Improved Time of Concentration Estimation on Overland Flow Surfaces Including Low-Sloped Planes

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Abstract: Time of concentration (T_c) is one of the most used time parameters in hydrologic analyses. As topographic slope (S_o) approaches zero, traditional T_c estimation formulas predict large T_c. Based on numerical modeling and a review of relevant literature, a lower bound for slope (S_{lb}) of 0.1% was identified as a threshold below which traditional T_c estimation formulas become unreliable and alternate methods should be considered. In this study, slopes less than S_{lb} are defined as low slopes. Slopes equal to or exceeding S_{lb} are defined as standard slopes where traditional T_c estimation formulas are appropriate. A field study was conducted on a concrete plot with a topographic slope of 0.25% to collect rainfall and runoff data between April 2009 and March 2010 to support numerical modeling of overland flows on low-sloped planes. A quasi-two-dimensional dynamic wave model (Q2DWM) was developed for overland flow simulation and validated using published and observed data. The validated Q2DWM was used in a parametric study to generate T_c data for a range of slopes that were used to develop T_c regression formulas for standard slopes ($S_o \geq 0.1$ %) and low slopes ($S_o < 0.1\%$). **DOI: 10.1061/(ASCE)HE.1943-5584.0000830.** *© 2014 American Society of Civil Engineers.*

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Introduction

Without actually using the term "time of concentration" (T_c), the concept was first presented by Mulvany (1851) as the time at which discharge is the highest for a uniform rate of rainfall as the runoff from every portion of the catchment arrives at the outlet. It is the time needed for rain that falls on the most remote part of the catchment to travel to the outlet (Kuichling 1889). McCuen et al. (1984) stated that almost all hydrologic analyses require the value of a time parameter as input, and T_c is the most commonly used.

Even though T_c is a fundamental time parameter, the practical measurement of the time required to travel the entire flow path in a watershed was seldom attempted except by Pilgrim (1966). Because field measurement of the travel time is labor, time, and cost intensive, hydrograph analysis of observed or simulated discharges is often used to determine T_c.

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Determination of T_c using hydrograph analysis dates from Kuichling (1889), who stated "discharge from a given drainage area increases directly with the rainfall intensity until it reaches T_c". Hicks (1942) analyzed hydrographs from laboratory watersheds and computed T_c as the time from the beginning of rainfall to the time of equilibrium discharge. Izzard (1946) defined T_c from the beginning of a rainfall until the runoff reaches 97% of the input rate. Muzik (1974) defined T_c as the time to equilibrium discharge for his laboratory watersheds. Su and Fang (2004) determined T_c as the time from the beginning of effective rainfall to the time when flow reaches 98% of the equilibrium discharge. Wong (2005) considered T_c as the time from the beginning of effective rainfall to the time when flow reaches 95% of the equilibrium discharge.

A number of empirical formulas were developed to estimate T_c, but the applicability of any formula for general use is constrained by lack of diversity in the data used to develop the formula (McCuen et al. 1984). Sheridan (1994) indicated that, after more than a century of development and evolution in hydrologic design concepts and procedures, the end-user is constrained by confusing choices of empirical formulas for estimating T_c for ungauged watersheds.

Most of the empirical formulas to estimate T_c use the reciprocal of topographic slope S_o. As S_o approaches zero (such as in the coastal plains of the southeastern United States, the Texas Gulf Coast, and the High Plains), the resulting prediction of T_c approaches infinity. If used in hydrologic design, such estimates result in underestimation of peak discharge. A hydrologic design based on underestimated discharge is prone to failure by hydraulic overloading. In the absence of proper estimates of time of concentration, analysts frequently choose arbitrary values that are based on local rules of thumb or engineering judgment. If the estimate is less than the actual time of concentration, then the resulting estimate of peak discharge will be greater than the correct value (overestimated), resulting in costly overdesign. However, underestimation of peak discharge resulting in underdesign is also possible if the analyst-selected time of concentration is less than the correct value.

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Such underestimates can result in failure of the drainage system, loss of lives, etc., with costs that exceed those of the overdesigned system. Therefore, appropriate estimation of T_c for low-sloped terrains is required and will increase confidence in design discharge estimates for those regions.

The development of a method for estimating T_c for low-sloped planes requires identification of a threshold, below which slope is defined as "low." Such a boundary (S_{lb}) represents a threshold below which traditional relations like Henderson and Wooding (1964) and Morgali and Linsley (1965) become unreliable when slope approaches zero. In this study, slopes less than S_{lb} (0.1%) are defined as low slopes for which alternate methods for T_c estimation should be considered. Slopes equal to or greater than S_{lb} are defined as standard slopes ($S_o \geq 0.1\,\%$) where traditional T_c estimation formulas are appropriate.

Based on the literature review and the results of numerical modeling, an effective lower bound of the topographic slope was established. A field study was conducted to collect rainfall and runoff data on a concrete plot with an average slope of 0.25% to extend the research database for relatively low-sloped planes. A quasi-two-dimensional dynamic wave model (Q2DWM) for overland flows was developed and validated using published and observed data. Based on the results of the validation studies, T_c values were calculated as the time from the beginning of effective rainfall to the time when discharge reaches 98% of the peak discharge. The Q2DWM was used to conduct a parametric study to extend the project dataset. Relationships between T_c and physically based input variables were developed for overland flow planes of standard slopes ($S_o \geq 0.1\,\%$). In the final step, authors developed a T_c estimation formula for overland flow planes with low slopes ($S_o < 0.1\,\%$) using an alternate slope ($S_o + S_{lb}$).

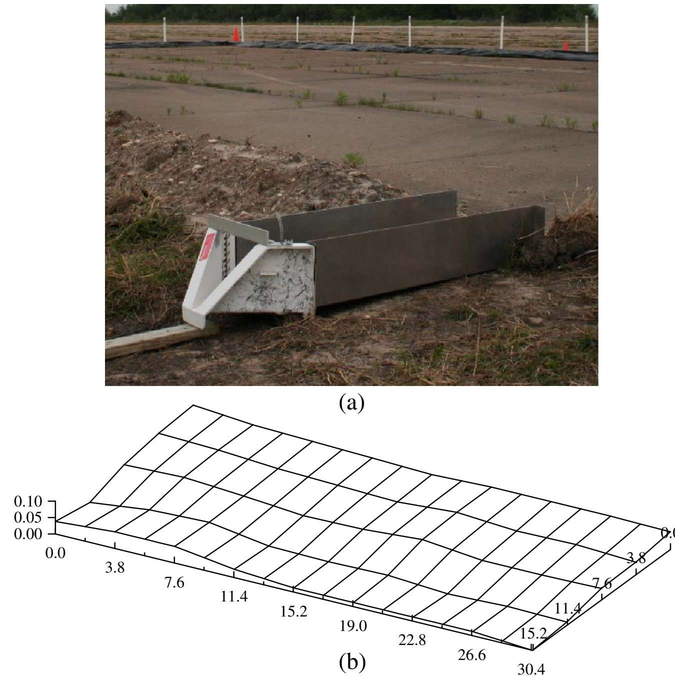

Fig. 1. Field study test site; the z-axis scale is magnified 20 times in comparison to the scale of x- or y-axis for better visualization of elevation changes: (a) airfield concrete runaway plot of 30.5 m by 15.2 m with H-flume at the outlet and tipping bucket rain gauge near the plot located at the Texas A&M University Riverside Campus; (b) digital elevation model of the concrete runaway plot

Field Study

Izzard (1946) and Yu and McNown (1964) conducted laboratory and field studies to investigate travel time and runoff characteristics of overland flow. Izzard used rectangular asphalt and turf surfaces 1.8 m (6 ft) wide, 3.7 to 21.9 m (12 to 72 ft) long, with slopes ranging from 0.1 to 4%. Rainfall was simulated using sprinklers that produced intensities from 41.9 to 104.1 mm/h (1.65 to 4.10 in./h). Izzard used runoff hydrographs to find T_c as the time from the beginning of a rainfall until the runoff reaches 97% of the input rate. Yu and McNown (1963) reported runoff hydrographs measured at an airfield watershed in Santa Monica, CA. Runoff was measured during simulated rainfall events with intensities varying from 6.4 to 254 mm/h (0.25 to 10.0 in./h) from three concrete surfaces 152.4 m (500 ft) long and 0.9 m (3 ft) wide, with slopes of 0.5, 1.0, and 2.0%. Li and Chibber (2008) conducted field experiments on five surfaces, bare clay, lawn, pasture, concrete, and asphalt, using a rainfall simulator. The test watersheds were 9.1 m (30 ft) long and 1.8 m (6 ft) wide, with slopes ranging from 0.24 to 0.48%. T_c was defined as the time required for the runoff hydrograph to reach peak discharge. Fifty-three events (Li and Chibber 2008) were used to derive an estimation formula for T_c with S_o in the denominator.

For the study reported herein, a field study was conducted using a concrete plot with slope of 0.25% to extend the research database for relatively low-sloped planes. Researchers at Texas A&M University instrumented a concrete plot to record rainfall and runoff. The plot is located at the Texas A&M University Riverside Campus on an abandoned airstrip taxiway [Fig. 1(a)]. The plot is surrounded by soil berms of 178 mm (7 in.) tall to form a watershed boundary. Fig. 1(a) is an image of the concrete plot looking upslope along the greater diagonal. The tipping-bucket rain gauge and the 0.23 m (0.75 ft) H-flume located at the outlet are visible in the image. The plot survey was conducted by recording elevation differences every 3.80 m (12.50 ft) with a vertical resolution of 0.30 mm (0.001 ft) with respect to the outlet [Fig. 1(b)]. The slope along the diagonal from the far corner to the outlet of the rectangular plot is 0.25%. Fig. 1(b) is a digital elevation model (perspective view) of the plot where the scale in z-axis is magnified 20 times in comparison to the scale of the x- or y-axis.

Stage (water-surface elevation) of flow in the H-flume [Fig. 1(a)] was measured using an ISCO bubbler flow module connected to an ISCO sampler (http://www.isco.com/). The flow module records a flow depth observation in the H-flume at 0.30 mm (0.001 ft) resolution every minute. The ISCO tipping-bucket rain gauge records rainfall depths at 0.25 mm (0.01 in.) resolution once each minute. The instruments were manually connected and powered before each forecasted rainfall event. The ISCO sampler was triggered to store data when rainfall intensity exceeded 0.25 mm/h (0.01 in./h) or the flow depth in the H-flume was greater than 0.90 mm (0.003 ft).

During the study period, 27 rainfall events were recorded. The 24 events listed in Table 1 were used during the numerical model calibration and verification. Three events were excluded because outlet discharges exceeded what could be attributed to incoming rainfall. This mismatch was attributed to the sediment transported to the H-flume when the high-intensity rainfall eroded the boundary berm. Such sediment deposited in the H-flume increased the depth readings and introduced an uncorrectable bias.

Recorded flow depths were adjusted when the bubbler flow module read false initial flow depth. This false reading occurred during an initial dry period, or when two consecutive rainfall events

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Table 1. Total Rainfall Depth, Total Rainfall Duration, Maximum Rainfall Intensity, Total Runoff Volume, and Runoff Coefficient for 24 Rainfall Events Measured on Concrete Surface for Field Study

Events	Total rainfall depth (mm)	Total rainfall duration (h)	Maximum rainfall intensity (mm/h)[a]	Total runoff volume (m^3)	Volumetric runoff coefficient
04/12/2009	8.18	1.58	34.14	2.22	0.58
04/18/2009	22.40	3.33	34.14	7.11	0.68
04/25/2009	59.39	4.58	89.61	25.55	0.93
04/27–28/2009	7.11	2.92	12.80	2.08	0.63
04/28/2009	11.38	4.42	38.40	4.20	0.79
07/20/2009	47.64	1.92	76.81	18.69	0.84
09/10/2009	14.58	1.50	68.28	3.56	0.53
09/11–12/2009	38.40	14.00	17.07	13.06	0.73
09/13/2009	76.20	1.50	102.41	12.44	0.35
09/23–24/2009	6.05	11.92	4.27	1.85	0.66
09/24/2009	6.40	1.92	12.80	2.55	0.86
10/09/2009	55.83	8.17	55.47	24.54	0.95
10/11/2009	13.16	4.17	25.60	5.63	0.92
10/13/2009	36.63	5.50	85.34	13.67	0.80
10/21–22/2009	27.74	11.83	34.14	11.94	0.93
10/26/2009	7.47	3.92	8.53	2.54	0.73
11/20–22/2009	21.34	24.67	12.80	9.55	0.96
12/01–02/2009	30.58	8.25	12.80	11.76	0.83
01/28–29/2010	70.05	5.00	81.08	30.42	0.94
02/08/2010	9.25	1.42	46.94	3.80	0.89
03/01–02/2010	13.51	16.08	29.87	5.81	0.93
03/08–09/2010	8.53	8.42	34.14	3.29	0.83
03/16–17/2010	19.91	26.83	8.53	7.96	0.86
03/24–25/2010	8.53	1.00	59.74	3.13	0.79

[a]Time interval used to compute rainfall intensity was 5 min.

occurred in a short interval of time. These initial readings were considered offsets and subtracted from subsequent depths. Adjusted depths in the H-flume were converted to discharges using the rating curve provided by the flume manufacturer, Free Flow, Inc. (http://freeflowinc.com/).

Total runoff volume for each event was computed from observed discharges and compared to total rainfall volume. Early in development of the dataset, it was discovered that recorded total rainfall volumes were less than observed total runoff volumes. Habib et al. (2001) found that the rainfall intensity measured by tipping-bucket rain gauge could be erroneous at the 1-min interval readings, but the errors were significantly reduced at the 5-min and 10-min interval readings. Therefore, rainfall data were adjusted. A total-catch (container) rain gauge was installed at the test plot to record total event rainfall depths at 1 mm resolution to confirm rainfall depths recorded using the tipping-bucket rain gauge. The readings from the tipping-bucket rain gauge were also compared to data from the weather station at Riverside, Bryan, TX (KTXBRYAN19), which is located about 1.6 km from the test site. The weather station uses Davis Vantage Pro2 weather instrument to record cumulative rainfall volume every 10 min in real time. The comparison of rainfall data recorded using the tipping-bucket rain gauge, container rain gauge, and Davis Vantage Pro2 instruments at the weather station indicated a systematic underrecording by the tipping-bucket rain gauge. The event-rainfall data collected at the container rain gauge matched the measurements from the weather station (coefficient of determination $R^2 = 0.99$ and the slope of the regression line is 0.98). The event-rainfall data recorded by tipping-bucket rain gauge correlated well with the data recorded from the weather station ($R^2 = 0.96$ and the slope of the regression line is 0.72). Therefore, rainfall data recorded with the tipping-bucket rain gauge were aggregated into 5-minute-interval data and then were adjusted by dividing the data by 0.72, the slope of the regression line.

Twenty-four rainfall-runoff events monitored and used during this study are summarized in Table 1. Total rainfall depths ranged from 6.0 to 76.2 mm (0.2 to 3.0 in.) and rainfall durations ranged from 1 to 27 h. Observed maximum 5-min rainfall intensities varied from 4.3 to 102.4 mm/h (0.2 to 4.0 in./h). Total runoff volume (Table 1) was computed from the runoff hydrograph. The volumetric runoff coefficient (Table 1), the total runoff divided by total rainfall (Dhakal et al. 2012), was computed. The effective rainfall depth, one of the input data to Q2DWM, is derived by multiplying the volumetric runoff coefficient with the gross rainfall depth. Rainfall and runoff data collected during the field study were used to validate the performance of the Q2DWM for watersheds with low slopes as discussed subsequently.

Quasi-Two-Dimensional Dynamic Wave Model

Overland flow has been simulated using one- and two-dimensional (1D or 2D) kinematic or diffusion wave models (Henderson and Wooding 1964; Woolhiser and Liggett 1967; Singh 1976; Yen and Chow 1983; Abbott et al. 1986; Chen and Wong 1993; Wong 1996; Jia et al. 2001; Ivanov et al. 2004) and dynamic wave models (Morgali and Linsley 1965; Yeh et al. 1998; Su and Fang 2004). Both kinematic and diffusion wave models have been used to simulate surface water movement (Kazezyılmaz-Alhan and Medina 2007; López-Barrera et al. 2012) in hydrologic-hydraulic models. The kinematic wave model is frequently used for the development of T_c formulas (Wong 2005). Woolhiser and Liggett (1967) introduced a kinematic wave number for evaluating the validity of the kinematic wave assumption for simulating flow over a sloping plane with lateral inflow. McCuen and Spiess (1995) suggested that the kinematic wave assumption should be limited to kinematic wave number $nL/\sqrt{S_o} < 100$ where n, L, and S_o are Manning's roughness coefficient, length, and slope of the plane, respectively. Therefore, the kinematic wave model may not be suitable for overland flow planes with low slopes.

Hromadka and Yen (1986) developed a quasi-2D diffusion hydrodynamic model (DHM) to incorporate the pressure effects neglected by the kinematic approximation. Even though the diffusion wave approximation is fairly accurate for most overland flow conditions (Singh and Aravamuthan 1995; Moramarco and Singh 2002; Singh et al. 2005), it is inaccurate for cases in which the inertial terms play prominent roles such as when the slope of the surface is small (Yeh et al. 1998). In this study, a quasi-2D dynamic wave model, Q2DWM, was developed by modifying the quasi-2D DHM for simulating overland flow on low-sloped planes. The local and convective acceleration terms neglected in DHM were included in Q2DWM because they can be significant for overland flow on low-sloped planes in comparison to other terms.

The governing equations of DHM (Hromadka and Yen 1986) and Q2DWM were solved using a two-dimensional square grid system (Fig. 2) and the integrated finite difference version of the nodal domain integration method (Hromadka and Yen 1986). Each cell has four intercell boundaries in the north, east, south, and west directions. Each cell is represented using bed elevation (z_p in Fig. 2), flow depth (h_p), and Manning's roughness coefficient n. The quasi-2D DHM (Hromadka and Yen 1986) and Q2DWM solve the one-dimensional equation of motion, Eq. (1), along four directions in the east-west and north-south directions independently for each computation cell (Fig. 2) first and then solve the continuity Eq. (2)

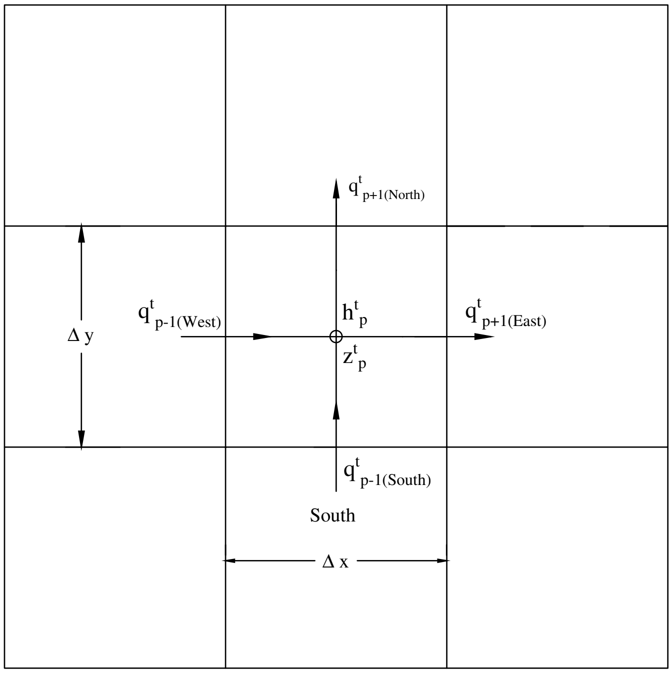

Fig. 2. Two-dimensional Q2DWM finite difference grids surrounding the cell j, k in the Cartesian computational domain, where q is flow rate (flux) between adjacent cells, h and z are water depth and bottom elevation for the cell

$$\frac{\partial q_j}{\partial t} + \frac{\partial}{\partial j}\left(\frac{q_j^2}{h}\right) + gh\left(\frac{\partial H}{\partial j} + S_{fj}\right) = 0 \tag{1}$$

$$\Sigma \frac{\partial q_j}{\partial j} + \frac{\partial h}{\partial t} = i \tag{2}$$

where j varies from 1 to 4, 1 for north, 2 for east, 3 for south, and 4 for west direction, q_j is the flow rate per unit width in the j direction, i is the effective rainfall intensity as a source term, S_{fj} is the friction slope in j direction, g is the gravitational acceleration, and H and h are the water-surface elevation and flow depth in each computational cell as functions of time t. The water surface elevation H is given by Eq. (3)

$$H = h + z \tag{3}$$

where z is the bottom elevation of the computational cell. Both h and z are defined at the cell center, and fluxes (q_j), and friction slopes (S_{fj}) are defined at the intercell boundaries (Fig. 2). Writing Eq. (1) in velocity form, we get

$$\frac{\partial v_j}{\partial t} + v_j\frac{\partial v_j}{\partial j} + g\left(\frac{\partial H}{\partial j} + S_{fj}\right) = 0 \tag{4}$$

The friction slope (S_{fj}) in Eq. (4) is approximated from Manning's equation (Akan and Yen 1981)

$$v_j = \frac{k_n}{n}h^2 S_{fj}^{1/2} \quad \text{or} \quad S_{fj} = \left(\frac{v_j n}{k_n h^{2/3}}\right)^2 \tag{5}$$

where $k_n = 1$ (SI units) or 1.49 (FPS units). The average values of h and n of the two adjacent cells in the j direction are used for Eq. (5).

Hromadka and Yen (1986) defined a dimensionless momentum factor, m_j, which represents the sum of first two acceleration terms in Eq. (4) after dividing by g

$$m_j = \frac{1}{g}\left[\frac{\partial(v_j)}{\partial t} + v_j\frac{\partial v_j}{\partial j}\right] = a_{lj} + a_{cj} \tag{6}$$

where a_{lj} and a_{cj} are dimensionless local and convective accelerations, respectively. Using m_j from Eq. (6), Eq. (4) is written as

$$S_{fj} = -\left(\frac{\partial H}{\partial j} + m_j\right) \tag{7}$$

Using Eq. (7) with Eq. (5), the velocity in each direction (j) can be calculated as

$$v_j = -K_j\left(\frac{\partial H}{\partial j} + m_j\right) \tag{8}$$

where K_j is conduction parameter computed as (Hromadka and Yen 1986)

$$K_j = \frac{k_n}{n}h^{2/3}\frac{1}{\left(\frac{\partial H}{\partial j} + m_j\right)^{1/2}} \tag{9}$$

Richardson and Julien (1994) studied the acceleration terms of the Saint-Venant equations for overland flow under stationary and moving storms. The local acceleration during the rising limb of a hydrograph and the convective acceleration after equilibrium can be estimated as

$$a_{lj} = \frac{\beta - 1}{gt^{(2-\beta)}}\alpha i^{(\beta-1)} \tag{10}$$

$$a_{cj} = \frac{\beta - 1}{\beta g X^{(2/\beta-1)}}\alpha^{2/\beta} i^{(2-2/\beta)} \tag{11}$$

where $\alpha = S_{fj}^{0.5}/n$, $\beta = 5/3$ (Richardson and Julien 1994), i is rainfall intensity in m/s, and X is the distance in m from its boundary along each j direction. During the rising limb of a hydrograph, the space derivatives are comparatively small, and the local acceleration [Eq. (10)] is dominant. As the time t increases or flow approaches equilibrium, time derivatives in Eq. (4) vanish, and the convective acceleration [Eq. (11)] is dominant (Richardson and Julien 1994).

After the velocity or the flow rate in each j direction is solved, the flow depth is updated using continuity Eq. (2). Eq. (2) was derived from the conservation of mass or volume in each cell, e.g., the cell p in Fig. 2. The difference form of Eq. (2) can be written as

$$h_p^t = h_p^{t-1} - \Delta t\left(\Sigma\frac{q_j}{\Delta j}\right) + i\Delta t \tag{12}$$

where superscripts $t-1$ and t stand for the previous and new time step. Eq. (12) was solved explicitly for each cell. Rainfall input (i) was converted from effective rainfall intensity (after removing any rainfall losses) to a depth change in each cell at each time step to model its contribution to the flow hydraulics. In Eq. (12), Σq_j is the sum of q_{east}, q_{west}, q_{south}, and q_{north} (Fig. 2). For quasi-2D DHM (Hromadka and Yen 1986) and Q2DWM, Δx (or Δj) is equal to Δy for each square cell (Fig. 2).

For the Q2DWM, the time step Δt is dynamically updated based on the minimum and the maximum time steps (Δt_{min} and Δt_{max}), where Δt_{min} is an input parameter and Δt_{max} is dynamically updated using Eq. (13). At each time step, after velocity and flow depth are solved for all cells in the simulation domain, the maximum velocity (v_{max}) of all the cells in the simulation domain and its corresponding flow depth ($h_{v\max}$) where v_{max} occurs are

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Table 2. Time of Concentration (T_c) and Peak Discharge (Q_p) Estimated from Published Experimental Data and Modeled Using Q2DWM for Published Overland Flow Planes Including Q_p Estimated Using Rational Method, Input Parameters, and Model Performance Parameters

T_c (min)		$Q_p(\times 10^{-3}\ m^3/s)$								
Experimental data	Model	Rational Method	Experimental data	Model	L (m)	S_o (%)	n	i (mm/h)	Ns	RMSE ($\times 10^{-3}\ m^3/s$)
3.2[a]	3.0	0.091	0.090	0.091	3.7	2.0	0.013	49.0	0.87	0.008
8.0[a]	7.9	0.518	0.518	0.518	21.9	0.1	0.013	46.5	0.99	0.022
6.3[a]	6.5	1.045	1.048	1.045	21.9	0.1	0.013	93.7	0.99	0.031
6.7[a]	6.4	1.096	1.099	1.096	21.9	0.1	0.013	98.3	0.98	0.059
4.6[b]	4.1	2.439	2.435	2.438	152.4	2.0	0.011	189.0	0.98	0.116
11.7[b]	10.8	0.648	0.663	0.649	152.4	0.5	0.011	50.3	0.98	0.031
22.6[b]	21.3	0.656	0.658	0.655	152.4	0.5	0.035	50.8	0.99	0.024
16.9[b]	14.9	0.278	0.280	0.279	152.4	0.5	0.011	21.6	0.95	0.024

Note: Input (controlling) variables for the experimental overland flow planes are L = length in meters, S_o = slope in percent, n = Manning's roughness coefficient, and i = rainfall intensity in mm/h. Model performance parameters are Ns = Nash-Sutcliffe coefficient and RMSE = root mean square error.
[a]Experimental data from Izzard and Augustine (1943).
[b]Experimental data from Yu and McNown (1964).

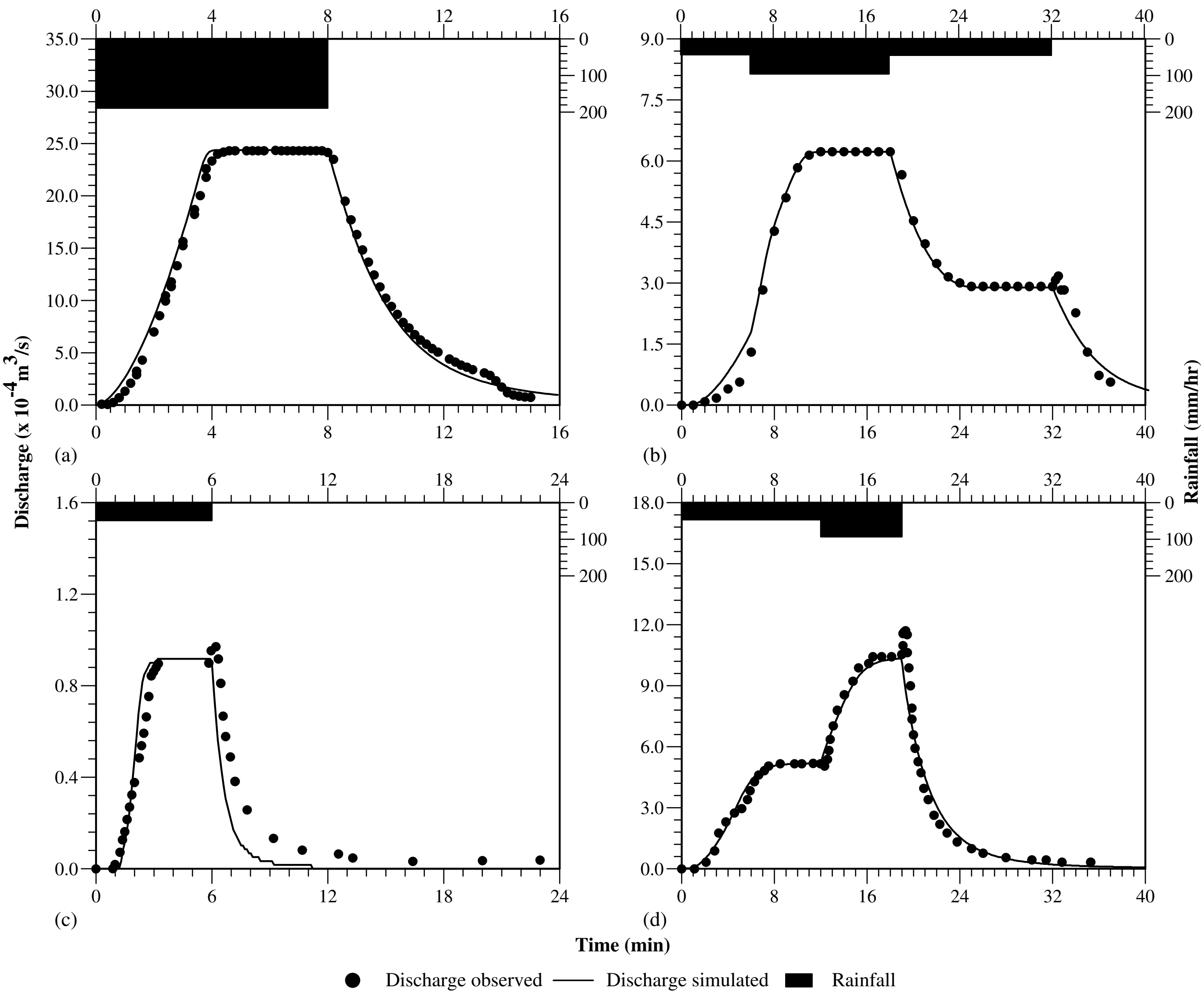

Fig. 3. Observed rainfall hyetographs and observed and simulated hydrographs for (a) concrete surface 152.4 m long and 0.3 m wide with slope of 2%; (b) concrete surface 76.8 m long and 0.9 m wide with slope of 0.5%; (c) asphalt pavement 3.7 m long and 1.8 m wide with slope of 2%; (d) concrete surface 21.9 m long and 1.8 m wide with slope of 0.1% [observed data presented in (a) and (b) are from Yu and McNown 1963 and in (c) and (d) from Izzard and Augustine 1943]

determined. Similarly, the maximum flow depth ($h_{\max}$) of all the cells and its corresponding velocity ($v_{h\max}$) where $h_{\max}$ occurs are determined. The variables $v_{\max}$ and v_{hmax} are calculated from the sum of average of east-west (x-velocity) and average of north-south (y-velocity). Hence, the maximum time step $\Delta t_{\max}$ is computed as

$$\Delta t_{\max} = Cr \times \mathrm{Min}\left(\frac{\Delta x}{v_{\max} + \sqrt{gh_{v\max}}}, \frac{\Delta x}{v_{h\max} + \sqrt{gh_{\max}}}\right) \quad (13)$$

where Cr is the Courant number (Courant et al. 1967), a numerical stability criterion, the limit of which is taken as 0.1 for the authors' low-sloped study. The model starts with $\Delta t_{\min}$, and increases at 5% of $\Delta t_{\min}$ at each computational cycle until the time step is just smaller than or equal to $\Delta t_{\max}$ calculated by Eq. (13).

The Q2DWM advances in time explicitly for all the cells in the domain until the specified simulation ending time is reached and simulates quasi-2D overland flow coupled with a simple rainfall loss model. For validation with the experimental data, an initial abstraction was used to remove rainfall at or near the beginning of rainfall event that did not produce runoff, and then the fractional loss model (FRAC) was used (McCuen 1998). The FRAC model (Thompson et al. 2008) assumes that the watershed converts a constant fraction (proportion) of each rainfall input into an excess rainfall. The constant runoff fraction used was a volumetric runoff coefficient (Dhakal et al. 2012). However, for parametric study effective rainfall is an input to the model.

Model Validation Using Published Data from Previous Studies

The Q2DWM was first validated using published data. The Los Angeles District of the U.S. Army Corps of Engineers conducted an extensive experimental rainfall-runoff study on three separate concrete channels (Yu and McNown 1963). Yu and McNown (1963) reported runoff hydrographs from different combinations of slope, roughness, and rainfall intensity (using an artificial rainfall simulator). Hydrographs simulated using Q2DWM matched observed hydrographs well (Table 2). Two example comparisons are shown in Figs. 3(a) and 3(b). Observed and simulated hydrographs from a concrete surface of 152.4 m (500 ft) by 0.3 m (1 ft) with a slope of 2% and of 76.8 m (252 ft) by 0.3 m (1 ft) with a relatively low slope of 0.5% are shown in Figs. 3(a) and 3(b), respectively. The hyetograph for the experiment presented in Fig. 3(a) was a rainfall intensity of 189 mm/h (7.44 in./h) with a duration of 8 min. The hyetograph for the event depicted in Fig. 3(b) was a variable rainfall intensity of 43.2 mm/h (1.70 in./h) for first 6 min, then 95.8 mm/h (3.77 in./h) from 6 to 18 min, and finally 44.5 mm/h (1.75 in./h) for the remaining portion of the storm with a total duration of 32 min.

Izzard and Augustine (1943) analyzed runoff data from paved and turf surfaces collected by the Public Roads Administration in 1942. Their objective was to study the hydraulics of overland flow using a rainfall simulator. The data were collected in three phases. The data used in Fig. 3 are from the first phase, which comprised smooth asphalt or concrete paved surfaces. Observed and simulated hydrographs for a 3.7 m (12 ft) long and 1.8 m (6 ft) wide asphalt pavement with slope of 2% for a 6-min uniform rainfall intensity of 49.0 mm/h (1.93 in./h) and a 21.9 m (72 ft) long and 1.8 m (6 ft) wide concrete surface with slope of 0.1% for a variable rainfall intensity of 46.5 mm/h (1.83 in./h) for 12 min, then 93.0 mm/h (3.65 in./h) for 12 to 19 min are shown in Figs. 3(c) and 3(d), respectively (Izzard and Augustine 1943).

Hydrographs were simulated using 1 ft by 1 ft cell size and Manning's roughness coefficient of 0.011 for concrete and 0.013 for asphalt surfaces. The Nash-Sutcliffe coefficient (Ns) and root mean square error (RMSE) were used to evaluate Q2DWM

Table 3. Peak Discharge (Q_p) and Time to Peak (T_p) Measured and Simulated Using Q2DWM and Nash-Sutcliffe Coefficient (Ns) and Root Mean Square Error (RMSE) for 24 Rainfall Events Observed on Concrete Plot

	Measured		Simulated			
Events	Q_{pm}[a] ($\times 10^{-3}$ m^3/s)	T_{pm} (h)	Q_{ps}[b] ($\times 10^{-3}$ m^3/s)	T_{ps} (h)	Ns	RMSE ($\times 10^{-3}$ m^3/s)
04/12/2009	0.720	0.33	0.729	0.42	0.95	0.045
04/18/2009	0.615	2.67	0.795	0.50	0.77	0.109
04/25/2009	2.447	2.92	3.553	2.83	0.51	0.535
04/27–28/2009	0.301	0.58	0.326	0.58	0.83	0.030
04/28/2009	0.718	4.00	1.108	4.00	0.82	0.071
07/20/2009	2.721	1.67	4.161	1.67	0.76	0.360
09/10/2009	1.813	0.50	1.149	0.58	0.83	0.198
09/11–12/2009	0.718	6.00	0.678	6.00	0.96	0.037
09/13/2009	2.411	1.08	3.458	1.00	0.86	0.262
09/23–24/2009	0.218	1.25	0.188	1.42	0.76	0.019
09/24/2009	0.385	1.42	0.505	1.42	0.87	0.037
10/09/2009	1.798	0.58	2.372	0.75	0.93	0.137
10/11/2009	0.493	1.08	0.766	0.33	0.86	0.044
10/13/2009	2.194	5.33	3.458	5.33	0.70	0.267
10/21–22/2009	0.974	11.00	1.136	11.00	0.92	0.079
10/26/2009	0.374	0.75	0.366	0.67	0.86	0.031
11/20–22/2009	0.414	20.83	0.521	21.00	0.80	0.045
12/01–02/2009	0.658	6.67	0.884	6.67	0.85	0.076
01/28–29/2010	3.262	3.50	3.831	3.67	0.69	0.453
2/08/2010	0.724	0.83	0.996	0.50	0.78	0.107
03/01–02/2010	0.710	3.00	0.878	3.00	0.86	0.057
03/08–09/2010	0.670	8.17	1.200	8.08	0.64	0.074
03/16–17/2010	0.534	13.08	0.577	12.67	0.86	0.049
03/24–25/2010	0.718	0.33	1.160	0.33	0.72	0.098

[a]Subscript m stands for measured values.
[b]Subscript s stands for simulated values.

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performance. Legates and McCabe (1999) demonstrated that Ns is a parameter to measure goodness of fit between modeled and observed data. Bennis and Crobeddu (2007) concluded that, for a hydrograph simulation, a good agreement between the simulated and the measured data is achieved when Ns exceeds 0.7. Hydrographs simulated using Q2DWM were compared with eight experimental hydrographs from Izzard and Augustine (1943) and Yu and McNown (1963). The average Ns was 0.97 (ranged from 0.87 to 0.99 in Table 2), and average RMSE was $0.04 \times 10^{-3}\ \mathrm{m^3/s}$ (ranged from 0.008 to $0.116 \times 10^{-3}\ \mathrm{m^3/s}$ in Table 2). These statistics indicate close agreement between measured and simulated hydrographs.

Model Validation Using Observations from Current Field Study

Measured rainfall-runoff data were used to validate the performance of the Q2DWM for catchments with relatively low slope and with elevation variations in two dimensions. Simulated hydrographs matched observed hydrographs well (Table 3). Four example comparisons are shown in Fig. 4. Rainfall intensities measured from rainfall events (Fig. 4) were more variable compared with the artificial rainfalls shown in Fig. 3. Both measured and simulated hydrographs showed response to rainfall intensity variation, for example, the event on September 11–12, 2009, [Fig. 4(c)]. Simulated and measured peak discharges (Q_p) and time-to-peak (T_p) are listed in Table 3 and compared in Fig. 5 for all 24 events. There are two relatively large disagreements between simulated and measured T_p in Fig. 5 because the initial rainfall abstractions, used in the simple rainfall loss model for Q2DWM, were less than the actual initial abstractions for these events.

Q2DWM simulations were based on 3.81 m (12.5 ft) square cells [Fig. 1(b)] with a Manning's roughness coefficient of 0.02. Cell sizes finer than 3.81 m were tested but did not improve model results. Aggregated observed hyetographs with a 5-min interval were used as model input. The model boundary condition at the outlet is crucial to overland flow simulation. Su and Fang (2004) developed estimation formulas of T_c for low-sloped planes with 100 and 20% opening at the outlet boundary. In the field study, the surrounding boundaries of the rectangular plot were closed

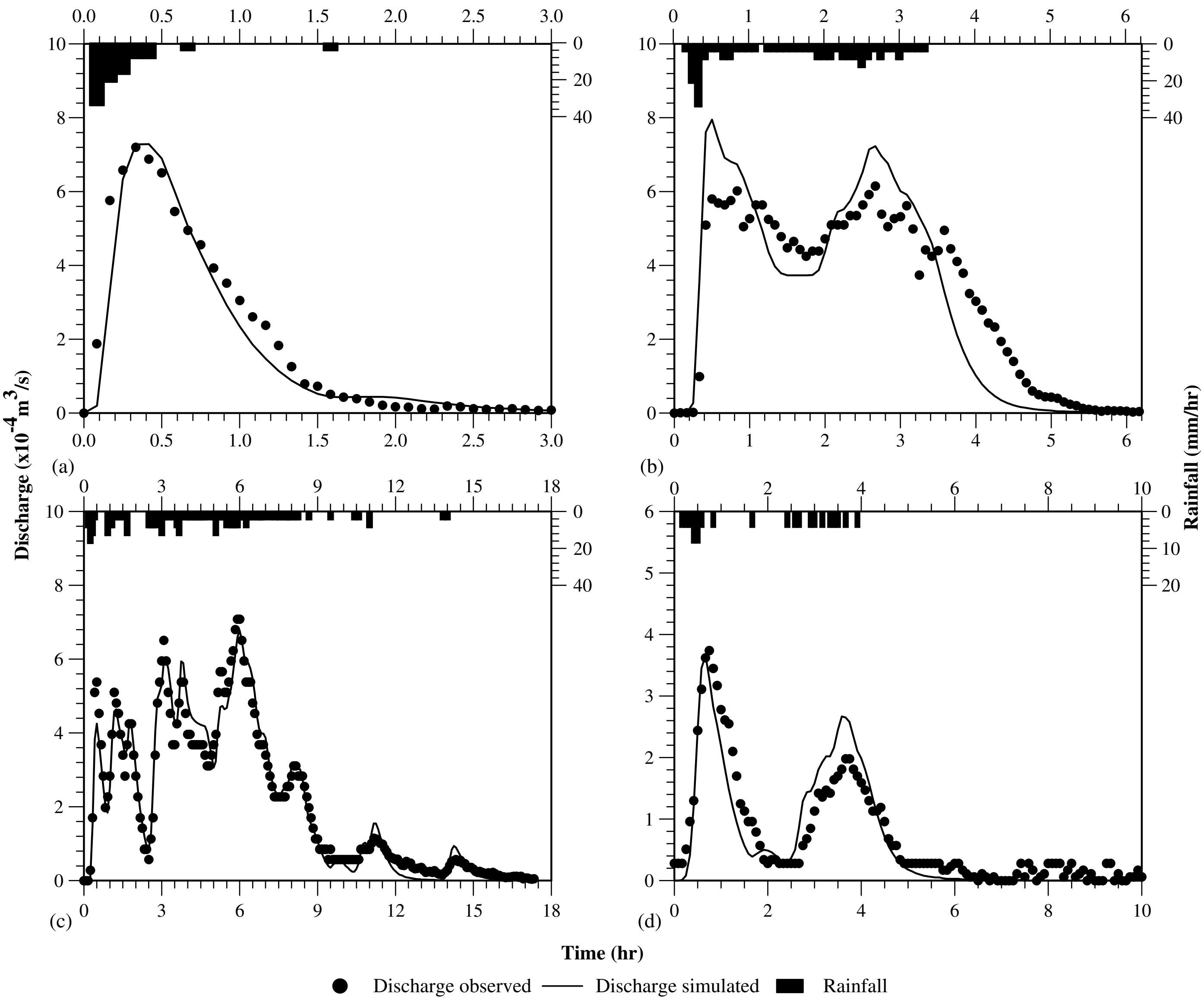

Fig. 4. Observed rainfall hyetographs and observed and simulated hydrographs on the concrete plot located at Texas A&M University for the events on (a) April 12, 2009; (b) April 18, 2009; (c) September 11–12, 2009; (d) October 26, 2009

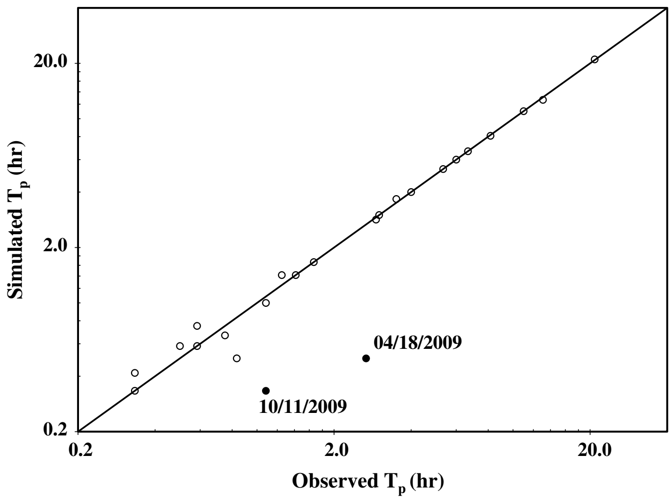

Fig. 5. Simulated time to peak (T_p) using Q2DWM versus observed T_p for 24 rainfall events on the concrete plot (Fig. 1)

using soil berms (Fig. 1) except an opening through the 0.75-ft H-flume. The H-flume is a specially shaped open-channel flow section designed to restrict the channel width from 0.434 to 0.023 m (1.425 to 0.075 ft) and create a critical flow condition for flow measurement. Therefore, the boundary condition at the outlet was critical flow for a rectangular opening. A calibrated opening width of 0.122 m (0.4 ft) for the 3.81 m (12.5 ft) computational cell size was used.

The Ns and RMSE statistics developed for 24 simulated hydrographs are listed in Table 3. The average Ns was 0.81 and the average RMSE was 0.13×10^{-3} m^3/s. These results indicate an acceptable match between measured and simulated hydrographs; therefore, Q2DWM can be used to estimate response for watershed with standard ($S_o \geq 0.1\,\%$) and low slopes ($S_o < 0.1\,\%$) for uniform and variable rainfall intensities.

Estimation of Time of Concentration

There is no practical method to directly measure T_c in the field or laboratory. Therefore, the indirect approach of analyzing the discharge hydrograph is the viable method to estimate T_c. For the study reported herein, T_c is defined as the time from the beginning of effective rainfall to the point when the runoff reaches 98% of the peak discharge under a constant rainfall rate. This approach is similar to those used by Izzard (1946), Su and Fang (2004), and Wong (2005). For the parametric study, the peak discharge was calculated using the rational formula (Kuichling 1889). When the discharge approaches equilibrium from a constant rainfall supply, the time rate of change of discharge is nearly zero and could fluctuate (in response to numerical diffusion and unsteady flow nature), especially for low-sloped overland flows. This sensitivity at "computational equilibrium" makes the determination of the practical equilibrium time difficult (McCuen 2009) and prone to error. Therefore, T_c was not estimated as the equilibrium time, but the time to 98% of the peak discharge.

Peak discharges calculated using the rational formula, modeled using Q2DWM, and measured just before rainfall cessation are listed in Table 2. Peak discharges calculated from above three methods are almost the same (Table 2), and absolute relative difference between two peaks is less than 2%. T_c values were derived from Q2DWM simulated hydrographs for planes with slopes of 0.1, 0.5 (relatively low slope), and 2% (standard slope), rainfall intensity from 21.6 to 189.3 mm/h (0.85 to 7.45 in./h), roughness from 0.011 to 0.035, and plane length from 3.7 to 152.4 m (12 to 500 ft). T_c values extracted from Q2DWM simulated hydrographs agree well with T_c derived from published experimental hydrographs. The average error of T_c is 0.6 min with a standard deviation of 0.7 min. Therefore, Q2DWM produces T_c results that commensurate with observations and is considered valid for the subsequent parametric study.

Table 4. Dimensionless Low-Slope Bound (S_{lb}) where "Low-Slope" Behavior is in Effect, Which is Recommended in Published Literature and Current Study

S_{lb} (%)	Methods	Reference(s)
0.1	Classification of data	Yates and Sheridan (1973)
0.5	Observed data analysis	Capece et al. (1988)
0.5	Physical model experiments	De Lima and Torfs (1990)
0.1	Classification of data	Sheridan (1994)
0.2	Numerical model experiments	Van der Molen et al. (1995)
0.05	Numerical model experiments	Su and Fang (2004)
0.5	Physical model experiments	Li et al. (2005), and Li and Chibber (2008)
0.2	Numerical model experiments	Cleveland et al. (2008)
0.3	Observed data analysis	Cleveland et al. (2011)
0.1	Numerical model experiments	Current study

Identification of Lower-Bound Slope (S_{lb})

Developing appropriate equations to estimate T_c for overland flow on low-sloped planes requires a definition of what constitutes "low-slope." Yates and Sheridan (1973) conducted one of the first studies on flow measurement techniques in low-sloped watersheds. They considered flow measurement in streams with slopes less than 0.1% to be difficult and discussed hydrologic methods for those slopes. Capece et al. (1988) reported that delineation of watersheds with topographic slope less than 0.5% was difficult. Both Capece et al. (1988) and Sheridan et al. (2002) suggested that present hydrologic methods require modifications to improve performance for such "flatland" watersheds because the "flatland" energy and flow velocities are relatively small. Sheridan (1994) concluded that flow length was sufficient to explain hydrograph time parameters and precluded the use of topographic slope for "flatland" in the time parameter estimates. Sheridan (1994) classified channel slopes of 0.1–0.5% as stream networks of low-sloped systems. Van der Molen et al. (1995) used numerical experiments to conclude that water depth at the upper boundary is finite when slope is 0.2%. More recently, Su and Fang (2004) used a two-dimensional numerical model to examine the variation of T_c with plot slope, length, roughness coefficient, and rainfall and concluded that there is less variation of T_c for slopes less than 0.05%. Li et al. (2005) and Li and Chibber (2008) analyzed laboratory data and reported that the contribution of the slope to hydrograph time response is negligible for topographic slopes less than 0.5%. Cleveland et al. (2008) computed travel times using a particle tracking model based on an equation similar to Manning's equation. They reported that uncertainty in their prediction model increased substantially when they included watersheds of slopes of 0.02–0.2%. Cleveland et al. (2011) used the variation of dimensionless water-surface slope with Manning's roughness coefficient, n, provided by Riggs (1976) to examine the relation between them. They concluded that the relation between n and water-surface slope changed when the slope is less than 0.3%. This result can be considered another source for the low-slope threshold. In summary, most of the researchers considered the low-slope threshold to be between 0.1 and 0.5% (Table 4).

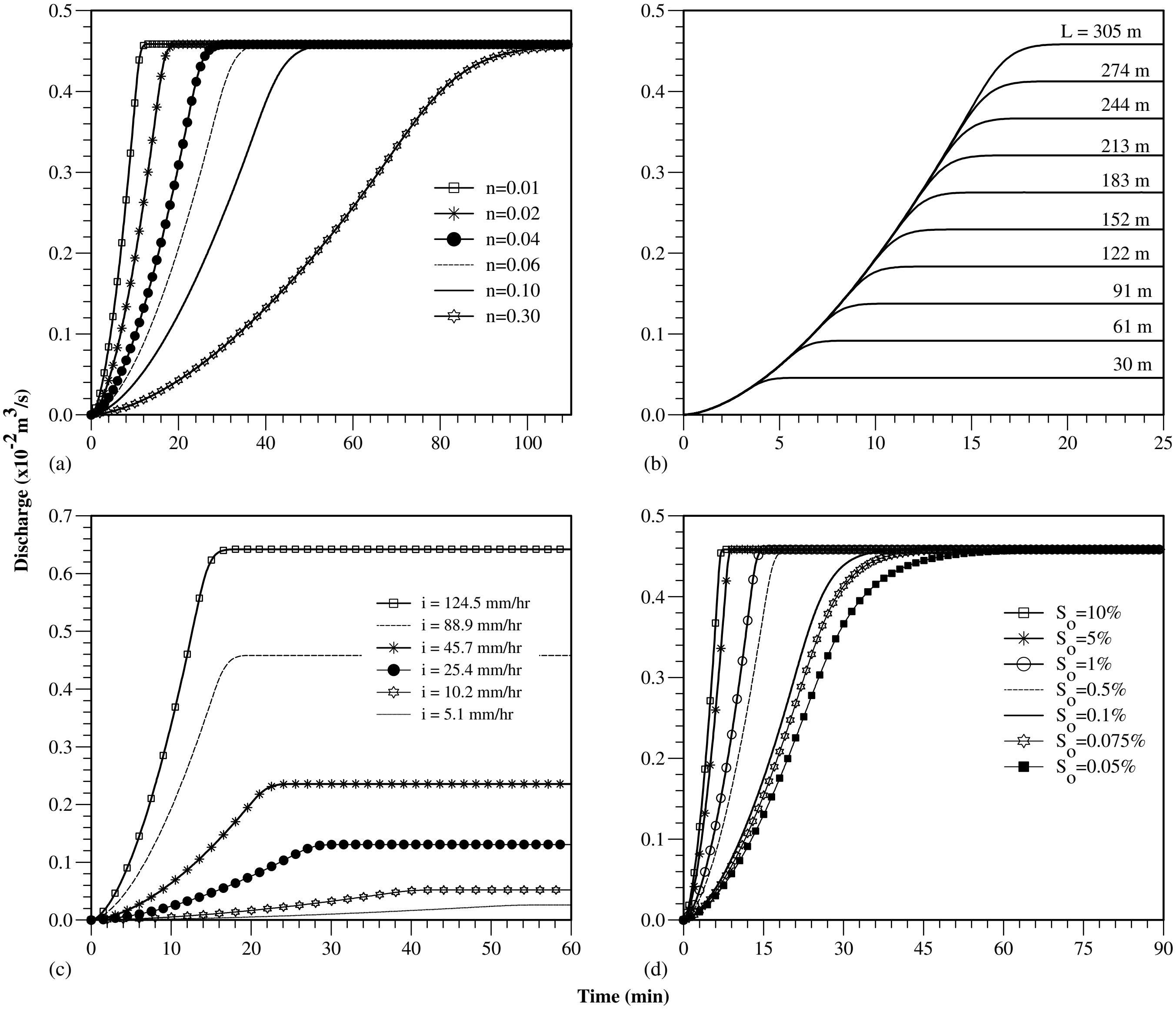

Fig. 6. Equilibrium *S*-hydrographs simulated using Q2DWM on impervious overland flow planes with (a) constant L, S_o, and i, and varying n; (b) constant n, S_o, and i, and varying L; (c) constant L, S_o, and n, and varying i; (d) constant L, n, and i and varying S_o

Related studies provide an insight into the definition of low slope. However, except for Su and Fang (2004), most evaluated the variation of slope with hydrologic variables other than T_c. To further examine the variation of T_c with slope, the authors conducted a series of Q2DWM numerical experiments to test the threshold slope for T_c estimations by varying S_o while retaining constant values of n, i, and L [$n = 0.02$, $i = 88.9$ mm/h (3.5 in./h), and $L = 305$ m (1000 ft)]. Simulated Q2DWM hydrographs for varying topographic slopes are shown in Fig. 6(d). Simulated hydrographs for slopes less than 0.1% are substantially different from those with greater slopes. Estimated T_c values versus S_o for two sets of numerical experiments are shown in Fig. 7: case (i) for $L = 305$ m (1000 ft), $n = 0.02$, $i = 88.9$ mm/h (3.5 in./h); and case (ii) for $L = 90$ m (300 ft), $n = 0.035$, $i = 25.4$ mm/h (1 in./h). The regression lines were derived for slopes greater than 0.1% (Fig. 7). When the slope is less than 0.1%, T_c values depart from the corresponding regression line ($S_o \geq 0.1\%$). Based on these numerical experiments, S_{lb}, a lower bound for topographic slope, can be established at 0.1%, which agrees reasonably well with the values recommended by others (Table 4). Inappropriate estimates of T_c are likely to arise if T_c equations such as Henderson and Wooding (1964) or Morgali and Linsley (1965) are used where slope is less than 0.1%, as shown in Fig. 7. The T_c equation commonly used in TR-55 by the Natural Resources Conservation Service (NRCS) for sheet flow (NRCS 1986) was derived from Morgali and Linsley (1965).

Parametric Study for the Time of Concentration of Overland Flow

Yen (1982) stated "overland and channel flows are in separate but connected hydraulic systems." Kibler and Aron (1983) reported that improved estimates of T_c are achieved if overland and channel flow are considered separately. Therefore, using the lower-bound slope (0.1%), a parametric study was conducted to develop estimating tools for standard ($S_o \geq 0.1\%$) and low-sloped ($S_o < 0.1\%$) overland flows where channel flows are negligible.

Development of empirical equations for T_c estimation dates from the 1940s, when Kirpich (1940) computed T_c for a watershed using channel length and average channel slope. For overland

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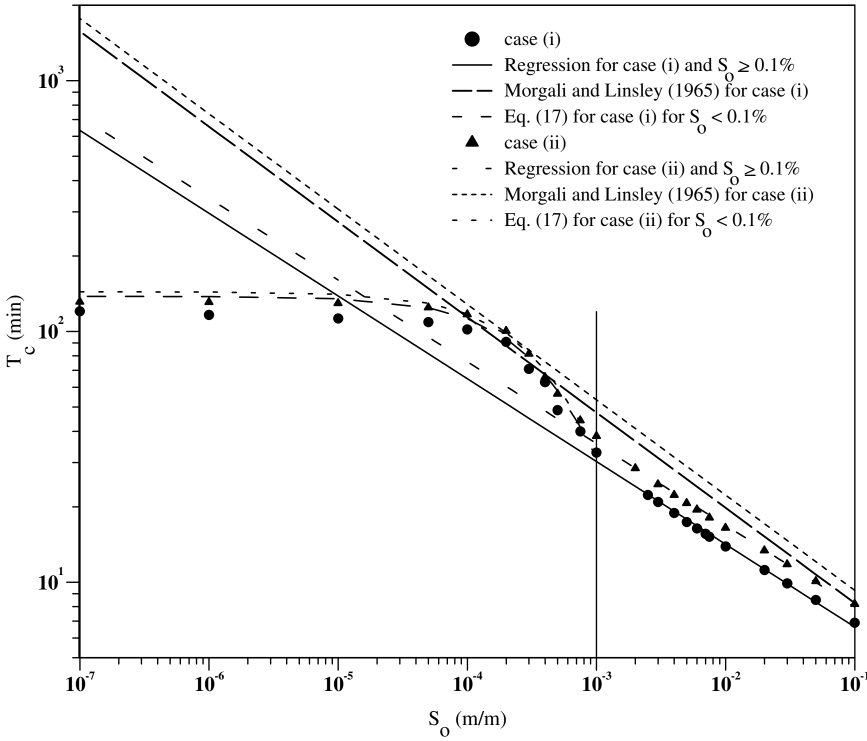

Fig. 7. Time of concentration (T_c) estimated using Q2DWM for overland flow planes at different slopes: case (i) $L = 305$ m, $n = 0.02$, $i = 88.9$ mm/h; and case (ii) $L = 90$ m, $n = 0.035$, $i = 24.4$ mm/h; linear regressions were developed for T_c data for planes with slope ≥0.1% (or $S_o = 0.001$); T_c predicted using Eq. (17) and the formula of Morgali and Linsley (1965) for cases (i) and (ii) are displayed for comparison

flows, Izzard (1946), Morgali and Linsley (1965), Woolhiser and Liggett (1967), and Su and Fang (2004) derived estimation formulas using length L, slope S_o, and Manning's roughness coefficient n of the overland flow plane, and rainfall intensity i as input variables.

More than 750 T_c values were estimated from hydrographs simulated using Q2DWM by varying the four physically based input variables, L, S_o, n, and i, to extend the dataset available for analysis. The input variable L was varied from 5 to 305 m (16 to 1000 ft), S_o from 0.001 to 10%, n from 0.01 to 0.80, and i from 2.5 to 254 mm/h (0.1 to 10.0 in./h). Hydrographs were simulated holding the three variables constant and varying the fourth by 10–20%. Example S-hydrographs from these simulations are displayed in Fig. 6. When n was varied from 0.01 to 0.30 for $L = 305$ m (1000 ft), $S_o = 0.5\%$, and $i = 88.9$ mm/h (3.5 in./h), T_c increased from 11.4 to 94.9 min [Fig. 6(a)]. Similarly, T_c increases as L increases [Fig. 6(b)], decreases as i increases [Fig. 6(c)], and increases as S_o decreases [Fig. 6(d)].

Five hundred fifty Q2DWM runs were conducted to obtain a database for developing an estimation formula for standard slopes ($S_o \geq 0.1\%$). A generalized power relation [Eq. (14)] was chosen for developing the regression equation

$$T_c = C_1 L^{k_1} S_o^{k_2} n^{k_3} i^{k_4} \tag{14}$$

where L is in m, S_o is in m/m, i is in mm/h, and C_1, k_1, k_2, k_3, and k_4 are regression parameters. Eq. (14) was log-transformed and nonlinear regression was used to estimate parameter values. The resulting equation is

$$T_c = 8.67 \frac{L^{0.541} n^{0.649}}{i^{0.391} S_o^{0.359}} \tag{15}$$

where T_c is in minutes, and other variables are as previously defined. Regression results are presented in Table 5. Statistical results indicate that the input variables L, S_o, n, and i have a high level of significance with p-value <0.0001 (Table 5) and are critical variables in the determination of T_c. The regression parameters (C_1, k_1, k_2, k_3, and k_4) have less standard errors and small ranges of variation at the 95% confidence interval (Table 5).

Values predicted with Eq. (15) compare well with those from formulas developed by Henderson and Wooding (1964) and Morgali and Linsley (1965), as shown in Fig. 8. Furthermore, the predicted values compare well with estimates from Q2DWM numerical experiments (Fig. 8). The coefficients of determination

Table 5. Parameter Estimates for the Independent Variables of Time of Concentration (T_c) Estimation Formula (15) for Standard Slopes ($S_o \geq 0.1\%$)

Parameter	Parameter estimate	95% confidence limits		Standard error	t-value	p-value
$Ln(C_1)$	2.160	2.103	2.217	0.029	74.6	<0.0001
k_1 for L	0.542	0.533	0.551	0.005	119.8	<0.0001
k_2 for S_o	−0.359	−0.366	−0.352	0.003	−105.0	<0.0001
k_3 for n	0.649	0.642	0.655	0.003	198.9	<0.0001
k_4 for i	−0.391	−0.399	−0.384	0.004	−100.7	<0.0001

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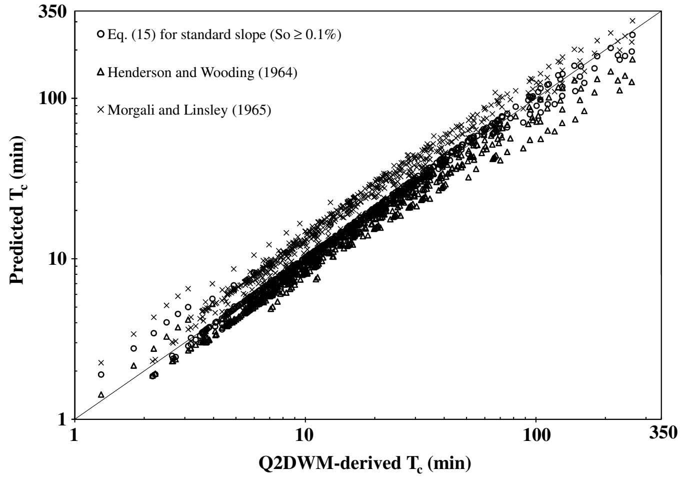

Fig. 8. Time of concentration (T_c) of overland flow planes predicted using regression Eq. (15) and the formulas of Henderson and Wooding (1964) and Morgali and Linsley (1965) versus T_c developed from numerical experiments using Q2DWM for standard slopes ($S_o \geq 0.1\%$)

R^2 and RMSE for Eq. (15), formulas of Henderson and Wooding (1964) and Morgali and Linsley (1965) are similar ($R^2 > 0.94$, as shown in Table 6).

Three additional estimation formulas were explored and developed using combinations of input variables and compared with the formulas described above. One option for a T_c estimation formula is to use the quotient $L/\sqrt{S_0}$ as a combined input variable. This combination was used for T_c formulas developed by Kirpich (1940), Johnstone and Cross (1949), and Linsley et al. (1958). The variable $L/\sqrt{S_0}$ is derived from application of Manning's equation for estimating overland flow velocity. The second option of combined variables considered is the product nL that is related to the total resistance length of the overland flow. The third option explored is to use the quotient $nL/\sqrt{S_0}$ that is related to Manning's equation. Estimation formulas of T_c using combined input variables were developed using nonlinear regression and are presented in Table 6. Estimation formulas using the combined variables performed as well as Eq. (15) and had R^2 values greater than 0.94. The p-value reported in Table 6 was developed between T_c and all input variables in each regression equation. These formulas are highly significant because the p-value for each formula is less than 0.0001 (Table 6). The p-values for the correlation between T_c and each of above three combined variables were developed and are each less than 0.0001. Therefore, these combined variables can also be considered as critical input variables in the determination of T_c. Based on these results, the regression equations developed in this study and those of Henderson and Wooding (1964) and Morgali and Linsley (1965) are acceptable for estimating T_c of overland flow on planes with standard slope ($S_o \geq 0.1\%$).

Time of Concentration for Low-Sloped Overland Flow

Using the equations presented in Table 6, the resulting estimates of T_c grow without bound as topographic slope S_o approaches zero. Therefore, an alternate formulation, Eq. (16) using the combined slope ($S_o + S_{lb}$) was chosen for planes with $S_o < 0.1\%$

$$T_c = C_2 L^{k_5} (S_o + S_{lb})^{k_6} n^{k_7} i^{k_8} \tag{16}$$

where C_2, k_5, k_6, k_7, and k_8 are constants derived from nonlinear regression. Using the Q2DWM dataset for low-sloped planes, the resulting regression equation is

$$T_c = \frac{L^{0.563} n^{0.612}}{11043.81 i^{0.304} (S_o + S_{lb})^{2.139}} \tag{17}$$

where T_c is in minutes, the low-slope threshold S_{lb} is 0.1%, and other variables in SI units are as previously defined.

Use of the offset S_{lb} in Eq. (17) allows computation of T_c in low- and zero-sloped conditions. For Eq. (17), the input variables L, ($S_o + S_{lb}$), n, and i are critical input variables for determination of T_c, presenting a high level of significance with p-value <0.0001 (Table 7). R^2 and RMSE for Eq. (17) are 0.87 and 16.9 min, respectively, when results from Eq. (17) are compared to T_c dataset (Fig. 9). Normalized RMSE (RMSE divided by the range of T_c values) is 6% for Eq. (17).

Comparing Eq. (15) for standard slopes with Eq. (17) for low slopes, regression constants or exponents of L, n, and i are similar, but the exponent of S_o (0.3–0.4 in Table 6) is much smaller than the exponent of ($S_o + S_{lb}$), which is 2.139 in Eq. (17). This is because a combined slope ($S_o + S_{lb}$) was used in Eq. (17) instead of topographic slope S_o. It is worth noting that Eq. (17) has a large

Table 6. Statistical Error Parameters for T_c Estimation Formulas Previously Published and Developed in Current Study for Standard Slopes ($S_o \geq 0.1\%$)

Source or function	Formula	R^2	RMSE[a] (min)	p-value[b]
Henderson and Wooding (1964)	$T_c = 6.98L^{0.60}n^{0.60}/(i^{0.40}S_o^{0.3})$	0.936	14.9	—
Morgali and Linsley (1965)	$T_c = 7.05L^{0.593}n^{0.605}/(i^{0.388}S_o^{0.38})$	0.962	11.3	—
$T_c = f(L, S_o, n, i)$, Eq. (15)	$T_c = 8.67L^{0.541}n^{0.649}/(i^{0.391}S_o^{0.359})$	0.974	6.4	<0.0001
$T_c = f(nL, S_o, i)$	$T_c = 5.89(nL)^{0.617}/(i^{0.400}S_o^{0.358})$	0.953	8.7	<0.0001
$T_c = f(L/\sqrt{S_o}, n, i)$	$T_c = 9.84n^{0.659}(L/\sqrt{S_o})^{0.596}/i^{0.392}$	0.946	8.9	<0.0001
$T_c = f(nL/\sqrt{S_o}, i)$	$T_c = 6.82(nL/\sqrt{S_o})^{0.633}/i^{0.398}$	0.939	10.5	<0.0001

[a]Statistical parameters R^2 and RMSE were developed against T_c data generated from 550 Q2DWM model runs for the parametric study.
[b]The p-value reported herein was developed between T_c and all input variables in each regression equation.

Table 7. Parameter Estimates for the Independent Variables of Time of Concentration (T_c) Estimation Formula (17) for Low Slopes ($S_o < 0.1\%$)

Parameter	Parameter estimate	95% confidence limits		Standard error	t-value	p-value
$Ln(C_2)$	−9.310	−10.288	−8.331	0.496	−18.77	<0.0001
k_5 for L	0.563	0.517	0.609	0.023	24.08	<0.0001
k_6 for ($S_o + S_{lb}$)	−2.139	−2.281	−1.997	0.072	−29.74	<0.0001
k_7 for n	0.612	0.575	0.648	0.019	32.77	<0.0001
k_8 for i	−0.304	−0.354	−0.254	0.025	−11.98	<0.0001

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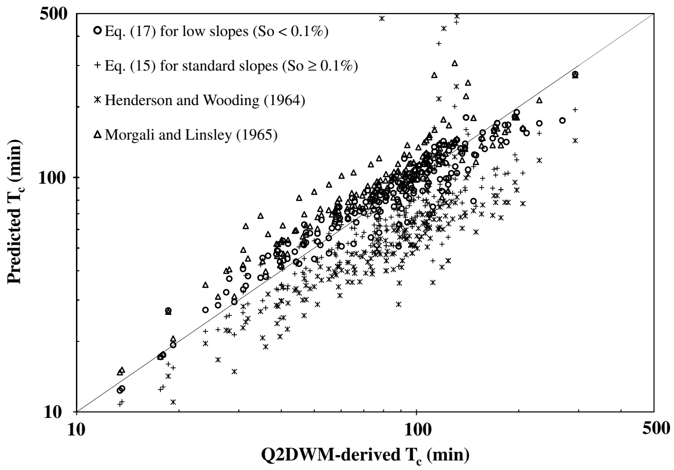

Fig. 9. Time of concentration (T_c) of overland flow planes predicted using regression Eqs. (17) and (15) and the formulas of Henderson and Wooding (1964) and Morgali and Linsley (1965) versus T_c developed from numerical experiments using Q2DWM for low slopes ($S_o < 0.1\%$)

coefficient in the denominator. The combination of large coefficient and large exponent for $(S_o + S_{lb})$ in the denominator produces T_c values that are acceptable in low-sloped planes.

When topographic slope S_o is much smaller than S_{lb}, e.g., $S_o < 0.005\%$, predicted T_c using Eq. (17) changes only slightly as S_o approaches zero, which is displayed in Fig. 7. This result also indicates that Eq. (17) agrees well with the data for two example cases in Fig. 7. Furthermore, this result corroborates those of previous studies (Sheridan 1994; Su and Fang 2004; Li et al. 2005; Li and Chibber 2008), concluding that negligible change occurs in T_c at low topographic slopes. Predicted T_c values from Eq. (17) correlated reasonably well with low-sloped T_c dataset ($R^2 = 0.87$, Fig. 9). However, T_c values predicted using Eq. (15) and formulas of Henderson and Wooding (1964) and Morgali and Linsley (1965) have very weak correlations with the same dataset, i.e., R^2 varied from 0.17 to 0.23 and RMSE from 144 to 716 min, indicating less of the variance is captured by these formulas.

Summary and Conclusions

A combination of field monitoring and numerical studies was performed to develop an ancillary dataset to further evaluate time of concentration, T_c, for overland flow, especially for low-sloped planes. The field study was conducted on a concrete plot with recording rain gauge and flow measurement equipment to extend the research database for relatively low-sloped planes of 0.25%. Rainfall and runoff data were recorded for 27 events between April 2009 and March 2010.

A quasi-two-dimensional dynamic wave model, Q2DWM, was developed to simulate runoff hydrographs for standard ($S_o \geq 0.1\%$) and low-sloped planes ($S_o < 0.1\%$). Q2DWM was validated using data from published studies and collected at the experimental watershed. The average Nash-Sutcliffe coefficients were 0.97 and 0.82 for published and field data, respectively. The validated Q2DWM model was used in a parametric study to generate T_c data for a range of slopes and other input variables (length L, roughness coefficient n, and rainfall intensity i) that were used to develop T_c regression formulas for standard and low slopes. In the authors' parametric study, T_c was defined as the time from the beginning of effective rainfall to the time when the flow reaches 98% of peak discharge. Classical formulas like Henderson and Wooding (1964) and Morgali and Linsley (1965) for estimating T_c deviate from modeled values where the watershed topographic slope is less than about 0.1%. This value (0.1%) is termed the lower-bound slope, S_{lb}. Slopes less than S_{lb} are defined as low slopes; those equal to or greater than S_{lb} are defined as standard slopes ($S_o \geq 0.1\%$).

During the parametric study, n was varied from 0.01 to 0.80, L from 5 to 305 m (16 to 1000 ft), i from 2.5 to 254 mm/h (0.1 to 10.0 in./h), and S_o from 0.0001 to 10%. Seven hundred fifty Q2DWM runs were conducted. Four regression equations (Table 6) were developed for T_c estimation of overland flow planes for standard slopes ($S_o \geq 0.1\%$). Formulas developed in this study and by Henderson and Wooding (1964) and Morgali and Linsley (1965) for standard slopes performed poorly in predicting T_c for low slopes with R^2 from 0.17 to 0.23. However, Eq. (17), which resulted from the regression analysis of 200 Q2DWM-derived low-sloped T_c dataset, performed reasonably well, with an R^2 of 0.87. Eq. (17) was developed for overland flow on low-sloped planes using $S_o + S_{lb}$ in place of topographic slope S_o. This equation is recommended for estimating T_c where topographic slopes are low ($S_o < 0.1\%$).

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Notation

The following symbols are used in this paper:

a_{cj}, a_{lj} = convective and local accelerations;
C_r = Courant number;
C_1, C_2 = regression coefficients;
g = acceleration due to gravity in meters/seconds^2;
H = water surface elevation in meters;
h = flow depth in meters;
$h_{\max}$ = maximum flow depth in meters of all cells in the domain;
h_p^{t-1}, h_p^t = flow depth at cell P in meters at time step $t-1$ and t;
$h_{v\max}$ = corresponding flow depth in meters where $v_{\max}$ occurs in the domain;
i = rainfall intensity in m/s or mm/h;
j = subscript that stands for flow direction (east, west, north, and south);
K_j = conduction parameter in j direction;
k_n = 1 (SI units) or 1.49 (FPS units);
$k_1, \ldots, k_8$ = regression constants for power functions of T_c estimation formulas;
L = plot length in meters;
m_j = dimensionless momentum quantity in j direction;
Ns = Nash-Sutcliffe coefficient;
n = Manning roughness coefficient;
p = arbitrary cell number;
Q_p = peak discharge in m^2/s or cubic meter seconds;
Q_{pm} = measured peak discharge in cubic meter seconds;
Q_{ps} = simulated peak discharge in cubic meter seconds;
q_{east}, q_{north}, q_{south}, q_{west} = flow rates per unit width in m^2/s in east, north, south and west direction;
q_j = flow rates per unit width in m^2/s in j direction;

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RMSE = root mean square error between observed and simulated discharges in cubic meter seconds;
R^2 = coefficient of determination;
S_{fj} = frictional slope in meters/meters in j direction;
S_{lb} = lower bound topographic slope in meters/meters;
S_o = topographic slope in meters/meters;
T_c = time of concentration;
T_{cm} = measured time of concentration in minutes;
T_{cs} = simulated time of concentration in minutes;
T_p = observed time to peak in minutes or hours;
T_{pm} = measured time to peak in hours;
T_{ps} = simulated time to peak in hours;
t = time in seconds;
$t-1$ = previous time step;
$t+1$ = next time step;
$v_{h\max}$ = corresponding flow velocity in m/s where $h_{\max}$ occurs in the domain;
v_j = flow velocity in m/s in j direction;
$v_{\max}$ = maximum flow velocity in m/s of all cells in the domain;
X = distance in meters from its boundary along each j direction;
z = bottom elevation in meters;
α = parameter given by $S_{fj}^{0.5}/n$;
$\beta = 5/3$;
Δt = time step in seconds;
$\Delta t_{\max}$ = maximum time step in seconds;
$\Delta t_{\min}$ = minimum time step in seconds;
Δj = spacing in j direction; and
Δx, Δy = spacing in x or y direction.

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