Cleveland #12

Bear, J., 1972. Dynamics of Fluids in Porous Media, Elsiver, New York.

6.6 Flow Nets and Ground Water Contour Maps

6.6.1 The φ–ψ Flow Net

The solution of problems of flow through porous media is usually given in the form of the piezometric head distributions $\varphi = \varphi(x, y, z)$ for steady flow, or $\varphi = \varphi(x, y, z, t)$ for unsteady flow. From these distributions we may define *equipotential* (*or isopiestic*)

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surfaces $\varphi = \varphi(x, y, z) =$ const in three-dimensional flow, or equipotential curves $\varphi = \varphi(x, y) =$ const in two-dimensional flow (say, in the xy plane). In unsteady flow, these surfaces or curves change with time, but at any given instant we have a definite family of surfaces or curves.

As explained in section 6.5, the statement of a problem of flow through a porous medium can be also made, and its solution presented, in terms of the stream function. In three-dimensional flow we have two stream functions λ and χ (par. 6.5.3), with surfaces $\lambda = \lambda(x, y, z) =$ const, being families of stream surfaces. In two-dimensional flow, in the xy plane, for example, we have the stream function Ψ (or $\psi = \Psi/K$ in an isotropic medium), with curves $\Psi =$ const (or $\psi =$ const) as streamlines.

In two-dimensional flow, a plot of equipotentials and streamlines is called a *flow net*. Examples of flow nets are given in figures 7.10.2, 7.10.3, 7.10.4, and 7.10.5. In all these cases the flow net is obtained analytically. In section 7.10 graphic techniques for deriving the flow net are explained. The use of models and analogs of various types for deriving the flow net is explained in chapter 11. Of special interest is the electric analog of the electrolytic tank type (par. 11.5.1).

The relationships between the functions φ and ψ, or Φ and Ψ are discussed in detail in paragraph 6.5.5. In an isotropic homogeneous medium we may use either the pair φ, ψ or the pair Φ, Ψ. In each case the *two families of curves* $\varphi =$ *const and* $\psi =$ *const* (or $\Phi =$ const and $\Psi =$ const) *are mutually orthogonal.* When the medium is not isotropic or is inhomogeneous, only φ and Ψ can be employed. In an isotropic medium, according to (6.5.49), the family of curves $\varphi =$ const is orthogonal to the family of curves $\Psi =$ const. When the medium is homogeneous but anisotropic, it is always possible to employ the technique described in section 7.4 first to transform the given domain into an equivalent isotropic one, to obtain the flow net in this transformed domain and then to transform the flow net back to the original domain. Equation (7.4.26) is applicable here. The result is a flow net in which *streamlines are not orthogonal* to equipotentials. Obviously, this results from the fact that in anisotropic media $\mathbf{q}$ and $\mathbf{J}$ are not colinear (sec. 5.6). Figure 6.6.1 shows a non-orthogonal flow net in an anisotropic medium; numbers on equipotentials are ratios φ/H in percents, while numbers on streamlines give Ψ/Q in percents.

* Figure 6.6.2a shows a portion of a flow net with two streamtubes in a homogeneous isotropic medium. It is customary to draw the flow net such that the difference $\Delta\varphi$ between any two adjacent equipotentials is constant. The difference $\Delta\psi$ between any two adjacent streamlines is also constant (equal to the discharge $\Delta Q/K$ through the streamtube). Because streamlines behave as impervious boundaries of the streamtube:

Not necessary, but convenient

$$** \qquad \Delta Q = K\,\Delta n_1(\Delta\varphi/\Delta s_1) = K\,\Delta n_2(\Delta\varphi/\Delta s_2)$$

where ΔQ is the discharge per unit thickness (dim L^2T^{-1}). Hence:

$$\Delta n_1/\Delta s_1 = \Delta n_2/\Delta s_2, \tag{6.6.1}$$

i.e., in a homogeneous medium, the ratio $\Delta n/\Delta s$ of the sides of the rectangles must remain constant throughout the flow net.

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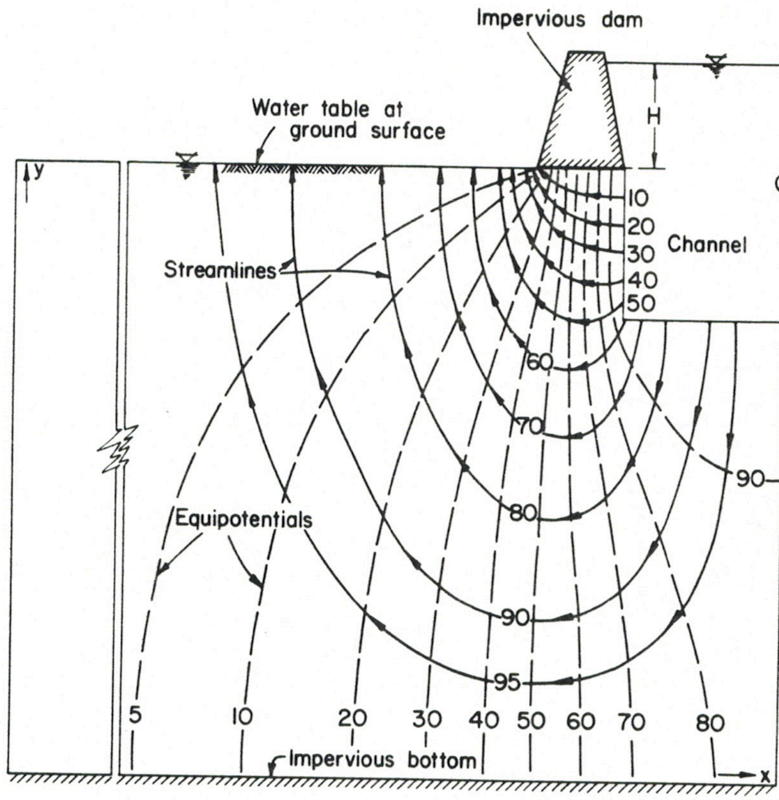

FIG. 6.6.1. Flow-net in an anisotropic medium (Todd and Bear, 1959).

When the medium is inhomogeneous, we have:

$$K_1 \Delta n_1(\Delta\varphi/\Delta s_1) = K_2 \Delta n_2(\Delta\varphi/\Delta s_2); \qquad K_1/(\Delta s_1/\Delta n_1) = K_2/(\Delta s_2/\Delta n_2) \qquad (6.6.2)$$

and the ratio between the sides of the curvilinear rectangles varies. If in a flow net $\Delta n \simeq$ const (i.e., streamlines are nearly parallel) we have $K_1/K_2 = \Delta s_1/\Delta s_2$ or $K_1/K_2 = (\Delta\varphi_2/\Delta s_2)/(\Delta\varphi_1/\Delta s_1)$. This means that the hydraulic conductivity is inversely proportional to the hydraulic gradient. Therefore, in such a case, the region of wide contour spacing corresponds to a higher hydraulic conductivity, whereas contours will be crowded in regions of lower hydraulic conductivity.

In many cases of homogeneous media, however, the flow net is drawn so that approximate curvilinear squares are formed (fig. 6.6.2b). In this case $\Delta s_i = \Delta n_i$ everywhere. This is done only because in some graphic methods it is easier to draw curvilinear squares than to draw curvilinear rectangles. In the case of a flow net made up of squares:

$$\Delta Q = K\,\Delta\varphi. \qquad (6.6.3)$$

In certain cases (e.g., when the electrolytic tank analog is used) the flow net is drawn such that we have m streamtubes, each carrying the same discharge ΔQ, and n equal drops in piezometric head from the highest φ ($= \varphi_{max}$) to the lowest φ ($= \varphi_{min}$) in the flow domain. Both m and n are integers. Then:

$$\Delta\varphi = (\varphi_{\max} - \varphi_{\min})/n; \qquad \Delta Q = Q_{\text{total}}/m.$$

$$Q_{\text{total}} = m\,\Delta Q = mK\,\Delta n\frac{\Delta\varphi}{\Delta s} = mK\,\Delta n\frac{\varphi_{\max} - \varphi_{\min}}{n\,\Delta s} = \frac{m}{n}\frac{\Delta n}{\Delta s}K(\varphi_{\max} - \varphi_{\min}) \quad (6.6.4)$$

or:

$$\frac{\Delta n}{\Delta s} = \frac{n}{m}\frac{Q_{\text{total}}}{K(\varphi_{\max} - \varphi_{\min})}. \quad (6.6.5)$$

Again, the ratio $\Delta n/\Delta s$ (in general $\neq 1$) remains constant throughout the flow domain. Sometimes n and m (not necessarily integers) are chosen such that squares with $\Delta s = \Delta n$ are obtained. Then:

$$Q_{\text{total}} = \frac{m}{n}K(\varphi_{\max} - \varphi_{\min}). \quad (6.6.6)$$

In a zoned porous medium, where K varies abruptly from K_1 to K_2 along a specified boundary, we have refraction of both equipotentials and streamlines across this boundary (par. 7.1.10). In plane flow through a porous medium of constant thickness, K should be replaced by the transmissivity $T = Kb$, with Q referring to the discharge through the entire thickness.

It is important to emphasize that when a homogeneous isotropic flow domain has boundaries on which the boundary conditions are given in terms of φ, and the flow is steady, described by the Laplace equation, the flow net is independent of the hydraulic conductivity. It depends only on the geometry of the flow domain. This is also true when one of the boundaries is a phreatic surface.

6.6.2 The Ground Water Contour Map

We shall use here the term ground water flow (or aquifer flow) in the sense discussed in section 6.4, i.e., a flow that is essentially horizontal. In general, the only information that we have about what happens in the aquifer is the piezometric heads observed and recorded at observation wells. In a phreatic aquifer these heads give the elevation

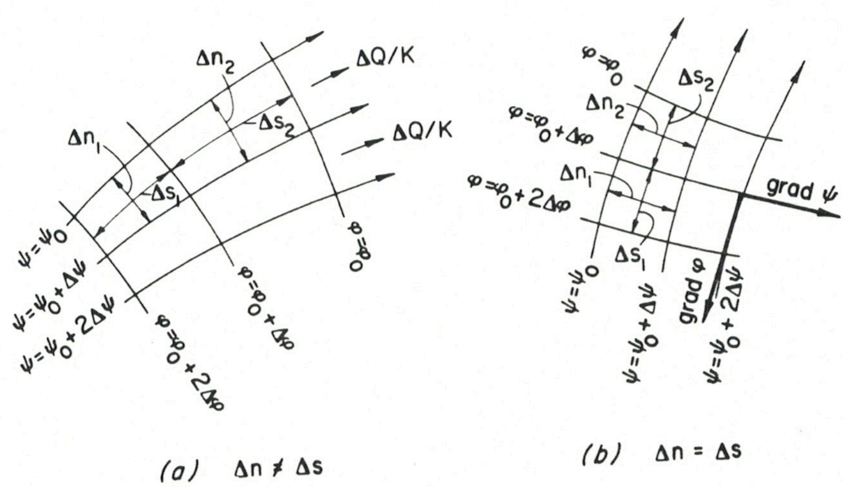

FIG. 6.6.2. A portion of a flow-net.

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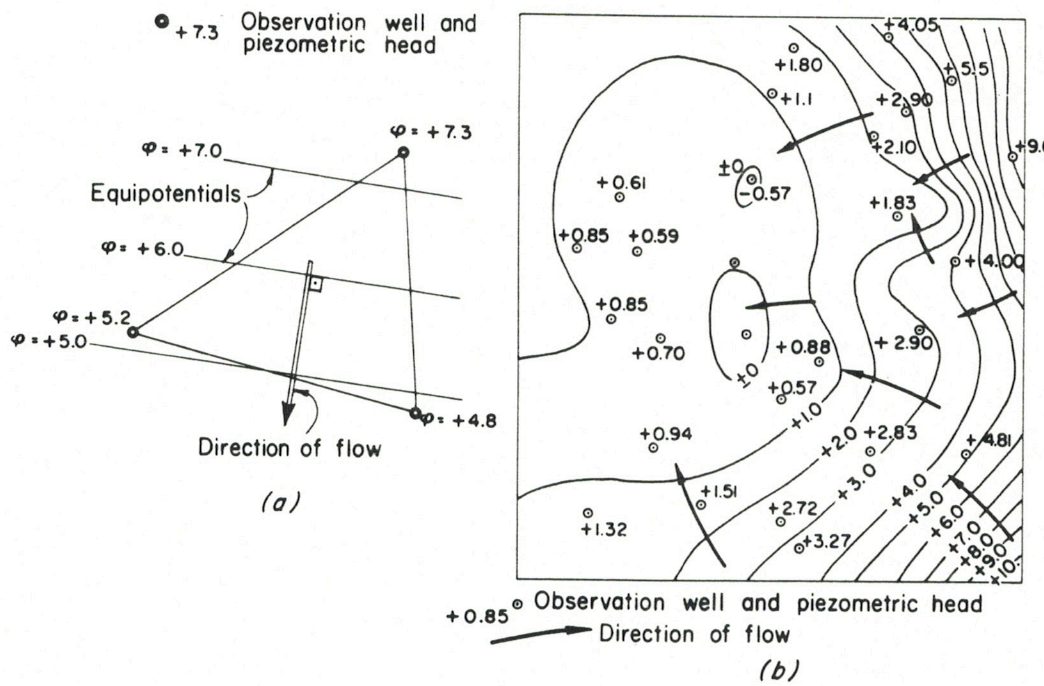

FIG. 6.6.3. Ground water contour map with arrows indicating directions of flow.

of the water table at the observation wells (neglecting the capillary fringe and employing the Dupuit assumption (chap. 8)). Except for special cases, such as when impervious faults are known to be present in the aquifer, we may assume that the piezometric head is a continuous function of the plane coordinates. Hence, using the information on values of φ at discrete points (observation wells), smooth equipotential curves $\varphi = \text{const}$ are drawn, usually by linear interpolation, over the flow domain enclosed between the observation wells. Care must be taken to use only the information from wells tapping the aquifer being considered. An example of drawing equipotential curves, also called *ground water contours*, is given in figures 6.6.3a and 6.6.3b. The contours give the shape of the piezometric surface (or phreatic surface in a water table aquifer).

Once the equipotentials are drawn, a family of streamlines that are everywhere orthogonal to the ground water contours (assuming that the aquifer is isotropic)* can be drawn. In certain cases the information on piezometric heads is insufficient to draw closely spaced contours that permit the exact drawing of streamlines. In such cases we draw on the contour map only arrows that indicate direction of flow (fig. 6.6.3b).

In unsteady flow the contour map gives an instantaneous picture of what happens in the aquifer. When interpreting a contour map (as to regions of recharge and discharge, etc.), care must be taken to distinguish between steady and unsteady situations. For example, figure 6.6.4 shows a mound in the water table of a phreatic aquifer. If this map describes steady flow, there *must be* a source of water inside the area enclosed by the + 5 contour curve. If, however, the flow is unsteady and this is only an instantaneous picture that changes with time, the water leaving the area *

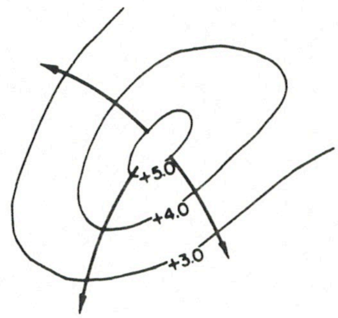

FIG. 6.6.4. A mound in the phreatic surface.

along the streamlines is taken from storage within the aquifer itself, producing a ubiquitous drop in the water table.

In reservoir engineering contours are usually drawn of pressure rather than of piezometric head.