or nozzle. In practical computation these expressions are modified by the introduction of a discharge coefficient which allows for variations from ideal behaviour and in particular for flow contraction effects associated with the geometry of the orifice or nozzle.

The admission of air to a pipeline through an air valve consequent on a drop in pipeline pressure below atmospheric pressure provides a practical example of the foregoing compressible flow behaviour from the water engineering field, as illustrated in Fig. 2.11. In this instance the external pressure (P_1) remains constant at atmospheric pressure. The inflow of air through the valve orifice increases with the decrease in the internal pipeline pressure (P_2) in accordance with flow eqn (2.40) until the critical pressure ratio (eqn (2.43)) is reached. Therefore, any further drop in internal pressure does not cause a corresponding increase in the rate of inflow since the latter now depends only on P_1, that is, atmospheric pressure, which is constant. An orifice operating under such conditions is sometimes described, for obvious reasons, as being 'choked'.

2.9 Resistance to fluid flow

The resistance to fluid flow arises primarily from the drag forces exerted on flowing fluids by the solid boundary surfaces of flow conduits. This drag results from the fact that there is zero slippage or relative movement at the contact interface between a flowing fluid and a solid surface, resulting in high shear rates in the adjacent boundary fluid layer. This shear deformation is manifested as a spatial velocity gradient in a direction normal to the boundary surface, decreasing in magnitude with distance from the boundary.

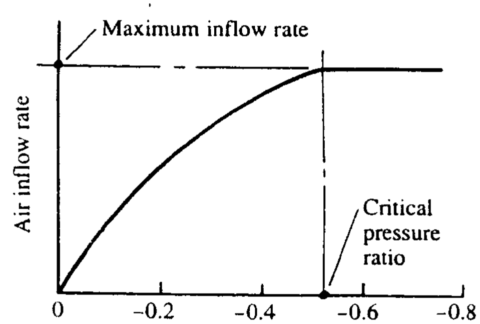

Fig. 2.11 Air entry rate for an air valve.

The existence of a velocity gradient implies a causative shear stress, which is essential to maintain flow and which is a measure of the resistance to flow. Where the flow is laminar, that is, where there is no turbulence in the flow, the local shear stress/velocity gradient ratio is a constant. This constant is by definition the fluid viscosity μ.

Where, however, flow conditions are turbulent, as is generally the case in civil engineering hydraulics, the correlation of shear stress and velocity gradient becomes more complex, being a flow property rather than a fluid property. The nature of turbulent flow and flow resistance in turbulent boundary layers is discussed in Chapter 3.

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