CE 5361 Surface Water Hydrology
Unit Hydrographs - I
Background

Contents

1 Rainfall-runoff process

A conventional approach to watershed hydrology is illustrated in Figure 1. Precipitation falls on the watershed, and various processes convert a fraction of the rainfall into direct, immediately observed runoff, the remainder either seeps into the soil or returns into the atmosphere. Typically as engineers we are concerned with the magnitude, timing, and quality of the runoff component at some location on the watershed (the outlet).

If actual input (hyetograph) and output (hydrograph) data are available, the transformation can be analyzed by a variety of hydrologic models many are relatively simple, others complex. This kind of examination is called *analysis*. If one of the two data streams is missing then the activity of predicting behavior is called *synthesis*. A great deal of engineering design relies on synthetic methods (i.e. estimating behavior without data). Usually in hydrologic studies the missing element is the discharge hydrograph for the reasons already explained, this hydrologic variable is far more difficult to collect than rainfall.

To perform an analysis or synthesis at the event time scale (as opposed to continuous simulation which is an altogether different beast), the analyst (us) will need to do the following:

1. Define the watershed or area that actually contributes to runoff out the location of interest.
2. Define the hyetograph to be used (historical, or a design storm).
3. Design storms are often by prescription and usually are related to some probability level.

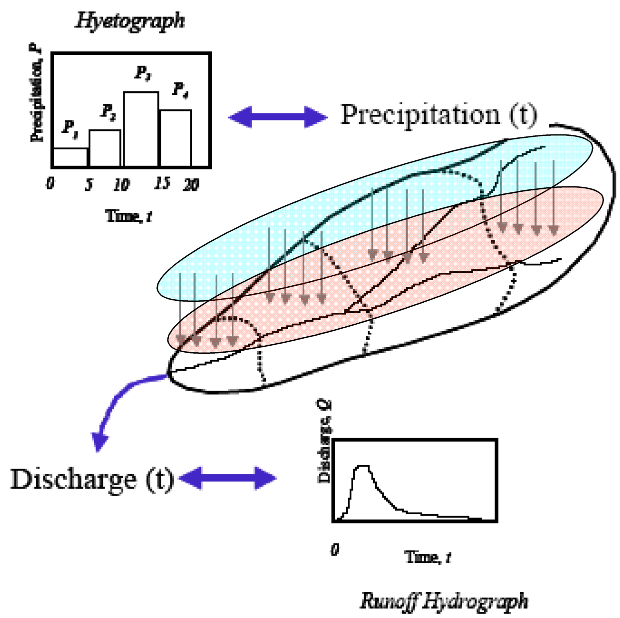

Figure 1: Watershed process

4. Determine the transformation from rainfall to runoff. This transformation requires the selection of loss models [1] and hydrograph models.
5. If the problem is analysis: Select a loss model and hydrograph transformation model; select model parameters that explain observed data.
6. If the problem is synthesis: Select a loss model and hydrograph transformation model; guess model parameters and hope they explain future behavior (obviously if we never make measurements in the future we really cannot evaluate how well we did).
7. In either case at the end of the exercise one must present, evaluate, and use the results. For instance in design applications: Size culverts, compute forces, etc. In regulatory applications: Establish floodplain limits, determine suitable land use, etc. In operations applications: Establish reservoir release values (i.e. flood pre-release); issue warnings; barricade streets in anticipation of flooding; issue boil water warnings; etc.

[1] For example, evaporation models, infiltration models or simply loss

2 Hydrograph Models: The Unit Hydrograph

The unit hydrograph is the principal model that temporally redistributes the excess rainfall (water available for runoff) to an outlet location. The other models in use are the modified rational-method. Nearly all hydrologic models incorporate some kind of unit hydrograph (possible non-linear) to convert excess rainfall into runoff.

This concept is key to understand hydrologic models (in contrast to hydraulic models). To reiterate: hydrologic modeling is essentially the systematic application of the following procedure:

1. Identify the watershed and location(s) of outlet points of interest.
2. Identify the rainfall signal going into the watershed.
3. Account for losses (loss model – very important!). $Input - Losses = Excess$
4. $Excess$ is not redistributed in time, that's the job of the hydrograph model.
5. Apply a hydrograph model to redistribute in the time domain the excess rainfall to the outlet. There are a variety of approaches. What is really going on in any approach (from strictly hydrologic to fluid dynamics approaches) it relating the distance, speed (of water), and time. The unit hydrograph is one such tool.

A unit hydrograph (UH) is defined as the runoff hydrograph that results from one unit of excess rainfall depth uniformly distributed over the entire watershed over one unit of time. An instantaneous unit hydrograph (IUH) is the unit hydrograph produced when the excess rainfall is applied over a very short time period.

The unit hydrograph concept is credited to Sherman in 1932 (Sherman, 1932), although the concept was in likely in use prior to that time. In his paper he illustrated a procedure to construct direct runoff hydrographs from a sequence of rainfall units by addition of ordinates of unit hydrographs lagged by the duration of the individual rainfall durations. Upon close examination, one concludes that Shermans procedure is graphical convolution of responses to different input weights. Subsequent efforts by many other authors codified these ideas, and UH theory today is essentially the application of linear-systems theory to the rainfall runoff process (Dooge, 1973; Chow, et al, 1988). In the 1970s, Chow and others worked on development of linear systems theory applications to hydrologic modeling. Chapter 7 in Chow, et al (1988) is an overview of that work.

The unit hydrograph (UH) response can be expressed as

$$q(t) = \int_0^T r(\tau) f(t-\tau) d\tau \qquad (1)$$

where where $q(t)$ is unit discharge from a basin at time t, $r(t)$ is an input function that represents either rainfall or excess rainfall, $f(t-\tau)$ is a response function (the unit hydrograph), and T is the duration of the input. Equation 1 assumes that basins respond as linear systems and this assumption is the main criticism of unit hydrograph theory. Despite this criticism, unit hydrographs are used to estimate streamflow from relatively small basins, typically for engineering purposes and often produce reasonable results. With the linearity assumption, the response, $f(t-\tau)$, has the same properties as a probability density function specifically, it integrates to unity on the range $(-\infty, \infty)$, and $f(t-\tau) \geq 0$ for any values of $(t-\tau)$.

Traditionally the UH is expressed in discrete space (not continuous like above) as

$$Q_n = \sum_{m=1}^{n<=M} P_m U_{n-m} \tag{2}$$

where U_n is the unit response function (actually a weight), and P_m is the excess precipitation (L) for period m.

The two equations differ slightly in that Equation 1 is scaled by the watershed area (that is multiply the response by drainage area to generate the DRH) while Equation 2 is already in proper scale.

The unitgraph, then, is a simple linear model that has some embedded assumptions:

1. Excess rainfall has a constant intensity within the effective duration,
2. Effective rainfall is uniformly distributed spatially,
3. Time base of runoff (period of time that direct runoff exceeds zero) resulting from an effective rainfall of specific duration is constant,
4. The ordinates of direct runoff of a constant base time are directly proportional to the total amount of direct runoff represented by each hydrograph, and
5. For a particular watershed, the size of the direct runoff hydrograph for two effective rainfall pulses is in direct proportion to the relative size of the pulses.

In actuality, these assumptions are often not true, particularly for small watersheds, which have a tendency to be non-linear in response. However, the unit hydrograph approach is usually good enough to obtain engineering estimates for design purposes.

Of great practical importance is the impulse-response function in Equation 1. This function is the IUH, if one knows the response function (or the set of weights in the discrete model), then one can predict the runoff hydrograph for any excess rainfall sequence (hyetograph) applied to the watershed.

Historically the response functions have been treated as statistical distributions although researchers have linked simplified physics to the distributions (Nash, 1958; Leinhard, 1971).

Linking a series of reservoirs in a feed forward (cascade) fashion Nash (1958) developed his IUH. The Nash model, gamma-hydrograph, and Pearson Type III hydrograph are identical distributions (under certain circumstances). Lienhard and Meyer (1967) showed that the gamma family of distributions can be explained using statistical-mechanical principles, establishing a rigorous physical basis for IUHs. Cleveland and others (2006) used a hybrid statistical-mechanical approach and particle tracking based on a quadratic kinematic model to directly predict unit hydrographs from watershed elevation data - further reinforcing the that the unit hydrograph is indeed a physics based model.

The unit hydrograph, or unitgraph, is defined to be the hydrograph of runoff resulting from a unit pulse of runoff, or effective precipitation, with a specific duration in time. Units of measurement vary with the particular procedure being applied, but in general, the depth of runoff is defined to be one unit (dimensionless) and the rate of runoff of the unitgraph has units of $\frac{L^2}{t}$. Some authors define the unit graph to have a depth of runoff of one (watershed) inch or one (watershed) centimeter. This definition leads to dimensional difficulty when convoluting a unitgraph with an effective rainfall hyetograph. But, acknowledgement of the alternative definition is important for historical reasons.

The unit hydrograph procedure should be limited to watershed drainage areas that are less than about 2,000 square miles. Concept can be (and is) applied in integrated arrangements of sub-watersheds and combined using stream-routing, storage-routing, and hydraulic routing technology (e.g. HEC-HMS, HEC-RAS, SWMM) to analyze complex problems. If storm patterns are thought to impact runoff hydrographs, then the watershed are be subdivided into smaller sub-watersheds and each of those subjected to a hydrograph analysis. The development of the procedure has been documented many times.

Unit hydrographs are developed for a specific watershed using two basic approaches. If unit rainfall-runoff data are available, then numerous techniques can be applied to estimate a unit hydrograph from the measurements (*analysis*). If no data are available, then methods of synthetic hydrology must be applied(texitsynthesis).

3 Synthetic Unit Hydrographs

Synthetic hydrology refers to development of unit hydrographs when site-specific data do not exist; That is, when there are no measurements from the watershed being studied. Methods of regionalization are used transfer known hydrographs (or other hydrologic entities) from location where measurements are available to those where the technology is needed. This procedure is done so frequently that hydrologists often dont even think about it, or the implications and errors associated with transposition or regionalization.

One procedure for synthetic hydrograph generation is to analyze a series of hydrographs for a watershed and correlate the hydrographs to watershed characteristics. These correlations are used to predict the hydrograph for some watershed whose characteristics are known, but for which no records are available the classic problem faced by highway design engineers. The literature on synthetic unit hydrographs is vast. The reader is referred to any modern hydrology textbook (McCuen, 1998) or even earlier literature (Dooge, 1973) for more information n synthetic unit hydrographs.

Computer programs such as HEC-1, HEC-HMS (Ref:), TR-20 (NRCS ref), SWMM and HSPF (U.S. EPA) all incorporate synthetic unit hydrographs to predict watershed response to various engineered and natural conditions, and the respective user manuals provide a decent overview of synthetic hydrology.

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