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Optimization of Monitoring Well Installation Time and Location During Aquifer Decontamination

SANG-IL LEE and PETER K. KITANIDIS*
Department of Civil Engineering, Dongguk University, 3-26 Pil-dong, Jung-gu, Seoul, 100–273, Korea

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Abstract. An important question in the systematic and objective design of general-purpose groundwater quality monitoring networks is how to evaluate quantitatively the information they provide. However, many applications require the design of a groundwater monitoring network in conjunction with remedial action at a subsurface contamination site. In such a case, it is conceptually clear what is a successful network: One that reduces the net cost of meeting the objectives of cleanup. Uncertainty entails a cost because the natural management response to uncertainty is overdesign for the sake of conservatism ('safety factor'). The additional information that the network provides must lead to cost reductions that outweigh its cost. This paper presents a method to determine the installation time and location of an additional monitoring well while the aquifer is being cleaned up. While rates of pumping and treatment are determined by the dual control method (a method for optimization with incomplete information) candidate well locations are ranked according to a 'cost-to-go' index that measures the costs expected until the goals of remediation are met. This index accounts for the cost associated with uncertainty about the system and thus is useful in appraising the value of information from new measurements in the context of the specific cleanup effort. The usefulness of the method is illustrated through application to a hypothetical two-dimensional aquifer with uncertain initial estimates of the system parameters and variables. Monte Carlo simulations demonstrate the cost effectiveness of solution obtained through this method.

Key words: groundwater quality monitoring, remediation, dual control, cost-to-go function

1. Introduction

Pump-and-treat systems for the cleanup or containment of contaminated groundwater are designed and operated on the basis of information about the concentration distribution and the movement of contaminants. Monitoring of the subsurface environment through direct sampling from monitoring and pumping wells or using geophysical techniques can provide some of this information. Until recently, most monitoring wells were placed based on the experience and intuition of the geohydrologist, regulatory constraints, and convenience.

Systematic approaches for the selection of the most appropriate locations have only recently appeared in the literature. However, most of these works have dealt with the placement of observation wells for general monitoring purposes, i.e., without specifying how the information will be used. Since it is not possible to

* Department of Civil Engineering, Stanford University, Stanford, CA 94305, U.S.A.

quantify the monetary or other discernible benefits from their operation, statistical measures, such as the mean square error of estimation of concentration or some other quantity of general interest, are used as surrogates. The general idea is that the smaller the mean square error, the higher the value of the information obtained from the monitoring network. Little has been done to establish the relation between tangible benefits and statistical surrogates and there are always questions about which statistical measure to use as a performance criterion.

Few works have dealt with methodologies that optimize a monitoring network while considering that the basic objective is to clean up an aquifer in a cost-effective fashion. Typically, the responsible party must pay for the decontamination and the monitoring needed to characterize the site, design the cleanup, and demonstrate that water quality criteria are met. The average per Superfund site cost of characterization, assessment, and remedial action design is about $ 1.7 million, and the cost of the remedial action, usually a pump-and-treat system, is about $ 12.4 million (Department of Energy, 1988). So, the challenge is how to design a monitoring network that minimizes the total cost of meeting these objectives. It is of little use to design a 'general purpose' monitoring system that meets some vague accuracy criteria when the objective is pretty much clear: To demonstrate that certain decontamination objectives have been met successfully and to do so in a cost-effective way.

One of the most important questions in field applications is when and where to add monitoring wells during cleanup for purposes of evaluating performance or selecting an alternative remedial action (EPA, 1988). The hydrogeologic properties and the distribution of the contaminants are complex and inadequately known; one can always find good reasons for asking for more monitoring wells or more samples, if one is simply interested in reducing the estimation variance. However, some of these funds can be used more effectively for remediation or plume containment.

In the following, after a brief review of the literature on groundwater quality monitoring network design, we present our methodology. The objective is to select when and where to add a monitoring well to the existing network before or during remediation to minimize the expected overall cost. Remediation strategies are determined by the dual control method developed elsewhere (Andricevic and Kitanidis, 1990; Lee and Kitanidis, 1991). In this approach, we minimize the total expected cost of operation subject to reliability constraints. The benefits of a monitoring network are evaluated in definite terms, to the extent that the cost of drilling wells, testing samples, pumping, treating, etc., can be appraised. We will apply the method to a hypothetical two-dimensional aquifer for which we will present and discuss the effects on performance of additional monitoring.

2. Literature Review

The design of effective groundwater quality monitoring networks is an area of great practical significance and active current research. This section reviews works

Table I. Comparison of methods in selected groundwater quality monitoring literature

Author	Method	Objective function	Domain	Transport Eq.
Loaiciga (1988)	BNP	Min. fixed cost + estimation error	2D	Not applied
Loaiciga (1989)	MIP	Min. variance of estimation error	2D	Used
Hsueh and Rajagopal (1988)	IP	Max. information coeff. based on detection probability	2D	Not applied
Massmann and Freeze (1987a, b)	Enumeration	Max. net present value (= benefit-cost-risk)	2D	Used
Meyer and Brill (1988)	IP	Max. number of plumes detected	2D	Used
Knopman and Voss (1989)	Multi-objective programming	Max. prediction difference Min. estimation variance Min. cost	1D	Used
Graham and McLaughlin (1989a, b)	Variance reduction	Min. concentration variance	2D	Used
Tucciarelli and Pinder (1991)	QL	Min. pumping and measurement cost	2D	Used
Lee and Kitanidis (this work)	Dual control	Min. pumping and measurement cost	2D	Used

BNP = Binary Nonlinear Programming, IP = Integer Programming, MIP = Mixed IP
QL = Quasi-Linearity Algorithm.

representative of many ways this complex problem can be addressed with no claim of complete coverage. For a more detailed review of the literature, readers are referred to Loaiciga et al. (1992). In Table I we summarize the key features of the reviewed methods.

Loaiciga (1988) selected the best sampling sites among a predetermined set of possible well locations. He chose as his objective function the sum of the well installation cost plus an expected loss associated with the estimation error of the concentration average over the domain. He also assumed a known shape of the contaminant plume so that there was no need to solve the solute transport equation. He formulated the network design problem as a binary nonlinear program. Loaiciga (1989) formulated the optimal sampling plan for groundwater quality monitoring as a mixed integer programming problem. A sampling plan consisted of the number and locations of sampling sites as well as the sampling frequency.

He minimized the variance of estimation error subject to resource availability and unbiasedness constraints, accounting for changes in concentration through the advection-dispersion equation.

Hsueh and Rajagopal (1988) used a 0-1 integer programming model for deciding what and where to sample. They were concerned with groundwater quality over a large state-wide aquifer, not specifically with a plume in a single site. Thus, no site-specific information such as hydraulic conductivity was used, nor any estimation or simulation models. An 'information coefficient', based on detection probabilities, significance of health and ecological effects, and the size of nearby populations, was minimized to select monitoring wells among two hundred possible well locations.

Massmann and Freeze (1987a,b) detailed a comprehensive framework for design of a landfill operation. Their objective was to maximize the net present value of a stream of benefits minus costs. Monitoring contributes to the objective function by reducing the probability of failure, or equivalently, increasing the probability of detection. They conducted Monte Carlo simulations to determine the probability of detection, given a certain monitoring network. This study presented a framework for decision analysis without dealing with technical issues of optimization and estimation.

Meyer and Brill (1988) developed a method for the optimal placement of wells in a monitoring network using simulation models jointly with optimization methods. Contaminant transport simulation provides information about the location of plumes while an optimization model locates a given number of wells to maximize the probability of detection. Significant computational requirements turn out to limit the applicability of the method.

Knopman and Voss (1989) formulated the same problem as a multi-objective problem. They considered the following objectives: model discrimination to identify the most descriptive mathematical model of transport, parameter estimation accuracy, and cost. A one-dimensional solute-transport problem was considered.

The variance reduction approach (Rouhani, 1985; Rouhani and Hall, 1988) adds to the network the groundwater sampling site that reduces the most the variance of estimation error associated with a set of established sampling locations. An 'information response function' was used to select the location of each additional measurement, then a type of economic gain function was used to determine the number of new sites. Graham and McLaughlin (1989a, b) located new monitoring wells in areas where the concentration variance is highest. They found that a sequential groundwater quality monitoring program which evolves over time could provide better predictions, for a fixed budget, than a less flexible program which specifies well locations before samples are collected. Other algorithms based on the variance reduction approach are also available (McKinney and Loucks, 1992; Woldt and Bogardi, 1992).

The methodology by Tucciarelli and Pinder (1991) can consider the effect of measurements on groundwater remediation. They determined pumping rates by minimizing the summation of pumping and measurement costs subject to chance constraints on concentration. The log-transmissivity covariance matrix, updated through new measurements, was related to the concentration covariance matrix using the transport equations and first-order analysis. Increased confidence in estimating concentrations contributed to reduction of the pumping rates, through the decrease in the magnitude of the stochastic part of the chance constraints. Problems like when and where to put monitoring wells were not addressed.

3. Aquifer Remediation Strategies

Optimal scheduling for a pump-and treat system can be done by combining groundwater flow and contaminant transport simulation with some optimization scheme. Recent studies have attempted to incorporate the effects of flow and transport modeling uncertainty into the optimal scheduling process. The dual control method has unique advantages in that it simultaneously determines the optimal pump-and-treat schedule, estimates uncertain aquifer parameters, and accounts for the need for hedging (Andricevic and Kitanidis, 1990; Lee and Kitanidis, 1991). With readers referred to the above works for a detailed description of the methodology, here we briefly explain how optimal pumping rates are determined for aquifer remediation using the dual control.

First, the equations of groundwater flow and solute transport are transformed into a state-space form. Typically, the hydraulic head and the solute concentration at all nodes form the state of the system. Uncertain parameters and imperfectly modelled processes can also be incorporated in the state-space representation. Measurements regularly taken at the observation wells are expressed through the measurement equation, which is the relation between observations and state variables. Once a system model is developed, one can predict the impact of pumping (or injection) on the flow and contaminant distribution both in time and space. Since the system model accounts for uncertainty, it recognizes that the prediction of the pumping impact is subject to error. A filtering process estimates the future state of the system and calculates the estimation error (i.e., covariance matrix for heads and concentrations). The extended Kalman filter, in our case, propagates the state estimate and the covariance based on the flow and transport processes in the first step, and then updates them utilizing new measurements. More details about the implementation of the extended Kalman filter can be found in Anderson and Moore (1979).

With the formulations of the system and the state estimation process ready, the next concern is how to determine the most cost-effective pumping schedule while meeting the groundwater quantity and quality constraints. It is reasonable to select pumping rates that minimize the cost weighted by the probability that it will be incurred. In the dual control method, optimal pumping rate at a certain time stage is expressed as the sum of a deterministic pumping and a stochastic pumping term. The former is obtained by solving a deterministic optimization problem through constrained differential dynamic programming (DDP), and the latter is obtained by solving an analytic equation.

The procedure is initiated by assuming initial conditional mean of the state, its covariance and pumping schedule. The deterministic pumping policy is obtained through constrained DDP which minimizes the cost-to-go function (the expected value of the cost during the remainder of the operating horizon). Next, the analytic equation is solved to obtain the stochastic pumping rate. The first period pumping rate, sum of the deterministic and the stochastic pumping rate, is applied in real time. Measurements are collected and processed through the extended Kalman filter to update the conditional mean and the covariance. This completes one cycle, and the same process is repeated to obtain the pumping rate of the next time period.

Since the analytic equation for the stochastic pumping term involves the expected cost due to incomplete state estimation in the future, the solution is determined in a way that the current decision (i.e., pumping to be applied now) reduces the estimation error and thus the cost of operation. In order to reduce the estimation error, the pumping rate at the first few stages might be selected to learn from the response of the system ('probing effect'). The other aspect of the dual control is that it steers the system to the direction minimizing possible losses due to the discrepancy of the actual future state from the currently available best estimate ('caution effect'). Therefore, pumping rates are determined over time to optimize the criterion of overall performance based on the currently available information and the accounting of process dynamics and anticipated uncertainties and observations.

4. Adding a Monitoring Well

With the methodology to select pumping rates for aquifer decontamination in place, we now turn our attention to the problem of locating an additional monitoring well during cleanup. Performance criterion is the cost-to-go function, augmented by the cost of the new well. This way, the value of information from a new monitoring well is evaluated in a straightforward way as the reduction in the cost of operation during the remaining periods and this reduction is compared to the cost of the new well. The principle is that we can select the best time and location for an additional monitoring well by calculating and comparing the magnitude of the cost-to-go value at all time stages and for all candidate monitoring well locations.

The situation is as follows: Aquifer remediation may have been carried out previously according to the adaptive pumping schedule following the method described earlier (or, for that matter, any other method). At this stage, it is necessary to decide whether it is cost-effective to add a monitoring well now. If it is not, we proceed with the operation and we repeat the process in the next time period to determine whether it is cost-effective to install a well then.

The extra monitoring well will provide additional information about the system which will give better estimates of the state variables. But the new well will incur installation and sampling costs (see Figure 1). The total sampling cost (summation

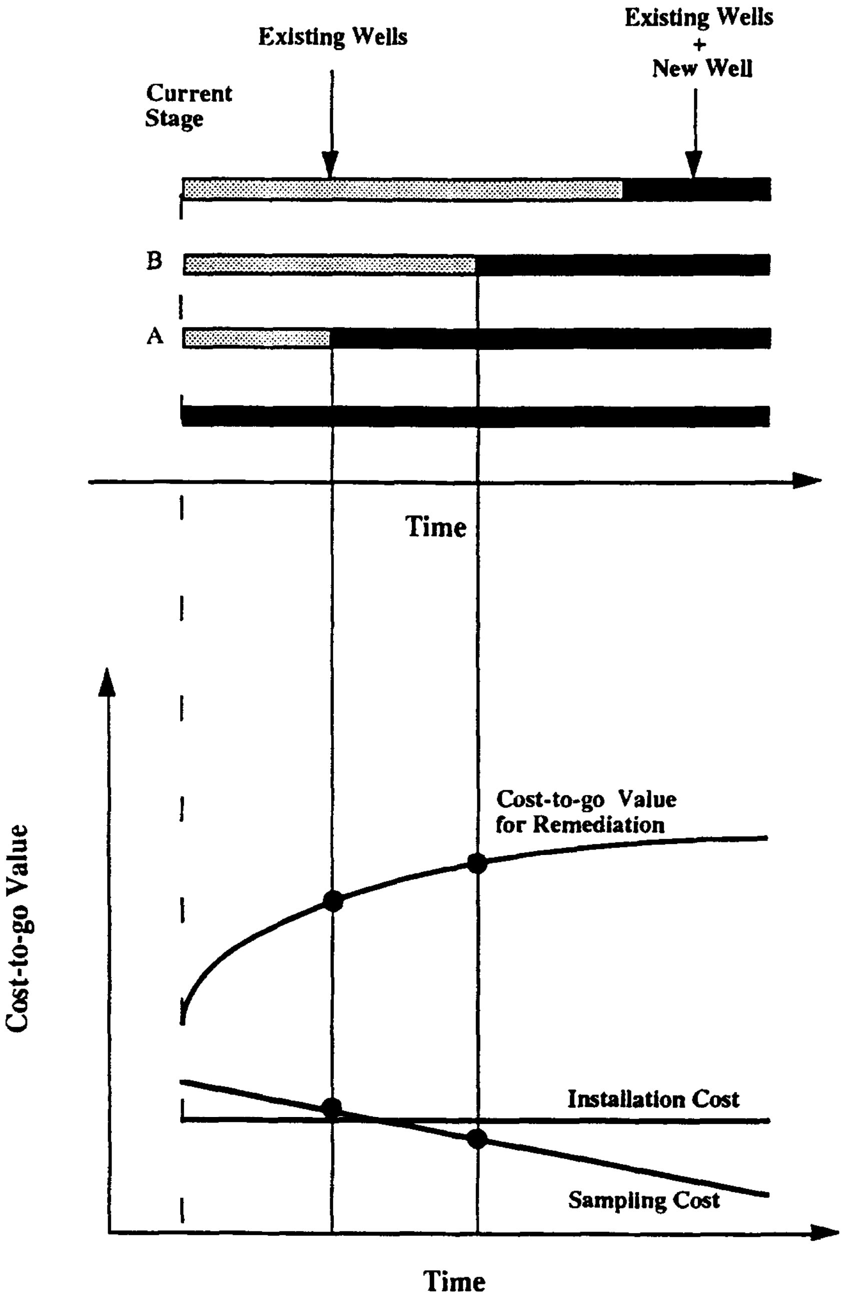

Figure 1. Costs associated with installing a monitoring well. Cost-to-go function (expected cost) for remediation increases when monitoring well is installed later since less information is available. The shape of the cost-to-go function for remediation is problem-specific.

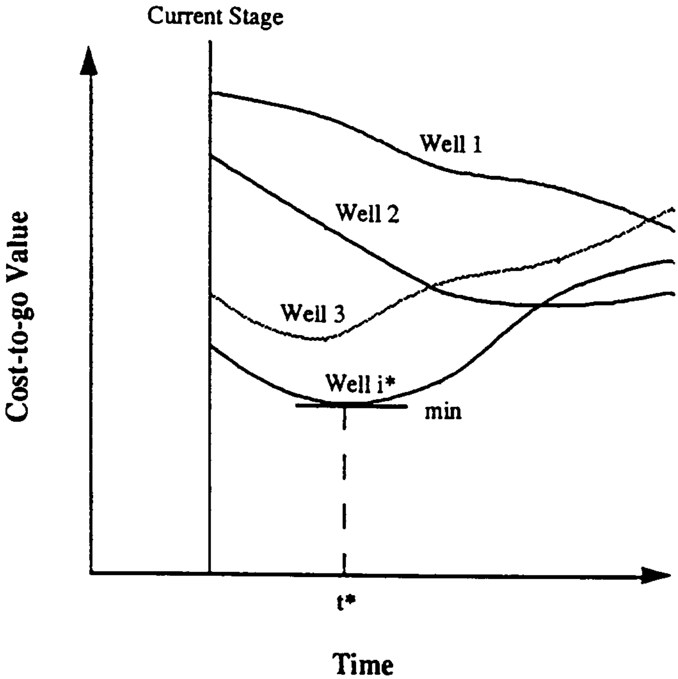

Figure 2. Determination of installation time and monitoring well location through minimization of the expected costs in the remaining periods. Time (t^*) and well (i^*) yielding minimum cost-to-go value is the choice at that stage.

of sampling costs from the time of installation to the end of the remediation period) will decrease if the new well is installed later, while the installation cost will remain the same regardless of the installation time. (Here, we do not account for the discount rate because we assume that the operating horizon is short. However, we can easily introduce this aspect of planning by using present values.) The installation time affects the remediation cost. The later the monitoring well is added, the higher the expected value of the remediation cost because the lack of information dictates conservatism. For example, if the extent of the plume is not known, the pumping rate must be higher to maintain a larger capture zone than if the boundaries of the plume were known with accuracy. Therefore, we must select the installation time and location of a monitoring well after a careful weighting of benefits and costs. Of course, the evaluation is based on the current knowledge about the system.

For every candidate monitoring well we compute the cost-to-go function at stage k, augmented by the installation and sampling cost of the new well assuming that a monitoring well is added at a certain stage in the future. The time and location for which the cost-to-go value is minimum is the most reasonable choice for installing a new monitoring well (see Figure 2). If the time chosen is not the current stage, the decision is postponed and the normal dual control is applied at that stage. In

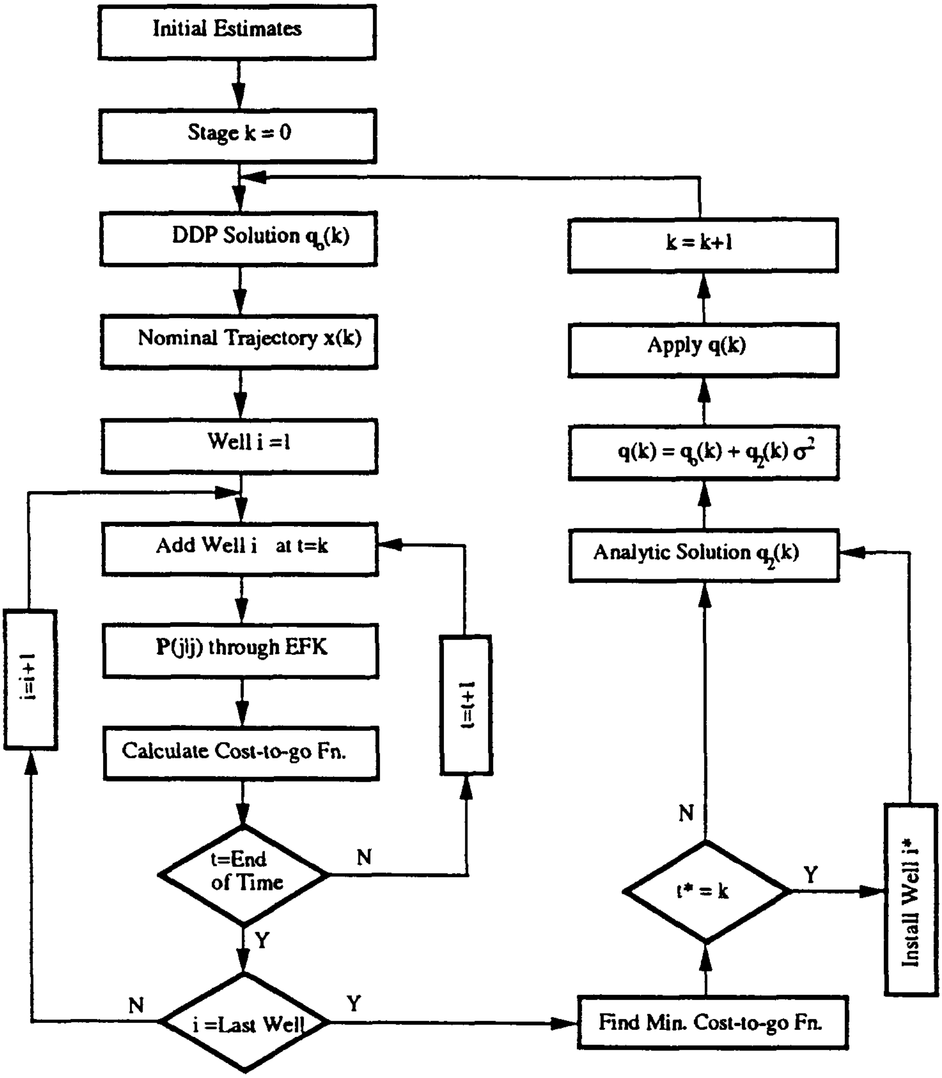

Figure 3. Implementation procedure for the decision of installation timing and location of a monitoring well.

practice, one does not need to compute the cost for all stages and all candidate well locations. As long as one knows that the best time is not now (the current stage), the decision can be postponed to the next stage.

The procedure for decision-making in monitoring well installation is summarized below and also shown in Figure 3.

1. Assume initial estimates of the aquifer parameters and pumping schedule.
2. Set $k = 0$. (Note that the procedure can be initiated at any stage k.)

3. Obtain the deterministic pumping policy, $q_0(k), k = k, \cdots, N-1$, using constrained DDP.
4. Calculate the nominal trajectory, $\mathbf{x}(k), k = k, \cdots, N$, using the deterministic pumping policy and the state-transition equation.
5. Choose one candidate monitoring well. Add to the measurements the head and concentration at the new monitoring well.
6. Propagate and update state estimation covariance using the extended Kalman filtering based on the nominal trajectory, the deterministic pumping policy, and the fact that measurements will be collected at all monitoring points including the new well.
7. Calculate the cost-to-go function.
8. Repeat steps 5 $\sim$ 7 assuming the well is installed at different stages. If a case is found that the cost-to-go function is lower than that of the current stage, go to step 9.
9. Choose another well and repeat steps 5 $\sim$ 8.
10. Find the time stage which gives the lowest cost-to-go value for all monitoring well locations.
11. If the resulting time stage is the current one, install the corresponding monitoring well now.
12. If not, calculate the stochastic pumping rate, $q_2(k)$. Pump and treat with pumping rate $q(k)$, $q_0(k)$ plus $q_2(k)$, and go to the next time stage.
13. Repeat steps 3 $\sim$ 12 until the decision is made.

Notice that the deterministic part of the cost-to-go function (excluding the installation and sampling cost of the new well) is not affected by the addition of a monitoring well because the deterministic cost-to-go function is computed on the assumption that the system is completely known. Therefore, additional information anticipated to be obtained through monitoring does not affect the value of the deterministic part of the cost-to-go function. Thus, one needs to calculate only the stochastic part of the cost-to-go function and the cost of monitoring. As explained earlier, the well installation cost is the same at any stage. Since the deterministic part of the remediation cost is not affected by the installation of monitoring wells, the optimization reduces to finding the best trade-off between the stochastic cost-to-go value of the remediation and the installation plus sampling cost. In other words, the value of increased information obtained through new measurements versus the cost for measurements.

It is worthwhile to clarify what we mean by 'information'. As used colloquially, the word information denotes 'knowledge' and the expression 'value of information' is used loosely. In the context of our analysis, however, these terms have precise meanings. Suppose that there are two possibilities, that the plume is in location A or in location B, and there is uncertainty about which of them is the true one. An observation that eliminates one of the two possibilities contains information. Information, therefore, is anything that resolves uncertainty or reduces

Table II. Parameters for aquifer simulation

Aquifer width	220 m
Aquifer length	400 m
Aquifer thickness	4.2 m
Storativity	2.2×10^{-4}
Effective porosity	0.3
Retardation factor	2.5
Longitudinal dispersivity	17 m
Transversal dispersivity	1.7 m
Recharge	2.0×10^{-3} m/d
Δx	20 m
Δy	20 m
Δt	11 days

the number of possibilities. Such reduction in the number of possibilities usually is beneficial. For example, it is less expensive to install and operate a hydraulic containment system that covers only location A instead of both locations. For us, the value of information is the reduction in the expected cost of operation.

In the following section we will demonstrate through a numerical example how this simple algorithm can be incorporated into an adaptive cleanup scheduling method to decide the best time and location of an additional monitoring well.

5. Numerical Example

5.1. Description of Case Study

For illustration purposes, we will apply the developed approach to a hypothetical two-dimensional confined aquifer that is identical to the one studied in Lee and Kitanidis (1991) (see Table II). The objective is to determine the most cost-effective cleanup strategy as well as to locate an additional monitoring well if it promises further cost reduction. Cleanup strategies are determined through the application of the dual control methodology and the decision when and where to add a monitoring well is made based on the cost-to-go criterion described in the previous section. The location of plumes and wells are shown in Figure 4. Figure 5 shows three zones of transmissivity.

Note that, in practice, the best estimates of the parameters may be smaller or larger than the true values. The initial estimates of transmissivities in the three zones differ from the true values. The best estimates (mean values) and estimation variances are given in Table III. Correlation between different zone estimates is neglected. For each zone, the true transmissivity value is given in the same table. Initial estimates of hydraulic heads and solute concentrations are tabulated in Table IV. They are taken as independent random variables with means and variances. The mean values of the estimates are assumed to be the true values.

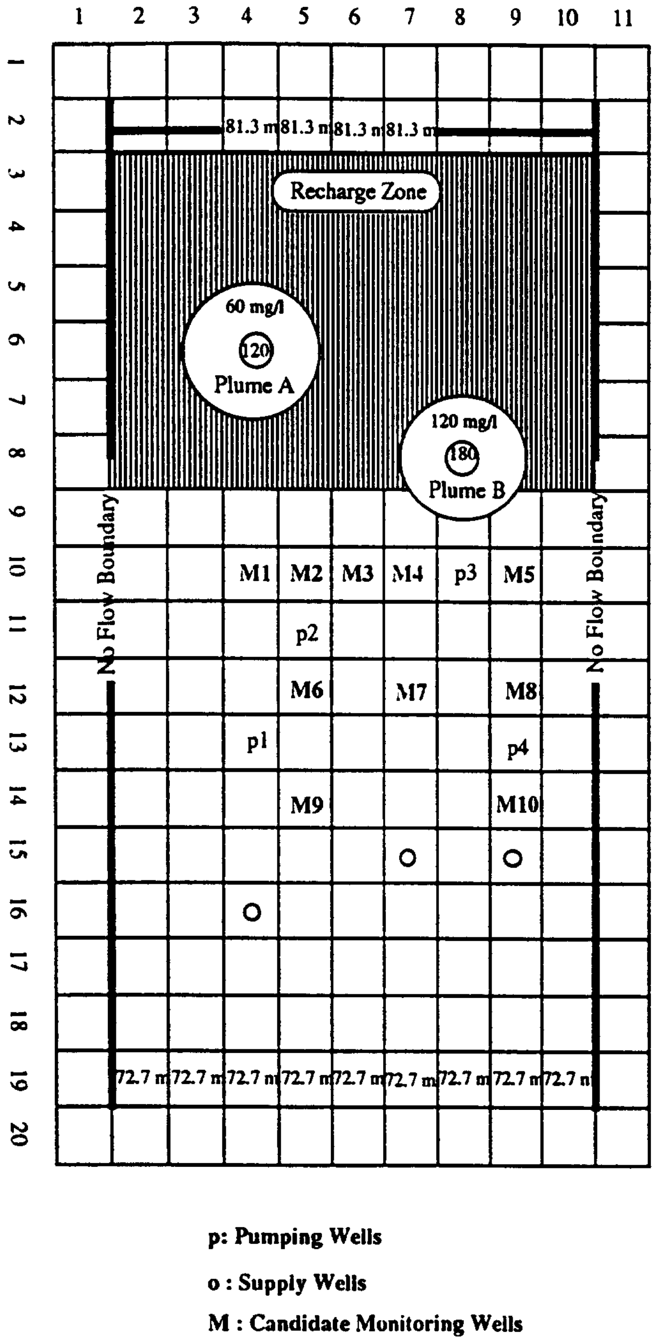

Figure 4. Contaminated aquifer with pumping, supply and candidate monitoring wells. There are 11 × 20 finite difference nodes ($\Delta x = \Delta y = 20$ m).

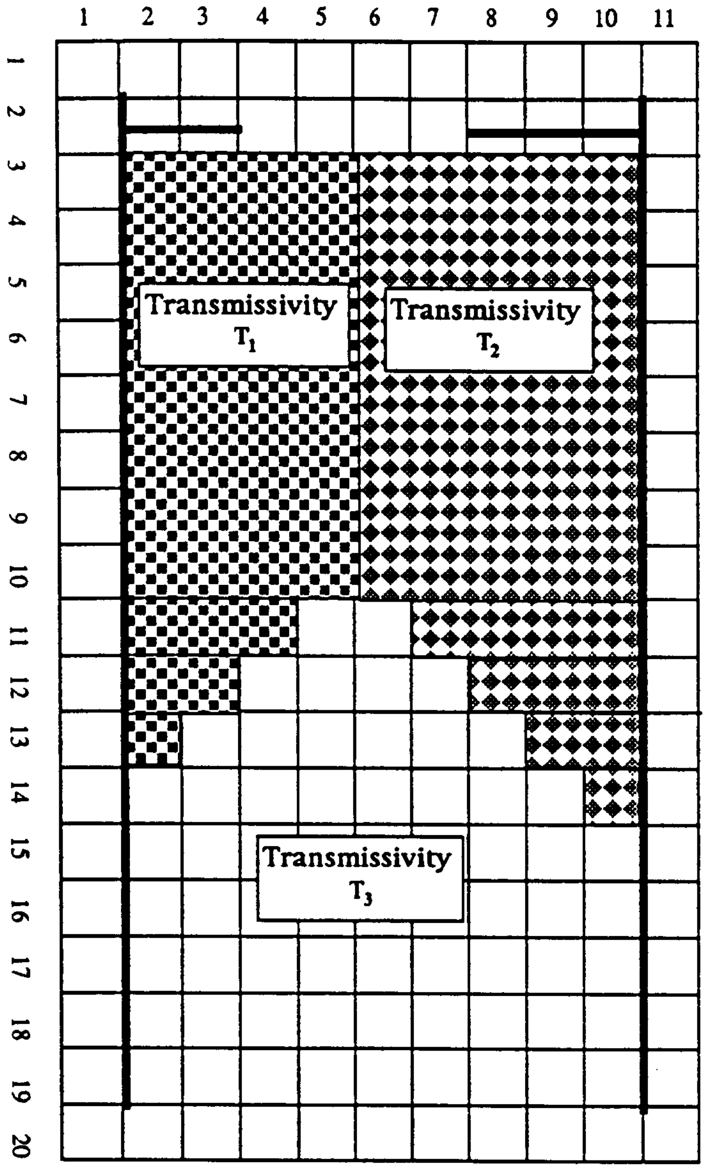

Figure 5. Three transmissivity zones. Initial estimates on transmissivity are in Table III.

Table III. Estimates for Transmissivity (m^2/day)

	Zone 1	Zone 2	Zone 3
Mean Value	18.4	4.9	130.4
Variance	30	5	110
True Value	23	7	142

The objective function without considering the augmentation of monitoring network, J_p, is

$$J_p = \min \quad E\left\{\sum_{k=0}^{46}[\mathbf{e}'\mathbf{q}(k)]^{0.6}(57.17 + 7.6[\{90\mathbf{e} - \mathbf{h}_p(k)\}'\mathbf{q}(k)]^{0.5})\right\} \quad (1)$$

expressed in million dollars. This function is interpreted as the stagewise operation and maintenance costs summed up over the whole time horizon. E denotes the expectation operator. $\mathbf{e}$ is a column vector of ones and $'$ is the transpose of a vector. Therefore, $\mathbf{e}'\mathbf{q}(k)$ implies total pumping rate. $\mathbf{h}_p(k)$ is the hydraulic head at the pumping well at time k. $90\mathbf{e} - \mathbf{h}_p(k)$ is the vector of head lifts at the pumping wells. Hydraulic lifts times pumping rate determines the electric power cost that is a major portion of operating cost. We use this objective function for purposes of illustrating the applicability of the optimization approach. In any real-world application, one should choose an objective function that is representative of the actual costs. The total time horizon (1.4 years) was divided into 47 stages.

Both decision and state variables are constrained by physical or regulatory limitations. Here, pumping rate (a decision variable) cannot be negative because we assume (as is often the case) that no recharge of the treated water is allowed. The solute concentration at a certain location and time should not exceed a standard negotiated with the regulatory agency. The hydraulic head is also subject to constraints; we want to avoid excessive pumping that may result in aquifer dewatering. In this work we neglected the capacity constraint on the pumps because we assumed that the originally installed pumps are powerful enough so that the capacity does not become a limiting factor.

Since hydraulic head and concentration cannot be predicted with certainty, constraints on these variables must be described probabilistically, through the distribution of possible values. Here, the following reliability constraints are imposed. The probability that concentration over the aquifer should meet the water quality standard (c^* = 10 mg/l) at the end of the time horizon, after decreasing with time (in this case, linear decrease), should not exceed some reliability level. Similarly, the probability of the hydraulic head dropping lower than the aquifer thickness (h^* = 50 m) at any time should be less than some reliability level. In this work, the reliability level is chosen as 95%. Thus:

$$q_j(k) > 0 \quad \text{for all } j \quad (2)$$

$$\text{Prob}\left\{c_i(k+1) \geq \frac{k(c^* - c_i(0))}{N-1} + c_i(0)|c_i(k)\right\} \leq 0.05 \quad \text{for all } i, k, \quad (3)$$

$$\text{Prob}\{h_i(k+1) \leq h^*|h_i(k)\} \leq 0.05 \quad \text{for all } i, k. \quad (4)$$

The information gathered is the hydraulic head and concentration data at pumping wells and water supply wells. Thus the measurement equation is

$$\mathbf{z}(k) = [h_i(k), c_i(k)] + \mathbf{v}(k), \quad i = \text{pumping and supply wells}, \quad (5)$$

Table IV. Initial estimates of head and concentration

Head (m)	Mean of Upstream Boundary	81.3
	Mean of Downstream Boundary	72.7
	Head Gradient	0.024
	Head Variance	9
Plume A (mg/l)	Range of Mean	60 ~ 120
	Variance	100
Plume B (mg/l)	Range of Mean	120 ~ 180
	Variance	300

where $\mathbf{v}(k)$ is the measurement error. System and measurement errors were generated according to the corresponding covariance matrices. In this example, they were assumed to be identity matrices for all times k, for illustration purpose. In practice, one may want to find appropriate covariance matrices by applying estimation methods that are available in the literature (such as Schlee *et al.* (1967) and Mehra (1970)).

5.2. Monte Carlo Simulations

We conducted Monte Carlo simulations to assess the effectiveness of the method. We generated statistically 20 realizations of initial transmissivities ($T_i(0)$) and initial state variables ($h_i(0), c_i(0)$) using the means and variances in Table III and Table IV and assuming normal distributions. Using the generated values, we proceeded to optimize the pumping rate and to select the location and time of an additional monitoring well. In reality, measurements would be collected from the contaminated site. In Monte Carlo simulations, however, measurement values were taken from a computer-simulated aquifer having true values of transmissivity, head and concentration in Table III and IV. For each realization, we determined from dual control the optimal pumping schedule and the total cost. In each realization, the devised pumping strategy successfully cleaned up the aquifer by the end of the time horizon and the average cost was given as 4.38 million dollars (Lee and Kitanidis, 1991).

Focusing on the monitoring problem, we are considering whether to add another monitoring well at some point in time while the remediation is taking place. Ten candidate monitoring well locations are shown in Figure 4.

Here is what we did for each realization. We must decide at stage k whether it is cost-effective to install a monitoring well at once or to defer the installation. In addition, if a well is to be installed, we need to pick the best location. We first obtain through DDP the deterministic part of the pumping rate over the remainder of the operating horizon. This part assumes that all state variables are known with certainty; consequently, it is affected only by the objective function, the equations that describe the flow and transport, and the other limitations on the system. We then use the deterministic controls to determine a nominal 'trajectory', i.e., an approximation of how the system will change over time. Next, we find the stochastic part of the pumping rate, which means the correction which accounts for uncertainty about the system. This correction depends additionally on the first two moments of the state variables and the type and accuracy of the anticipated observations. Even though we compute the optimal pumping rate for every remaining stage, we only apply the pumping rate for period k and discard the rest. We repeat the optimization for the remaining time periods.

To evaluate the benefits from an additional well, we modify the measurement equation to include the observation at the new well. Since the time of adding the well is a decision variable, we repeat the procedure many times, each time adding the well at a different time stage. Before installing the new well, head and concentration are measured only at the existing pumping and supply wells. At and after time t, the time stage when the new well is installed, head and concentration are measured at the existing wells and the new well. The criterion for selection of the location and time of an additional well is the value of the cost-to-go function.

The cost of well installation ($10 000/well) and for operation, such as well maintenance and sampling, ($500/period) accounts for the cost for monitoring, J_m.

$$J_m = 10,000 + 500(47 - k) \tag{6}$$

Therefore, the objective is now to minimize the total cost

$$J = J_p + J_m, \tag{7}$$

where J_p is defined in equation 1.

We repeated this procedure for 10 candidate well locations and we chose the one with the lowest cost. If the optimal time t^* is not the present, the installation of a well is postponed. Once the well was installed, then there is no going back and the dual control scheme is applied continuously until the end of the time horizon.

Table V summarizes the results of Monte Carlo analysis. Different timings and locations were resulted in for the installation of a monitoring well for 20 realizations. Distinct timing and location incurred different operating cost. Maximum savings of 0.59 million dollars were obtained by installing well 4 at the beginning of stage 16 (realizations # 8,9,10). Savings were calculated by comparison with the remediation cost using the dual control with existing wells. All 20 realizations yielded savings in total cost compared to the case without an additional monitoring well. This means that the value of information from the new well exceeds the cost of installation and sampling in this example.

The recommended time of well installation varied among realizations. The optimal stages were: 14, 15, and 16 with a frequency of 15%, 20%, and 65% of being selected as the best time to locate an additional well. It is noteworthy in this example that the optimal well location was uniquely determined by the optimal

Table V. Installation stage, monitoring well location, and savings due to well installation

#	Installation stage	Selected well No.	No. of samples	Sampling[a] cost	Overall[b] cost	Savings[c]	Relative savings (%)
1	14	5	33	0.0165	3.90	0.48	10.9
2	14	5	33	0.0165	3.92	0.46	10.5
3	14	5	33	0.0165	3.95	0.43	9.8
4	15	4	32	0.0160	3.81	0.57	13.0
5	15	4	32	0.0160	3.84	0.54	12.3
6	15	4	32	0.0160	3.85	0.53	12.1
7	15	4	32	0.0160	3.86	0.52	11.8
8	16	4	31	0.0155	3.79	0.59	13.4
9	16	4	31	0.0155	3.79	0.59	13.4
10	16	4	31	0.0155	3.79	0.59	13.4
11	16	4	31	0.0155	3.80	0.58	13.2
12	16	4	31	0.0155	3.81	0.57	13.0
13	16	4	31	0.0155	3.83	0.55	12.5
14	16	4	31	0.0155	3.84	0.54	12.3
15	16	4	31	0.0155	3.85	0.53	12.1
16	16	4	31	0.0155	3.85	0.53	12.1
17	16	4	31	0.0155	3.85	0.53	12.1
18	16	4	31	0.0155	3.86	0.52	11.8
19	16	4	31	0.0155	3.86	0.52	11.8
20	16	4	31	0.0155	3.86	0.52	11.8
Average	–	–	–	–	–	0.53	12.2

[a] Costs and savings are expressed in millions of dollars.
[b] Installation cost is always 10,000 dollars/well.
[c] Savings are calculated against the case with original wells.

installation time. For optimal installation stage 14, for instance, the choice was always well 5, for stages 15 and 16, well 4. The choice of well 4 or 5 seems reasonable; since plume B has higher concentration and there is more uncertainty about the original concentration estimate, well 4 or 5 should provide information that would reduce the cost of optimization the most.

The reason why the installation was postponed until about half a year (stages 14 $\sim$ 16, or 154 $\sim$ 176 days) in this particular example may be explained as follows: The decision depends on the remediation costs as well as installation and sampling costs. Early installation makes it possible to collect information for long periods. However, it also requires significant sampling costs. At early times one may not have enough information for an intelligent selection of the location of a monitoring well or additional measurements may not be informative enough to justify the cost. Only after half a year plumes migrate near to candidate monitoring wells and start providing valuable information about the aquifer. This is why the installation was deferred.

Table VI. Loss by installing a monitoring well. After stage 40, installation and sampling costs outweigh the value of information from the well (one sample run)

Stage	Well	Min. Loss[a]	Relative Loss(%)
40	7	0.001	0.02
41	7	0.003	0.07
42	8	0.006	0.14
43	8	0.007	0.16
44	8	0.008	0.18
45	8	0.009	0.21
46	8	0.009	0.21

[a] In millions of dollars

Near the end of the operating horizon, however, it may be too late to use the information obtained from monitoring for purposes of reducing the cost of remediation because by then the aquifer could have been characterized or decontaminated almost completely. To verify this theory, ten candidate monitoring wells were tested one at a time for every time stage. The realization had the same condition as the realization #10 which resulted in the maximum savings. Until stage 40, at least one well would result in savings if it were installed. After stage 40, however, the installation of a well would result in net loss because of the relatively large installation cost compared to the value of the information collected. Table VI shows that after stage 40 a minimum of 0.21% loss compared to the case of original wells may result from installing a monitoring well.

Remember that the actual cost J is affected by both pumping rate and hydraulic lift. Figure 6 shows how the new information from the additional monitoring well affects the pumping rate for the case of well 4 installation at the beginning of stage 16. The figure shows the summation of pumping rates at four pumping wells as a function of time. If we knew the system parameters without error, the optimal pumping rate would be nearly uniform for each pumping well to have the most cost-effective cleanup strategy. However, here the pumping generally decreases over time due to several reasons explained below.

First, the 'probing effect' of the dual control; pumping rates are initially high in order to excite the system and extract information about hydraulic properties and concentrations in the early stages. Second, the 'caution effect', i.e. the need to overpump to provide a 'safety factor' which ensures that the objectives of cleanup are met. Overpumping is higher at the early stages because the state of the system is less known. These two effects are correlated; Up to stage 15, pumping rates are identical since the two cases have identical observation sources. The extra monitoring well 4 added at the beginning of stage 16 provides additional information. Due to the additional information coming from the extra well, there

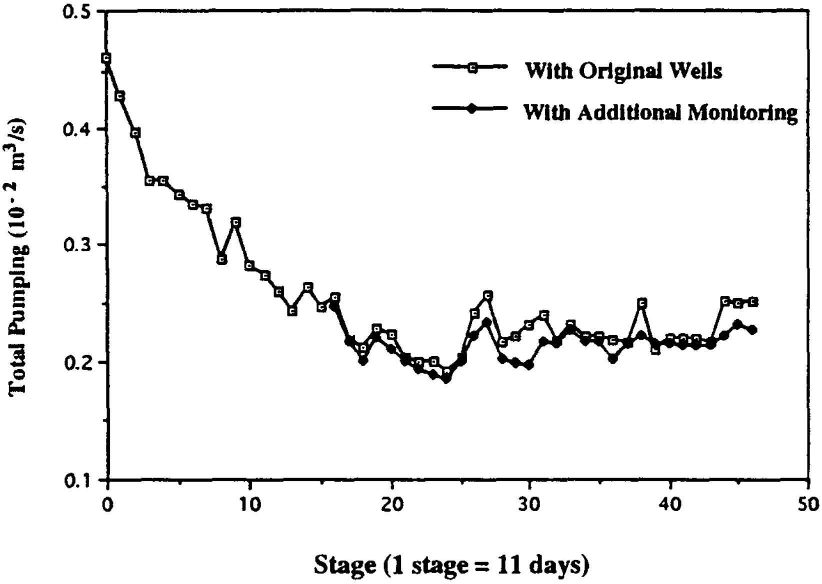

Figure 6. Change of total pumping rates with and without monitoring well (One sample run). Additional monitoring saves total pumping effort.

is less incentive to learn the system by pumping or to provide a safety factor. Less pumping resulted in less hydraulic lift and they both contributed to lowering the cost. The difference between pumping rates and thus the benefit of having an additional source of information became increasingly large with time. The third reason is the initially assumed transmissivity. The estimated transmissivity values were nearly always lower than the true values. Therefore, the cleanup task became feasible with less pumping than previously expected when higher transmissivities were observed from monitoring.

We must point out that, unlike the example presented here, it is generally possible that a new monitoring well could cause increase in pumping: This would be the case if additional information from the new well provides exceptionally bad news such as the existence of previously unknown plumes or that the hydrogeologic properties are so different from those originally estimated that the remediation system is really failing. However, on the average, an additional monitoring well should decrease the required pumping rate because more information means less need for caution. One can assure that this is the case by starting with conservatively large estimation variances for the system parameters.

Figure 7 displays the change of pumping rates at each well. Generally speaking, pumping wells 2 and 3 are more affected than the others since a new monitoring well (#4) is added nearby. In particular, significant reduction in pumping is observed for pumping well 3 which is the closest to the monitoring well. The activity of wells 1 and 4, in turn, increases as time increases. This means that the pumping effort is

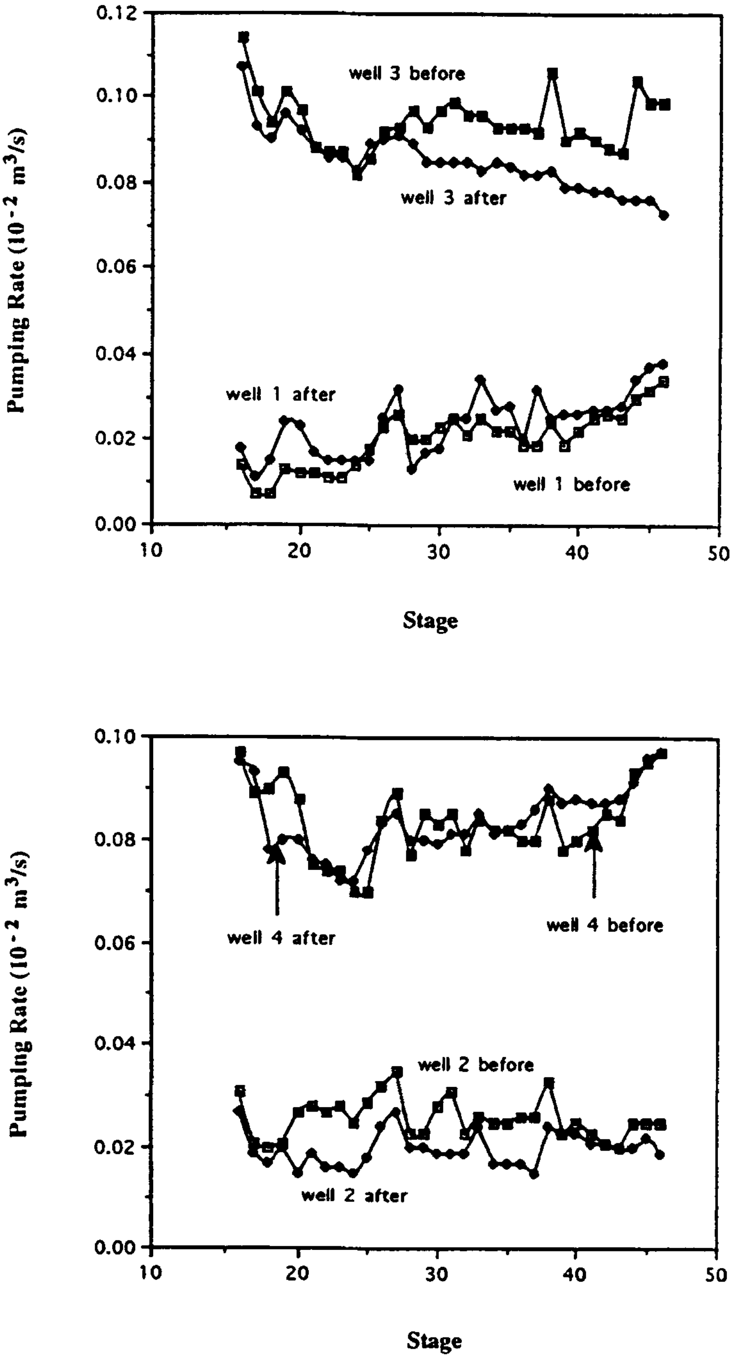

Figure 7. Change of pumping rates at four different wells with and without monitoring well (one sample run).

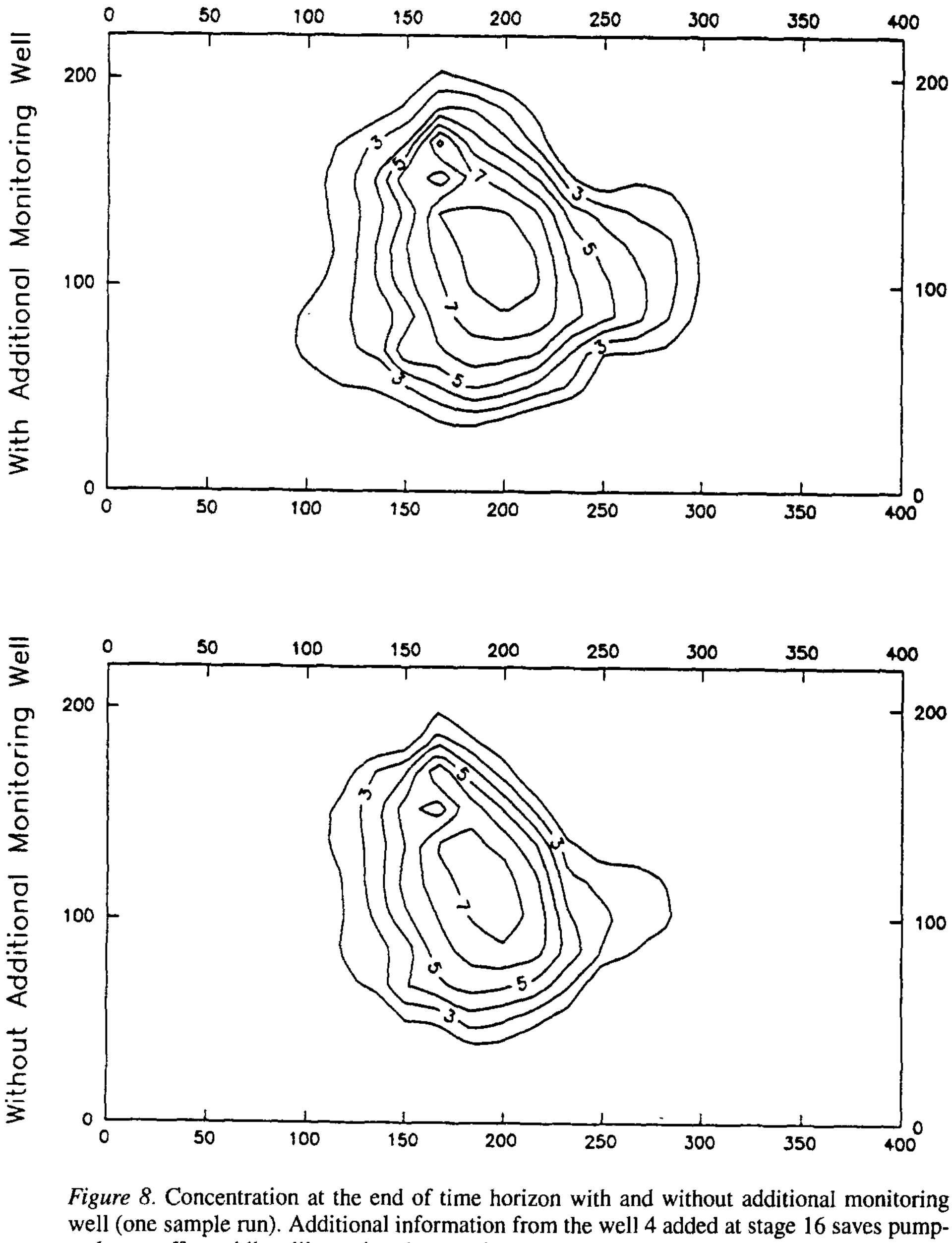

Figure 8. Concentration at the end of time horizon with and without additional monitoring well (one sample run). Additional information from the well 4 added at stage 16 saves pump-and-treat effort while still meeting the criteria.

redistributed by increasing the pumping rates at wells 1 and 4 as the plumes move toward the lower boundary.

Figure 8 compares the solute concentration at the end of the time horizon with the two different pumping schedules explained above (no additional well and well 4 added at stage 16). Both cases satisfy the water quality standard, while the additional monitoring well case can save a maximum of 13.4% of the total cost through more effective redistribution of pumping. The average savings over all realizations is 12.2% or $530 000. So significant a reduction in the remediation cost by adding a single monitoring well may not be representative of the real world; instead, it may have been a consequence of the cost function that we used and of our assumption that the pumping capacity is large enough that we have the freedom to adjust the pumping rate as needed to minimize pumping and treatment costs.

Nevertheless, it is apparent that the information from the new monitoring well is primarily responsible for the reduced total pumping which accounts for the cost reduction compared to the case of remediation with existing wells. Therefore, this example illustrates clearly the potential advantages of an integrated method that strives to optimize remediation and monitoring jointly. The methodology presented here could be used to determine a cost-effective remediation strategy under uncertainty.

6. Summary and Conclusions

Nowadays, groundwater quality monitoring is done at many hazardous waste sites that are being remediated or are considered for remediation. There, the key issue is how many monitoring wells to install and where to install them for purposes of appraising the progress achieved and of adjusting the pumping schedule to achieve the objectives of remediation with the least possible cost. Most of the methodologies for optimizing monitoring networks that are reported in the literature cannot optimize monitoring jointly with remediation.

In this paper, we presented a methodology to decide whether to add a monitoring well and if so to select when and where, while the site is being cleaned up through a pump-and-treat system. Our primary objective was to present the methodology and to illustrate its application on a relatively simple case. In this example, we made several assumptions that may not be valid for some real-world cases. For example, in the description of the processes we neglected kinetically controlled desorption and variability of concentrations in the vertical direction, both of which are important in many applications. Furthermore, we used a cost function that may not be defensible using a rigorous economic analysis. Nevertheless, none of these simplifications are essential limitations of this methodology. One could, for example, use three-dimensional representations of flow and transport with kinetic limitations of sorption and one could use any reasonable objective function.

Using uncertain estimates of aquifer parameters and state variables for a two-dimensional confined aquifer with two plumes, we computed pumping rates using the dual control method. Measurements of hydraulic head and concentration at pumping and supply wells contributed to a better characterization of the system. We selected pumping rates at four pumping wells and compared the expected total cost for different proposed times and locations of a monitoring well (that is, under consideration to be added to the system). The well location and the time period with the minimum cost-to-go value (expected value of expenses in the remaining periods) were selected.

To test this methodology and to illustrate its potential usefulness, twenty Monte Carlo simulations were performed for given initial estimates of the system. The decision on timing and location for an extra monitoring well was made by finding the minimum value of the expected cost of operation augmented by installation and sampling cost. For the example, the timing for well installation varied between 154 $\sim$ 176 days after the start of the cleanup action, about one-third of the total operation period. A considerable reduction in total cost was achieved by installing one additional monitoring well.

We also illustrated that the installation of an additional monitoring well does not necessarily pay off in the sense of resulting in savings in total remediation cost. In our example, installation of a monitoring well at too late a period (after stage 40) is not cost effective. That is, the additional information gathered from the well, even though it does reduce the uncertainty about the estimates of the state variables, is not always useful enough to compensate for the installation and sampling costs.

Although we focused on the case of a single additional monitoring well, the methodology can be extended. One can consider to add as many monitoring wells as it makes sense economically. Multiple well locations can be determined using the same procedure without any major change in the methodology itself.

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