Numerical Evaluation of Arbritary Areas on a Map (Numerical Planimetry)

Purpose: Calculation of plane areas is a common necessity in hydrology. Calculation of volumes (cut-and-fill) is a common necessity in Civil Engineering.

Method: A simple method involves clockwise numerical integration of a polygon that closely approximates the area of interest. The volume calculations extend the approach to include a uniform depth (height) variable that is factored into the area calculation to produce a volume. Certain commercial software performs most of these calculations automatically, but the method is simple enough to employ a spreadsheet for non-routine use.

Procedure:

1. Superimpose a coordinate system onto the area of interest.
2. Represent the boundary of the area by a polygon. Determine the x-y coordinates of the nodes (vertices) of the polygon.
3. Determine the maximum and minimum x,y values for the area. This procedure will determine 4 ordered pairs: x_{min},y_{min},x_{max},y_{max}. These 4 values define the bounding rectangle that can be used to scale the area.
4. Starting from the left-most node, label the nodes 1,2,3 ... N where node 1 is the node whose x-coordinate is the same as xmin. Label the nodes in a clockwise manner.
5. Calculate the area of the trapezoidal panel between to adjacent nodes (1 to 2; 2 to 3; 3 to 4 ; ...) using the trapezoidal formula. $A_{panel} = (x_{i+1} - x_i)*(1/2\ (y_{i+1} - y_i) + y_i)$
6. The sum of the panel areas will be the approximation for the arbritary area.

Examples:

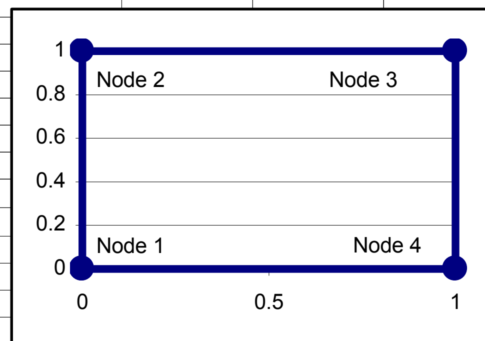
1. Area of a rectangle

Solution

	A	B	C	D
1	Area Calculations by Polygon Approximation			
2				
3				
4	Node #	X	Y	Area
5	1	0	0	
6	2	0	1	0
7	3	1	1	1
8	4	1	0	0
9	1	0	0	0
10	Sum			1

=(B6-B5)*(C5+0.5*(C6-C5))

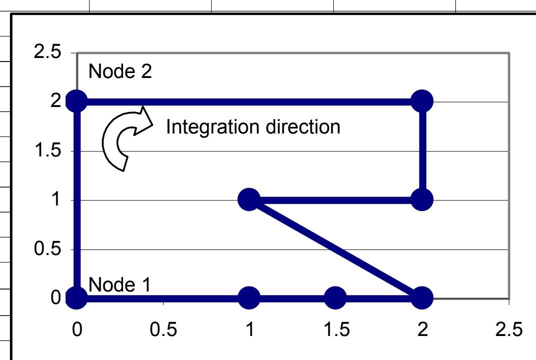
=SUM(D5:D9)



2. Area of a rectangle with a triangle cut-out.

	A	B	C	D
1	Area Calculations by Polygon Approximation			
2				
3				
4	Node #	X	Y	Area
5	1	0	0	
6	2	0	2	0
7	3	2	2	4
8	4	2	1	0
9	5	1	1	-1
10	6	2	0	0.5
11	7	1.5	0	0
12	8	1	0	0
13	1	0	0	0
14	Sum			3.5

=(B6-B5)*(C5+0.5*(C6-C5))



3. Arbitrary area of a watershed

Consider an arbitrary area (a watershed) whose area we wish to calculate. One can determine the area by planimetry (use a special tool to determine plane areas), or use the numerical method above. One simply draws a polygon that closely approximates the boundary of the watershed. One also needs to specify a distance scale (miles, feet, etc.) so that the final area can be represented in useful units.

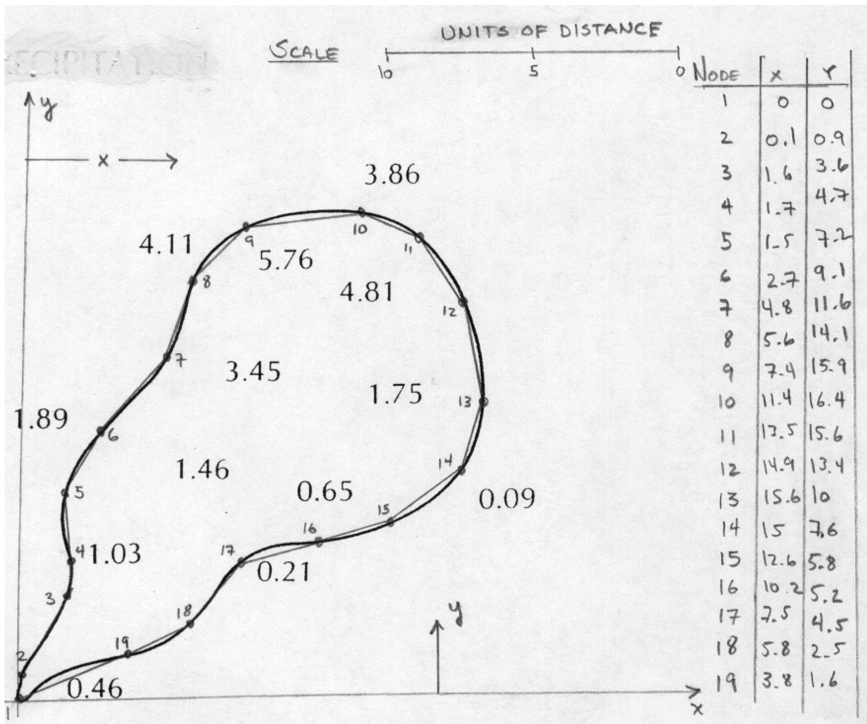


Figure 3. Illustrative watershed area. Distance units not specified

<u>Solution</u>

Figure 3 shows the polygon and node numbering system used in the calculation. Note that the first and last node must be the same otherwise the area is not closed and the method will produce a meaningless answer. Figure 4 is the spreadsheet calculations. If the distance units were kilometers, the resulting calculated area is 137 square kilometers.

	A	B	C	D	E	F	G	H	I
1	Area Calculations by Polygon Approximation								
2									
3									
4	Node #	X	Y	Area					
5	1	0	0						
6	2	0.1	0.9	0.045					
7	3	1.6	3.6	3.375					
8	4	1.7	4.7	0.415					
9	5	1.5	7.2	-1.19					
10	6	2.7	9.1	9.78					
11	7	4.8	11.6	21.735					
12	8	5.6	14.1	10.28					
13	9	7.4	15.9	27					
14	10	11.4	16.4	64.6					
15	11	13.5	15.6	33.6					
16	12	14.9	13.4	20.3					
17	13	15.6	10	8.19					
18	14	15	7.6	-5.28					
19	15	12.6	5.8	-16.08					
20	16	10.2	5.2	-13.2					
21	17	7.5	4.5	-13.095					
22	18	5.8	2.5	-5.95					
23	19	3.8	1.6	-4.1					
24	1	0	0	-3.04					
25	Sum			137.385					
26									
27		=(B6-B5)*(C5+0.5*(C6-C5))							
28									

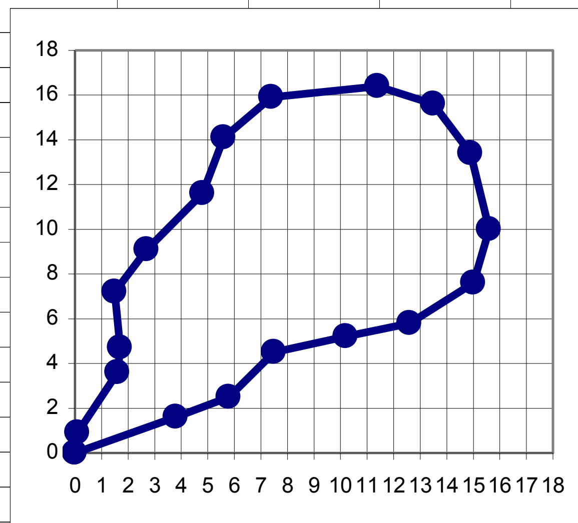


Figure 4. Solution to watershed area calculation

4. Total volume of rainfall over a watershed.

The same methods above can be used to determine the total volume of some distributed variable over an area. The iso-heyetal method of estimating uniform rainfall depth is a example. First one prepares a contour map of the variable of interest over the area of interest. Next one selects panels of the contour map that will represent sub-areas that have a uniform value of the depth variable. Lastly one determines the area of the sub-areas, and computes the volume as the product of the sub-area and the uniform depth value. This technique is illustrated with the map below. The accuracy of the procedure is increased as the panels are made to represent smaller and smaller variations in the depth variable.

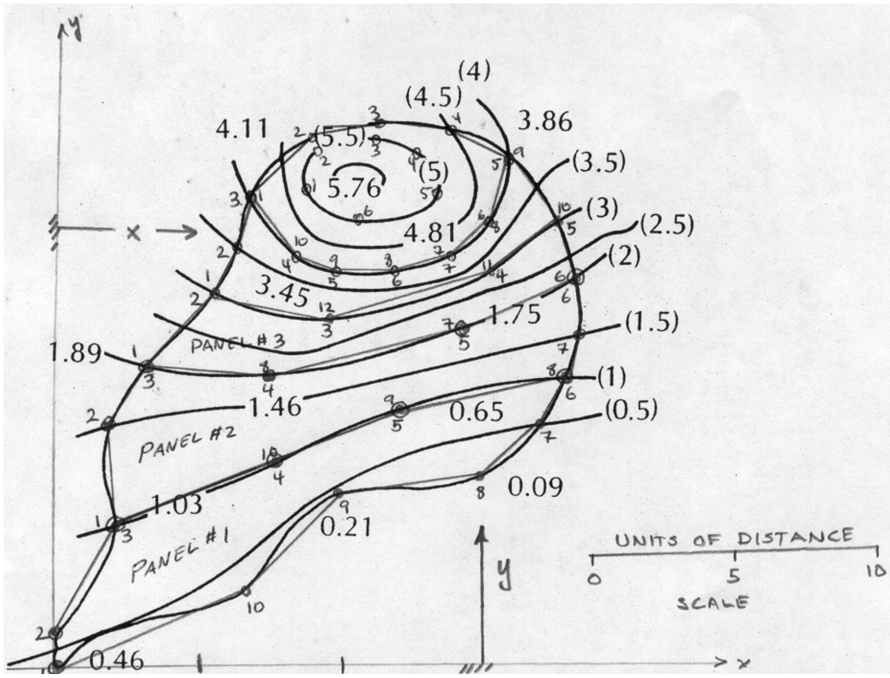


Figure 5. Isoheyetal panels for rainfall volume calculation

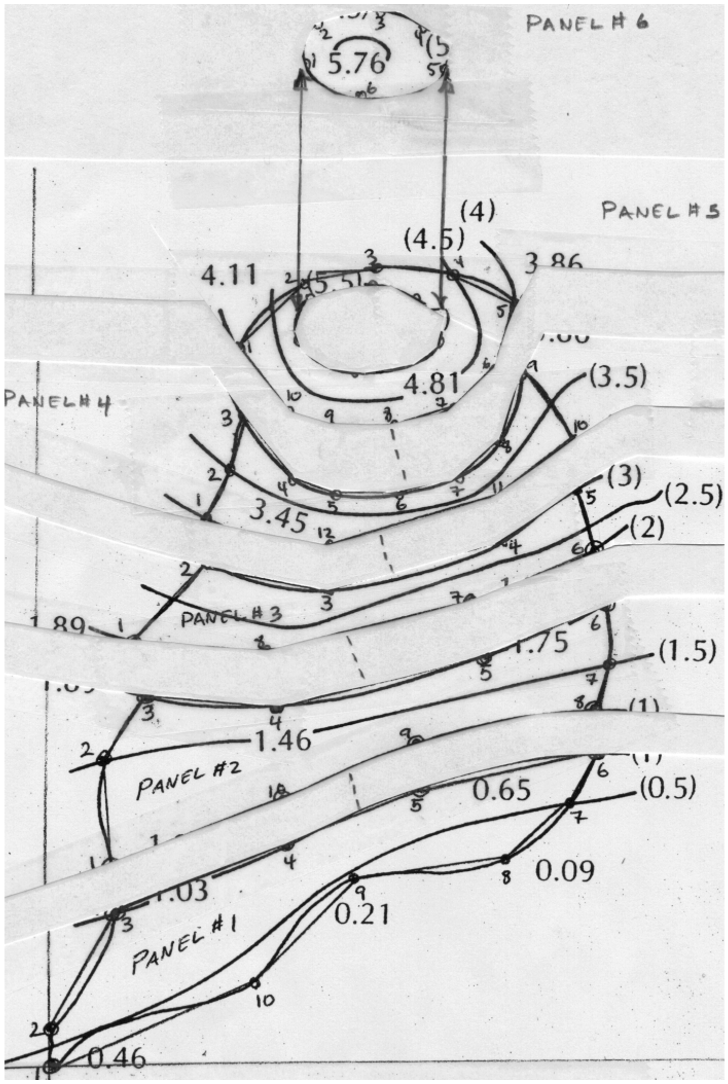


Figure 6. Exploded view of panels.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Area Calculations by Polygon Approximation														
3	Panel #1					Panel #2					Panel #3				
4	Node #	X	Y	Area	Depth	Node #	X	Y	Area	Depth	Node #	X	Y	Area	Depth
5	1	0	0			1	2	5			1	3.1	10.7		
6	2	0	1.2	0		2	1.9	8.4	-0.67		2	5.5	13	28.44	
7	3	2	5	6.2		3	3.1	10.7	11.46		3	9.5	12	50	
8	4	7.5	7.1	33.28		4	7.5	10.3	46.2		4	15.1	13.7	71.96	
9	5	12	8.8	35.78		5	14	11.7	71.5		5	17.5	15.5	35.04	
10	6	17.8	10	54.52		6	18	13.5	50.4		6	18	13.5	7.25	
11	7	16.9	8.4	-8.28		7	18.1	11.6	1.255		7	14	11.7	-50.4	
12	8	14.8	6.6	-15.8		8	17.8	10	-3.24		8	7.5	10.3	-71.5	
13	9	9.8	6	-31.5		9	12	8.8	-54.5		1	3.1	10.7	-46.2	
14	10	6.5	2.5	-14		10	7.5	7.1	-35.8						
15	1	0	0	-8.13		1	2	5	-33.3						
25	Sum			52.09	0.5	Sum			53.34	1.5	Sum			24.59	2.5
29	Volume			26.05					80					61.48	
31	Volume all Panels			400.5											
32	Area all Panels			193											
34	Effective Uniform Depth				2.076										

	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD
3	Panel #4					Panel #5					Panel #5				
4	Node #	X	Y	Area	Depth	Node #	X	Y	Area	Depth	Node #	X	Y	Area	Depth
5	1	5.5	13			1	6.8	16.5			1	8.6	16.5		
6	2	6.2	14.6	9.66		2	8.7	18.4	33.16		2	9	17.9	6.88	
7	3	6.8	16.5	9.33		3	11.1	19	44.88		3	11	18.2	36.1	
8	4	8.3	14.2	23.03		4	13.6	18.7	47.13		4	12.4	17.8	25.2	
9	5	9.7	13.8	19.6		5	15.2	17.8	29.2		5	13.2	16.5	13.72	
10	6	11.7	13.8	27.6		6	15	15.4	-3.32		6	10.5	15.5	-43.2	
11	7	13.7	14.3	28.1		7	13.7	14.3	-19.3		1	8.6	16.5	-30.4	
12	8	15	15.4	19.31		8	11.7	13.8	-28.1						
13	9	15.2	17.8	3.32		9	9.7	13.8	-27.6						
14	10	17.5	15.5	38.3		10	8.3	14.2	-19.6						
15	11	15.1	13.7	-35		1	6.8	16.5	-23						
16	12	9.5	12	-72											
17	1	5.5	13	-50											
25	Sum			21.24	3.5	Sum			33.41	4.5	Sum			8.3	5.5
27						Panel 5 Volume			150.3						
28						Panel 6 Volume			37.35						
29				74.32			Volume (5-6)		113					45.65	

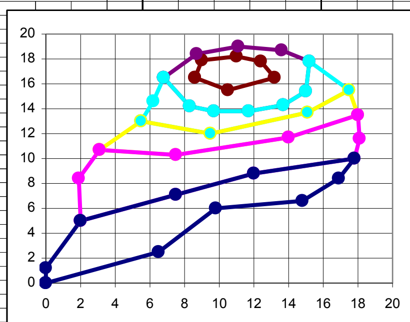


Figure 7. Solution to watershed rainfall example.

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