TABLE 4–4 Approximate Expected Rainfall Depths for Selected Durations, Return Periods, and Locations

Return Period (yr)	Duration	Atlanta	New York	Chicago	Los Angeles	Phoenix
		Depth (in.) for:				
2	15 min	1.0	1.0	0.9	0.5	0.5
2	1 hr	1.8	1.4	1.4	0.8	0.9
2	24 hr	4.2	3.5	2.8	3.5	2.0
100	15 min	1.9	1.7	1.5	1.9	1.0
100	1 hr	3.5	3.0	2.7	2.2	3.0
100	24 hr	8.0	6.7	5.7	9.0	5.0
Mean	Annual	48	43	34	16	8

tion in New York. The 24-hr recorded maximum of about 46 in. (Philippines) is almost seven times larger than the 24-hr event that can be expected in New York once every 100 years. The 24-hr recorded maximum in the Philippines is almost the same as the annual average rainfall in New York.

These comparisons show that rainfall depths show considerable variation with duration, frequency, and location. Given the importance of rainfall to hydrologic design, it is important to recognize typical values of point rainfalls and appreciate the variations that can be expected.

4.2.7 Storm-Event Isohyetal Patterns

Where many rain gages are located in a region, the measured depths of rainfall can be used to draw isohyets for total rainfall depths of a storm. Even in cases where a dense network of rain gages exists, considerable skill is required to draw the isohyets. The resulting maps are used frequently in assessing flood damages, especially in litigation following major storms.

Figure 4–5 shows the storm isohyets for the August 1, 1985, event in Cheyenne, Wyoming. The largest depth occurred around the state capitol. The isohyets for actual storms are often characterized by irregular shapes, as evidenced by the pattern of Figure 4–5.

4.3 ESTIMATING MISSING RAINFALL DATA

Measured precipitation data are important to many problems in hydrologic analysis and design. Because of the cost associated with data collection, it is very important to have complete records at every station. Obviously, conditions sometimes prevent this. For gages that require periodic observation, the failure of the observer to make the necessary visit to the gage may result in missing data. Vandalism of recording gages is another problem that results in incomplete data records, and instrument failure because of mechanical or electrical malfunctioning can result in missing data. Any such causes of instrument failure reduce the length and information content of the precipitation record.

Certainly, rainfall records are important. Rainfall data are an important input to hydrologic designs, whether measured storm event data or synthetic data based on characteristics of measured data. A number of federal and state agencies, most notably the National

Weather Service, have extensive data collection networks. They collect and analyze data to provide those who need such information with reasonably complete data records and accurate data summaries, such as *IDF* curves. Those involved in legal cases, such as for flooding that may be the result of human-made watershed modifications, also require accurate and complete rainfall records. Those involved in hydrologic research also require precipitation records to test models and evaluate hydrologic effects. Where parts of records are missing, it may be desirable, even necessary, to estimate the missing part of the record.

A number of methods have been proposed for estimating missing rainfall data. The station-average method is the simplest method. The normal-ratio and quadrant methods provide a weighted mean, with the former basing the weights on the mean annual rainfall at each gage and the latter having weights that depend on the distance between the gages where recorded data are available and the point where a value is required. The isohyetal method is the fourth alternative.

4.3.1 Station-Average Method

The station-average method for estimating missing data uses n gages from a region to estimate the missing point rainfall, $\hat{P}$, at another gage:

$$\hat{P} = \frac{1}{n} \sum_{i=1}^{n} P_i \tag{4–8}$$

in which P_i is the catch at gage i. Equation 4–8 is conceptually simple, but may not be accurate when the total annual catch at any of the n regional gages differ from the annual catch at the point of interest by more than 10%. Equation 4–8 gives equal weight to the catches at each of the regional gages. The value $1/n$ is the weight given to the catch at each gage used to estimate the missing catch.

Example 4–4

As an example, consider the following data:

Gage	Annual P (in.)	Storm Event P (in.)
A	42	2.6
B	41	3.1
C	39	2.3
X	41	?

The storm-event catch at gage X is missing. Ten percent of the annual catch at gage X is 4.1 in., and the average annual catch at each of the three regional gages is within ± 4.1 in.; therefore, the station-average method can be used. The estimated catch at the gage with the missing storm event total is

$$\hat{P} = \frac{1}{3}(2.6 + 3.1 + 2.3) = 2.67 \text{ in.} \tag{4–9}$$

Using this method requires knowledge of the average annual catch, even though this information is not used in computing the estimate $\hat{P}$.

4.3.2 Normal-Ratio Method

The normal-ratio method is conceptually simple, but it differs from the station-average method of Equation 4–8 in that the average annual catch is used to derive weights for the rainfall depths at the individual stations. The general formula for computing is $\hat{P}$:

$$\hat{P} = \sum_{i=1}^{n} w_i P_i \tag{4–10}$$

in which w_i is the weight for the rainfall depth P_i at gage i. The weight for station i is computed by

$$w_i = \frac{A_x}{nA_i} \tag{4–11}$$

in which A_i is the average annual catch at gage i, A_x is the average annual catch at station X, and n is the number of stations. The weights will not sum to 1 unless the mean of the average annual catches for the n stations equals the average annual catch at gage X. The sum of the weights will equal the ratio of the average annual catch at gage X to the mean of the average annual catches for the n gages. This is rational because, if gage X has a lower average annual catch, we would then expect the catch for an individual storm to be less than the mean of the storm event catches for the gages. When the average annual catches for all of the gages are equal, the station-average and normal-ratio methods yield the same estimate. When the variation in the average annual catches is small (less than 10%), the two methods provide estimates that are nearly similar, and the difference in the estimates does not warrant use of the additional information in the form of the average annual catches. When the average annual catches differ by more than 10%, the normal-ratio method is preferable; such differences might occur in regions where there are large differences in elevation (for example, regions where orographic effects are present) or where average annual rainfall is low but has high annual variability.

The conceptual basis of the normal-ratio method may be more apparent if Equations 4–10 and 4–11 are rewritten as

$$\frac{P_x}{A_x} = \frac{1}{n}\sum_{i=1}^{n}\frac{P_i}{A_i} = \bar{p} \tag{4–12}$$

The ratio P_i/A_i is the proportion for gage i of the mean annual catch that occurs in the specific storm; thus the right-hand side of Equation 4–12 is the average proportion. This average proportion is then used as the proportion at gage X where amount P_x was not recorded, and the estimate of P_x is found by multiplying the average proportion, $\bar{p}$, by the average annual catch at gage X, A_x. The value of $\bar{p}$ is considered to be the best estimate of the true proportion p of the catch during the storm for which the recorded value is missing.

Example 4–5

To illustrate the normal-ratio method, consider the following data:

Gage	Average Annual Rainfall (in.)	Total Storm Rainfall (in.)
A	41	2.4
B	37	2.3
C	46	3.1
X	40	?

The storm-event catch for gage X is missing and can be estimated using the data from the table. The average annual rainfall at the four gages is 41 in. Ten percent of this is 4.1 in. Since the annual rainfall at gage C is more than 4.1 in. from the mean of 40 in., the normal-ratio method should be used. The estimate from the normal-ratio method is

$$\begin{aligned}\hat{P} &= w_A P_A + w_B P_B + w_C P_C \\ &= \frac{A_x}{nA_A} P_A + \frac{A_x}{nA_B} P_B + \frac{A_x}{nA_C} P_C \\ &= \left[\frac{40}{3(41)}\right](2.4) + \left[\frac{40}{3(37)}\right](2.3) + \left[\frac{40}{3(46)}\right](3.1) \\ &= 0.325(2.4) + 0.360(2.3) + 0.290(3.1) = 2.51 \text{ in.}\end{aligned} \tag{4–13a}$$

Whereas the station-average method would yield an estimate of 2.60 in., Equation 4–13a indicates a value of 2.51 in. with the normal-ratio method. The latter value is less than the average o the other stations because the mean annual catch at gage X is less than the mean of the averag annual catches at gages A, B, and C (i.e., 40 in. versus 41.33 in.).

For the data used with Equation 4–13 the storm rainfall estimated with the normal-rati method expressed in the form of Equation 4–10 is

$$\begin{aligned}\hat{P} &= 40\left[\frac{1}{3}\left(\frac{2.4}{41} + \frac{2.3}{37} + \frac{3.1}{46}\right)\right] \\ &= 40\left[\frac{1}{3}(0.05854 + 0.06216 + 0.06739\right] \\ &= 40(0.06270) = 2.51 \text{ in.}\end{aligned} \tag{4–14}$$

Thus the proportion at the gages with a known storm catch ranged from 0.059 to 0.067, with mean proportion of 0.0627; this mean proportion is used as the best estimate of the proportio for the gage with the missing storm event rainfall. The estimate equals the estimate obtaine from Equations 4–10 and 4–11.

4.3.3 Isohyetal Method

The isohyetal method is another alternative for estimating missing rainfall data. The proc dure is essentially the same as that used for the isohyetal method when it is applied to th problem of estimating mean areal rainfall (see Section 4.5.3). The location and catch for eac gage are located on a map and used to draw lines of equal catch (that is, isohyets) for the storm duration of interest. The location of the gage for which data are missing is then plotted on the map and the catch estimated by interpolation within the isohyets. Of course, the accuracy of the estimate will depend on the number of gages used to draw the isohyets, the homogeneity of the meteorological conditions that generated the storm, and, if they exist, orographic effects.

Example 4–6

The isohyetal pattern shown in Figure 4–5 can be used to illustrate the isohyetal method. If a rain gage were located at the confluence of Diamond and Crow Creeks and the data for the August 1, 1985, storm event were missing, the isohyets would suggest a total storm depth of 3.5 in. In some parts of the isohyetal map, interpolation is not wholly objective; for example, in the area about the point defined by a latitude and longitude of 41°10′N and 104°45′W, the isohyet for 2 in. deviates considerably from the isohyet for 3 in. and there is no isohyet shown for 1 in. A similar problem of interpolation occurs near the intersection of Routes 25 and 85.

4.3.4 Quadrant Method

The station-average method does not account for either the closeness of the other gages to the location of the missing measurement or the density of the raingage network. The normal-ratio method requires information in addition to the storm event catches at all gages; namely, the average annual catch at each gage is required. The isohyetal method requires a reasonably dense network of gages in order to accurately construct the isohyets. Each of the methods has disadvantages.

The quadrant method is an alternative to these other methods. Once again, the estimated catch $\hat{P}$ is a weighted average of catches at other gages, and Equation 4–10 is used. The method is based on two assumptions: (l) catches at gages that are located close to each other are not independent estimates of the catch at the unknown point, and therefore, all gages are not necessary; and (2) the weight assigned to a gage used to estimate $\hat{P}$ should decrease as the distance between the gage and the point where an estimate is required increases.

To estimate P, the region is divided into four quadrants using north-south and east-west lines that intersect at the point where the catch is missing (point X in Figure 4–8). The coordinates of each gage with respect to the center location are determined and used to compute the distance between the gages and the center point. The quadrant method uses only the point in each quadrant that is closest to the center location (that is, point X); this attempts to ensure that the gages used to estimate P are somewhat independent. The weights are defined as a function of the reciprocal of the square of the distance between the gage and the center location X, $1/d_i^2$. If there is a gage in each quadrant, the weight for quadrant i is

$$w_i = \frac{1/d_i^2}{\sum_{j=1}^{4}\left(\frac{1}{d_j^2}\right)} \tag{4–15}$$

where d_i is the distance between gage i and the center location.

In some cases, there may not be a gage in one or more quadrants or a gage may be too far from the center point to have a significant weight. In such cases, the summation in the de-

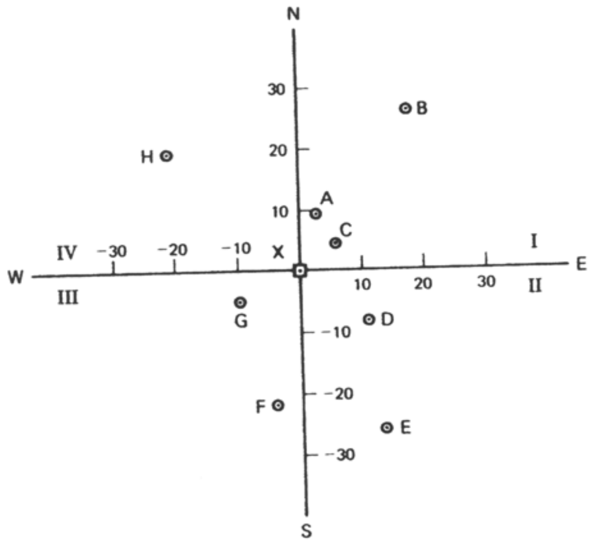


FIGURE 4–8 Rain gage location for estimating storm rainfall by using the quadrant method.

nominator of Equation 4–15 will be over those quadrants that include a gage that yields a significant weight.

The delineation of quadrants does not have to be based on a north-south, east-west axis system. If there are hydrometeorological reasons for a different axis system, such as that caused by orographic effects, a different system can be used. In such cases, the reasons for adopting the alternative system should be clearly stated. There may even be cases where a non-Cartesian coordinate system is used, where justified by hydrometeorological conditions.

Example 4–7

The data of Figure 4–8 and Table 4–5 can be used to illustrate the quadrant method. Eight gages with known catches are located in the region around the point where the catch at a gage is miss-

TABLE 4–5 Calculation of Total Storm Rainfall Using the Quadrant Method[a]

		Coordinates							
Quadrant	Gage	N-S	E-W	d_i^2	d_i	$(1/d_i^2) \times 10^3$	P_i (in.)	w_iP_i	w_i
I	*A*	9	3	90	9.49		3.9		
	B	26	18	1000	31.26		1.4		
	C	4	6	52	7.21*	19.231	3.7	2.102	0.568
II	*D*	–8	11	185	13.60*	5.405	1.6	0.254	0.159
	E	–26	14	872	29.53		0.2		
III	*F*	–22	–4	500	22.36		0.9		
	G	–5	–10	125	11.18*	8.000	3.0	0.708	0.236
IV	*H*	19	–21	802	28.32*	1.247	0.9	0.033	0.037
						33.883		3.097	

*Asterisks indicate gages selected to compute the weights.

ing. The distance is computed from the coordinates and one point from each quadrant selected (points C, D, G, and H). Only these four gages are used to compute the weights and estimate P. Since gage C is closest to point X, it has the largest weight. It is of interest to note that gage A is not used to estimate P even though it is the second closest gage to point X; it is not used because gage C is in the same quadrant and closer to point X. Even though gage H is much farther from point X, it is used to estimate P because it is in a different quadrant and, therefore, assumed to be an independent estimate of the rainfall at point X. The weights for gages C, D, G, and H are 0.568, 0.159, 0.236, and 0.037, respectively. The sum of the weights equals 1.0. The weighted estimate of the rainfall equals 3.10 in.

4.4 GAGE CONSISTENCY

Estimating missing data is one problem that hydrologists need to address. A second problem occurs when the catch at rain gages is inconsistent over a period of time and adjustment of the measured data is necessary to provide a consistent record. A consistent record is one where the characteristics of the record have not changed with time. Adjusting for gage consistency involves the estimation of an effect rather than a missing value. An inconsistent record may result from any one of a number of events; specifically, adjustment may be necessary due to changes in observation procedures, changes in exposure of the gage, changes in land use that make it impractical to maintain the gage at the old location, and where vandalism frequently occurs.

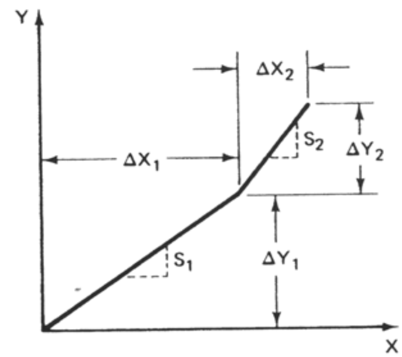
Double-mass-curve analysis is the method that is used to check for an inconsistency in a gaged record. A *double-mass curve* is a graph of the cumulative catch at the rain gage of interest versus the cumulative catch of one or more gages in the region that has been subjected to similar hydrometeorological occurrences and are known to be consistent. If a rainfall record is a consistent estimator of the hydrometeorological occurrences over the period of record, the double-mass curve will have a constant slope. A change in the slope of the double-mass curve would suggest that an external factor has caused changes in the character of the measured values. If a change in slope is evident, then the record needs to be adjusted, with either the early or later period of record adjusted. Conceptually, adjustment is nothing more than changing the values so that the slope of the resulting double-mass curve is a straight line.

The method of adjustment is easily understood with the schematic of Figure 4–9. The double-mass curve between the cumulative regional catch X and the cumulative catch at the gage Y where a check for consistency is needed is characterized by two sections, which are denoted in Figure 4–9 by the subscripts 1 and 2. The slopes of the two sections, S_1 and S_2, can be computed from the cumulative catches:

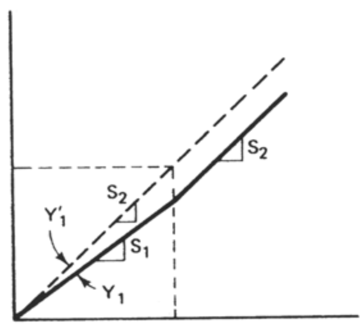
$$S_i = \frac{\Delta Y_i}{\Delta X_i} \tag{4–16}$$

in which S_i is the slope of section i, ΔY_i is the change in the cumulative catch for gage Y between the endpoints of the section i, and ΔX_i is the change in the cumulative catch for the sum of the regional gages between the endpoints of section i.

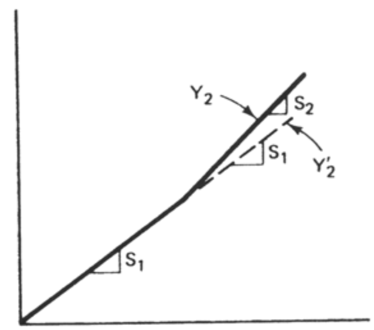
Either section of the double-mass curve can be adjusted for consistency. If the gage has been permanently relocated, it would be of interest to adjust the initial part of the record (that



(a) Notation



(b) Adjustment of lower section



(c) Adjustment of upper section

FIGURE 4–9 Adjustment of a double-mass curve for gage consistency.

is, part 1 of Figure 4–9a) so that it is consistent with the latter part of the record (that is, part 2 of Figure 4–9a) and the data that will be collected in the future. If the gage has been relocated only temporarily or if the exposure at the gage adversely affected the catch for a brief period, it would be of interest to adjust the latter part of the record so that it would be consistent with the initial part of the record; this adjustment will make the latter part of the record consistent with both the initial part of the record and the data that will be collected in the future.

If it is of interest to adjust the initial part of the record of station Y, then S_2, which is based on Y_2 and X_2, is correct, and it is necessary to adjust the Y_1 values. The adjustment will provide an adjusted series of data, which will be denoted as y_i. If S_2 is correct, the slope S_1 should be removed from Y_1 and replaced with slope S_2; this adjustment is made by

$$y_1 = \frac{S_2}{S_1}Y_1 \tag{4–17}$$

in which the ratio S_2/S_1 is the adjustment factor. The adjustment is shown in Figure 4–9b. By multiplying each value in the Y_1 series by the adjustment factor of Equation 4–17, a series y_1 that has the same slope as the upper section, Y_2, will be produced.

The procedure for adjusting the upper section of the double-mass curve is similar to the adjustment of the lower section. The intent is to replace the slope of the upper section with the slope of the lower section (see Figure 4–9c). The adjusted values are computed by

$$y_2 = \left(\frac{S_1}{S_2}\right) Y_2 \tag{4–18}$$

Dividing by S_2 removes the effect of the existing slope and replaces it with S_1 through multiplication by S_1. The series y_2 will have slope S_1.

Example 4–8

The data of Table 4–6 can be used to illustrate the use of Equation 4–17 to adjust the lower section of a double-mass curve. Gage H was permanently relocated after a period of 3 yr (at the end of 1981); thus it is necessary to adjust the recorded values from 1979 through 1981. The double-mass curve is shown in Figure 4–10, with the cumulative catch at three gages in the region plotted against the cumulative catch at the gage of interest, gage H. The slope for the 1979–1981 period is

$$S_1 = \frac{99 - 0}{229 - 0} = 0.4323$$

The slope from 1982 to 1986 is

$$S_2 = \frac{230 - 99}{573 - 229} = 0.3808$$

Using Equation 4–17, the adjusted values from 1979 through 1981 can be computed using

$$h_1 = \left(\frac{0.3808}{0.4323}\right) H_1 = 0.8809 H_1 \tag{4–19}$$

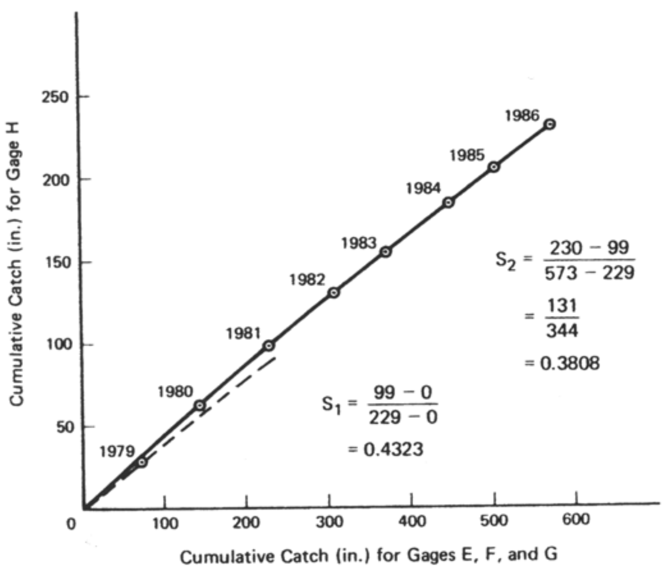


FIGURE 4–10 Adjustment of the lower section of a double-mass curve.