

Streamflow from Rainfall by Unit-Graph Method

Observed runoff following isolated one-day rainfall forms basis of computation—Method applicable to rainfalls of any intensity or duration

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BY MAKING USE of a single observed hydrograph, one due to a storm lasting one day, it is possible to compute for the same watershed the runoff history corresponding to a rainfall of any duration or degree of intensity. From the known hydrograph the "unit" graph must be determined, representing 1 in. of runoff from a 24-hour rainfall. The daily ordinates of the unit graph can then be combined in accordance with the variation in daily precipitation figures so as to show the runoff from a storm of any length.

Following a storm, the hydrograph representing the flow in the main-stream channel shows the runoff increasing to a maximum point and then subsiding to the value it had before the storm. For a single storm the graph is generally of a triangular shape, with the falling stage taking never less and usually two or more times as long as the rising stage. For the same drainage area, however, there is a definite total flood period corresponding to a given rainfall, and all one-day rainfalls, regardless of intensity, will give the same length of base of the hydrograph.

If a given one-day rainfall produces a 1-in. depth of runoff over the given drainage area, the hydrograph showing the rates at which the runoff occurred can be considered a unit graph for that watershed. As the area under a hydrograph for any time period represents the total volume of runoff in that time, it follows that the area under the unit graph, if expressed in inches of depth over one square mile, or inch-miles, is numerically equal to the area of the watershed.

Determining the unit graph

Based on these laws it follows that for any observed rainfall, during a unit of time, the ordinates of runoff vary directly with the depth of runoff on the drainage area. For example, let there be an observed hydrograph of runoff due to rainfall of 3 in. during a unit of time of one day, over a drainage area of 1,000 square miles. Measurement of the area of this hydrograph shows 2,000 inch-miles of runoff. The volume of rainfall, however, is 3,000 inch-miles. Therefore, the runoff is $66\frac{2}{3}$ per cent,

or 2 in., and the observed graph represents a 2-in. runoff applied in 24 hours. The unit graph for this area, then, is one having the same base but ordinates one-half as great as those on the observed graph. This is the procedure for determining a unit graph for any drainage area. The graph is a constant for any particular drainage area, but drainage areas of different physical characteristics give radically different forms.

A topography with steep slopes and few pondage pockets gives a graph with a high sharp peak and a short time period. A flat country with large pondage pockets gives a graph with a flat rounded peak and a long time period.

Application of unit graph

After a unit graph has been constructed for a particular area it may be used to compute a hydrograph of runoff for this area for any individual storm or sequence of storms of any duration or intensity over any period of time. The principle to use in applying the unit graph is to follow the summation process of nature. For example, consider a case where the unit graph is known and data are at hand for the rainfall for two consecutive days. Estimate the percentages of runoff for each of these days. This will give a hydrograph for each of these days of rain. Call them graphs *A* and *B*. The runoff on the first day equals the runoff of the first day of graph *A*. The runoff on the second day equals the runoff on the second day of graph *A* plus the runoff of the first day of graph *B*. The runoff on the third day equals the runoff on the third day of graph *A* plus the runoff of the second day of graph *B*. If the unit graph has a total period of T_0 days, the total flood period for this two-day storm will be $T_0 + 1$ days. For a six-day storm it will be $T_0 + 5$, etc.

The foregoing process of development of a graph for a continuous rainfall at a uniform rate is illustrated in Fig. 1. The triangle *OPQ* with base $T_0 = 6$ is the graph of runoff due to a rain during a unit of time—say one day. That rain produces runoff for six days. The average rate of runoff for each day is represented by the ordinates *a*, *b*, *c*, *d*, *e* and *f* in the triangle *OPQ*. The same amount of runoff due to a rain on the second day produces the graph indicated by the first dotted line above

OPQ. A continued rain with the same daily depth of runoff produces successively the additional dotted graphs. At the end of the fifth day of such continuous rain, with uniform depths of runoff for each day, the runoff graph *ORS* will be formed. The peak at *R* will be the maximum rate of runoff. Further

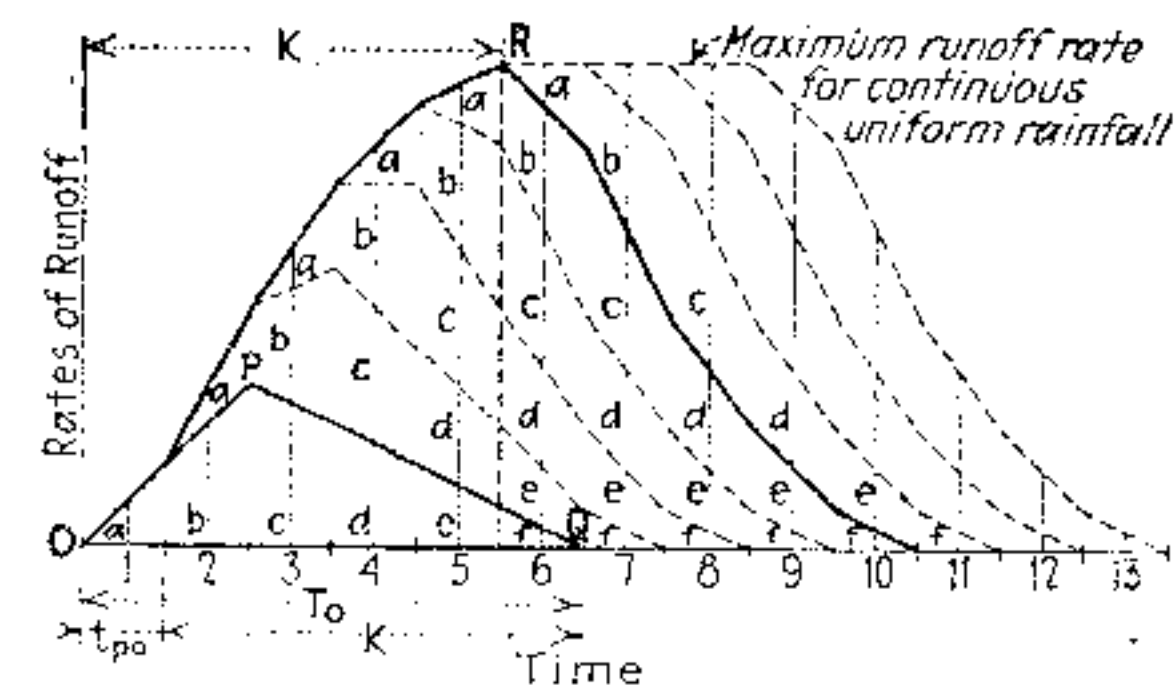


Fig. 1—Simple hydrograph of runoff from a continuous uniform rain, when the unit graph is triangular.

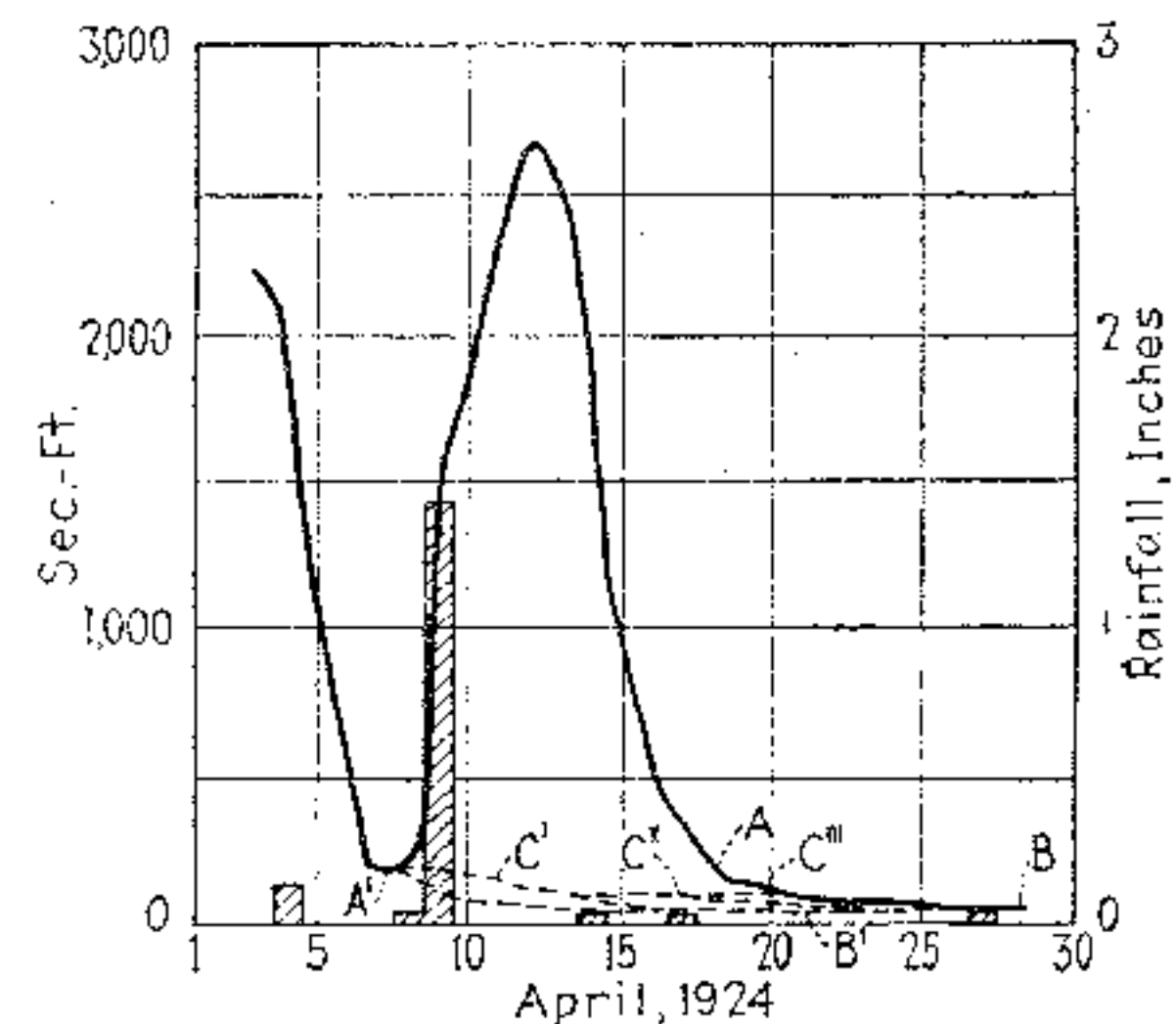


Fig. 2—At Plumfield, Ill., on the Big Muddy River, there was a fairly well-isolated rain of 1.42 in. on April 9, 1924, yielding a hydrograph with ordinates proportional to those of the unit graph.

similar rainfall will result in a continuation of this maximum runoff rate up to the duration of this storm.

Let T_0 = Base of a runoff graph for a rain of duration t_{pu} . This can be derived from an observed hydrograph.

t_{pu} = Unit time of precipitation (one day or one hour).

t_p = Duration of any rainfall.

K = Concentration period.

$K = T_0 - t_{pu}$.

Let T = Total flood period or base of the graph for any rainfall of duration t_p .

Then $T = K + t_p$.

When $t_p = K$, then $T = 2K$, provided the accumulated depth of runoff (depth of rain — losses) is a constant during each hour of t_p .

When the duration of rainfall, t_p , is less than the concentration period K , then T is always longer than the calculated time of transit from the most

TABLE I — COMPUTATION OF UNIT GRAPH FOR BIG MUDDY RIVER, PLUMFIELD, ILL.

Date, April, 1924	Observed Runoff, Sec.-Ft.	Deduction Base Flow, Sec.-Ft.	Net Runoff, Sec.-Ft.	Unit Graph, Sec.-Ft.	Day
9	1,440	150	1,290	1,950	1
10	1,850	140	1,710	2,590	2
11	2,360	130	2,230	3,370	3
12	2,680	120	2,560	3,870	4
13	2,440	100	2,340	3,540	5
14	1,720	90	1,630	2,470	6
15	940	80	860	1,310	7
16	478	80	398	610	8
17	309	80	229	350	9
18	193	90	103	160	10
19	140	90	50	80	11
20	106	106	0
Total, sec.-ft.-days.....			13,400	20,300	
Total, inch-miles.....			500	753	

remote point of the drainage area, generally by 50 to 100 per cent. This fact is due to pondage by the innumerable little detention reservoirs that cover the surface as well as to the effect of channel constrictions in the main stream. In some flat drainage areas with a great deal of pondage the concentration period may be much longer than the possible duration of storms of high intensity. Pondage holds back part of the rainfall until the accumulated depth balances the net rainfall rate and releases it after the rain stops. The inflow and outflow from these detention reservoirs at the end of the concentration period are equal, and there is no further pondage available. Surface pondage must not be confused with surface pocket storage due to sink holes. The latter collect rainfall at the beginning of the storm but do not release it as surface runoff.

Hydrograph for varying rates of rainfall

The ordinates of all graphs of runoff for unit time, t_{po} , are directly proportional to the net depths s of rainfall in that same unit time.

This follows because the bases of the graphs T_o are equal. The area of a graph₁ equals Σ ordinates₁ \times t_{po} , and the area of a graph₂ equals Σ ordinates₂ \times t_{po} . Also, the area of graph₁, as it is a measure of the entire volume of runoff, equals the drainage area \times s_1 and the area of graph₂ equals the drainage area \times s_2 .

$$\frac{\Sigma \text{ ordinates}_1 \times t_{po}}{\Sigma \text{ ordinates}_2 \times t_{po}} = \frac{D.A. \times s_1}{D.A. \times s_2}$$

$$\text{or } \frac{\text{ordinates}_1}{\text{ordinates}_2} = \frac{s_1}{s_2}$$

If the ordinates considered are those for a unit graph, $s_2 = 1$ in., and we have the following rule:

The ordinates for any graph of runoff for a unit time t_{po} are equal to the corresponding ordinates of the unit graph multiplied by the given net depth of rainfall. Conversely, the ordinates of a unit graph are each equal to the corresponding ordinates of a given graph (for unit time) divided by the given net depth of rainfall.

In Fig. 1, s was constant day after day. If s varied from day to day, we

would have on successive days s_1, s_2, s_3 , etc., and the ordinates on successive days would be $a_1, b_1, c_1; a_2, b_2, c_2; a_3, b_3, c_3$; etc.

A resultant graph would be formed in the same way as the one in Fig. 1, but it would not be a smooth curve. It would reflect all the variations of rainfall.

Instead of the graphical construction, as in Fig. 1, a tabulation of successive ordinate figures can be used to give the values for the resulting hydrograph. This method of computing a hydrograph for any period or sequence of rainfalls from a single unit graph will be illustrated later by examples of specific cases.

Base flow

Base flow, on a hydrograph covering an extended period of time, is indicated by the flow line that continues during a dry period long after the total flood period of the preceding rain has ceased. The point where the total flood period due to surface runoff ceases and only groundwater or base flow continues is not capable of exact determination. However, it can be closely approximated by inspection of surface-runoff hydrographs terminating a dry period. The base flow exists and is a part of almost any observed hydrograph of runoff. In evolving a unit graph by the procedure heretofore mentioned it is necessary first to subtract the base flow from the observed hydrograph in order to arrive at the net hydrograph due solely to the rainfall. The base flow can be estimated by consulting the available flow records made during dry periods and selecting a base-flow line that followed a rainfall period similar to the period preceding the graph in question. It is preferable that the unit graph should be derived from a graph due to a rain of high intensity. The base flow then is a small percentage of the surface runoff, and any error due to estimated base flow is slight.

Unit graph constructed for the Big Muddy River

The drainage area of the Big Muddy River at Plumfield, Ill., is 753 square miles. Daily rainfall records of the U. S. Weather Bureau are available at Mount Vernon, in the center of the upper portion, and at Benton, near the center of the lower portion of the drainage area. The records of the Mount Vernon station apply to 43 per cent of the area, and the Benton records apply to 57 per cent. The average daily flow of the Big Muddy at Plumfield is contained in the water-supply papers of the U. S. Geological Survey.

The derivation of a unit graph is obtained preferably from an observed hydrograph due to a single isolated 24-hour rainfall of high intensity, without any material rainfall either during the runoff period or just preceding it.

A search of the records shows a fair

example of such a rainfall on April 9, 1924. The record of daily precipitation at this time was as follows:

Date — April, 1924	4	8	9	14	16	17	27
Mt. Vernon, in.....	0.16	0.94	0.08	0.04	0.05
Benton, in....	0.09	0.05	1.80	0.01	0.02	0.08
Weighted average, in....	0.12	0.03	1.42	0.03	0.03	0.06

Fig. 2 shows the runoff during this period and the average daily rainfalls. In Table I, column 3 lists the runoff due to rains prior and subsequent to the rain of April 9, which must be deducted from the daily runoff figures to give the desired net runoff due solely to the rain of April 9.

The values to be deducted are derived in the following manner: In Fig. 2, starting at the low stage of April 7, draw the line $A' B'$ with daily ordinates of flow equal to those on the line $A B$. $A' B'$ then represents the runoff and base flow as it tapers off from stage A' or A , provided no subsequent rain occurs. There were, however, three small rains of 0.03 in. each, on April 8, 14 and 17. The runoff from these small rains was appreciable in this month, as shown by the runoff record of 193 sec.-ft. on April 7 and 226 sec.-ft. on April 8. This increase of 33 sec.-ft. was due to the 0.03-in. rain on April 8. By reference to the main graph (Fig. 2) it is found that for this drainage area the peak comes on the fourth day, and runoff ceases about the tenth day after the end of the rain. The net peak flow in Fig. 2 is about 2,500 sec.-ft. for a rain of 1.42 in. By proportion the peak flow for a rain of 0.03 in. would be 53 sec.-ft. The smaller rain, however, has a smaller runoff. Call it a peak of 40 sec.-ft. The runoff of the 0.03-in. rain then may be represented by a triangular graph with base of ten days and height of 40 sec.-ft. on the fourth day. Now add successively, in Fig. 2, three such small graphs to the ordinates of the line $A' B'$ with their peaks as shown at C', C'' and C''' . The ordinates to the line $A' C' C'' C'''$ now represent the deductions, and numerically they form column 3 of Table I. Column 2 minus column 3 gives the net runoff due solely to the rain of April 9.

The next step is to determine the figures for a unit graph for this drainage area. It will have the same time base as the graph represented by column 4. The ordinates, however, will be in proportion to the ordinates of column 4 as 1 in. is to the runoff depth due to the rain of 1.42 in. This runoff depth is found as follows: The sum of the average daily runoffs in column 4 is 13,400 sec.-ft.-days. This total volume of runoff equals 500 inch-miles. The total volume of rainfall was 1.42 in. \times 753 square miles of drainage area, or 1,070 inch-miles. Hence:

$$\text{Percentage of runoff} = \frac{500}{1,070} = 46.8 \text{ per cent.}$$

Fig. 4 shows the computed hydrograph of flow of the Delaware River at Port Jervis, N. Y., for August, 1928. This is an example of runoff from topography with relatively steep slopes. The drainage area is 3,070 square miles. The unit graph for Port Jervis (Fig. 6) was derived in a manner similar to that presented in detail for the Big Muddy River, based on a 2.58-in. rainfall on Nov. 16, 1926.

The observed runoff, according to the U.S.G.S. record, was 85 per cent of the rainfall during that storm. As a check on the unit graph thus derived, another graph was computed from the average

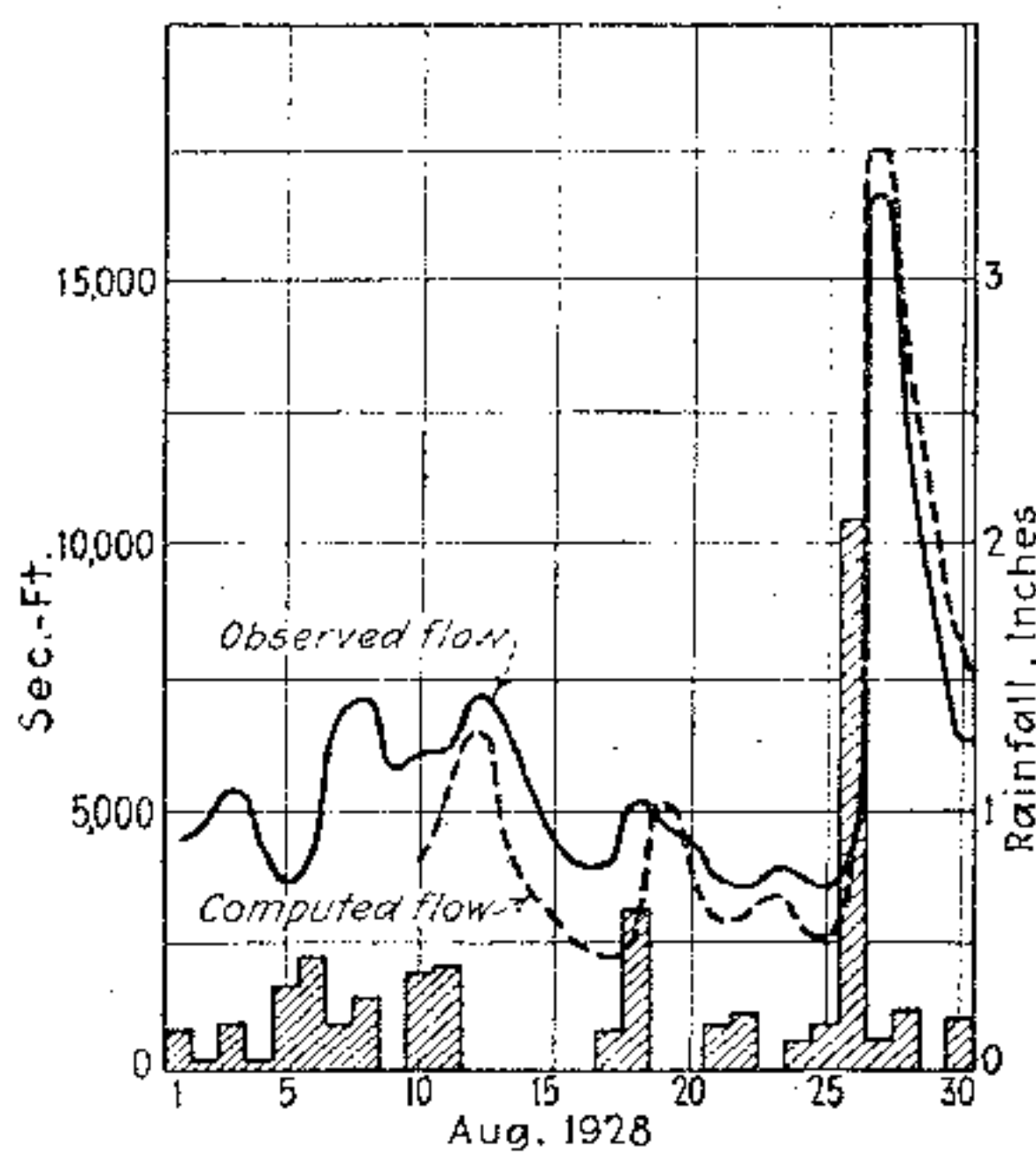


Fig. 4—Computed and observed flows of the Delaware River at Port Jervis, N. Y., reflect the quick runoff due to the steep slopes of the watershed.

rainfall of 1.79 in. on Oct. 6, 1926, when the recorded runoff was only 23.8 per cent of the rainfall. This unit graph gave a 24-hour peak rate of 23,900 sec.-ft., compared with 24,900 sec.-ft. of the former unit graph. Similar comparisons on other drainage basins indicate that this degree of accuracy may be obtained by the method, regardless of the amount of rain in a unit of time or the percentage of runoff.

Fig. 5 is a hydrograph of the observed flow on the Sangamon River at Oakford, Ill., during March, April and May, 1927, together with the hydrograph computed from an observed unit graph due to rain on March 14, 1922.

In Fig. 6 unit graphs for a number of drainage basins are shown. They reflect the effects of shape, size, topography and pondage of different basins, each with a 24-hour rainfall producing a 1-in. depth of runoff. When no streamflow records from a particular area are available, it is possible to derive a unit graph by analogy from unit graphs from similar basins with like topographical characteristics.

The ordinates and time intervals of the unit graphs for two similar watersheds of different sizes are proportional to the square root of the watershed areas, provided the difference in size is

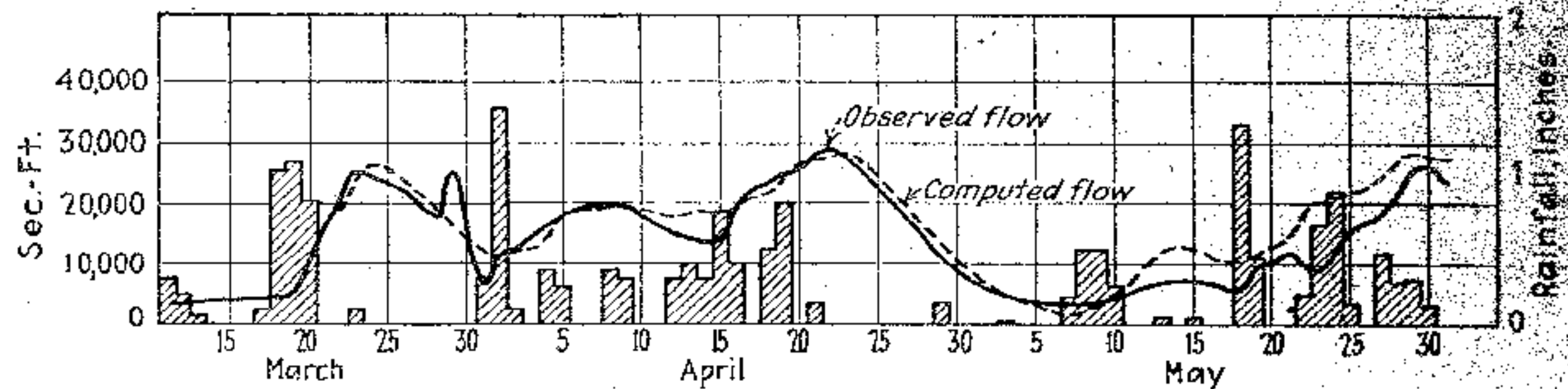


Fig. 5—Close agreement between the computed and observed hydrographs for the Sangamon River at Oakford, Ill., for a three-month period in 1927. Computed flow was based on data recorded in 1922.

not great. When the difference in areas is large, a correction factor must be applied. Referring to Fig. 6, all of these unit graphs can be reduced by analogy to unit graphs for any similar basins—say for a similar area of 1,500 square miles in each case. The peak rates of flow for two of these cases would then be as follows:

	Area, Sq. Miles	Peak Runoff, Sec.-Ft.	For Similar Area of 1,500 Sq. Miles, Sec.-Ft.
Port Jervis...	3,070	25,100	17,100
Taylorville...	510	4,900	6,500

This suggests the possibility of developing a series of unit graphs covering an extreme range of drainage-basin characteristics and sizes. Such a series of unit graphs would in effect constitute a range of runoff-rate coefficients to be used in connection with given rainfalls and the seasonal percentage of rainfall runoff.

Percentage of runoff

Runoff is not primarily a percentage of rainfall. As Robert E. Horton has pointed out, it is the residual amount after deducting losses—interception, pocket storage, evaporation and infiltration—from rainfall. The writer has, however, followed the customary method, used in discussions of runoff from sewerage areas, of expressing runoff as a percentage of the rainfall. Confining ourselves to surface runoff as compared with groundwater outflow seepage, or base flow, we find that the percentage of runoff increases with the rate and duration of precipitation. The percentage is also increased by the occurrence of previous precipitations. It varies with the season according to the temperature and amount of vegetation, the topography, soil and conditions causing pocket storage and pondage.

Percentages of runoff, considered only in relation to a single one of the foregoing factors, appear very erratic. If, however, the observations are confined to a single area or to closely similar areas and if they are segregated according to the seasons, the data will be quite consistent. If, in addition, cognizance is taken of the effect of prior precipitation, then the percentages of runoff will be in harmonious accord.

The figures for percentage of runoff

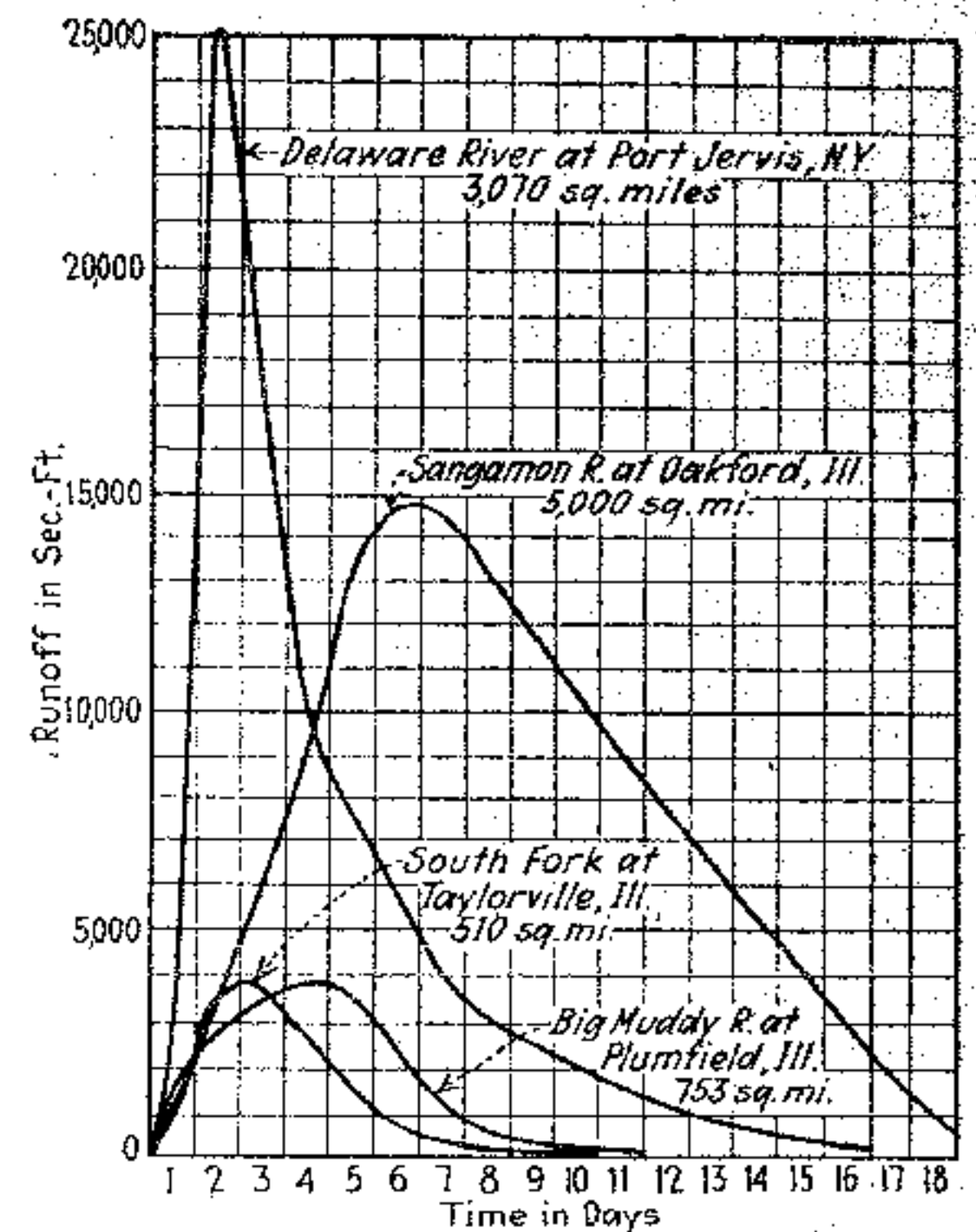


Fig. 6—Unit graphs for different watersheds reflect the variations in shape, size, topography and pondage.

used here in computing the unit graph were obtained for a given drainage area and a rainfall of known amount. With several such figures applying to the same calendar month, a curve was drawn showing runoff per cent plotted against rainfall intensity. Fig. 7 shows a series of such curves for various watersheds. Each curve is applicable to a certain month only. In preparing the diagram the effect of previous rainfall was taken into account by adding a certain proportion of it to that of the day in question, according to the factors given in Table III.

TABLE III—PROPORTION OF PREVIOUS RAINFALL TO BE ADDED IN DETERMINING PER CENT RUNOFF

Number of Intervening Dry Days	Proportion of Previous Rainfall to Be Added to Given Day
0	1.00
1	0.8
2	0.6
3	0.5
4	0.4
5	0.3
6	0.2
7	0.2
8	0.1
9	0.1
10	1/11
11	1/12

For example: Given a record of average daily rainfalls in April on the Big Muddy drainage basin as follows: April 28, 2 in.; 26, 1 in.; 24, 0.5 in.; and 10, 1 in. Required the runoff on April 28: by rule in Table III we have 2 in. +

1 in. \times 0.8 + 0.5 in. \times 0.6 + 1 in. \times $\frac{1}{10}$ = 3.16 in. Referring to diagram Fig. 7, we find that the ordinate for 3.16 in. on the Big Muddy in April is 70 per cent. The runoff from the rain of April 28 is therefore 2 in. \times 70 per cent = 1.4 in. It will be noted that the effect of rains two or three weeks prior to the rain in question have slight effect on the percentage of runoff.

The principle of the foregoing is rational, but the rule is empirical and only roughly approximate. In Table II use has been made of the foregoing in de-

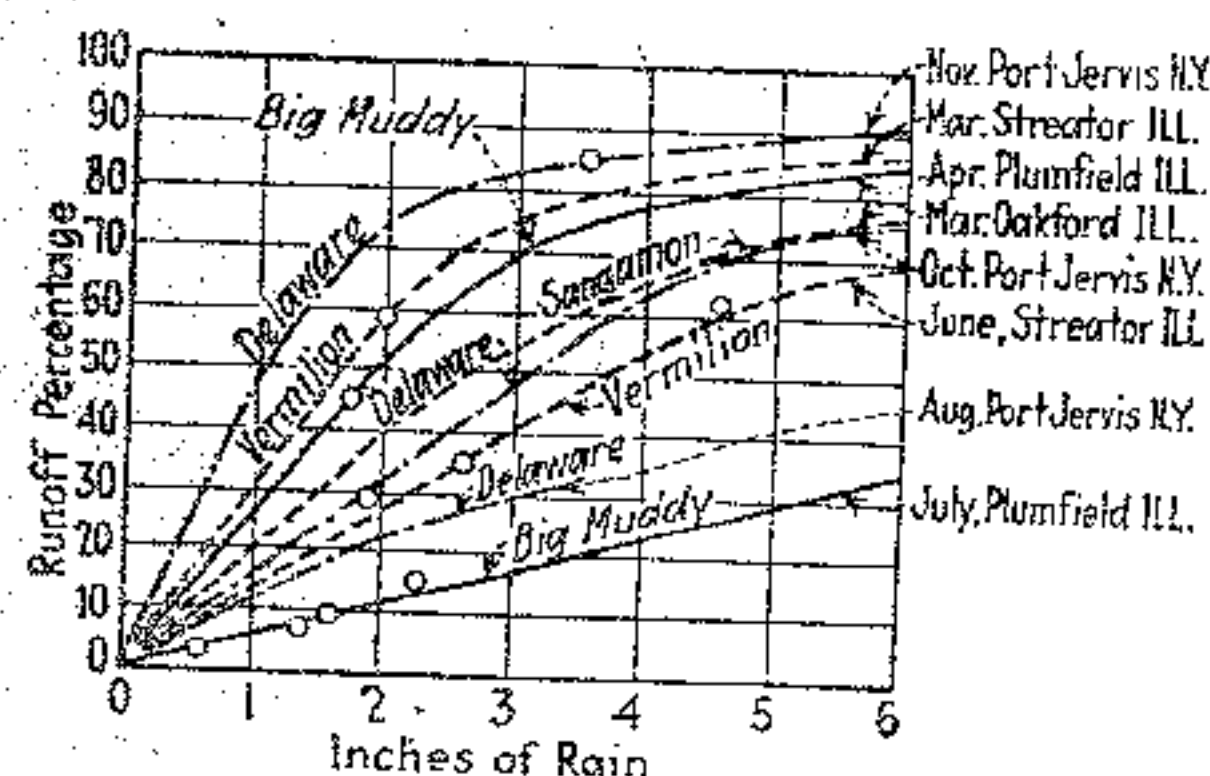


Fig. 7—For any calendar month the percentage of rainfall appearing as runoff varies consistently with the quantity of precipitation.

termining the column headed "Per Cent of Runoff," in the absence of more exact data. It agrees fairly well with the facts in most cases. To use Fig. 7, first find the summation of rainfall by the above rule. The ordinate in Fig. 7 corresponding to this summation of rainfalls gives the per cent runoff, and this should be applied to the rainfall of the given day.

Accuracy of the method

The accuracy of computed runoff is limited by the accuracy of the estimated percentage of runoff, as heretofore explained. The observed hydrograph reflects all the numerous factors influencing runoff from rainfall on a given area. The unit graph should be based upon a uniform depth of rainfall over the entire area. The unit graph for the Big Muddy River was based on a rain of 1.80 in. at the lower end and 0.94 in. at the upper end. This unit graph therefore has excessive rates of runoff for the rising stage and insufficient rates for the falling stage. This is reflected in the computed curve in Fig. 3. After a trial application like this it is possible to adjust the unit graph, and with the rectified graph to compute the runoff as in Table II, obtaining somewhat closer agreement with the observed runoffs.

The published records of rainfall by the U. S. Weather Bureau are for total precipitation in a calendar day of 24 hours. The published data of runoff by the U.S.G.S. are likewise average-flow records for this 24-hour period. This leads to the use of a 24-hour time unit as the basis for the unit graph. In general this suffices. There are, however, relatively small drainage areas with steep slopes and high velocity of flow where the 24-hour unit will not

suffice and a shorter time unit is necessary. The writer has found this to be true in the case of upper tributaries of the Delaware River like the West Branch at Hales Eddy and the East Branch at Fish Eddy, N. Y. The effect of storage by water-power dams will modify the observed flow as contrasted with the computed flow. This effect is very apparent at Hales Eddy, and to a lesser degree affects the comparison of observed and computed flow at Port Jervis, N. Y., as shown in Fig. 4.

The unit-graph method in runoff problems offers a number of improvements over existing procedures. Present applications of the rational method are confined to small areas and to assumed rainfalls during the entire period of concentration. In the unit-graph

method the runoff from any sequence of rainfalls may be analyzed. The relative effects of long-continued rains of low intensity and short storms of high intensity may be compared. The important effect of surface and channel pondage for storms of less than the concentration period is automatically included by the application of a unit graph derived from an observed hydrograph. Runoff rates from hypothetical storms can be computed in exactly the same manner as was illustrated for actual rainfalls in Table II.

The unit-graph method enables a runoff record to be computed when only limited streamflow data are available. This application is of special service in investigations of drainage, flood control, water power and water supply.

Systematic Maintenance Essential for Fire Hydrants and Valves

BECAUSE of the relative infrequency with which they are operated, regular inspection of the valves and hydrants of a waterworks system is essential if they are to be maintained in good working order at all times. Well-formulated rules for the guidance of the maintenance staff were laid down by W. H. Durbin, superintendent of the Terre Haute Water Works Corp., at the meeting of the Indiana Section of the American Water Works Association at Lafayette on March 9, 1932.

All valves should open in the same direction, and if necessary, new stems and nuts should be installed so as to bring about a condition of uniformity. The usual result of having the two types of valve in the system is unsatisfactory service, twisted stems and higher maintenance costs. The necessity of uniformity cannot be too strongly emphasized, said Mr. Durbin.

Maintaining valves in Terre Haute

It is the practice in Terre Haute to inspect each valve once a year. A permanent record card has entered upon it the date of inspection, the condition of the valve and valve box, the name of the inspector and other information of a pertinent nature. The usual inspection consists of placing the valve key upon the nut and turning it several times to determine if it operates freely, also to make certain that the valve is fully open. If the valve does not work freely and forcing appears inadvisable to the inspector, that fact is noted on the card so that the valve can be given proper attention later. The operation of a valve is often made more difficult by the drying out of the packing. Where this is the case considerable relief can be obtained by

removing the stuffing-box gland, loosening up the packing and lubricating it. All valve boxes that are found to be covered are adjusted to the grade of the street.

At one time it was the practice at Terre Haute to build brick wells or vaults for every valve located in a paved street irrespective of its size. Experience has demonstrated that the cost of maintaining the wells and especially the iron cover is far in excess of any expense incurred in the valves themselves. It is very seldom necessary to repack a valve, and where this is needed it is usually more economical to dig up the street than to maintain the well. Should the stem become broken or bent, even with the ordinary well, it has often been impossible to make the needed repairs without removing the cover and possibly some of the brickwork. Under heavy automobile traffic it is almost necessary to use a malleable lid to prevent breakage, and this adds to the cost still further. For a gear-operated valve the vault is thought to be advisable, but not for a straight operating valve 16 in. or less in diameter.

Inspecting and lubricating hydrants

Fire hydrants serve a highly important purpose in the distribution system, and it is absolutely necessary that they be always in first-class working order. At Terre Haute two general hydrant inspections are made each year. At these times the hydrant is opened and closed, packed if necessary, the revolving nut is lubricated, nozzles are properly leaded and drains are opened. In addition to these two general inspections, frequent visits are made during the winter months. Where the groundwater plane is above