CE 3354 Engineering Hydrology

Lecture 21: Groundwater Hydrology Concepts – Part 1

Outline

- Porous Media Concepts
  - Porosity, Yield, Average Linear Velocity
  - Heads and gradients
- Storage
  - Confined and Unconfined
- Darcy's Law
  - Permeability

Groundwater Hydrology

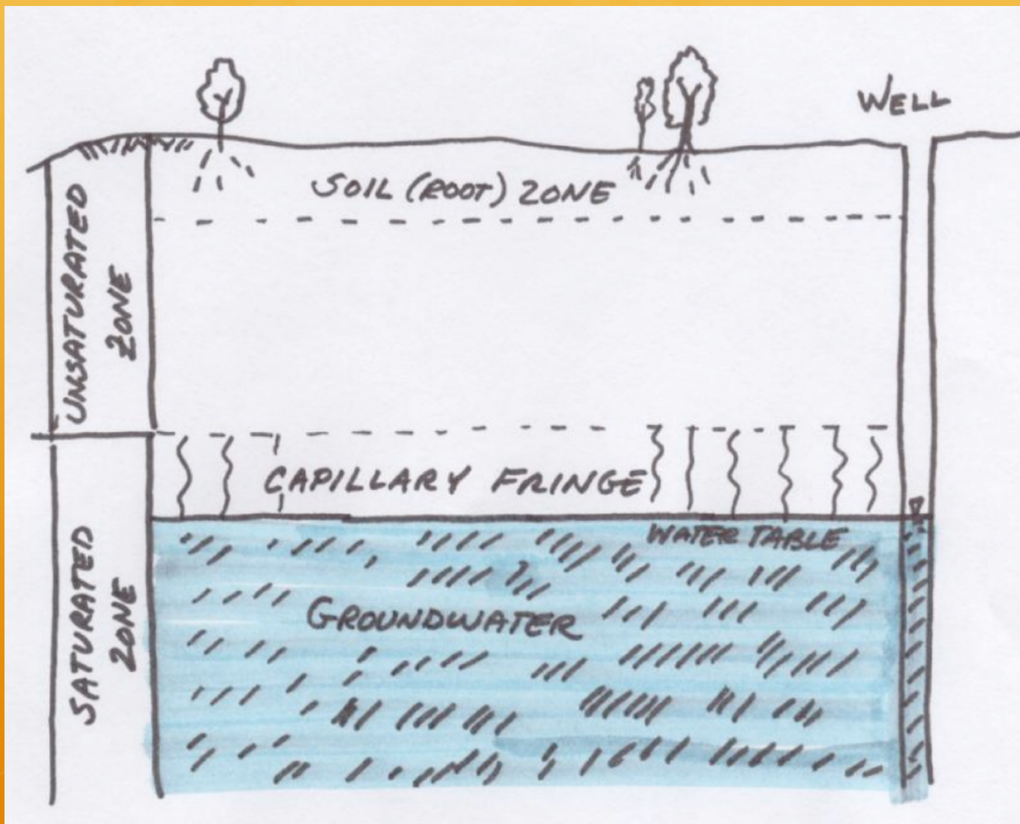
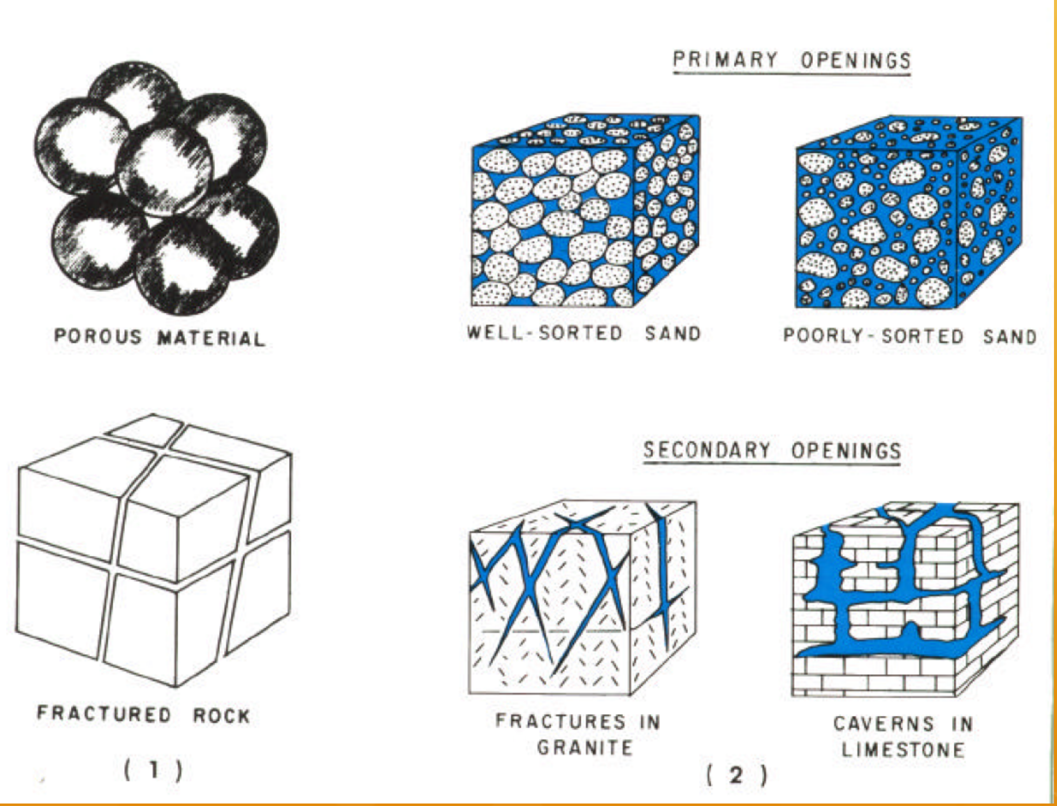


Figure 1: Soil profile sketch (adapted from Heath 1989)

- Groundwater hydrology is the study of water beneath the surface of the Earth

Porosity

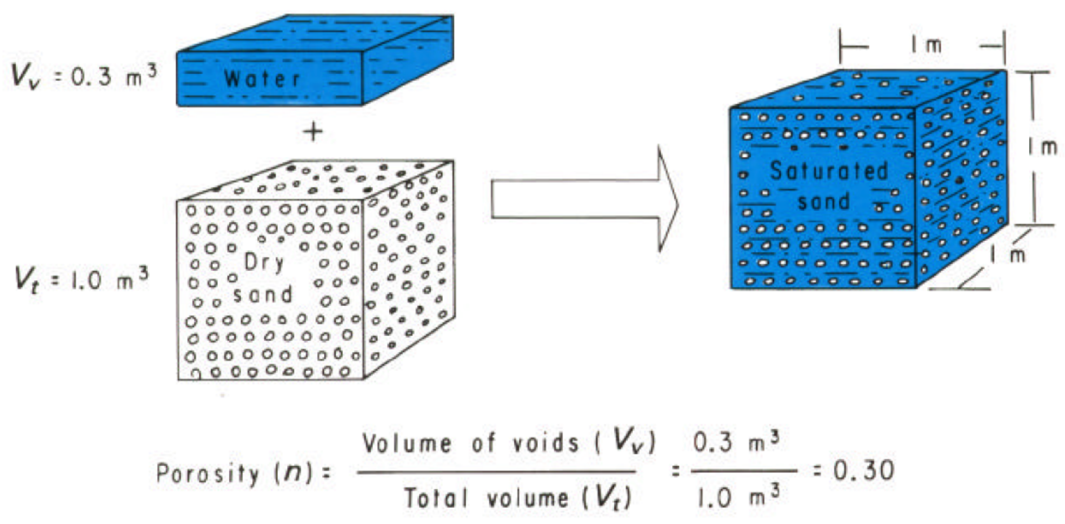
- Groundwater usually is found in porous media (not underground rivers).
- A porous medium is comprised of solid space and void or pore space.
  - Liquids and gasses are found in the pore space, the solid matrix forms the physical structure of aquifers and other geologic formations of interest.
- The ratio of total pore volume to bulk medium volume is the porosity



$$n = \frac{V_{voids}}{V_{bulk}}$$

Porosity

- Range in values large for geologic materials



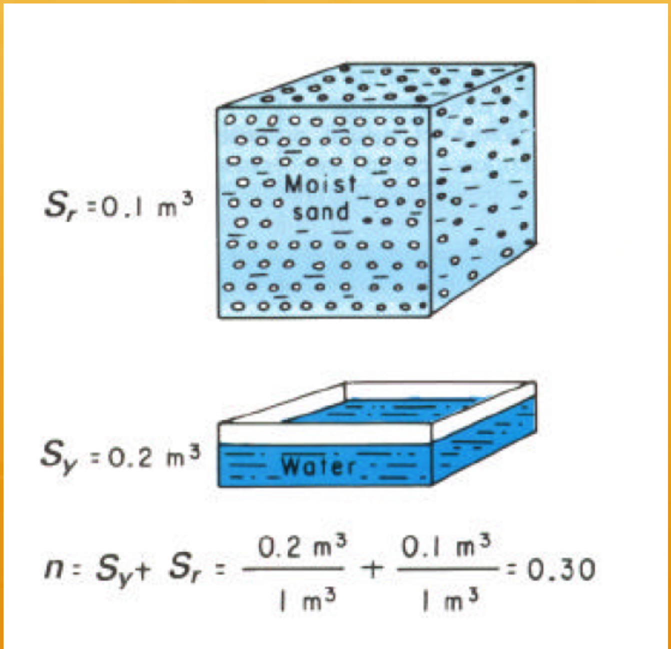
Soil Description	Porosity Range
Volcanic, pumice	0.80-0.90
Peat	0.60-0.80
Silt	0.35-0.60
Clay	0.35-0.55
Loess	0.40-0.55
Sand, dune	0.35-0.45
Sand, fine	0.25-0.55
Sand, coarse	0.30-0.45
Gravel, coarse	0.25-0.35
Sand and gravel	0.20-0.35
Till	0.25-0.45
Siltstone	0.25-0.40
Sandstone	0.25-0.50
Volcanic, vesicular	0.10-0.50
Volcanic, tuff	0.10-0.40
Limestone	0.05-0.55
Schist	0.05-0.50
Basalt	0.05-0.35
Shale	0.01-0.10
Volcanic, dense	0.01-0.10
Igneous, dense	0.01-0.05
Salt bed	0.005-0.03

Specific Yield

- A concept related to porosity is the specific yield of a material
- S_y is the amount of water that will drain from a porous medium under the influence of gravity.
- S_r is the amount of water left behind in the material and is called the specific retention

$$S_r = \frac{V_{retained}}{V_{bulk}}$$

$$S_y = \frac{V_{drained}}{V_{bulk}}$$



The sum of the two terms is the porosity ($S_y + S_r = n$).

Specific Yield

- The specific yield is important in water supply as it represents the amount of water that can drain to wells.
- Thus when making groundwater reservoir estimates the water in storage should be based on the specific yield and not porosity.
- Two related terms are:
  - water content
  - and saturation

Material	S_y	S_r
Soil	0.40	0.15
Clay	0.02	0.48
Sand	0.22	0.03
Gravel	0.19	0.01
Limestone	0.18	0.02
Sandstone	0.06	0.05
Granite	0.0009	0.0001
Basalt	0.08	0.03

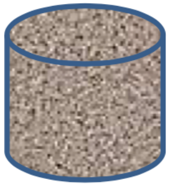
$$\theta_{wc} = \frac{V_{water}}{V_{bulk}}$$

$$S_w = \frac{V_{water}}{V_{void}}$$

Measuring Porosity

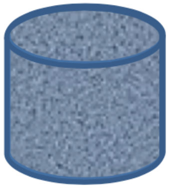
- Porosity is measured in the laboratory from small samples by gravimetric methods and fluid displacement methods.
- At the field scale, porosity is measured by geophysical tools calibrated to local geologic media.
  - Resistivity logging
  - Acoustic logging
  - Neutron logging

Weight dry



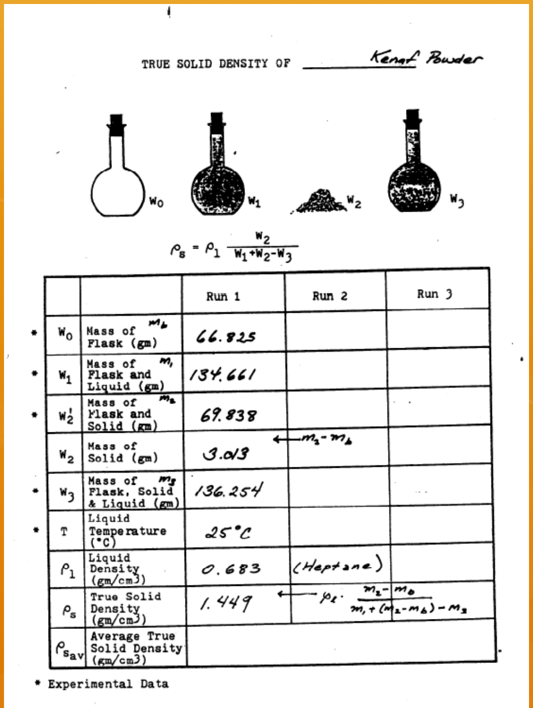
W_{dry} = 20 gms

Weight saturated



W_{sat} = 22.5 gms

$$n = \frac{M_{water}}{V_{bulk}} \frac{1}{\rho_{water}}$$



TRUE SOLID DENSITY OF Kenaf Powder

$$\rho_s = \rho_1 \frac{W_2}{W_1 + W_2 - W_3}$$

			Run 1	Run 2	Run 3
*	W_0	Mass of Flask (gm) m_b	66.825		
*	W_1	Mass of Flask and Liquid (gm) m_1	134.661		
*	W_2'	Mass of Flask and Solid (gm) m_2	69.838		
	W_2	Mass of Solid (gm)	3.013	← $m_2 - m_b$	
*	W_3	Mass of Flask, Solid & Liquid (gm) m_3	136.254		
*	T	Liquid Temperature (°C)	25°C		
	ρ_1	Liquid Density (gm/cm3)	0.683	(Heptane)	
	ρ_s	True Solid Density (gm/cm3)	1.449	← $\rho_l \cdot \frac{m_2 - m_b}{m_1 + (m_2 - m_b) - m_3}$	
	$\rho_{s_{av}}$	Average True Solid Density (gm/cm3)			

* Experimental Data

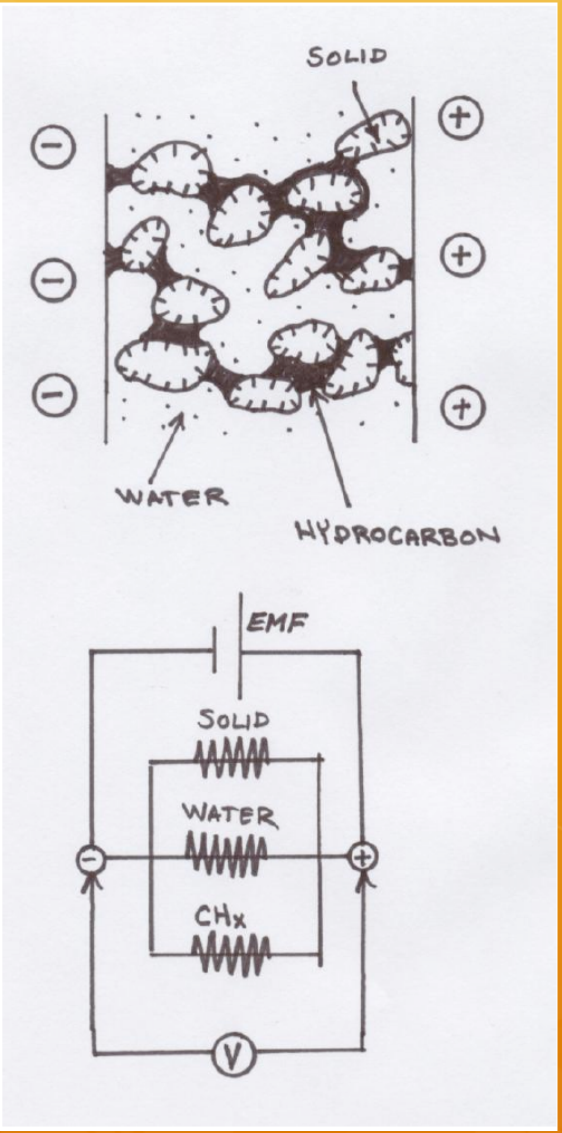
$$n = 1 - \frac{\rho_{bulk}}{\rho_{solids}}$$

Measuring Porosity

- At the field scale, porosity is measured by geophysical tools calibrated to local geologic media.
  - Resistivity logging
  - Acoustic logging
  - Neutron logging

$$F = \frac{R_{100}}{R_A}$$

$$F = \frac{S_w \bullet R_{100}}{R_m}$$



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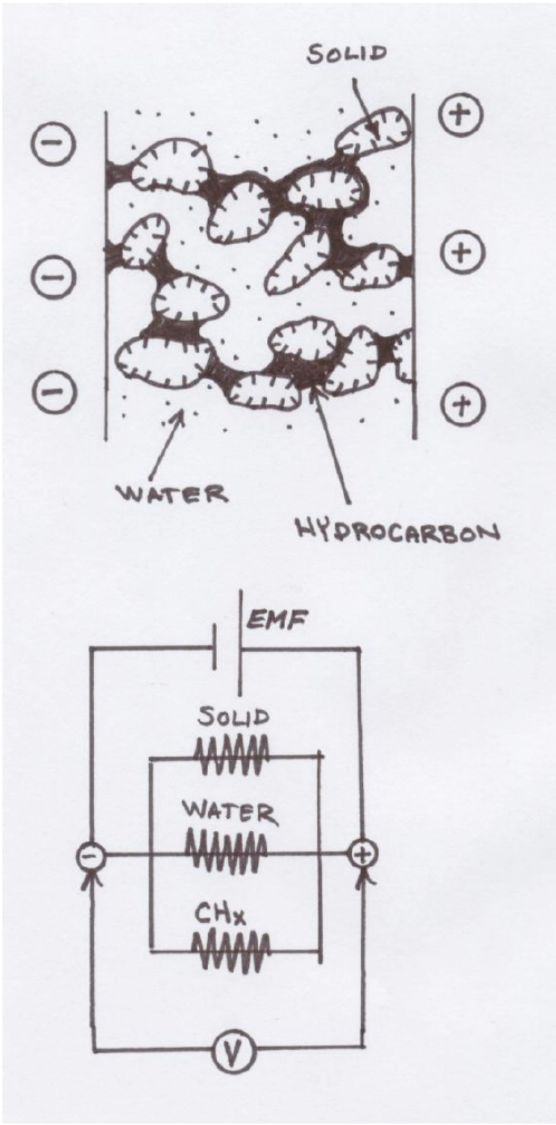

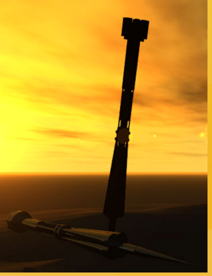

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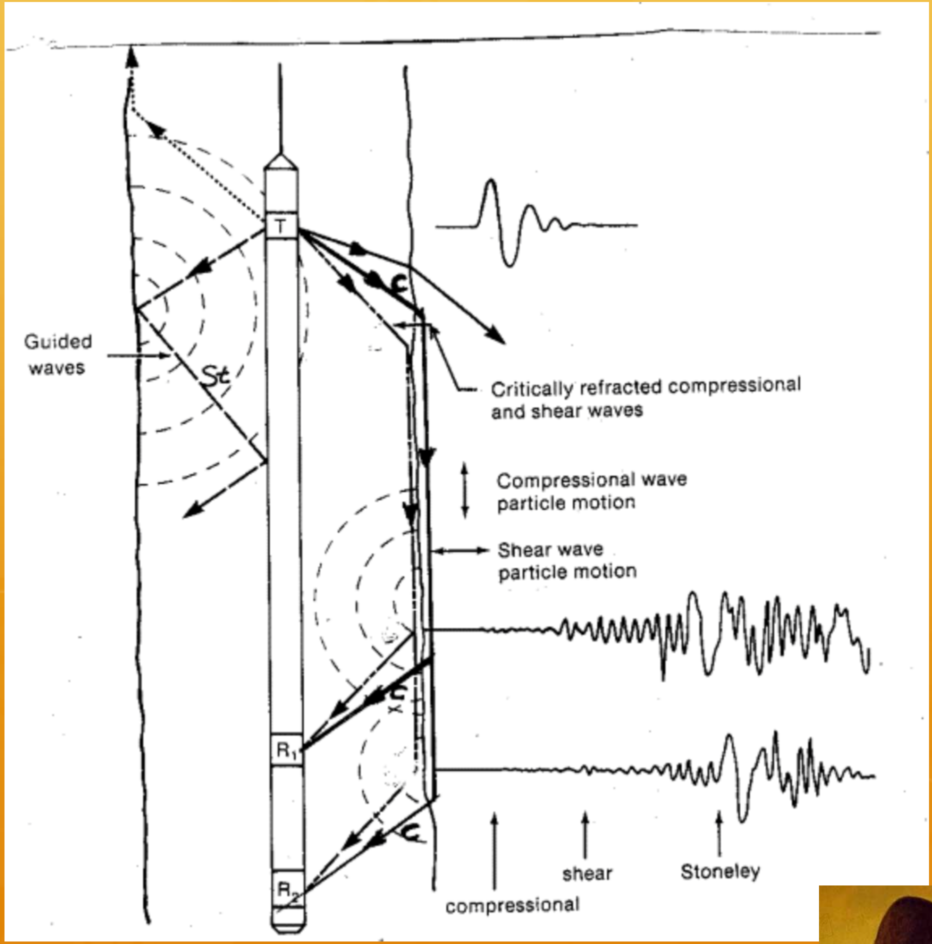
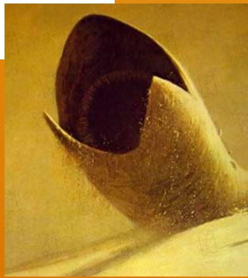


$$\Delta t_{measured} = n\frac{1}{v_{fluid}} + (1-n)\frac{1}{v_{solids}}$$

$$n = \frac{\Delta t_{measured} - \Delta t_{solid}}{\Delta t_{fluid} - \Delta t_{solid}}$$



Measuring Porosity

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Material	v (feet/sec)
Water	5,000-5,400
Clay (saturated)	5,000-6,000
Shale	6,000-16,000
Sandstone	15,000-18,000
Limestone	19,000-21,000
Dolomite	21,000-24,000

$$\Delta t_{measured} = n \frac{1}{v_{fluid}} + (1 - n) \frac{1}{v_{solids}}$$

$$n = \frac{\Delta t_{measured} - \Delta t_{solid}}{\Delta t_{fluid} - \Delta t_{solid}}$$

Measuring Porosity

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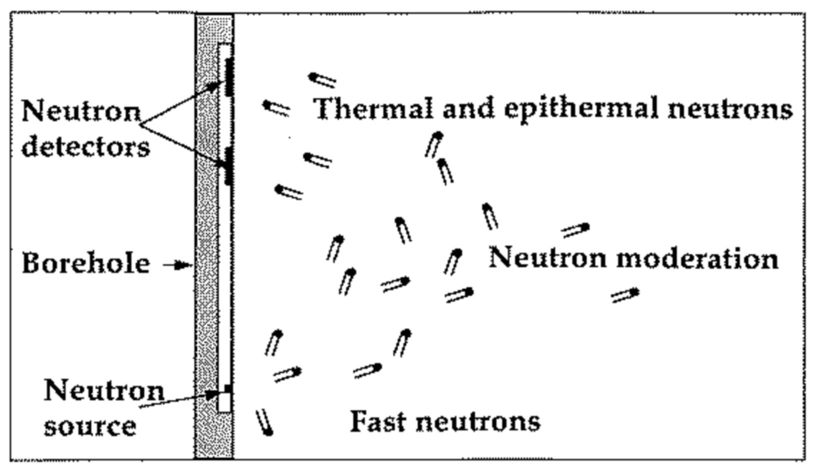


***Neutron Survey*:** A neutron log is based on the principle that hydrogen is a good neutron reflector.

Since water is a dense source of hydrogen (11% by mass) it will scatter high-energy neutrons much more than a material that is a poor neutron reflector (moderator)

A neutron log measures the energy of neutrons scattered by a material.

High backscatter => mostly water; thus high porosity.

Low backscatter => mostly mostly solid; thus low porosity.

The neutron log must be calibrated against local media

Average Linear Velocity

- The discharge Q divided by cross sectional flow area A in a pipe or open channel is the velocity V
- In groundwater, some of the area is solid, so the porosity enters the equation

$$u = \frac{Q}{A}\frac{1}{n}$$

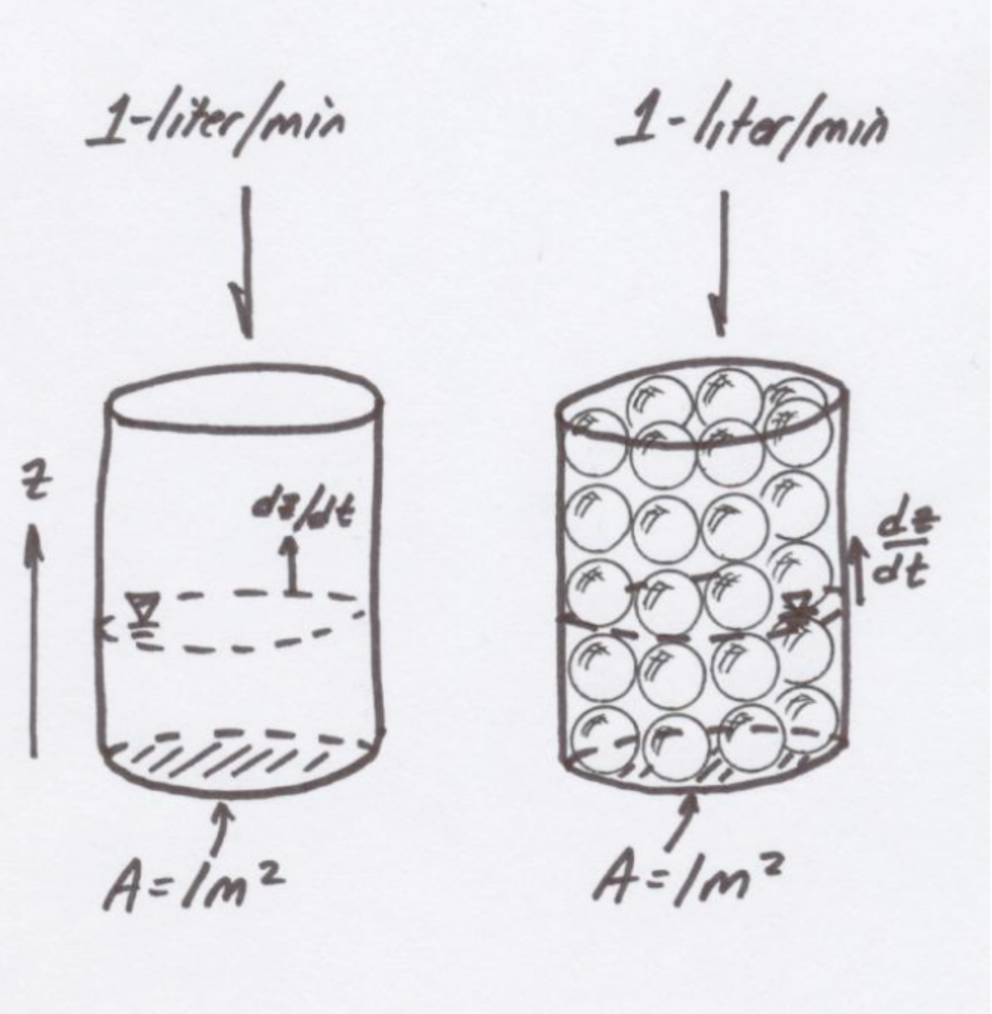
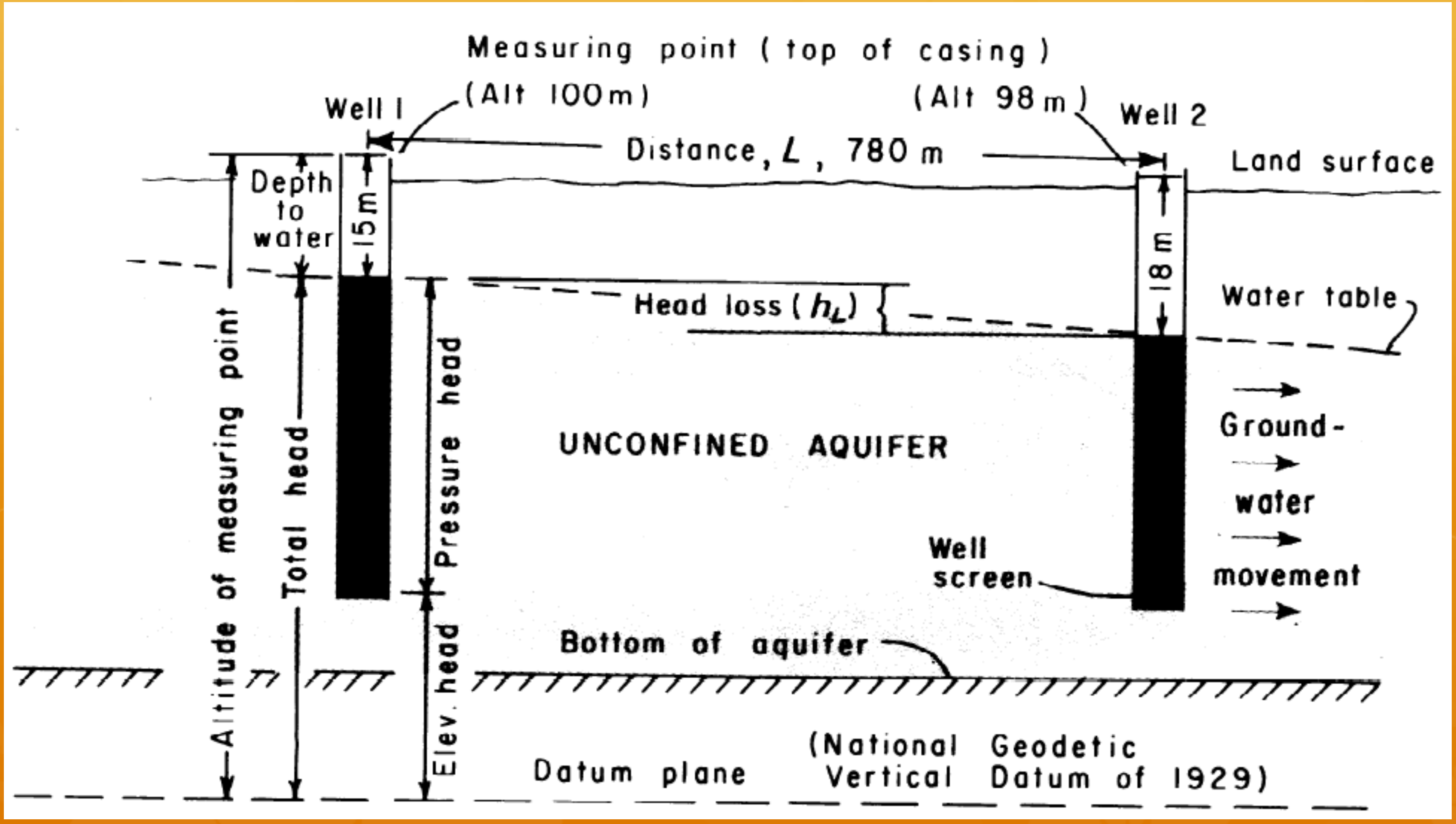


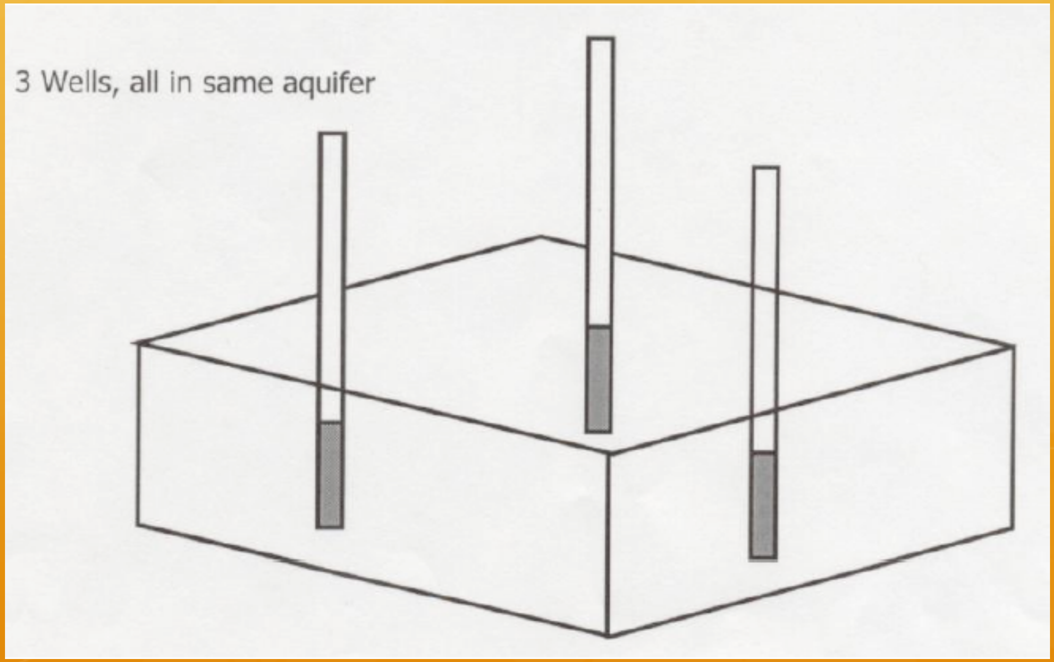
Figure 3: Garbage can with and without marbles

Energy and Head

- Hydraulic gradient is change in head per unit distance



Energy and Head



- Piezometric surface in the vicinity of the three wells can be approximated by a plane.
- All three wells sample the same aquifer unit.
- Wells measure vertically averaged head
- Head measurements at same time

3-Well Gradient Example

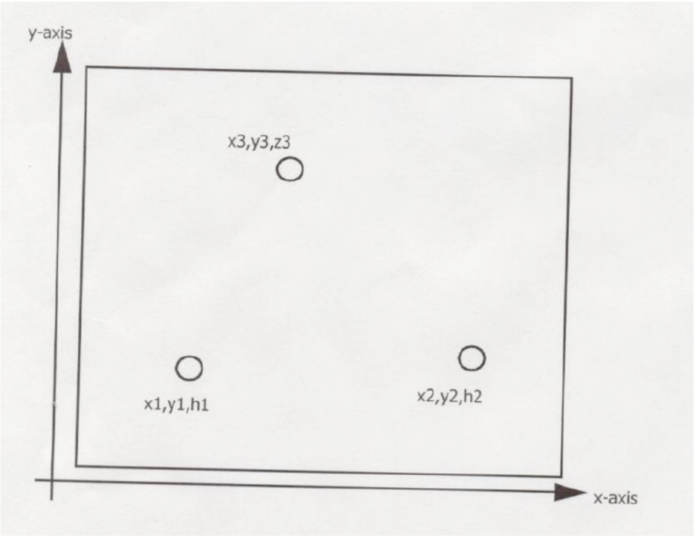


Figure 5: Three wells plotted on an arbitrary coordinate system

1. Plot locations and water levels of each well as depicted on Figure 5.
2. Select an origin to measure x and y distances for each well location.
3. Solve the equation of the plane by solving the system of linear equations for all wells simultaneously - you are solving for the unknown coefficients a,b, and c in:

$$h_1 = ax_1 + by_1 + c \quad (13)$$
$$h_2 = ax_2 + by_2 + c$$
$$h_3 = ax_3 + by_3 + c$$

4. Once you have the values for a, b, and c, the hydraulic gradient is the vector $\overline{\nabla h} = -a_i - b_j$ where i and j are the unit vectors in the x and y directions. The magnitude of the gradient is $\sqrt{a^2 + b^2}$. This gradient is also the flow direction if the porous media is homogeneous and isotropic.

3 wells monitor water levels in a confined aquifer. Well A is 3000-feet south of well B. Well C is 2000 feet due West of well B. The land surface elevations and depth to water for each well are in Table 4. Find the hydraulic gradient (direction and magnitude).

Table 4: Well Data for Three Wells

Well ID	Top of Casing (feet)	Depth to Water (feet)	Head (feet)
Well A	480.0	40.0	
Well B	610.0	140.0	
Well C	545.0	85.0	

3-Well Gradient Example

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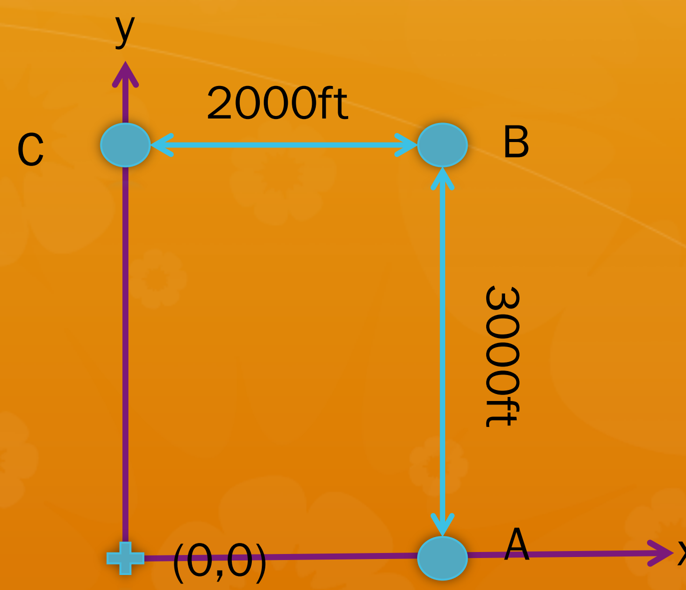
H8

	A	B	C	D	E	F	G
1			TOC	DTW	HEAD	X	Y
2	Well	A	480	40	440	2000	0
3	Well	B	610	140	470	2000	3000
4	Well	C	545	85	460	0	3000
5							



3-Well Gradient Example

Instructions: Enter data for three wells in shaded area above left. Spreadsheet solves linear system and computes gradient. Plot shows gradient direction.

Save and Exit

Groundwater Hydrology Gradient Spreadsheet
Field Data from Three Wells

	x	y	head
Well A	2000	0	440
Well B	2000	3000	470
Well C	0	3000	460

Hydraulic	-0.005	<-i.
Gradient	-0.01	<-j.

A-Matrix			b-vector	x-vector
2000	0	1	440	0.005
2000	3000	1	470	0.01
0	3000	1	460	430
A-Inverse				
0	0.0005	-0.0005		
-0.0003333	0.00033	0		
1	-1	1		

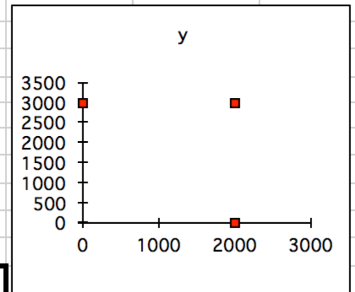
Head	+	0	0	<-x
Function	+	0	0	<-y
	+	430		
		430	<= h(x,y)	

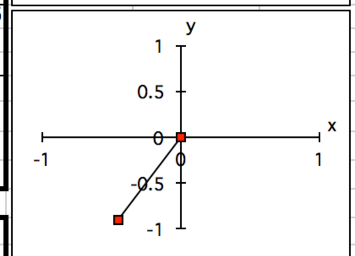
This spreadsheet prepared by
Theodore G. Cleveland, Ph.D., P.E.

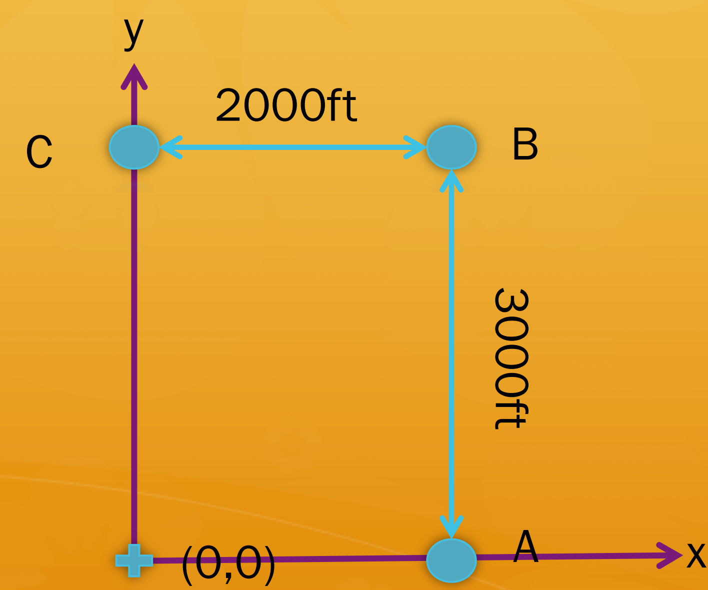
Hydraulic Gradient Vector Magnitude	
Magnitude=	0.01118

i=	j=		
0	0	Hydraulic Gradient	
-0.4472136	-0.8944	Direction Cosines	
		x-dir.=>	-0.4472
		y-dir.=>	-0.8944

MODEL1 | Sheet1



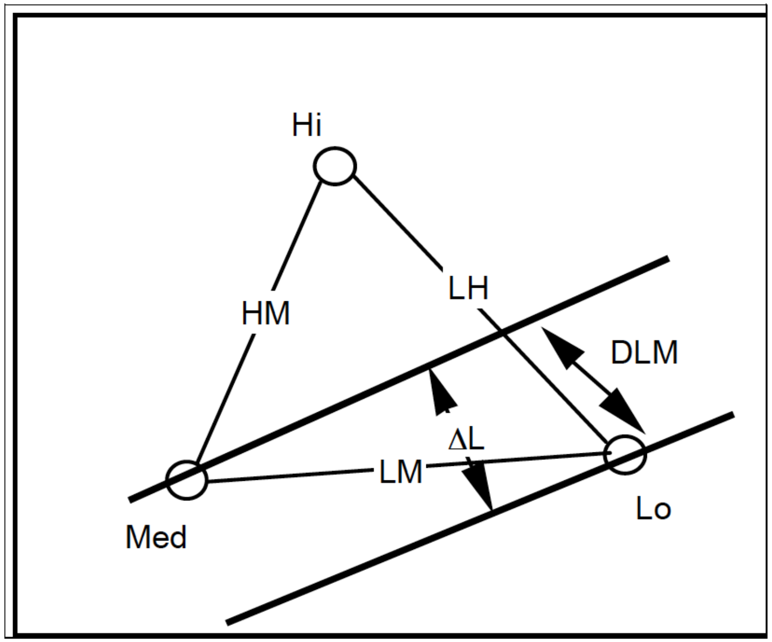




3-Well Gradient Example

<u>Graphical method:</u>

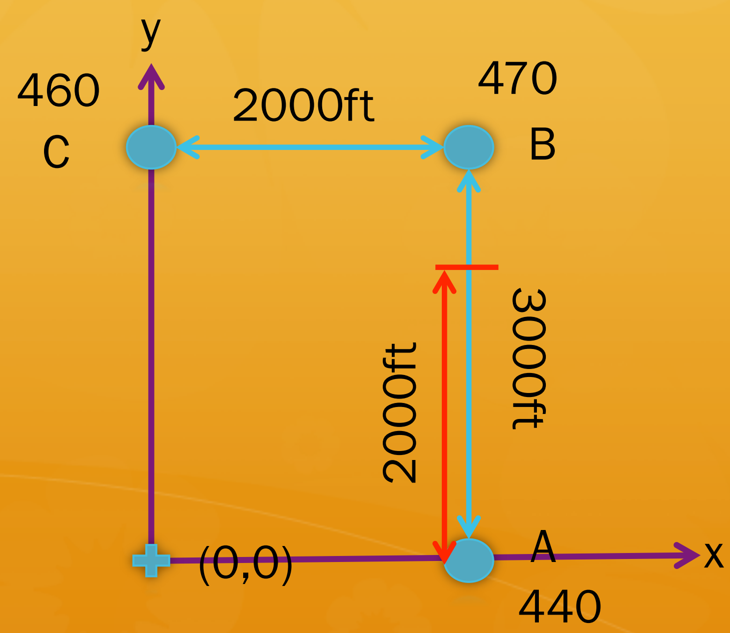
a) Plot locations and static water level (SWL) elevations of each well.



b) Identify High, Medium, and Low SWL elevations. (Shown as Hi, Med, and Lo in picture)

c) Calculate point between High and Low wells where SWL = Medium Value from :

$$DLM = \frac{\text{Med-Lo}}{\text{Hi-Lo}} * LH.$$



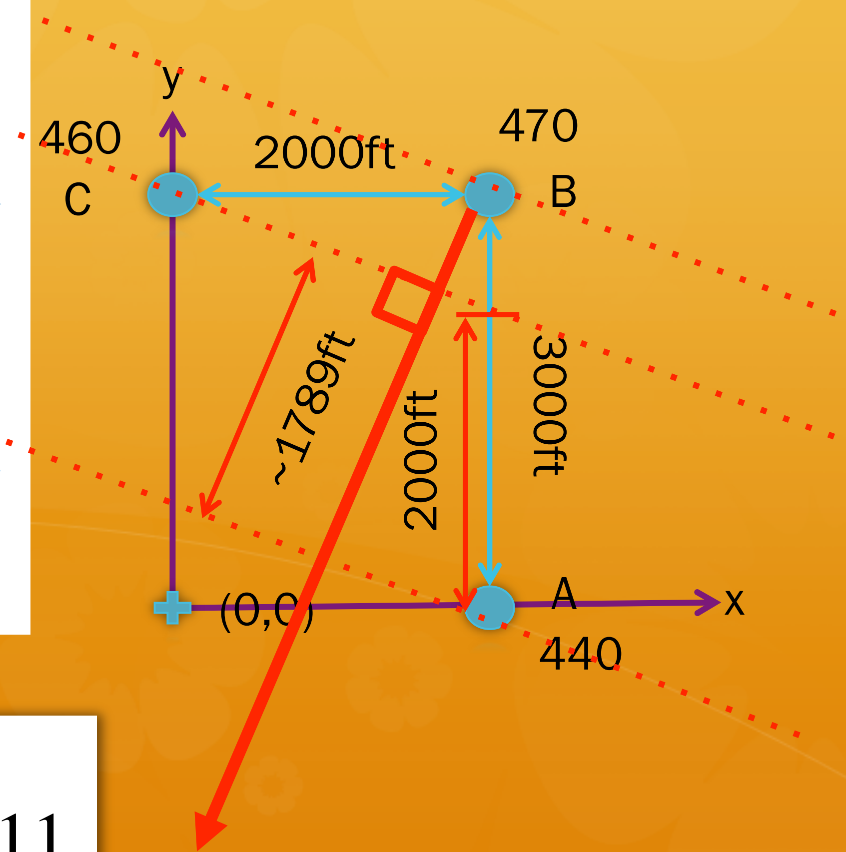
3-Well Gradient Example

Hi = head value at high well
Med = head value at medium well
Lo = head value at low well
LH = distance from High well to Low well.

d) Draw equipotential line connecting points of equal head from Medium well to DLM along segment LH.

e) Draw gradient orientation line perpendicular to this equipotential line.

f) Compute hydraulic gradient as: Δh/ΔL where Δh is the change in head between Medium well and Low Well, and ΔL is the distance between the medium well equipotential line and the low well equipotential line along the gradient orientation line.



$$\frac{\Delta h}{\Delta L} = \frac{460\,ft - 440\,ft}{1789\,ft} = 0.011$$

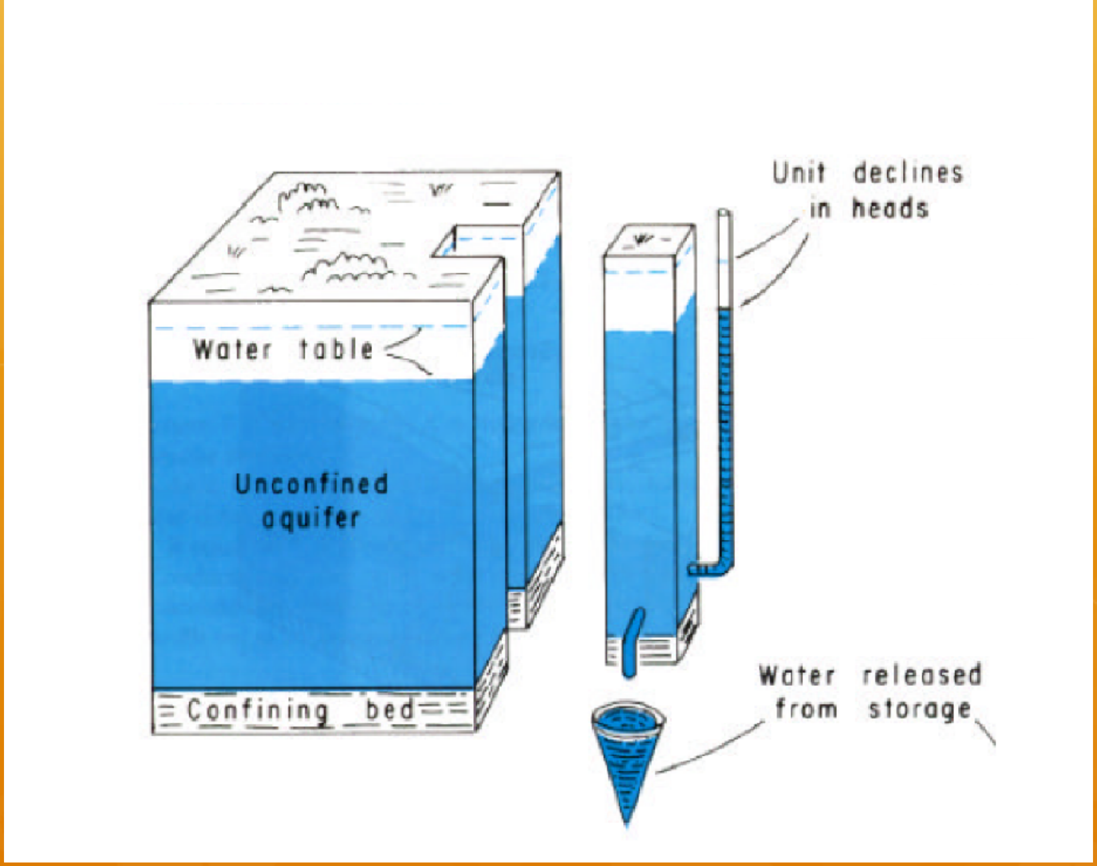
Groundwater Storage

- Storage refers to the ability of a porous medium to store water within its bulk.
- The mechanisms of storage are:
  - draining and filling of the pore space
  - compression of the water, and
  - compression of the solids.
- In an unconfined aquifer the draining and filling of the pore space is the most significant mechanism.
- In a confined aquifer, the compression and decompression of the solids structure is the primary mechanism of storage.

Unconfined Aquifer

- Storage in Unconfined is by drain/fill pore space.

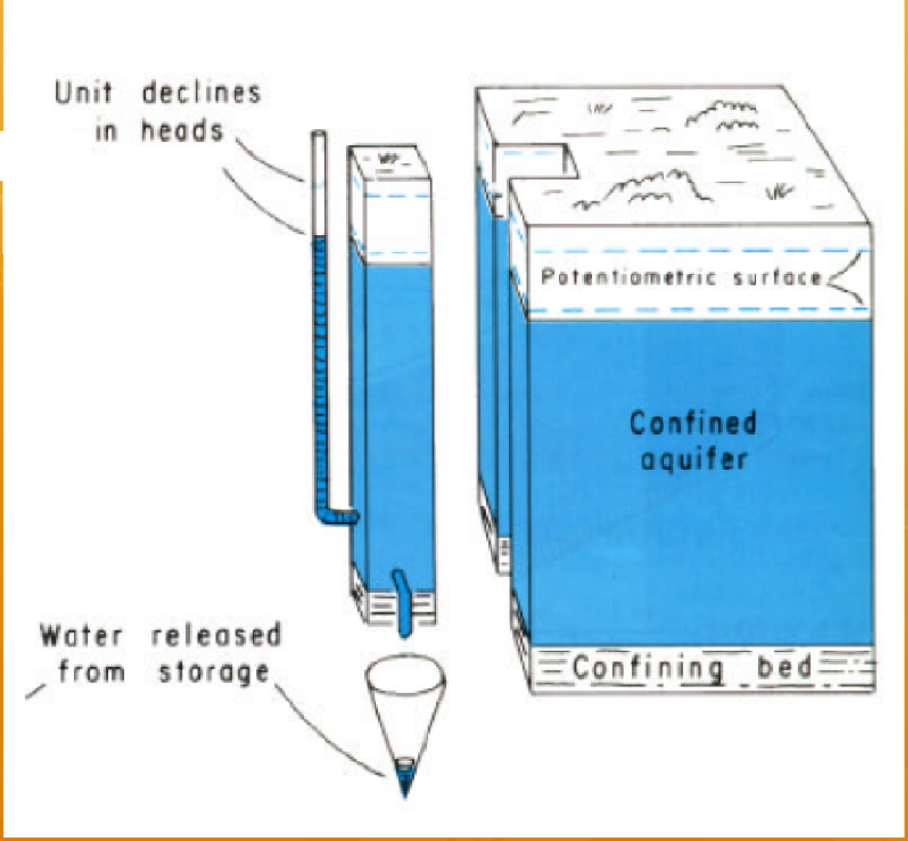
$$S = \frac{V_{water}}{\Delta h \ A}$$



Confined Aquifer

- Storage in confined is by compression/ decompression of the aquifer and water

$$S = \frac{V_{water}}{\Delta h \ A}$$



Specific Storage

- Storage per unit thickness of aquifer is called the specific storage

$$S_s = \frac{V_{water}}{\Delta h \ A \ b}$$

$$S = \int_0^b S_s(z) \, dz$$

If the specific storage is a constant, then the result is simply $S = S_s \, b$.

Estimating Storage

- Estimate by making head measurements at two different times and apply the storage equation

$$S = \frac{\Delta V_{water}}{\Delta h \, \Delta x \, \Delta y}$$

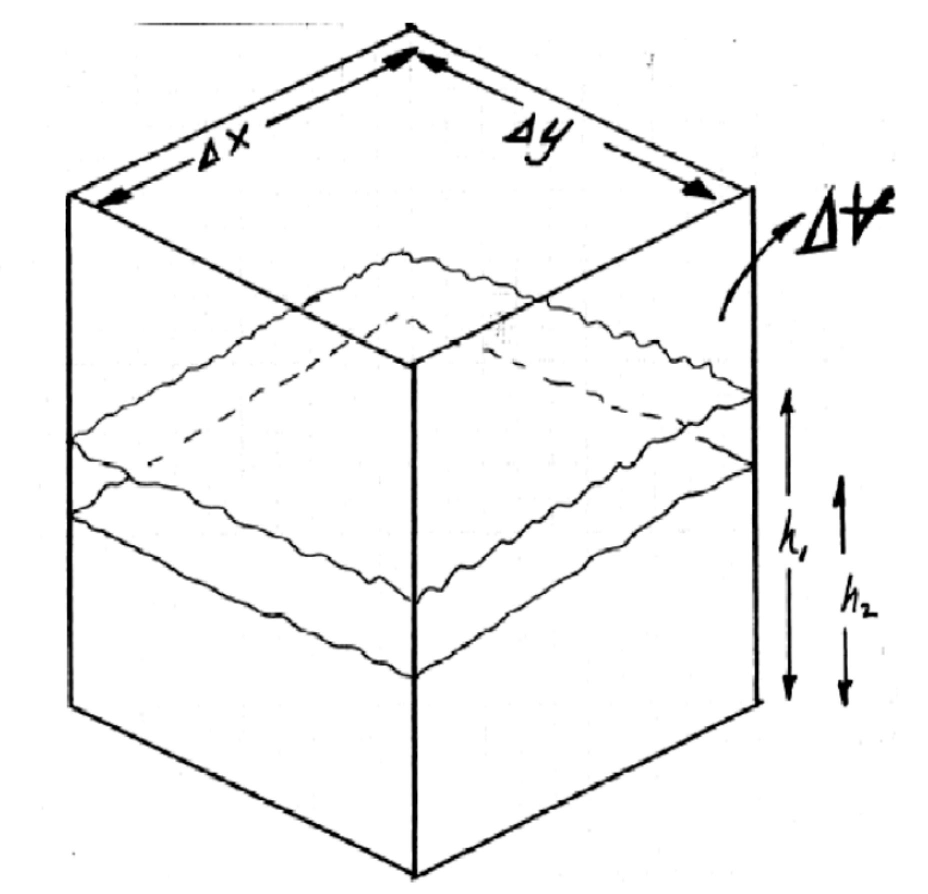


Figure 3: Storage in a single aquifer block

Estimating Storage

- Same idea for multiple blocks
- Estimate each block and then average

$$S = \frac{\sum_{i=1}^{num.blocks} \Delta\, V_i}{\sum_{i=1}^{num.blocks} \Delta h_i\ \Delta x_i\ \Delta y_i}$$

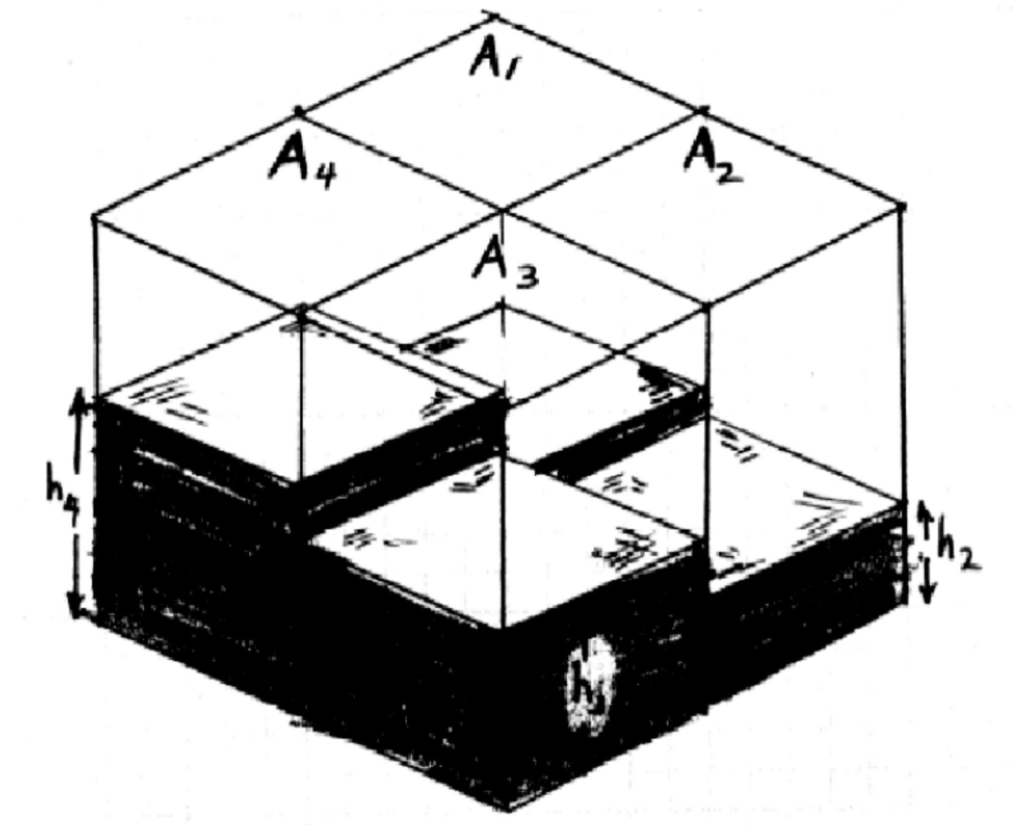


Figure 4: Aquifer comprised of many blocks

Estimating Storage

- Use of groundwater elevation maps

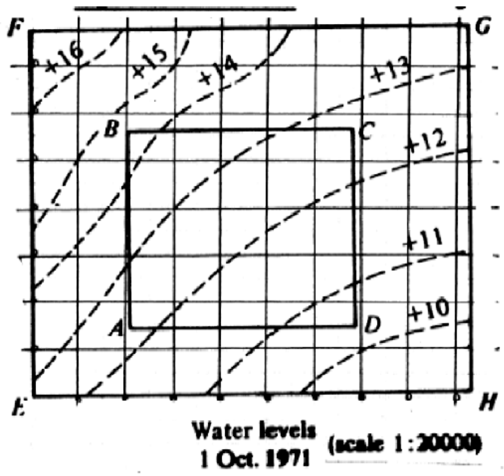


Figure 5: Water level map at Year 1

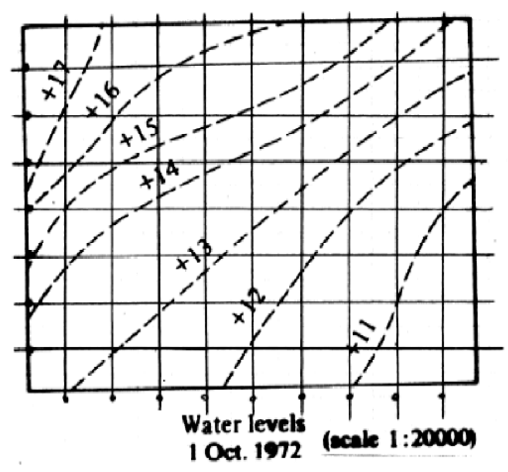


Figure 6: Water level map at Year 2

$$S = \frac{\Delta\, V_{EFGH}}{\sum_{i=1}^{num.blocks} \Delta h_i \; \Delta x_i \; \Delta y_i}$$

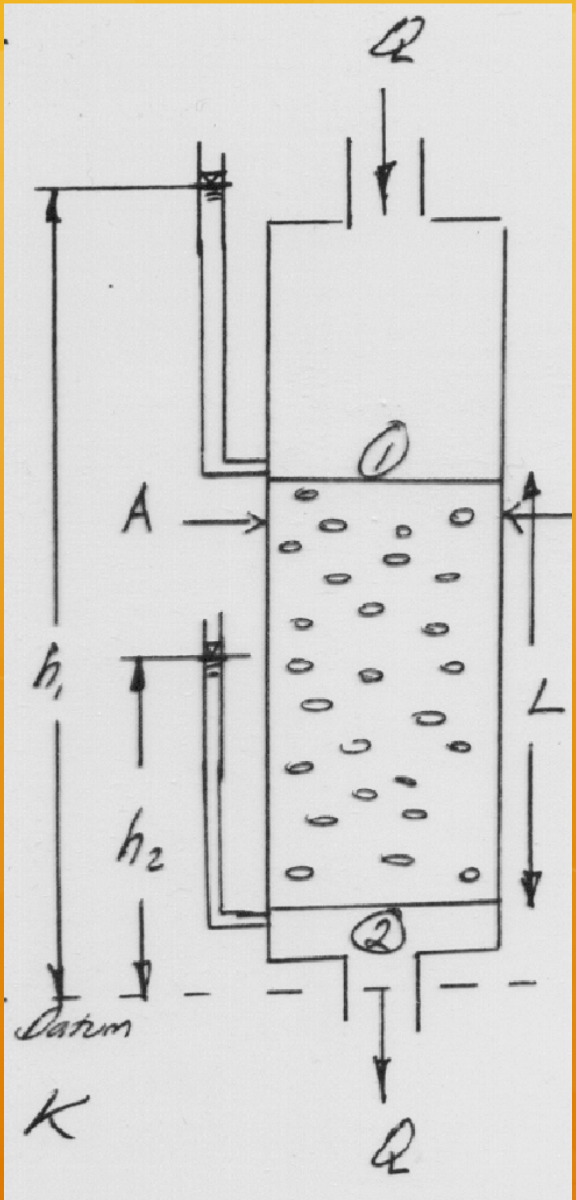
Darcy's Law

- Permeability refers to the ease which water can flow through a porous material under a specified gradient.
- Permeable materials offer little resistance, while impermeable materials offer a lot of resistance.

Darcy's Law

- Established experimentally 1856
- Total discharge through a filter, Q, was proportional to:
  - cross sectional area of flow, A,
  - head loss h1 – h2.
- Q, was inversely proportional to:
  - the length of the filter column, L.

$$Q \propto A \, \frac{h_1 - h_2}{L}$$

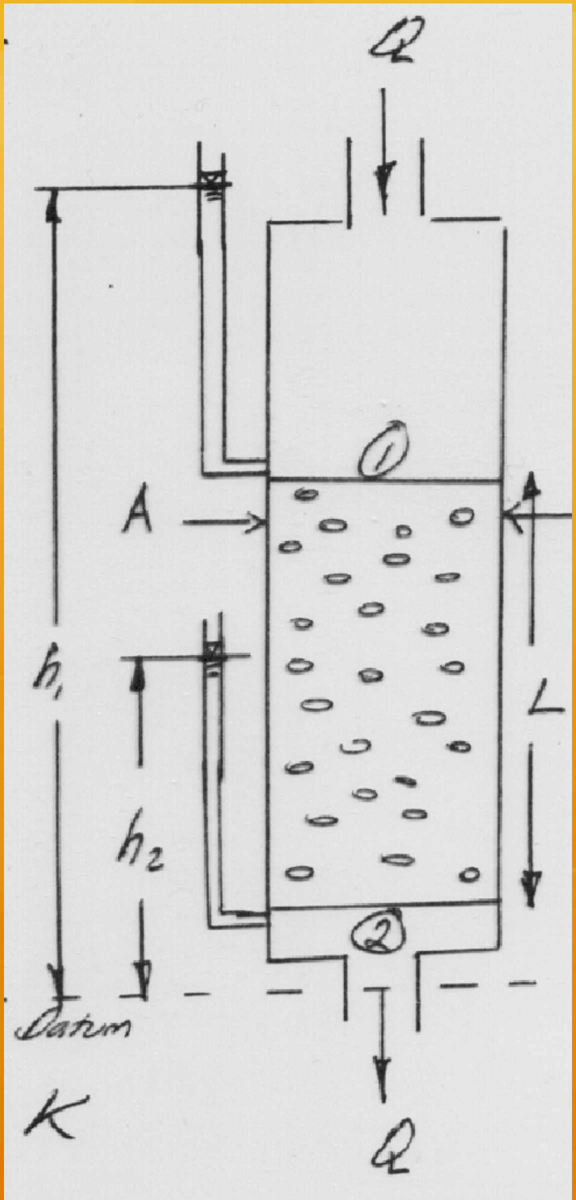


Darcy's Law

- The constant of proportionality is called the hydraulic conductivity
  - Permeability is sometimes used interchangeably
  - In reservoir engineering the permeability is related to K, but not numerically identical.

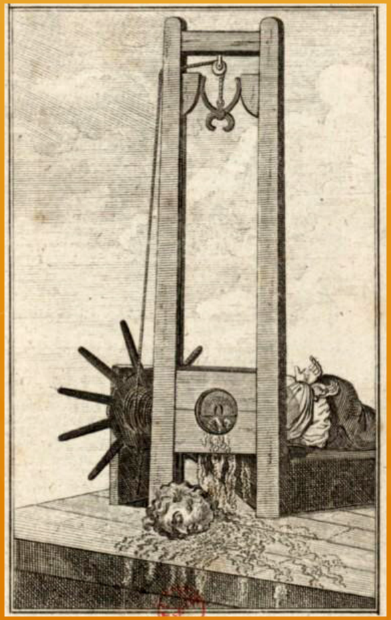
$$Q = KA\,(\frac{h_1 - h_2}{L})$$

$$K = \frac{k\rho\ g}{\mu}$$



Measuring Permeability

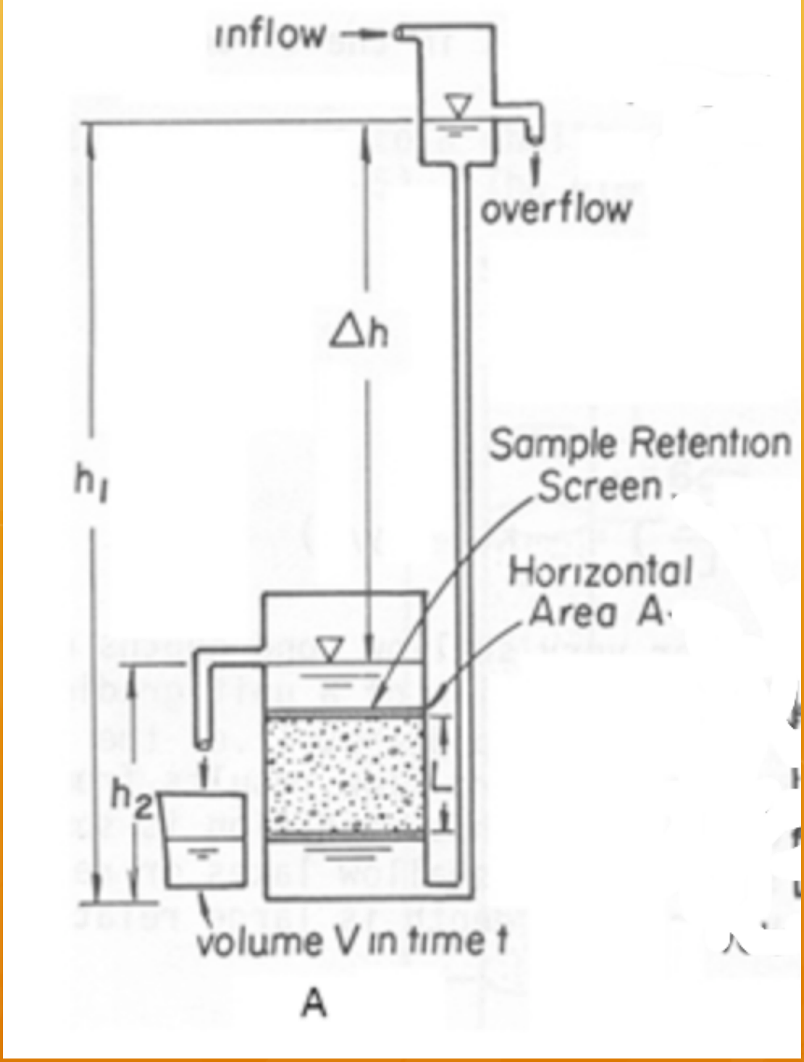
- Permeameters
  - Constant head
  - Falling head



Constant Head Permeameter

- A sample is placed in the permeameter
- Constant head gradient is maintained across the sample.
- Flow rate is measured
- Darcy's law applied:

$$K = \frac{Q}{A}\frac{L}{h_1 - h_2}$$



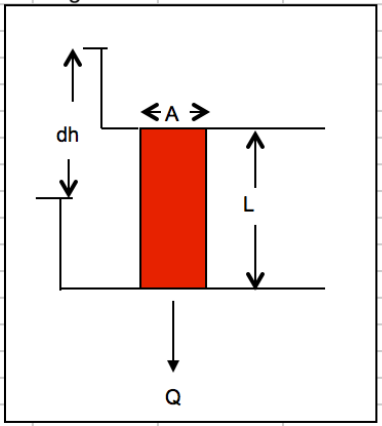
Constant Head Permeameter

- Spreadsheet model to make computations

Model2_CHPerm.xls

	A	B	C	D
1	Constant Head Permeameter			
2	Temperature (oC)		20	
3				
4	Data			Cell Formulas
5	Area (A)	225	cm^2	
6	Length (L)	25	cm	
7	Volume(V)	50	cm^3	
8	Time (t)	456	sec	
9	Head Loss (dh)	15	cm	
10				
11	density	0.998203	g/cm^3	
12	viscosity	0.01005	g/(sec cm)	
13	gravitational	980	cm/sec^2	
14	acceleration			
15				
16	Computed Values			B5/B6
17	Flow(Q)	0.109649123	=V/t	
18				(B15/B3)*(B4/B7)
19	Hydraulic (K)	8.1E-04	cm/sec	
20	Conductivity			
21				B17*B10/(B9*B11)
22	Intrinsic (k)	8.34435E-09	cm^2	
23	Permeability			

Diagram

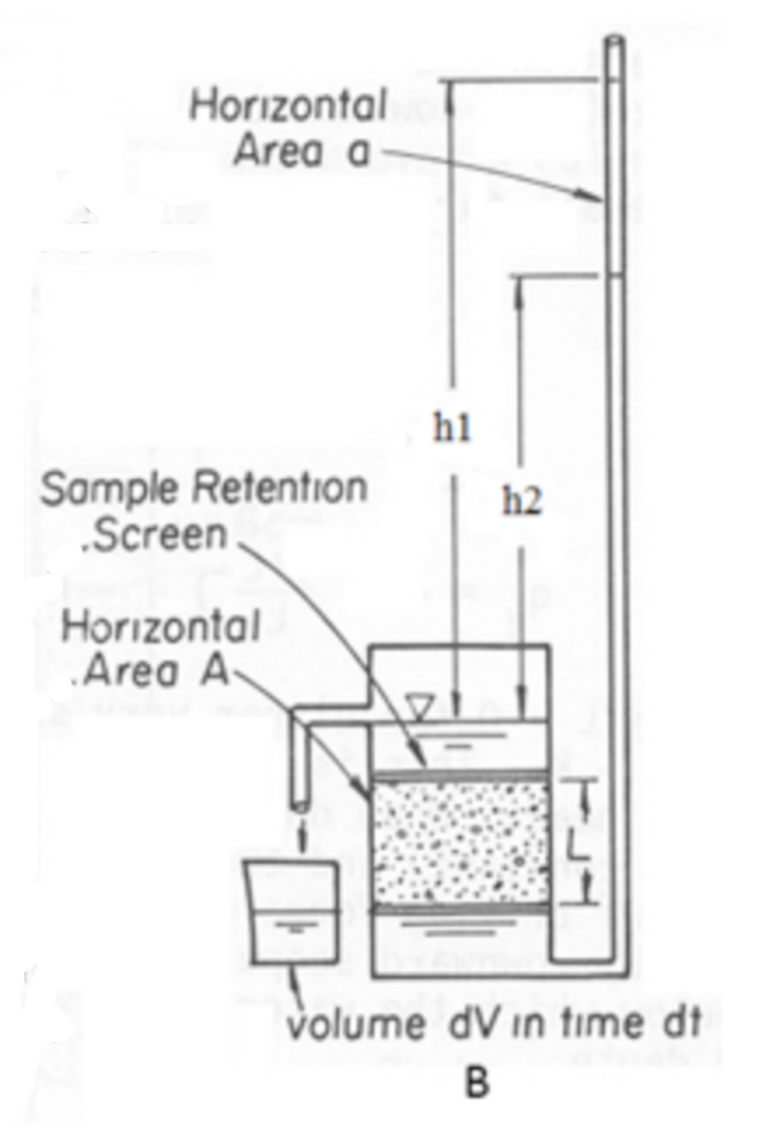


$$K = \frac{Q}{A}\,\frac{L}{h_1 - h_2}$$

Falling Head Permeameter

- The head is measured at the inlet of the sample as the height of water in the tube above the sample
- Change in this height with time is the flow rate
- Head and the flow rate vary with time.

$$Q(t) = \frac{K A \, h(t)}{L}$$



Falling Head Permeameter

- Volume balance

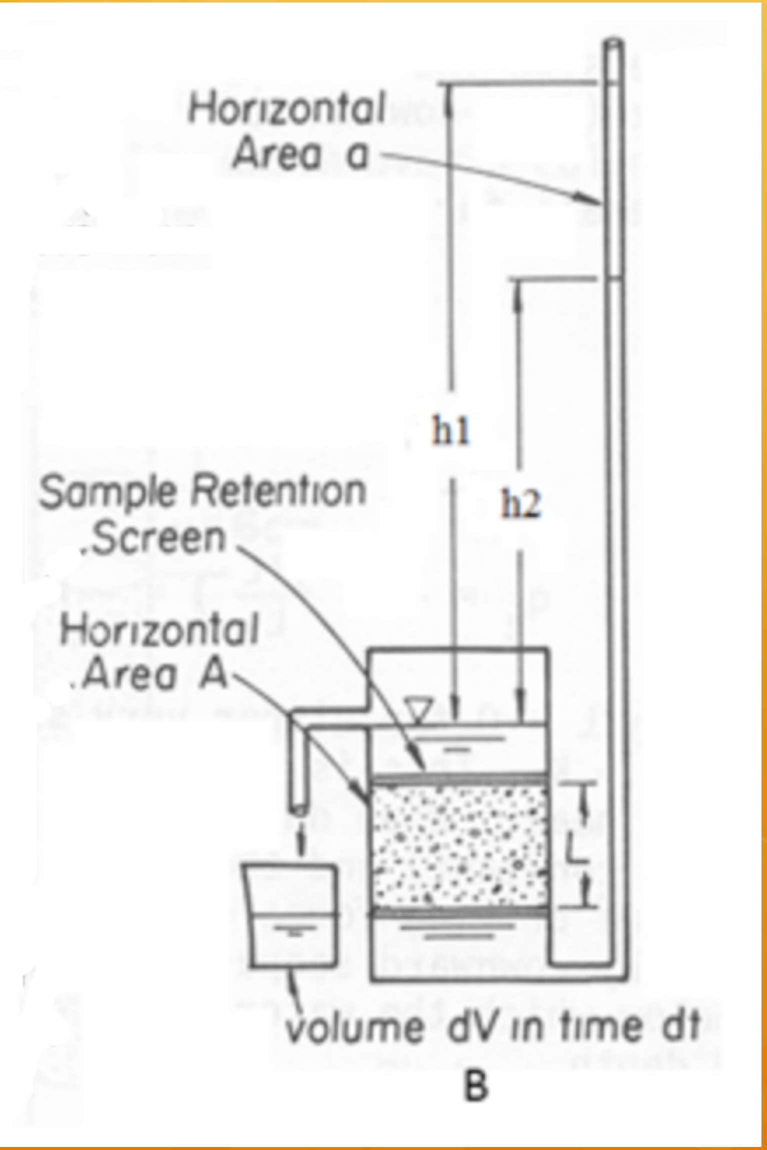
$$Q(t) = -a\frac{dh(t)}{dt} = \frac{K A\, h(t)}{aL}$$

- Separate and integrate

$$\int \frac{dh(t)}{h} = -\frac{K A}{aL}\int dt$$

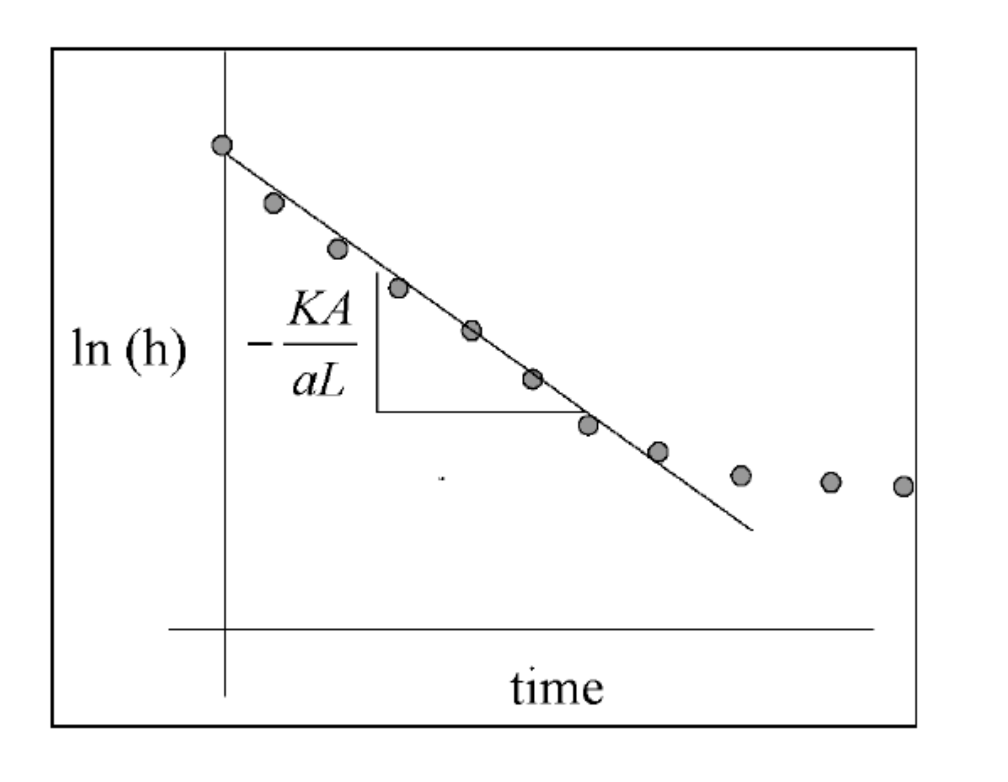
- Simplify

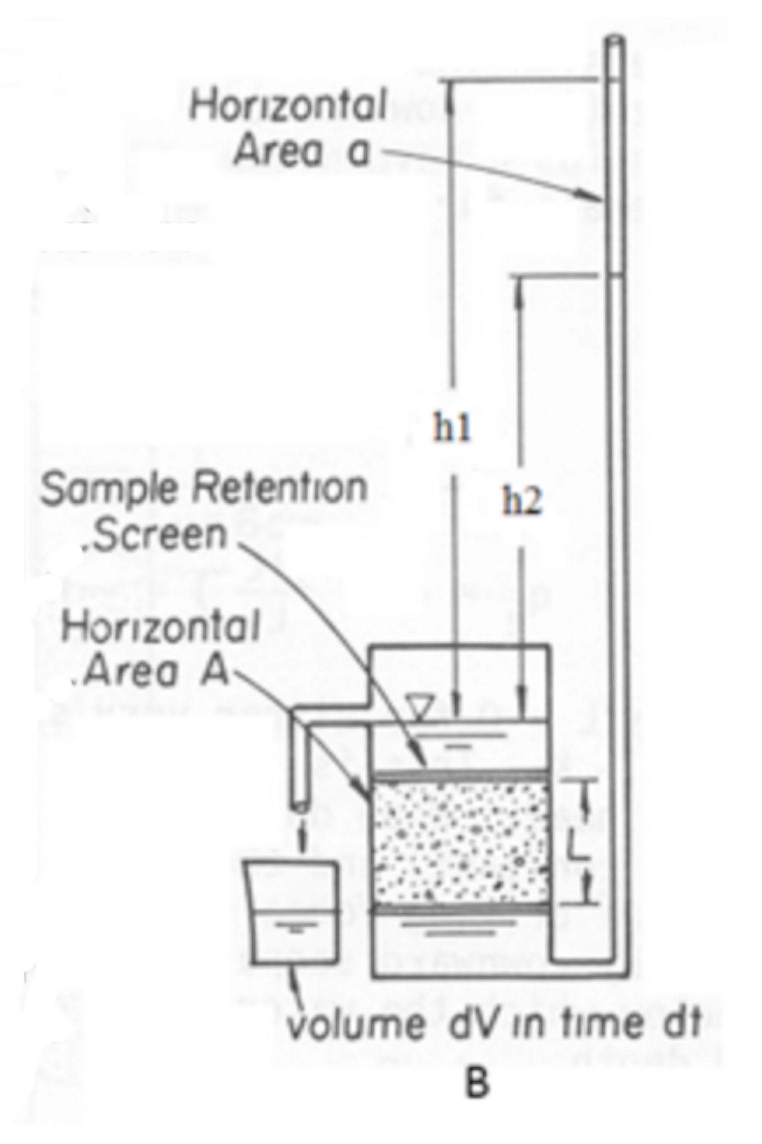
$$ln(\ h(t)) = ln(\ h_0) - \frac{K A\ t}{aL}$$



Falling Head Permeameter

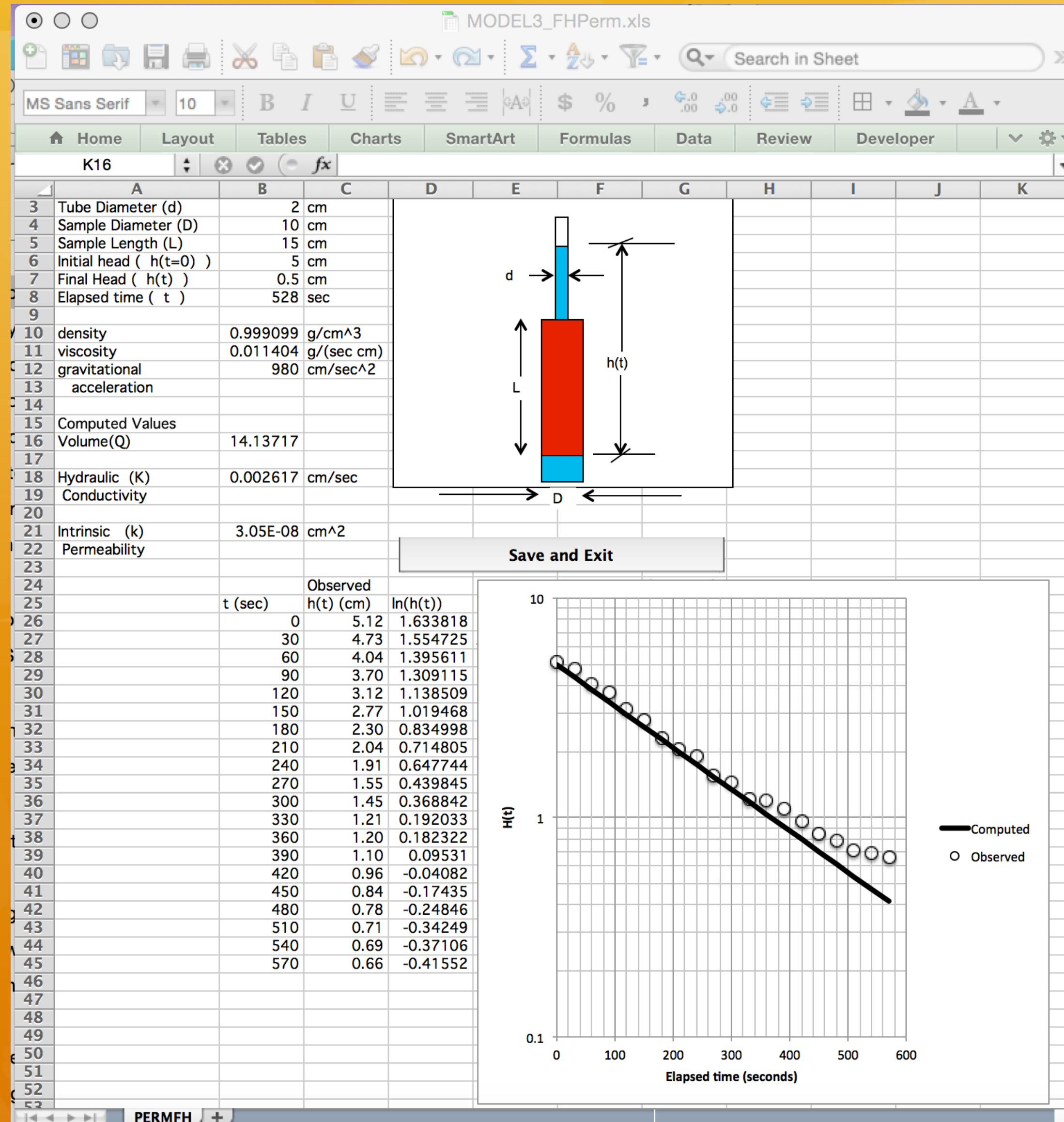
- Analysis by plotting and find slope of line in log-linear space





Falling Head Permeameter

- Spreadsheet tool to help with the analysis. Use trial-error with K to get computed to fit observed



Tube Diameter (d)	2	cm
Sample Diameter (D)	10	cm
Sample Length (L)	15	cm
Initial head (h(t=0))	5	cm
Final Head (h(t))	0.5	cm
Elapsed time (t)	528	sec
density	0.999099	g/cm^3
viscosity	0.011404	g/(sec cm)
gravitational acceleration	980	cm/sec^2
Computed Values		
Volume(Q)	14.13717	
Hydraulic (K) Conductivity	0.002617	cm/sec
Intrinsic (k) Permeability	3.05E-08	cm^2

t (sec)	Observed h(t) (cm)	ln(h(t))
0	5.12	1.633818
30	4.73	1.554725
60	4.04	1.395611
90	3.70	1.309115
120	3.12	1.138509
150	2.77	1.019468
180	2.30	0.834998
210	2.04	0.714805
240	1.91	0.647744
270	1.55	0.439845
300	1.45	0.368842
330	1.21	0.192033
360	1.20	0.182322
390	1.10	0.09531
420	0.96	-0.04082
450	0.84	-0.17435
480	0.78	-0.24846
510	0.71	-0.34249
540	0.69	-0.37106
570	0.66	-0.41552

Field Methods

Measurements - Field Methods

In principle the measurement of hydraulic conductivity in the field should simply extend concepts of the permeameter devices; however the measurement of flow rates in field applications is not trivial.

Wells and pumps are used in aquifer pumping tests, a subject of later discussion. If the flow rate is known in the field, and two wells are known to be located on the same flow line, then Darcys law can be applied directly to infer the hydraulic conductivity from head measurements at two wells, using concepts identical to the constant head permeameter.

Infiltration and auger hole tests can be used to infer hydraulic conductivity in a manner analogous to the falling head permeameter test.

Next Time

- Direct Application of Darcy's Law
- Steady flow solutions
  - Rectilinear flow
  - Flow to wells