CE 5361 Surface Water Hydrology
Notes

Infiltration

Water that soaks into the ground and thereby enters the soil structure is considered removed from the runoff process at the time it enters the soil. The process by which this occurs is called infiltration. This process is the first step in a lengthy and vital process, the interaction of soil, water, air, and plant life.

The soil matrix in its simplest form consists of particles of soil (minerals) loosely packed together in such a way that there are void spaces (pores). The pores are filled by either air or by water. If the voids are completely filled with water, the soil is said to be saturated. If a volume of saturated soil overlies something that does not block flow, some of the water contained in it will drain away, and some will remain trapped in the pore spaces in the soil by capillary forces. The size of the pore spaces in natural soils is such that capillary forces are important in the movement of water through them. The amount of water that drains through is called gravitational water, and is particular to the soil, as is the amount retained. The water retained balances forces between gravity and capillarity, and maintains equilibrium.

Plants have roots that penetrate the upper soil layers, and remove water held there by capillarity. The upper layers of soil are thereby unbalanced, having capillary potential available to take up water. When rain falls on such a soil, there are initially two forces driving water into the soil- gravity and capillarity. If sufficient rain is available, the upper layers of the soil will become saturated, and the water will proceed downward. Due to the dual forces of capillarity and gravity, the initial rate of uptake of water may be quite high. As the capillary force is satisfied, gravity becomes the only force and the rate of uptake reaches an equilibrium value, as gravitational water drains through the soil.

The progress of this phenomenon can be shown graphically [sketch on board]. Initially, the rate of infiltration is quite high, and it decays to a steady-state value as a first-order function of time.

Mathematically, there have been a number of relationships proposed to represent this progress. Collectively these equations represent different loss models. They are all attempts to explain a complex phenomenon currently beyond our understanding at any but the smallest scales. What follows are the set of more common models in use.

Horton's Model

Hortons equation is one of the simpler, and is presented here:

$$q(t) = f_c + (f_o - f_c)e^{-kt} \tag{1}$$

where $q(t)$ = the infiltration rate at time t, in length/time; f_c = the equilibrium infiltration rate, in length/time; f_o = the initial infiltration rate, in length/time; k = a decay constant, particular to the soil, in $time^{-1}$; t = time.

The integral of Hortons equation with time is the volume of rainfall that infiltrates during an event.

$$I(t) = \int_0^t q(\tau)d\tau \tag{2}$$

where $I(t)$ is cumulative infiltration at time t, a depth.

The parameters in Hortons equation can be determined for any give soil by infiltrometer tests. The parameters in Hortons equation can be determined for any give soil by infiltrometer tests. A graph of Hortons equation for $f_c = 0.53 in/hr$, $f_o = 3 in/hr$, and $k = 4.182 hr^{-1}$ is shown in Figure 1. When rain begins, infiltration also begins, and is the initial value (denoted f_o in Hortons equation) is quite high (in this case, $3.0 in/hr$). At that time, infiltration rate usually exceeds the intensity of rainfall. As the infiltration rate drops, it at some point intersects with the rate of rainfall. After that point, rainfall rate will exceed infiltration rate. With no runoff, water would begin to pond on the soil surface. The time from beginning of rain until that point is reached is called the time to ponding. After some time, the infiltration rate has dropped until it approaches an equilibrium value, represented by f_c (in this case, $0.53 in/hr$). After that time, gravity alone is the force driving the infiltration process.

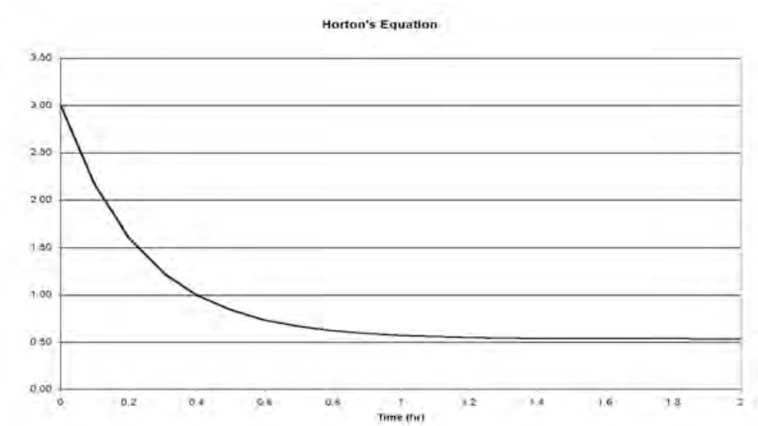

Figure 1: Horton's model using supplied parameters

Phi-index model

The ϕ-index is a simple infiltration model used in hydrology. The method assumes that the infiltration capacity is a constant ϕ (in/hr). With corresponding observations of a rainfall hyetograph and a runoff hydrograph, the value of ϕ can in many cases be easily guessed. Field studies have shown that the infiltration capacity is greatest at the start of a storm and that it decreases rapidly to a relatively constant rate. The recession time of the infiltration capacity may be as short as 10 to 15 minutes. Therefore, it is not unreasonable to assume that the infiltration capacity is constant over the entire storm duration. When the rainfall rate exceeds the capacity, the loss rate is assumed to equal the constant capacity, which is called the phi (ϕ) index. When the rainfall is less than the value of ϕ, the infiltration rate is assumed to equal to the rainfall intensity.

Mathematically, the phi-index method for modeling losses is described by

$q(t) = i(t)$, for $i(t) < \phi$
$q(t) = \phi$, for $i(t) > \phi$

where $q(t)$ is the loss rate,$i(t)$ is storm rainfall intensity, $t$is time, and ϕ is a constant. If measured rainfall-runoff data are available, the value of ϕ can be estimated by separating base flow from the total runoff volume, computing the volume of direct runoff, and then finding the value of ϕ that results in the volume of effective rainfall being equal to the volume of direct runoff. A statistical mean phi-index can then be computed as the average of storm event phi values.

Green-Ampt Model

The Green-Ampt model needs some context before presentation. The falling head permeameter is used when the hydraulic conductivity of the porous material is small. In this case the constant head permeameter cannot easily generate a high enough gradient to produce measurable flow in a reasonable amount of time.

Figure 2 is a schematic diagram of a falling head permeameter. A sample of porous medium is placed the device. The length of the sample is L, the cross sectional area of the sample is A. A smaller area tube rises above the sample. This tube provides the driving force required to move water through the porous sample with reasonably small water volume. The area of this tube is a. The head is measured at the inlet of the sample as the height of water in the tube above the sample, $h(t)$. $Q(t)$ is the varying rate of flow through the sample. Observe that in a falling head permeameter, both the head and the flow rate vary with time.

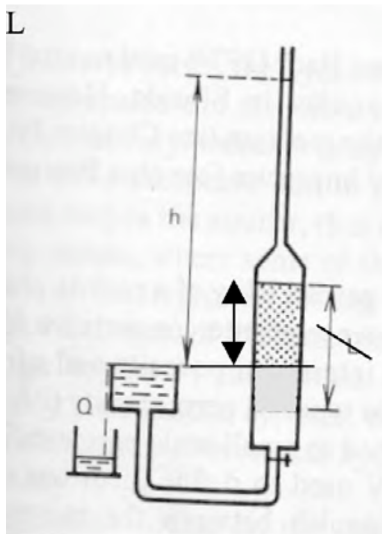

Figure 2: Falling head permeameter schematic. From DeMarsilly, 1986 – need cite

Darcys law for this situation is

$$Q(t) = \frac{KA\ h(t)}{L} \tag{3}$$

A volume balance for the device leads alternate expression for discharge rate, and equating these two expressions gives a differential equation for flow in the porous medium.

$$Q(t) = -a\frac{dh(t)}{dt} = \frac{KA\ h(t)}{aL} \tag{4}$$

Separation of variables produces an expression that can be integrated to relate head and time.

$$\int \frac{dh(t)}{h} = -\frac{KA}{aL}\int dt \tag{5}$$

Integration of this equation will provide a formula that relates $h(t)$ and t in terms of hydraulic conductivity, tube and sample areas and the length of the sample. The result of integration and evaluation of the constant of integration produces an equation that describes the falling head permeameter

$$ln(\ h(t)) = ln(\ h_0) - \frac{KA\ t}{aL} \tag{6}$$

This equation is linear in time, so that a plot of the logarithm of h versus time should be a straight line, the slope of the line is proportional to the hydraulic conductivity, K.

Now, back to runoff. On a watershed the two areas a and A are the same, but not all the water is forced into the ground (sample), some runs off - and to do so it must pond a little.

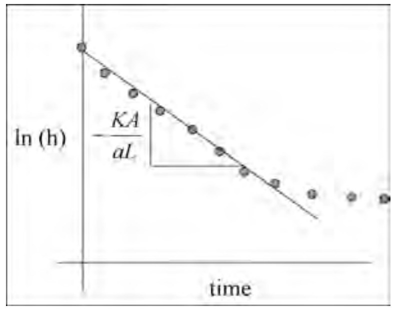

Figure 3: Falling head permeameter results (typical)

Figure 4 is a schematic of such a process over three different times. In the middle panel, there is some ponding, and some water has infiltrated into the soil.

The amount infiltrated is $\frac{V}{A} = nz$.

The rate is then simply the speed of the wetting front $\frac{dv}{dt} = n\frac{dz}{dt}$ where $v = \frac{V}{A}$.

Darcy's law from the ground surface to the wetting front is $\frac{Q}{A} = q = K\frac{H+h_c+z}{z}$.

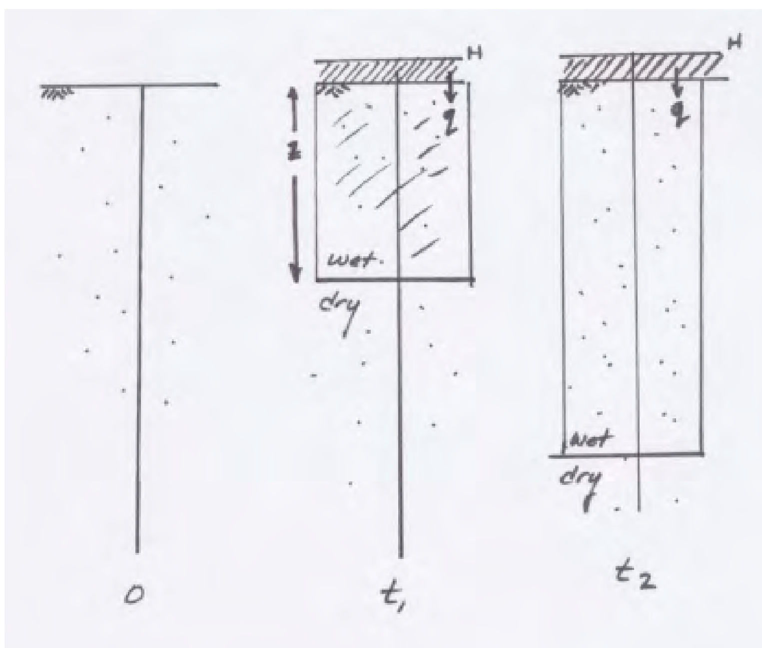

Figure 4: Watershed infiltration schematic

FInally equating these two rates yields a differential equation of the progress of the wetting front.

$$n\frac{dz}{dt} = K\frac{H + h_c + z}{z} \tag{7}$$

Proceeding as before with the permeameter one separates and integrates as

$$\frac{dz}{H + h_c + z} = \frac{K}{n}dt \tag{8}$$

A partial fraction expansion of the left term is

$$\frac{H + h_c + z}{H + h_c + z}dz - \frac{H + h_c}{H + h_c + z}dz = \frac{K}{n}dt \tag{9}$$

Integrate both sides

$$z - (H + h_c)ln(\ (H + h_c + z)) = \frac{K}{n}t + \ C \tag{10}$$

Evaluate the constant of integration at $t = 0, z = 0$

$$z + (H + h_c)ln((H + h_c) - (H + h_c + z)) = \frac{K}{n}t \tag{11}$$

Rearrange a bit

$$z + (H + h_c)ln(\frac{(H + h_c)}{(H + h_c + z)}) = \frac{K}{n}t \tag{12}$$

Now make a substitution $I(t) = nz$

$$I(t) = nz = n[(\frac{K}{n}t)\ \ -\ \ (H + h_c)ln(\frac{(H + h_c)}{(H + h_c + I(t)/n)})] \tag{13}$$

After further rearrangement becomes:

$$I(t) = Kt + (H + h_c)ln(1 + \frac{I(t)}{(H + h_c)(n)}) \tag{14}$$

This equation is of the same general form as the falling head permeameter with the following differences. $I(t)$ is the cumulative infiltration depth is analogous to the sample length and would be the depth into the ground that the infiltration penetrates, n is the porosity (in this equation I have assumed that the soil is dry ahead of the water and fully saturated after). The above equation is evaluated recursively to determine infiltration depth, and any precipitation depth not infiltrated is assumed to runoff during that time.

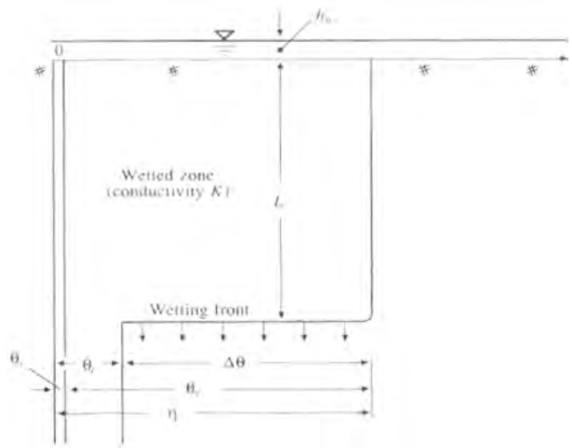

Figure 5: Variables in the Green-Ampt infiltration model. The vertical axis is the distance from the soil surface; the horizontal axis is the moisture content of the soil. (Source: Applied Hydrology by Chow/Maidement/Mays 1988)

Now for classical Green-Ampt

Green & Ampt (1911) proposed the simplified picture of infiltration shown in Figure 5. The wetting front is a sharp boundary dividing soil below with moisture content ϕ_i from saturated soil done with moisture content ϕ_e above. The wetting front has penetrated to a depth L in time t since infiltration began. Water is ponded to a small depth h_0 on the soil surface. The method computes total infiltration rate at the end of time t. The Green-Ampt solution is essentially identical to the wet/dry case just presented except $n = \phi_e - \phi_i$ and $H + h_c = h_0 + \psi$

$$I(t) = Kt + (h_o + \psi_f)ln(1 + \frac{I(t)}{(h_o + \psi_f)(\phi_e - \phi_i)}) \tag{15}$$

This method gives the total amount of infiltration in the soil at the end of a particular storm event. Depending on this value and the total amount of precipitation, we can calculate the amount of runoff. Green-Ampt is the preferred method for use in Harris County studies (as per TSARP white papers). Either presentation is adequate what is needed is K,n, and h_c or K,$\nabla\phi$, and ψ_f.

My personal preference is the K,n, and h_c form because the parameters are (in my opinion) easier to estimate, but most models (HEC-HMS, SWMM, etc.) use the h_c or K,$\nabla\phi$, and ψ_f, form.

SCS Curve Number Method

For historical reasons and because the CN method is still in common use it is examined here.The curve number approach is based on a volume balance (just like the constant fraction approach). The derivation is included in the bibliography as National Engineering Handbook Chapter 10.

The curve number is a solution to:

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \tag{16}$$

where Q is the cumulative runoff (integral of the hydrograph),P is the cumulative rainfall (integral of the hyetograph). Drainage area is used to make units consistent; either divide Q by area to get a depth, or multiply P by area to get volume). S is the retention (a storage term).

The curve number, CN, is simply a transformation of S

$$CN = \frac{1000}{S + 10} \tag{17}$$

The CN values for a large number of land coverages (hydrologic soil complex coverage number; or runoff curve number) were tabulated by the NRCS many years ago. It is of note that the method was intended for agricultural use. Equations similar in structure to the NRCS equations above can be constructed by assuming the watershed operates as a linear reservoir and such an analysis provides some understanding of CN (as other than just a tabulation). When viewed in such a fashion the CN is like a response or residence time parameter and represents how long in dimensionless time it takes the watershed to reach an equilibrium storage condition where the precipitation that enters leaves. Large values (90+) are fast responding watersheds, small values (50-ish) are slow responding watersheds.

Figure 6 is the typical graphical representation from NRCS sources.

Some Thoughts on CN Approach and relationship to simplified physics

The CN approach is in wide use, hence there is some justification for its coverage here as a loss model.

The NRCS CN method is a well-documented empirical procedure. The method can be related to a physical explanation assuming a linear reservoir model, but the author is unsure if this approach was how the procedure was originally developed. Chapter 10 of the NEH is a derivation of the CN procedure and is required reading for the course.

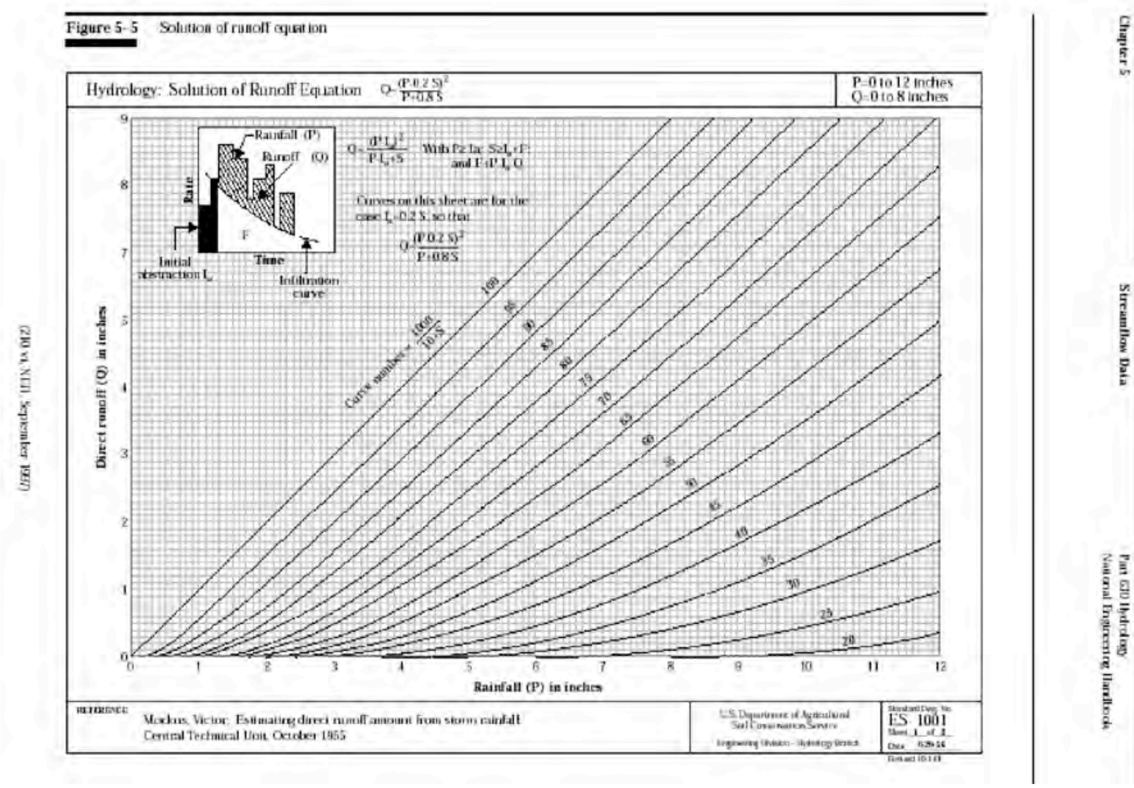

Figure 6: CN Diagram NEH Chapter 10

A physical plausibility argument is as follows:

Assume a watershed can be represented as a linear reservoir, where the discharge is proportional to accumulated storage. Using the CN variable names and definitions it is possible to construct a discharge function that is a decaying exponential. The decay rate conveys similar information as the curve number, that is it relates how much cumulative precipitation must occur before the retention is satisfied and the ratio of actual to potential retention becomes one.

First the variable names associated with Figure 7 and their definitions from chapter 10, NEH.

I_a is the initial abstraction - it represents input rainfall that never appears as runoff and is removed at the beginning of an event.

F_a is the watershed retention - it represents the depth of water retained on the watershed after runoff begins.

S is the potential watershed retention after runoff begins - it represents the maximum possible depth of of water retained on the watershed if the rain goes on forever.

Q is the actual runoff depth.

P is the actual rainfall depth.

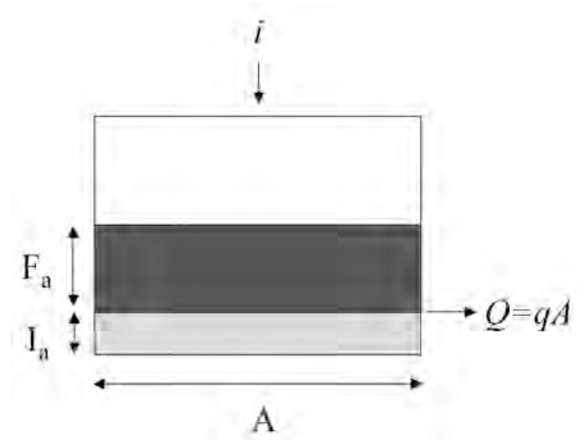

Figure 7: Linear Reservoir Representation

Figure 7 is a sketch of a watershed as a reservoir. The area is A, precipitation rate is i, the initial abstraction is Ia and the actual retention (accumulated depth on the watershed in excess of I_a) is F_a. The discharge in volume units is qA.

If one writes a mass balance on the watershed in terms of F_a, ASSUMING the initial abstraction is already satisfied (as in the NRCS definition of F_a above), then the resulting equation is

$$A\frac{dF_a}{dt} = iA - qA \tag{18}$$

Then if one normalizes by the watershed area, the result is

$$\frac{dF_a}{dt} = i - q \tag{19}$$

The units of precipitation and discharge are now L/T (depth per unit time). The units of accumulated depth are also length (thus the time derivative is L/T). If we ASSUME a linear response, that is the specific discharge is proportional to accumulated depth (F_a) we can postulate a model for the discharge as .

$$q = \frac{1}{\alpha}F_a \tag{20}$$

The term involving α is simply a constant.

Now substitute into the mass balance and solve for F_a and ASSUME that precipitation rate is a constant value one can arrive at:

$$q = \frac{1}{\alpha}F_a = i(1 - e^{-\frac{t}{\alpha}}) \tag{21}$$

In this case, time begins when the initial abstraction is satisfied.

Now examine the definitions of P and Pe in the NRCS documents. By defintion:

$$P_e = \int_0^t q(\tau)d\tau = it + i\alpha e^{-\frac{t}{\alpha}} - i\alpha \tag{22}$$

$$P = it + I_a + F_a \tag{23}$$

Now if we construct a chart that plots Equations 22 and 23 (and an equal value line) we can observe the impact of α on the cumulative runoff. For instance Figures 8 and 9 are two such figures with different values of α. From the figures we observe that the value of α impacts when the P_e line becomes parallel to the equal value line (i.e. the value of α is in some sense a time-to-equilibrium parameter).

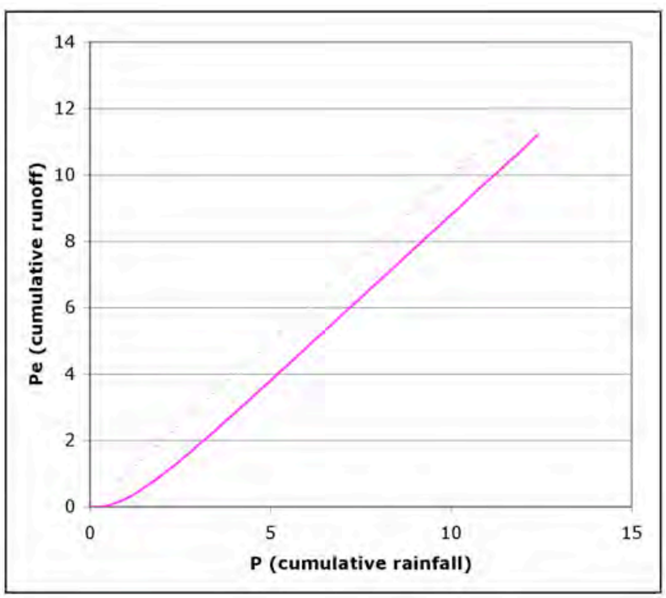

Figure 8: P_e versus P for $\alpha = 1$, $i = 1$, $S = 1$, $I_a = 0.2$. Note NRCS uses $I_a = 0.2S$ in their method.

Now if we compare the appearance of these two figures to 6 we can observe that the three charts convey the same kinds of curves, specifically the amount of precipitation accumulation required to produce constant runoff, as well as the conversion ratio between observed

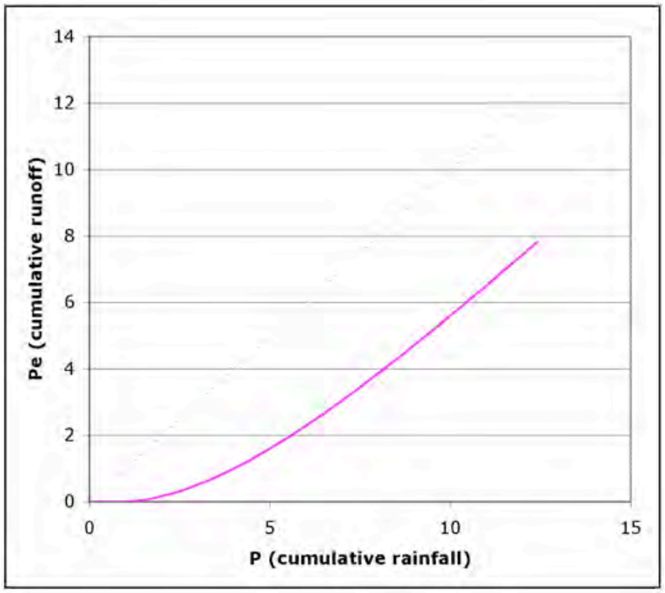

Figure 9: P_e versus P for $\alpha = 4$, $i = 1$, $S = 4$, $I_a = 0.8$. Note NRCS uses $I_a = 0.2S$ in their method.

cumulative precipitation and observed cumulative runoff. In the case of the linear-reservoir model, the information is conveyed by the parameter α which dimensionally is a residence time, while the curve number CN plays a similar role in the NRCS methodology.

Furthermore, for the two linear reservoir cases the following transformations seem to apply:

1. $\alpha = \frac{100-CN}{10}$ Thus the first curve is for CN=90; the second for CN=60.
2. $i\alpha = S$ Thus the product of the residence time and intensity is the maximum potential retention.
3. Runoff starts (in the time domain) at $t_r = \frac{I_a}{i} = \frac{\alpha}{5}$

It is my opinion that these relationships are not accidental and that the original NRCS CN method was based on the linear reservoir model of small agricultural watersheds. Obviously as a loss model we would be concerned with the difference, $P - P_e$. The point here is that even though the CN method is presented as empiricism, it does have backing in a simplified physics model of runoff.

The second, and perhaps more important point, is that the equations and algorithms in engineering books can usually be explained by simplified mathematics and (in my opinion) such understanding should be attempted before really putting too much faith into the equations[1].

Summary

This lecture presented the infiltration as a significant loss component. Several common loss models were presented including:

1. Proportional loss model - Empirical based, forces volume balance in cumulative space, related to rational method (the proportional are called runoff coefficients). All rainfall bursts contribute to runoff.
2. ϕ-index model - Empirical based, forces volume balance in cumulative space. Some rainfall bursts may not contribute to runoff.
3. Horton's model - Observational based. When used in hydrologic models may not preserve volume balances.
4. Green-Ampt model - Physics based. When used in hydrologic models may not preserve volume balances.
5. NRCS CN model - Not strictly a loss model; Empirical based, forces volume balance. Simplifies physics model can produce similar behavior under same assumptions as NRCS assumptions.

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[1]The equations thus explained may be just as wrong, but at least you have put some thought into their origin and use.