GROUND WATER

Computer Notes

Type-Curve Matching Using a Computer Spreadsheet

by Theodore G. Cleveland[a]

Abstract

This note shows how to construct and perform a graphical type-curve analysis using a computer spreadsheet. Formulas are provided to generate the axis shifting behavior that is required for the graphical fit. Arbitrary match points are obtained by trial-and-error (or by closed form solution) for use in the calculation of aquifer hydraulic parameters. The analysis is illustrated using a leaky aquifer type curve and a confined aquifer type curve. Macro programming is not required, only data entry and the ability to use plotting features of a computer spreadsheet. The approach can be adapted for parameter estimation using any tabulated type curve.

Introduction

Type-curve matching analysis is a convenient tool for interpreting aquifer hydraulics despite advances in computer technology that make simulation-optimization methods feasible. The Theis matching technique is a component of textbook chapters on well hydraulics (Fetter, 1994; Bedient et al., 1994; Domenico and Schwartz, 1990; Marsily, 1986), the subject of specialty books (Reed, 1980; Kruseman and deRidder, 1990; Dawson and Istok, 1991; Walton, 1991), and a component of short courses on aquifer pumping tests (NGWA, 1994). Type curves are also used to present the solutions of complicated mathematical expressions that apply to special aquifer hydraulics problems (Avci, 1994; Moench, 1993; Shan et al., 1992; Motz, 1991). The use of these curves for parameter estimation usually involves some modification of the Theis-matching technique.

This note shows how to perform a Theis-matching technique using a computer spreadsheet and tabulated type-curve data. The method is the "spreadsheet equivalent" of plotting time-drawdown data, drawing type curves, locating a match point, and performing the associated arithmetic to determine the formation constants. It evolved from a need to teach the type-curve matching techniques, without the tedium of obtaining graph paper and drawing scaled type curves, while being general enough to extend to other tabulated type curves.

This spreadsheet approach does not require macro programming, and it can be used to match type curves that are not supplied with commercial software programs. The approach is illustrated using the Hantush leaky well function and several data sets from a ground-water hydrology text as examples.

Spreadsheet Based Theis-Matching Method

The Hantush (Hantush, 1956) leaky well function is used to illustrate the spreadsheet approach. The model equations for a leaky aquifer with negligible aquitard storage are:

$$s[u] = \frac{Q}{4\pi T} W\left[u, \frac{r}{B}\right]; \quad u = \frac{r^2 S}{4Tt} \tag{1}$$

where s is the drawdown in the observation well, Q is the pumping well constant discharge rate, T is the aquifer transmissivity, S is the aquifer storage coefficient, r is the distance from the pumping well to the observation well, t is time since pumping began, $B = (Tb'/K')^{1/2}$ is the leakance factor, K′ is the aquitard hydraulic conductivity, and b′ is the aquitard thickness. The unknown aquifer parameters are T, S, and K′ (as determined from the leakance). This particular solution is valid when the following conditions are met (Hantush, 1956):

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Received October 1994, revised April 1995, accepted May 1995.

$$t > \frac{S\ 30\ r_w^2}{T\left[1 - \left(\frac{10\ r_w^2}{B}\right)^2\right]}, \quad \text{and} \quad \frac{r_w}{B} < 10 \qquad (2)$$

where r_w is the radius of the well, and t, S, T, and B are the same as previously defined.

Generally the manual type-curve analysis involves plotting the observation data on graph paper with the same scale as a published type curve, overlaying the type curve onto the data plot, adjusting the relative position of the two plots, identifying a match point, and performing some simple computations using values from the match point. The details of a manual type-curve analysis for this particular problem can be found in Walton (1962), or in Fetter (1994).

The principal effort is plotting the data and adjusting the two plots while maintaining parallel axes, and specialized commercial software (Geraghty & Miller, 1994; Scientific Software, 1994) is available to assist in the matching, but commercial software, as this note illustrates, is not necessary for analysts with computer spreadsheets. Furthermore, the spreadsheet approach is adaptable for type curves that commercial software may not include.

To perform the analysis on a spreadsheet, the type-curve tabulation that represents different values of the well function for different values of u and r/B is entered into a spreadsheet with the dimensionless group that contains time arranged in a vertical column, and the other dimensionless group arranged in a horizontal row along the top of the table, as shown on the upper portion of Figure 1. This data entry step is tedious, but only needs to be performed once for any given type curve. Afterwards, analysis only involves data entry of the test data.

A set of type curves is constructed by instructing the spreadsheet to produce an XY plot using the first column as the x-axis, the tabulation columns as different curves along the y-axis, and the top row as labels for each different type curve. Current (circa 1994-95) spreadsheets allow the user great flexibility in scale selection. The lower portion of Figure 1 is the plot of the type-curve tabulation using log-log scales, and plotting the x-axis in reverse order.

The test data-curve tabulation is then prepared on the same spreadsheet with time and drawdown data arranged as two columns. These two columns are shown on Figure 1 to the left of the plot, and the data are labeled as "Actual Pumping Test Data." The other test information is also included, in this case: discharge rate, aquitard thickness, and observation well distance. These data are also in Figure 1, below the test data. To the right of the "Actual Pumping Test Data" section is a section labeled "Shifted Test Values." These values are the values that are plotted as the data curve and are used to perform the analysis. The example spreadsheets in this note include conversions for the units shown on the figures; however, the method is applicable to any consistent set of units.

The log-log plot of the shifted data produces a curve of similar shape as the type curves, but displaced in the vertical and horizontal direction from the set of type curves. Figure 2 is a picture of the lower part of the spreadsheet that depicts this situation, with the shifted time-drawdown data superimposed on the type curves. The dot and cross symbols are used to identify the match point and are explained shortly.

In a manual analysis the analyst overlays the type-curve set onto the test data curve and adjusts the type-curve set up-and-down, left-and-right to fit the data; identifies match point(s) and performs associated computations to estimate the hydraulic parameters. The spreadsheet equivalent of adjusting the curves is accomplished using some simple computations.

The two displaced plots are related to the following system of equations:

$$\ln[s] = \ln\left[\frac{Q}{4\pi T}\right] + \ln\left[W\left(u, \frac{r}{B}\right)\right] \qquad (3)$$

$$\ln[t] = \ln\left[\frac{r^2 S}{4T}\right] + \ln\left[\frac{1}{u}\right] = \ln\left[\frac{r^2 S}{4T}\right] - \ln[u] \qquad (4)$$

The test data curve is a graph of the left-hand side of the two equations, the type curves are the far right terms of the two equations, and the shifts in axes are represented by the two middle terms of the equations. To permit shifting of axes on the spreadsheet plots, we appeal to the structure of these two equations, and actually plot "adjusted" time-drawdown data, where we make vertical and horizontal adjustments. The formulas to make these adjustments are:

$$s^* = (\text{shift}_v)\ s \longleftrightarrow \ln[s^*] = \ln[\text{shift}_v] + \ln[s] \qquad (5)$$

$$1/t^* = (\text{shift}_h)/t \longleftrightarrow -\ln[t^*] = \ln[\text{shift}_h] - \ln[t] \qquad (6)$$

When the horizontal and vertical shift parameters are set to one, the unshifted data are plotted. When different values are input, the adjusted data plot (t* vs. s*) shifts up-and-down, left-and-right relative to the type-curve plot depending on these values. The analyst then changes these values and observes changes in the relative position of the data and type curves to perform the match.

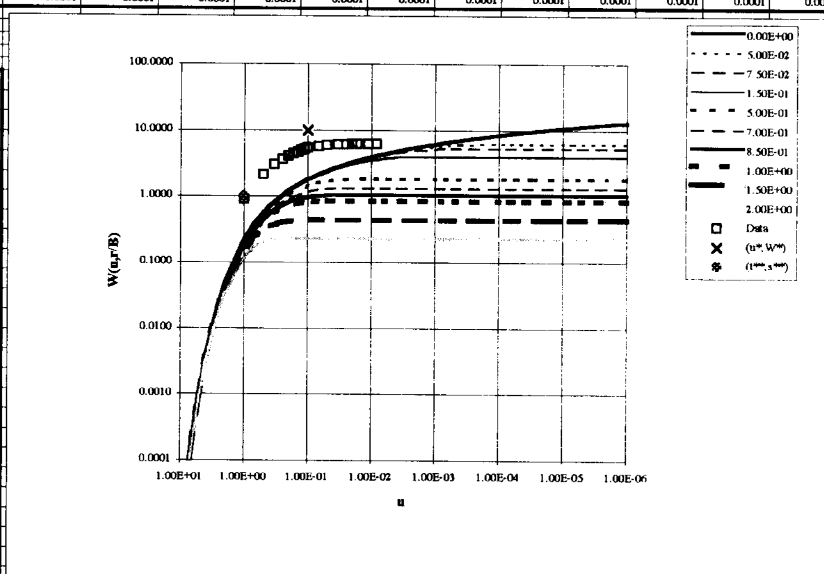
Figure 2 shows the bottom portion of the spreadsheet, the pumping test data are from Fetter (1994), pg. 271. The adjusted data points (t*, s*) are displaced from the type curves when the shift parameters are set to one, and represent the actual test data (t, s) plotted on the same scales as the type curves. In addition, two other points are plotted. These points are (u**, W**) and (t**, s**). They represent the analyst's choice of match point on the type curves and test data curve, respectively. These points are used later to locate a match point for subsequent calculations, and are plotted as the dot and cross symbols on the graph.

The data curve is shifted by trial-and-error relative to the type curves using different values of the shift parameters. Figure 3 shows the lower portion of the spreadsheet after the shift parameters have been adjusted. These values are entered by the analyst on the spreadsheet below the test data (labeled as item 2 on the spreadsheet). Decreasing the value of the horizontal shift parameter moves the test data curve to the right relative to the set of type curves. Decreasing the value of the vertical shift parameter moves the data curve downward relative to the set of type curves. Comparing Figure 2 with Figure 3, the values of the shift parameters have been changed from (1, 1) to (1.13, 0.2) indicating that the analyst shifted the data curve leftward and downward to find an acceptable match.

Once an acceptable type curve is located (Figure 3) the next step is placing a match point on both curves that overlay each other. The symbols for the match point in this note are indicated by t**, s**, u**, and W**. The matching task is accomplished by *choosing* a convenient (u**, W**) match pair on the type-curve axis and plotting this pair using a special symbol (shown as a cross on all the figures). This pair can be changed by the user to

Hantush Type Curve Spreadsheet																
Tabulated Type Curve Data								r/B								
u	0.00E+00	1.00E-03	5.00E-03	1.00E-02	2.50E-02	5.00E-02	7.50E-02	1.50E-01	3.00E-01	5.00E-01	7.00E-01	8.50E-01	1.00E+00	1.50E+00	2.00E+00	2.50E+00
1.00E-06	13.2383	13.0031	10.8283	9.4425	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
2.00E-06	12.5451	12.4240	10.8174	9.4425	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
3.00E-06	12.1397	12.0581	10.7849	9.4425	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
4.00E-06	11.8520	11.7905	10.7374	9.4422	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
5.00E-06	11.6289	11.5795	10.6822	9.4413	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
6.00E-06	11.4465	11.4053	10.6240	9.4394	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
7.00E-06	11.2924	11.2570	10.5652	9.4361	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
8.00E-06	11.1589	11.1279	10.5072	9.4313	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
9.00E-06	11.0411	11.0135	10.4508	9.4251	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
1.00E-05	10.9357	10.9109	10.3963	9.4176	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
2.00E-05	10.2426	10.2301	9.9530	9.2961	7.6111	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
3.00E-05	9.8371	9.8288	9.6392	9.1499	7.6101	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
4.00E-05	9.5495	9.5432	9.3992	9.0102	7.6069	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
5.00E-05	9.3263	9.3213	9.2054	8.8827	7.6000	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
6.00E-05	9.1440	9.1398	9.0426	8.7673	7.5894	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
7.00E-05	8.9899	8.9863	8.9027	8.6625	7.5754	6.2285	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
8.00E-05	8.8563	8.8532	8.7798	8.5669	7.5589	6.2284	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
9.00E-05	8.7386	8.7358	8.6703	8.4792	7.5402	6.2283	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
1.00E-04	8.6332	8.6308	8.5717	8.3983	7.5199	6.2282	5.4228	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
2.00E-04	7.9402	7.9390	7.9092	7.8192	7.2898	6.2173	5.4227	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
3.00E-04	7.5348	7.5340	7.5141	7.4534	7.0759	6.1848	5.4212	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
4.00E-04	7.2472	7.2466	7.2317	7.1859	6.8929	6.1373	5.4160	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
5.00E-04	7.0242	7.0237	7.0118	6.9750	6.7357	6.0821	5.4062	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
6.00E-04	6.8420	6.8416	6.8316	6.8009	6.5988	6.0239	5.3921	4.0601	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
7.00E-04	6.6879	6.6876	6.6790	6.6527	6.4777	5.9652	5.3745	4.0600	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
8.00E-04	6.5545	6.5542	6.5467	6.5237	6.3695	5.9073	5.3542	4.0599	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
9.00E-04	6.4368	6.4365	6.4299	6.4094	6.2716	5.8509	5.3317	4.0598	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
1.00E-03	6.3315	6.3313	6.3253	6.3069	6.1823	5.7965	5.3078	4.0595	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
2.00E-03	5.6394	5.6393	5.6363	5.6271	5.5638	5.3538	5.0517	4.0435	2.7449	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
3.00E-03	5.2349	5.2348	5.2329	5.2267	5.1845	5.0408	4.8243	4.0092	2.7448	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
4.00E-03	4.9482	4.9482	4.9467	4.9421	4.9105	4.8016	4.6335	3.9951	2.7444	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
5.00E-03	4.7261	4.7260	4.7249	4.7212	4.6960	4.6084	4.4713	3.8821	2.7428	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
6.00E-03	4.5448	4.5448	4.5438	4.5407	4.5197	4.4467	4.3311	3.8384	2.7398	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
7.00E-03	4.3916	4.3916	4.3908	4.3882	4.3702	4.3077	4.2078	3.7529	2.7350	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
8.00E-03	4.2591	4.2590	4.2583	4.2561	4.2404	4.1857	4.0980	3.6903	2.7284	1.8488	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
9.00E-03	4.1423	4.1423	4.1416	4.1396	4.1258	4.0772	3.9991	3.6302	2.7202	1.8487	1.3210	1.0485	0.8420	0.4276	0.2278	0.1247
1.00E-02	4.0379	4.0379	4.0373	4.0356	4.0231	3.9795	3.9091	3.5725	2.7104	1.8486	1.3207	1.0485	0.8420	0.4276	0.2278	0.1247
2.00E-02	3.3547	3.3547	3.3544	3.3536	3.3476	3.3264	3.2917	3.1158	2.5688	1.8379	1.3207	1.0484	0.8420	0.4276	0.2278	0.1247
3.00E-02	2.9591	2.9591	2.9589	2.9584	2.9545	2.9409	2.9183	2.8017	2.4110	1.8062	1.3177	1.0481	0.8420	0.4276	0.2278	0.1247
4.00E-02	2.6813	2.6812	2.6811	2.6807	2.6779	2.6680	2.6515	2.5655	2.2661	1.7603	1.3094	1.0465	0.8418	0.4276	0.2278	0.1247
5.00E-02	2.4679	2.4679	2.4678	2.4675	2.4653	2.4576	2.4448	2.3776	2.1371	1.7075	1.2955	1.0426	0.8409	0.4276	0.2278	0.1247
6.00E-02	2.2953	2.2953	2.2952	2.2950	2.2932	2.2870	2.2766	2.2218	2.0227	1.6524	1.2770	1.0362	0.8391	0.4276	0.2278	0.1247
7.00E-02	2.1508	2.1508	2.1508	2.1506	2.1491	2.1439	2.1352	2.0894	1.9206	1.5973	1.2551	1.0272	0.8360	0.4276	0.2278	0.1247
8.00E-02	2.0269	2.0269	2.0269	2.0267	2.0255	2.0210	2.0136	1.9745	1.8290	1.5436	1.2310	1.0161	0.8316	0.4275	0.2278	0.1247
9.00E-02	1.9187	1.9187	1.9187	1.9185	1.9174	1.9136	1.9072	1.8732	1.7460	1.4918	1.2054	1.0032	0.8259	0.4274	0.2278	0.1247
1.00E-01	1.8229	1.8229	1.8229	1.8227	1.8218	1.8184	1.8128	1.7829	1.6704	1.4422	1.1791	0.9890	0.8190	0.4271	0.2278	0.1247
2.00E-01	1.2227	1.2226	1.2226	1.2226	1.2222	1.2209	1.2186	1.2066	1.1602	1.0592	0.9284	0.8216	0.7148	0.4135	0.2268	0.1247
3.00E-01	0.9057	0.9057	0.9057	0.9056	0.9054	0.9047	0.9035	0.8969	0.8713	0.8142	0.7369	0.6706	0.6010	0.3812	0.2211	0.1240
4.00E-01	0.7024	0.7024	0.7024	0.7024	0.7022	0.7018	0.7010	0.6969	0.6809	0.6446	0.5943	0.5501	0.5024	0.3411	0.2096	0.1217
5.00E-01	0.5598	0.5598	0.5598	0.5598	0.5597	0.5594	0.5588	0.5561	0.5453	0.5206	0.4860	0.4550	0.4210	0.3007	0.1944	0.1174
6.00E-01	0.4544	0.4544	0.4544	0.4544	0.4543	0.4541	0.4537	0.4518	0.4441	0.4266	0.4018	0.3793	0.3543	0.2630	0.1774	0.1112
7.00E-01	0.3738	0.3738	0.3738	0.3738	0.3737	0.3735	0.3733	0.3719	0.3663	0.3534	0.3351	0.3183	0.2996	0.2292	0.1602	0.1040
8.00E-01	0.3106	0.3106	0.3106	0.3106	0.3106	0.3104	0.3102	0.3092	0.3050	0.2953	0.2815	0.2687	0.2543	0.1994	0.1436	0.0961
9.00E-01	0.2602	0.2602	0.2602	0.2602	0.2602	0.2601	0.2599	0.2591	0.2559	0.2485	0.2378	0.2280	0.2168	0.1734	0.1281	0.0881
1.00E+00	0.2194	0.2194	0.2194	0.2194	0.2194	0.2193	0.2191	0.2186	0.2161	0.2103	0.2020	0.1943	0.1855	0.1509	0.1139	0.0803
2.00E+00	0.0489	0.0489	0.0489	0.0489	0.0489	0.0489	0.0489	0.0488	0.0483	0.0477	0.0467	0.0456	0.0444	0.0394	0.0335	0.0271
3.00E+00	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130	0.0128	0.0126	0.0124	0.0122	0.0112	0.0100	0.0086
4.00E+00	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0038	0.0037	0.0037	0.0036	0.0036	0.0034	0.0031	0.0027
5.00E+00	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0011	0.0010	0.0010	0.0009
6.00E+00	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003
7.00E+00	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

Actual Pumping Test Data (from Fetter, pp 271)			Shifted Test Values	
t	s		t*	s*
1	0.9		1.0000	0.9000
2	2.15		0.5000	2.1500
3	3.05		0.3333	3.0500
4	3.64		0.2500	3.6400
5	4.07		0.2000	4.0700
6	4.52		0.1667	4.5200
7	4.74		0.1429	4.7400
8	5.02		0.1250	5.0200
9	5.21		0.1111	5.2100
10	5.53		0.1000	5.5300
15	5.72		0.0667	5.7200
20	5.97		0.0500	5.9700
30	6.12		0.0333	6.1200
40	6.2		0.0250	6.2000
50	6.25		0.0200	6.2500
60	6.27		0.0167	6.2700
90	6.29		0.0111	6.2900
120	6.29		0.0083	6.2900
Q	560	gpm		
	74.866	ft^3/min		
r	82	ft		
b'	9	ft		



(1) Select convenient u,W(u)** on type curve (shown as (u**,W**)).**

	0.1	<- u**
	10	<- W**

(2) Shift data point curve relative to type curve

	1	<- Shift-h	(Left-Right Shifting)
	1.0	<- Shift-v	(Up-Down Shifting)

(3) Adjust (t,s) to move match point (on data curve - shown as (t,s**))**

	1	<-t**	1.00	<- computed u*** (should agree with u** in step 1)
	1	<-s**	1.00	<-computed W*** (should agree with W** in step 1)

(4) Read (r/B) from type-curve

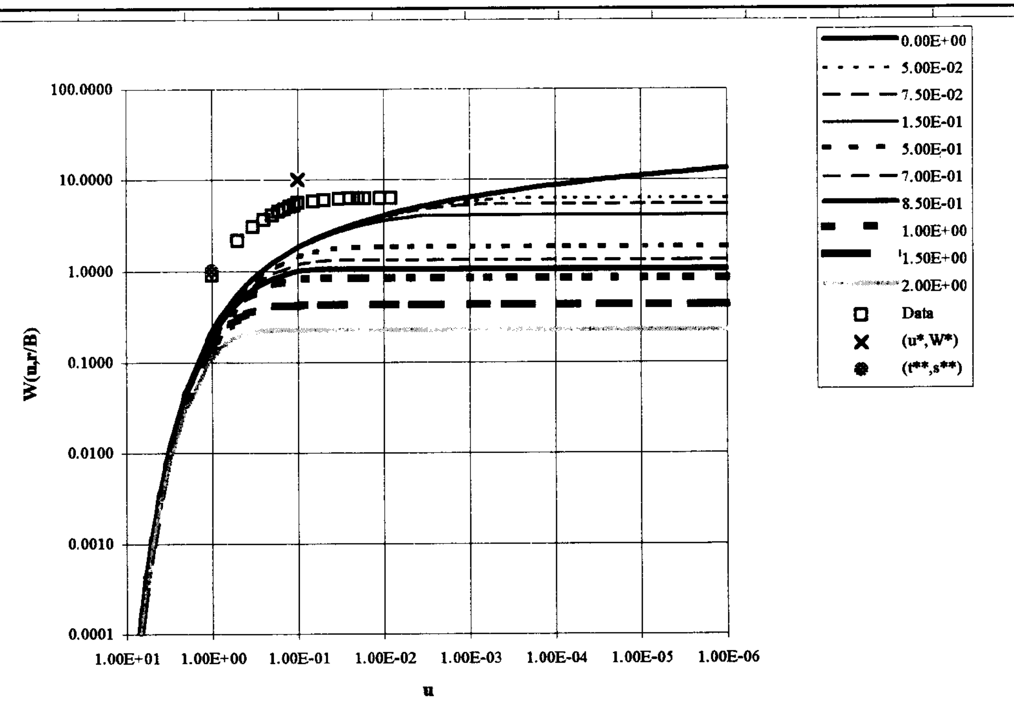
	0.8	<-r/B

(5) Compute Formation Constants

T=	59.5767	ft^2/min	85790	ft^2/day
S=	0.0035			
K'=	0.0510	ft^2/min	73.5	ft/day

Fig. 1. Leaky type-curve analysis spreadsheet. Upper portion is data from tabulated function values. Lower portion is field data and plots of both data and type-curve tabulations on the same scales.

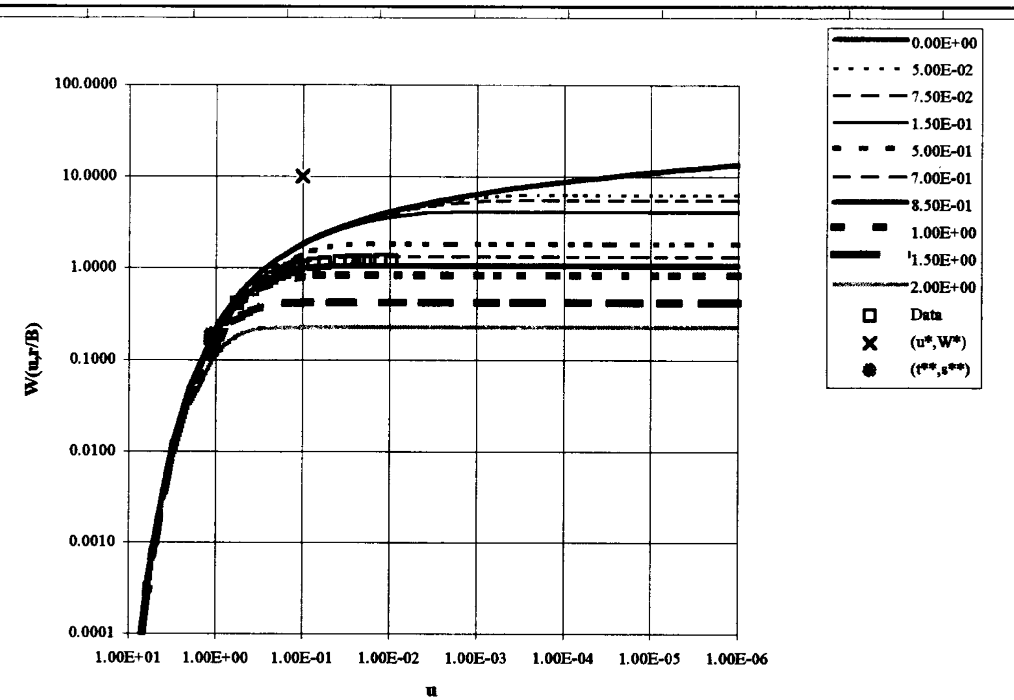
Actual			Shifted	
Pumping Test Data			Test Values	
(from Fetter, pp 271)				
t	s		t*	s*
1	0.9		1.0000	0.9000
2	2.15		0.5000	2.1500
3	3.05		0.3333	3.0500
4	3.64		0.2500	3.6400
5	4.07		0.2000	4.0700
6	4.52		0.1667	4.5200
7	4.74		0.1429	4.7400
8	5.02		0.1250	5.0200
9	5.21		0.1111	5.2100
10	5.53		0.1000	5.5300
15	5.72		0.0667	5.7200
20	5.97		0.0500	5.9700
30	6.12		0.0333	6.1200
40	6.2		0.0250	6.2000
50	6.25		0.0200	6.2500
60	6.27		0.0167	6.2700
90	6.29		0.0111	6.2900
120	6.29		0.0083	6.2900
Q	560	gpm		
	74.866	ft^3/min		
r	82	ft		
b'	9	ft		



(1) Select convienent u**,W(u)** on type curve (shown as (u**,W**)).				
	0.1	<- u**		
	10	<- W**		
(2) Shift data point curve relative to type curve				
	1	<- Shift-h	(Left-Right Shifting)	
	1.0	<- Shift-v	(Up-Down Shifting)	
(3) Adjust (t,s) to move match point (on data curve - shown as (t**,s**))				
	1	<-t**	1.00	<- computed u** (should agree with u** in step 1)
	1	<-s**	1.00	<-computed W** (should agree with W** in step 1)
(4) Read (r/B) from type-curve				
	0.8	<-r/B		
(5) Compute Formation Constants				
T=	59.5767	ft^2/min	85790	ft^2/day
S=	0.0035			
K'=	0.0510	ft^2/min	73.5	ft/day

Fig. 2. Leaky type-curve analysis spreadsheet; lower portion. Table shows field data and plot of data and type curves on same scale. Data are displaced relative to type-curve sets. Shift parameters are set to (1, 1); t, s** are set to (1, 1); u**, W** are set to (0.1, 10); r/B is set to 0.8.**

Actual			Shifted	
Pumping Test Data			Test Values	
(from Fetter, pp 271)				
t	s		t*	s*
1	0.9		1.1300	0.1710
2	2.15		0.5650	0.4085
3	3.05		0.3767	0.5795
4	3.64		0.2825	0.6916
5	4.07		0.2260	0.7733
6	4.52		0.1883	0.8588
7	4.74		0.1614	0.9006
8	5.02		0.1413	0.9538
9	5.21		0.1256	0.9899
10	5.53		0.1130	1.0507
15	5.72		0.0753	1.0868
20	5.97		0.0565	1.1343
30	6.12		0.0377	1.1628
40	6.2		0.0283	1.1780
50	6.25		0.0226	1.1875
60	6.27		0.0188	1.1913
90	6.29		0.0126	1.1951
120	6.29		0.0094	1.1951
Q	560	gpm		
	74.866	ft^3/min		
r	82	ft		
b'	9	ft		



(1) Select convienent u,W(u)** on type curve (shown as (u**,W**)).**				
	0.1	<- u**		
	10	<- W**		
(2) Shift data point curve relative to type curve				
	1.13	<- Shift-h	(Left-Right Shifting)	
	0.2	<- Shift-v	(Up-Down Shifting)	
(3) Adjust (t,s) to move match point (on data curve - shown as (t,s**))**				
	1	<-t**	1.13	<- computed u** (should agree with u** in step 1)
	1	<-s**	0.19	<-computed W** (should agree with W** in step 1)
(4) Read (r/B) from type-curve				
	0.8	<-r/B		
(5) Compute Formation Constants				
T=	59.5767	ft^2/min	85790	ft^2/day
S=	0.0035			
K'=	0.0510	ft^2/min	73.5	ft/day

Fig. 3. Leaky type-curve analysis spreadsheet; lower portion; data fitted to curve set. Table shows field data and plot of data and type curves on same scale. Data are "fitted" to type-curve sets. Shift parameters are determined by trial-and-error. Increasing vertical shift parameter moves data upward relative to type curves. Increasing horizontal shift parameter moves data to left relative to type-curve sets. Shift parameters set to (1.13, 0.2); other adjustable values unchanged from Figure 2.

reflect the needs of a particular problem. Once the (u**, W**) target is set, the analyst can then change values (t**, s**) in the spreadsheet. The spreadsheet automatically computes an adjusted data pair location on the (u, W) scale and plots (t**, s**) using another special symbol (shown as a dot on all the figures). These changes move the (t**, s**) match point relative to the (u**, W**) target point, while preserving the previously obtained axes shifts. Figure 4 shows the spreadsheet when the match point has been selected. Comparing Figure 3 to Figure 4 the values of (t**, s**) have been changed from the arbitrary values of (1, 1) to the match values of (11.3, 50). These values move the (t**, s**) match point so that it overlies the (u**, W**) match point.

Actually, the match point is uniquely determined once the analyst chooses (u**, W**), and the values (t**, s**) can be computed from:

$$t^{**} = shift_h / u^{**} \tag{7}$$

$$s^{**} = W^{**} / shift_v \tag{8}$$

For illustrative purposes, or teaching, it is worthwhile to simply find the (t**, s**) match by trial-and-error to maintain the analogy between the spreadsheet and manual method. Additionally, this particular relationship [equations (7) and (8)] may not necessarily hold for other type curves.

Figure 4 shows the result of matching these points. The arbitrary values for (u**, W**) were chosen as (0.1, 10). The matching values (t**, s**) from the data plots are (11.3, 50). Once the (t**, s**) pair overlies the target point, the user has identified the solution to the equation pair below:

$$\ln[s^{**}] = \ln\left[\frac{Q}{4\pi T}\right] + \ln\left[W^{**}\left(u, \frac{r}{B}\right)\right] \tag{9}$$

$$\ln[t^{**}] = \ln\left[\frac{r^2 S}{4T}\right] + \ln\left[\frac{1}{u^{**}}\right] \tag{10}$$

The user must read from the plot which value of r/B is appropriate and record this value prior to computing the aquitard hydraulic conductivity. The unknown aquifer parameters are then computed from the equation pair using:

$$T = \frac{Q}{4\pi s^{**}} W^{**}\left[u, \frac{r}{B}\right] \tag{11}$$

$$S = \frac{4Tu^{**}t^{**}}{r^2} \tag{12}$$

$$K' = \frac{Tb'(r/B)^2}{r^2} \tag{13}$$

Alternatively, the unknown parameters for the leaky type-curve example can be computed from:

$$T = \frac{Q shift_v}{4\pi} \tag{14}$$

$$S = \frac{4T shift_h}{r^2} \tag{15}$$

$$K' = \frac{Tb'(r/B)^2}{r^2} \tag{16}$$

By integrating equations (14), (15), and (16) into the spreadsheet, the parameter computations and solutions can be automatically included in the spreadsheet. These computations are included in the spreadsheet examples available from the author.

The result for the particular data shown is T = 1629 sq ft/day; S = 0.0008; K′ = 1.4 ft/day. The spreadsheet produces values containing more significant figures than usually justified by the test data, and the analyst will need to be aware of this fact and report the fitted parameters using an appropriate number of significant figures. In the figures various different numerical formats were used in the spreadsheet, and these can be adjusted as desired by the user. For instance, the user can specify that the drawdown data be recorded with four places after the decimal point as is often recorded by a datalogger.

Additional Results

The same spreadsheet above was used to determine the parameters for time-drawdown data taken from two examples in Fetter (1994). The first example is simply a different data set for a leaky aquifer; the second example is a data set for a confined aquifer. To perform a spreadsheet match for a confined aquifer using the Theis solution, the analyst can either create a new spreadsheet with the Theis type curve, or use the leaky type-curve spreadsheet above and match the r/B = 0 curve. Figure 5 shows the results of the Theis case. The type-curve set has been replotted so that only the r/B = 0 curve is shown.

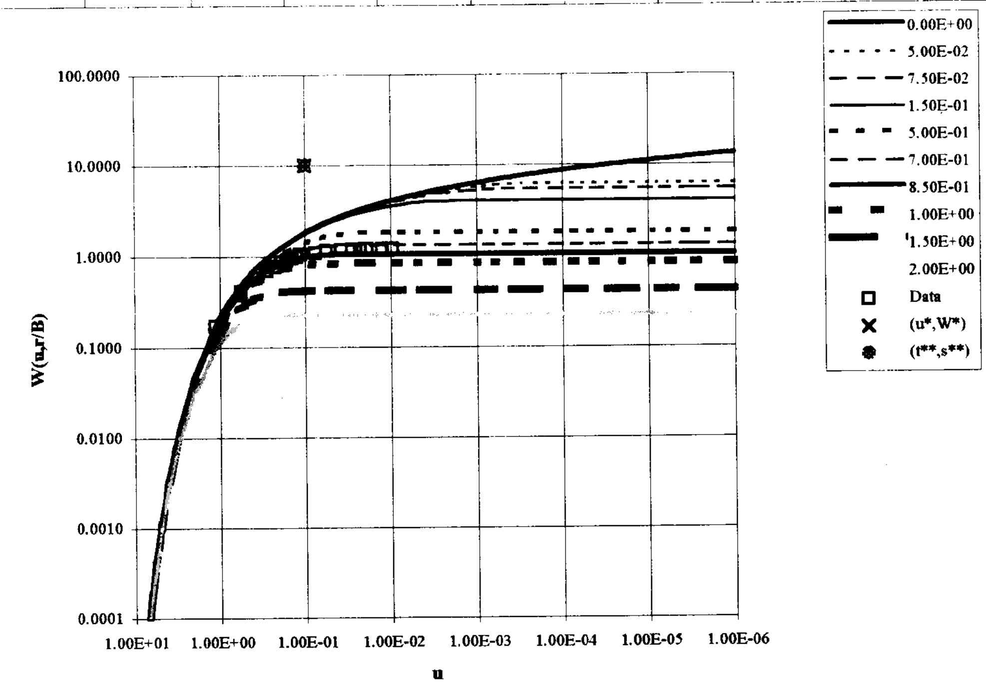
The table portion of the spreadsheet was unchanged; the time-drawdown data were simply entered into the same area of the spreadsheet as were the previous data. Because the data plot range will change, the user must either replot the data curve, or insert/delete rows in the spreadsheet before entering the new data. It is usually easier to insert or delete rows in the middle of the data range, and let the spreadsheet adjust the plots. If one accidentally deletes the first or last row of the data range it will disconnect the link between the data and the plot, and the user must replot the data range.

Table 1. Results of Spreadsheet Matching on Two Datasets

Data reference	*Reported values*	*Spreadsheet values*	*Remarks*
pg. 231, Fetter (1994)	$T = 200\ ft^2/day$ S = 0.00020 r/B = 0.22 K′ = 0.015 ft/day	$T = 199\ ft^2/day$ S = 0.00020 r/B = 0.22 K′ = 0.015 ft/day	Hantush model; u**, W** = 0.1, 10 $Shift_h = 3.3$ $Shift_v = 0.52$
pg. 223, Fetter (1994)	$T = 1400\ ft^2/day$ S = 0.000024	$T = 1393\ ft^2/day$ S = 0.0000227 r/B = 0 Spreadsheet gives Theis solution when data are matched to r/B = 0 curve.	Theis model; u**, W** = 0.1, 10 $Shift_v = 0.404$

Actual		Shifted	
Pumping Test Data		Test Values	
(from Fetter, pp 271)			
t	s	t*	s*
1	0.9	1.1300	0.1710
2	2.15	0.5650	0.4085
3	3.05	0.3767	0.5795
4	3.64	0.2825	0.6916
5	4.07	0.2260	0.7733
6	4.52	0.1883	0.8588
7	4.74	0.1614	0.9006
8	5.02	0.1413	0.9538
9	5.21	0.1256	0.9899
10	5.53	0.1130	1.0507
15	5.72	0.0753	1.0868
20	5.97	0.0565	1.1343
30	6.12	0.0377	1.1628
40	6.2	0.0283	1.1780
50	6.25	0.0226	1.1875
60	6.27	0.0188	1.1913
90	6.29	0.0126	1.1951
120	6.29	0.0094	1.1951

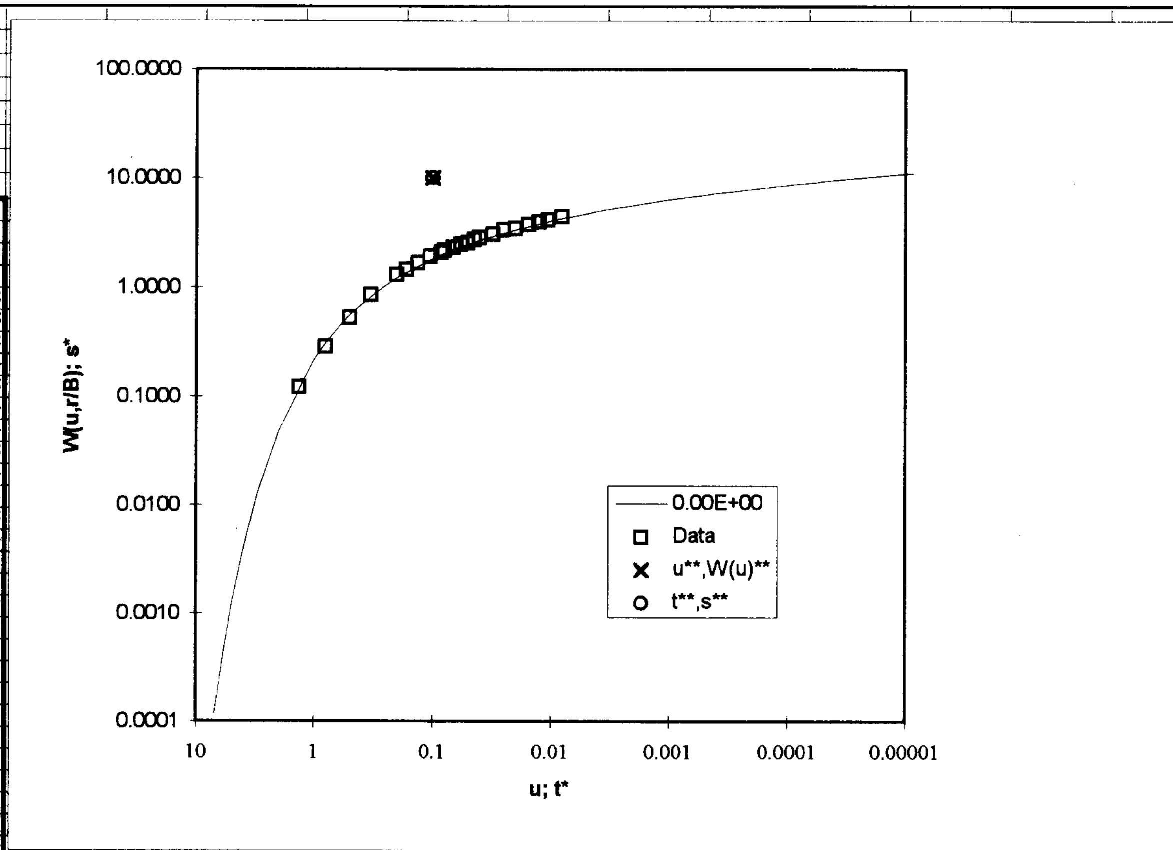
Q	560	gpm
	74.866	ft^3/min
r	82	ft
b'	9	ft



(1) Select convienent u,W(u)** on type curve (shown as (u**,W**)).**			
0.1	<- u**		
10	<- W**		
(2) Shift data point curve relative to type curve			
1.13	<- Shift-h	(Left-Right Shifting)	
0.2	<- Shift-v	(Up-Down Shifting)	
(3) Adjust (t,s) to move match point (on data curve - shown as (t,s**))**			
11.3	<-t**	0.10	<- computed u** (should agree with u** in step 1)
52.65	<-s**	10.00	<-computed W** (should agree with W** in step 1)
(4) Read (r/B) from type-curve			
0.8	<-r/B		

(5) Compute Formation Constants				
T=	1.1316	ft^2/min	1629	ft^2/day
S=	0.0008			
K'=	0.0010	ft^2/min	1.4	ft/day

Fig. 4. Leaky type-curve analysis spreadsheet; lower portion; data fitted; match point identified. Shift parameters set to (1.13, 0.2); t, s** set to (11.3, 52.65); r/B set to 0.8; u**, W** set to (0.1, 10). Parameters T, S, and K′ are computed automatically from equations (11), (12), and (13).**



Actual		Shifted	
Pumping Test Data		Test Values	
(from Fetter, pp 223)			
t	s	t*	s*
3	0.3	1.3333	0.1212
5	0.7	0.8000	0.2828
8	1.3	0.5000	0.5252
12	2.1	0.3333	0.8484
20	3.2	0.2000	1.2928
24	3.6	0.1667	1.4544
30	4.1	0.1333	1.6564
38	4.7	0.1053	1.8988
47	5.1	0.0851	2.0604
50	5.3	0.0800	2.1412
60	5.7	0.0667	2.3028
70	6.1	0.0571	2.4644
80	6.3	0.0500	2.5452
90	6.7	0.0444	2.7068
100	7	0.0400	2.8280
130	7.5	0.0308	3.0300
160	8.3	0.0250	3.3532
200	8.5	0.0200	3.4340
260	9.2	0.0154	3.7168
320	9.7	0.0125	3.9188
380	10.2	0.0105	4.1208
500	10.9	0.0080	4.4036

Q	225	gpm
	30.080	ft^3/min
r	824	ft
b'	0	ft

(1) Select convienent u**,W(u)** on type curve (shown as (u**,W**)).

	0.1	<- u**	
	10	<- W**	

(2) Shift data point curve relative to type curve

	4	<- Shift-h	(Left-Right Shifting)
	0.404	<- Shift-v	(Up-Down Shifting)

(3) Adjust (t,s) to move match point (on data curve - shown as (t**,s**))

	40	<-t**	0.10	<- computed u** (should agree with u** in step 1)
	24.75	<-s**	10.00	<-computed W** (should agree with W** in step 1)

(4) Read (r/B) from type-curve

	0	<-r/B

(5) Compute Formation Constants

T=	0.9672	ft^2/min	1393	ft^2/day
S=	2.27909E-05			
K'=	0.0000	ft^2/min	0.0	ft/day

Fig. 5. Confined aquifer analysis using leaky type-curve analysis spreadsheet. Time-drawdown data from Fetter (1994), p. 223. Only $r/B = 0$ type curve is plotted for Theis case. Shift parameters set to (4, 0.404); t^{**}, s^{**} set to (40, 24.75); r/B set to 0; u^{**}, W^{**} set to (0.1, 10). Parameters T, S, and K′ are computed automatically from equations (11), (12), and (13).

Table 1 shows the results of the spreadsheet approach with the values reported in Fetter for the two different data sets. The table shows that the spreadsheet matching procedure produces comparable results to the published results of manual plotting and fitting. The reader should be able to reproduce the results in the table using the listed match point values in the remarks column, and the referenced data sets.

Conclusion

This note shows how to construct and perform type-curve matching using the built-in graphics on a computer spreadsheet. The type-curve data are entered as a table, and plotted. The test data are entered as another table, and adjusted values are plotted using axis shifting parameters. Once an acceptable fit is established, the analyst chooses a convenient target point on the type curve and then adjusts an artificial match point on the test data curve to identify a Theis-type match point for subsequent computations. The estimated parameters using this approach agree with published estimates using the same data. The spreadsheet method preserves the nature of the graphical method, but removes the tedium of plotting data and drawing type curves.

The method illustrated is general enough to use for other type curves. Macro programming is not required, only data entry and the ability to use plotting features of a computer spreadsheet. Specialized software, other than a computer spreadsheet, is not required. An analyst can construct type-curve matching spreadsheets for any tabulated well function, in some cases eliminating the need for specialized software. The analyst should carefully check and plot all tabulated well function values to avoid errors, before relying on a newly developed spreadsheet.

Copies of the spreadsheets used in this note are available for free from the author by sending a formatted diskette and a self-addressed, stamped disk mailer. The spreadsheet shown was created in Excel, but can also be accomplished using Lotus 1-2-3. The author can provide either Excel 5.0 (Macintosh or PC) or Lotus 1-2-3 Version 4 (PC only) copies. The Lotus spreadsheet does not plot reverse type curves, so the curves will appear to be "flipped" in the horizontal plane.

— email author or download spreadsheet from www.egr.uh.edu/cleveland

email: cleveland@uh.edu

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