CHAPTER 3

GROUND WATER FLOW AND WELL MECHANICS

3.1 STEADY-STATE WELL HYDRAULICS

The case of steady flow to a well implies that the variation of head occurs only in space and not in time. The governing equations of flow presented in Section 2.4 can be solved for pumping wells in unconfined or confined aquifers under steady or unsteady conditions. Boundary conditions must be kept relatively simple and aquifers must be assumed to be homogeneous and isotropic in each layer. More complex geometries can be handled by numerical simulation models in two or three dimensions (Chapters 8, 9 and 10).

3.2 STEADY ONE-DIMENSIONAL FLOW

For the case of ground water flow in the x-direction in a confined aquifer, the governing equation becomes

$$\frac{d^2h}{dx^2} = 0 \tag{3.1}$$

and has the solution

$$h = -\frac{vx}{K} + h_0 \tag{3.2}$$

where $h = 0$ and $x = 0$ and $dh/dx = -v/K$, according to Darcy's law. This states that head varies linearly with flow in the x–direction.

The case of steady one-dimensional flow in an unconfined aquifer was presented in Section 2.5 using Dupuit's assumptions. The resulting variation of head with x is called the Dupuit parabola and represents the approximate shape of the water table for relatively flat slopes. In the presence of steep slopes near wells, the Dupuit approximation may be in error, and more sophisticated computer methods should be used.

3.3 STEADY RADIAL FLOW TO A WELL—CONFINED

The drawdown curve or cone of depression varies with distance from a pumping well in a confined aquifer (Figure 3.1). The flow is assumed two-dimensional for a completely penetrating well in a homogeneous, isotropic aquifer of unlimited extent. For horizontal flow, the above assumptions apply, and Q at any radius r equals, from Darcy's law,

$$Q = -2\pi rbK\frac{dh}{dr} \tag{3.3}$$

for steady radial flow to a well. Integrating after separation of variables, with $h = h_w$ at $r = r_w$ at the well, yields

$$Q = 2\pi Kb\frac{h - h_w}{\ln(r / r_w)} \tag{3.4}$$

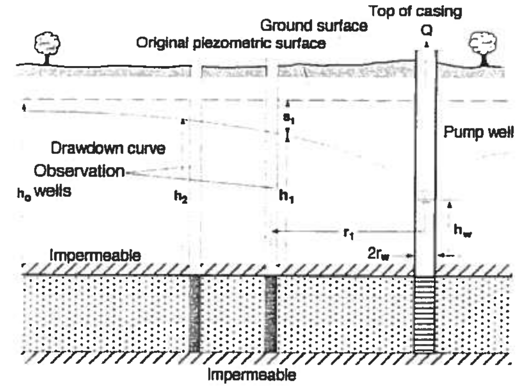

Figure 3.1 Radial flow to a well penetrating an extensive confined aquifer.

Equation (3.4) shows that h increases indefinitely with increasing r, yet the maximum head is h_0 for Figure 3.1. Near the well, the relationship holds and can be rearranged to yield an estimate for transmissivity T

$$T = Kb = \frac{Q}{2\pi(h_2 - h_1)}\ln\frac{r_2}{r_1} \tag{3.5}$$

by observing heads h_1 and h_2 at two adjacent observation wells located at r_1 and r_2, respectively, from the pumping well. In practice, it is often necessary to use unsteady-state analyses because of the long times required to reach steady state.

Example 3.1 DETERMINATION OF K AND T IN A CONFINED AQUIFER

A well is constructed to pump water from a confined aquifer. Two observation wells, OW–1 and OW–2, are constructed at distances of 100 m and 1000 m, respectively. Water is pumped from the pumping well at a rate of 0.2 m³/min. At steady state, drawdown s' is observed as 2 m in OW–2 and 8 m in OW–1. Determine the hydraulic conductivity K and transmissivity T if the aquifer is 20 m thick.

Solution. Given

Q = 0.2 m³/min,
r_2 = 1000 m,
r_1 = 100 m,

$s_2' = 2$ m,
$s_1' = 8$ m,
$b = 20$ m.

Equation (3.5) gives

$$T = Kb = \frac{Q}{2\pi(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right)$$

Knowing that $s_1' = h_0 - h_1$ and $s_2' = h_0 - h_2$, we have

$$T = Kb = \frac{Q}{2\pi(s_1' - s_2')}\left[\ln\left(\frac{r_2}{r_1}\right)\right] = \frac{0.2\text{m}^3/\text{min}}{(2\pi)(8\text{m} - 2\text{m})} \ln\left(\frac{1000\text{m}}{100\text{m}}\right)$$

$$T = 0.0122\text{m}^2/\text{min} = 2.04\text{cm}^2/\text{sec}$$

Then,

$$K = T/b = \frac{(2.04\text{cm}^2/\text{sec})}{(20\text{m})(100\text{cm}/1\text{m})}$$

$$\boxed{K = 1.02 \times 10^{-3}\text{cm/sec}}$$

3.4 STEADY RADIAL FLOW TO A WELL—UNCONFINED

Applying Darcy's law for radial flow in an unconfined, homogeneous, isotropic, and horizontal aquifer and using Dupuit's assumptions (Figure 3.2),

$$Q = -2\pi r K h \frac{dh}{dr} \tag{3.6}$$

Integrating, as before,

$$Q = \pi K \frac{h_2^2 - h_1^2}{\ln(r_2/r_1)} \tag{3.7}$$

Solving for K,

$$K = \frac{Q}{\pi(h_2^2 - h_1^2)} \ln \frac{r_2}{r_1} \tag{3.8}$$

where heads h_1 and h_2 are observed at adjacent wells located distances r_1 and r_2 from the pumping well, respectively.

Example 3.2 DETERMINATION OF *K* IN AN UNCONFINED AQUIFER

A fully penetrating well discharges 75 gpm from an unconfined aquifer. The original water table was recorded as 35 ft. After a long time period the water table was recorded as 20 ft MSL in an observation well located 75 ft away and 34 ft MSL at an observation well located 2000 ft away. Determine the hydraulic conductivity of this aquifer in ft/s.

Solution. Given

$Q = 75$ gpm,
$r_2 = 2000$ ft,
$r_1 = 75$ ft,
$h_2 = 34$ ft, and
$h_1 = 20$ ft.

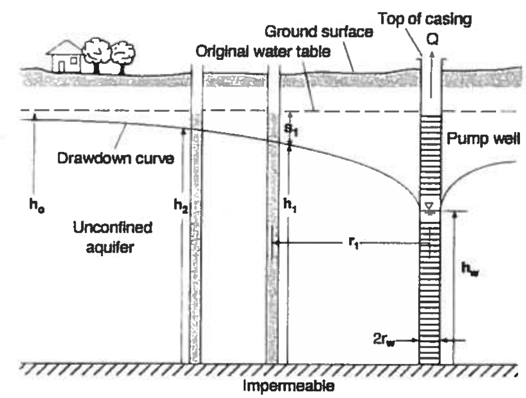

Figure 3.2 Radial flow to a well penetrating an extensive unconfined aquifer.

Equation (3.8) gives

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right)$$

$$= \frac{(75\text{gpm})(0.134\text{ft}^3/\text{gal})(1\text{min}/60\text{sec})}{(\pi)(34^2\text{ft}^2 - 20^2\text{ft}^2)} \ln\frac{2000\text{ft}}{75\text{ft}}$$

$$\boxed{K = 2.32\times10^{-4}\text{ft/s}}$$

3.5 WELL IN A UNIFORM FLOW FIELD

A typical problem in well mechanics involves a well pumping from a uniform flow field (Figure 3.3). A vertical section and plan view indicate the sloping piezometric surface and the resulting flow net. The ground water divide between the region that flows to the well and the region flowing by the well can be found from

$$-\frac{y}{x} = \tan\left(\frac{2\pi Kbi}{Q}y\right) \quad (3.9)$$

Equation (3.9) results from the superposition of radial and one-dimensional flow field solutions, where i is the original piezometric slope (gradient). It can be shown that

$$y_L = \pm\frac{Q}{2Kbi} \quad (3.10)$$

as $x \rightarrow \infty$, and the stagnation point (no flow) occurs at

$$x_S = -\frac{Q}{2\pi Kbi}, \quad y = 0 \quad (3.11)$$

Equations (3.10) and (3.11) may be applied to unconfined aquifers for cases of relatively small drawdowns, where b is replaced by h_0, the average saturated aquifer thickness. An important application of the well in a uniform flow field involves the evaluation of pollution sources and impacts on downgradient well fields and the potential for pumping and capturing a plume as it migrates downgradient. Chapter 13 addresses capture zone methods in more detail.

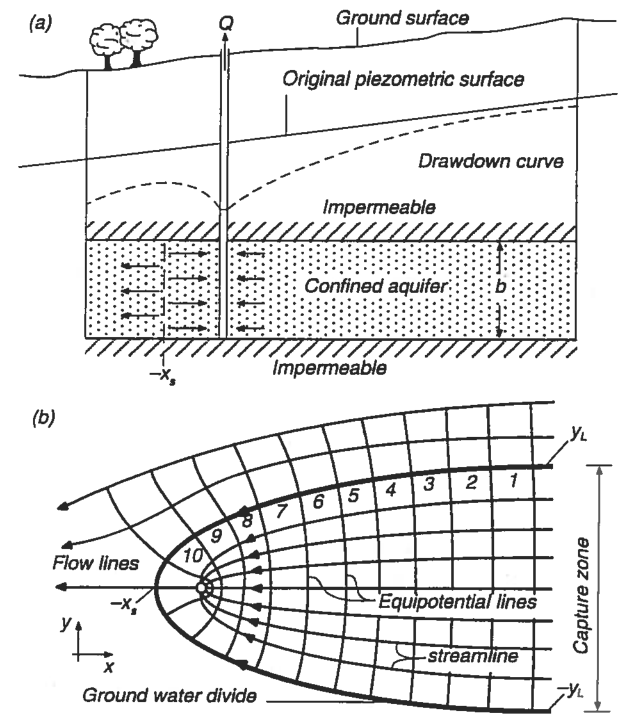

Figure 3.3 Flow to a well penetrating a confined aquifer having a sloping-plane piezometric surface. (a) vertical section. (b) Plan view.

3.6 MULTIPLE-WELL SYSTEMS

For multiple wells with drawdowns that overlap, the principle of superposition can be used. Drawdown at any point in the area of influence of several pumping wells is equal to the sum of drawdowns from each well in a confined aquifer.

Because Laplace's equation is linear, the superposition of drawdown effects is found by simple addition. Linear superposition is generally valid only for confined aquifers, since T does not change with drawdown. Thus, drawdown at any point in the area of influence of several pumping wells is equal to the sum of drawdown from each well. The above methods can be used for evaluating the effect of multiple wells in dewatering applications, well field effects, and for the case of unsteady well hydraulics. Several of the homework problems and example 3.6 demonstrate the use of linear superposition and well flow near boundaries.

The same principle applies for well flow near a boundary. Figure 3.4 shows the case for a well pumping near a fixed head stream, and Figure 3.5 shows an impermeable boundary. **Image wells** placed on the other side of the boundary at a distance x_w, can be used to represent the equivalent hydraulic condition. In one case, the image well is recharging at the

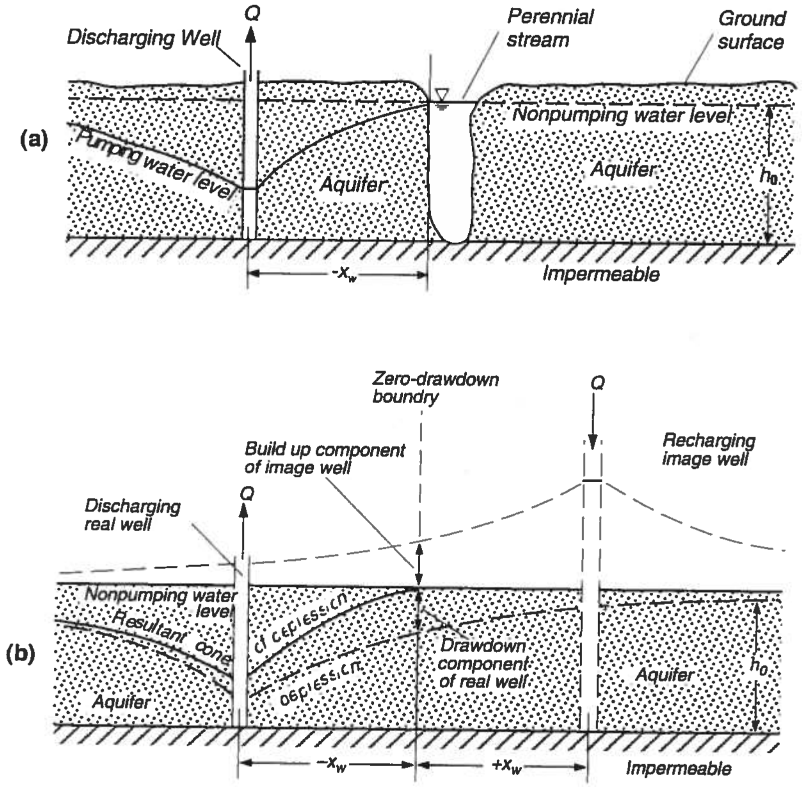

Figure 3.4 Sectional views. (a) Discharging well near a perennial stream. (b) Equivalent hydraulic system in an aquifer of infinite areal extent. Aquifer thickness h_0 should be very large compared with resultant drawdown near real well. Source: Ferris, et al., 1962.

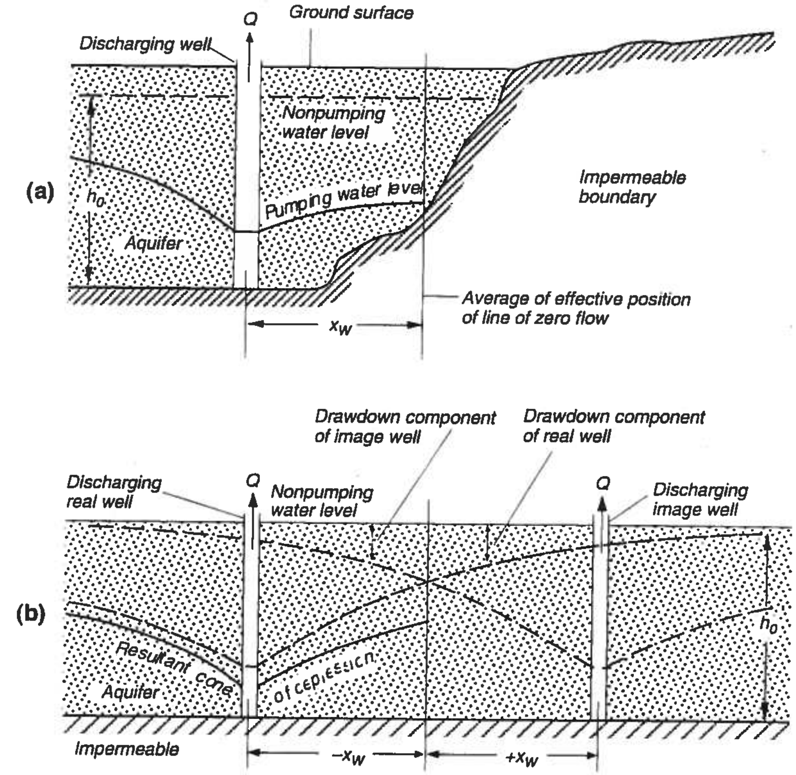

Figure 3.5 Sectional views. (a) Discharging well near an impermeable boundary. (b) Equivalent hydraulic system in an aquifer of infinite areal extent. Aquifer thickness h_0 should be very large compared with resultant drawdown near real well. Source: Ferris, et al., 1962.

same rate Q, and in another case it is pumping at rate Q. The summation of drawdowns from the original pumping well and the image well provides a correct boundary condition at distance x_w from the well. Thus, the use of image wells allows an aquifer of finite extent to be transformed into an infinite aquifer so that closed-form solution methods can be applied. Figure 3.6 shows a flow net for a pumping well and a recharging image well and indicates a line of constant head between the two wells. The steady-state drawdown s' at any point (x,y) is given by

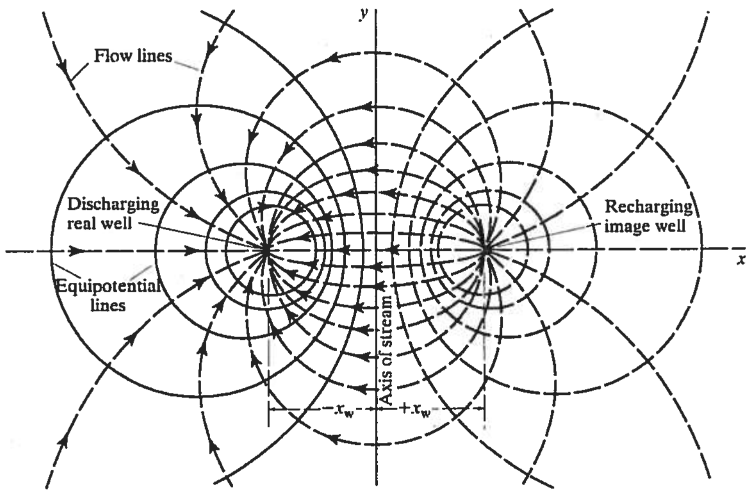

Figure 3.6 Flow net for a discharging real well and a recharging image well. Source: Ferris, et al,1962.

$$s' = \frac{Q}{4\pi T} \ln \frac{(x-x_w)^2 + (y-y_w)^2}{(x+x_w)^2 + (y-y_w)^2} \tag{3.12}$$

where $(\pm x_w, y_w)$ are the locations of the recharge and discharge wells (DeWiest, 1965). For the case shown in Figure 3.6, $y_w = 0$.

3.7 UNSTEADY WELL HYDRAULICS

3.7.1 The Theis Method of Solution

Because a well penetrating a confined aquifer of infinite extent is pumped at a constant rate, a drawdown occurs radially extending from the well. The rate of decline of head times the storage coefficient summed over the area of influence equals the discharge. The rate of decline decreases continuously as the area of influence expands. The governing ground water flow equation (see Eq. (2.27)) in plane polar coordinates is

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \tag{3.13}$$

where

h = head
r = radial distance
S = storage coefficient, and
T = transmissivity

Theis (1935) obtained a solution for Eq. (3.13) by assuming that the well is a mathematical sink of constant strength and by using boundary conditions $h = h_0$ for $t = 0$ and $h \rightarrow h_0$ as $r \rightarrow \infty$ for $t \geq 0$:

$$s' = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}}{u} du = \frac{Q}{4\pi T} W(u) \tag{3.14}$$

where s' is drawdown, Q is discharge at the well, and

$$u = \frac{r^2 S}{4Tt} \tag{3.15}$$

Equation (3.14) is known as the nonequilibrium, or Theis equation. The integral is written as $W(u)$ and is known as the exponential integral, or well function, which can be expanded as a series:

$$W(u) = -0.5772 - \ln(u) + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + \cdots \tag{3.16}$$

The equation can be used to obtain aquifer constants S and T by means of pumping tests at fully penetrating wells. It is widely used because a value of S can be determined, only one observation well and a relatively short pumping period are required, and large portions of the flow field can be sampled with one test.

The assumptions inherent in the Theis equation should be included since they are often overlooked:

1. The aquifer is homogeneous, isotropic, uniformly thick, and of infinite areal extent.
2. Prior to pumping, the piezometric surface is horizontal.
3. The fully penetrating well is pumped at a constant rate.

4. Flow is horizontal within the aquifer.
5. Storage within the well can be neglected.
6. Water removed from storage responds instantaneously with a declining head.

These assumptions are seldom completely satisfied for a field problem, but the method still provides one of the most useful and accurate techniques for aquifer characterization. The complete Theis solution requires the graphical solution of two equations with four unknowns:

$$s' = \frac{Q}{4\pi T} W(u) \tag{3.17}$$

$$\frac{r^2}{t} = \left(\frac{4T}{S}\right) u \tag{3.18}$$

The relation between W(u) and u must be the same as that between s' and r^2/t because all other terms are constants in the equations. Theis suggested a solution based on graphical superposition. Example 3.3 indicates how a plot of W(u) vs. u, called a **type curve,** is superimposed over observed time-drawdown data while keeping the coordinate axes parallel. The two plots are adjusted until a position is found by trial, such that most of the observed data fall on a segment of the type curve. Any convenient point is selected, and values of W(u), u, s' and r^2/t are used in Eqs. (3.17) and (3.18) to determine S and T (see Figure 3.7).

It is also possible to use Theis's solution for the case where several wells are sampled for drawdown simultaneously near a pumped well. Distance-drawdown data are then fitted to the type curve similar to the method just outlined.

Example 3.3 DETERMINATION OF *T* AND *S* BY THE THEIS METHOD

A fully penetrating well in a 25 m thick confined aquifer is pumped at a rate of 0.2 m^3/s for 1000 min. Drawdown is recorded vs. time at an observation well located 100 m away. Compute the transmissivity and storativity using the Theis method.

Solution. A plot of s' vs. r^2/t is made on log-log paper. This is superimposed on a plot of W(u) versus u, which is also on log-log paper. A point is chosen at some convenient point on the matched curve, and values for s', r^2/t, W(u) and u are read (see Figure 3.7 and the accompanying Tables 3.1 and 3.2).

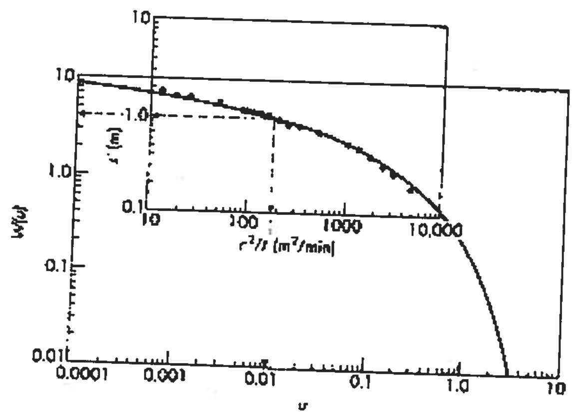

Figure 3.7 Theis curve compared to measured data.

TABLE 3.1 Radial flow to a well penetrating an extensive confined aquifer

Time (min)	s' (m)	Time (min)	s' (m)	Time (min)	s' (m)
1	0.11	20	0.71	90	1.11
2	0.20	30	0.82	100	1.15
3	0.28	40	0.85	200	1.35
4	0.34	50	0.92	400	1.55
6	0.44	60	1.02	600	1.61
8	0.50	70	1.05	800	1.75
10	0.54	80	1.08	1000	1.80

From the plot,

r^2/t = 180 m^2/min

s' = 1.0 m

u = 0.01

W(u) = 4.0

TABLE 3.2 Calculated values for the Well function

u	W(u)	u	W(u)	u	W(u)	u	W(u)	u	W(u)
1 e-10	22.45	2 e-08	17.15	3 e-06	12.14	4 e-04	7.25	5 e-02	2.47
2	21.76	3	16.74	4	11.85	5	7.02	6	2.3
3	21.35	4	16.46	5	11.63	6	6.84	7	2.15
4	21.06	5	16.23	6	11.45	7	6.69	8	2.03
5	20.84	6	16.05	7	11.29	8	6.55	9	1.92
6	20.66	7	15.9	8	11.16	9	6.44	1 e-01	1.823
7	20.5	8	15.76	9	11.04	1 e-03	6.33	2	1.223
8	20.37	9	15.65	1 e-05	10.94	2	5.64	3	0.906
9	20.25	1 e-07	15.54	2	10.24	3	5.23	4	0.702
1 e-09	20.15	2	14.85	3	9.84	4	4.95	5	0.56
2	19.45	3	14.44	4	9.55	5	4.73	6	0.454
3	19.05	4	14.15	5	9.33	6	4.54	7	0.374
4	18.76	5	13.93	6	9.14	7	4.39	8	0.311
5	18.54	6	13.75	7	8.99	8	4.26	9	0.26
6	18.35	7	13.6	8	8.86	9	4.14	1 e+00	0.219
7	18.2	8	13.46	9	8.74	1 e-02	4.04	2	0.049
8	18.07	9	13.34	1 e-04	8.63	2	3.35	3	0.013
9	17.95	1 e-06	13.24	2	7.94	3	2.96	4	0.004
1 e-08	17.84	2	12.55	3	7.53	4	2.68	5	0.001

Equation (3.17) gives

$$s' = \frac{Q}{4\pi T} W(u)$$

$$T = \frac{Q\,W(u)}{4\pi s'}$$

$$T = \frac{(0.2\text{m}^3/\text{sec})(4.0)}{(4\pi)(1.0\text{m})}$$

$$\boxed{T = 6.37\times10^{-2}\,\text{m}^2/\text{sec}}$$

Equation (3.18) gives

$$\frac{r^2}{t} = \frac{4Tu}{S}$$

$$S = \frac{4Tu}{r^2/t}$$

$$= \frac{(4)(6.37\times10^{-2}\,\text{m}^2/\text{sec})(0.01)}{(180\text{m}^2/\text{min})(1\,\text{min}/60\,\text{sec})}$$

$$\boxed{S = 8.49\times10^{-4}}$$

3.7.2 Cooper-Jacob Method of Solution

Cooper and Jacob (1946) noted that for small values of r and large values of t, the parameter u in Eq. (3.16) becomes very small so that the infinite series can be approximated by

$$s' = \frac{Q}{4\pi T}\left[-0.5772 - \ln\left(\frac{r^2 S}{4Tt}\right)\right] \tag{3.19}$$

Further rearrangement and conversion to decimal logarithms yields

$$s' = \frac{2.30Q}{4\pi T}\log\left(\frac{2.25Tt}{r^2 S}\right) \tag{3.20}$$

Thus, a plot of drawdown s' vs. logarithm of t forms a straight line, as shown in Figure 3.8. A projection of the line to $s' = 0$, where $t = t_0$, yields

$$0 = \frac{2.3Q}{4\pi T}\log\left(\frac{2.25Tt_0}{r^2 S}\right) \tag{3.21}$$

and it follows that, since log (1) = 0,

$$S = \frac{2.25Tt_0}{r^2} \tag{3.22}$$

Finally, by replacing s' by $\Delta s'$, where $\Delta s'$ is the drawdown difference of data per log cycle of t, Eq. (3.20) becomes

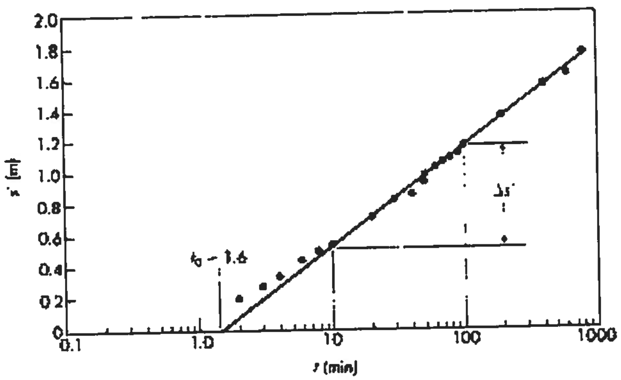

Figure 3.8 Cooper-Jacob method of analysis.

$$T = \frac{2.3Q}{4\pi\Delta s'} \tag{3.23}$$

The Cooper-Jacob method first solves for T with Eq. (3.23) and then for S with Eq. (3.22) and is applicable for small values of u (less than 0.01). Calculations with the Theis method were presented in Example 3.3, and the Cooper-Jacob method is used in Example 3.4.

Example 3.4 DETERMINATION OF T AND S BY THE COOPER-JACOB METHOD

Using the data given in Example 3.3, determine the transmissivity and storativity of the 25 m thick confined aquifer using the Cooper-Jacob method.

Solution. Values of s' and t are plotted on semilog paper with the t-axis logarithmic (see Figure 3.4). A line is fitted through the later time periods and is projected back to a point where $s' = 0$. This point determines t_0. $\Delta s'$ is measured over one log cycle of t.

From the plot,

$$t_0 = 1.6 \text{ min}$$
$$\Delta s' = 0.65 \text{ m}$$

Equation (3.23) gives

$$T = \frac{2.3Q}{4\pi\Delta s'}$$
$$= \frac{(2.3)(0.2\text{m}^3/\text{sec})}{(4\pi)(0.65\text{m})}$$

$$\boxed{T = 5.63\times10^{-2}\,\text{m}^2/\text{sec}}$$

Equation (3.22) gives

$$S = \frac{2.25Tt_0}{r^2}$$
$$= \frac{(2.25)(5.63\times10^{-2}\,\text{m}^2/\text{sec})(1.6\text{min})(60\text{sec}/1\text{min})}{(100\text{ m})^2}$$

$$\boxed{S = 1.22\times10^{-3}}$$

3.7.3 Slug Tests

Slug tests involve the use of a single well for the determination of aquifer formation constants. Rather than pumping the well for a period of time, as described above, a volume of water is suddenly removed or added to the well casing and observations of recovery or drawdown are noted through time. By careful evaluation of the drawdown curve and knowledge of the well screen geometry, it is possible to derive K or T for an aquifer.

Typical procedure for a slug test requires use of a rod of slightly smaller diameter than the well casing or a pump to evacuate the well casing. The simplest slug test method in a piezometer was published by Hvorslev (1951), who used the recovery of water level over time to calculate hydraulic conductivity of the porous media. Hvorslev's method relates the flow $q(t)$ at the piezometer at any time to the hydraulic conductivity and the unrecovered head distance, $H_0 - h$ in Figure 3.9, by

$$q(t) = \pi r^2 \frac{dh}{dt} = FK(H_0 - h) \tag{3.24}$$

where F is a factor that depends on the shape and dimensions of the piezometer intake. If $q = q_0$ at $t = 0$, then $q(t)$ will decrease toward zero as time increases. Hvorslev defined the basic time lag

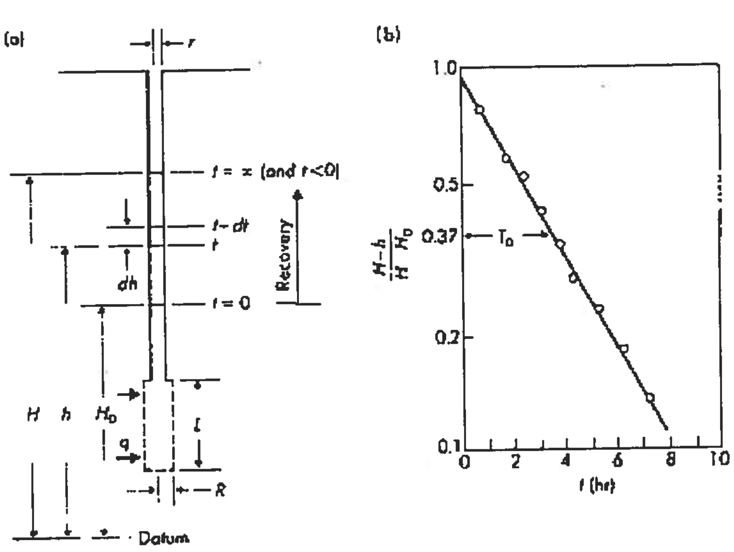

Figure 3.9 Hvorslev piezometer test. (a) Geometry. (b) Method of analysis.

$$T_0 = \frac{\pi r^2}{FK}$$

and solved Eq. (3.24) with initial conditions $h = H_0$ at $t = 0$. Thus

$$\frac{H-h}{H-H_0} = e^{-t/T_0} \tag{3.25}$$

By plotting recovery $(H-h)/(H-H_0)$ vs. time on semilog graph paper, we find that $t = T_0$, where recovery equals 0.37 (Figure 3.9). For piezometer intake length divided by radius (L/R) greater than 8, Hvorslev has evaluated the shape factor F and obtained an equation for K.

$$K = \frac{r^2 \ln(L/R)}{2LT_0} \tag{3.26}$$

Several other slug test methods have been developed by Cooper et al. (1967) and Papadopoulos et al. (1973) for confined aquifers. These methods are similar to Theis's in that a curve-matching procedure is used to obtain S and T for a given aquifer. A family of type curves $H(t)/H_0$ vs. Tt/r_c^2 was published for five values of the variable α, defined as $(r_s^2/r_c^2)S$ in Figure 3.10. Papadopoulos et al. (1973) added five additional values of α. The solution method is graphical and requires a semilogarithmic plot of measured $H(t)/H_0$ vs. t, where H_0 is the assumed initial excess head. The data are then curve-matched to the plotted type curves by horizontal translation until the best match is achieved (Figure 3.10), and a value of α is selected for a particular curve. The vertical time axis t, which overlays the vertical axis for $Tt/r_c^2 = 1.0$ is selected, and a value of T can then be found from $T = 1.0 r_s^2/t_1$. The value of S can be found from the definition of α. The use of the method is representative of the formation only in the immediate vicinity of the test hole and should be used with caution.

The most commonly used method for determining hydraulic conductivity in ground water investigations is the Bouwer and Rice (1976) slug test. While it was originally designed for unconfined aquifers, it can be used for confined or stratified aquifers if the top of the screen is some distance below the upper confining layer. The method is based on the following equation:

$$K = \frac{r_c^2 \ln(R_e/r_w)}{2L_e} \frac{1}{t} \ln \frac{y_0}{y_t} \tag{3.27}$$

where

- r_c = radius of casing,
- y_0 = vertical difference between water level inside well and water level outside at $t = 0$,
- y_t = vertical difference between water level inside well and water table outside (drawdown) at time t,
- R_e = effective radial distance over which y is dissipated, and varying with well geometry,
- r_w = radial distance of undisturbed portion of aquifer from centerline (usually thickness of gravel pack),
- L_e = length of screened, perforated, or otherwise open section of well, and
- t = time.

In the above equation, y and t are the only variables. Thus, if a number of y and t measurements are taken, they can be plotted on semi-logarithmic paper to give a straight line. The slope of the best-fitting straight line will provide a value for $[\ln(y_0/y_t)]/t$. All the other parameters in the above equation are known from well geometry, and K can be calculated. A point to note is that drawdown on the ground water table becomes increasingly sig-

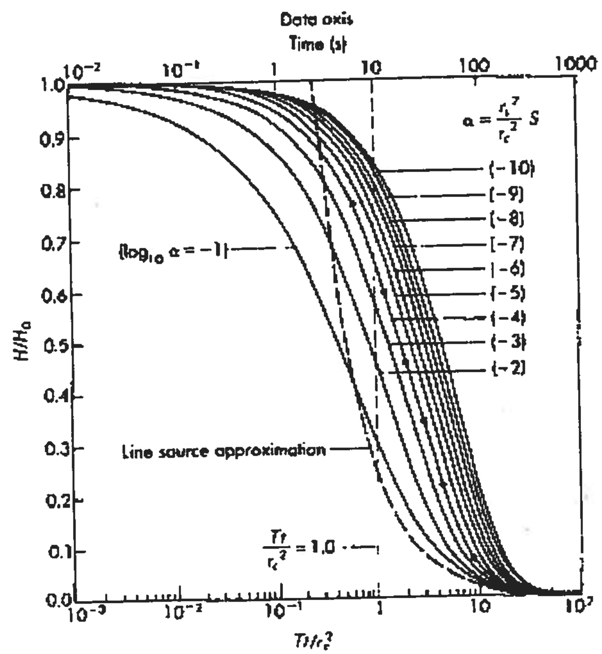

Figure 3.10 Papadopoulos slug test type curves. Source: Papadopoulos, et al., 1973. © 1973 American Geophysical Union.

nificant as the test progresses, and the points will begin to deviate from the straight line for large t and small y. Hence, only the straight line portion of the data must be used in the calculation for K.

Example 3.5. SLUG TEST METHOD

A screened, cased well penetrates a confined aquifer. The casing radius is 5 cm and the screen is 1 m long. A gravel pack 2.5 cm wide surrounds the well and a slug of water is injected that raises the water level by 0.28 m. The change in water level with time is as listed in the following table. Given that R_e is 10 cm, calculate K for the aquifer (see Figure 3.11a).

Solution. Data for y vs. t are plotted on semi-log paper as shown in Figure 3.11b. The straight line from y_0 = 0.28 m to y_t = 0.001 m covers 2.4 log cycles. The time

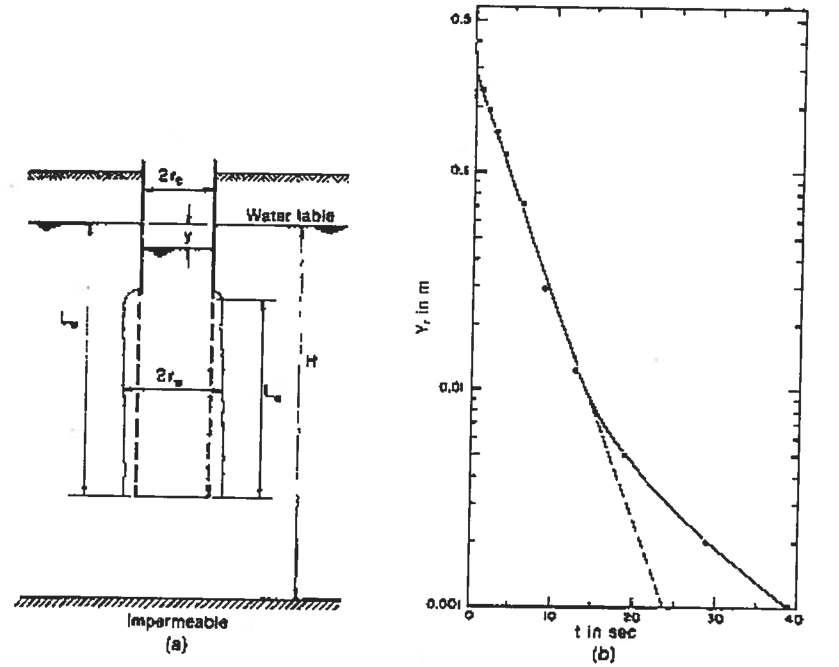

Figure 3.11 (a) Geometry and symbols for slug test on partially penetrating, partially screened well in an unconfined aquifer with gravel pack and/or developed zone around screen. (b) Graph of log Y_t versus t for slug test on well in Salt River bed, 27th Avenue, Phoenix, AZ. Source: Bouwer and Rice, 1976.© American Geophysical Union.

increment between the two points is 24 seconds. To convert the log cycles to natural log, a factor of 2.3 is used. Thus, $1/t\ln(y_0/y_t) = 2.3 \times 2.4/24 = 0.23$. Using this value in the Bouwer and Rice equation gives

$$K = \frac{(5\text{cm})^2 \ln(10\text{cm}/7.5\text{cm})}{2 \times 100\text{cm}}(0.23\text{sec}^{-1})$$

and

$$K = 8.27 \times 10^{-3} \text{ cm/s}$$

t (sec)	y_t (m)
1	0.24
2	0.19
3	0.16
4	0.13
6	0.07
9	0.03
13	0.013
19	0.005
20	0.002
40	0.001

Example 3.5 indicates how slug tests are applied to field data. It should be noted that slug tests are often used at hazardous waste sites since large volumes of contaminated water do not have to be dispersed, as in the case of a pump test. However, the pump test generally gives a better picture of overall hydraulic conductivity than does the slug test at a site.

3.7.4 Radial Flow in a Leaky Aquifer

Leaky aquifers represent a unique and complex problem in well mechanics. When a leaky aquifer is pumped, as shown in Figure 3.12, water is withdrawn both from the lower aquifer and from the saturated portion of the overlying aquitard. By creating a lowered piezometric surface below the water table, ground water can migrate vertically downward and then move horizontally to the well. While steady-state conditions in a leaky system are possible, a general nonequilibrium analysis for unsteady flow is more applicable and more often occurs in the field. When pumping starts from a well in a leaky aquifer, drawdown of the piezometric surface can be given by

$$s' = \frac{Q}{4\pi T} \mathrm{W}\left(u, \frac{r}{B}\right) \tag{3.28}$$

where the quantity r/B is given by

$$\frac{r}{B} = \frac{r}{\sqrt{T/(K'/b')}}$$

where T is transmissivity of the aquifer, K' is vertical hydraulic conductivity of the aquitard, and b' is thickness of the aquitard. Values of the function $W(u, r/B)$ have been tabulated by Hantush (1956) and have been used by Walton (1960) to prepare a family of type curves, shown in Figure 3.13. Equation (3.28) reduces to the Theis equation for $r/B = 0$. The method of solution for the leaky aquifer works in the same way as the Theis solution with a super-

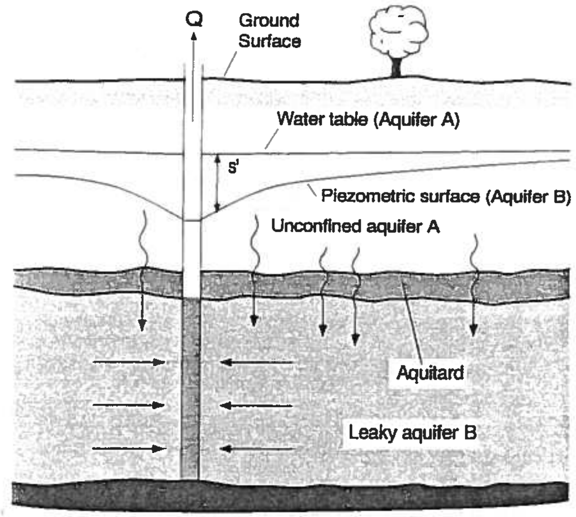

Figure 3.12 Well pumping from a leaky aquifer.

position of drawdown data on top of the leaky type curves. A curve of best fit is selected, and values of W, $1/u$, s', and t are found, which allows T and S to be determined. Finally, based on the value of r/B, it is possible to calculate K' and b'.

In general, leaky aquifers are much more difficult to deal with than confined or unconfined systems. But the method just described does provide a useful tool for evaluating leaky systems analytically. For more complex geologies and systems with lenses, a three-dimensional computer simulation may be employed to properly represent ground water flow. These types of models are described in detail in Chapter 10.

Example 3.6 APPLICATION OF THEIS

A 375-m square excavation is to be dewatered by the installation of four wells at the corners. Point A is in the middle and Point B is on one side equidistant from two of the wells. For an allowable pumping period of 24 hours, determine the pumping rate required to produce a minimum drawdown of 4 m everywhere within the limits of the excavation. The confined aquifer has a transmissivity of 2×10^{-4} m^2/s and a storage factor of 7×10^{-5}.

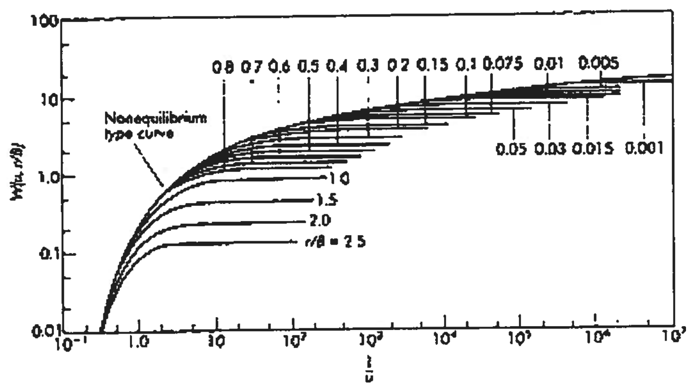

Figure 3.13 Type curves for analysis of pumping test data to evaluate storage coefficient and transmissivity of leaky aquifers. Source: Walton, 1960.

Solution. By symmetry, we expect the maximum drawdown to be at either A or B, so we must determine which of them is limiting. We will use the Theis equations (below) to determine the flow rate Q necessary to create a 4 m drawdown at A and B.

$$s=\left(\frac{Q}{4\pi T}\right)\mathrm{W}(u), \quad \text{and} \quad u=\frac{r^2 S}{4Tt}$$

a) Determine the required pumping rate for 4 m drawdown at A. Using r, S, T, and t, find u with the second equation above, then determine W(u) from Table 3.2, and solve the first equation for Q.

$$r=\sqrt{2}\,\frac{375}{2}= 265 \text{ m}$$

$$u=\frac{(265\text{m})^2(7\times10^{-5})}{(4)(2\times10^{-4}\text{m}^2/\text{sec})(24\text{hr})(3600\text{sec}/\text{hr})}=0.071$$

$$\mathrm{W}(u)=2.14$$

and each well contributes 25%,

$$Q=\frac{1}{4}\frac{s}{W(u)}(4\pi T)=\frac{(4\text{m})}{(4)(2.14)}(4)(\pi)(2\times10^{-4}\text{m}^2/\text{sec})$$

$$Q=1.17\times10^{-3}\ \text{m}^3/\text{s} = 4.23\ \text{m}^3/\text{hr for each well}$$

b) Determine drawdown at B using the flowrates calculated above. Drawdown at B is a combination of two wells 187.5 m from B and two wells at $r = \sqrt{187.5^2+375^2}$ m = 419 m from B. For the closer two wells:

$$u=\frac{(187.5\text{m})^2(7\times10^{-5})}{(4)(2\times10^{-4}\text{m}^2/\text{sec})(24\text{hr})(3600\text{sec}/\text{hr})}= 0.036$$

$$\mathrm{W}(u)=2.79$$

and the drawdown produced by the closer two wells is:

$$s=2\frac{(1.17\times10^{-3}\text{m}^3/\text{sec})}{(4)(\pi)(2\times10^{-4}\text{m}^2/\text{sec})}(2.79)= 2.60 \text{ m}$$

For the two farther wells,

$$u=\frac{(419\text{m})^2(7\times10^{-5})}{(4)(2\times10^{-4}\text{m}^2/\text{sec})(24\text{hr})(3600\text{sec}/\text{hr})}= 0.18 \Rightarrow \mathrm{W}(u) = 1.34$$

$$s=2\frac{(1.17\times10^{-3}\text{m}^3/\text{sec})}{(4)(\pi)(2\times10^{-4}\text{m}^2/\text{sec})}(1.34)= 1.25 \text{ m}$$

Summing over all four wells, the total drawdown at B = 2.60 + 1.25 = 3.85 m. Thus, the drawdown at B is less than at A, so requiring a 4 m drawdown at B will automatically meet the criteria at Point A, and over the entire site. Since s and Q are linearly related, multiplying the above calculated Q by (4 m/3.85 m) will give us a drawdown of 4 m at B. Therefore Q = 4.4 m³/hr will keep the construction site dry.

SUMMARY

Chapter 3 has provided a review of well mechanics under both steady-state and transient conditions. The principle of superposition was applied for multiple well systems and image well problems. The Theis method of solution was derived and several examples are shown. Slug test methods for single wells are covered in detail. Well mechanics solutions are important in the application of pump tests and capture zones, which are discussed in more detail in Chapters 10 and 13.