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Modified rational unit hydrograph method and applications

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The modified rational method (MRM) is an extension of the rational method to develop triangular and trapezoidal runoff hydrographs. A trapezoidal unit hydrograph (UH) was developed from the MRM for the case when the duration of rainfall is less than the time of concentration of the watershed and is called the modified rational unit hydrograph (MRUH). The MRUH method was applied to 1400 rainfall-runoff events at 80 watersheds in Texas. Application of the MRUH method involved three steps: (*a*) determination of rainfall excess using the runoff coefficient; (*b*) determination of the MRUH using drainage area and time of concentration; and (*c*) simulating event runoff hydrographs. The MRUH performed as well as the Gamma function UH, Clark-HEC-1 UH and NRCS curvilinear UH methods when the same rainfall loss model was used. The MRUH method can be applied to time-variable rainfall distributions and at watersheds with drainage areas greater than typically used for the rational method (a few hundred acres).

Notation

A	drainage area in (ha, acres, or km^2, or mile^2)
AI	cumulative area as a fraction of watershed area
C	runoff coefficient
C_{lit}	composite literature-based runoff coefficient
C_{vbc}	back-computed volumetric runoff coefficient
D	storm duration (min or h)
D_{w}	watershed equivalent diameter (km)
EF	Nash–Sutcliffe efficiency (dimensionless)
I	average rainfall intensity (mm/h or in/h) with the duration equal to time of concentration
i	gross rainfall intensity (mm/h or in/h)
$i_{\mathrm{e}} = Ci$	effective rainfall intensity (mm/h or in/h)
L	main channel length (mile)
m_{o}	dimensional correction factor (1·008 in imperial units, 1/360 = 0·00278 in SI units)
$Q(t)$	direct runoff hydrograph (DRH) ordinates derived by discrete convolution (m^3/s or ft^3/s)
Q_{p}	peak discharge of DRH (m^3/s or ft^3/s)
QB	relative error in observed and simulated DRH peak discharges
$\bar{Q}_{\mathrm{pm}}$	mean of the modelled DRH peak discharges (subscript 'm' stands for modelled)
$\bar{Q}_{\mathrm{po}}$	mean of the observed DRH peak discharges (subscript 'o' stands for observed)
Q_{pD}	peak discharge of the modified rational method's DRH for the case when $D < T_{\mathrm{c}}$
Q_{pR}	peak discharge of the rational method (m^3/s or ft^3/s)
Q_{pUG}	peak discharge of the Gamma unit hydrograph (GUH) (m^3/s or ft^3/s)
Q_{pUM}	peak discharge of the modified rational unit hydrograph (MRUH) (m^3/s or ft^3/s)
Q_{pUN}	peak discharge of the Natural Resources Conservation Service unit hydrograph NRCS UH (m^3/s or ft^3/s)
$Q_{\mathrm{uG}}(t)$	GUH ordinates (m^3/s or ft^3/s)
$Q_{\mathrm{uI}}(t)$	instantaneous unit hydrograph (IUH) ordinates (m^3/s or ft^3/s)
$Q_{\mathrm{uM}}(t)$	MRUH ordinates (m^3/s or ft^3/s)
R^2	coefficient of determination
RRMSE	the root mean squared error of DRH ordinates normalised by observed Q_{p}
S	main channel slope (ft/mile)
S_{o}	channel slope (m/m or ft/ft) for equations in Appendix 2

TB	relative error in observed and simulated DRH times to peaks
T_c	time of concentration (min or h)
TI	fraction of time of concentration
T_p	time to peak of DRH (min or h)
T_{pU}	time to peak of UH (min or h)
T_{pUG}	time to peak of the GUH (min or h)
T_{pUN}	time to peak of the NRCS UH (min or h)
W	watershed width (km)
α	shape parameter of GUH

1. Introduction

The rational method was originally developed for estimating peak discharge Q_{pR} for sizing drainage structures, such as storm drains and culverts (Kuichling, 1889). The Q_{pR} (in m^3/s or ft^3/s) is computed using

1. $$Q_{pR} = m_o CIA$$

where C is the runoff coefficient (dimensionless), I is the average rainfall intensity (mm/h or in/h) over a critical period of storm duration (i.e. time of concentration T_c), A is the drainage area (hectares or acres), and m_o is the dimensional correction factor (1/360 = 0·00278 in SI units, 1·008 in Imperial units). Kuichling (1889) and Lloyd-Davies (1906) are credited with independent development of the rational method (Singh and Cruise, 1992).

Incorporation of detention basins to mitigate effects of urbanisation on peak flows requires design methods to include the volume of runoff as well as the peak discharge (Rossmiller, 1980). Poertner (1974) developed the modified rational method (MRM) to use when designing hydraulic structures involving storage. The MRM approximates a direct runoff hydrograph (DRH) resulting from a design storm as being either triangular or trapezoidal in shape (Smith and Lee, 1984; Viessman and Lewis, 2003; Walesh, 1989) depending on the relation between the storm duration D and time of concentration T_c. Smith and Lee (1984) revisited the rational method that implied a rectangular response function, which is an instantaneous unit hydrograph (IUH), and developed DRHs using IUH for both constant and variable rainfall intensity events. Singh and Cruise (1992) analysed the rational formula using a systems approach and concluded that watershed's IUH is a rectangular distribution with the base time equal to T_c of the watershed if a watershed can be represented as a linear, time-invariant system. They used the convolution to derive the S-hydrograph and D-hour unit hydrograph (UH) from application of the rational method. Guo (2000, 2001) developed a rational hydrograph method (RHM) for continuous, time-variable rainfall events. Bennis and Crobeddu (2007) developed an improved RHM for small urban catchments using a rectangular impulse response function. However, with the exception of Smith and Lee (1984) and Bennis and Crobeddu (2007), all studies related to MRM consider MRM producing DRHs from constant rainfall distributions (Rossmiller, 1980; Viessman and Lewis, 2003). All of the methods were developed and tested for small watersheds with limited data. Similarly, none of the studies has tested the sensitivity of the proposed methods to C and T_c.

In this study, MRM was applied to develop a trapezoidal UH that is termed the modified rational unit hydrograph (MRUH). The purposes of the study were: (*a*) to evaluate the applicability of the method to watersheds of size greater than typically used with either the rational method or the MRM (that is, a few hundred acres); and (*b*) to study the effects of the runoff coefficient and the time of concentration on prediction of DRHs when the MRUH method is used. The MRUH method was used to compute DRHs for 1400 rainfall-runoff events at 80 watersheds in Texas, USA. The DRHs obtained from the MRUH were compared with those obtained from three other UH models: the Clark UH developed for the HEC−1 generalised basin (Clark, 1945; USACE, 1981), Gamma function UH for Texas watersheds (Pradhan, 2007), and Natural Resources Conservation Service (NRCS) curvilinear UH (NRCS, 1972).

2. Modified rational unit hydrograph

First, let us revisit the MRM. If $D = T_c$, the resulting DRH from the MRM is triangular with a peak discharge $Q_p = Q_{pR} = CIA$ at time $t = T_c$; that is case (a) in Figure 1. If $D > T_c$, the resulting DRH is trapezoidal with a constant maximum discharge $Q_p = CIA$ from time D to T_c; that is case (b) in Figure 1. The linear rising and falling limbs have a duration of T_c, as shown in Figure 1 (e.g. from Viessman and Lewis, 2003; Walesh, 1989). If $D < T_c$, then the resulting DRH is trapezoidal with a constant maximum discharge of Q_{pD} (Equation 2) from the end of the storm duration D to T_c as reported by Smith and Lee (1984) and Walesh (1989)

2. $$Q_{pD} = CIA(D/T_c) = Q_{pR}(D/T_c)$$

Smith and Lee (1984) and Singh and Cruise (1992) noted that if the rate of change of the contributing area is constant so that the accumulated tributary area increases and decreases linearly and symmetrically with the time, then the IUH or impulse response function (Chow *et al.*, 1988) $Q_{uI}(t)$ is of rectangular shape given by

3. $$Q_{uI}(t) = \frac{dA}{dt} = \frac{A}{T_c} \quad (0 < t < T_c)$$

Using the rectangular response function (Equation 3), Smith and Lee (1984) and Singh and Cruise (1992) derived the resulting DRH ordinates $Q(t)$ by convolution as

4. $$Q(t) = \int_0^t i_e(\tau)\, Q_{uI}(t-\tau)\, d\tau$$

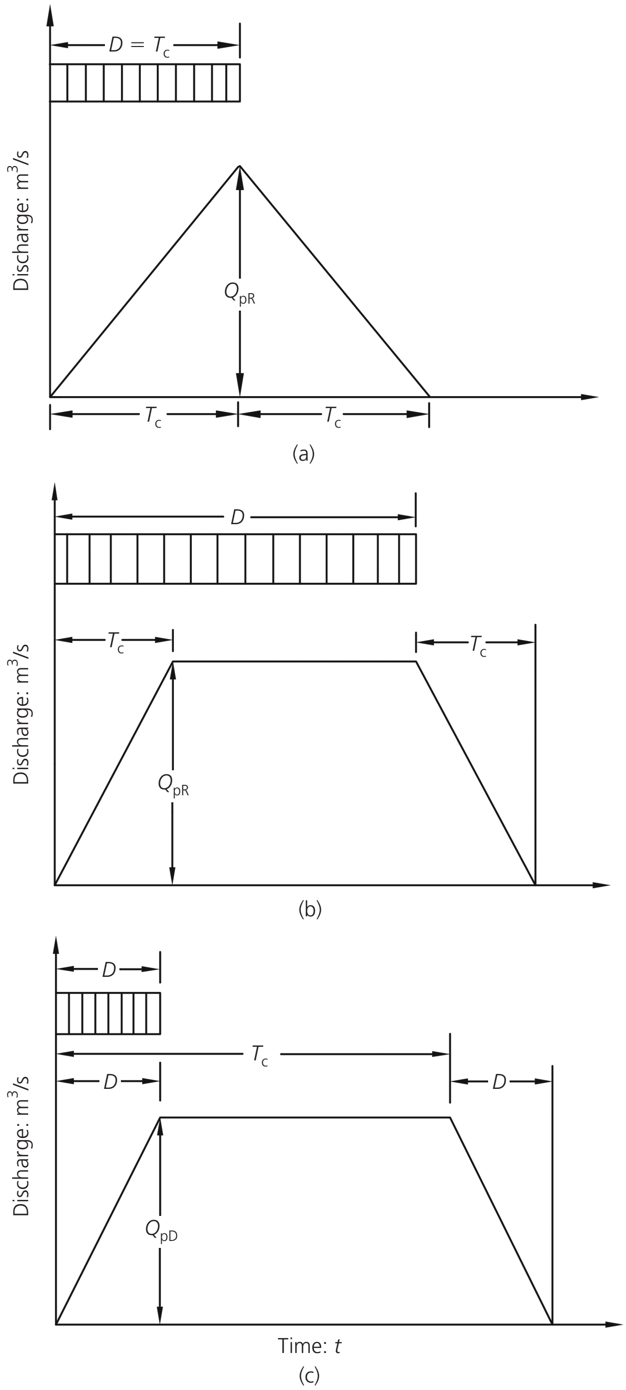

Figure 1. The modified rational hydrographs or DRHs for three different cases: (a) $D = T_c$, (b) $D > T_c$, and (c) $D < T_c$

where τ is the time with respect to which the integration is carried out and $i_e(\tau) = Ci$ is the effective rainfall intensity with i as gross rainfall intensity. Two types of DRHs, triangular and trapezoidal shape (Figure 1), were obtained from Equation 4 for constant rainfall intensity, depending on the storm duration.

Using MRM's DRH (case C in Figure 1) for a D-h rainfall event, the modified rational unit hydrograph or MRUH can be developed if DRH's ordinates are divided by the effective rainfall depth (i.e. $C\ I\ D$) based on the UH derivation method (Viessman and Lewis, 2003). The MRUH is trapezoidal in shape with constant peak discharge $Q_{pUM} = Q_{pD}/(C\ I\ D) = A/T_c$ from D to T_c. The time base for the MRUH is $D + T_c$ and MRUH ordinates can be computed from Equation 5:

$$Q_{uM}(t) = \frac{A}{T_c}\frac{t}{D} \qquad 0 \leqslant t \leqslant D$$

$$Q_{uM}(t) = \frac{A}{T_c} \qquad D \leqslant t \leqslant T_c$$

5. $$Q_{uM}(t) = \frac{A}{T_c}\frac{T_c + D - t}{D} \qquad T_c \leqslant t \leqslant T_c + D$$

The D-h MRUH results from a constant excess rainfall intensity of $1/D$ in/h over D h and has a peak discharge of A/T_c in ft^3/s when drainage area A is in acres and T_c is in hours for 1 in of rainfall excess (taking into account that 1 acre in/h is nearly equal to 1 ft^3/s). If SI units are used (drainage area A in ha and rainfall intensity in mm/h), the peak discharge from the MRUH should be equal to $A/(360T_c)$ in m^3/s for 1 mm of rainfall excess. Three examples of the MRUH developed for three watersheds used in this paper are shown in Figure 2. It is worth mentioning that cases (a), (b) and (c) of the MRM in Figure 1 are DRHs and none is UH, although cases (b) and (c) have the same shape as MRUH in Figure 2.

The assumption and restriction for the application of the rational method and original MRM include constant rainfall intensity throughout the storm duration (Rossmiller, 1980) and for small catchments, that is drainage areas less than 0·8 km^2 or 200 acres (TxDOT, 2002). Application of the MRUH method involves three steps as stated in the abstract. Because the MRUH method is an UH method, the approach establishes a continuity of hydrograph-development methods from very small watersheds to relatively large watersheds. The UH for a watershed can be used to predict the DRH for any given rainfall excess hyetograph (constant or time-variable rainfall distribution) using the UH discrete convolution (Chow *et al.*, 1988; Viessman and Lewis, 2003). In summary, application of the MRUH method is straightforward and similar to the application of other UH methods using discrete convolution; the assumption and restriction for the MRM are no longer necessary, which will be demonstrated through this study.

The MRUH method was first tested using rainfall-runoff data obtained for concrete surfaces from Yu and McNown (1964). The first dataset was based on a test bed with an area of 152·4 m by 0·3 m (500 ft by 1 ft), surface slope of 0·02 (dimensionless), and constant rainfall intensity. The second

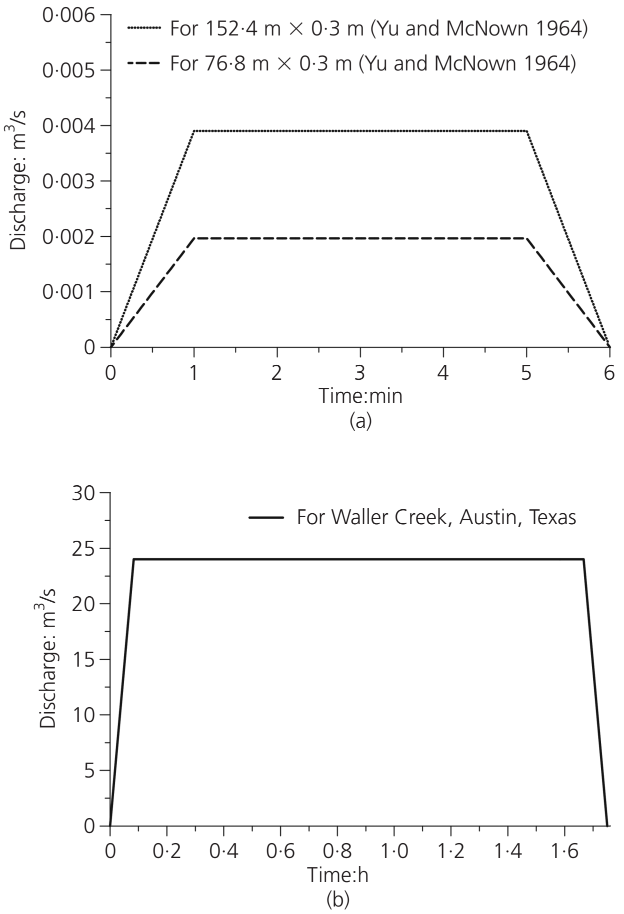

Figure 2. The MRUHs developed for: (a) two laboratory settings from Yu and McNown (1964) and (b) for the watershed associated with USGS streamflow-gauging station 08157000 Waller Creek, Austin, Texas. T_c values used for MRUHs were computed using the Kirpich method (Equation 18)

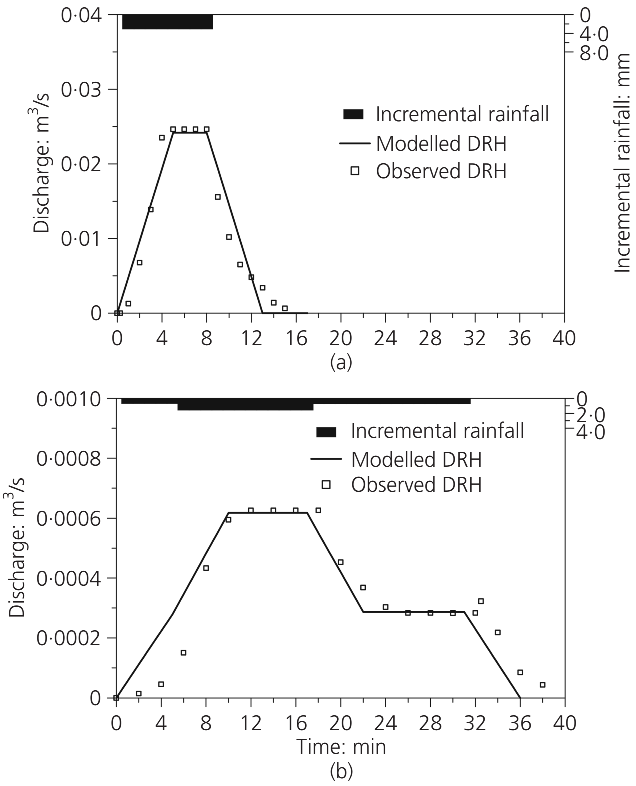

Figure 3. Incremental rainfall hyetograph and observed and modelled DRHs using the MRUHs for the two laboratory tests on concrete surfaces: (a) 152·4 m × 0·3 m with 2% slope and (b) 76·8 m × 0·3 m with 0·5% slope reported by Yu and McNown (1964)

dataset was based on a test bed with an area of 76·8 m by 0·3 m (250 ft by 1 ft), surface slope of 0·005, and variable rainfall intensity. The T_c of about 5 min was computed using the Kirpich method (Kirpich, 1940) for both experiments. A trapezoidal 1 min MRUH was developed for each experiment (Figure 2(a)). The runoff coefficient was taken to be unity. For both cases, the modelled DRHs using MRUH match the observed DRHs well (Figure 3).

The Nash–Sutcliffe efficiency *EF* (Equation 12) was 0·93 and 0·80 for the experiments using the constant (Figure 3(a)) and time-variable rainfall intensity (Figure 3(b)), respectively. According to Bennis and Crobeddu (2007), a good agreement between the simulated and the measured data is reached when *EF* is higher than 0·7 for hydrograph simulation; therefore, the large *EF* values given above indicated a good fit between modelled and observed DRHs for both experiments.

3. Applications of the MRUH method in Texas watersheds

3.1 Watersheds studied and rainfall-runoff database

Watershed data taken from a larger dataset (Asquith *et al.*, 2004) accumulated by researchers from the United States Geological Survey (USGS) Texas Water Science Center, Texas Tech University, University of Houston and Lamar University were used for this study. Location and geographic distribution of the stations are shown in Figure 4. The drainage areas of the 80 study watersheds ranged from approximately 0·8 to 65·0 km^2 (0·3 to 25 $mile^2$), with a median value of 15·8 km^2 (6·1 $mile^2$); 50 watersheds (62·5% of the 80 watersheds) have drainage areas less than 20 km^2 (7·7 $mile^2$). The stream slope of study watersheds ranged from 0·0026 to 0·0196 (dimensionless), with a median value of 0·0079. The main channel lengths estimated were approximately 2–80 km (1·2–49·7 miles). The percentage of impervious area (IMP) of the 80 study watersheds ranged from 0·0 to 74·0%, with a median value of 26·0%. About 40% of the watersheds were rural watersheds with IMP less than 5%, and about 29% of the watersheds were urbanised with IMP greater than 60%.

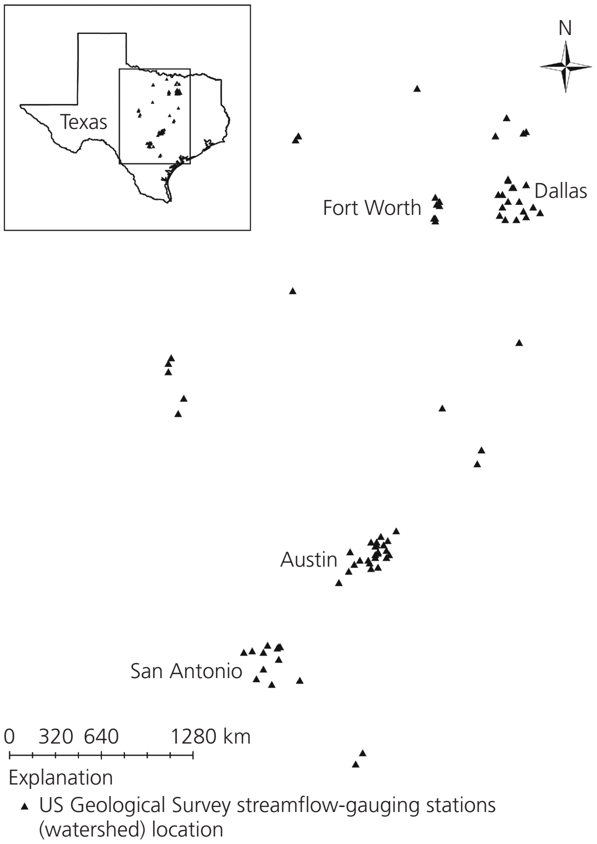

Figure 4. Map showing the USGS streamflow-gauging stations (triangles) associated with the watershed locations in Texas, USA

The rainfall-runoff dataset comprised about 1400 rainfall-runoff events recorded during 1959–1986. Event rainfall depths ranged from 3·56 mm (0·14 in) to 489·20 mm (19·26 in), with a median value of 57·66 mm (2·27 in). About 41 and 86% of the events had a storm depth less than 50·8 mm (2 in) and 101·6 mm (4 in), respectively. The base flow separations for observed runoff hydrographs were not done. This is because the majority of the gauging stations are on small ephemeral streams; base flow represents a small component of the total flow at the station. The streamflow for the watershed frequently was zero at the beginning of the storms (Asquith *et al.*, 2004).

3.2 Time of concentration and runoff coefficients

Time of concentration, T_c, and the runoff coefficient, C, are the required parameters for the MRUH method. The T_c values were estimated by Fang *et al.* (2008) using four empirical equations (see Appendix 2): (1) Williams equation (Williams, 1922); (2) Kirpich equation (Kirpich, 1940); (3) Johnstone and Cross equation (Johnstone and Cross, 1949); and (4) Haktanir and Sezen equation (Haktanir and Sezen, 1990).

The excess rainfall or the net rainfall is obtained from the product of the incremental rainfall and C (the volumetric interpretation, Dhakal *et al.*, 2012), similar to Smith and Lee (1984). Two estimates of C were examined for the application of the MRUH method. The first C is a watershed composite, literature-based coefficient (C_{lit}) derived from land-use information for the watershed and published values of C_{lit} for appropriate land uses (Dhakal *et al.*, 2012). The second C is a back-computed, volumetric runoff coefficient (C_{vbc}) determined by preserving the runoff volume using observed rainfall and runoff data. C_{vbc} was estimated by the ratio of total runoff depth to total rainfall depth for an individual observed storm event. The determination and comparison of C_{lit} and C_{vbc} for the study watersheds was documented by Dhakal *et al.* (2012).

3.3 DRHs derived using the MRUH method

For the 80 Texas watersheds, observed rainfall hyetograph and runoff hydrograph data were tabulated using a time interval of 5 min. Therefore, a 5 min MRUH was developed for each of the 80 study watersheds. The 5 min MRUH duration was less than T_c for all study watersheds.

The observed and simulated DRHs for the event on 8 July 1973 at the USGS streamflow-gauging station 08157000 Waller Creek, Austin, Texas are presented in Figure 5 as an illustrative example. The watershed drainage area was 5·72 km^2 (2·21 mile^2). The C_{vbc} was 0·29. The T_c values estimated using the Kirpich, Haktanir and Sezen, Johnstone and Cross, and Williams equations were 1·7, 2·2, 1·4 and 3·4 h, respectively. Peak discharges Q_{pUM} of the 5 min MRUH using 1 in (or 25·4 mm) rainfall excess for the watershed are 23·7, 18·3, 28·8 and 11·9 m^3/s using T_c values estimated from the Kirpich, Haktanir and Sezen, Johnstone and Cross, and Williams equations, respectively. Figure 2(b) shows an example MRUH for the watershed developed using T_c estimated from the Kirpich method (Equation 18); and the other three MRUHs developed from other T_c methods are trapezoids with different peaks and time bases ($D + T_c$), but the area under each trapezoid is the same because MRUH is a UH. The duration of the rainfall event was 19 h. Three distinct rainfall episodes resulted in three distinct peaks. These were reasonably represented by the DRHs derived from the MRUH using T_c estimated by the Kirpich, Haktanir and Sezen, and Johnson and Cross equations. The DRH developed from the MRUH using the Williams equation appears to over-estimate T_c for the watershed, and discharge peaks of the DRH were then underestimated (Figure 5). When the MRUHs were developed using T_c values estimated from the Kirpich, Haktanir and Sezen, Johnstone and Cross, and Williams equations, the *EF* (Equation 14) values derived between observed DRH and modelled DRHs using the above corresponding MRUHs are 0·83, 0·86, 0·70 and 0·63, respectively. Simulated times to peak (T_p) agree reasonably well with observed values (Figure 5) when using T_c estimated by Kirpich, Haktanir and Sezen, and Johnson and Cross equations for the MRUHs. However, using T_c estimated by the Williams equation for the MRUH resulted in the computed T_p exceeding the observed T_p.

Different combinations of T_c and C were used for applications of the MRUH method to predict the DRHs and to determine the

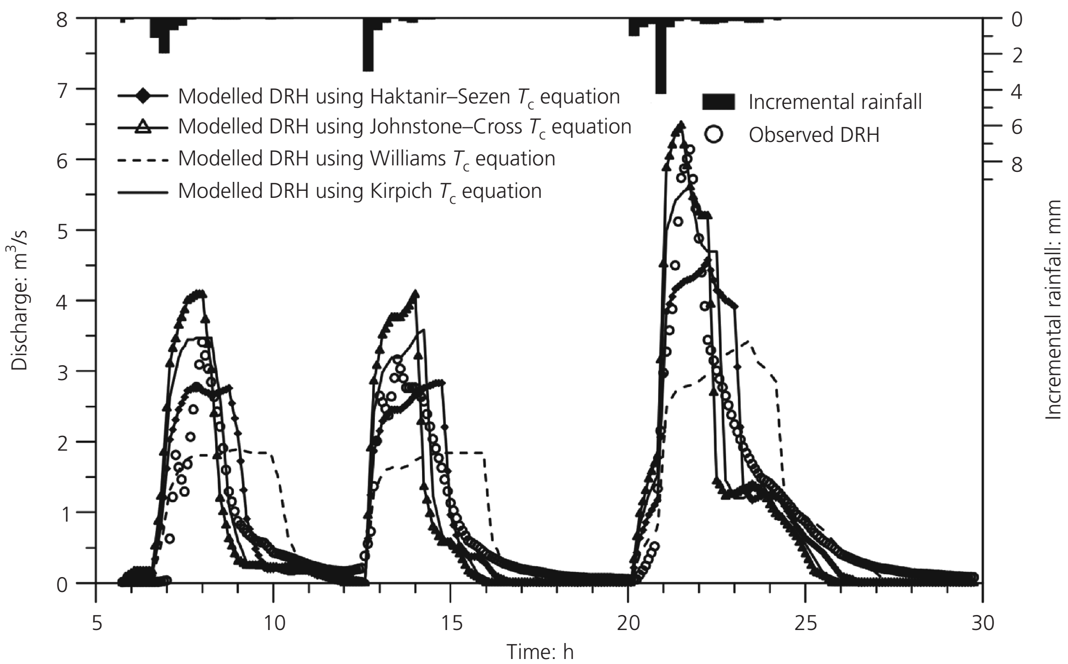

Figure 5. Incremental rainfall hyetograph for the event on 8 July 1973 and observed and modelled DRHs using the MRUHs with T_c estimated by four empirical equations for the watershed associated with the USGS streamflow-gauging station 08157000 Waller Creek, Austin, Texas

sensitivity of the DRH peak discharges (Q_p) to different T_c and C values. Five combinations of T_c and C were used.

(*a*) T_c estimated using Haktanir and Sezen equation and C_{vbc}.
(*b*) T_c estimated using Johnstone and Cross equation and C_{vbc}.
(*c*) T_c estimated using Williams equation and C_{vbc}.
(*d*) T_c estimated using Kirpich equation and C_{vbc}.
(*e*) T_c estimated using Kirpich equation and C_{lit}.

Figure 6 is a plot of the observed and computed DRH peaks using C_{vbc} and T_c values calculated using the four different empirical equations. In comparison to observed Q_p modelled Q_p using T_c estimated from the Haktanir and Sezen, Johnstone and Cross and Kirpich equations not only graphically look alike (Figure 6), but also are similar with respect to three statistical parameters (Table 1): coefficient of determination R^2; Nash–Sutcliffe efficiency *EF*; and relative error in peak *QB* (defined in Appendix 1). The results for *EF* using the Williams equation are inferior to the others. The fraction of modelled Q_p results that are within 1/3 of a log-cycle from the 1: 1 line are summarised in Table 1 and ranged from 67·5% (Williams equation) to 88·7% (Johnstone and Cross equation) of total events. Fractions of storms with *QB* less than $\pm$ 50% (Cleveland *et al.*, 2006) are listed in Table 1 for applications of the MRUH method with four combinations of T_c and C. Using T_c estimated from the Kirpich equation and C_{vbc} resulted in 75% of storms with *QB* less than $\pm$ 50%. Parameter C_{vbc} (back-computed from rainfall and runoff data) results in the preservation of event runoff volume, and the Kirpich equation provides reliable estimations on watershed T_c values (Fang *et al.*, 2008). Ideally, computed and observed peaks should plot precisely along the equal value line (black line in Figure 6). However, the UH is a mathematical model that is an incomplete description of the complexity of the combination of the rainfall-runoff process and runoff dynamics. Therefore, the relatively simple approach cannot fully capture the nuances of watershed dynamics and deviations from this ideal (the equal-value line) are expected. For example, Asquith and Roussel (2009) computed mean residual standard error about 1/3 of a log-cycle for annual peak discharges at 638 streamflow gauging stations in Texas.

The observed T_p and computed T_p values of DRHs predicted using C_{vbc} and T_c values calculated using the four different empirical equations were compared using three error parameters R^2, *EF* and relative error in time to peak *TB* (Equation 16). Parameter T_c, estimated from the Haktanir and Sezen, Johnstone and Cross, and Kirpich equations, produces the similar values of the quantitative measures: R^2, *EF*, median value of *TB* and fraction of storms with *TB* less than $\pm$ 50% (Table 1). The T_p results using the Williams equation seem to be slightly inferior to the others with respect to median value of *TB* and percentage of storms within $\pm$ 1/3 of a log cycle (Table 1). In summary, for predicting Q_p and T_p, use of T_c estimated from the Williams equation for the MRUH produces less accurate results than those computed using the Kirpich, Haktanir and Sezen, and Johnstone and Cross equations.

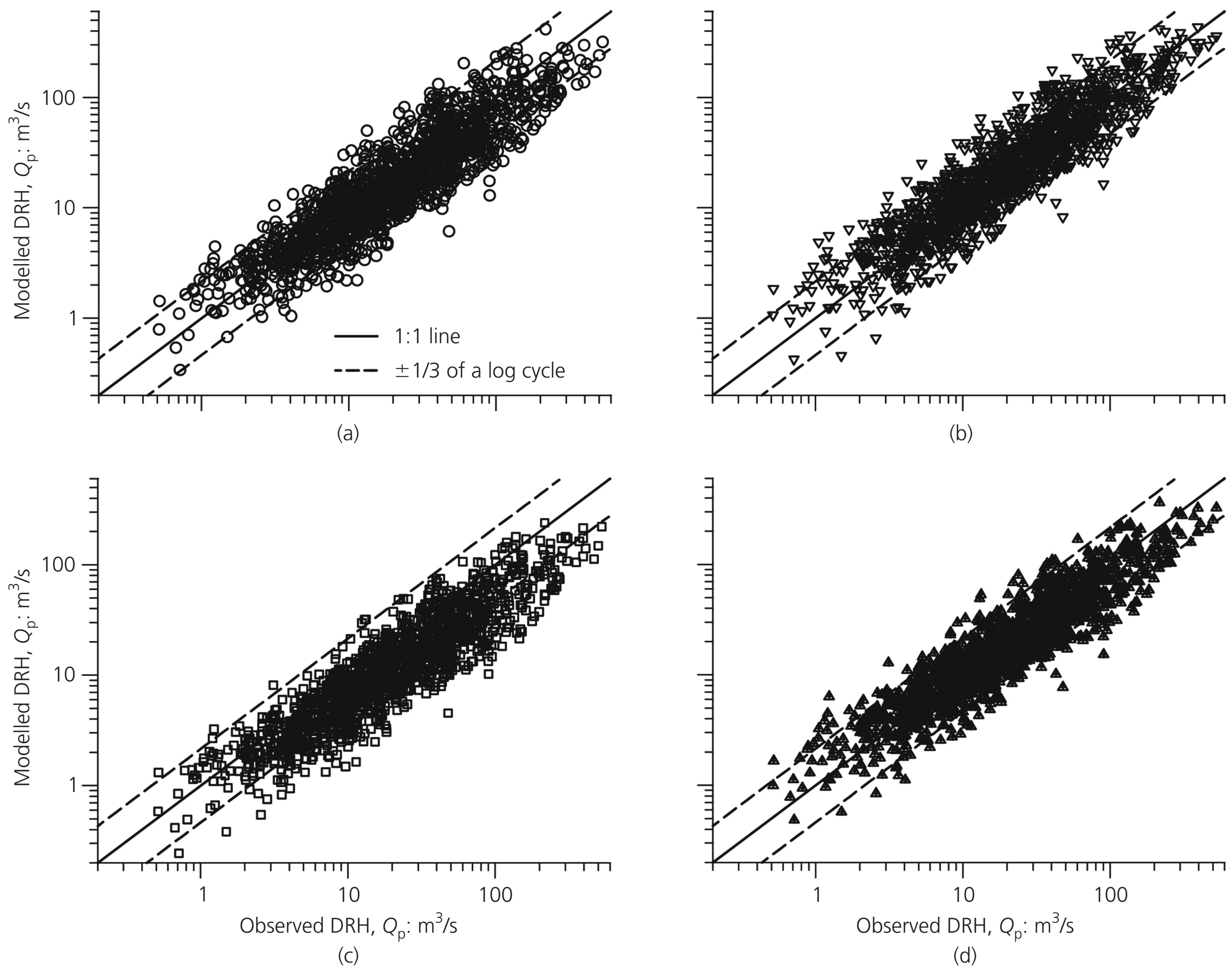

Figure 6. Modelled plotted against observed DRH peak discharges Q_p for 1400 rainfall-runoff events in 80 Texas watersheds. Modelled DRH peaks were developed using event C_{vbc} and MRUHs with T_c estimated using four different methods: (a) Haktanir and Sezen equation, (b) Johnstone and Cross equation, (c) Williams equation, and (d) Kirpich equation

Simulated Q_p results obtained from the MRUH method using the forward-computed (literature-based) runoff coefficient C_{lit} are compared against the Q_p results obtained using the back-computed runoff coefficient C_{vbc} (Figure 7). For both the cases, T_c values were estimated using the Kirpich equation. For the peak discharges predicted using C_{lit}, most of the values are above the equal value line (1: 1 line). Q_p results computed using C_{vbc} are superior to those using C_{lit} with respect to all statistical measures used to assess goodness of fit (Table 2). Use of C_{lit} tends to generate estimates of Q_p that exceed expected values (observations) when the C_{lit} values are interpreted as volumetric coefficients. In contrast, there is no difference in five quantitative measures between the observed and predicted T_p values (Table 2), regardless of which runoff coefficient is used. Hence, the simulation results of Q_p are more sensitive to the choice of C or rainfall loss model than to the choice of T_c. Furthermore, the T_p results are not related to C when the MRUH method was used and controlled by the time-variable rainfall distribution.

A sensitivity analysis was performed to evaluate the sensitivity of the DRH derived from the MRUH method to T_c and C. A rainfall event on 7 May 1972 for the USGS streamflow-gauging station 08178600 Salado Creek, San Antonio (24·88 km^2 or 9·61 $mile^2$) was selected for the analysis. The T_c used for the MRUH was varied from −50 to +50% of T_c estimated from the Kirpich equation. Similarly, the C used for rainfall loss was varied from −50 to +50% of C_{vbc}. The EF computed between the observed DRH and modelled DRH derived from the MRUH method using C_{vbc} and T_c estimated from the Kirpich equation was 0·89. The changes in EF values as a result of the changes in T_c and C for the sensitivity analysis are presented in Table 3. The change in EF ranged from 0·01 to −0·22 for ± 50% change in T_c. Similarly, the change in EF ranged from 0·02 to −0·66 for ± 50% change in C. This analysis further supports the above conclusion that DRH value derived using the MRUH method are more sensitive to the choice of C than to the choice of T_c.

Statistical parameters	Using the Haktanir and Sezen equation*	Using the Johnstone and Cross equation†	Using the Williams equation‡	Using the Kirpich equation§
R^2 for Q_p	0·75	0·80	0·75	0·80
EF for Q_p	0·66	0·79	0·48	0·73
Median value of QB	−0·19	0·00	−0·41	−0·10
Fraction of storms with $-0.5 \leqslant QB \leqslant 0.5$	0·70	0·72	0·60	0·75
Percentage of storms within ± 1/3 of a log cycle (Q_p)	82·4	88·7	67·5	88·6
R^2 for T_p	0·75	0·72	0·74	0·73
EF for T_p	0·74	0·71	0·74	0·72
Median value of TB	0·00	−0·05	0·10	−0·01
Fraction of storms with $-0.5 \leqslant TB \leqslant 0.5$	0·72	0·73	0·65	0·72
Percentage of storms within ± 1/3 of a log cycle (T_p)	82·1	80·5	78·2	82·3

* T_c computed using the Haktanir and Sezen equation ranged from 0·8 to 6·5 h in the study watersheds, with median and mean values of 2·6 and 2·9 h, respectively.
† T_c computed using the Johnstone and Cross equation ranged from 0·7 to 5·0 h in the study watersheds, with median and mean values of 1·7 and 1·9 h, respectively.
‡ T_c computed using the Williams equation ranged from 1·2 to 11·7 h in the study watersheds, with median and mean values of 4·0 and 4·5 h, respectively.
§ T_c computed using the Kirpich equation ranged from 0·6 to 7·1 h in the study watersheds, with median and mean values of 2·2 and 2·4 h, respectively.

Table 1. Quantitative measures of the success of the DRH Q_p and T_p modelled using C_{vbc} and MRUHs with T_c estimated using four equations

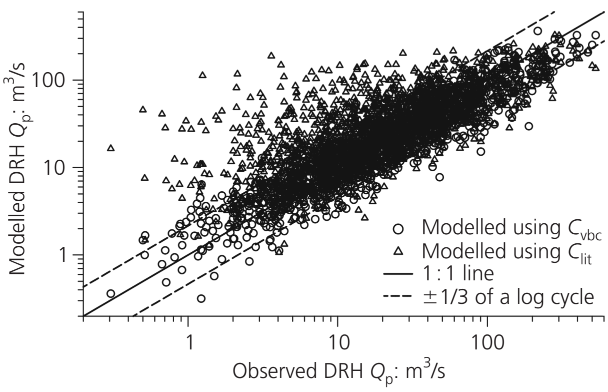

Figure 7. Observed and modelled DRH peak discharges developed using C_{vbc} (circles) and C_{lit} (triangles) and MRUHs with T_c estimated using the Kirpich equation for 80 Texas watersheds

4. Comparison of DRHs from different UH methods

In addition to the MRUH, three other UH models – UH developed using the Clark IUH method (Clark, 1945) with the generalised basin shape of HEC−1 (USACE, 1981), the NRCS UH (NRCS, 1972), and the Gamma function UH (GUH) for Texas watersheds (Pradhan, 2007) – were used to develop the DRH for each rainfall-runoff event in the database for the comparison.

Regression equations were developed for 5 min GUH parameters: Q_{pUG} (in ft^3/s) and T_{pUG} (in h) for Texas watersheds (Pradhan, 2007)

$$T_{pUG} = 0.55075\,A^{0.26998}\,L^{0.42612}\,S^{-0.06032} \tag{6}$$

$$Q_{pUG} = 93.22352\,A^{0.83576}\,L^{-0.326}\,S^{0.5} \tag{7}$$

where A is drainage area in square miles, L is main channel length in miles, and S is main channel slope (ft/mile, elevation difference in feet divided by main channel length in miles). The ordinates of the GUH can be obtained from (Viessman and Lewis, 2003)

$$Q_{uG}(t) = Q_{pUG}(t/T_{pUG})^{\alpha}\mathrm{e}^{\left[1-(t/T_{pUG})\right]^{\alpha}} \tag{8}$$

where α is the shape parameter of GUH, which is determined from Q_{pUG} and T_{pUG} (Aron and White, 1982).

The IUH method of Clark (1945) is based on the time−area curve method (Bedient and Huber, 2002). A synthetic time−area curve derived from a generalised basin shape was used to implement

Statistical parameters	Using C_{vbc}	Using C_{lit}
R^2 for Q_p	0·80	0·44
EF for Q_p	0·73	0·42
Median value of QB	−0·10	0·45
Fraction of storms with $-0{\cdot}5 \leqslant QB \leqslant 0{\cdot}5$	0·75	0·45
Percentage of storms within ± 1/3 of a log cycle (Q_p)	88·6	63·0
R^2 for T_p	0·73	0·73
EF for T_p	0·72	0·72
Median value of TB	−0·01	−0·01
Fraction of storms with $-0{\cdot}5 \leqslant TB \leqslant 0{\cdot}5$	0·72	0·72
Percentage of storms within ± 1/3 of a log cycle (T_p)	82·3	82·3

Table 2. Quantitative measures of the success of the DRH Q_p and T_p modelled using MRUH with T_c estimated using the Kirpich equation and C estimated using two different methods (C_{vbc} and C_{lit})

Change in T_c: %	Change in EF	Change in C: %	Change in EF
−50	−0·18	−50	−0·27
−25	−0·02	−25	−0·02
−10	0·01	−10	0·02
10	−0·03	10	−0·06
25	−0·09	25	−0·21
50	−0·22	50	−0·66

Table 3. Sensitivity (change in EF) of DRH derived from MRUH on T_c and C for the rainfall event on 7 May 1972 for the USGS streamflow-gauging station 08178600 Salado Creek, San Antonio, Texas

Clark's IUH in HEC-1 (USACE, 1981). The equations for the time–area curve are

9. $$AI = 1{\cdot}414\ TI^{1{\cdot}5},\ \ 0 \leqslant TI \leqslant 0{\cdot}5$$

10. $$1 - AI = 0{\cdot}414\ (1 - TI)^{1{\cdot}5},\ \ 0{\cdot}5 < TI < 1$$

where AI is the cumulative area as a fraction of watershed area and TI is fraction of T_c.

The NRCS curvilinear UH was developed in the late 1940s (NRCS, 1972). The Q_{pUN} for the NRCS UH is computed by approximating the UH with a triangular shape having base time of $8/3T_{pUN}$ and unit area (Viessman and Lewis, 2003)

11. $$Q_{pUN} = \frac{484A}{T_{pUN}}$$

where Q_{pUN} is ft^3/s and A is the drainage area in mile^2.

UHs developed using all four models, including the MRUH, for the watershed associated with the USGS streamflow-gauging station 08048520 Sycamore Creek in Fort Worth are shown in Figure 8(a). The shape of the MRUH is trapezoidal, whereas the UHs from the Clark-HEC-1, the Gamma, and the NRCS methods are curvilinear. The UH peak discharge from each model is different (Figure 8(a)). However, the area under the UH curves is the same. This is because each UH corresponds to 1 in of a uniform excess rainfall over a 5 min duration (one impulse).

Gamma, Clark-HEC-1 and NRCS UHs developed for each watershed were applied to the 1400 rainfall-runoff events in the database to generate DRHs using discrete UH convolution (Chow *et al.*, 1988). The values of C_{vbc} determined for each event were used. Parameter T_c, which was determined using the Kirpich method (Kirpich, 1940), was used for those methods that require T_c. As an illustrative example, observed and simulated DRHs for the rainfall event on 28 July 1973 at the USGS streamflow-gauging station 08048520 (Sycamore Creek in Fort Worth, Texas) by the four models (base flow was assumed to be zero) is presented in Figure 8(b). The watershed area is 45·66 km^2 (17·63 mile^2), T_c is 3·96 h from the Kirpich method, and C_{vbc} is 0·20. Simulated peak discharges from the four UH methods are different, but comparable. For the particular example shown in Figure 8(b), the MRUH and the Clark-HEC-1 model appear to perform better than the other UH models with regard to prediction of Q_p. The T_p, simulated values using the four methods agree reasonably well with the observed value (Figure 8(b)). Furthermore, the area under the four simulated DRHs matches that of the observed curve because the event C_{vbc} was used.

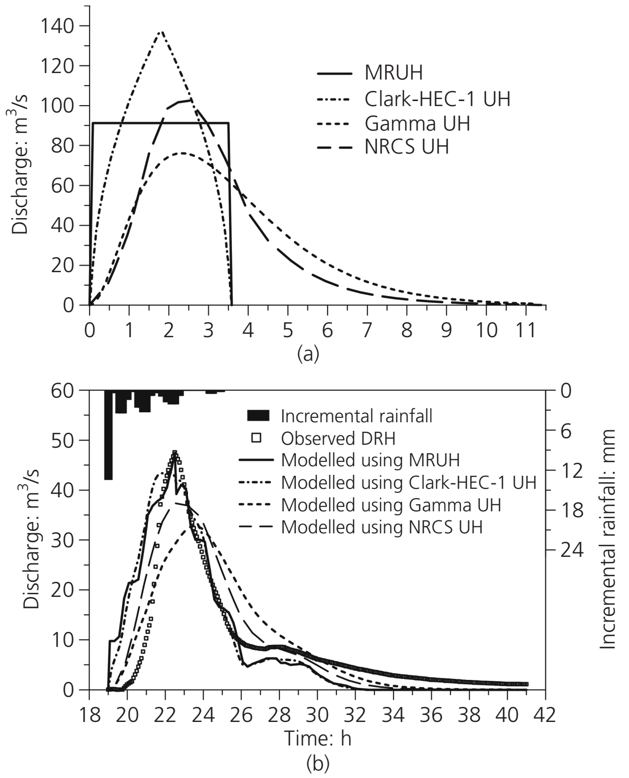

Figure 8. (a) Modified rational, Gamma, Clark-HEC-1, and NRCS UHs developed for the watershed associated with USGS streamflow-gauging station 08048520 Sycamore, Fort Worth, Texas; and (b) rainfall hyetograph, observed and modelled DRHs using the four different UHs for the rainfall event on 28 July 1973 for the same watershed

Simulated DRH ordinates derived from all the four UH models were compared with observed DRH ordinates for each rainfall event, and the root mean squared error of the DRH ordinates normalised by observed Q_p (RRMSE, Equation 12) was calculated for each event and then averaged for all the events in the same watershed. A statistical summary of averaged normalised root mean squared errors for 80 study watersheds is presented in Table 4. All the four UH models behave similarly to predict DRHs based on statistical parameters in Table 4, and Figure 8(b) shows one example to illustrate the similarity of DRHs derived from these UH models.

Statistical parameters	Using MRUH	Using Gamma UH	Using Clark-HEC-1 UH	Using NRCS UH
Maximum	1·78	1·61	1·95	1·74
Minimum	0·25	0·19	0·23	0·22
Mean	0·61	0·61	0·62	0·57
Median	0·52	0·53	0·53	0·51

Table 4. Statistical summary of watershed-averaged root mean squared errors between modelled and observed DRHs normalised by observed peak discharges

The observed and modelled Q_p results from all four UH models developed using C_{vbc} and T_c from the Kirpich method are presented in Figure 9 for all 1400 events. Modelled Q_p results from all the four UH models are similar (Figure 9). Based on the three statistical measures (RRMSE, R^2, and EF) the authors concluded that all the four UH models perform similarly in predicting DRH Q_p and T_p (Table 5) after considering possible errors in DRH prediction. Fractions and percentages of storms for each model meeting the tolerances of QB and TB are also listed in Table 5 and show that all the models perform similarly. However, the GUH developed for Texas watersheds perform slightly worse than the other three UH models (Table 5) in predicting DRH Q_p.

5. Summary and conclusions

The MRM is an extension of the rational method to produce simple triangular and trapezoidal DRHs that have been used in some engineering applications. MRM's DRH for $D < T_c$ was used to derive a trapezoidal UH termed the modified rational UH or MRUH. The MRUH method was applied at 80 watersheds in Texas to determine the DRHs for 1400 rainfall-runoff events. The purposes were: (1) to evaluate the applicability of the MRUH method when applied to watersheds of larger size (0·8–65·0 km^2 or 0·3–25 mile^2), and (2) to study the effects of C and T_c on prediction accuracy of the MRUH method on DRH ordinates, DRH Q_p and DRH T_p. Three other UH models; the Clark (using HEC–1's generalised basin equations), the Gamma, and the NRCS UHs were used to compute the DRH for each rainfall-runoff event in the same database. Simulated peak discharges of DRHs from MRUH and the other three UHs agree reasonably well with observed values. The drainage area of the study watersheds (0·8–65·0 km^2 or 0·3–25 mile^2) is greater than that usually accepted for rational method application (0·8 km^2 or 0·3 mile^2).

Three general conclusions for the study are: (1) being a UH, the MRUH method can be applied to time-variable rainfall events and for watersheds with drainage areas greater than typically used with either the rational method or the MRM (a few hundred

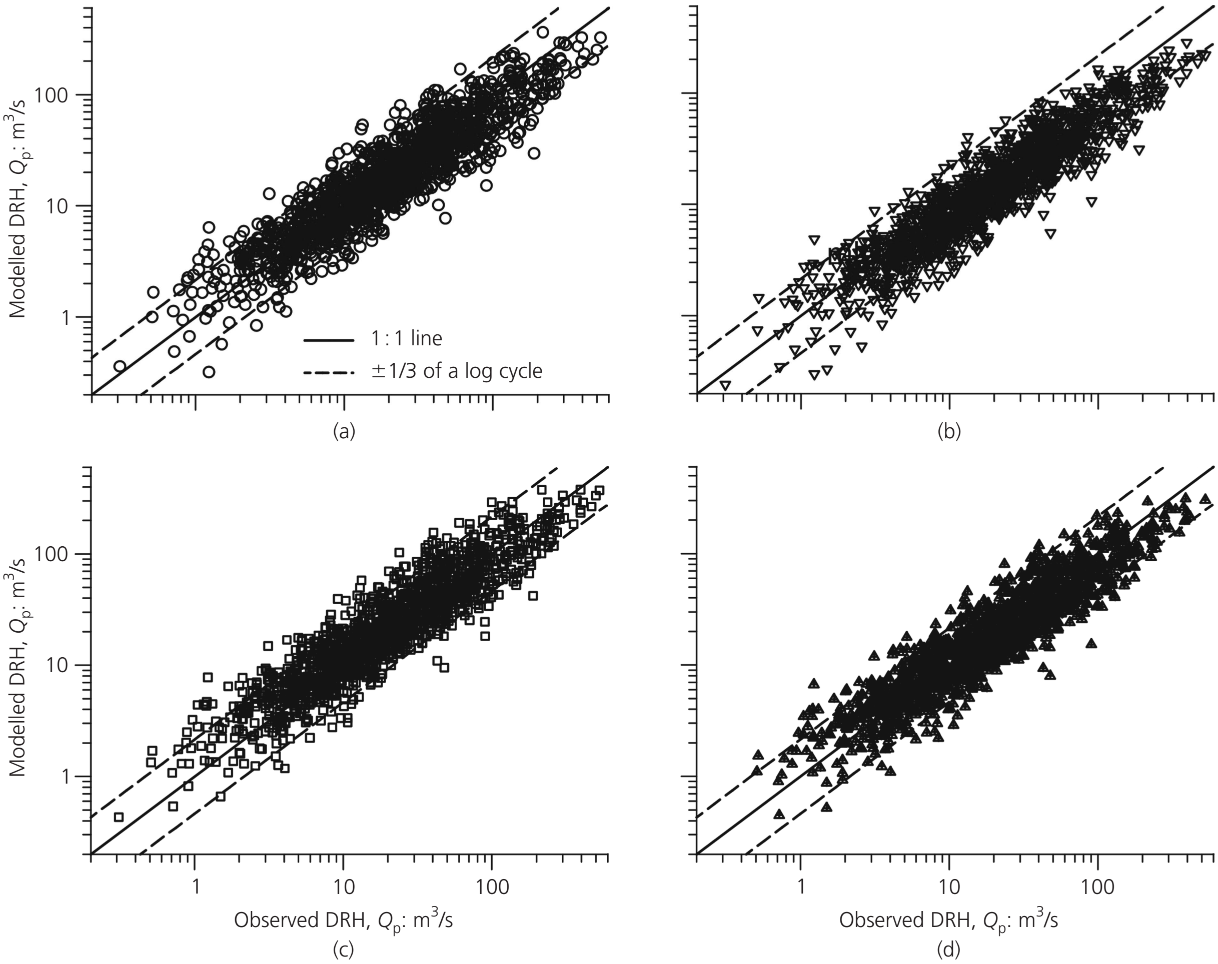

Figure 9. Observed and modelled DRH peak discharges using: (a) MRUH, (b) Gamma UH, (c) Clark-HEC-1 UH and (d) NRCS UH for 1400 rainfall-runoff events in 80 Texas watersheds

Statistical parameters	Using MRUH	Using Gamma UH	Using Clark-HEC-1 UH	Using NRCS UH
R^2 for Q_p	0·80	0·82	0·81	0·83
EF for Q_p	0·73	0·63	0·79	0·76
Median value of *QB*	−0·10	−0·32	0·02	−0·12
Fraction of storms with $-0{\cdot}5 \leqslant QB \leqslant 0{\cdot}5$	0·75	0·71	0·71	0·77
Percentage of storms within ± 1/3 of a log cycle (Q_p)	88·6	80·6	88·5	90·9
R^2 for T_p	0·73	0·73	0·71	0·71
EF for T_p	0·72	0·72	0·70	0·70
Median value of *TB*	−0·01	0·03	−0·02	0·00
Fraction of storms with $-0{\cdot}5 \leqslant TB \leqslant 0{\cdot}5$	0·72	0·73	0·75	0·75
Percentage of storms within ± 1/3 of a log cycle (T_p)	82·3	84·1	81·8	82·4

Table 5. Quantitative measures of the success of DRH Q_p and T_p modelled using four UH models for 1400 rainfall-runoff events in 80 Texas watersheds

acres); (2) the MRUH performs about as well as other UH methods used in this study for predicting Q_p and T_p of the DRH, so long as the same rainfall loss model is used; (3) modelled peak discharges from application of the MRUH method are more sensitive to the selection of C and less sensitive to T_c. In predicting peak discharges and DRHs for engineering design, rainfall loss estimation results in greater uncertainty and contributes more model errors than variations of UH methods and model parameters for UH.

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Appendix 1: Statistical measures to evaluate model performance

Five statistical measures were used to analyse modelled DRH results against observed ones. They are the root mean squared error (RMSE) of the DRH ordinates normalised by observed DRH Q_p, – that is, relative RMSE or RRMSE, the coefficient of determination R^2, the Nash–Sutcliffe efficiency EF, the relative error in peak QB, and the relative error in time to peak TB (Cleveland *et al.*, 2006; Loague and Green, 1991; Zhao and Tung, 1994)

12. $$\text{RRMSE} = \frac{\left[\sum_{j=1}^{N} (Q(t)_{\text{m}j} - Q(t)_{\text{o}j})^2/N\right]^{0\cdot5}}{Q_{\text{po}}}$$

13. $$R^2 = \left[\frac{\sum_{i=1}^{n} (Q_{\text{po}i} - \bar{Q}_{\text{po}})(Q_{\text{pm}i} - \bar{Q}_{\text{pm}})}{\sqrt{\sum_{i=1}^{n} (Q_{\text{po}i} - \bar{Q}_{\text{po}})^2}\sqrt{\sum_{i=1}^{n} (Q_{\text{pm}i} - \bar{Q}_{\text{pm}})^2}}\right]^2$$

14. $$EF = \frac{\sum_{i=1}^{n} (Q_{\text{po}i} - \bar{Q}_{\text{po}})^2 - \sum_{i=1}^{n} (Q_{\text{pm}i} - Q_{\text{po}i})^2}{\sum_{i=1}^{n} (Q_{\text{po}i} - \bar{Q}_{\text{po}})^2}$$

15. $$QB = \frac{Q_{\text{pm}i} - Q_{\text{po}i}}{Q_{\text{po}i}}$$

and

16. $$TB = \frac{T_{\text{pm}i} - T_{\text{po}i}}{T_{\text{po}i}}$$

where $Q(t)_{\text{m}j}$ is the modelled DRH ordinate (subscript m stands for modelled), $Q(t)_{\text{o}j}$ is the observed DRH ordinate (subscript o stands for observed), N is the number of DRH ordinates for an event, $Q_{\text{pm}i}$ is the modelled Q_p for the event i, $Q_{\text{po}i}$ is the observed Q_p, n is the number of observations, $\bar{Q}_{\text{pm}}$ and $\bar{Q}_{\text{po}}$ are the mean values of the modelled and observed peak discharges, $T_{\text{pm}i}$ is the modelled T_p, and $T_{\text{po}i}$ is the observed T_p.

Appendix 2: Empirical equations used to estimate T_c

Four empirical equations Williams (1922), Kirpich (1940), Johnstone and Cross (1949) and Haktanir and Sezen (1990) used to estimate T_c (in min) by Fang *et al.* (2008) are given respectively below:

17. $$T_c = 16{\cdot}32LA^{0\cdot4}/(D_w S_o^{0\cdot2})$$

18. $$T_c = 3{\cdot}978L^{0\cdot77}S_o^{-0\cdot385}$$

19. $$T_c = 3{\cdot}258(L/S_o)^{0\cdot5}$$

20. $$T_c = 26{\cdot}85L^{0\cdot841}$$

where L is the channel length in km, D_w is the watershed equivalent diameter in km, W is the watershed width in km, A is the area in km^2, and S_o is the channel slope in m/m or ft/ft (dimensionless).

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