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What Is Watershed Runoff?

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Abstract. A nonlinear mathematical model, starting with the integral of an infiltration capacity function, is developed to analytically equate the difference between rainfall and runoff to hydrologic variables. Only the three independent variables—storm rainfall, duration, and soil moisture—are used, and an equation is evolved in which the identity of the coefficients is kept intact and unusually good statistical control is maintained. The coefficients of the equation appear to be stable over a range of watershed sizes and conditions. The equation strongly indicates that runoff usually originates from a small, but relatively consistent, part of the watershed. The function can be manipulated to show a 'function of apparent watershed infiltration capacity.' This function characterizes the infiltration capacity of that portion contributing to runoff, on the average, and should prove to be a useful infiltration capacity index with which watersheds can be compared. The equation itself provides insight into why in situ measurements of infiltration capacity seldom agree with the capacity determined from rainfall-runoff data. It also indicates why storm runoff frequently is not linear with respect to causative factors.

INTRODUCTION

Infiltration, by definition, is the movement of water through the soil surface into the ground. The concept of infiltration is easily understood. Measuring and understanding the physical factors controlling the infiltration process, however, is exceedingly difficult.

The rate of infiltration of precipitation, in general, equals the rate of precipitation as long as the infiltration capacity of the soil is not exceeded. When this capacity is exceeded, the excess precipitation is either stored temporarily on the soil or begins to run off. Since an understanding of the surface runoff process is basic to many problems in engineering, agriculture, municipal planning, etc., and since surface runoff is closely related to infiltration capacities, there has been a considerable effort expended in trying to evaluate measures of infiltration capacity.

Measurements of infiltration capacity have generally followed two directions. Actual measurements of infiltration capacity are made in situ using artificial sprinkling devices or small surface runoff plots. Indirect determinations of infiltration capacity are made from measurements of storm precipitation and watershed runoff. These indirect determinations often result in more than an index of the infiltration capacity, since results frequently vary from storm to storm and seldom agree with in situ measurements. It is important to note, however, that the infiltration capacity obtained from these indirect determinations represents an average for the watershed, whereas the in situ or plot measurements approach a point, or local, measure of infiltration capacity. Point measures can vary considerably from the average.

Indirect measurements of infiltration capacity have generally been hampered by the fact that this capacity is not a constant value. It is a complex function, principally of soil moisture but also of many other factors. It may change considerably during the course of a single storm. Computations of either mathematical infiltration capacity functions or indices have generally been limited to individual storms. An analytic approach to the computation of an infiltration capacity function has been lacking because available techniques for this type of problem solution usually required linear or linear-transformable equations.

With the availability of electronic computers, the limitations of simplicity and linearity have been removed as requirements of analytic solutions. Computer techniques are now available that will evaluate almost any equation that can be devised for data adjustment. This paper shows the development of a complex function of watershed infiltration capacity using analytic techniques. This development starts with an available empirical model and continues with improvements based on the results obtained in fitting the model to actual data.

The Infiltration Process

The role of infiltration in the hydrologic cycle was probably first recognized by *Horton* [1933]. This infiltration pertained to the passage of water through the soil surface. Once the water is in the soil its further movement is defined as percolation. The maximum rate at which water can enter the soil is dependent on many factors. Horton recognized a maximum capacity and a minimum capacity. The maximum capacity for any given rain occurs at the beginning of the rain. This rate decreases rapidly at first because of changes in the structure of the surface soil and increases in surface soil moisture, and then gradually approaches a somewhat stable minimum. The decay in the capacity results from such processes as change in surface soil moisture, rain packing of the soil, closing of sun checks, swelling of crumb structure, in-washing, and breaking down of the crumb structure. The minimum infiltration capacity approaches the percolation rate of the soil profile. Under conditions of saturation the percolation rate and, in turn, the infiltration capacity can be limited by the permeability of some least pervious soil horizon.

Horton defined the infiltration capacity, $f(p)$, as the maximum rate at which a given soil can absorb precipitation as it falls. The actual infiltration during a storm is equal to the capacity only when the rainfall intensity equals or exceeds $f(p)$. Horton proposed that the relation between $f(p)$ and the rainfall duration can be expressed by

$$f(p) = f_c + (f_0 - f_c)e^{-kT} \quad (1)$$

where

$f(p)$ = instantaneous infiltration capacity in inches per hour at time T.
f_c = minimum infiltration capacity.
f_0 = maximum infiltration capacity at time zero.
T = time in hours.
e = Napierian base.
k is a positive constant.

If this infiltration capacity function can be considered constant for a given set of physical watershed conditions (i.e., cover, soil, state of tilth, etc.), the integral of this function over time would determine the maximum amount of precipitation that could be infiltrated in a given time. If the storm rainfall is intense enough to exceed the infiltration capacity during the entire storm, the difference between the storm rainfall and this volume resulting from the integration would be approximately equal to the surface runoff volume. Thus it should be possible to determine the coefficients in this equation from rainfall-runoff data. Evaluating this equation from data, however, would necessitate using nonlinear fitting techniques since the function cannot be easily linearized.

Earlier work in the Tennessee Valley Authority (TVA) has shown the feasibility of using the multivariate technique of component analysis in the solution of complex equations by the method of nonlinear least squares [*Snyder*, 1962]. The technique for solution by nonlinear least squares may be found under the name of 'method of differential corrections' [*Nielsen*, 1957]. Whenever the composite technique of nonlinear least squares and component analysis has been tried in TVA, successful evaluation of mathematical models has been achieved. All the solutions of the various forms of the developed equation were obtained from an available special-purpose IBM 704 program [*TVA*, 1963].

Development of an Infiltration Capacity Function

Basic data. The data used in the development of the watershed infiltration capacity function were collected at the Western North Carolina Cooperative Research Project located near Waynesville and Asheville, North Carolina. This project has been described in various progress reports as well as in special reports on the project [*TVA*, 1960*a*]. The purpose of this small watershed research project is to determine water-land relationships for agricultural development of some of the principal soils and crops in western North Carolina. In accomplishing the project objectives, information is being obtained on the effects of single agricultural practices on (*a*) runoff, (*b*) soil moisture, and (*c*) groundwater levels. The program involves basic research in the development of hydrologic relationships.

The four experimental watersheds are located in the Blue Ridge subregion on the southern Appalachian soils region. The soils are all acid and low in available phosphorus and they vary in their content of potassium. Subsoils are usually permeable and well drained. Average slopes in the watersheds range between 22 and 38 per cent, some slopes being as steep as 60 per cent. The watersheds are generally bowl-shaped, varying in size from 3.7 to 5.6 acres. A metal cutoff wall is installed to a predominantly metamorphic bedrock at the lower end of each watershed. Continuous discharge measurements are made with either a 2-foot H-type flume or a 1½-foot San Dimas flume. Surface runoff measurements are also made on 0.05-acre subplots within some of the watersheds.

The development of the infiltration capacity function was based on data obtained when the watersheds were in either a moderately or heavily grazed pasture cover. These two covers were used so that changes in the structure of the soil surface resulting from either the impact of high-energy rainfall or tillage would be minimized in the initial stages of the development.

Initial assumptions. Various investigators have shown that the initial infiltration capacity is a function of soil moisture. As the soil moisture at the beginning of a storm becomes higher, the initial infiltration capacity becomes lower, approaching the percolation rate as a condition of saturation is reached. If the infiltration capacity function of a watershed can be considered relatively constant under a given watershed regime, it should be possible to use soil moisture to predict the initial infiltration capacity or, in other words, the beginning point of the function. If, in addition, storm runoff can be considered to be the flow that results when rainfall exceeds the infiltration capacity, it should be possible to evaluate a watershed infiltration capacity function from conventional hydrologic measurements. The extent to which that portion of flow known as subsurface flow, or interflow, may invalidate this assumption will be discussed later.

The soil moisture used in this study is an index computed on a special-purpose IBM 704 program [*TVA*, 1963]. The basic concept of this index is a bookkeeping system of daily values of soil moisture balance. An upper- and a lower-level soil moisture index are computed. The upper-level index, which was used in the equations that follow, has been found to agree well with near-surface measurements of soil moisture.

The initial trial to develop a watershed infiltration capacity function was based on Horton's infiltration capacity equation. The area under the infiltration capacity curve for time T is a volumetric loss. If storm runoff can be considered an excess that results when the infiltration capacity is exceeded, this loss should equal the difference between rainfall and runoff.

$$L = R - RO = \int_0^D (c + be^{-nT})\, dT \quad (2)$$

where

L = loss in inches.
R = storm rainfall in inches.
RO = storm runoff in inches.
D = storm duration in hours.
c, b, and n are coefficients to be determined. They correspond to f_c, $(f_0 - f_c)$, and k, respectively, in (1).

Integration of the equation in this form, however, starts at the maximum infiltration capacity, or initially dry condition, whereas the initial infiltration capacity for a given storm might be considerably less than the over-all maximum. In fact, when the ground is saturated, the maximum infiltration capacity at the beginning of a storm may not be much different from the minimum capacity (c). The equation, therefore, needs to be modified to incorporate soil moisture in such a manner that it indicates the time-beginning point of the equation or, in other words, the lower limit of integration. This can be done easily because the exponential function has the characteristic of being piecewise continuous in the interval $0 \leqq t \leqq T$; therefore, the exponential function can be divided into a finite number of subintervals in each of which $F(t)$ is continuous and possesses finite left- and right-hand limits at every point $0 \leqq t \leqq T$. If a time-soil moisture functional relationship can be established to determine the time equivalence of the computed soil moisture index, this soil moisture function can be added directly in order to determine the beginning time in the infiltration capacity equation, thus allowing the equation to vary from storm to storm. With this provision, the equation would be

$$L = \int_{f(sm)}^{D+f(sm)} (c + be^{-nT})\, dT \qquad (3)$$

where $f(sm)$ = function of soil moisture.

Integrating, we get

$$L = cT - (b/n)e^{-nT} \Big|_{f(sm)}^{D+f(sm)} \qquad (4)$$

The functional scaling relationship between time and the soil moisture index is unknown. The function must, however, be able to relate the index, which varies from zero for dry conditions to 1.0 for wet conditions, to an equivalent time in hours, and probably should allow for a nonlinear relation. Further, the time equivalence of the soil moisture parameter should be zero for storms that occur when the soil is dry (*sm* index equals zero). A polynomial relation can satisfy these two conditions so that

$$f(sm) = mS + gS^2 \qquad (5)$$

where S = soil moisture index and m and g are coefficients.

Substituting the functional form for $f(sm)$ given in (5) for the limits of integration in (4) yields

$$L = cD + (b/n)\{\exp[-n(mS + gS^2)] - \exp[-n(D + mS + gS^2)]\} \qquad (6)$$

Figure 1 graphically shows the relationship of the variables.

The equation in this form was fitted to watershed hydrologic data. The only storms included in the data were continuous, with relatively high rainfall intensities throughout. It was hoped, at this stage, that if any storms were included that contained some rainfall at intensities below the infiltration capacity the prediction error resulting from the fitting could be used to further modify the storm list. The results obtained in fitting this equation to watershed 1 pasture data are shown in equation 1, Table 1. Theoretically, the value of the polynomial soil moisture–time function at a soil moisture value of 1.0 should be approximately equal to the length of time it takes the exponential function to become insignificant. In other words, the exponential function ideally should drop out of the equation for soil moisture values of 1.0. Equation 1, Table 1, however, indicates that at a soil moisture value of 1.0 the time equivalent of the soil moisture is negative; this does not have a physical interpretation. In view of the low degree of adjustment achieved in fitting the equation, as indicated by a multiple correlation coefficient $r = 0.61$, it appears that the additional degree of freedom allowed by the polynomial time–soil moisture function was used to adjust for some of the large residual error. Since the polynomial relationship cannot be evaluated, a linear time–soil moisture relationship should be adequate.

Modification for interception losses. It was apparent from the results of fitting the first

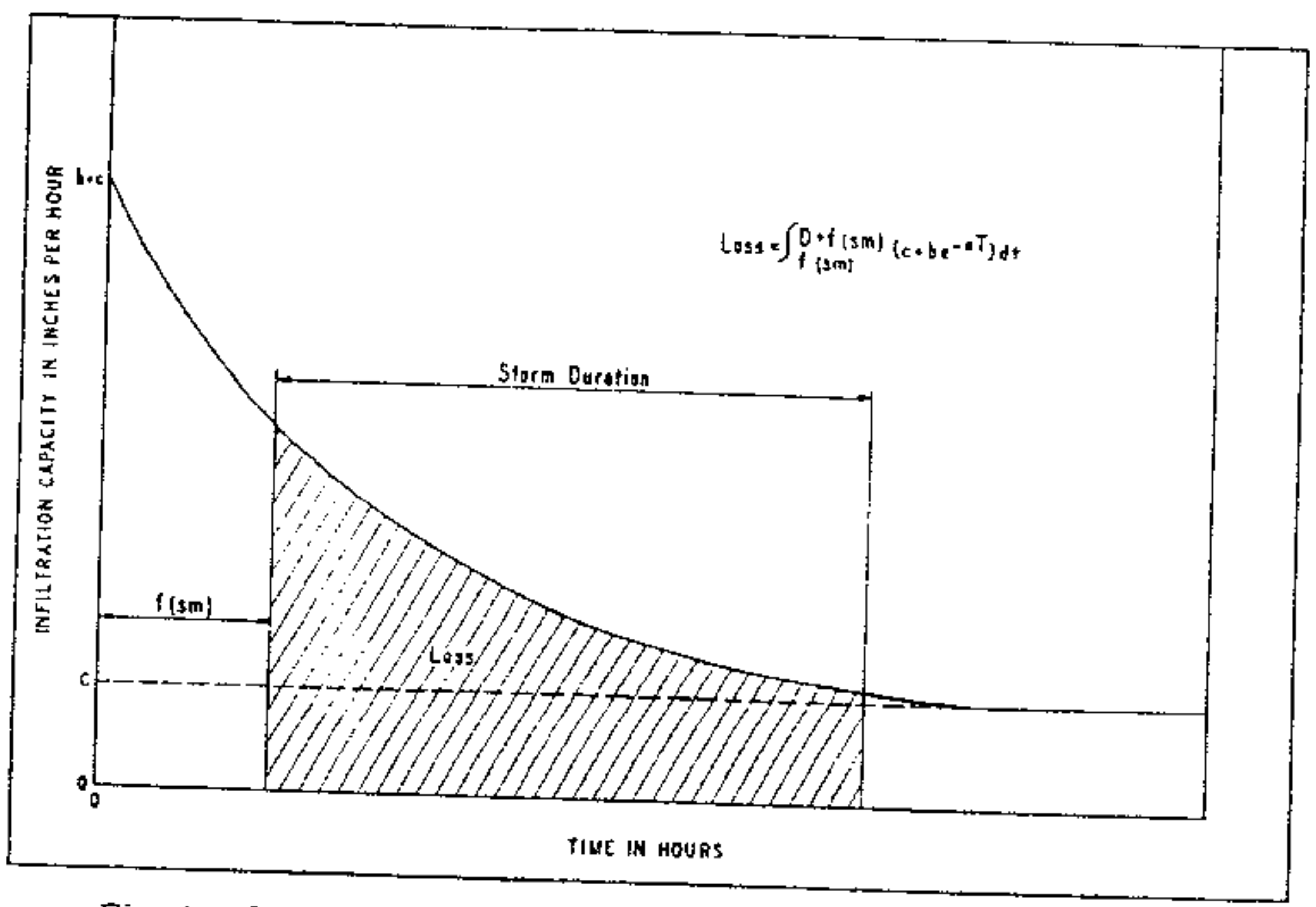

Fig. 1. Graphical representation of infiltration capacity function.

TABLE 1. Preliminary Equation Solutions

Equation 1 (watershed 1)

$$L = 0.138D + (0.0002/1.69)\{\exp[-1.69(7.90S - 12.18S^2)] - \exp[-1.69(D + 7.90S - 12.18S^2)]\}$$

Standard deviation $Se = 0.36$
Multiple correlation coefficient $r = 0.61$

Equation 2 (watershed 1)

$$L = 0.378 - 0.0767S + 0.0901D + (1.358/4.322)\{\exp[-(4.322 \times 1.189S)] - \exp[-4.322(D + 1.189S)]\}$$

$Se = 0.34$
$r = 0.66$

Equation 3 (watershed 5)

$$L = 0.266 - 0.068S + 0.026D + (3.87/2.82)\{\exp[-(2.82 \times 0.518S)] - \exp[-2.82(D + 0.518S)]\}$$

$Se = 0.39$
$r = 0.65$

Equation 4 (watershed 1)

$$L = -0.117 + 0.099S + 0.0016D + (0.853/5.433) \cdot \{\exp[-(5.433 \times 0.296S)] - \exp[-5.433(D + 0.296S)]\} + 0.948R$$

$Se = 0.0178$
$r = 0.99925$

where

L = loss in inches.
D = storm duration in hours.
S = soil moisture index.
R = storm rainfall in inches.
e = Napierian base.

equation to data that some major factor involved in the infiltration capacity relationship had been ignored. This factor could be interception. Interception losses represent a reduction in the effective rainfall, or throughfall. Studies by Wollny [*Baver,* 1938] in 1880 showed that rainfall interception by plants could amount to a substantial percentage of the rainfall. Since interception losses are not dependent on the duration of the storm, the equation cannot in its present form adjust for this type of loss. Adjustment could be made by the addition of a constant to the equation. It is probable, however, that this interception loss is not a constant; rather it is high when conditions are dry and less when conditions are wet. To approximate this condition it was assumed that the interception loss varied with the soil moisture as follows:

$$f(i) = a - gS \qquad (7)$$

where $f(i)$ = function of interception, S = soil moisture, and a and g are coefficients. The assembled equation is now

$$L = a - gS + cD + (b/n)(e^{-nmS} - e^{-n(D+mS)}) \qquad (8)$$

In this form the equation was again fitted to watershed 1 and, in addition, to watershed 5 data. The results of these two fittings are shown in Table 1, equations 2 and 3, respectively. The coefficients of both of these equations are reasonable. The indicated interception $(a - gS)$ at a soil moisture value of zero is 0.38 and 0.27 inch, respectively, on watersheds 1 and 5. For wet conditions where soil moisture equals 1.0, the interception values decrease slightly to 0.30 and 0.20 inch, respectively. The minimum infiltration capacity (c) is 0.09 in./hr on watershed 1 and 0.026 in./hr on watershed 5, and the maximum infiltration capacity for these two watersheds $(b + c)$ is 1.45 and 3.9 in./hr, respectively. These values are in substantial agreement with other measurements made on these watersheds and also agree with published data.

Despite the apparent reasonable magnitudes of the coefficients, the degree of adjustment of this equation is low for both sets of data. Figure 2, a residual error plot of watershed 1 data, shows that the degree of adjustment is low because a prediction bias exists that is related to the dependent variable loss.

Modification for partial area runoff. The

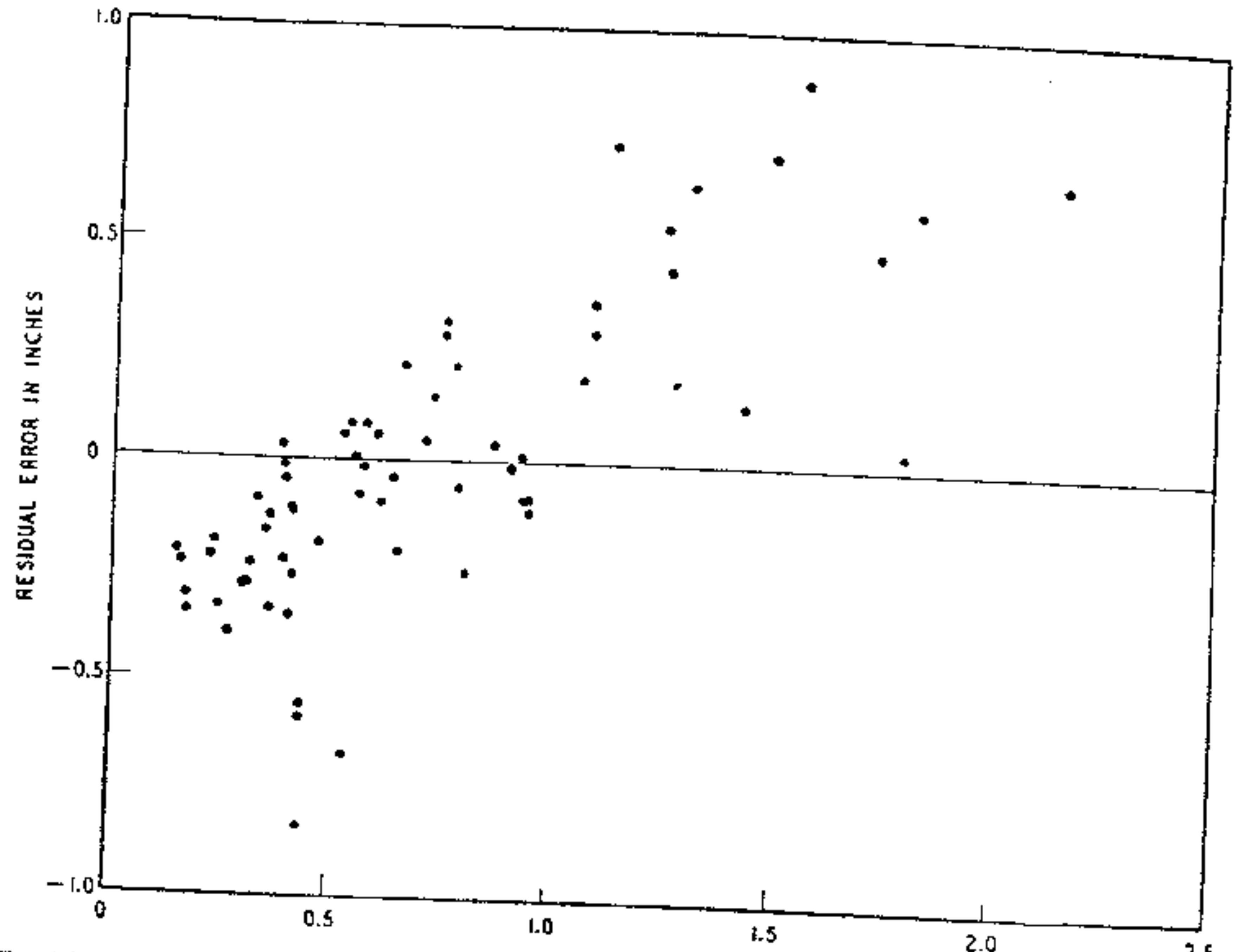

Fig. 2. Residual error of prediction versus observed loss for Waynesville watershed 1 (equation 2, Table 1).

residual error bias shown in Figure 2 in itself provides no insight as to the cause of the bias. Large residual error (R_e = observed − predicted) results when either the observed data are in error or the model does not fit the data. This could mean that either the runoff does not occur according to the infiltration capacity concept or the model itself is incorrect. Since the observed hydrologic data are considered reasonably correct, the model must be revised.

The consistency of the bias shown in Figure 2 suggests that a linear relationship can be fitted between the residual error and loss, the dependent variable of the equation. If this equation were linear, the bias could not have occurred, since the addition of such a linear loss function to the equation, in effect, amounts to a simple rescaling of each of the coefficients of the independent variables, a solution which would have been found in the fitting process. However, since the equation is nonlinear a simple rescaling of all the independent variables is not possible. This same effect can be achieved if the dependent variable, loss, is rescaled by including either storm rainfall or storm runoff as an independent variable in the equation. This fact is developed further in a following section. Actually, storm rainfall was selected as the rescaling independent variable because it is considerably larger than runoff and therefore more proportional to the loss. The equation with the linear rainfall term R added (the intercept at zero rainfall will be absorbed into the constant a of the equation) is

$$L = a - gS + cD + (b/n)(e^{-nmS} - e^{-n(D+mS)}) + hR \qquad (9)$$

The results of fitting this equation to watershed 1 pasture data are shown in Table 1, equation 4. A high degree of adjustment was achieved by the equation, as indicated by a multiple correlation coefficient of 0.99925. The very low standard error, 0.02 inch, is within an acceptable range for hydrologic data. The sign and values of all of the coefficients except the g term are within reasonable limits. The reversed sign of the coefficient g, however, does not have physical significance. It indicates that the interception loss increases with soil moisture and that the equation is probably overdetermined. This term can therefore be omitted from the equation. The physical significance of the term h will be developed in the following section.

The final form of the infiltration capacity equation is

$$L = a + cD + (b/n)(e^{-nmS} - e^{-n(D+mS)}) + hR \qquad (10)$$

TABLE 2. Equation Solutions

Watershed	Cover	Condition	Season	Coefficients a	c	b	n	m	h	Statistics r	Se
Watershed 1	Pasture		All	−0.032	0.0014	0.391	8.61	0.035	0.954	0.9995	0.015
Watershed 5	Pasture		All	−0.139	−0.045	1.025	1.52	0.61	0.765	0.975	0.113
Parker Branch	Complex	Calibration	Summer	−0.016	0.0045	0.394	3.31	0.21	0.782	0.995	0.040
Parker Branch	Complex	Evaluation	Summer	−0.017	0.006	0.287	2.86	0.47	0.870	0.993	0.052
Parker Branch	Complex	Calibration	Dry summer	0.001	0.011	0.351	3.55	0.53	0.772	0.996	0.048
Parker Branch	Complex	Calibration	Wet summer	0	0.002	0.258	3.91	0.37	0.862	0.996	0.018
Parker Branch	Complex	Evaluation	Dry summer	−0.006	0.011	0.133	5.59	0.10	0.921	0.9997	0.015
Parker Branch	Complex	Evaluation	Wet summer	−0.022	−0.128	0.307	.133	2.45	0.739	0.969	0.084
Parker Branch	Complex	Evaluation	Winter	−0.176	0.005	1.923	2.56	0.41	0.559	0.950	0.108
Middle Creek	Complex		Summer	−0.059	0.0006	0.785	4.58	0.12	0.856	0.985	0.104
Middle Creek	Complex		Winter	0.022	0.009	0.427	2.01	0	0.612	0.976	0.143
Watershed 2	Pasture	Moderately grazed	All	−0.0002	0.0009	0.487	4.07	0.43	0.849	0.9925	0.060
Watershed 2	Pasture	Heavily grazed	All	0.0044	0.0031	0.606	4.31	0.29	0.793	0.9905	0.066
Copper Basin 1 W	36%		All	−0.021	0.043	1.13	2.73	0.21	0.142	0.888	0.122

In this form the equation has been fitted to several sets of data from the single-practice western North Carolina watersheds as well as several other watersheds.

Interpretation of Results

Table 2 shows the results of fitting the final equation to data from various watersheds. The Parker Branch watershed, described in various progress reports and special reports on the project [*TVA*, 1960*b*], is a 1.51-mi² research area located in the same general region as the western North Carolina project watersheds. This complex-practice agricultural watershed was calibrated hydrologically from 1953 to 1955. Starting in 1955 the farmers were given an intensive farm development program designed to achieve optimum economic well-being of the people. Evaluation of the effects of this program was begun in 1958 and ended in 1962. The Middle Creek watershed is a 32.7-mi² area within Hiwassee River watershed. This area is in the Great Valley of east Tennessee. This agricultural complex-practice watershed was gaged from 1944 through 1961 to demonstrate the effect of TVA's applicable regional development integrated on a single watershed. The Copper Basin area in the southeastern corner of Tennessee was denuded primarily as a result of open-air copper ore roasting in the late 19th century. Watershed 1W, a 5-acre area, was gaged from 1943 through 1951 as part of a project to determine the effect of an erosion control program. Cover on this watershed, which was left untreated, never exceeded 36 per cent of the area. These areas are shown on the location map, Figure 3.

The coefficients of the equation for the various solutions shown in Table 2, with a few exceptions which will be explained later, show surprising consistency. This consistency generally remains despite the fact that the drainage areas for the watersheds vary from 3.7 acres to 32.7 mi², are located from the Blue Ridge physiographic region to the Great Valley, and vary in

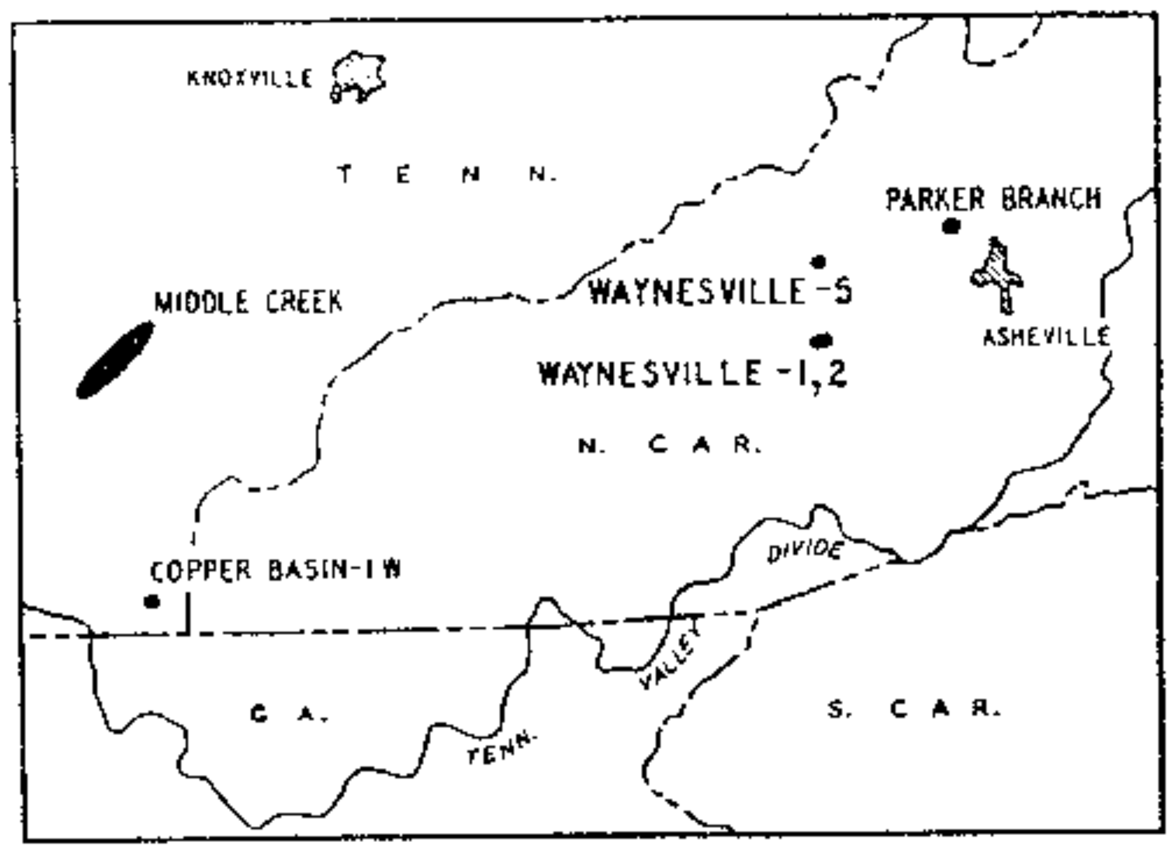

Fig. 3. Location map.

covers from single-practice pasture, to denudation, to complex practice. The storms from the Middle Creek watershed have been divided into summer and winter seasons and in addition to that division, the Parker Branch data have been subdivided into storms with significant rainfall within the three days before the storm, i.e. initial wetness and dryness.

The indicated minimum infiltration capacity for most data sets is of the order of 0.01 in./hr or less; the maximum rate is approximately ½ in./hr. This consistency also generally follows through the coefficient n which determines the shape of the exponential decay function and through the coefficient m which determines the time equivalent of soil moisture. In most cases the multiple correlation coefficient is unusually high, whereas the standard error is very low relative to the mean dependent variable, the loss, and even the mean runoff.

Despite the consistency of the coefficients and the high degree of adjustment achieved by the equation, most of the indicated infiltration capacities are low in comparison with published data. The reason for this can be better understood by studying the coefficient h. Rearranging the equation will show the rescaling nature of this term

$$\text{Loss} = R - hR - RO = a + cD + (b/n)(e^{-nmS} - e^{-n(D+mS)}) = f(ip) \quad (11)$$

$$\text{or} \quad R(1 - h) - RO = f(ip)$$

where $f(ip)$ = function of infiltration capacity.

If the reduction in rainfall indicated by the term $(1 - h)$ is real, there is an implication that not all of the storm rainfall is effective in producing runoff, if runoff is to occur according to the infiltration capacity concept. It is not obvious at this point why this is so.

The dependent variable could also have been rescaled by using runoff as an independent variable and an equally good fit of the equation would have been achieved. This can be shown if the equation is rearranged

$$R - \frac{RO}{(1 - H)} = \frac{1}{(1 - h)} f(ip) \quad (12)$$

This rearrangement indicates that if it were possible to rescale storm runoff by a relatively constant factor, the resulting rainfall-runoff relation would occur according to the infiltration capacity concept. Runoff, unlike rainfall, can be rescaled mathematically. The observed volume of watershed runoff in cubic feet per second times one day (dsf) is converted to watershed inches by dividing by the drainage area (DA) and an appropriate constant (K) as in (13)

$$\text{inches} = \frac{dsf}{DA \times K} \quad (13)$$

If both sides of (13) are divided by the term $(1 - h)$ this term becomes more meaningful.

$$\frac{\text{inches}}{(1 - h)} = \frac{dsf}{(1 - h) DA \times K} \quad (14)$$

Storm runoff can be scaled upward in converting to the watershed inch unit if the effective drainage area is reduced. The term $(1 - h)$ is actually a measure of this effective runoff-producing area of a watershed. It has been the conversion to the watershed inch runoff unit with its implication of total watershed area contribution that has been complicating these studies. If either the total storm rainfall is reduced to an amount that fell over the effective runoff-producing area or the runoff is scaled upward to the watershed inch unit over a reduced drainage area, the resulting rainfall-runoff relation does occur according to the infiltration capacity concept. The effective runoff-producing area of a watershed, then, is not the same as that delineated by the topographical divide.

That the entire watershed may not contribute to runoff during moderate-size storms is not surprising. *Amorocho and Orlob* [1961] and *Moldenhauer et al.* [1960] had indications of this possibility, and the *Southeastern Forest Experiment Station* [1961] estimated that at the Coweeta Hydrologic Laboratory in western North Carolina no more than about 40 per cent of the drainage area of their watersheds contributes to storm runoff even in a relatively heavy rainfall area. It is the low percentage of the total area that does contribute and the apparent consistency of the size of this area on some watersheds that are surprising. For the equation solutions shown in Table 2, the term $(1 - h)$ indicates an average contribution that varied from a low of only 4.6 per cent of watered 1 to a maximum of 85.8 per cent of Copper Basin 1W.

Verification of the magnitude of the computed effective runoff-producing area of a watershed is difficult on the Parker Branch or Middle Creek watersheds because of the complex patterns of land use. Ideally, to verify that the term $(1 - h)$ is a measure of the effective runoff-producing area of a watershed, the equation should be fitted to data obtained from watersheds with known runoff-producing areas. One way to do this would be to use relatively impervious watersheds such as those in urban areas. For this type of watershed the coefficient h should have small values, so that $(1 - h)$ approaches unity. Lacking this type of data, some verification of this interpretation of the term $(1 - h)$ can be made using data from the western North Carolina project watersheds and from Copper Basin 1W watershed. These two projects represent extremes in land-use practices. Figure 4 shows a verification plot of the residual error versus the loss for watershed 1. The bias has been removed from the error, leaving a more normal pattern, and the errors themselves are reduced to acceptable values. The more normal error pattern indicates that with the dependent variable rescaled the equation does adjust to these data. The relatively small remaining error indicates that the percentage of watershed contributing to runoff is relatively constant over the range of storms included in the data set. The storm precipitation within this data set includes storm rainfall amounts varying from 0.16 in. to 2.28 in. and intensities up to 2.6 in./hr. The coefficient h indicates that on this watershed for storms in the light-to-moderate range, runoff occurs on the average from only 4.6 per cent of the watershed, or, conversely, the storms within the range of these data did not exceed the infiltration capacity of about 95 per cent of this watershed. On this watershed, installation of a sheet-metal cutoff wall has resulted in a permanently swampy area above the flume. The swampy area covers about 1 to 2 per cent of the watershed. It is evident, by the quick response of the streamflow discharge to changes in rainfall intensity and the unusually low yield of storm rainfall, that most of the storm runoff that was measured from this watershed under pasture cover came from this swampy area. The equation indicates that most of the storm runoff results from an area about twice the size of the swampy portion of the watershed.

If the concept that, on the average, runoff usually results from a relatively fixed percentage of the watershed area is valid, further verification should be possible on watershed 2. On this watershed three subplots are maintained to measure surface runoff. These 1/20-acre subplots are located around the watershed about ⅓ to ½ of the distance up to the divide. Some subplot surface runoff was measured from about ⅓ of the storms included in this data set. This seems to contradict the result of the equation that indicates that average runoff from the watershed results from only 15 per cent of the area. The amount of storm runoff recorded, however, supports the interpretation. Only during one of the storms included in the set was a substantial volume of runoff recorded from all three plots. In general, surface runoff from these plots amounted to less than 0.01 inch, and seldom was runoff recorded from all three plots during a given storm. The only time substantial

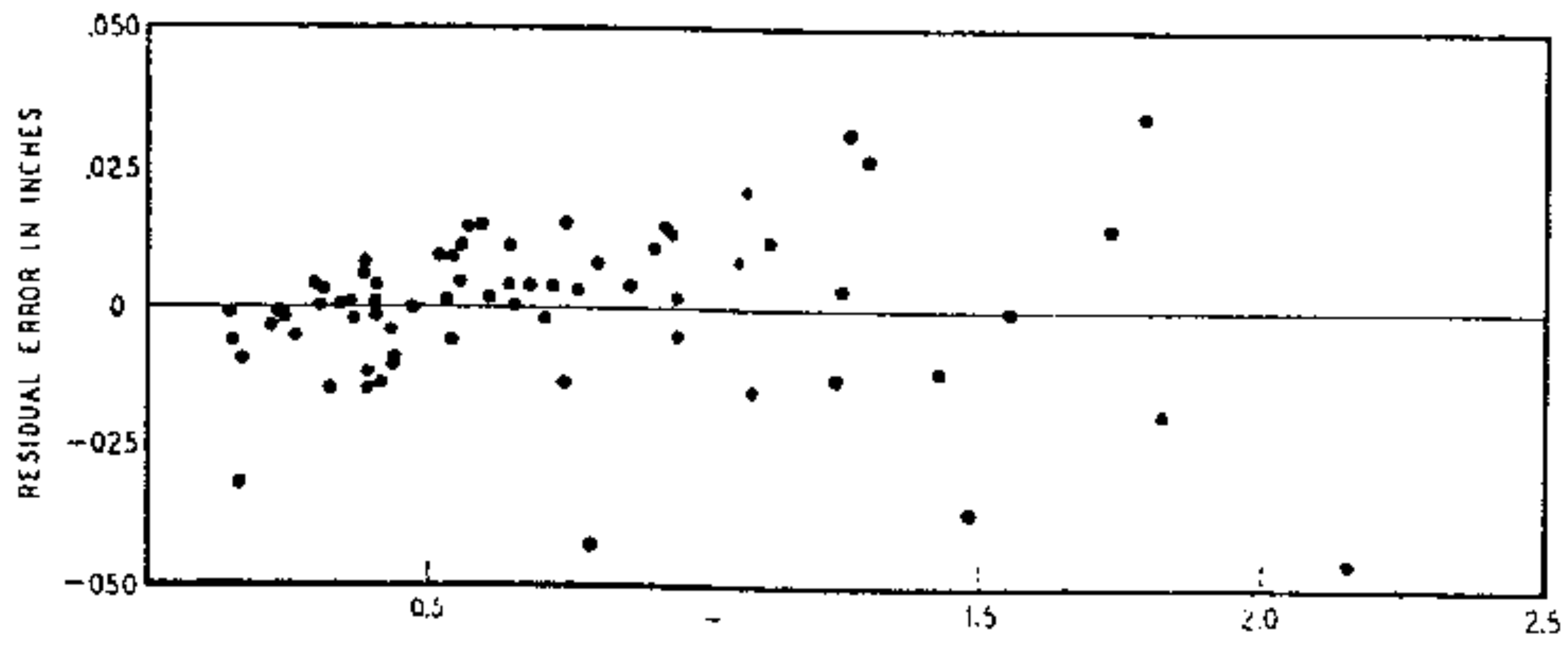

Fig. 4. Residual error of prediction versus observed loss, Waynesville Watershed 1.

runoff was measured at the plots, the storm was very intense.

Some intuitive judgment concerning the reasonableness of the term h that results from fitting the equation to Copper Basin 1W watershed data should be possible. This watershed, with 64 per cent of the area completely denuded, is subject to relatively heavy, rapid runoff through an advanced system of gullies that completely dissect the area. Streamflow is intermittent and most of the runoff can be considered surface runoff. Under these conditions, runoff should occur from a large percentage of the watershed, and the indicated infiltration capacities should be relatively low since crusting of the soil surface results from frequent, intense storms. The coefficient h for the Copper Basin 1W data (Table 2) indicates that the average watershed area contributing to runoff was 86 per cent. The indicated infiltration capacities, however, are not low. They are actually among the highest of the various sets of data used. In fact, these are the only indicated infiltration capacities, of all those determined by the equation, that are reasonable. This reasonableness suggests that the indicated infiltration capacity values are related to the amount of the watershed contributing to runoff.

The residual error distribution that results from fitting this equation to any of the data sets generally follows a seasonal pattern. High runoff (negative predictions) occurs during the winter season and low runoff occurs during the summer. This separation suggests that during the winter, when the amount of moisture stored in the soil profile is high, runoff probably occurs from a somewhat larger portion of the watershed than during the summer when this contributing area is smaller. The fitting technique sets the equation at what might be considered an average condition. This interpretation helps to explain some of the exceptions to the consistency of the coefficients found in Table 2. Inconsistencies were found in the fittings when the data contained a disproportionately high number of winter storms with 'apparently' high runoff, or when the actual infiltration capacities of the watershed were low, so that occasional high runoff resulted when more than the average watershed area contributed during a very intense storm. Both these conditions occur at watershed 5. A negative, uninterpretable coefficient for the minimum infiltration capacity resulted. The storms that were used for the Parker Branch evaluation period study, labeled 'summer data with wet antecedent conditions,' on the other hand, contained many events with high runoff that resulted because the moisture storage capacity was reduced when the upper soil horizons were near saturation. The equation resulting from these data is an almost random array of coefficients. This result must occur when large variations in the runoff occur because of conditions external to the equation. This result did not occur with the Copper Basin 1W fitting because the data were relatively 'homogeneous.' Further modifications of the equation will have to be made to account for, at least, the effect of changes in the volume of moisture stored within the soil on runoff.

If the infiltration capacity function that has been developed is to measure the watershed infiltration capacity, storm surface runoff should have been used rather than total storm runoff. Present methods of hydrograph analysis, however, do not permit reliable separation of the hydrograph into surface and subsurface flow. Because total storm runoff has been used, the losses predicted by using the function will be somewhat lower than they would have been if surface runoff only had been used. However, the degree to which this equation adjusts for a fairly wide range of hydrological events indicates that these two components of storm runoff, combined to evaluate the equation, are not incompatible as data. If this is true, the function should be a relatively stable index of the infiltration capacity of a watershed.

An Apparent Infiltration Capacity Function

It would be helpful if the coefficients that result from fitting the equation had more reasonable values. It was noted above that the magnitude of the coefficients is related to the size of the contributing area. Equation 12 shows how the coefficients might be rescaled to remove the effect of the size of the contributing area on their magnitude. By dividing through the equation by the term $(1 - h)$, the coefficients a, c, and b are scaled upward. This has the effect of scaling the coefficients of the infiltration capacity function upward to the equivalent value they would have had if runoff had occurred from the

TABLE 3. Apparent Watershed Infiltration Capacities

Watershed	Cover	Condition	Season	Coefficient a	c	b	n	m
Watershed 1	Pasture		All	−0.70	0.03	8.5	8.61	0.035
Parker Branch	Complex	Calibration	Summer	−0.07	0.02	1.8	3.31	0.21
Parker Branch	Complex	Evaluation	Summer	−0.13	0.05	2.2	2.86	0.47
Parker Branch	Complex	Calibration	Dry summer	0.004	0.05	1.5	3.55	0.53
Parker Branch	Complex	Calibration	Wet summer	0.0	0.014	1.9	3.91	0.37
Parker Branch	Complex	Evaluation	Dry summer	−0.08	0.14	1.7	5.59	0.10
Parker Branch	Complex	Evaluation	Winter	−0.31	0.009	3.4	2.56	0.41
Middle Creek	Complex		Summer	−0.41	0.004	5.45	4.58	0.12
Middle Creek	Complex		Winter	0.06	0.023	1.1	2.01	0
Watershed 2	Pasture	Moderately grazed	All	−0.001	0.006	3.2	4.07	0.43
Watershed 2	Pasture	Heavily grazed	All	0.021	0.015	2.9	4.31	0.29
Copper Basin 1W	36%		All	−0.025	0.050	1.32	2.73	0.21

entire watershed in a manner similar to that from the contributing area. The shape coefficient n and the time–soil moisture coefficient m remain unchanged. Table 3 shows the computed coefficients of the apparent infiltration capacity for the equation solutions included in Table 2. The solutions for watershed 5 and Parker Branch that broke down have not been included for reasons previously explained.

The value of the minimum and maximum apparent infiltration capacities (c and $c + b$, respectively) as shown in Table 3 are not too different across the set of solutions. The minimum capacity ranged from a low of 0.006 in./hr on the moderately grazed pasture of watershed 2 to 0.14 in./hr for all summer storms during the calibration period at Parker Branch. The maximum rate ranged from 8.53 in./hr on the watershed 1 pasture to 1.12 in./hr for winter storms on Middle Creek. The coefficients of the equation fitted to Copper Basin 1W data are well within the range of values obtained from other watersheds where considerably less of the watershed contributed to runoff. This consistency indicates that these coefficients should be a measure of the infiltration capacity of that area of the watershed that on the average contributes to runoff. Considering that the equation contains only the three independent variables—storm duration, rainfall, and a soil moisture index—and that no restrictions are imposed on the magnitudes of the coefficients of the equations, the range of coefficients obtained in fitting this equation is surprisingly consistent. This consistency seems to hold over watersheds ranging in size from 3.7 acres to 32.7 mi^2 and for watersheds with decidedly different topography, agricultural practices, and subclassifications of storms within a data set. It would appear, then, that the apparent watershed infiltration capacity function is a relatively stable measure with which the runoff, or, more directly, the loss, from a watershed may be characterized.

Conclusions

The stepwise development of a hydrologic equation to evaluate apparent watershed infiltration capacity has been presented. This equation can be useful both as a hydrologic tool and as a means of developing a better understanding of the storm runoff process.

The results obtained from fitting the equation to various sets of data indicate that storm runoff, at least in the geographic region of the study, frequently occurs from only a small part of the watershed area. This was found to be true on small test watersheds in pasture cover, and it appears to be true also for larger watersheds with complex land-use patterns. These results also seem to indicate that the size of the runoff-contributing area may not vary much under normal conditions. Logically, the size of the area must change as the moisture storage capacity of the soil changes, and in addition, the size of this area is also related to rainfall intensity. However, it appears that these two factors do not cause large variations in the size of the runoff-producing area of many watersheds except under unusual conditions. If this is true, the concept of partial area storm runoff helps to explain the nonlinear nature of many hydrologic relations. Mathematical models relating storm rainfall and modifying variables to runoff, peak discharge, or other dependent variables frequently underpredict large events, particularly if a wide range of storms is used. A typical storm list contains a large number of small-to-moderate events where the runoff-contributing part of the watershed is a relatively small area. Because of the number of these smaller events, the model is forced to fit this typical event. When during an extreme storm runoff does occur from a much larger area within the watershed, the small increase in the measured independent variables seldom justifies the large increase in the dependent variable. Actually, it appears that different relationships can be justified for large and small events.

The results from this study also help to explain why infiltration studies based on rainfall-runoff data seldom agree with in situ measurements. If storm runoff usually occurs from only a small part of a watershed, the infiltration capacity of a greater part of this watershed is seldom exceeded during normal storms. It appears, then, that in situ measurements of infiltration capacity will always equal or exceed measurements obtained from rainfall-runoff data. The two techniques will yield similar results only when either the in situ measurements are made on the runoff-contributing area or the function of apparent infiltration capacity is evaluated for relatively impervious or saturated areas.

The function that was developed should be useful as a means of estimating storm runoff. Because the equation was constructed logically, most of the coefficients have a physical interpretation and thus are relatively stable and reproducible. This stability is an important factor if the function is ever to be used for predicting hydrologic events on an ungaged watershed. Although only three independent variables are used, the standard errors that result from fitting are generally less than 10 per cent of the mean of the dependent variable.

The equation in its present form has a somewhat limited application. On small test areas it will work well only with data that do not contain too many storms with unusually high runoff. On larger areas it appears to fit consistently, probably because of the integrating effect of many factors. To be useful, the model requires further improvements, and these improvements are currently under development. From a practical point of view, and judging from results obtained thus far, the model should adjust for increased runoff resulting from reductions in soil moisture storage, changes in agricultural cover, and extreme rainfall intensities. If these factors can be incorporated in the model successfully, a far better understanding of watershed runoff should result.

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