from: Viessman,W., Knapp, J.W., Lewis, G. L., and Harbaugh, T.E. 1977. "Introduction to Hydrology" 2ed. IEP Publishers, New York, 704p.

8

Groundwater Hydrology

8-1 Introduction

The amount of water stored below ground in the United States exceeds by a significant amount all above ground storage in streams, rivers, reservoirs, and lakes including the Great Lakes.[6] This enormous reservoir sustains streamflow during precipitation-free periods and constitutes the major source of fresh water for many arid localities. Figure 8-1 indicates the distribution and nature of primary groundwater areas of the United States.

The quantification of the volume and rate of flow of groundwater in various regions is an exceedingly difficult task because volumes and flow rates are determined to a considerable extent by the geology of the region. The character and arrangement of rocks and soils are important factors, and these are often highly variable within a groundwater reservoir. An additional difficulty is the inability to measure directly many critical geologic and hydraulic reservoir characteristics.

In spite of these predicaments, hydrologists are continually developing new techniques for measurement and analysis that are contributing to an extensive body of knowledge in the field of groundwater hydrology. Many practical problems can be adequately solved by employing these techniques. This chapter presents the fundamentals of flow in a porous medium and shows how they are applied to the solution of various hydrologic problems.

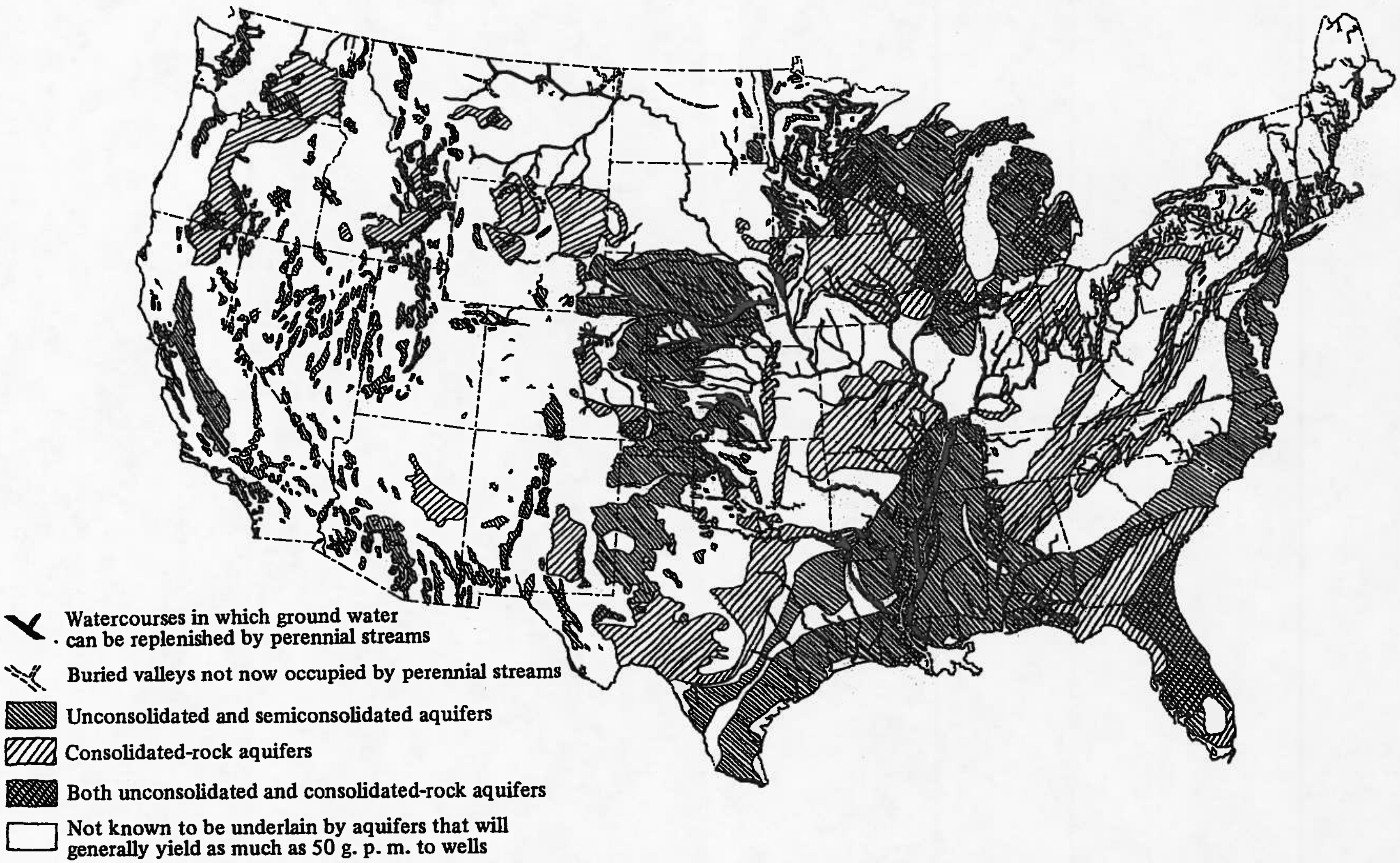


Fig. 8-1. *Groundwater areas in the United States. Patterns show that areas underlaid by aquifers are generally capable of yielding to individual wells 50 gpm or more of water containing not more than 2000 ppm of dissolved solids (includes some areas where more highly mineralized water is actually used). (From H. E. Thomas, "Underground Sources of Water"* Water, The Yearbook of Agriculture, *Washington, D.C.: U.S. Department of Agriculture 1955.)*

8-2 Groundwater Flow—General Properties

Understanding the movement of groundwater requires a knowledge of the time and space dependency of the flow, the nature of the porous medium and fluid, and the boundaries of the flow system.

Groundwater flows are usually three-dimensional. Unfortunately, the solution of such problems by analytic methods is intensely complex unless the system is symmetric.[2] In other cases, space dependency in one of the coordinate directions may be so slight that assumption of two-dimensional flow is satisfactory. Many problems of practical importance fall into this class. Sometimes one-dimensional flow can be assumed, thus further simplifying the solution.

Fluid properties such as velocity, pressure, temperature, density, and viscosity often vary in time and space. When time dependency occurs, the issue is termed an *unsteady flow problem* and solutions are usually difficult. On the other hand, situations where space dependency alone exists are *steady flow problems*. Only homogeneous (single-phase) fluids will be considered here. For a discussion of multiple phase flow, Ref. 2 is recommended.

Boundaries to groundwater flow systems may be fixed geologic structures or free water surfaces that are dependent for their position on the state of the flow. A hydrologist must be able to define these boundaries mathematically if he is to solve groundwater flow problems.

Porous media through which groundwaters flow may be classified as isotropic, anisotropic, heterogeneous, homogeneous, or several possible combinations of these. An *isotropic* medium has uniform properties in all directions from a given point. *Anisotropic* media have one or more properties that depend on a given direction. For example, permeability of the medium might be greater along a horizontal plane than along a vertical one. *Heterogeneous* media have nonuniform properties of anisotropy or isotropy, while *homogeneous* media are uniform in their characteristics.

8-3 Subsurface Distribution of Water

Groundwater distribution may be generally categorized into zones of aeration and saturation. The saturated zone is one in which all voids are filled with water under hydrostatic pressure. In the zone of aeration, the interstices are filled partly with air, partly with water. The saturated zone is commonly called the *groundwater zone*. The zone of aeration may ideally be subdivided into several subzones. Todd classifies these as follows.[4]

1. *Soil water zone.* A soil water zone begins at the ground sur-

face and extends downward through the major root band. Its total depth is variable and dependent upon soil type and vegetation. The zone is unsaturated except during periods of heavy infiltration. Three categories of water classification may be encountered in this region: hygroscopic water, which is adsorbed from the air; capillary water, held by surface tension; and gravitational water, which is excess soil water draining through the soil.

2. *Intermediate zone.* This belt extends from the bottom of the soil-water zone to the top of the capillary fringe and may change from nonexistence to several hundred feet in thickness. The zone is essentially a connecting link between a near-ground surface region and the near-water-table region through which infiltrating fluids must pass.

3. *Capillary zone.* A capillary zone extends from the water table (Figure 8-2) to a height determined by the capillary rise that can be generated in the soil. The capillary band thickness is a function of soil texture and may fluctuate not only from region to region but also within a local area.

4. *Saturated zone.* In the saturated zone, groundwater fills the pore spaces completely and porosity is therefore a direct measure of storage volume. Part of this water (specific retention) cannot be removed by pumping or drainage because of molecular and surface tension forces. Specific retention is the ratio of volume of water retained against gravity drainage to gross volume of the soil.

Water that can be drained from a soil by gravity is known as the *specific yield.* It is expressed as the ratio of the volume of water that can be drained by gravity to the gross volume of the soil. Values of

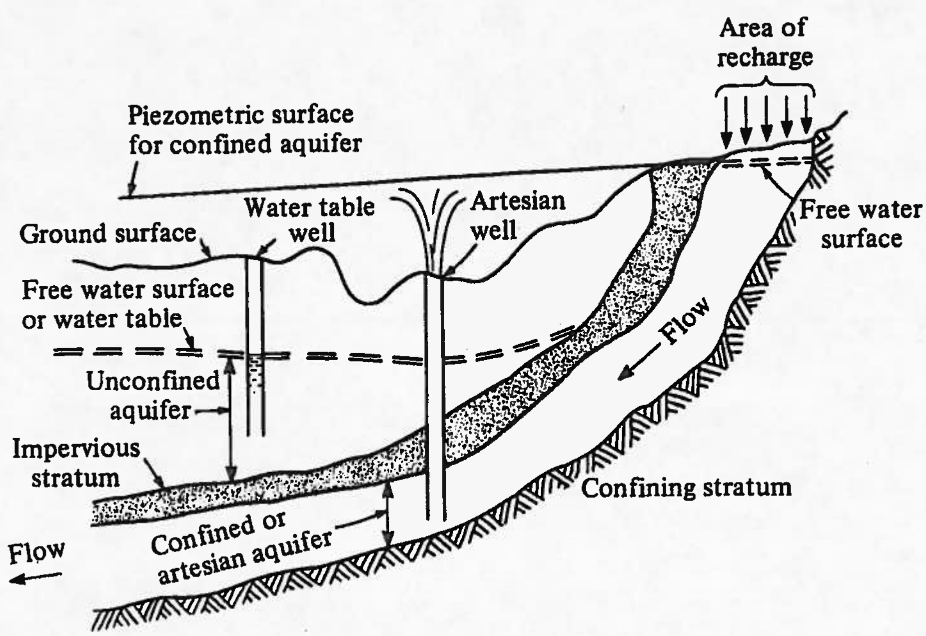


Fig. 8-2. Aquifer classifications.

specific yield depend upon the soil particle size, shape and distribution of pores, and degree of compaction of the soil. Average values for alluvial aquifers range from 10 to 20 %. Meinzer and others have developed procedures for determining the specific yield.[46]

8-4 Geologic Considerations

The determination of groundwater volumes and flow rates requires a thorough knowledge of the geology of a groundwater basin. In bedrock areas, hydrologic characteristics of the rocks, that is, their location, size, orientation, and ability to store or transmit water, must be known. In unconsolidated rock areas, basins often contain hundreds to thousands of feet of semiconsolidated to unconsolidated fill deposits that originated from the erosion of headwater areas. Such fills often contain extensive quantities of stored water. The characteristics of these basin fills must be evaluated.

A knowledge of the distribution and nature of geohydrologic units such as *aquifers, aquifuges,* and *aquicludes* is essential to proper planning for development or management of groundwater supplies. In addition, bedrock basin boundaries must be located and an evaluation made of their leakage characteristics.

An aquifer is a water-bearing stratum or formation that is capable of transmitting water in quantities sufficient to permit development. Aquifers may be considered as falling into two categories, confined and unconfined, depending upon whether or not a water table or free surface exists under atmospheric pressure. Storage volume within an aquifer is changed whenever water is recharged to, or discharged from, an aquifer. In the case of an unconfined aquifer this may be easily determined as

$$\Delta S = S_y \Delta V \tag{8-1}$$

where

ΔS = the change in storage volume
S_y = the average specific yield of the aquifer
ΔV = the volume of the aquifer lying between the original water table and the water table at some later specified time

For saturated, confined aquifers, pressure changes produce only slight modifications in the storage volume. In this case, the weight of the overburden is supported partly by hydrostatic pressure and somewhat by solid material in the aquifer. When hydrostatic pressure in a confined aquifer is reduced by pumping or other means, the load on the aquifer increases, causing its compression, with the result that some water is forced out. Decreasing the hydrostatic pressure also causes a small expansion which in turn produces an additional release

S_c = .0001 – .001

of water. For confined aquifers, water yield is expressed in terms of a *storage coefficient* S_c, defined as the volume of water an aquifer takes in or releases per unit surface area of aquifer per unit change in head normal to the surface. Figure 8-2 illustrates the classifications of aquifers.

In addition to water-bearing strata exhibiting satisfactory rates of yield, there are also nonwater-bearing and impermeable strata that may contain large quantities of water but whose transmission rates are not high enough to permit effective development. An aquifuge is a formation impermeable and devoid of water; an aquiclude is an impervious stratum.

8-5 Fluctuations in Groundwater Level

Any circumstance that alters the pressure imposed on underground water will also cause a variation in the groundwater level. Seasonal factors, changes in stream and river stages, evapotranspiration, atmospheric pressure changes, winds, tides, external loads, various forms of withdrawal and recharge, and earthquakes all may produce fluctuations in the water table level or piezometric surface, depending upon whether the aquifer is free or confined.[4] It is important that an engineer concerned with the development and utilization of groundwater supplies be aware of these factors. He should also be able to evaluate their importance relative to operation of a specific groundwater basin.

8-6 Groundwater-Surfacewater Relationships

Notwithstanding that water resource development has often been based on the predominant use of either surface or groundwaters, it must be emphasized that these two components of the total water resource are interdependent. Changes in one component can have far-reaching effects on the other. Coordinated development and management of the combined resource is critical. Linkage between surface and groundwaters should be investigated in all regional studies so that adverse effects can be noted if they exist and opportunities for joint management understood.

In Chapter 4 it was shown how surface stream flows are sustained by the groundwater resource, and it was also pointed out that groundwaters are replenished by infiltration derived from precipitation on the earth's surface.

Underground reservoirs often are extensive and can serve to store water for a multitude of uses. If withdrawals from these reservoirs consistently exceed recharge, *mining* occurs and ultimate depletion of the resource results. By properly coordinating the use of surface and groundwater supplies, optimum regional water resource development seems most likely to be assured. Several studies directed toward this coordinated use have been initiated.[1,5,18]

8-7 Hydrostatics

Water located in pore spaces of a saturated medium is under pressure (called *pore pressure*) which can be determined by inserting a piezometer in the medium at a point of interest. If location A (Fig. 8-3) is considered, it can be seen that pore pressure is given by

$$p = h_a \gamma \tag{8-2}$$

where

p = the pore pressure (gauge pressure)
h_a = the head measured from the point to the water table
γ = the specific weight of water

Pore pressure is considered positive or negative, depending upon whether the pressure head is measured above (positive) or below (negative) the point under consideration. If an arbitrary datum is established, the total head or piezometric head above the datum is

$$P_p = z + h \tag{8-3}$$

where P_p is known as the piezometric potential. In Fig. 8-3 this is equal to $h_a + z_a$ for point A in the saturated zone and $z_b - h_b$ for point B in the unsaturated zone. The term h_a is the pore pressure of A while $-h_b$ denotes tension or vacuum (negative pore pressure) at B.

8-8 Groundwater Flow

Analogies can be drawn between flow in pipes under pressure and in fully saturated confined aquifers. The flow of groundwater with a free

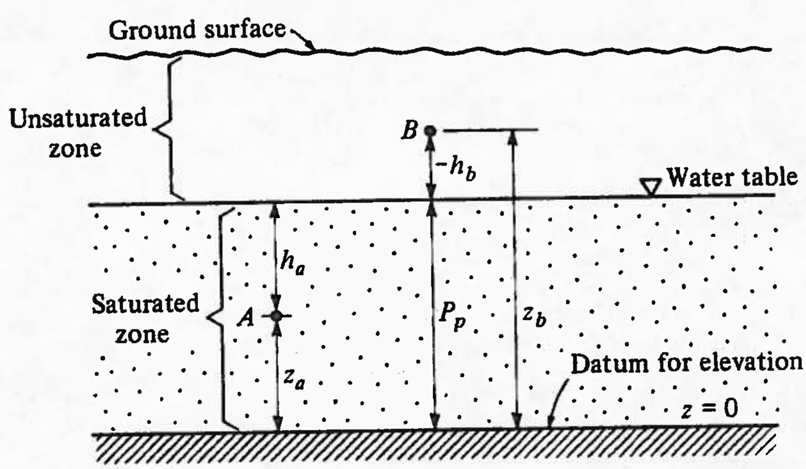


Fig. 8-3. *Definition sketch showing hydrostatic pressures in a porous medium.*

surface is also similar to that in an open channel. A major difference is the geometery of a groundwater system flow channel as compared with common hydraulic pipe flow or channel systems. The problem can easily be recognized by envisioning a discharging cross section composed of a number of small openings, each with its own geometry, orientation, and size so that the flow velocity issuing from each pore varies in both magnitude and direction. Difficulties in analyzing such systems are apparent. Computations are usually based on macroscopic averages of fluid and medium properties over a given cross-sectional area.

Unknown quantities to be determined in groundwater flow problems are density, pressure, and velocity if constant temperature conditions are assumed to exist. In general, water is considered incompressible, so the number of working variables is reduced. An exception to this is discussed later relative to the storage coefficient for a confined aquifer. Primary emphasis here will be placed on the flow of water in a saturated porous medium.

8-9 Darcy's Law

Darcy's law for fluid flow through a permeable bed is stated as[19]

$$Q = -KA\frac{dh}{dx} \tag{8-4}$$

where

A = the total cross-sectional area including the space occupied by the porous material
K = the hydraulic conductivity of the material
Q = the flow across the control area A

In Eq. 8-4,

$$h = z + \frac{p}{\gamma} + C \tag{8-5}$$

where

h = the piezometric head
z = the elevation above a datum
p = the hydrostatic pressure
C = an arbitrary constant

If the specific discharge $q = (Q/A)$ is substituted in Eq. 8-4,

$$q = -K\frac{d}{dx}\left(z + \frac{p}{\gamma}\right) \tag{8-6}$$

Note that q also equals the porosity n multiplied by the pore velocity

V_p. Darcy's law is widely used in groundwater flow problems. Several applications will be illustrated in later sections.

Darcy's law is limited in applicability to cases where the Reynolds number is on the order of 1. For Reynolds numbers less than 1, Darcy's law may be considered valid. Deviations from Darcy's law have been shown to occur at Reynolds numbers as low as 2, depending upon such factors as grain size and shape. The Reynolds number N_R is defined herein as

$$N_R = \frac{\rho q d}{\mu} \tag{8-7}$$

where

q = the specific discharge
d = the mean grain diameter
ρ = fluid density
μ = dynamic viscosity

For many conditions of practical importance (zones lying adjacent to collecting devices are an exception), Darcy's law has been found to apply.

Of special interest is the fact that the Darcy equation is analogous to Ohm's law

$$i = \left(\frac{1}{R}\right) E \tag{8-8}$$

where

i = the current
R = the resistance
E = the voltage

Current and velocity are analogous as are K and $1/R$, and E, and dh/dx. The similarity of the two equations is the basis for electric analog models of groundwater flow systems.[4,38]

Example 8-1 Water temperature in an aquifer is 60°F and the rate of water movement = 1.2 ft/day. The average particle diameter in the porous medium is 0.08 in. Find the Reynolds number and indicate whether Darcy's law is applicable.

Equation 8-7 gives the Reynolds number as

$$N_R = \frac{\rho q d}{\mu}$$

This may also be written as

$$N_R = \frac{q d}{\nu}$$

From Table 1 in Appendix B, ν is found to be 1.21×10^{-5} ft²/sec. Converting the velocity q into units of ft/sec gives $q = 1.2/86{,}400 = 1.39 \times 10^{-5}$. The mean grain diameter in ft = 0.08/12 = 0.0067. Substituting these values in the equation, we obtain

$$N_R = \frac{1.39 \times 10^{-5} \times 0.0067}{1.21 \times 10^{-5}}$$

$$= 0.0077$$

Since $N_R < 1.0$, Darcy's law does apply.

8-10 Permeability

The hydraulic conductivity K is an important parameter that is often separated into two components, one related to the medium, the other to the fluid. The product

$$k = Cd^2 \tag{8-9}$$

called the *specific* or *intrinsic permeability*, is a function of the medium only. In Eq. 8-9, d represents the mean grain diameter of the particles; and C is a constant shape factor associated with packing, size distribution, and other factors.[2,4] Using this definition, hydraulic conductivity, also known as the *coefficient of permeability*, can be written

$$K = \frac{k\gamma}{\mu} \tag{8-10}$$

Dimensions of intrinsic permeability are L^2. Since values of k given as ft² or cm² are extremely small, a unit of measure known as the *darcy* has been widely adopted.

$$1 \text{ darcy} = 0.987 \times 10^{-8} \text{ cm}^2 \quad \text{or} \quad 1.062 \times 10^{-11} \text{ ft}^2$$

Several ways of expressing hydraulic conductivity are reported in the literature. The U.S. Geological Survey has defined the standard coefficient of permeability K_s as the number of gallons per day of water passing through 1 ft² of medium under a unit hydraulic gradient at a temperature of 60°F. Another measure, called the *field coefficient of permeability* K_f, is defined as

$$K_f = K_s\left(\frac{\mu_{60}}{\mu_f}\right) \tag{8-11}$$

where

μ_{60} = the dynamic viscosity of water at 60°F
μ_f = the dynamic viscosity at the prevailing field temperature

500 Ogalla 480 gal/day-ft²

The temperature effect on density is neglected, since it is usually quite small over the range of groundwater temperatures encountered in practice.

It is often convenient to use the coefficient of transmissivity

$$T = K_f b \tag{8-12}$$

where

K_f = the field hydraulic conductivity
b = the saturated depth of the aquifer

Table 8-1 gives the values of the intrinsic permeability and the standard coefficient of permeability for several classes of materials. Considerable variation within divisions can occur; hence a careful geologic survey should accompany all groundwater studies.

Example 8-2 Laboratory tests of an aquifer material give a standard coefficient of permeability $K_s = 3.78 \times 10^2$ gpd/ft². If the prevailing field temperature is 50°F, find the field coefficient of the permeability K_f.

Using Eq. 8-11, we obtain

$$K_f = K_s\left(\frac{\mu_{60}}{\mu_f}\right)$$

From Table 1, Appendix B, the kinematic viscosity at 60°F = 1.21×10^{-5} ft²/sec and at 50°F it is 1.41×10^{-5} ft²/sec. For constant density,

$$K_f = \frac{3.78 \times 10^2 \times 1.21 \times 10^{-5}}{1.41 \times 10^{-5}}$$

and

$$K_f = 3.24 \times 10^2 \text{ gpd/ft}^2$$

Table 8-1 *Some Values of the Standard Coefficient of Permeability and Intrinsic Permeability for Several Classes of Materials*

Material	Approximate Range K_s (gal/day/ft²)	Approximate Range k (darcys)
Clean gravel	10^6–10^4	10^5–10^3
Clean sands; mixtures of clean gravels and sands	10^4–10	10^3–1
Very fine sands; silts; mixtures of sands, silts, clays; stratified clays	10–10^{-3}	1–10^{-4}
Unweathered clays	10^{-3}–10^{-4}	10^{-4}–10^{-5}

8-11 Velocity Potential

Potential theory is directly applicable to groundwater flow computations. The *velocity potential* ϕ is a scalar function of time and space. The potential is defined by

$$\phi(x, y, z) = -K\left(z + \frac{p}{\gamma}\right) + C \tag{8-13}$$

where C is an arbitrary constant. By definition, its derivative with respect to any given direction is the velocity of flow in that direction. Thus it may be written that

$$u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y} \qquad w = \frac{\partial \phi}{\partial z} \tag{8-14}$$

where

u, v, and w = velocities in the x, y, and z directions, respectively

and K is assumed constant. In vector notation this becomes

$$V = \text{grad } \phi = \nabla\phi \tag{8-15}$$

with V the combined velocity vector and

$$\text{grad } \phi = \frac{\partial \phi}{\partial x}\,\mathbf{i} + \frac{\partial \phi}{\partial y}\,\mathbf{j} + \frac{\partial \phi}{\partial z}\,\mathbf{k} = \nabla\phi \tag{8-16}$$

8-12 Hydrodynamic Equations

The determination of values for the variables u, v, w, and h is the target of most groundwater flow problems. The first three variables are the specific discharge components in the x, y, and z directions, respectively, while h is the total head at a specified point in the flow domain. To effect a solution, four equations involving these variables are needed. These are the equations of motion in each direction plus the continuity equation.

The equations of motion are based on Newton's second law,

$$F = ma \tag{8-17}$$

where

F = the force
m = the mass
a = the acceleration

Considering forces acting on a fluid element, accelerations in the three coordinate directions may be determined according to Eq. 8-17. If frictionless flow is assumed (reasonable for many cases of flow in porous media), the body forces plus the surface force (pressure) must be equivalent to the total force in each direction. In the manner of Harr,[3] the following equations (Euler's equations) in the three coordinate directions are obtained:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{8-18}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} \tag{8-19}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} - g \tag{8-20}$$

where X, Y, and Z are body forces per unit mass in each coordinate direction. For steady flow [u, v, w, and $h, \neq f(t)$] the first terms in the left-hand side of each equation vanish. With laminar groundwater flow in the range of validity of Darcy's law, velocities are small (often on the order of 5 ft/yr to 5 ft/day).[4] Thus for steady laminar flow, Eqs. 8-18 through 8-20 reduce to

$$X = \frac{1}{\rho}\frac{\partial p}{\partial x} \qquad Y = \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad Z = \frac{1}{\rho}\frac{\partial p}{\partial z} + g \tag{8-21}$$

In most groundwater flow problems the velocity head is negligible; thus p may be given as $\rho g(h - z)$. Then Eq. 8-21 becomes

$$X = g\frac{\partial h}{\partial x} \qquad Y = g\frac{\partial h}{\partial y} \qquad Z = g\frac{\partial h}{\partial z} \tag{8-22}$$

Remembering that Darcy's law defines $\partial h/\partial x = -u/K$, and so on, it follows that

$$X = -\frac{gu}{K} \qquad Y = -\frac{gv}{K} \qquad Z = -\frac{gw}{K} \tag{8-23}$$

For steady laminar flow, the body forces are linear functions of velocity and Eqs. 8-18 to 8-20 may be written as

$$g\frac{\partial h}{\partial x} = -g\frac{u}{K} \tag{8-24}$$

$$g\frac{\partial h}{\partial y} = -g\frac{v}{K} \tag{8-25}$$

$$g\frac{\partial h}{\partial z} = -g\frac{w}{K} \tag{8-26}$$

where

$$u = -\frac{K\partial h}{\partial x}$$

$$v = -\frac{K\partial h}{\partial y} \tag{8-27}$$

$$w = -\frac{K\partial h}{\partial z}$$

This demonstrates that the equations of motion fit Darcy's law for steady laminar flow.

The continuity equation may be stated as[20]

$$\frac{\partial \rho}{\partial t} + \partial\frac{(\rho u)}{\partial x} + \partial\frac{(\rho v)}{\partial y} + \partial\frac{(\rho w)}{\partial z} = 0 \tag{8-28}$$

This equation is valid for a compressible fluid with time-dependent properties. In steady compressible flow the first term becomes zero, and for steady incompressible flow the equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{8-29}$$

Now since $u = \partial\phi/\partial x$, and so on, Eq. 8-29 becomes

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{8-30}$$

which is known as the *Laplace equation.* With steady state laminar flow, groundwater motion is completely described by the continuity equation subject to appropriate boundary conditions.

If the hydraulic conductivity K is constant, Eq. 8-30 can be written as

$$\nabla^2 h = 0 \tag{8-31}$$

the expression of steady incompressible flow in a homogeneous isotropic porous medium.

For unsteady flow, the compressibility of both aquifer and water are pertinent. Consider a small element of porous medium that has a volume $\Delta x\ \Delta y\ \Delta z$. Then the term in a continuity equation representing a change in storage is defined by

$$\frac{\partial(\rho n\ \Delta x\ \Delta y\ \Delta z)}{\partial t} \tag{8-32}$$

Presupposing that compressive forces are predominant in the vertical (z) direction, lateral changes can be neglected. Thus in terms of the element described, only Δz is considered variable. A storage expression written as the sum of three terms involving partial derivatives of the variables Δz, ρ, and porosity n is[2]

$$\frac{\partial(\rho n\ \Delta x\ \Delta y\ \Delta z)}{\partial t} = \left(n\rho\frac{\partial(\Delta z)}{\partial t} + \rho\,\Delta z\,\frac{\partial n}{\partial t} + n\,\Delta z\,\frac{\partial \rho}{\partial t}\right)\Delta x\ \Delta y \tag{8-33}$$

The three elements on the right can be expressed in terms of pore pressure p, the aquifer compressibility α, and the fluid compressibility β.[2,4]

Fluid compressibility is defined as the reciprocal of its bulk modulus of elasticity. It is given by[4]

$$\beta = -\frac{\partial V/V}{\partial p} \tag{8-34}$$

where

V = the volume
p = the pore pressure

If the piezometric surface of a confined aquifer is lowered a distance of one unit, the amount of water released from a column of aquifer of unit horizontal cross-sectional area is defined as the storage coefficient S. This is analogous to the specific yield S_y of an unconfined aquifer. Obviously, in Eq. 8-34 S is equivalent to ∂V. Further, if the aquifer column is of height b, $V = b$. The change in pressure ∂p is equivalent to the negative product of the change in head (one unit) and specific weight of water. Making these substitutions in Eq. 8-34, we find that

$$\beta = \frac{S}{\gamma b} \tag{8-35}$$

Now if the aquifer material is considered elastic, that is, if Δz and n can be modified, the volume change can be expressed in terms of alteration in the density of the material due to the difference in packing. Thus

$$\left(\frac{\partial V}{V}\right) = -\left(\frac{\partial \rho}{\rho}\right) \tag{8-36}$$

Introducing Eqs. 8-35 and 8-36 into Eq. 8-34 gives

$$\partial \rho = \frac{\rho S}{b\gamma}\,\partial p \tag{8-37}$$

Next, substituting this expression for $\partial \rho$ in Eq. 8-28, we obtain

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = -\frac{\rho S}{b\gamma}\,\frac{\partial p}{\partial t} \tag{8-38}$$

The left-hand side of this equation can be expanded to

$$\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \left(u\,\frac{\partial \rho}{\partial x} + v\,\frac{\partial \rho}{\partial y} + w\,\frac{\partial \rho}{\partial z}\right) \tag{8-39}$$

The second term is normally very small compared with the first and can be neglected. The validity of this assumption improves as the flow angle decreases. Using Eq. 8-39 and the foregoing assumption, Eq. 8-38 becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{S}{b\gamma}\frac{\partial p}{\partial t} \tag{8-40}$$

or if isotropic conditions prevail,

$$K\nabla^2 h = \frac{S}{b\gamma}\frac{\partial p}{\partial t} \tag{8-41}$$

since from Eq. 8-27 $u = -K\,\partial h/\partial x$, and so on. Inserting γh for p and the transmissivity T for Kb produces

$$\nabla^2 h = \frac{S}{T}\frac{\partial h}{\partial t} \tag{8-42}$$

which is the general equation for unsteady flow in a confined aquifer of constant thickness b.

The storage coefficient S and the transmissivity are commonly called the *formation constants* of a confined aquifer. For an unconfined aquifer Eq. 8-42 reverts to

$$\nabla^2 h = \frac{S}{Kb}\frac{\partial h}{\partial t} \tag{8-43}$$

since b is a function of the change in head. The unsteady flow equation for an unconfined aquifer is nonlinear in form. The solution of such an equation is discussed by Jacob.[22] Where variations in saturated thickness of unconfined aquifers are minor, Eq. 8-42 may be used as an approximation.[4]

For unconfined aquifers, the right-hand side of Eq. 8-43 is often negligible so that the equation

$$\nabla^2 h = 0 \tag{8-31}$$

is frequently valid for both steady and unsteady flow.

8-13 Flowlines and Equipotential Lines

Many problems of practical interest in groundwater hydrology can be considered two-dimensional flow problems. The equation of continuity for steady incompressible flow in an isotropic medium then becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8-44}$$

and

$$\nabla^2 h = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \tag{8-45}$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{8-46}$$

The Laplace equation is satisfied by two conjugate harmonic functions ϕ and ψ.[3,4] Curves $\phi(x, y)$ = constant are orthogonal to the curves $\psi(x, y)$ = constant. The function $\phi(x, y)$ is the velocity potential, the function $\psi(x, y)$ is known as the *stream function* and is defined by

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \tag{8-47}$$

Substituting Eq. 8-47 into Eq. 8-44 yields

$$\frac{\partial^2\psi}{\partial x\,\partial y} - \frac{\partial^2\psi}{\partial y\,\partial x} = 0 \tag{8-48}$$

It has already been shown that

$$u = \frac{\partial\phi}{\partial x} \qquad v = \frac{\partial\phi}{\partial y}$$

so we can write

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \qquad \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \tag{8-49}$$

These are known as the *Cauchy-Riemann equations*. The stream function satisfies both the equation of continuity and the equations of Cauchy-Riemann. It can also be shown that the Laplace equation is satisfied and therefore[2,3]

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \tag{8-50}$$

Refer now to Fig. 8-4. If V is a velocity vector tangent to a particle

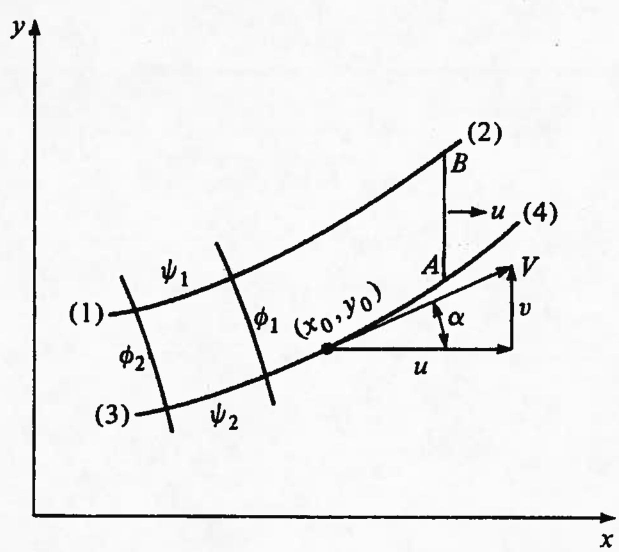


Fig. 8-4. *Definition sketch for a stream function.*

flow path 3–4, then it can be decomposed into two components u and v.[20] By geometry of the figure

$$\frac{v}{u} = \frac{dy}{dx} = \tan \alpha \tag{8-51}$$

and thus

$$v\,dx - u\,dy = 0 \tag{8-52}$$

If Eqs. 8-47 are substituted into Eq. 8-51, then

$$\frac{\partial \psi}{\partial x}\,dx + \frac{\partial \psi}{\partial y}\,dy = 0 \tag{8-53}$$

The total differential $d\psi$ is equal to zero, and ψ must be a constant. A series of curves $\psi(x, y)$ equal to a succession of constants can be drawn and will be tangent at all points to the velocity vectors. These curves trace the flow path of a fluid particle and are known as *streamlines* or *flowlines*. An important property of the stream function is demonstrated with the aid of Fig. 8-4. Consider the flow crossing a vertical section AB between streamlines defined as ψ_1 and ψ_2. If the discharge across the section is designated as Q, it is apparent that

$$Q = \int_{\psi_2}^{\psi_1} u\,dy \tag{8-54}$$

or

$$Q = \int_{\psi_2}^{\psi_1} d\psi \tag{8-55}$$

and

$$Q = \psi_1 - \psi_2 \tag{8-56}$$

Equation 8-56 illustrates the important property that flow between two streamlines is constant. Streamline spacing reveals the relative magnitudes of flow velocities between them. Higher values are associated with narrower spacings, and vice versa.

The curves in Fig. 8-4 designated as ϕ_1 and ϕ_2, called *equipotential lines*, are determined by velocity potentials $\phi(x, y) =$ constant. These curves interesect the flowlines at right angles, illustrated in the following way. The total differential $d\phi$ is given by

$$d\phi = \frac{\partial \phi}{\partial x}\,dx + \frac{\partial \phi}{\partial y}\,dy \tag{8-57}$$

Substituting for terms $\partial\phi/\partial x$ and $\partial\phi/\partial y$ their equivalents u and v makes

$$u\,dx + v\,dy = 0 \tag{8-58}$$

and

$$\frac{dy}{dx} = -\frac{u}{v} \tag{8-59}$$

Thus equipotential lines are normal to flowlines. The system of flowlines and equipotential lines forms a flow net.

One significant point of difference between ϕ and ψ functions is that equipotential lines exist only when the flow is irrotational. For two-dimensional flow the condition of irrotationality is said to exist when the z component of vorticity ζ_z is zero, or

$$\zeta_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \tag{8-60}$$

Proof of this is given by Eskinazi.[20] Substituting for u and v in Eq. 8-60 in terms of ϕ, we obtain

$$\frac{\partial^2 \phi}{\partial x\, \partial y} - \frac{\partial^2 \phi}{\partial y\, \partial x} = 0 \tag{8-61}$$

This indicates that when the velocity potential exists, the criterion for irrotationality is satisfied.

Once either streamlines or equipotential lines in a flow domain are determined, the other is automatically known because of the relationships in Eq. 8-49. Thus

$$\psi = \int \left(\frac{\partial \phi}{\partial x}\, dy - \frac{\partial \phi}{\partial y}\, dx\right) \tag{8-62}$$

and

$$\phi = \int \left(\frac{\partial \psi}{\partial y}\, dx - \frac{\partial \psi}{\partial x}\, dy\right)$$

It is enough then to determine only one of the functions, since the other can be obtained using relations Eq. 8-62. The complex potential given by

$$w = \phi + i\psi \tag{8-63}$$

where i is the square root of -1 is widely used in analytic flow net analyses.[2,3] Of special importance is the fact that

$$\nabla^2 w = \nabla^2 \phi + i \nabla^2 \psi = 0 \tag{8-64}$$

satisfies the conditions of continuity and irrotationality simultaneously.

Equations presented in this section have been limited to the case of two-dimensional flow. Extension to three dimensions would be obtained in a similar fashion.

8-14 Boundary Conditions

To solve groundwater flow problems it is necessary that appropriate boundary conditions be specified. Some of the more commonly encountered ones are described in this section; more comprehensive discussions will be found elsewhere.[13,23]

Boundary conditions discussed can be categorized as follows: impervious boundaries, surfaces of seepage, constant head boundaries, and lines of seepage (free surfaces).

Impervious boundaries may be man-made objects such as concrete dams, rock strata, or soil strata that are highly impervious. In Fig. 8-5 the impervious boundary AB represents such a limit. Since flow cannot cross an impervious boundary, velocity components normal to it vanish and the impervious boundary is a streamline. In other words, at the boundary, ψ = constant.

Next look at the upstream face of the earth dam BC. At any point of elevation y along BC the pressure can be assumed hydrostatic, or

$$p = \gamma(h - y) \tag{8-65}$$

The definition of a velocity potential states that

$$\phi = -K\left(\frac{p}{\gamma} + y\right) + C \tag{8-66}$$

Substituting for pressure in Eq. 8-66 yields

$$\phi = -K\left(\frac{\gamma(h - y)}{\gamma} + y\right) + C \tag{8-67}$$

and

$$\phi = -Kh + C \tag{8-68}$$

Thus for a constant reservoir level h and an isotropic medium,

$$\phi = \text{constant}$$

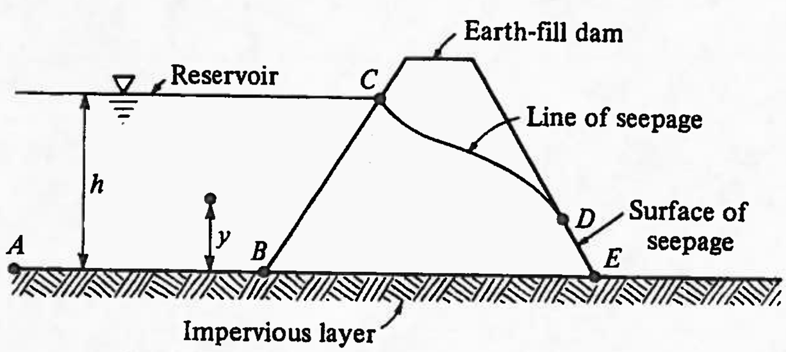


Fig. 8-5. *Some common boundary conditions.*

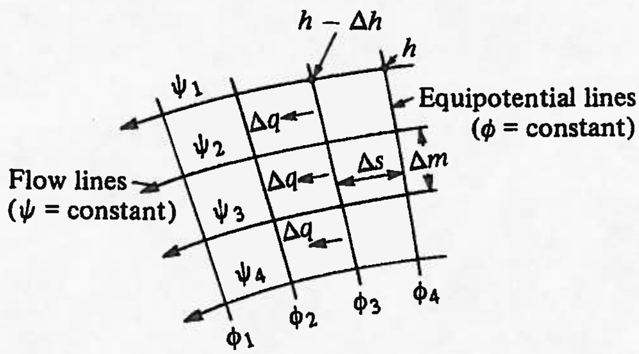


Fig. 8-6. *Segment of an orthogonal flow net.*

and surface BC, often termed a ***reservoir boundary,*** is an equipotential line.

The free surface or line of seepage CD in Fig. 8-5 is seen to be a boundary between the saturated and unsaturated zones. Since flow does not occur across this boundary, it is obviously also a streamline. Pressure along this free surface must be constant, and therefore along CD

$$\phi + Ky = \text{constant} \tag{8-69}$$

This is a linear relationship in ϕ, and therefore equal vertical falls along CD must be associated with successive equipotential drops. One important groundwater flow problem is to determine the location of the line of seepage.

The surface of seepage DE of Fig. 8-5 represents the location at which water seeps through the downstream face of the dam and trickles toward point E. The pressure along DE is atmospheric. The surface of seepage is neither a flowline nor an equipotential line.

8-15 Flow Nets

Flow nets, or graphical representations of families of streamlines and equipotential lines, are widely used in groundwater studies to determine quantities, rates, and directions of flow. The use of flow nets is limited to steady incompressible flow at constant viscosity and density for homogeneous media or for regions that can be compartmentalized into homogeneous segments. Darcy's law must be applicable to the flow conditions.

The manner in which a flow net can be used in problem solving is best explained with the aid of Fig. 8-6. This diagram shows a portion of a flow net constructed so that each unit bounded by a pair of streamlines and equipotential lines is approximately square. The reason for this will be clear later.

A flow net can be determined exactly if functions ϕ and ψ are known beforehand. This is often not the case, and as a result, graphically constructed flow nets have been much used. The preparation of a flow net requires application of the concept of square elements and adherence to boundary conditions. Graphical flow nets are usually difficult for a beginner to create, but with reasonable practice an acceptable net can be drawn. Various mechanical methods for graphical flow net construction are presented in the literature and will not be discussed here.[3,23]

After a flow net has been constructed, it can be analyzed using geometry of the net and by applying Darcy's law.

Remembering that $h = (p/\gamma + z)$, we find that Fig. 8-6 shows that the hydraulic gradient G_h between two equipotential lines is given by

$$G_h = \frac{\Delta h}{\Delta s} \tag{8-70}$$

Then applying Darcy's law, in the manner of Todd,[4] the flow increment between adjacent streamlines is

$$\Delta q = K\,\Delta m\left(\frac{\Delta h}{\Delta s}\right) \tag{8-71}$$

where Δm represents the cross-sectional area for a net of unit width normal to the plane of the diagram. If the flow net is constructed in an orthogonal manner and composed of approximately square elements,

$$\Delta m \approx \Delta s$$

and

$$\Delta q = K\,\Delta h \tag{8-72}$$

Now if there are n equipotential drops between the equipotential lines, it is evident that

$$\Delta h = \frac{h}{n}$$

where h is the total head loss over the n spaces. If the flow is divided into m sections by the flowlines, then the discharge per unit width of the medium will be

$$Q = \sum_{i=1}^{m} \Delta q = \frac{Kmh}{n} \tag{8-73}$$

When the medium's hydraulic conductivity is known, the discharge can be computed using Eq. 8-73 and a knowledge of flow net geometry.

Where the flow net has a free surface or line of seepage, the entrance and exit conditions given in Fig. 8-5 will be useful. A more comprehensive discussion of these conditions is given in Ref. 24.

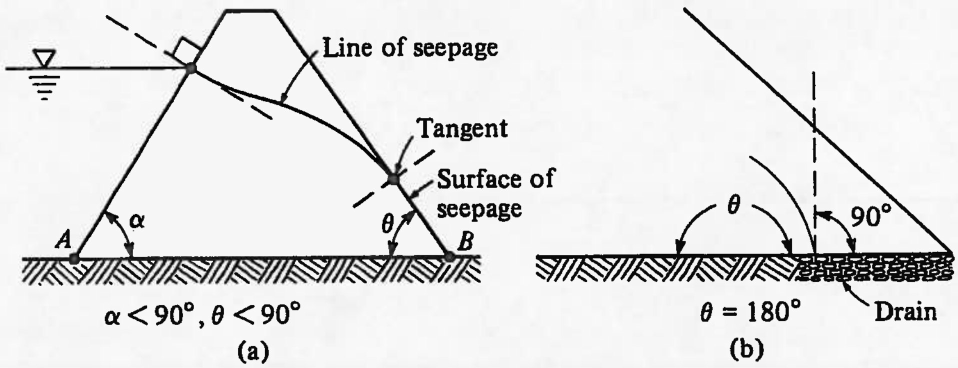


Fig. 8-7. *Some entrance and exit conditions for the line of seepage. (After A. Casagrande, "Seepage Through Dams," in* Contributions to Soil Mechanics, 1925–1940, *Boston: Boston Society of Civil Engineers, 1940.)*

Some trouble arises in flow net construction at locations where the velocity becomes infinite or vanishes. Such points are known as ***singular points*** and according to DeWiest may be placed in three separate categories.[2] In the first classification flowlines and equipotential lines do not intersect at right angles. Such a situation often occurs when a boundary coincides with a flowline; point A in Fig. 8-7 is an example.

The second classification has a discontinuity along the boundary that abruptly changes the slope of the streamline. In Fig. 8-8, points A, B, and C represent such discontinuities. At points A and C the velocity is infinite, while at point B it is zero. If the angle of discontinuity measured in a counterclockwise direction inside the flow field is less than 180°, the velocity is zero; if larger than 180°, it is infinite. The angle at A is 270°, for example.

The third category includes the case where a source or sink exists in the flow net. Under these circumstances the velocity is infinite, since squares of the flow net approach zero size as the source or sink

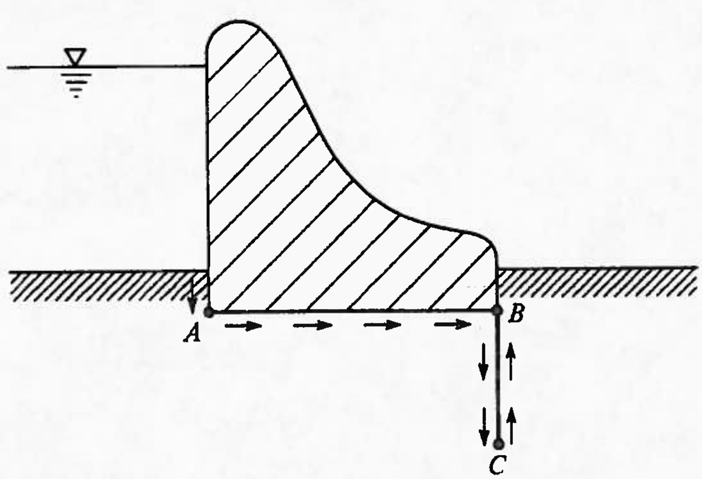


Fig. 8-8. *Flowline slope discontinuities.*

is approached. Wells and recharge wells represent sinks and sources in a practical sense and will be discussed later.

8-16 Variable Hydraulic Conductivity

It is common for flow within a porous medium of one hydraulic conductivity to enter another region with a different hydraulic conductivity. When such a boundary is crossed, flowlines are refracted. The change in direction that occurs can be determined as a function of the two permeabilities involved in the manner of Todd and DeWiest.[2,4] Figure 8-9 illustrates this.

Consider two soils of permeabilities K_1 and K_2 which are separated by the boundary LR shown in Fig. 8-9. The direction of the flowlines before and after crossing the boundary is defined by angles θ_1 and θ_2.

For continuity to be preserved, the velocity components in media K_1 and K_2, which are normal to the boundary, must be equal, since the cross-sectional area at the boundary is AB for a unit depth. Using Darcy's law and noting the equipotential drops h_a and h_b,

$$K_1 \frac{\Delta h_a}{AC} \cos \theta_1 = K_2 \frac{\Delta h_b}{BD} \cos \theta_2 \tag{8-74}$$

From the geometry of the figure it is apparent that

$$AC = AB \sin \theta_1$$

$$BD = AB \sin \theta_2$$

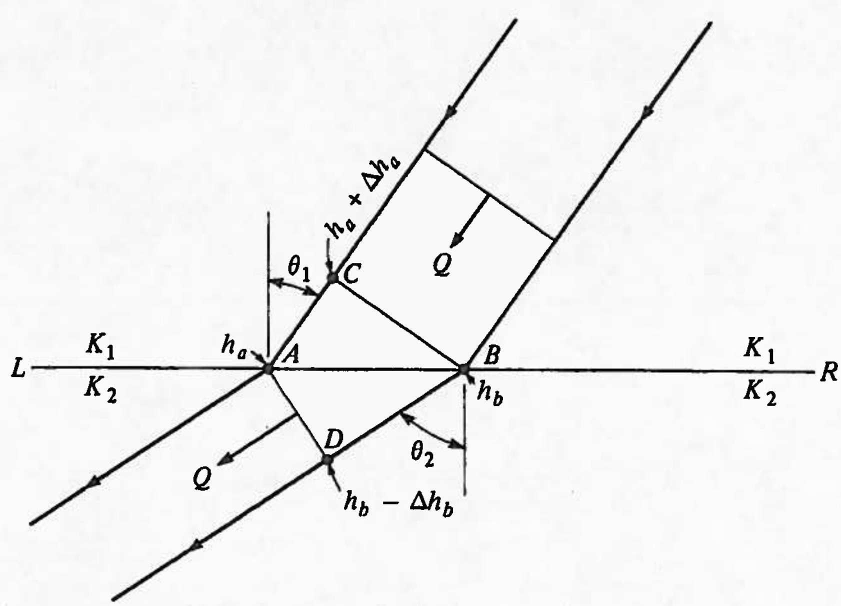


Fig. 8-9. Flowline refraction.

The head loss between A and B is shown on the figure to be equal to both Δh_a and Δh_b, and since there can be only a single value,

$$\Delta h_a = \Delta h_b$$

Introducing these expressions in Eq. 8-74 produces

$$\frac{K_1}{\tan\theta_1} = \frac{K_2}{\tan\theta_2} \tag{8-75}$$

For refracted flow in a saturated porous medium, the ratio of the tangents of angles formed by the intersection of flowlines with normals to the boundary is given by the ratio of hydraulic conductivities. As a result of refraction, the flow net on the K_2 side of the boundary will no longer be squares if the equipotential line spacing DB is maintained. To adjust the net on the K_2 side, the relation

$$\frac{\Delta h_b}{\Delta h_a} = \frac{K_1}{K_2} \tag{8-76}$$

can be used where $\Delta h_b \neq \Delta h_a$.

Equipotential lines are also refracted in crossing permeability boundaries. The relationship for this is

$$\frac{K_1}{K_2} = \frac{\tan\alpha_2}{\tan\alpha_1} \tag{8-77}$$

where α is the angle between the equipotential line and a normal to the boundary of permeability.[2]

8-17 Anisotropy

In many cases hydraulic conductivity is dependent upon the direction of flow within a given layer of soil. This condition is said to be anisotropic. Sedimentary deposits often fit this aspect, with flow occurring more readily along the plane of deposition than across it. Where the permeability within a plane is uniform but very small across it as compared to that along the plane, a flow net can still be used after proper adjustments are made. A discussion of this is given elsewhere.[2,3,33] Nonhomogeneous aquifers require special consideration but may sometimes be analyzed by using representative or average parameters. A detailed study is outside the scope of this book.[2,3,38]

8-18 Dupuit's Theory

Groundwater flow problems in which one boundary is a free surface can be analyzed on the basis of Dupuit's theory of unconfined flow. This theory is founded on two assumptions made by Dupuit in 1863.[14] First, if the line of seepage is only slightly inclined, streamlines may be considered horizontal and, correspondingly, equipotential lines will be essentially vertical. Second, slopes of the line of seepage and the hydraulic gradient are equal. When field conditions are known to be satisfactorily represented by these assumptions, the results obtained according to Dupuit's theory compare very favorably with those arrived at by more rigorous techniques.

Figure 8-10 is useful in translating the foregoing assumptions into a mathematical statement. Consider an element given in the figure which has a base area $dx\,dy$ and a vertical height h. Writing the continuity equation in the x direction and considering steady flow to be the case,

$$\text{inflow}_{x_0} = \text{velocity}_{x_0} \times \text{area}_{x_0} \tag{8-78}$$

The velocity at $x = 0$ is given by Darcy's law as

$$u_{x_0} = -K\frac{\partial h}{\partial x} \tag{8-79}$$

Thus the discharge across the element at $x = 0$ is

$$Q_0 = -K\frac{\partial h}{\partial x}\,h\,dy \tag{8-80}$$

The outflow at $x = dx$ is obtained by a Taylor's series expansion as

$$Q_{dx} = -K\frac{\partial h}{\partial x}\,h\,dy + dx\frac{\partial}{\partial x}\left(-K\frac{\partial h}{\partial x}\,h\,dy\right) + \cdots \tag{8-81}$$

Subtracting the outflow from the inflow if K is considered constant, we obtain

$$I_x - O_x = K\,dx\,dy\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) \tag{8-82}$$

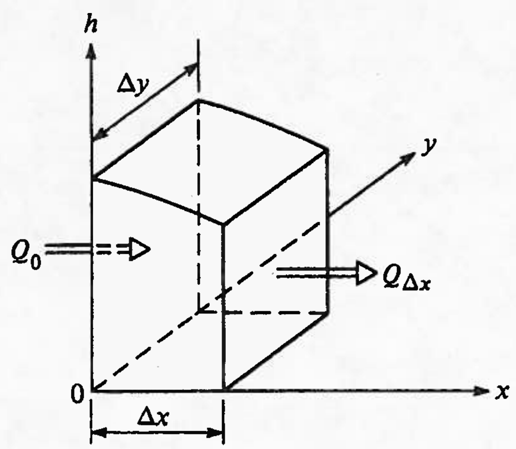


Fig. 8-10. Definition sketch for development of Dupuit's equation.

or

$$I_x - O_x = \frac{K\,dx\,dy}{2}\frac{\partial}{\partial x}\left(\frac{\partial h^2}{\partial x}\right) \tag{8-83}$$

where dx and dy are considered fixed lengths. A similar consideration in the y direction yields

$$I_y - O_y = \frac{K\,dx\,dy}{2}\frac{\partial}{\partial y}\left(\frac{\partial h^2}{\partial y}\right) \tag{8-84}$$

Assuming that there is no movement in the vertical direction, these are the only components of the inflow and outflow. Further, still dealing with steady flow, the change in storage must be zero. As a result,

$$\frac{K\,dx\,dy}{2}\frac{\partial}{\partial x}\left(\frac{\partial h^2}{\partial x}\right) + \frac{K\,dx\,dy}{2}\frac{\partial}{\partial y}\left(\frac{\partial h^2}{\partial y}\right) = 0 \tag{8-85}$$

and since $(K\,dx\,dy)/2$ is constant, this reduces to

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = 0 \tag{8-86}$$

or

$$\nabla^2 h^2 = 0 \tag{8-87}$$

Consequently, according to Dupuit's assumptions, Laplace's equation for the function h^2 must be satisfied.[25]

In the particular case where recharge is occurring as a result of infiltrated water reaching the water table, a simple adjustment may be made to Eq. 8-86. If the recharge intensity (dimensionally LT^{-1}) is specified as R, then the total recharge to the element of Fig. 8-10 will be $R\,dx\,dy$ and the continuity equation for steady flow becomes

$$K\frac{dx\,dy}{2}\left(\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2}\right) + R\,dx\,dy = 0 \tag{8-88}$$

or more simply,

$$\nabla^2 h^2 + \frac{2}{K}R = 0 \tag{8-89}$$

Now, applying Dupuit's theory to the flow problem illustrated on Fig. 8-11b, and assuming one-dimensional flow in the x direction only, the discharge per unit width of the aquifer given by Darcy's law is

$$Q = -Kh\frac{dh}{dx} \tag{8-90}$$

In this instance h is the height of the line of seepage at any position x

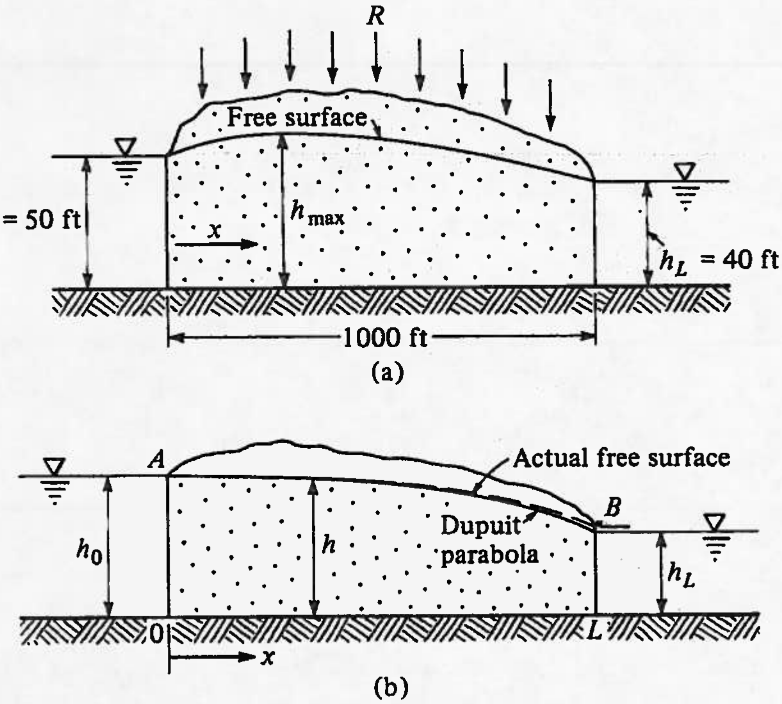


Fig. 8-11. *Steady flow in a porous medium between two water bodies: (a) free surface with infiltration; and (b) free surface without infiltration.*

along the impervious boundary. For the one-dimensional example considered here, Eq. 8-86 becomes

$$\frac{d^2h^2}{dx^2} = 0 \tag{8-91}$$

Upon integration,

$$h^2 = ax + b \tag{8-92}$$

where a and b are constants.

Then for boundary conditions at $x = 0$, $h = h_0$,

$$b = h_0^2 \tag{8-93}$$

Differentiation of Eq. 8-92 yields

$$2h\frac{dh}{dx} = a \tag{8-94}$$

Also from Darcy's equation, $h\,dh/dx = -Q/K$. Making this substitution,

$$a = \frac{-2Q}{K} \tag{8-95}$$

and inserting the values of the constants in Eq. 8-92, we obtain

$$h^2 = -2\frac{Q}{K}x + h_0^2 \tag{8-96}$$

This is the equation of a free surface. It is a parabola (often called *Dupuit's parabola*). If the existence of a surface of seepage at B is ignored, and noting that at $x = L$, $h = h_L$, we find that Eq. 8-96 becomes

$$h_L^2 = -\frac{2QL}{K} + h_0^2 \tag{8-97}$$

or

$$Q = \frac{K}{2L}(h_0^2 - h_L^2) \tag{8-98}$$

which is known as the *Dupuit equation.*

Example 8-3 Refer to Fig. 8-11a. Given the dimensions shown and a recharge intensity R of 0.01 ft/day, find the discharge at $x = 1000$ ft using Dupuit's equation. Assume that $K = 8$.

Solution
Note that

$$\frac{dQ}{dx} = R$$

or

$$Q = Rx + C$$

At $x = 0$,

$$Q = Q_0$$

therefore,

$$Q = R_x + Q_0$$

Also,

$$Q = -Kh\frac{dh}{dx}$$

$$-Kh\frac{dh}{dx} = Rx + Q_0$$

Integrating yields

$$\left.\frac{-Kh^2}{2}\right|_{h_0}^{h_L} = \left.\frac{Rx^2}{2}\right|_0^L + \left.Q_0x\right|_0^L$$

and inserting the limits,

$$\frac{-K(h_L^2 - h_0^2)}{2} = \frac{RL^2}{2} + Q_0L$$

$$Q_0 = \frac{K(h_0^2 - h_L^2)}{2L} - \frac{RL}{2}$$

Then since $Q = Rx + Q_0$,

$$Q = R\left(x - \frac{L}{2}\right) + \frac{K(h_0^2 - h_L^2)}{2L}$$

$$R = 0.01 \times 7.5 = 0.075 \text{ gpd/ft}^2$$

$$Q = 0.075(1000 - 500) + \frac{8(50^2 - 40^2)}{2000}$$

$$= 0.075 \times 500 + \frac{8 \times 900}{2000}$$

$$= 37.5 + 3.6$$

$$= 41.1 \text{ gpd/ft}^2$$

8-19 Methods for Developing Groundwater Supplies

Development of groundwater supplies is accomplished mainly through wells or infiltration galleries. Many factors are involved in the performance of these collection works, and a thorough knowledge of groundwater flow mechanics and regional geology is essential. Some groundwater flow problems can be solved by applying relatively simple mathematical tools. Other problems require more rigorous analyses. Graphical studies and model analyses are also widely employed.

Flow to Wells

A well system can be considered as composed of three elements —the well structure, pump, and discharge piping.[15] The well itself contains an open section through which water enters and a casing to transport the flow to the ground surface. The open section is usually a perforated casing or slotted metal screen permitting water to enter and at the same time preventing collapse of the hole. Occasionally, gravel is placed at the bottom of the well casing around the screen.

When a well is pumped, water is removed from the aquifer immediately adjacent to the screen. Flow then becomes established at locations some distance from the well in order to replenish this withdrawal. Because of flow resistance offered by the soil, a head loss results and the piezometric surface adjacent to the well is depressed, producing a cone of depression (Fig. 8-12), which spreads until equilibrium is reached and steady state conditions are established.

The hydraulic characteristics of an aquifer (which are described by the storage coefficient and aquifer permeability) can be determined by laboratory or field tests. The three most commonly used field methods are the application of tracers, the use of field permeameters, and aquifer performance tests.[4] A discussion of aquifer performance tests will be given here along with the development of flow equations for wells.[11,15,16]

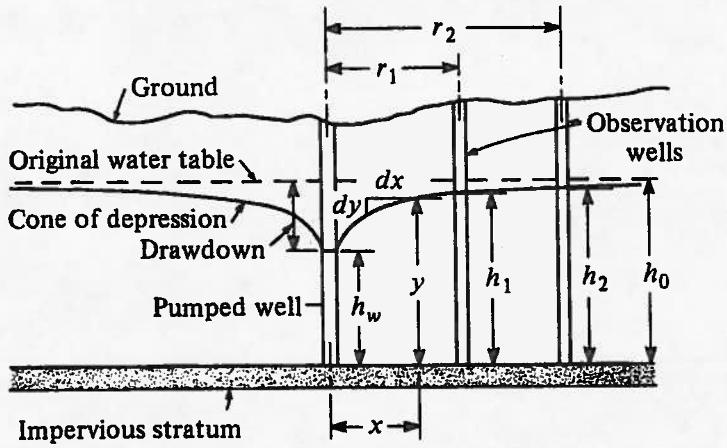


Fig. 8-12. *Well in an unconfined aquifer.*

Aquifer performance tests may be either (1) equilibrium or (2) nonequilibrium tests. In (1) the cone of depression must be stabilized for a flow equation to be derived. For (2) the derivation includes a condition that steady state conditions have not been reached. Adolph Thiem published the first performance tests based on equilibrium conditions in 1906.[8]

8-20 Steady Unconfined Radial Flow Toward a Well

The basic equilibrium equation for an unconfined aquifer can be derived using the notation of Fig. 8-12. Here flow is assumed to be radial; the original water table is considered to be horizontal; the well is presumed to fully penetrate the aquifer of infinite aereal extent; and steady state conditions must prevail. Then flow toward the well at any distance x away must equal the product of the cylindrical element of area at that section and the flow velocity. With Darcy's law this becomes

$$Q = 2\pi x y K_f \frac{dy}{dx} \tag{8-99}$$

where

$2\pi xy$ = the area through any cylindrical shell, in ft^2 with the well as its axis

K_f = the hydraulic conductivity (ft/sec)

dy/dx = the water table gradient at any distance x

Q = the well discharge (ft^3/sec)

Integrating over the limits specified, we find that

$$\int_{r_1}^{r_2} Q \frac{dx}{x} = 2\pi K_f \int_{h_1}^{h_2} y\, dy \tag{8-100}$$

$$Q \log_e \frac{r_2}{r_1} = \frac{2\pi K_f(h_2^2 - h_1^2)}{2} \tag{8-101}$$

and

$$Q = \frac{\pi K_f(h_2^2 - h_1^2)}{\log_e (r_2/r_1)} \tag{8-102}$$

Converting K_f to the field units of gpd/ft², Q to gpm, and $\log_e$ to $\log_{10}$, we can rewrite Eq. 7-102 as

$$K_f = \frac{1055Q \log_{10} (r_2/r_1)}{h_2^2 - h_1^2} \tag{8-103}$$

If the drawdown in the well does not exceed one-half of the original aquifer thickness h_0, reasonable estimates of Q or K_f can be obtained by using Eq. 8-102 or 8-103, even if the height h_1 is measured at the well periphery where $r_1 = r_w$, the radius of the well boring.

Example 8-4 An 18-in. well fully penetrates an unconfined aquifer of 100-ft depth. Two observation wells located 100 and 235 ft from the pumped well are known to have drawdowns of 22.2 and 21 ft, respectively. If the flow is steady and $K_f = 1320$ gpd/ft², what would be the discharge?

Solution
Equation 8-102 is applicable, and for the given units this is

$$Q = \frac{K(h_2^2 - h_1^2)}{1055 \log_{10} (r_2/r_1)}$$

$$\log_{10} (r_2/r_1) = \log_{10} (235/100) = 0.37107$$

$$h_2 = 100 - 21 = 79 \text{ ft}$$

$$h_1 = 100 - 22.2 = 77.8 \text{ ft}$$

$$Q = \frac{1320(79^2 - 77.8^2)}{1055 \times 0.37107}$$

$$= 634.44 \text{ gpm}$$

8-21 Steady Confined Radial Flow Toward a Well

The basic equilibrium equation for a confined aquifer can be obtained in a similar manner, using the notation of Fig. 8-13. The same assumptions apply. Mathematically, the flow in ft³/sec is found from

$$Q = 2\pi x m K_f \frac{dy}{dx} \tag{8-104}$$

Integrating, we obtain

$$Q = 2\pi K_f m \frac{h_2 - h_1}{\log_e (r_2/r_1)} \tag{8-105}$$

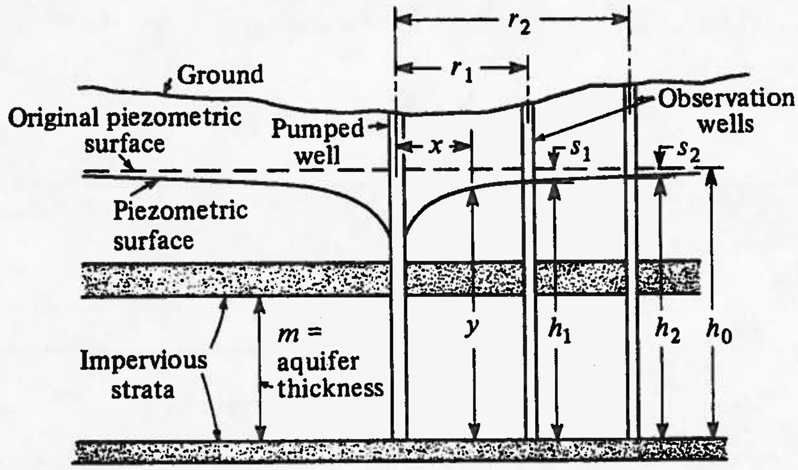


Fig. 8-13. Radial flow to a well in a confined aquifer.

The coefficient of permeability may be determined by rearranging Eq. 8-105 to the form

$$K_f = \frac{528Q \log_{10}(r_2/r_1)}{m(h_2 - h_1)} \tag{8-106}$$

where

Q = gpm
K_f = the permeability (gpd/ft^2)
r and h = ft

Example 8-5 Determine the permeability of an artesian aquifer being pumped by a fully penetrating well. The aquifer is 90 ft thick and composed of medium sand. The steady state pumping rate is 850 gpm. The drawdown of an observation well 50 ft away is 10 ft; in a second observation well 500 ft away it is 1 ft.

Solution

$$K_f = \frac{528Q \log_{10}(r_2/r_1)}{m(h_2 - h_1)}$$

$$= \frac{528 \times 850 \times \log_{10}(10)}{90 \times (10 - 1)}$$

$$= 554 \text{ gpd/ft}^2$$

8-22 Well in a Uniform Flow Field

For a steady state well in a uniform flow field where the original piezometric surface is not horizontal, a somewhat different situation from that previously assumed prevails. Consider the artesian aquifer shown in Fig. 8-14. The heretofore assumed circular area of influence becomes distorted in this case. A solution is possible by applying potential theory; by using graphical means; or, if the slope of the

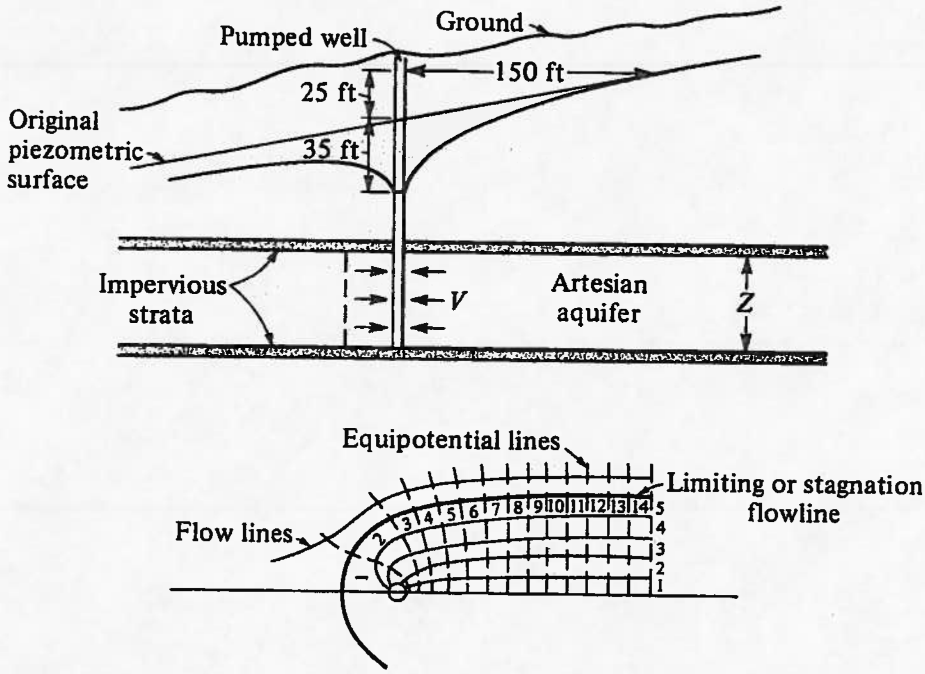


Fig. 8-14. *Well in a uniform flow field and flow net definition.*

piezometric surface is very slight, Eq. 8-105 may be employed without serious error.

Figure 8-14 provides a graphical solution to a uniform flow field problem. First, an orthogonal flow net consisting of flowlines and equipotential lines must be constructed. This should be done so that the completed flow net will be composed of a number of elements that approach little squares in shape. Once the net is complete, it can be analyzed by considering the net geometry and using Darcy's law in the manner of Todd.[4]

Example 8-6 Find the discharge to the well of Fig. 8-14 by using an applicable flow net. Consider the aquifer to be 35 ft thick, $K_f = 3.65 \times 10^{-4}$ fps, and other dimensions as shown.

Solution

Using Eq. 8-73, we find that

$$q = \frac{Kmh}{n}$$

where

$$h = (35 + 25) = 60 \text{ ft}$$

$$m = 2 \times 5 = 10$$

$$n = 14$$

$$q = \frac{3.65 \times 10^{-4} \times 60 \times 10}{14}$$

$$= 0.0156 \text{ cfs per unit thickness of the aquifer}$$

The total discharge Q is thus

$$Q = 0.0156 \times 35 = 0.55 \text{ cfs or } 245 \text{ gpm}$$

8-23 Well Fields

When more than one unit in a well field is pumped, there is a composite effect on the free water surface. This consequence is illustrated by Fig. 8-15 in which the cones of depression are seen to overlap. The drawdown at a given location is equal to the sum of the individual drawdowns.

If within a particular well field, pumping rates of the pumped wells are known, the composite drawdown at a point can be determined. In like manner, if the drawdown at one point is known, the well flows can be calculated.

If the drawndown at a given point is designated as m, and subscripts $1, 2, \ldots, n$ are used to relate this drawdown to a particular well (for example, m_1 refers to the drawdown for W_1) for the total drawdown m_T at some location,[4]

$$m_T = \sum_{i=1}^{n} m_i \tag{8-107}$$

The number of wells, their rate of pumping, and well-field geometry and characteristics determine the total drawdown at a specified location.

Again considering Eq. 8-102, we obtain

$$h_0^2 - h^2 = \frac{Q}{\pi K} \log_e \left(\frac{r_0}{r}\right) \tag{8-108}$$

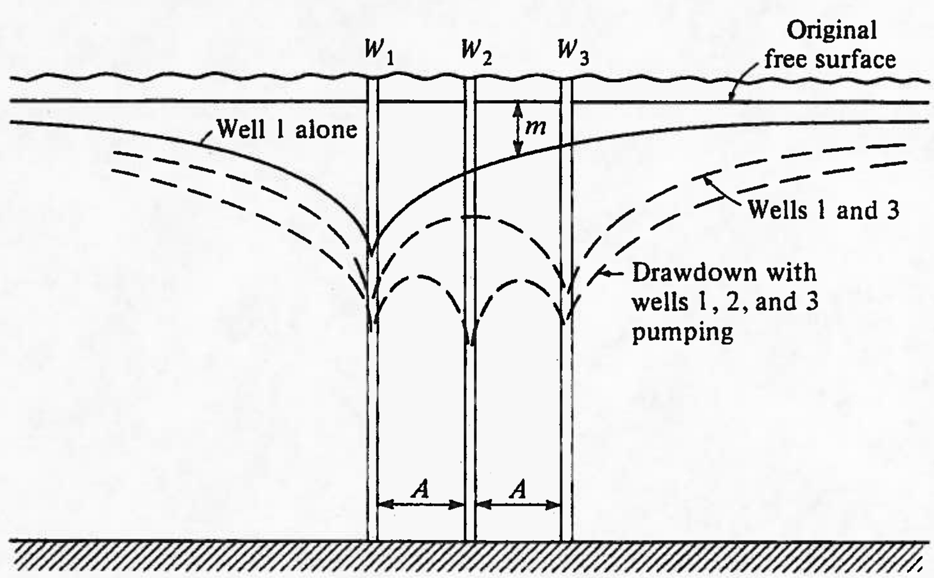


Fig. 8-15. Combined effect of pumping several wells at equal rates.

It can be seen that the drawdown for a well pumped at rate Q can be computed if h_0, r_0, and r are known. It follows then from Eq. 8-107 that for n pumped wells in an unconfined aquifer,

$$h_0^2 - h^2 = \sum_{i=1}^{n} \frac{Q_i}{\pi K} \log_e \left(\frac{r_{0i}}{r_i}\right) \tag{8-109}$$

where

h_0 = the original height of the water table
h = the combined effect height of the water table after pumping n wells
Q_i = the flow rate of the ith well
r_{0i} = distance of the ith well to a location at which the drawdown is considered negligible
r_i = the distance from well i to the point at which the drawdown is being investigated

Todd indicates that values of r_0 used in practice often range from 500 to 1000 ft.[4] The impact of this assumption is softened because Q in Eq. 8-108 is not very sensitive to r_0. Equation 8-109 should be used only where drawdowns are relatively small.

For flow in a confined aquifer the expression for combined drawdown becomes

$$h_0 - h = \sum_{i=1}^{n} \frac{Q_i}{2\pi Km} \log_e \left(\frac{r_{0i}}{r_i}\right) \tag{8-110}$$

Equations for well flow covering a variety of particular well-field patterns are reported in the literature.[4] Those given here are applicable for steady flow in a homogeneous isotropic medium.

8-24 The Method of Images

Some groundwater flow problems subjected to boundary conditions negating the direct use of radial flow equations can be transformed into infinite systems fitting these equations by applying the method of images.[15,22,49]

When a stream is located near a pumped well and the stream and aquifer are interconnected, the drawdown curve of a pumped well may be affected as shown in Fig. 8-16. Another boundary condition often affecting the drawdown of a well is an impervious formation that limits the extent of the aquifer. The cone of depression of a pumped well is not affected until the boundary is intersected. After that, the shape of the drawdown curve will be changed by the boundary. Boundary effects can frequently be evaluated by means of so-called "image wells." The boundary condition is replaced by either a recharging or a discharging well which is pumped or re-

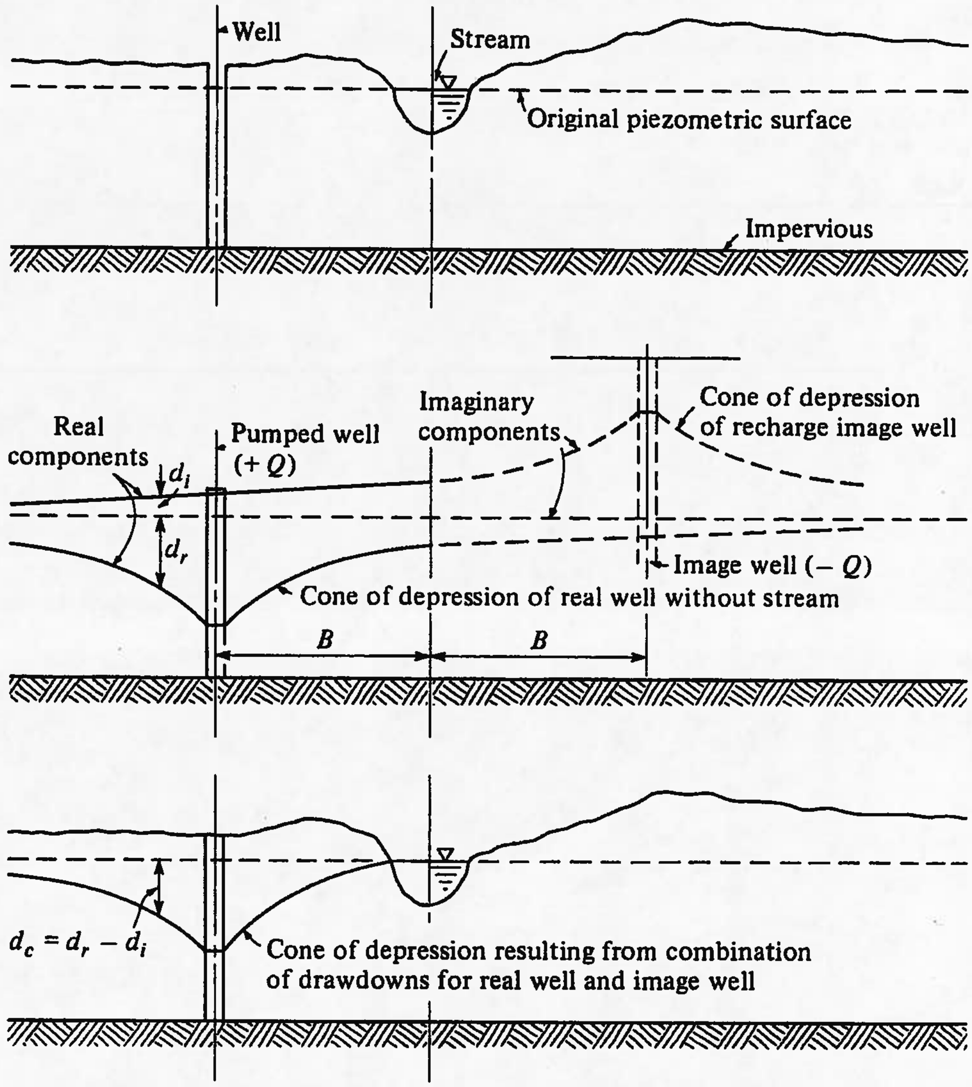


Fig. 8-16. *Drawdown in a pumping well whose aquifer is connected to a stream.*

charged at a rate equivalent to that of the pumped well. That is, in an infinite aquifer, drawdowns of the real and image wells would be identical. The image well is located at a distance from the boundary equal to that of the real well but on the opposite side (Fig. 8-16). Streams are replaced by recharge wells while impermeable boundaries are supplanted by pumped image wells. Computations for the case of a well and impervious boundary directly follow the procedures outlined under the section on well fields. For the well and stream system, the recharge image well is considered to have a negative discharge. The heads are then added according to this sign convention.

The procedure for combining drawdown curves of real and image wells to obtain an actual drawdown curve is illustrated graphically for the example shown in Fig. 8-16. More detailed information on other cases can be found elsewhere.[2,49]

8-25 Unsteady Flow

When a new well is first pumped, a large portion of the discharge comes directly from the storage volume released as the cone of depression develops. Under these circumstances the equilibrium equations overestimate permeability and therefore the yield of the well. When steady state conditions are not encountered—as is usually the situation in practice—a nonequilibrium equation must be used. Two approaches can be taken, the rather rigorous method of C. V. Theis or a simplified procedure such as that proposed by Jacob.[9,10]

In 1935 Theis published a nonequilibrium approach that takes into consideration time and storage characteristics of the aquifer.[9] His method utilizes an analogy between heat transfer described by the Biot-Fourier law, and groundwater flow to a well. Theis states that the drawdown (s) in an observation well located at a distance r from the pumped well is given by

$$s = \frac{114.6Q}{T} \int_u^\infty \frac{e^{-u}}{u}\, du \tag{8-111}$$

where

T = transmissibility (gpd/ft)
Q = discharge (gpm)

and

$$u = \frac{1.87 r^2 S_c}{Tt} \tag{8-112}$$

where

S_c = the storage coefficient
t = time in days since the start of pumping

The integral in Eq. 8-111 is usually known as the *well function* of u and commonly written as $W(u)$. It may be evaluated from the infinite series

$$W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2 \times 2!} + \frac{u^3}{3 \times 3!} + \cdots \tag{8-113}$$

The basic assumptions employed in the Theis equation are essentially the same as those in equation 8-102 except for the nonsteady state condition. Some values of this function are given in Table 8-2.

Equations 8-111 and 8-112 can be solved by comparing a log-log

Table 8-2 *Values of W(u) for Various Values of u*

u	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.000012
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

Source: After L. K. Wenzel, "Methods for Determining Permeability of Water Bearing Materials with Special Reference to Discharging Well Methods," U.S. Geological Survey, Water-Supply Paper 887, Washington, D.C., 1942.

plot of u versus $W(u)$ known as a *type curve,* with a log-log plot of the observed data r^2/t versus s. In plotting type curves, $W(u)$ and s are ordinates, u and r^2/t are abscissas. The two curves are superimposed and moved about until segments coincide. In this operation the axes must remain parallel. A coincident point is then selected on the matched curves and both plots marked. The type curve then yields values of u and $W(u)$ for the desired point. Corresponding values of s and r^2/t are determined from a plot of the observed data. Inserting these values in Eqs. 8-111 and 8-112 and rearranging, values for transmissibility T and storage coefficient S_c can be found.

Often this procedure can be shortened and simplified. When r is small and t large, Jacob found that values of u are generally small.[10]

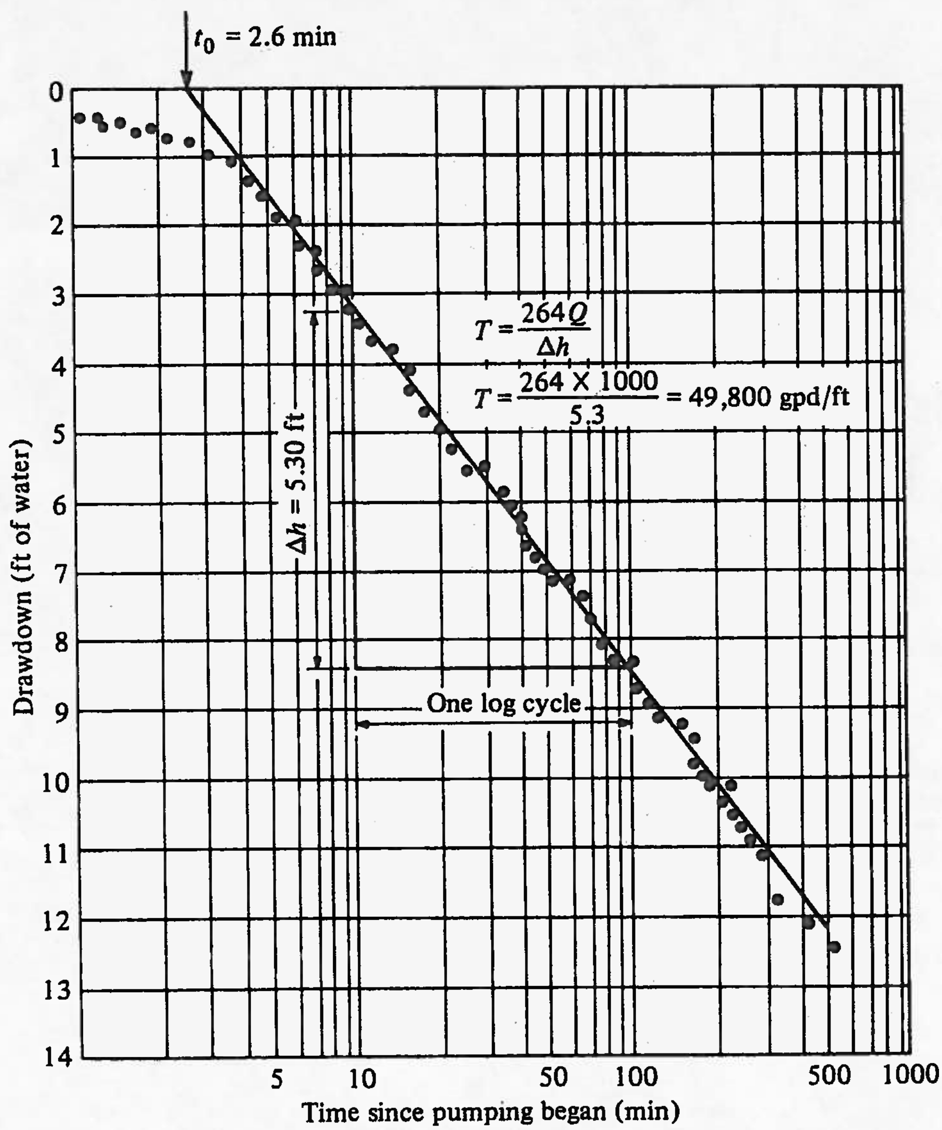


Fig. 8-17. *Pumping test data, Jacob method.*

Thus terms in the series of Eq. 8-113 beyond the second one become negligible and the expression for T becomes

$$T = \frac{264Q(\log_{10} t_2 - \log_{10} t_1)}{h_0 - h} \tag{8-114}$$

which can be further reduced to

$$T = \frac{264Q}{\Delta h} \tag{8-115}$$

where

Δh = the drawdown per log cycle of time $[(h_0 - h)/(\log_{10} t_2 - \log_{10} t_1)]$
Q = well discharge (gpm)
h_0 and h = as defined in Fig. 8-13
T = the transmissibility (gpd/ft)

Field data on drawdown $(h_0 - h)$ versus t are drafted on semilogarithmic paper. The drawdown is plotted on an arithmetic scale, Fig. 8-17. This plot forms a straight line whose slope permits computing formation constants using Eq. 8-115 and

$$S_c = \frac{0.3Tt_0}{r^2} \tag{8-116}$$

with t_0 the time corresponding to zero drawdown.

Example 8-7 Using the following data, find the formation constants for an aquifer using a graphical solution to the Theis equation. Discharge equals 540 gpm.

Distance from Pumped Well, r (ft)	r^2/t	Average Drawdown, s (ft)
50	1,250	3.04
100	5,000	2.16
150	11,250	1.63
200	20,000	1.28
300	45,000	0.80
400	80,000	0.51
500	125,000	0.33
600	180,000	0.22
700	245,000	0.15
800	320,000	0.10

Solution

Plot s versus r^2/t and $W(u)$ versus u as shown in Fig. 8-18. Determine the match point as noted and compute S_c and T using Eqs. 8-111 and 8-112,

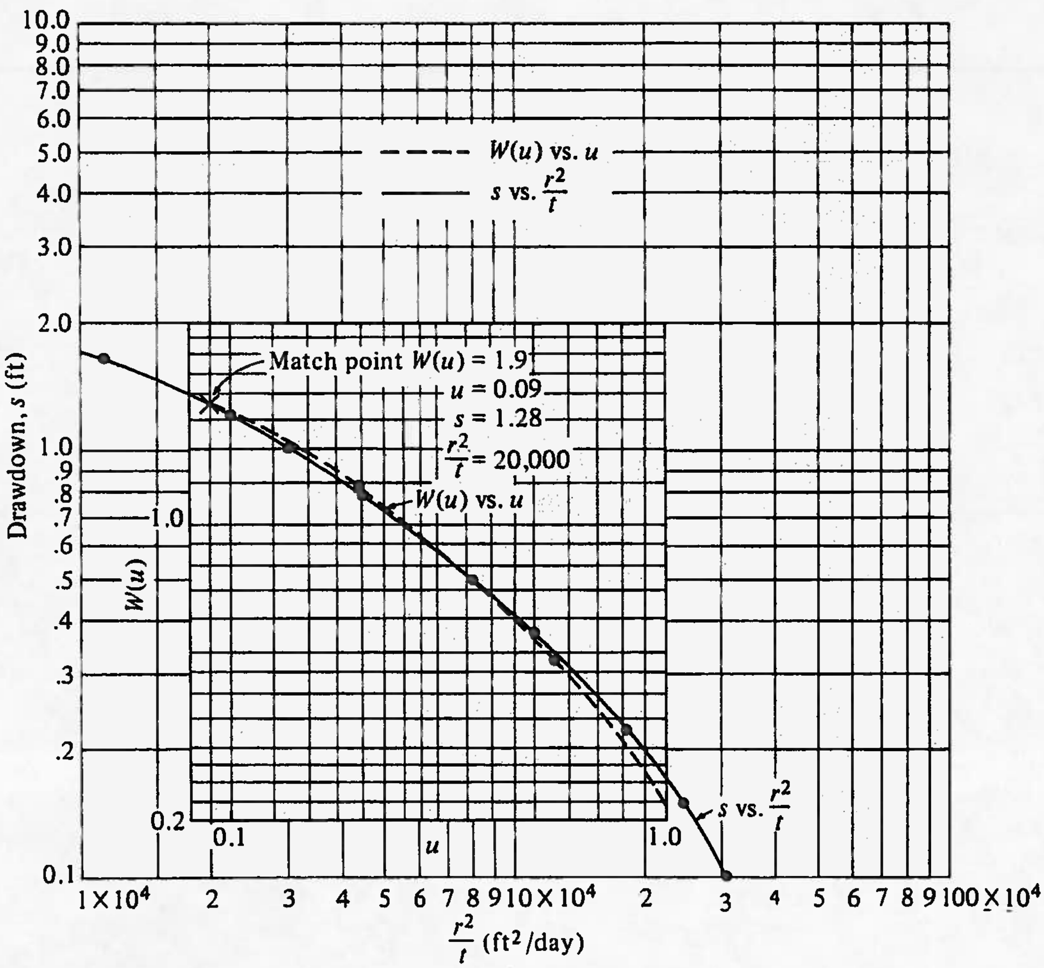


Fig. 8-18. *Graphical solution to Theis' equation.*

$$T = \frac{114.6Q}{s} W(u)$$

$$= \frac{114.6 \times 540}{1.28} \times 1.9 = 99{,}500 \text{ gpd/ft}$$

$$S_c = \frac{uT}{1.87r^2/t}$$

$$= \frac{0.09 \times 99{,}500}{1.87 \times 20{,}000} = 0.240$$

Example 8-8 Using the data given in Fig. 8-17, find the coefficient of transmissibility T and storage coefficient S_c for an aquifer. Given $Q = 1000$ gpm and $r = 300$ ft.

Solution
Find the value of Δh from the graph, 5.3 ft. Then by Eq. 8-115

$$T = \frac{264Q}{\Delta h} = \frac{264 \times 1000}{5.3}$$

$$= 49{,}800 \text{ gpd/ft}$$

Using Eq. 8-116, we find that

$$S_c = \frac{0.3Tt_0}{r^2}$$

Note from Fig. 8-16 that $t_0 = 2.6$ min. Converting to days, we find that this becomes

$$t_0 = 1.81 \times 10^{-3} \text{ days}$$

and

$$S_c = \frac{0.3 \times 49{,}800 \times 1.81 \times 10^{-3}}{(300)^2}$$

$$= 0.0003$$

8-26 Leaky Aquifers

The foregoing analyses have dealt with free aquifers or those confined between impervious strata. In reality, many cases exist wherein the confining strata are not completely impervious and water is actually transferred from them to the productive aquifer. The flow regime is altered and computations must include leakage. Since about 1930, leaky aquifers have been the subject of research by investigators such as De Glee, Jacob, Hantush, DeWiest, Walton, and others.[27–36,38] A thorough treatment of their work is beyond the scope of this book; interested readers should consult the indicated references.

8-27 Partially Penetrating Wells

In many actual situations there is only partial penetration of the well. The question then arises as to the applicability of procedures developed previously for full penetration.

Numerous studies of this problem have been conducted.[13,37,39] In 1957 Hantush reported that steady flow to a well just penetrating an infinite leaky aquifer becomes very nearly radial at a distance from the well of about 1.5 times the aquifer thickness.[39] As depth of penetration increases, the approach to radial flow becomes increasingly apparent. Therefore, computation of drawdowns for partially penetrating wells are made using equations for total penetration with relative safety, provided that the distance from the pumped well is greater than 1.5 times the aquifer thickness. At points closer to the well, it is frequently possible to use a flow net or other relationships developed for this region.[2,4,38]

8-28 Salt-Water Intrusion

The contamination of fresh groundwater by the intrusion of salt water often presents a serious quality problem. Islands and coastal regions are particularly vulnerable. Aquifers located inland sometimes contain highly saline waters as well. Fresh water is lighter than salt water (specific gravity of the latter is about 1.025) and forms a fresh water layer above the underlying salt water. This equilibrium is disturbed when an aquifer is pumped, since salt water replaces the fresh water removed. Under equilibrium conditions, a drawdown of 1 ft in a fresh water table corresponds to a rise of about 40 ft by salt water. Wells subjected to salt water intrusion obviously have limited pumping rates.

Recharge wells have been drilled in coastal areas to maintain a head sufficient to preclude sea water intrusion, a practice employed effectively in Southern California.

8-29 Computers and Numerical Methods in Groundwater Hydrology

Many advances in the application of computers and numerical methods have taken place since the late 1940s.[41–45]

Electric analogs solve a wide variety of groundwater flow problems,[2,43,45] and consist essentially of a resistance-capacitance network. Table 8-3 indicates the manner in which components of the electric analog and the actual flowfield are related.

Digital computers have also proved to be versatile tools for use in groundwater studies.[2,44,47] The applicable mathematical model is usually written in finite difference or finite element form. Groundwater simulation models will be discussed further in Chapter 10.

Table 8-3 *Elements of a Groundwater Reservoir and an Electric Analog Compared*

Groundwater Reservoir Component	Corresponding Electric Analog Component
Hydraulic conductivity	Resistivity
Aquifer storage	Capacitance
Head	Voltage
Volumetric flow rate	Amperage

Table 8-4 Some Important Forms of Recharge and Discharge

Recharge	Discharge
Seepage from streams, ponds, lakes	Seepage to lakes, streams, springs
Subsurface inflows	Subsurface outflows
Infiltrated precipitation	Evapotranspiration
Water recharged artificially	Pumping or other artificial means of collection

8-30 Groundwater Basin Development

To utilize groundwater resources efficiently while simultaneously permitting the maximum development of the resource, equilibrium must be established between withdrawals and replenishments. Economic, legal, political, social, and water quality aspects require full consideration.

Lasting supplies of groundwater will be assured only when long-term withdrawals are balanced by recharge during the corresponding period. The potential of a groundwater basin can be assessed by employing the water budget equation,

$$\sum I - \sum O = \Delta S$$

where the inflow $\sum I$ includes all forms of recharge, the total outflow $\sum O$ includes every kind of discharge, and ΔS represents the change in storage during the accounting period. The most significant forms of recharge and discharge are those listed in Table 8-4.

A groundwater hydrologist must be able to estimate the quantity of water that can be economically and safely produced from a groundwater basin in a specified time period. He should also be competent to evaluate the consequences of imposing various rates of withdrawal on an underground supply.

Development of groundwater basins should be based on careful study, since groundwater resources are finite and exhaustible. If the various types of recharge balance the withdrawals from a basin over a period of time, no difficulty will be encountered. Excessive drafts, however, can deplete underground water supplies to a point where economic development is not feasible. The mining of water will ultimately deplete the entire supply.

Problems

8-1 What is the Reynolds number for flow in a soil when the water temperature is 50°F, the velocity 0.6 ft/day, and the mean grain diameter 0.08 in.?

8-2. A 12-in. well fully penetrates a confined aquifer 100 ft thick. The coefficient of permeability is 600 gpd/ft^2. Two test wells located 40 and 120 ft away show a difference in drawdown between them of 9 ft. Find the rate of flow delivered by the well.

8-3. Determine the permeability of an artesian aquifer being pumped by a fully penetrating well. The aquifer composed of medium sand is 130 ft thick. The steady state pumping rate is 1300 gpm. The drawdown in an observation well 65 ft away is 12 ft, and in a second observation well 500 ft away 1.2 ft. Find K_f in gpd/ft^2.

8-4. Consider a confined aquifer with a coefficient of transmissibility $T = 700$ ft^3/(day)(ft). At $t = 5$ min the drawdown = 5.1 ft; at 50 min, $s = 20.0$ ft; at 100 min, $s = 26.2$ ft. The observation well is 60 ft from the pumping well. Find the discharge of the well.

8-5. Assume that an aquifer being pumped at a rate of 300 gpm is confined and pumping test data are given as follows. Find the coefficient of transmissibility T and the storage coefficient S. Assume $r = 55$ ft.

Time since pumping started (min)	1.3	2.5	4.2	8.0	11.0	100.0
Drawdown s (ft)	4.6	8.1	9.3	12.0	15.1	29.0

8-6. Given the following data:

$Q = 60{,}000$ ft^3/day $\quad t = 30$ days, $r = 1$ ft
$T = 650$ ft^3/(day)(ft) $\quad S_c = 6.4 \times 10^{-4}$

Assume this to be a nonequilibrium problem. Find the drawdown s. Note for

$u = 8.0 \times 10^{-9} \quad W(u) = 18.06$
$u = 8.2 \times 10^{-9} \quad W(u) = 18.04$
$u = 8.6 \times 10^{-9} \quad W(u) = 17.99$

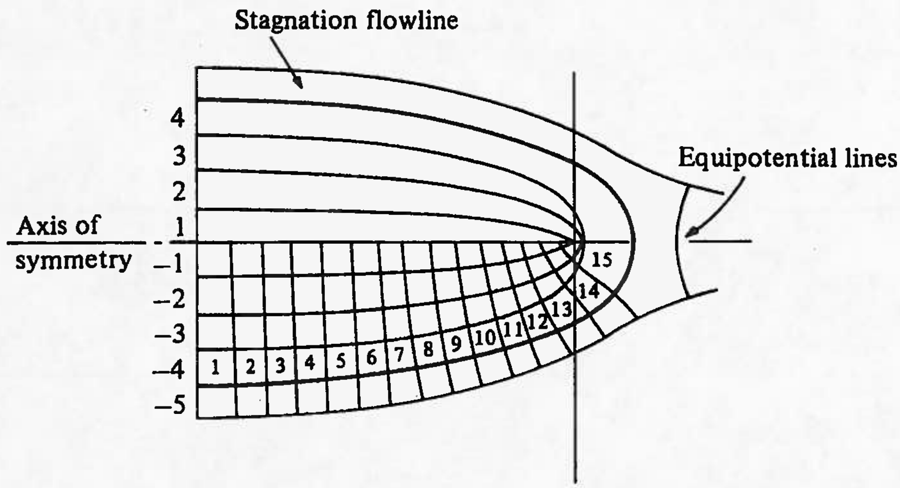
8-7. The water temperature in an aquifer is 58°F and the rate of water movement 1.2 ft/day. The average particle diameter in a porous medium is 0.06 in. Find the Reynolds number and indicate whether Darcy's law applies.

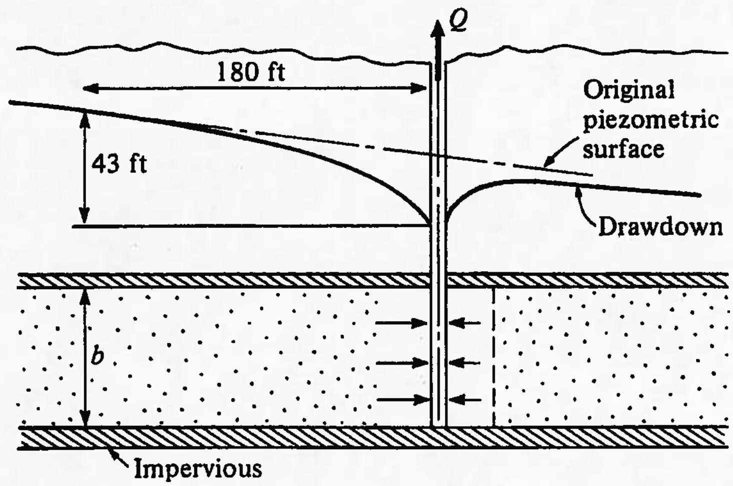
8-8. A laboratory test of a soil gives a standard coefficient of permeability $K_s = 3.78 \times 10^2$ gpd/ft^2. If the prevailing field temperature is 60°F, find the field coefficient of permeability K_f.

8-9. An 18-in. well fully penetrates an unconfined aquifer 100 ft deep. Two observation wells located 90 and 235 ft from the pumped well are known to have drawdowns of 22.5 and 20.6 ft, respectively. If the flow is steady and $K_f = 1300$ gpd/ft^2, what would be the discharge?

8-10. A confined aquifer 80 ft deep is being pumped under equilibrium conditions at a rate of 700 gpm. The well fully penetrates the aquifer. Water levels in observation wells 150 and 230 ft are 95 and 97 ft, respectively. Find the field coefficient of permeability.

8-11. Given the well and flow net data in the following figure, find the discharge using a flow net solution. The well is fully penetrating and the confined aquifer 50 ft thick; $K_f = 2.87 \times 10^{-4}$ ft/sec.





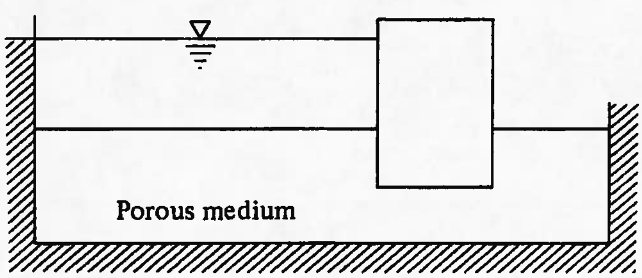
8-12. A well is pumped at the rate of 500 gpm under nonequilibrium conditions. For the data listed, find the formation constants S and T. Use the Theis method.

r^2/t	Average Drawdown, h (ft)
1,250	3.24
5,000	2.18
11,250	1.93
20,000	1.28
45,000	0.80
80,000	0.56
125,000	0.38
180,000	0.22
245,000	0.15
320,000	0.10

8-13. Given a well pumping at a rate of 590 gpm. An observation well is located at $r = 180$ ft. Find S and T using the Jacob method for the following test data.

Drawdown (ft)	Time (min)
0.43	26
0.94	78
1.08	99
1.20	131
1.34	173
1.46	218
1.56	266
1.63	303
1.68	331
1.71	364
1:85	481
1.93	573
2.00	661
2.06	732
2.12	843
2.15	926
2.20	1034
2.23	1134
2.28	1272
2.30	1351
2.32	1419
2.36	1520
2.38	1611

8-14. By employing a finite difference method, find and plot the flowline in the following figure for $\psi = 0.5$.



8-15. A 24-in. diameter well penetrates the full depth of an unconfined aquifer. The original water table and a bedrock aquifuge were located 50 ft and 150 ft, respectively, below the land surface. After pumping at a rate of 1700 gpm continuously for 1920 days, equilibrium drawdown conditions were established, and the original water levels in observation wells located 1000 and 100 ft from the center of the pumped well were lowered 10 and 20 ft, respectively. (a) Determine the field permeability (gpd/ft^2) of the aquifer.

(b) For the same well, zero drawdown occurred outside a circle with a 10,000-ft radius measured from the center of the pumped well. Inside the circle, the average drawdown in the water table was observed to be 10 ft. Determine the coefficient of storage of the aquifer.

8-16. A well fully penetrates the 100-ft depth of a saturated unconfined aquifer. The drawdown at the well casing is 40 ft when equilibrium conditions are established using a constant discharge of 50 gpm. What is the drawdown when equilibrium is established using a constant discharge of 66 gpm?

8-17. After a long rainless period, the flow in Wahoo Creek decreases by 8 cfs from Memphis downstream 8 mi to Ashland. The stream penetrates an unconfined aquifer, where the water table contours near the creek parallel the west bank and slope to the stream by 0.00020, while on the east side the contours slope away from the stream toward the Lincoln wellfield at 0.00095. Compute the transmissivity of the aquifer knowing $Q = TIL$ where I is the slope and L is the length.

8-18. The time-drawdown data for an observation well located 296 ft from a pumped artesian well (500 gpm) are given in the following table. Find the coefficient of storage (ft^3 of water/ft^3 of aquifer) and the transmissivity (gpd/ft) of the aquifer by the Theis Method. Use 3×3 cycle log paper.

Time (hr)	Drawdown (ft)	Time (hr)	Drawdown (ft)
1.9	0.28	9.8	1.09
2.1	0.30	12.2	1.25
2.4	0.37	14.7	1.40
2.9	0.42	16.3	1.50
3.7	0.50	18.4	1.60
4.9	0.61	21.0	1.70
7.3	0.82	24.4	1.80

8-19. Over a 100-mi^2 surface area, the average level of the water table for an unconfined aquifer has dropped 10 ft because of the removal of 128,000 area-ft of water from the aquifer. Determine the storage coefficient for the aquifer. The specific yield is 0.2 and the porosity is 0.22.

8-20. Over a 100-mi^2 surface area, the average level of the piezometric surface for a confined aquifer in the Denver area has declined 400 ft as a result of long-term pumping. Determine the amount of water (acre-ft) pumped from the aquifer. The porosity is 0.3 and the coefficient of storage is 0.0002.

References

1. Clark, J. W., W. Viessman, Jr., and M. J. Hammer, *Water Supply and Pollution Control*, 2nd ed. (New York: Thomas Y. Crowell Company, 1965).
2. DeWiest, R. J. M., *Geohydrology* (New York: John Wiley & Sons, Inc., 1965).
3. Harr, M. E., *Groundwater and Seepage* (New York: McGraw-Hill Book Company, 1962).

4. Todd, D. K., *Groundwater Hydrology* (New York: John Wiley & Sons, Inc., 1960).
5. Buras, Nathan, "Conjunctive Operation of Dams and Aquifers," *ASCE, J. Hydr. Div. Proc.*, 89, No. HY6 (Nov. 1963).
6. Ferris, J. G., "Ground Water," *Mech. Eng.*, Jan. 1960.
7. Thomas, H. E., "Underground Sources of Water," in *Water, The Yearbook of Agriculture*. Washington, D.C.: U.S. Department of Agriculture, 1955.
8. Thiem, G., *Hydrologische Methodern* (Leipzig: Gebhardt, 1906), p. 56.
9. Theis, C. V., "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground Water Storage," *Trans. Am. Geophys. Union*, 16 (1935): 519–524.
10. Cooper, H. H., Jr., and C. E. Jacob, "A Generalized Graphical Method for Evaluating Formation Constants and Summarizing Well-Field History," *Trans. Am. Geophys. Union*, 27 (1946): 526–534.
11. Hoffman, John F., "How Underground Reservoirs Provide Cool Water for Industrial Uses," *Heating, Piping, and Air Conditioning*, Oct. 1960.
12. Meinzer, O. E., "Outline of Methods for Estimating Groundwater Supplies," U.S. Geological Survey, Water-Supply Paper 638-C, Washington, D.C., 1932.
13. Muskat, M., *The Flow of Homogeneous Fluids Through Porous Media* (Ann Arbor, Mich.: J. W. Edwards, Inc., 1946).
14. Dupuit, Jules, *Etudes théoriques et pratiques sur le mouvement des eau dans les canaux de couverts et à travers les terrains perméables*, 2nd ed. (Paris: Dunod, 1863).
15. Hoffman, John F., "Field Tests Determine Potential Quantity, Quality of Ground Water Supply," *Heating, Piping, and Air Conditioning*, Aug. 1961.
16. Hoffman, John F., "Well Location and Design," *Heating, Piping, and Air Conditioning*, Aug. 1963.
17. Richter, R. C., and R. Y. D. Chun, "Artificial Recharge of Ground Water Reservoirs in California," *Proc. ASCE, J. Irrigation and Drainage Div.*, 85, No. IR4 (Dec. 1959).
18. Clendenen, F. B., "A Comprehensive Plan for the Conjunctive Utilization of a Surface Reservoir with Underground Storage for Basin-Wide Water Supply Development: Solano Project California," Doctorate Eng. thesis, University of California, Berkeley, 1959.
19. Darcy, Henri, *Les fontaines publiques de la ville de Dijon* (Paris: V. Dalmont, 1856).
20. Eskinazi, Salamon, *Principles of Fluid Mechanics* (Boston: Allyn and Bacon, Inc., 1962).
21. Kaplan, W., *Advanced Calculus* (Reading, Mass.: Addison-Wesley Publishing Company, Inc., 1952).
22. Jacob, C. E., "Flow of Groundwater," in Hunter Rouse (ed.), *Engineering Hydraulics* (New York: John Wiley & Sons, Inc., 1950).
23. Taylor, D. W., *Fundamentals of Soil Mechanics* (New York: John Wiley & Sons, Inc., 1948).
24. Casagrande, A., "Seepage Through Dams," in *Contributions to Soil Mechanics, 1925–1940* (Boston: Boston Society of Civil Engineers, 1940).

25. Polubarinova-Kochina, P. Ya. *Theory of Groundwater Movement* (Princeton, N.J.: Princeton University Press, 1962).
26. Wenzel, L. K., "Methods for Determining Permeability of Water Bearing Materials with Special Reference to Discharging Well Methods," U.S. Geological Survey, Water-Supply Paper 887, Washington, D.C., 1942.
27. De Glee, G. J., *Over Grondwaterstromingen by Wateronttrekking by middel van Plutten* (Delft: T. Waltman, Jr., 1930, p. 175).
28. Jacob, C. E., "Radial Flow in a Leaky Artesian Aquifer," *Trans. Am. Geophys. Union,* 27 (1946): 198–205.
29. Hantush, M. S., "Plain Potential Flow of Groundwater with Linear Leakage," Ph.D. dissertation, University of Utah, 1949.
30. Hantush, M. S., and C. E. Jacob, "Nonsteady Radial Flow in an Infinite Leaky Aquifer and Nonsteady Green's Functions for an Infinite Strip of Leaky Aquifer," *Trans. Am. Geophys. Union,* 36 (1955): 95–112.
31. Hantush, M. S., and C. E. Jacob, "Flow to an Eccentric Well in a Leaky Circular Aquifer," *J. Geophys. Res.*, 65 (1960): 3425–3431.
32. Hantush, M. S., "Analysis of Data from Pumping Tests in Leaky Aquifers," *Trans. Am. Geophys. Union,* 37 (1956): 702–714.
33. Hantush, M. S., "Modification of the Theory of Leaky Aquifers," *J. Geophys. Res.,* 65 (1960): 3713–3725.
34. DeWiest, R. J. M., "On the Theory of Leaky Aquifers," *J. Geophys. Res.*, 66 (1961): 4257–4262.
35. DeWiest, R. J. M., "Flow to an Eccentric Well in a Leaky Circular Aquifer with Varied Lateral Replenishment," *Geofis. Pura e Aplic.*, 54 (1963): 87–102.
36. Walton, W. C., "Leaky Artesian Aquifer Conditions in Illinois," Report of Investigation No. 39, Illinois State Water Survey, 1960.
37. Kirkham, D., "Exact Theory of Flow into a Partially Penetrating Well," *J. Geophys. Res.*, 64 (1959): 1317–1327.
38. Walton, William C., *Groundwater Resource Evaluation* (New York: McGraw-Hill Book Company, 1970).
39. Hantush, M. S., "Nonsteady Flow to a Well Partially Penetrating an Infinite Leaky Aquifer," *Proc. Isaqi Sci. Soc.*, 1 (1957): 10–19.
40. Todd, David K., "Ground Water Has To Be Replenished," *Chem. Eng. Prog.*, 59, No. 11 (November 1963).
41. Stallman, R. W., "From Geologic Data . . . To Aquifer Analog Models," *Geotimes*, 5, No. 5 (Apr. 1961).
42. Remson, Irwin, Charles A. Appel, and Raymond A. Webster, "Groundwater Models Solved by Digital Computer," *Proc. ASCE, J. Hydr. Div.*, 91, No. HY 3 (May 1965).
43. Stallman, Robert W., "Electric Analog of Three-Dimensional Flow to Wells and Its Application to Unconfined Aquifers," United States Department of the Interior Geological Survey, Open File Report, July 26, 1961.
44. Fayers, F. J., and J. W. Sheldon, "The Use of a High-Speed Digital Computer in the Study of the Hydrodynamics of Geologic Basins," *J. Geophys. Res.*, 67, No. 6 (June 1962): 2421–2431.
45. Brown, Russell H., "Progress in Ground Water Studies with the Electric-Analog Model," *JAWWA* (Aug. 1962): 943–958.

46. Meinzer, O. E., "The Occurrence of Groundwater in the United States," U. S. Geological Survey, Water-Supply Paper No. 489, 1923.
47. Robertson, J. M., *Hydrodynamics in Theory and Application* (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1965).
48. Shahbazi, M., and D. K. Todd, "Analytic Techniques for Determining Ground Water Flow Fields," Water Resources Center Contribution No. 117, Hydraulic Laboratory, University of California, Berkeley, Aug. 1967.
49. Wisler, C. O., and E. F. Brater, *Hydrology* (New York: John Wiley & Sons, Inc., 1959).