CE 3354 Engineering Hydrology

Lecture 14: Unit Hydrographs

OUTLINE

Unit Hydrographs

- Theory
- Data Analysis to construct a UH
  - Chapter 7 CMM

UNIT HYDROGRAPHS

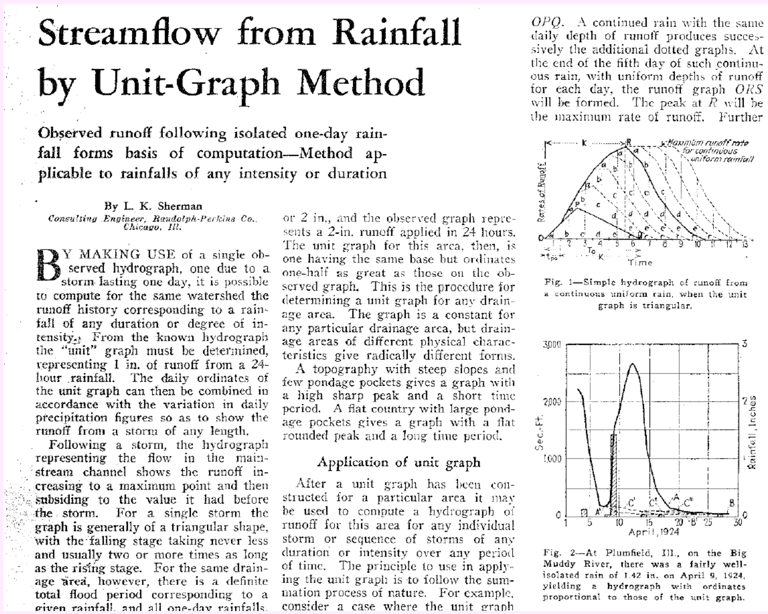
What is a unit hydrograph?

How are they used?

How are they built from data (analysis)?

How are they built when data do not exist (synthesis)?

WHAT IS A UNIT HYDROGRAPH?



Streamflow from Rainfall by Unit-Graph Method

Observed runoff following isolated one-day rainfall forms basis of computation—Method applicable to rainfalls of any intensity or duration

By L. K. Sherman
Consulting Engineer, Randolph-Perkins Co., Chicago, Ill.

BY MAKING USE of a single observed hydrograph, one due to a storm lasting one day, it is possible to compute for the same watershed the runoff history corresponding to a rainfall of any duration or degree of intensity. From the known hydrograph the "unit" graph must be determined, representing 1 in. of runoff from a 24-hour rainfall. The daily ordinates of the unit graph can then be combined in accordance with the variation in daily precipitation figures so as to show the runoff from a storm of any length.

Following a storm, the hydrograph representing the flow in the main-stream channel shows the runoff increasing to a maximum point and then subsiding to the value it had before the storm. For a single storm the graph is generally of a triangular shape, with the falling stage taking never less and usually two or more times as long as the rising stage. For the same drainage area, however, there is a definite total flood period corresponding to a given rainfall, and all one-day rainfalls ... or 2 in., and the observed graph represents a 2-in. runoff applied in 24 hours. The unit graph for this area, then, is one having the same base but ordinates one-half as great as those on the observed graph. This is the procedure for determining a unit graph for any drainage area. The graph is a constant for any particular drainage area, but drainage areas of different physical characteristics give radically different forms.

A topography with steep slopes and few pondage pockets gives a graph with a high sharp peak and a short time period. A flat country with large pondage pockets gives a graph with a flat rounded peak and a long time period.

Application of unit graph

After a unit graph has been constructed for a particular area it may be used to compute a hydrograph of runoff for this area for any individual storm or sequence of storms of any duration or intensity over any period of time. The principle to use in applying the unit graph is to follow the summation process of nature. For example, consider a case where the unit graph ...

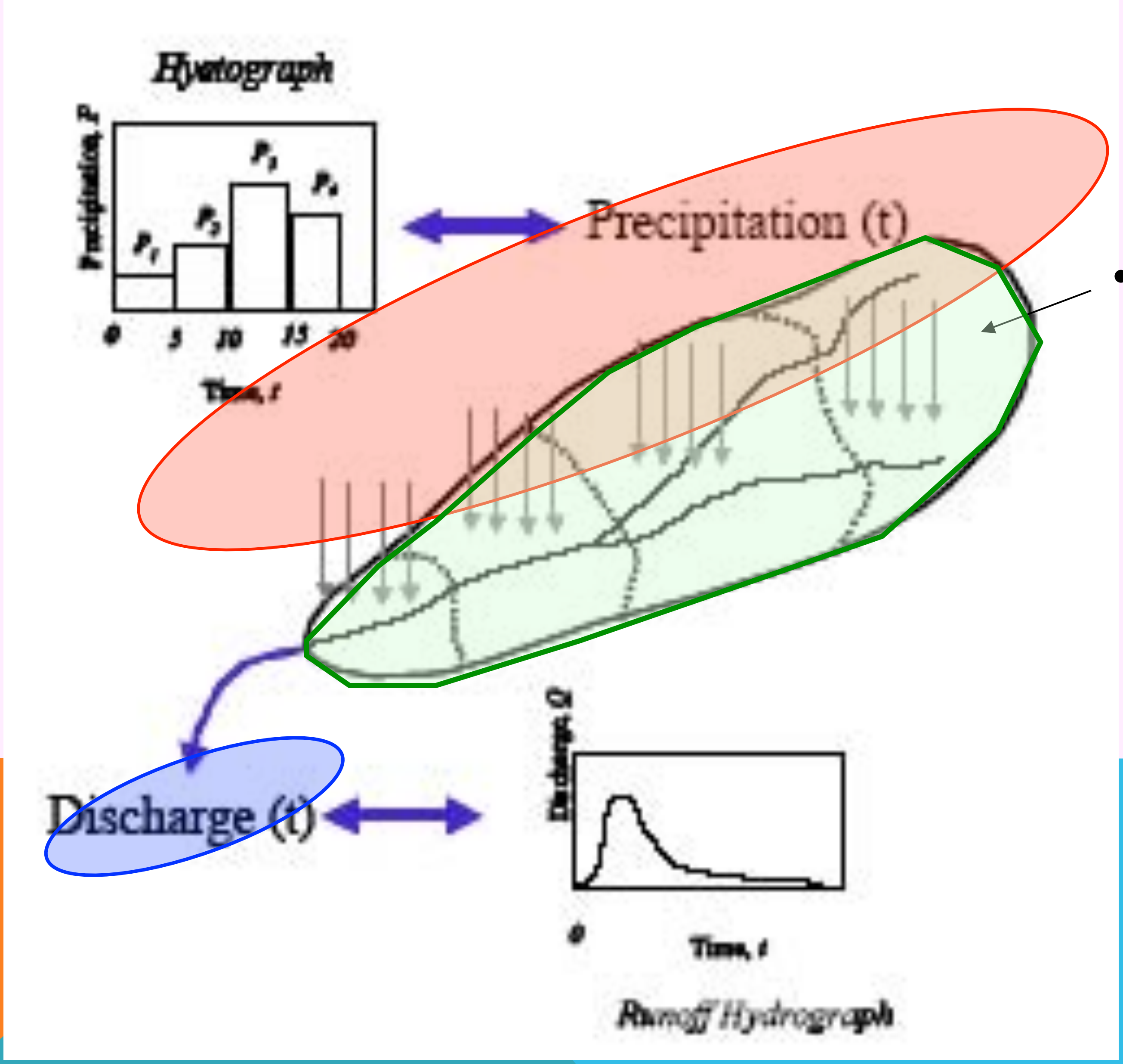
OPQ. A continued rain with the same daily depth of runoff produces successively the additional dotted graphs. At the end of the fifth day of such continuous rain, with uniform depths of runoff for each day, the runoff graph *ORS* will be formed. The peak at *R* will be the maximum rate of runoff. Further

Fig. 1—Simple hydrograph of runoff from a continuous uniform rain, when the unit graph is triangular.

Fig. 2—At Plumfield, Ill., on the Big Muddy River, there was a fairly well-isolated rain of 1.42 in. on April 9, 1924, yielding a hydrograph with ordinates proportional to those of the unit graph.

- **Used to explain the time re-distribution of excess precipitation on a watershed**
- **Represents the response of the watershed at the outlet to a unit depth of EXCESS precipitation**
  - EXCESS implies some kind of loss model is applied to the raw precipitation
  - Time re-distribution implies some kind of transfer behavior is applied
- **L. K. Sherman 1932 is credited with seminal publication of the concept**
  - Read the document in AdditionalReadings

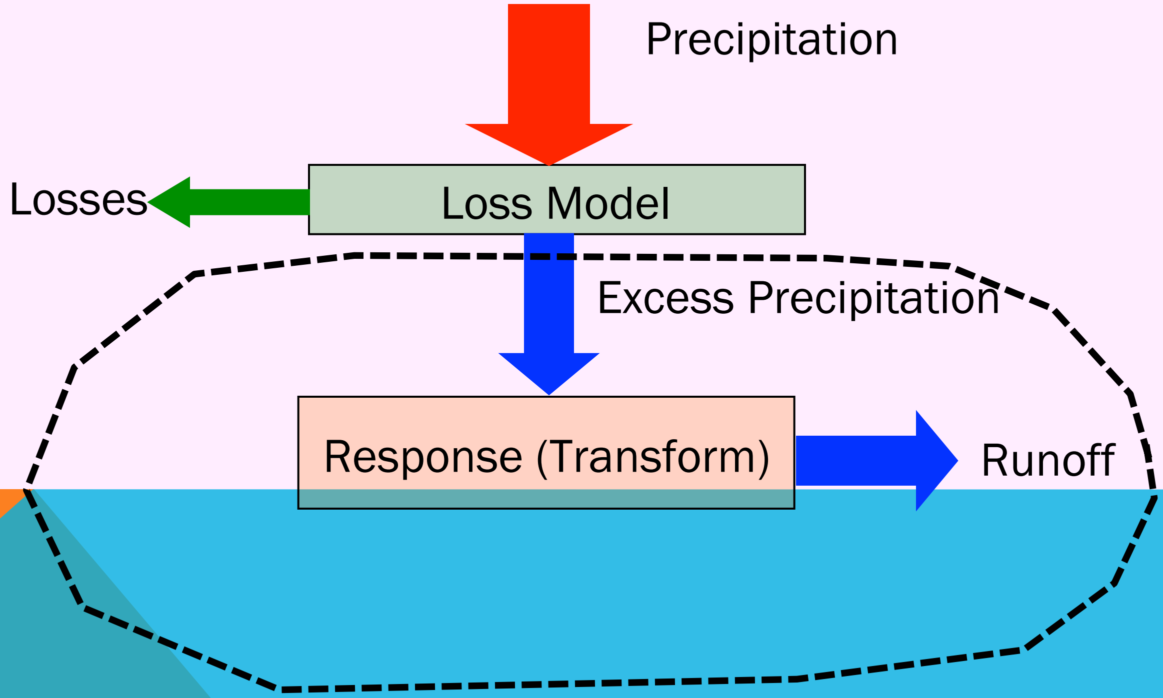
RESPONSE MODEL



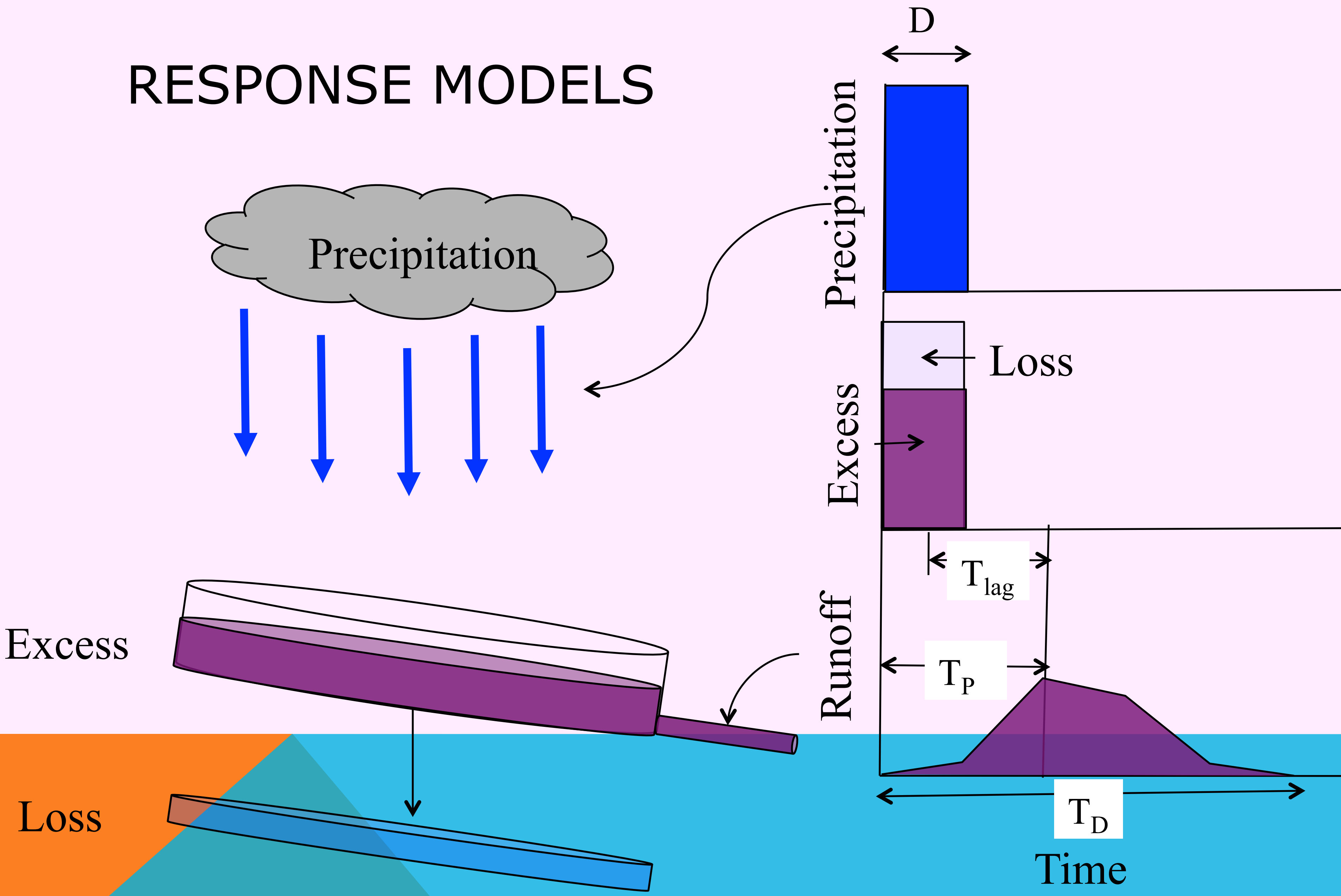
- Watershed
  - Losses
  - Transformation
  - Storage
  - Routing

RESPONSE MODEL

Response models convert the excess precipitation signal into a direct runoff hydrograph at the point of interest



RESPONSE MODELS



HYETOGRAPHS

Typically divided into three components

- Initial abstraction
- Loss
- Excess
  - This component becomes direct runoff

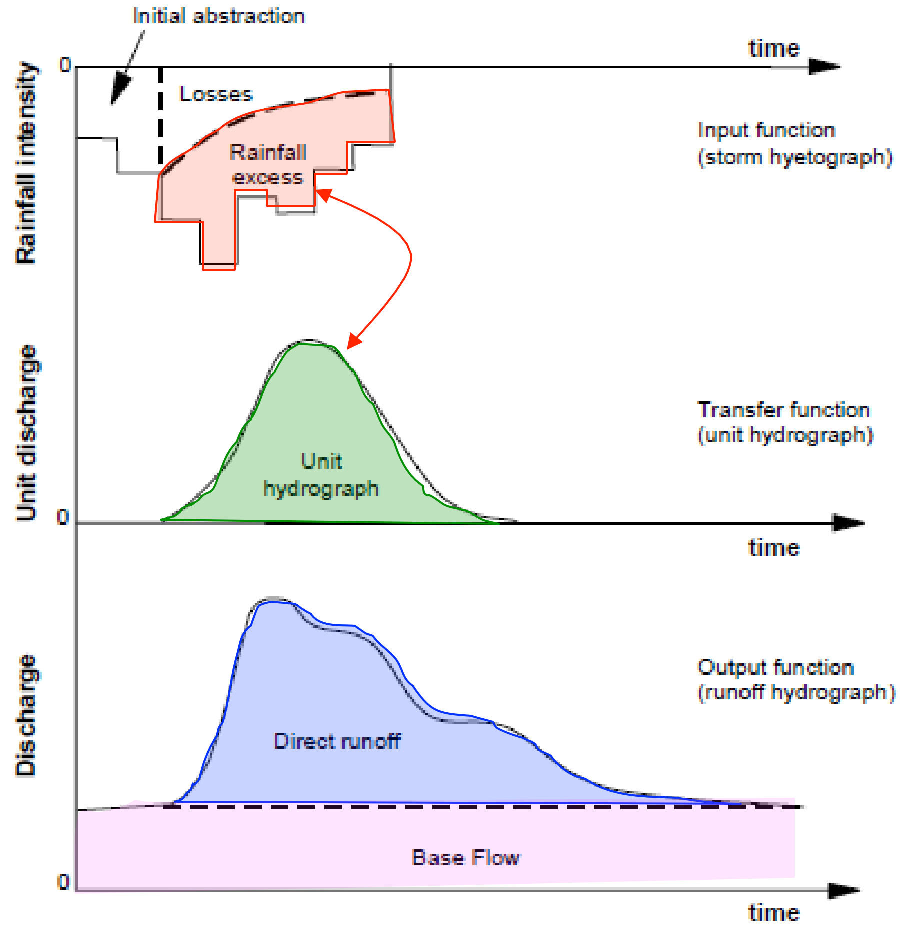


Figure 6.1. Rainfall/runoff as the system process

UNIT HYDROGRAPH - EXAMPLE

- The example is from CMM pp 216-223
- Least-Squares Approach

Unit Hydrograph (Classical Example)				
Time		Depth	Flow	
0.5	1	1.06	428	
1	2	1.93	1923	
1.5	3	1.81	5297	
2	4		9131	
2.5	5		10625	
3	6		7834	
3.5	7		3921	
4	8		1846	
4.5	9		1402	
5	10		830	
5.5	11		313	

Figure 7: Data for UH application example

UNIT HYDROGRAPH - EXAMPLE

- Build an equation array using the unknown unit weights and the known EXCESS precipitation depths,

$$\begin{bmatrix} P_1 & & & \\ P_2 & P_1 & & \\ & & \ddots & \\ & & & P_M \end{bmatrix} \bullet \begin{bmatrix} U_1 \\ U_2 \\ \\ U_{N-M+1} \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ \vdots \\ Q_N \end{bmatrix}$$

Figure 9: UH Equation Array (Vector-Matrix)

UNIT HYDROGRAPH – EXAMPLE

5.1.1 Back-substitution

Straightforward. Solve each equation successively (back substitute) for U .
$U_1 = Q_1/P_1 = 428/1.06 = 404cfs/in.$
$U2 = (Q2 - P_2U_1)/P1 = (1923 - 1.93 * 404)/1.06 = 1079cfs/in.$
And so on . . . .

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Unit Hydrograph (Back-Substitute)																					
2	Observations						[P]												[U]	[Q*]	[Q]-[Q*]	
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11					
4	0.5	1	1.06	428		1	1.06	0	0	0	0	0	0	0	0	0	0		403.77	428	-0.000228	403.774
5	1	2	1.93	1923		2	1.93	1.06	0	0	0	0	0	0	0	0	0		1079	1923.02	-0.023434	1078.98
6	1.5	3	1.81	5297		3	1.81	1.93	1.06	0	0	0	0	0	0	0	0		2343.1	5297	0	2343.11
7	2	4	0	9131		4	0	1.81	1.93	1.06	0	0	0	0	0	0	0		2505	9130.5	0.5025618	2505.5
8	2.5	5	0	10625		5	0	0	1.81	1.93	1.06	0	0	0	0	0	0		1461	10624.3	0.6560813	1461.66
9	3	6	0	7834		6	0	0	0	1.81	1.93	1.06	0	0	0	0	0		453	7833.96	0.04	453.04
10	3.5	7	0	3921		7	0	0	0	0	1.81	1.93	1.06	0	0	0	0		379.5	3920.97	0.03	379.53
11	4	8	0	1846		8	0	0	0	0	0	1.81	1.93	1.06	0	0	0		276.9	1845.88	0.121	277.021
12	4.5	9	0	1402		9	0	0	0	0	0	0	1.81	1.93	1.06	0	0		170.5	1402.04	-0.042	170.458
13	5	10	0	830		10	0	0	0	0	0	0	0	1.81	1.93	1.06	0		-0.47	829.756	0.2442	-0.2258
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0	1.81	1.93	1.06		5.32	313.337	-0.3371	4.9829
15																						
16										Back substitute (lazy way)												
17										(1) Guess U												
18										(2) Compute Q*												
19										(3) Compute difference												
20										(4) Adjust U one-by-one to get difference small												
21										(5) Stop when difference is small percent of actual Q												
22										(6) If SOLVER works on machine - can automate - crashes on Mac (Microsoft hates Mac!)												

Figure 10: Backsubstitution in a spreadsheet

UNIT HYDROGRAPHS – EXAMPLE

- Observe that if the linear system has full ranked matrix (rows=columns) and non-zero diagonal, one could just solve the resulting linear equation for the unitgraph weights
- Probably better than manual back-substitution which is error prone
    - Many instances the system is over-determined – more equations than unknowns and an optimization technique is usually applied

UNIT HYDROGRAPHS – EXAMPLE

5.1.2 Optimization Approach - Solving the Normal Equations

The least squares solution to the matrix equation is
, $[\mathbf{U}] = [[\mathbf{P}^T][\mathbf{P}]]^{-1}[\mathbf{P}^T][\mathbf{Q}]$

Again using a spreadsheet the result is displayed in Figure 11. This method also sometimes fails, but it can be completely automated (no brains required - unless it fails then a lot of brains are required to figure out what went wrong).

Three other approaches in common practice are optimization using linear programming (Danzig's algorithm) - excess and deficits are summed and minimized; non-linear programming (essentially a variation of the least-squares, but can constrain solution space); and pattern searching (also a constrained approach).

UNIT HYDROGRAPHS – EXAMPLE

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	Unit Hydrograph (Least Squares Example)																		
2	Observations						[P]												[U]
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11		
4	0.5	1	1.06	428		1	1.06	0	0	0	0	0	0	0	0	0	0		403.774
5	1	2	1.93	1923		2	1.93	1.06	0	0	0	0	0	0	0	0	0		1078.98
6	1.5	3	1.81	5297		3	1.81	1.93	1.06	0	0	0	0	0	0	0	0		2343.15
7	2	4	0	9131		4	0	1.81	1.93	1.06	0	0	0	0	0	0	0		2505.44
8	2.5	5	0	10625		5	0	0	1.81	1.93	1.06	0	0	0	0	0	0		1460.75
9	3	6	0	7834		6	0	0	0	1.81	1.93	1.06	0	0	0	0	0		452.74
10	3.5	7	0	3921		7	0	0	0	0	1.81	1.93	1.06	0	0	0	0		380.425
11	4	8	0	1846		8	0	0	0	0	0	1.81	1.93	1.06	0	0	0		275.774
12	4.5	9	0	1402		9	0	0	0	0	0	0	1.81	1.93	1.06	0	0		170.931
13	5	10	0	830		10	0	0	0	0	0	0	0	1.81	1.93	1.06	0		0.89846
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0	1.81	1.93	1.06		1.77518
15																			
16							[P]-transpose												
17																			
18						1	1.06	1.93	1.81	0	0	0	0	0	0	0	0		
19						2	0	1.06	1.93	1.81	0	0	0	0	0	0	0		
20						3	0	0	1.06	1.93	1.81	0	0	0	0	0	0		
21						4	0	0	0	1.06	1.93	1.81	0	0	0	0	0		
22						5	0	0	0	0	1.06	1.93	1.81	0	0	0	0		
23						6	0	0	0	0	0	1.06	1.93	1.81	0	0	0		
24						7	0	0	0	0	0	0	1.06	1.93	1.81	0	0		
25						8	0	0	0	0	0	0	0	1.06	1.93	1.81	0		
26						9	0	0	0	0	0	0	0	0	1.06	1.93	1.81		
27						10	0	0	0	0	0	0	0	0	0	1.06	1.93		
28						11	0	0	0	0	0	0	0	0	0	0	1.06		

=MMULT(MINVERSE(MMULT(G18:Q28,G4:Q14)),MMULT(G18:Q28,D4:D14))

=TRANSPOSE(G4:

Figure 11: Least-Squares Minimization (by Normal Equations) in a spreadsheet

USING UNIT HYDROGRAPHS

The whole point is to use the unitgraph to predict the response to a different storm (either real or a design storm)

Just getting the unit graph is meaningless unless we intend to use it. In the present example, if we use the unitgraph for different rainfall signals we can predict the direct runoff hydrograph for these events.

For example suppose we wish to evaluate the DRH for

P=[2.00,3.00,1.00]

P=[5.00,0.00,0.00]

P=[0.00,0.00,5.00]

Then we simply evaluate the matrix equation [Q]=[P][U] with different [P] matrices. (Results in figures)

USING UNIT HYDROGRAPHS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Unit Hydrograph (Least Squares Example)																			
2	Observations						[P]												[U]	[Q*]
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11			
4	0.5	1	2	428		1	2	0	0	0	0	0	0	0	0	0	0		403.774	807.55
5	1	2	3	1923		2	3	2	0	0	0	0	0	0	0	0	0		1078.98	3369.3
6	1.5	3	1	5297		3	1	3	2	0	0	0	0	0	0	0	0		2343.15	8327
7	2	4	0	9131		4	0	1	3	2	0	0	0	0	0	0	0		2505.44	13119
8	2.5	5	0	10625		5	0	0	1	3	2	0	0	0	0	0	0		1460.75	12781
9	3	6	0	7834		6	0	0	0	1	3	2	0	0	0	0	0		452.74	7793.2
10	3.5	7	0	3921		7	0	0	0	0	1	3	2	0	0	0	0		380.425	3579.8
11	4	8	0	1846		8	0	0	0	0	0	1	3	2	0	0	0		275.774	2145.6
12	4.5	9	0	1402		9	0	0	0	0	0	0	1	3	2	0	0		170.931	1549.6
13	5	10	0	830		10	0	0	0	0	0	0	0	1	3	2	0		0.89846	790.36
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0	1	3	2		1.77518	177.18

Unit Response from original data

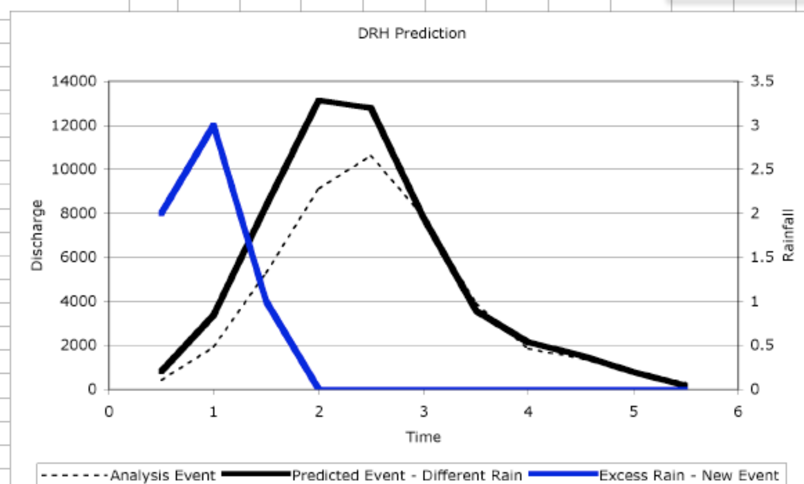


Figure 14: DRH using P=[2.00,3.00,1.00]

USING UNIT HYDROGRAPHS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Unit Hydrograph (Least Squares Example)																			
2	Observations						[P]												[U]	[Q*]
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11			
4	0.5	1	5	428		1	5	0	0	0	0	0	0	0	0	0	0		403.774	2018.9
5	1	2	0	1923		2	0	5	0	0	0	0	0	0	0	0	0		1078.98	5394.9
6	1.5	3	0	5297		3	0	0	5	0	0	0	0	0	0	0	0		2343.15	11716
7	2	4	0	9131		4	0	0	0	5	0	0	0	0	0	0	0		2505.44	12527
8	2.5	5	0	10625		5	0	0	0	0	5	0	0	0	0	0	0		1460.75	7303.8
9	3	6	0	7834		6	0	0	0	0	0	5	0	0	0	0	0		452.74	2263.7
10	3.5	7	0	3921		7	0	0	0	0	0	0	5	0	0	0	0		380.425	1902.1
11	4	8	0	1846		8	0	0	0	0	0	0	0	5	0	0	0		275.774	1378.9
12	4.5	9	0	1402		9	0	0	0	0	0	0	0	0	5	0	0		170.931	854.65
13	5	10	0	830		10	0	0	0	0	0	0	0	0	0	5	0		0.89846	4.4923
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0	0	0	5		1.77518	8.8759

Unit Response from original data

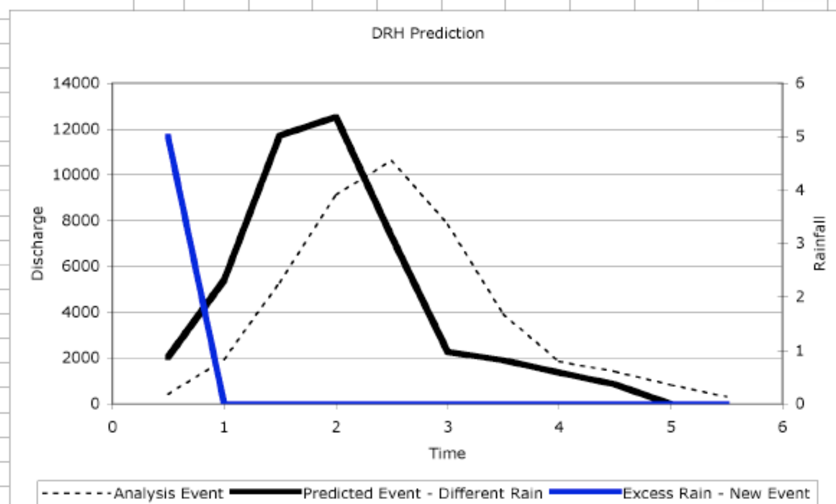


Figure 15: DRH using P=[5.00,0.00,0.00]

USING UNIT HYDROGRAPHS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Unit Hydrograph (Least Squares Example)																			
2	Observations						[P]												[U]	[Q*]
3	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11			
4	0.5	1	0	428		1	0	0	0	0	0	0	0	0	0	0	0		403.774	0
5	1	2	0	1923		2	0	0	0	0	0	0	0	0	0	0	0		1078.98	0
6	1.5	3	5	5297		3	5	0	0	0	0	0	0	0	0	0	0		2343.15	2018.9
7	2	4	0	9131		4	0	5	0	0	0	0	0	0	0	0	0		2505.44	5394.9
8	2.5	5	0	10625		5	0	0	5	0	0	0	0	0	0	0	0		1460.75	11716
9	3	6	0	7834		6	0	0	0	5	0	0	0	0	0	0	0		452.74	12527
10	3.5	7	0	3921		7	0	0	0	0	5	0	0	0	0	0	0		380.425	7303.8
11	4	8	0	1846		8	0	0	0	0	0	5	0	0	0	0	0		275.774	2263.7
12	4.5	9	0	1402		9	0	0	0	0	0	0	5	0	0	0	0		170.931	1902.1
13	5	10	0	830		10	0	0	0	0	0	0	0	5	0	0	0		0.89846	1378.9
14	5.5	11	0	313		11	0	0	0	0	0	0	0	0					1.77518	856.39

Unit Response from original data

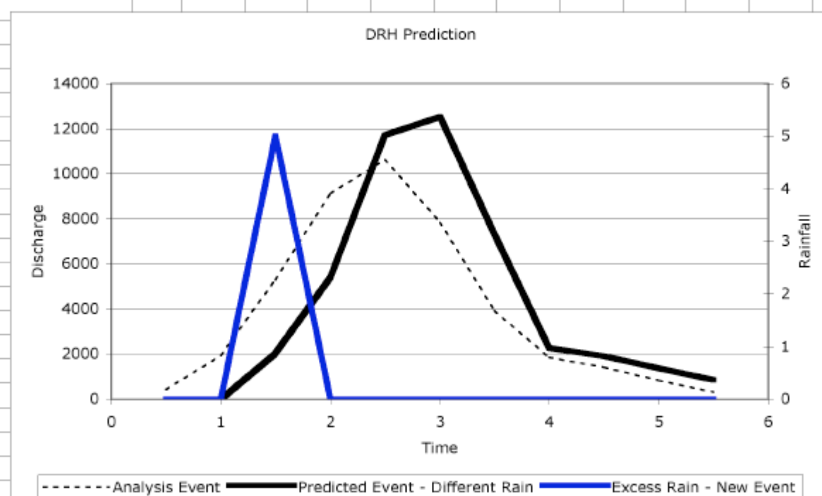


Figure 16: DRH using P=[0.00,0.00,5.00]

PARAMETRIC UNIT HYDROGRAPHS

- Parametric UH are simply functions with parameters (like F=ma) where if we have the parameters the hydrograph can be reconstructed using the function rather than keeping around a bunch of unit response values
- Will illustrate using same example

A simple transfer function used in this example is $U(t) = K\ (\frac{t}{T})^N\ exp(-\frac{t}{T})$. The unknown parameters are K,N, and T. The parameter T is a timing parameter and essentially locates the peak of the discharge it is analogous in concept to T_p or T_c or T_L depending on how the equation is constructed. Parameters K is related to the drainage area, and N is related basin shape, but like timing they cannot be easily be inferred from maps etc. The main difference here, is that the unit weights are values of the transfer function at different locations in time, so instead of just 9 unit weights we actually have as many as needed for a given case, and this information makes changing the time base simpler.

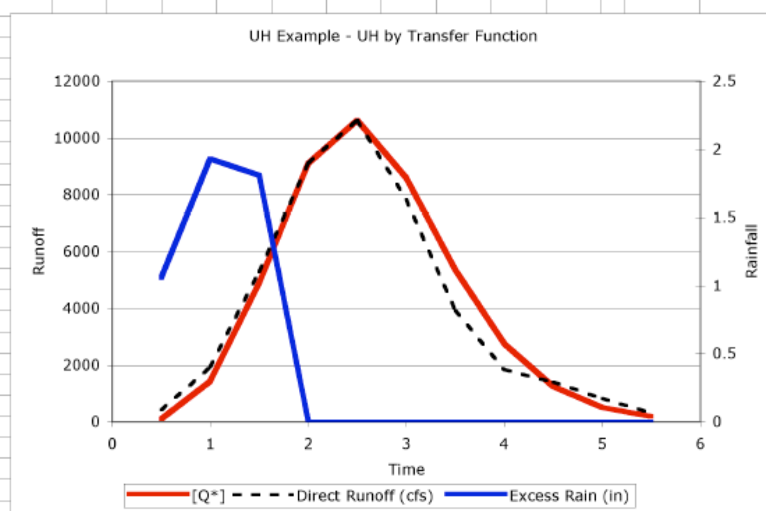
PARAMETRIC UNIT HYDROGRAPHS

- Parametric UH are simply functions with parameters (like F=ma) where if we have the parameters the hydrograph can be reconstructed using the function rather than keeping around a bunch of unit response values
- Will illustrate using same example

To estimate K,N, and T we simply construct the [Q]=[P][U] model where [U] is given by the above function at the correct times, then adjust K,N, and T to minimize the differences between the observed [Q] and the model [Q]. Figure 17 is an example of the approach. I choose to minimize the sum of squared differences at the peak as the merit function, but one could choose others. The process of using a minimization procedure to estimate the parameters (and ultimately the unit weights) is often called de-convolution.

PARAMETRIC UNIT HYDROGRAPHS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
1	Unit Hydrograph (Transfer Function)																	K	31.931			
2												U(t)=K*(t/T)^N*exp(-(t/T))						N	5.7815			
3																		T	0.3052			
4																						
5																						
6	Observations						[P]												[U]	[Q*]	[Q]-[Q*]	Bias
7	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)			1	2	3	4	5	6	7	8	9	10	11					
8	0.5	1	1.06	428		1	1.06	0	0	0	0	0	0	0	0	0	0		107.63	114.084	313.916	0.73345
9	1	2	1.93	1923		2	1.93	1.06	0	0	0	0	0	0	0	0	0		1150.7	1427.44	495.5613	0.2577
10	1.5	3	1.81	5297		3	1.81	1.93	1.06	0	0	0	0	0	0	0	0		2331.6	4887.1	409.902	0.07738
11	2	4	0	9131		4	0	1.81	1.93	1.06	0	0	0	0	0	0	0		2391.2	9117.33	13.67158	0.0015
12	2.5	5	0	10625		5	0	0	1.81	1.93	1.06	0	0	0	0	0	0		1688.6	10625	6.73E-11	6.3E-15
13	3	6	0	7834		6	0	0	0	1.81	1.93	1.06	0	0	0	0	0		941.73	8585.17	-751.1711	-0.0959
14	3.5	7	0	3921		7	0	0	0	0	1.81	1.93	1.06	0	0	0	0		446.27	5346.89	-1425.892	-0.3637
15	4	8	0	1846		8	0	0	0	0	0	1.81	1.93	1.06	0	0	0		187.72	2764.82	-918.8209	-0.4977
16	4.5	9	0	1402		9	0	0	0	0	0	0	1.81	1.93	1.06	0	0		72.088	1246.46	155.5369	0.11094
17	5	10	0	830		10	0	0	0	0	0	0	0	1.81	1.93	1.06	0		25.765	506.209	323.7912	0.39011
18	5.5	11	0	313		11	0	0	0	0	0	0	0	0	1.81	1.93	1.06		8.6889	189.416	123.5837	0.39484
19																						
20							(1) Guess K,N,T													Average Bias		0.09169
21							(2) Compute Q*															
22							(3) Compute difference															
23							(4) K,N,T to get difference small															
24							(5) Stop when average difference is small percent of actual Q															
25							(6) If SOLVER works on machine - can automate - crashes on Mac (Microsoft hates Mac!)															



VALUE OF PARAMETRIC UNIT HYDROGRAPHS

- Fewer values to keep track of
- Simple extension of time-base
- If the parameters can be associated with watershed metrics (Slope, MCL, soil properties, shape, etc.) the resulting model is called a synthetic unit hydrograph
  - Called synthetic because response can be synthesized from the metrics rather than from analyzing observations (which we may not have in cases of practical interest)

EXTENDED TIME BASE

Unit Hydrograph (Transfer Function)

U(t)=K*(t/T)^N*exp(-(t/T))

K	31.931
N	5.7815
T	0.3052

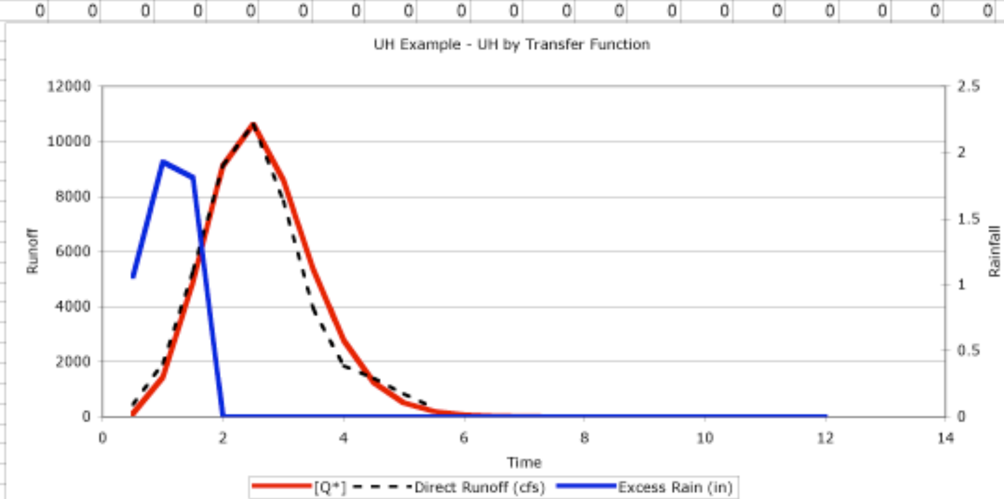
Observations

[P]

Row	A Time(hrs)	B Time (increment)	C Excess Rain (in)	D Direct Runoff (cfs)	F	G 1	H 2	I 3	J 4	K 5	L 6	M 7	N 8	O 9	P 10	Q 11
8	0.5	1	1.06	428	1	1.06	0	0	0	0	0	0	0	0	0	0
9	1	2	1.93	1923	2	1.93	1.06	0	0	0	0	0	0	0	0	0
10	1.5	3	1.81	5297	3	1.81	1.93	1.06	0	0	0	0	0	0	0	0
11	2	4	0	9131	4	0	1.81	1.93	1.06	0	0	0	0	0	0	0
12	2.5	5	0	10625	5	0	0	1.81	1.93	1.06	0	0	0	0	0	0
13	3	6	0	7834	6	0	0	0	1.81	1.93	1.06	0	0	0	0	0
14	3.5	7	0	3921	7	0	0	0	0	1.81	1.93	1.06	0	0	0	0
15	4	8	0	1846	8	0	0	0	0	0	1.81	1.93	1.06	0	0	0
16	4.5	9	0	1402	9	0	0	0	0	0	0	1.81	1.93	1.06	0	0
17	5	10	0	830	10	0	0	0	0	0	0	0	1.81	1.93	1.06	0
18	5.5	11	0	313	11	0	0	0	0	0	0	0	0	1.81	1.93	1.06
19	6	12	0	0	12	0	0	0	0	0	0	0	0	0	1.81	1.93
20	6.5	13	0	0	13	0	0	0	0	0	0	0	0	0	0	1.81
21	7	14	0	0	14	0	0	0	0	0	0	0	0	0	0	0
22	7.5	15	0	0	15	0	0	0	0	0	0	0	0	0	0	0
23	8	16	0	0	16	0	0	0	0	0	0	0	0	0	0	0
24	8.5	17	0	0	17	0	0	0	0	0	0	0	0	0	0	0
25	9	18	0	0	18	0	0	0	0	0	0	0	0	0	0	0
26	9.5	19	0	0	19	0	0	0	0	0	0	0	0	0	0	0
27	10	20	0	0	20	0	0	0	0	0	0	0	0	0	0	0
28	10.5	21	0	0	21	0	0	0	0	0	0	0	0	0	0	0
29	11	22	0	0	22	0	0	0	0	0	0	0	0	0	0	0
30	11.5	23	0	0	23											
31	12	24	0	0	24											

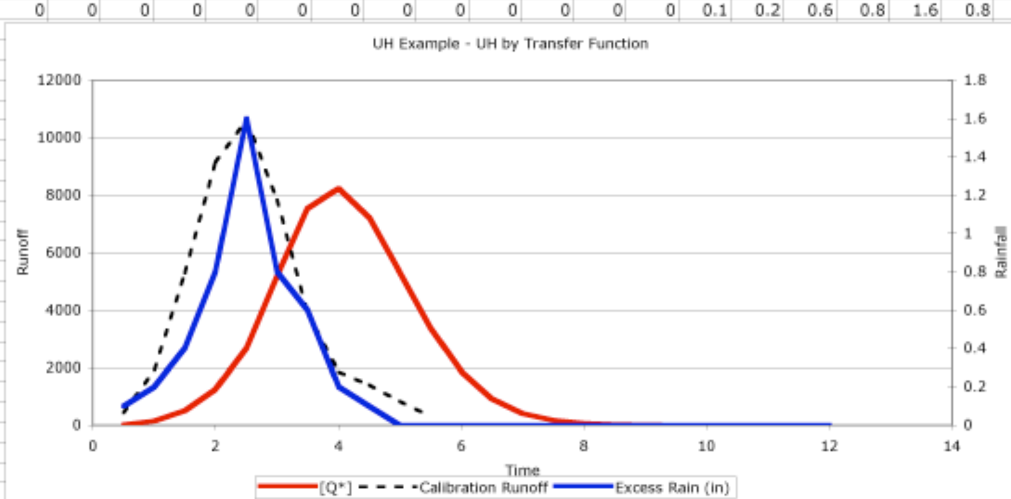
Row	R 12	S 13	T 14	U 15	V 16	W 17	X 18	Y 19	Z 20	AA 21	AB 22	AC 23	AD 24
8	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0
19	1.06	0	0	0	0	0	0	0	0	0	0	0	0
20	1.93	1.06	0	0	0	0	0	0	0	0	0	0	0
21	1.81	1.93	1.06	0	0	0	0	0	0	0	0	0	0
22	0	1.81	1.93	1.06	0	0	0	0	0	0	0	0	0
23	0	0	1.81	1.93	1.06	0	0	0	0	0	0	0	0
24	0	0	0	1.81	1.93	1.06	0	0	0	0	0	0	0
25	0	0	0	0	1.81	1.93	1.06	0	0	0	0	0	0
26	0	0	0	0	0	1.81	1.93	1.06	0	0	0	0	0
27	0	0	0	0	0	0	1.81	1.93	1.06	0	0	0	0
28	0	0	0	0	0	0	0	1.81	1.93	1.06	0	0	0
29	0	0	0	0	0	0	0	0	1.81	1.93	1.06	0	0
30									0	1.81	1.93	1.06	0
31									0	0	1.81	1.93	1.06

Row	AG [U]	AH [Q*]	AI [Q]-[Q*]
8	107.63	114.084	313.916
9	1150.7	1427.44	495.5613
10	2331.6	4887.1	409.902
11	2391.2	9117.33	13.67158
12	1688.6	10625	6.73E-11
13	941.73	8585.17	-751.1711
14	446.27	5346.89	-1425.892
15	187.72	2764.82	-918.8209
16	72.088	1246.46	155.5369
17	25.765	506.209	323.7912
18	8.6889	189.416	123.5837
19	2.7929	66.3649	-66.36489
20	0.8623	22.0313	-22.0313
21	0.2572	6.99213	-6.992127
22	0.0745	2.13623	-2.136233
23	0.021	0.63172	-0.631718
24	0.0058	0.1816	-0.181604
25	0.0016	0.05093	-0.050933
26	0.0004	0.01398	-0.013977
27	0.0001	0.00376	-0.003762
28	3E-05	0.001	-0.000995
29	7E-06	0.00026	-0.000259
30	2E-06	6.7E-05	-6.66E-05
31	4E-07	1.7E-05	-1.69E-05



EXTENDED TIME BASE – DIFFERENT STORM

	A	B	C	D	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC	AD	AG	AH
4																															
5																															
6	Observations					[P]																								[U]	[Q*]
7	Time(hrs)	Time (increment)	Excess Rain (in)	Direct Runoff (cfs)		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24		
8	0.5	1	0.1	428	1	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	107.63	10.7626
9	1	2	0.2	1923	2	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1150.7	136.593
10	1.5	3	0.4	5297	3	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2331.6	506.345
11	2	4	0.8	9131	4	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2391.2	1251.81
12	2.5	5	1.6	10625	5	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1688.6	2672.47
13	3	6	0.8	7834	6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	941.73	5180.81
14	3.5	7	0.6	3921	7	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	446.27	7536.99
15	4	8	0.2	1846	8	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	187.72	8238.64
16	4.5	9	0.1	1402	9	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	72.088	7231.13
17	5	10	0	830	10	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25.765	5322.8
18	5.5	11	0	313	11	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	0	8.6889	3376.98
19	6	12	0	0	12	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0	2.7929	1869.23
20	6.5	13	0	0	13	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0	0.8623	915.214
21	7	14	0	0	14	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0	0.2572	403.219
22	7.5	15	0	0	15	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0	0.0745	162.576
23	8	16	0	0	16	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0	0.021	60.8779
24	8.5	17	0	0	17	0	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0	0.0058	21.4296
25	9	18	0	0	18	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0	0.0016	7.16082
26	9.5	19	0	0	19	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0	0.0004	2.28937
27	10	20	0	0	20	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	0	0.0001	0.70473
28	10.5	21	0	0	21	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	0	3E-05	0.20994
29	11	22	0	0	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0.2	0.6	0.8	1.6	0.8	0.4	0.2	0.1	0	0	7E-06	0.06078
30	11.5	23	0	0	23																				0.8	0.4	0.2	0.1	0	2E-06	0.01716
31	12	24	0	0	24																				1.6	0.8	0.4	0.2	0.1	4E-07	0.00474



PARAMETRIC UNIT HYDROGRAPH PROPERTIES

The unit hydrograph (UH) response can be expressed as

$$\frac{Q(t)}{A} = \int_0^T r_{xs}(\tau)u(t-\tau)d\tau \qquad (4)$$

where where $\frac{Q(t)}{A}$ is the specific discharge from a basin at time t ("the *response*", $r(t)$ is an input function that represents excess rainfall (" the *excitation*"), $u(t-\tau)$ is the kernel function, in this case the unit hydrograph, and T is the duration of the input. Equation 4 assumes that basins respond as linear systems and this assumption is the main criticism of unit hydrograph theory. Despite this criticism, unit hydrographs are used to estimate streamflow from relatively small basins, typically for engineering purposes and often produce reasonable results. With the linearity assumption, the kernel, $u(t-\tau)$, has the same properties as a probability density function specifically, it integrates to unity on the range $(-\infty, \infty)$, and $f(t-\tau) \geq 0$ for any values of $(t-\tau)$.

HYDROGRAPHS

FHWA-NHI-02-001 Highway Hydrology

- Chapter 6, Section 6.1

Systems Approach

- Input : Hyetograph
- Transfer : Unit Hydrograph
- Output : Total Runoff Hydrograph

HYDROGRAPHS

Hydrograph Analysis

- Measured rainfall and runoff to infer the transfer function.
- Implies: Have **DATA**.

Hydrograph Synthesis

- Physical properties of watershed used to postulate the transfer function.
- Actual measurements not required – Produces an **ESTIMATE**

PARAMETRIC UNIT HYDROGRAPHS

HEC-HMS has several different UH models available (eg. NRCS DUH, Clark, etc.).

These models are described by "parameters"

- NRCS (Tp or Tc)
- Clark (Tc, R)

Selection of the parameters selects the shape of the UH and the time base (or time to peak).

The analyst can also enter an empirical (user specified) unit hydrograph.

DEVELOPING UNIT HYDROGRAPHS (ANALYSIS)

Follow FHWA methods

- Essentially back-substitution or OLS

Use HEC-HMS and parametric UH, adjust values of parameters to fit observed runoff

- Less tedious

Use HEC-HMS and user supplied UH, adjust values of UH ordinates (and re-scale to maintain a unit response) to fit observed runoff

- Less tedious

PARAMETRIC UNIT HYDROGRAPHS IN HMS

- Present an example that uses
  - NRCS DUH
  - Clark Unit Hydrograph
- Useful parametric models are listed below along with their parameters.
  - NRCS DUH : T_{lag} (Timing only)
  - Clark : T_c, and R (Timing and a storage-delay)
  - Snyder : T_{lag}, C_p (Timing and a peak rate factor)
  - Gamma : (User-Specified) T_c, K (Timing and a shape factor)

SYNTHESIZING UNIT HYDROGRAPHS

- Synthesis does not use rainfall-runoff data.
- Uses measurements on the watershed to postulate parameters of a parametric unit hydrograph.

EXAMPLE

HEC-HMS SIMULATION

DEVELOPING A UNIT HYDROGRAPH

PURPOSE

- Illustrate using HEC-HMS to develop a unit-hydrograph
  - Parametric UHs
  - User-specified UHs

LEARNING OBJECTIVES

- Learn how to use HMS to construct a parametric Unit Hydrograph
  - Use trial-and-error to select parameters that fit observations.
- Learn how to supply an arbitrary user-specified Unit Hydrograph to HMS

PROBLEM STATEMENT

- Determine the UH parameters for the precipitation and direct runoff for the watershed of Example 6.2 in FHWA "Highway Hydrology"
  - Use FHWA rainfall and direct runoff values as input and runoff.

BACKGROUND AND DATA

Watershed Properties

- AREA = $0.39 mi^2$
- SLOPE = Unknown
- CN = Unknown
- Precipitation = Given
- Direct Runoff = Given

Example 6.2(CU). Figure 6.7(CU) shows a 1-hour rainfall intensity hyetograph. The total volume of rainfall is:

$$P = \sum_{j=1}^{4} i_j \Delta t = \Delta t \sum_{j=1}^{4} i_j = \frac{15\ min}{60\ min/h}(0.24 + 0.47 + 0.51 + 0.12) = 0.335\ in$$

The total runoff hydrograph is also shown in Figure 6.7(CU).

The first step is to compute the base flow. The convex method of Section 6.1.4.1 will be used. Since the runoff begins to increase at the start of the second interval, the initial slope of the base flow function will equal the slope in the first 15-minute interval: 0.4 ft^3/s per 15 minutes. Since the peak of the hydrograph occurs at a storm time of 75 minutes, the initial portion of the base flow function will be extended from a storm time of 15 minutes to a time of 75 minutes. Using the decrease of 0.4 ft^3/s per 15 minutes produces the base flow rates shown in column 3 of Table 6.1(CU). Since there is a noticeable change of slope on the falling limb of the total runoff hydrograph at a storm time of 135 minutes, this will be used as the inflection point; direct runoff will end at a time of 135 minutes. Thus, the second leg of the base flow function can be represented by a linear segment between storm times of 75 and 135 minutes with a slope of:

$$slope = \frac{(7.1 - 2.6)\ ft^3/s}{(135 - 75)\ min} = 0.075\ ft^3/s \text{ per minute}$$

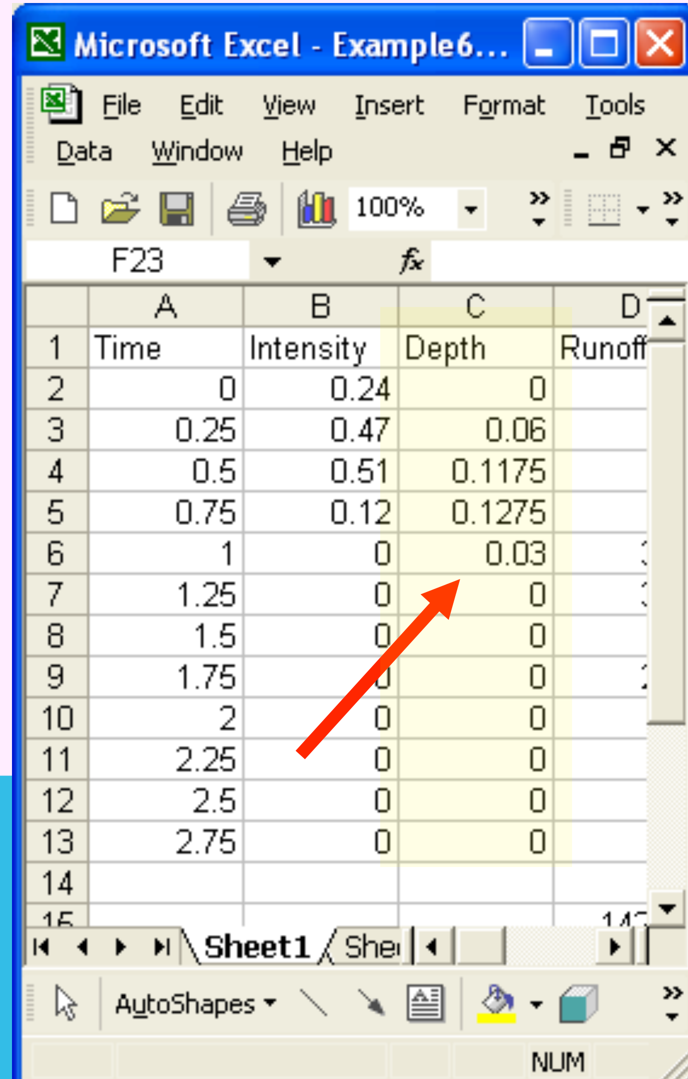
or 1.125 ft^3/s per 15-minute interval. Because the inflection point has a higher discharge than the base flow at the time to peak, the slope is positive. This slope is used to compute the base flow function for the interval from 75 to 135 minutes. Beyond the inflection point, all of the total runoff is assumed to be base flow. Values for the base flow are given in column 3 of Table 6.1(CU).

The base flow is subtracted from the total runoff to give the direct-runoff hydrograph (column 4 of Table 6.1(CU)). The volume of direct runoff can be computed using the trapezoidal rule in Equation 6.7:

$$V = \sum_{i=1}^{n} \Delta t \left(\frac{q_i + q_{i+1}}{2} \right) \qquad (6.7)$$

BACKGROUND AND DATA

Precipitation



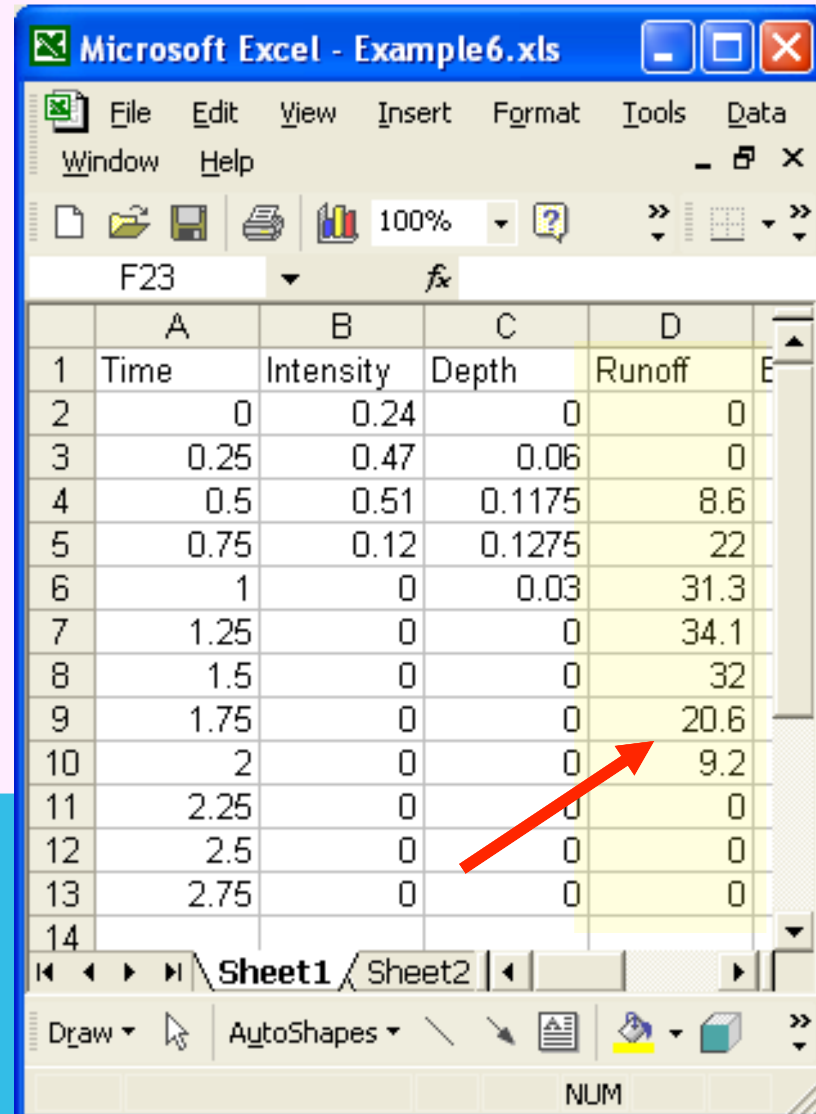
Microsoft Excel - Example6...

	A	B	C	D
1	Time	Intensity	Depth	Runoff
2	0	0.24	0	
3	0.25	0.47	0.06	
4	0.5	0.51	0.1175	
5	0.75	0.12	0.1275	
6	1	0	0.03	
7	1.25	0	0	
8	1.5	0	0	
9	1.75	0	0	
10	2	0	0	
11	2.25	0	0	
12	2.5	0	0	
13	2.75	0	0	
14				

BACKGROUND AND DATA

Direct Runoff

- Baseflow already separated



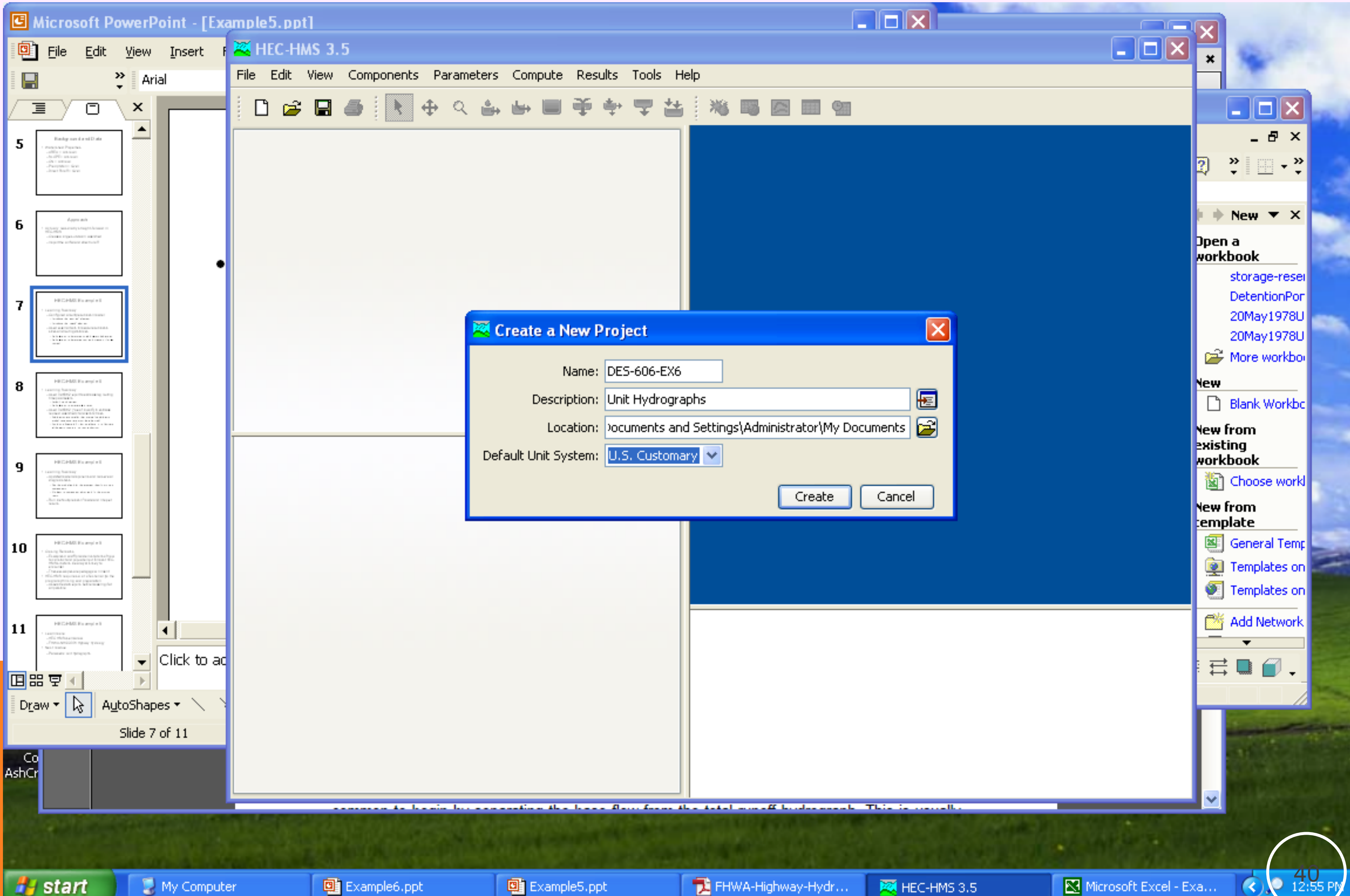
	Time	Intensity	Depth	Runoff
1	Time	Intensity	Depth	Runoff
2	0	0.24	0	0
3	0.25	0.47	0.06	0
4	0.5	0.51	0.1175	8.6
5	0.75	0.12	0.1275	22
6	1	0	0.03	31.3
7	1.25	0	0	34.1
8	1.5	0	0	32
9	1.75	0	0	20.6
10	2	0	0	9.2
11	2.25	0	0	0
12	2.5	0	0	0
13	2.75	0	0	0

APPROACH

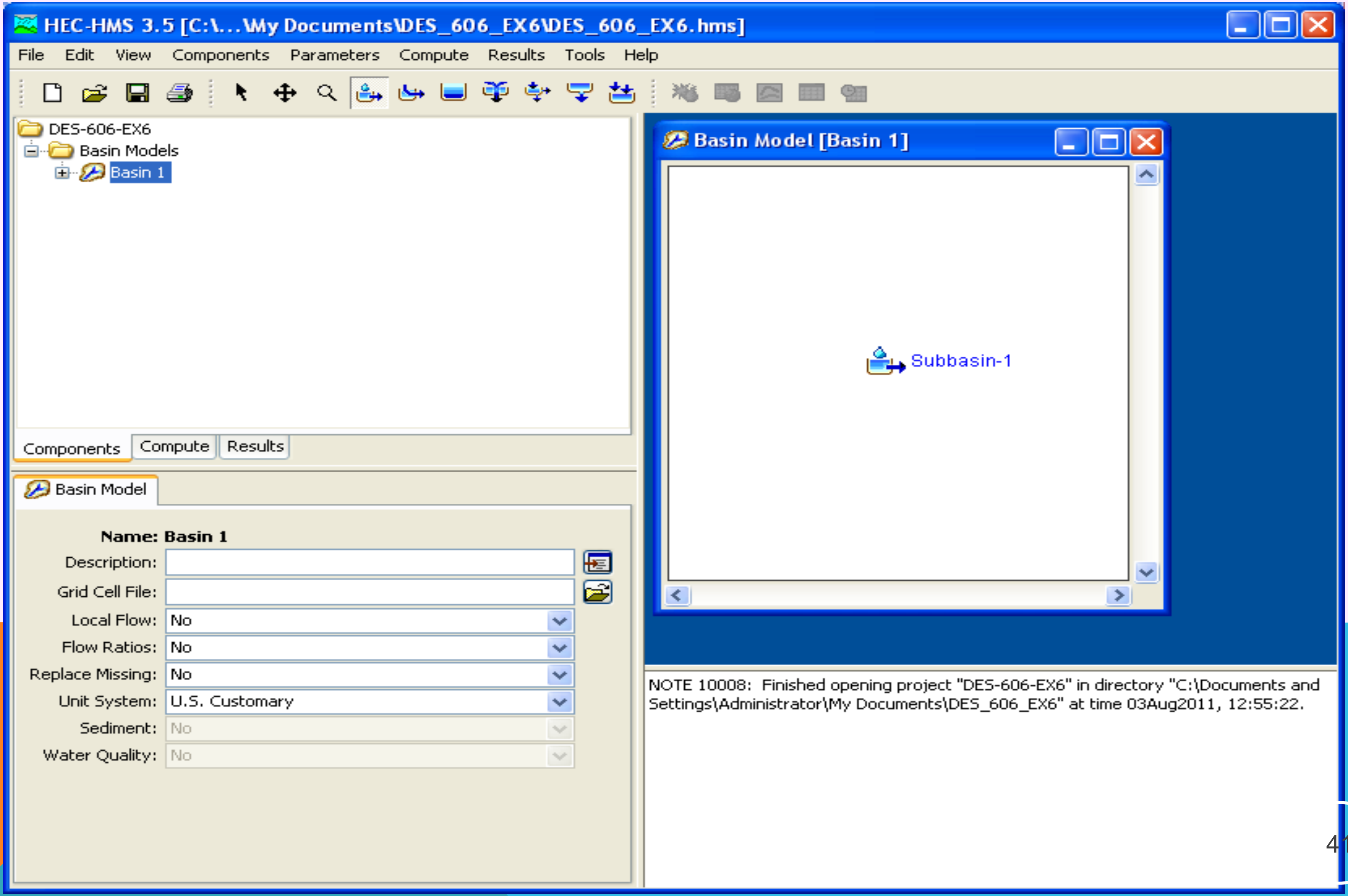
Actually reasonably straight-forward in HEC-HMS

- Create a single sub-basin watershed.
- Import the rainfall and direct runoff.
- Adjust loss and UH models to fit observations.
- Interpret and report results.

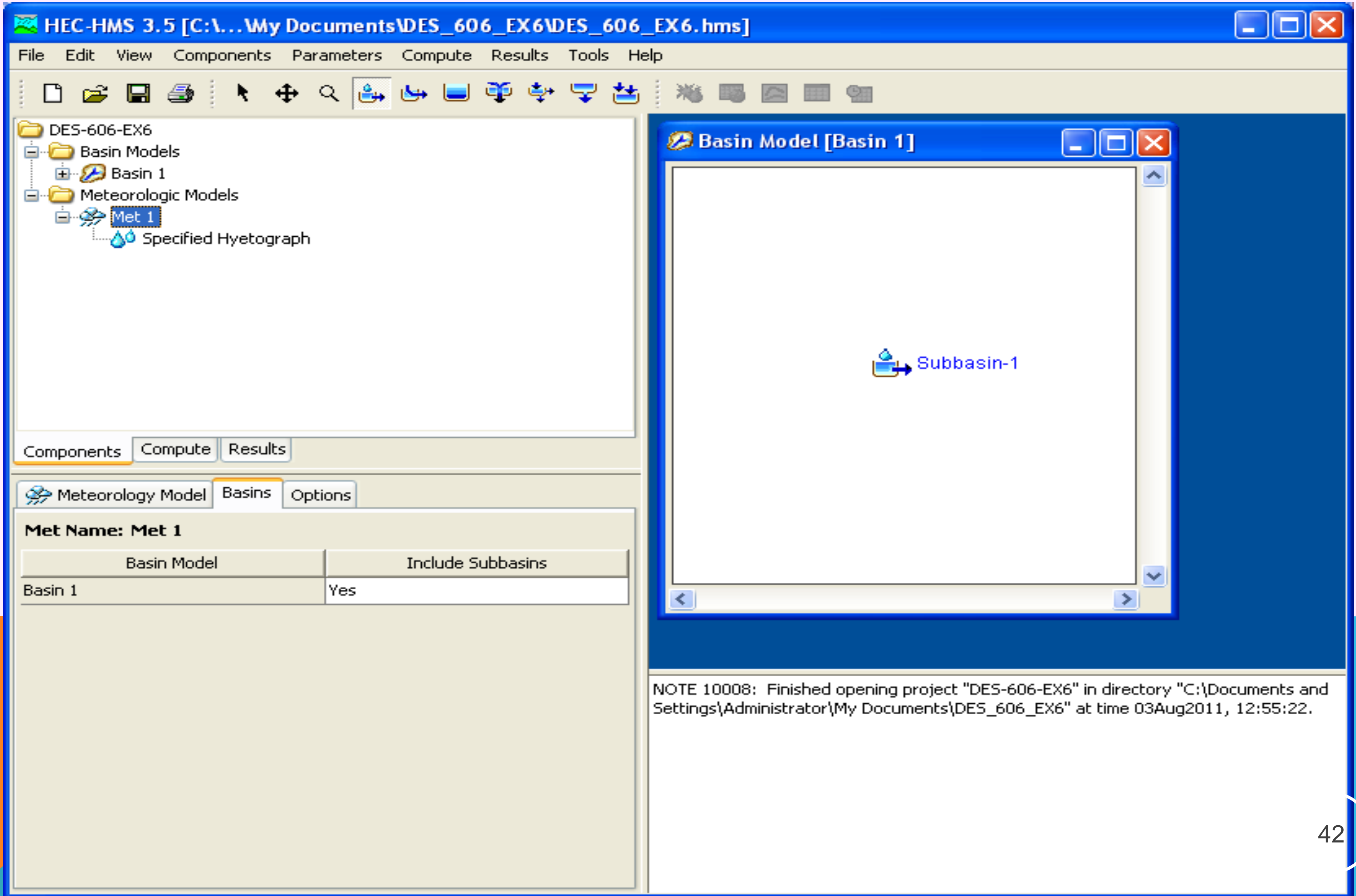
APPROACH – CREATE NEW PROJECT



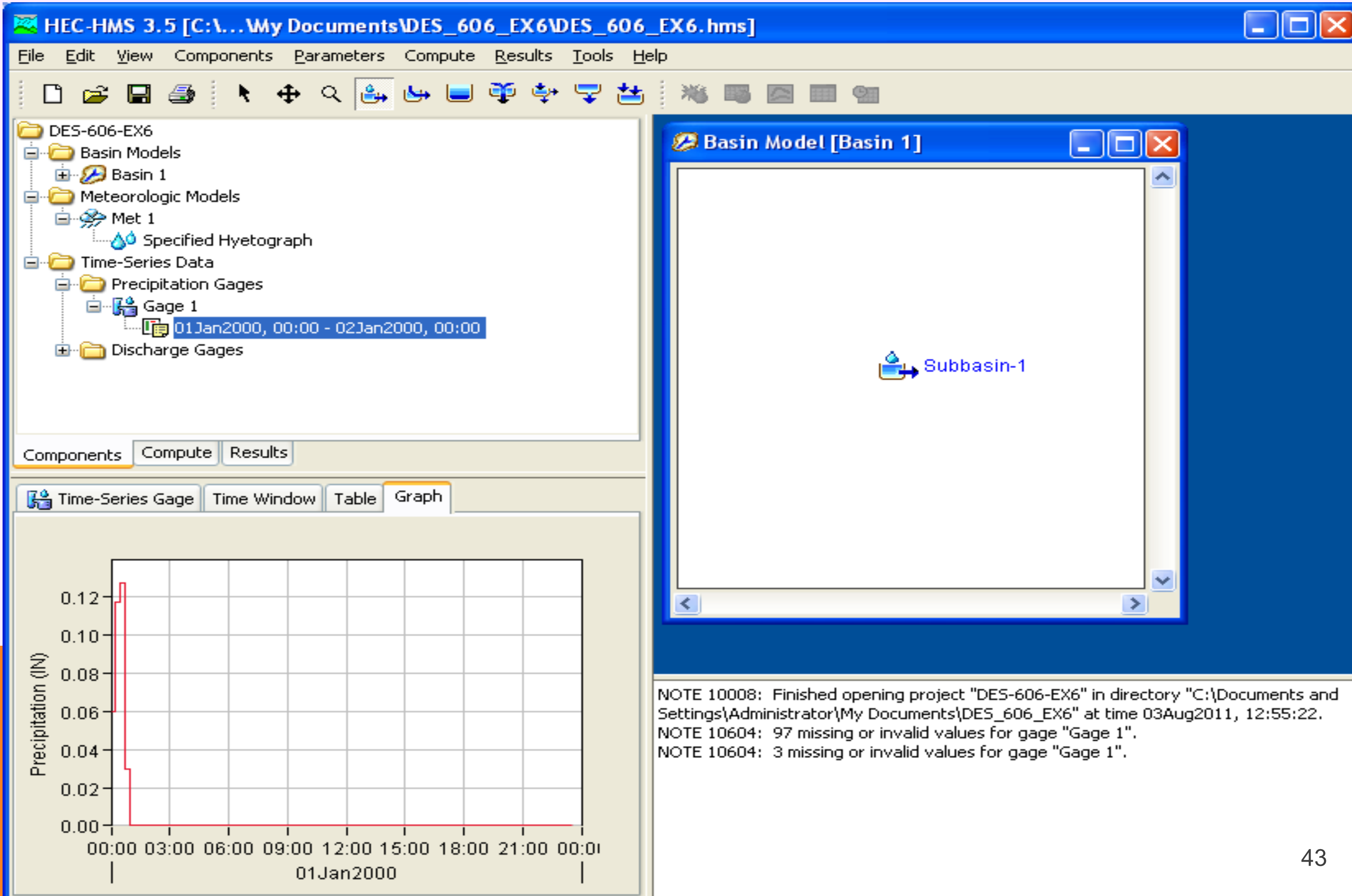
APPROACH – CREATE BASIN MODEL



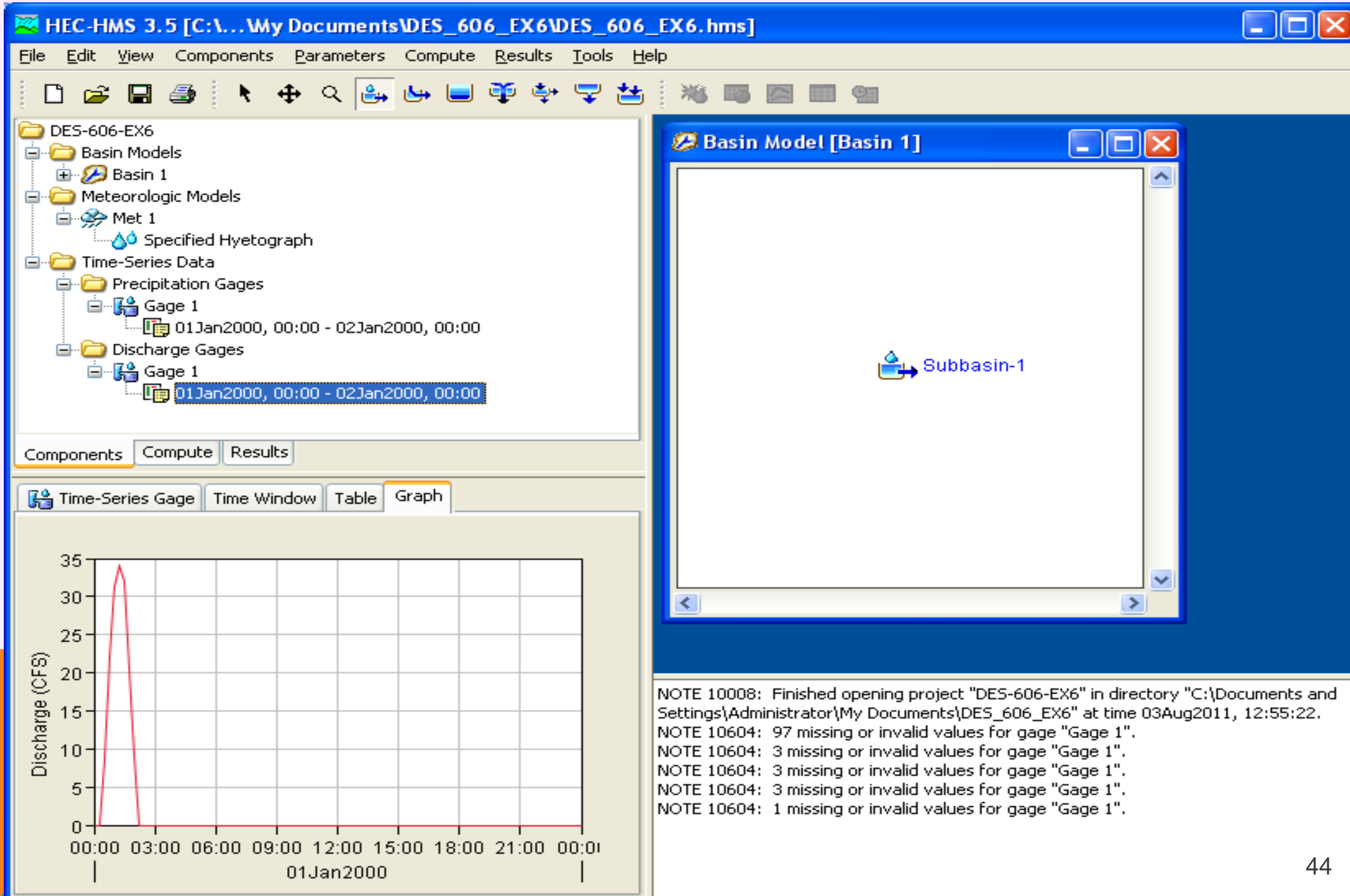
APPROACH – METEROLOGICAL MODEL



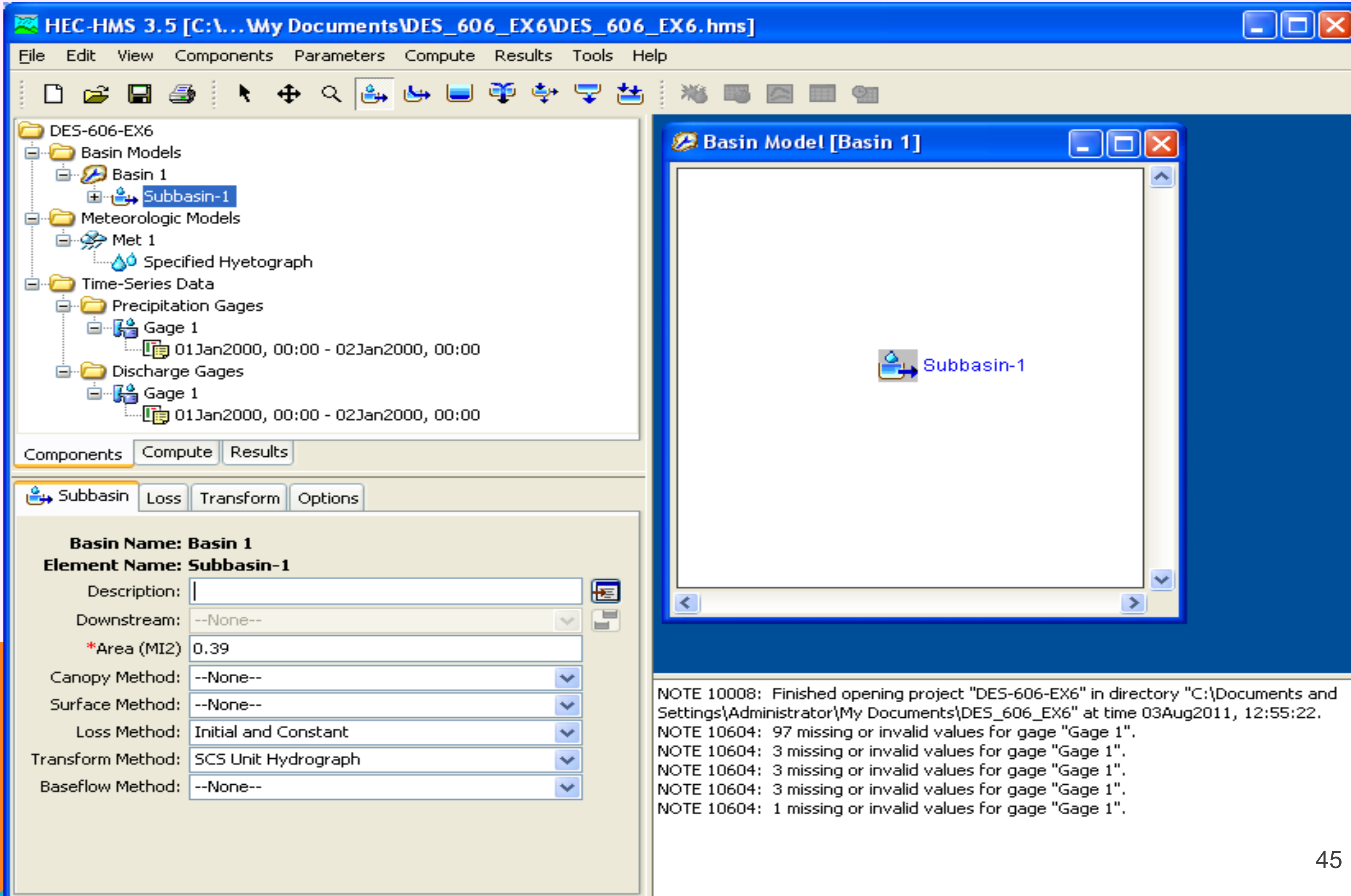
APPROACH – HYETOGRAPH TIME SERIES



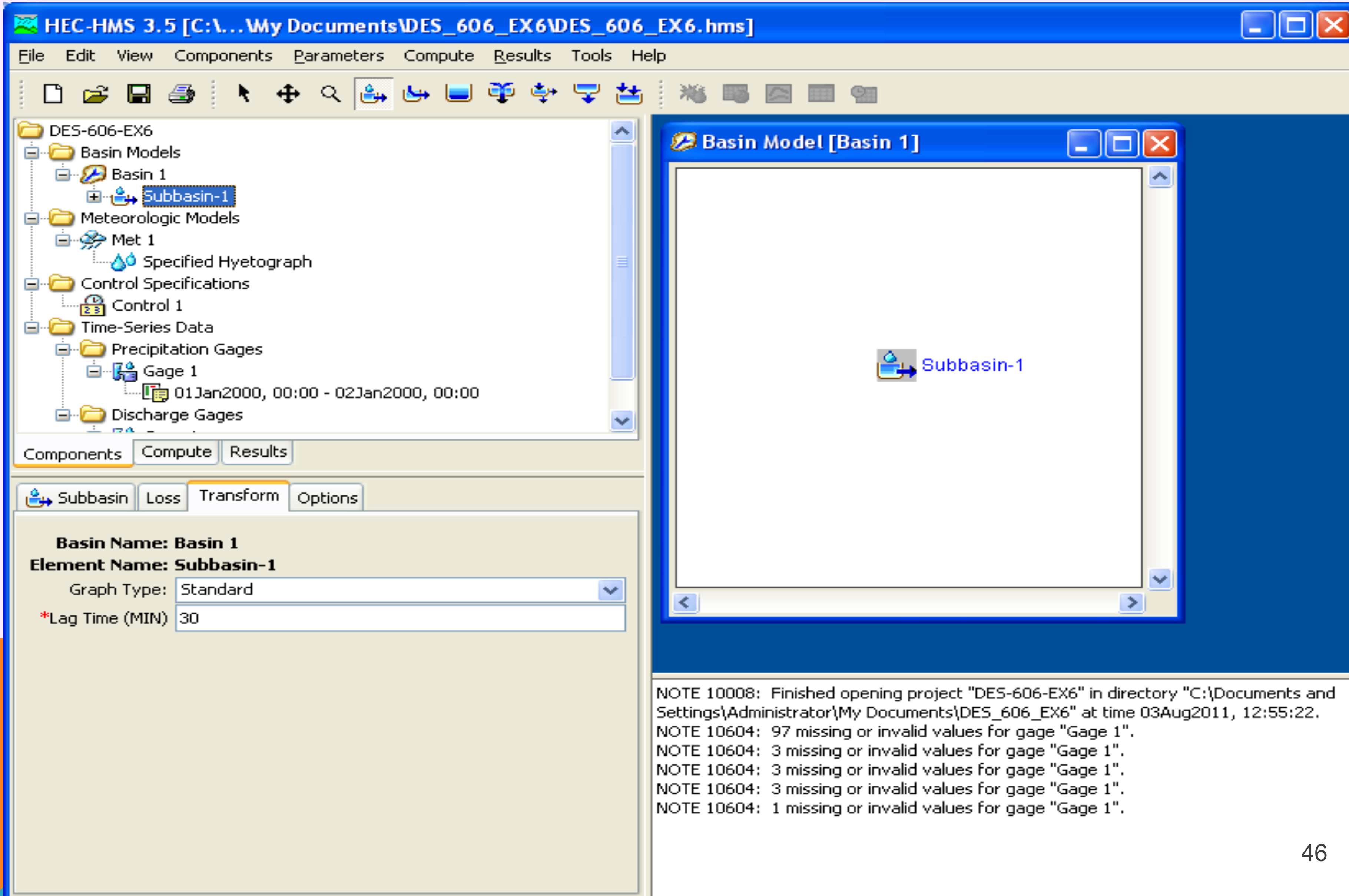
APPROACH – HYDROGRAPH TIME SERIES



APPROACH – BASIN MODEL AREA = 0.39MI^2

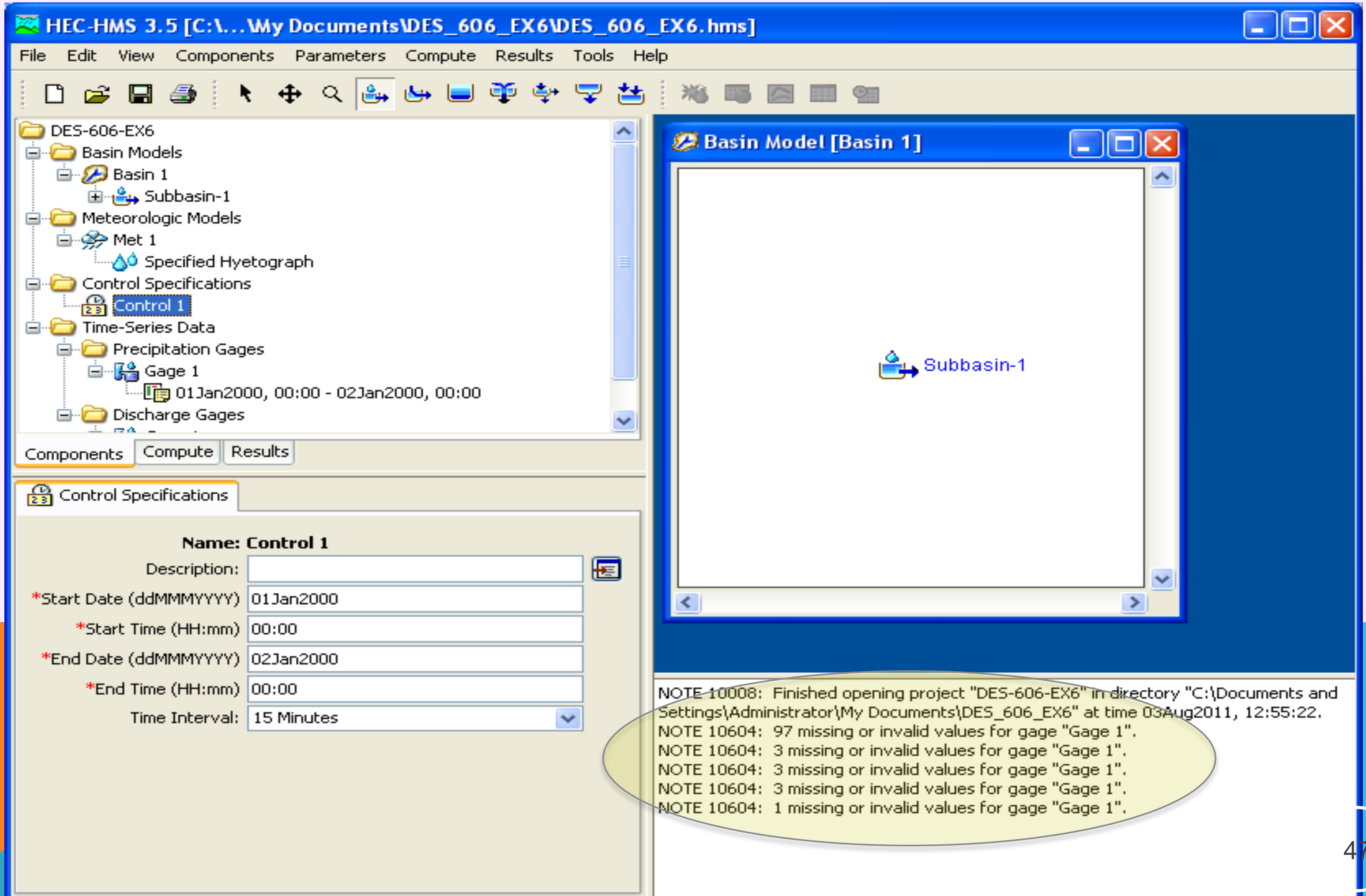


APPROACH – SET A LAG TIME (30 MIN)



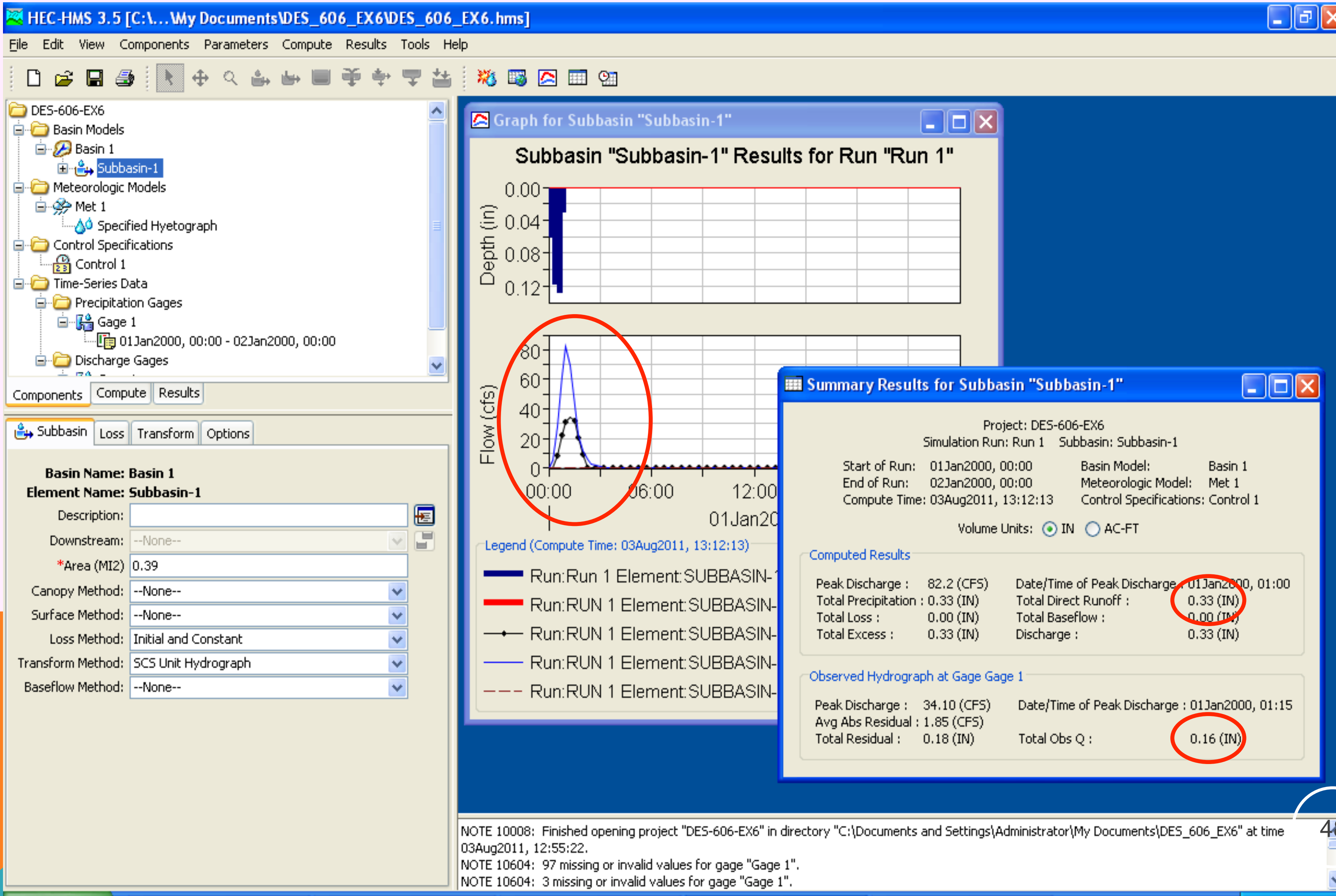
APPROACH – CONTROL SPECIFICATIONS

- Catch the missing warnings, fix, and run



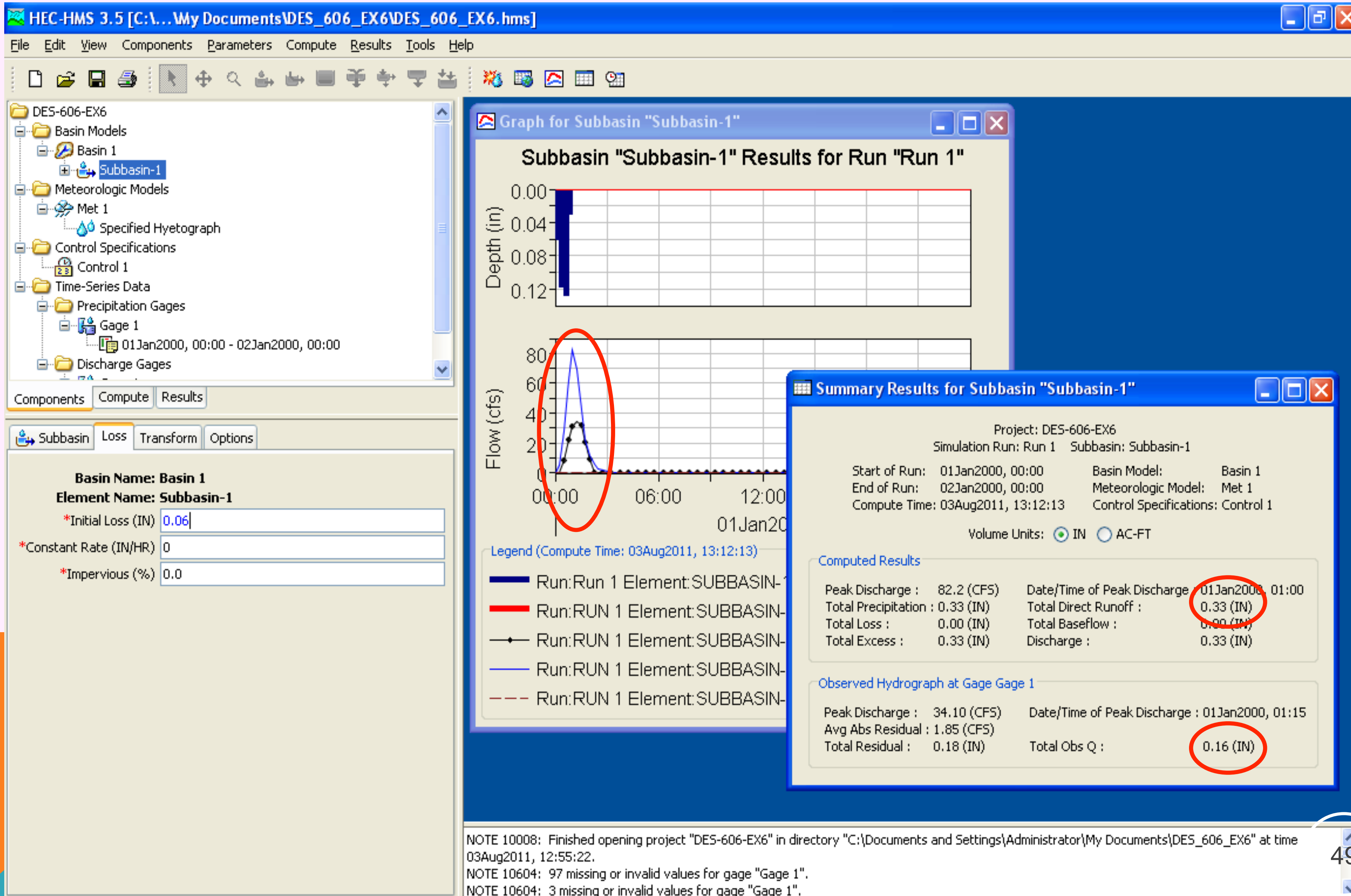
APPROACH – FIRST RUN

First simulation – volumes mismatch, need a loss model



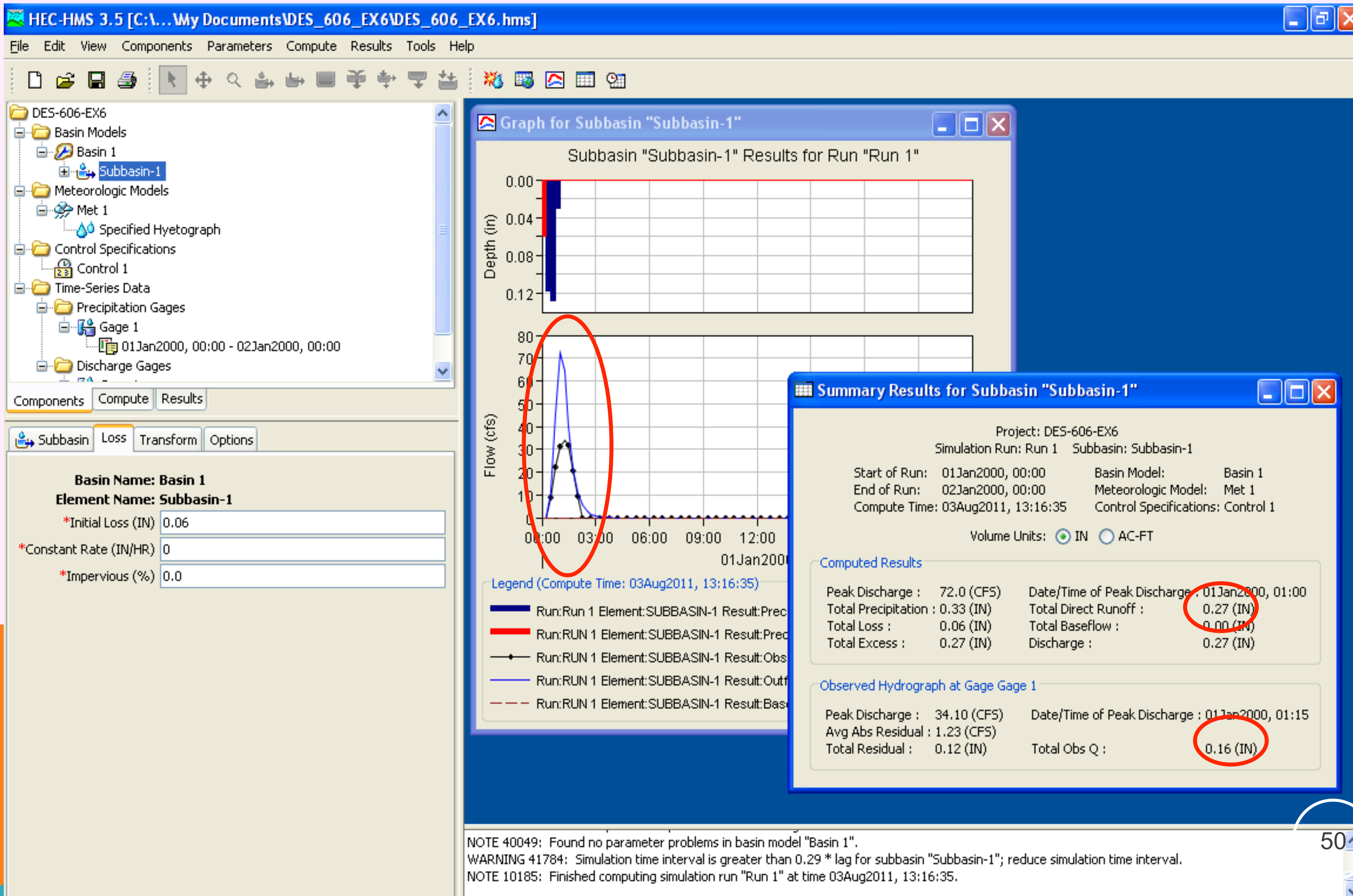
APPROACH – PICK A LOSS MODEL

Ia = 0.06 in, Cl =0.0



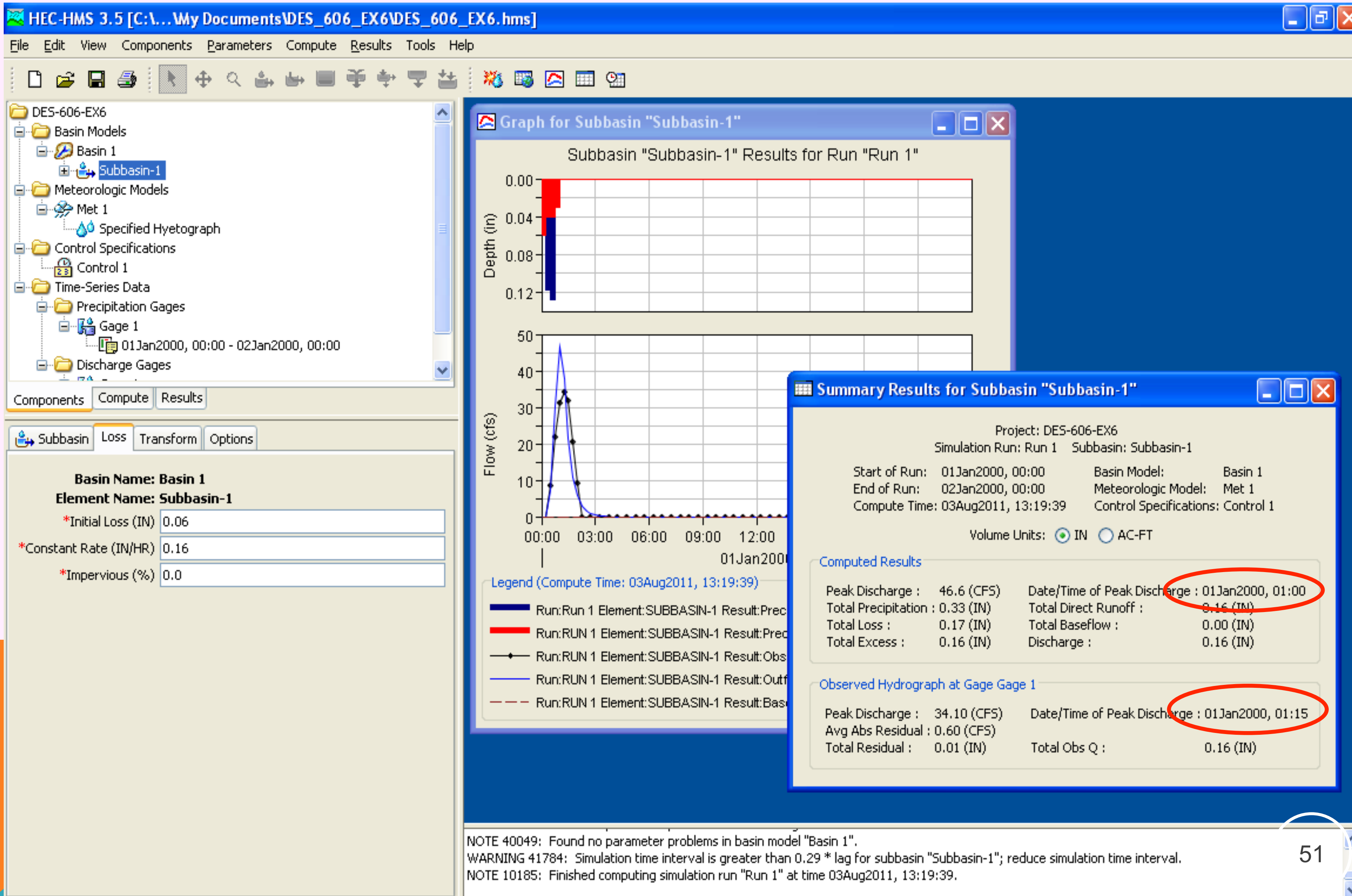
APPROACH

Second run, still volume mismatch. Increase the CI value.



APPROACH

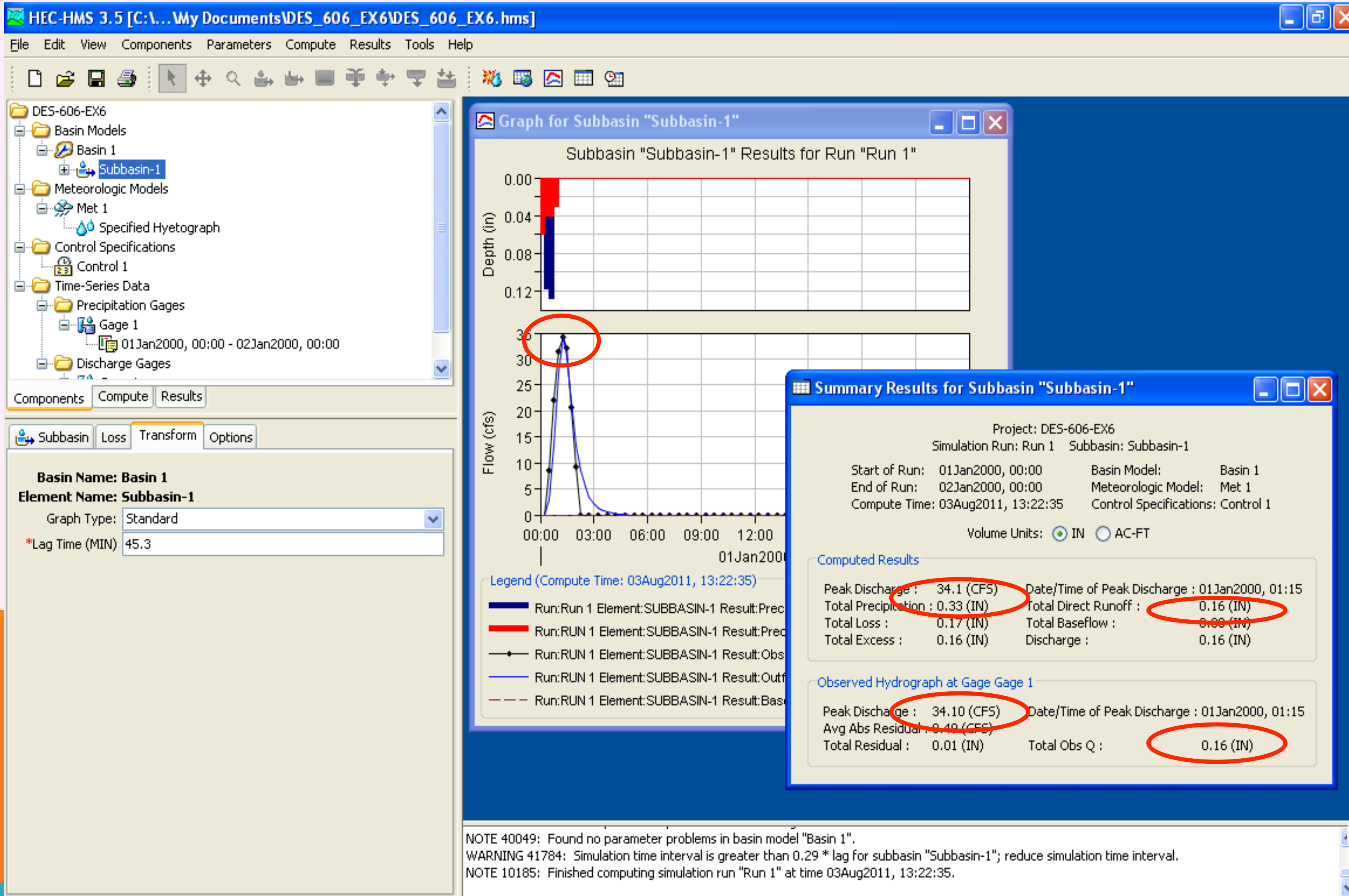
Volumes match adequately.



APPROACH

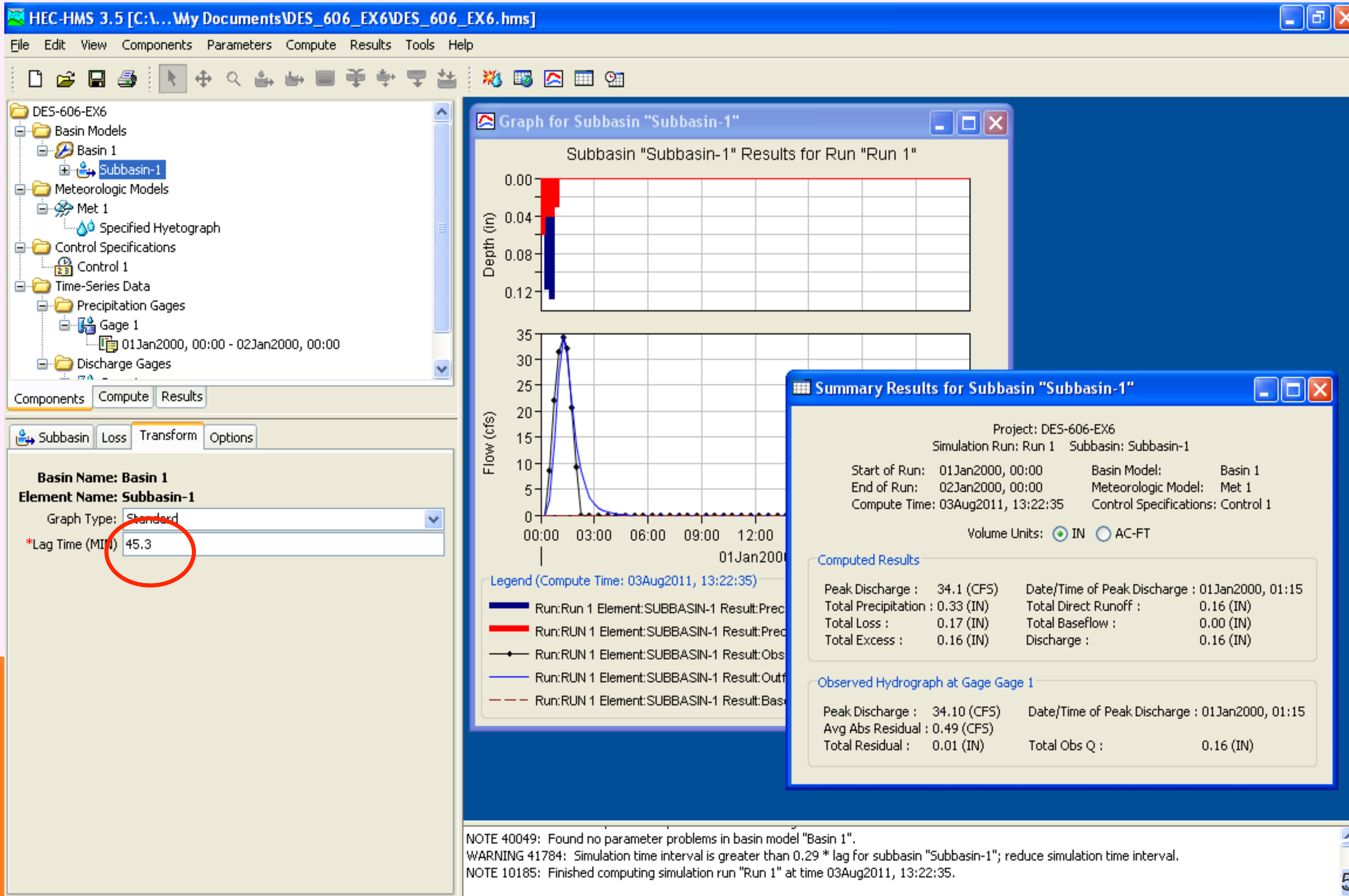
Several runs adjusting lag time.

- Peaks match, times good



APPROACH

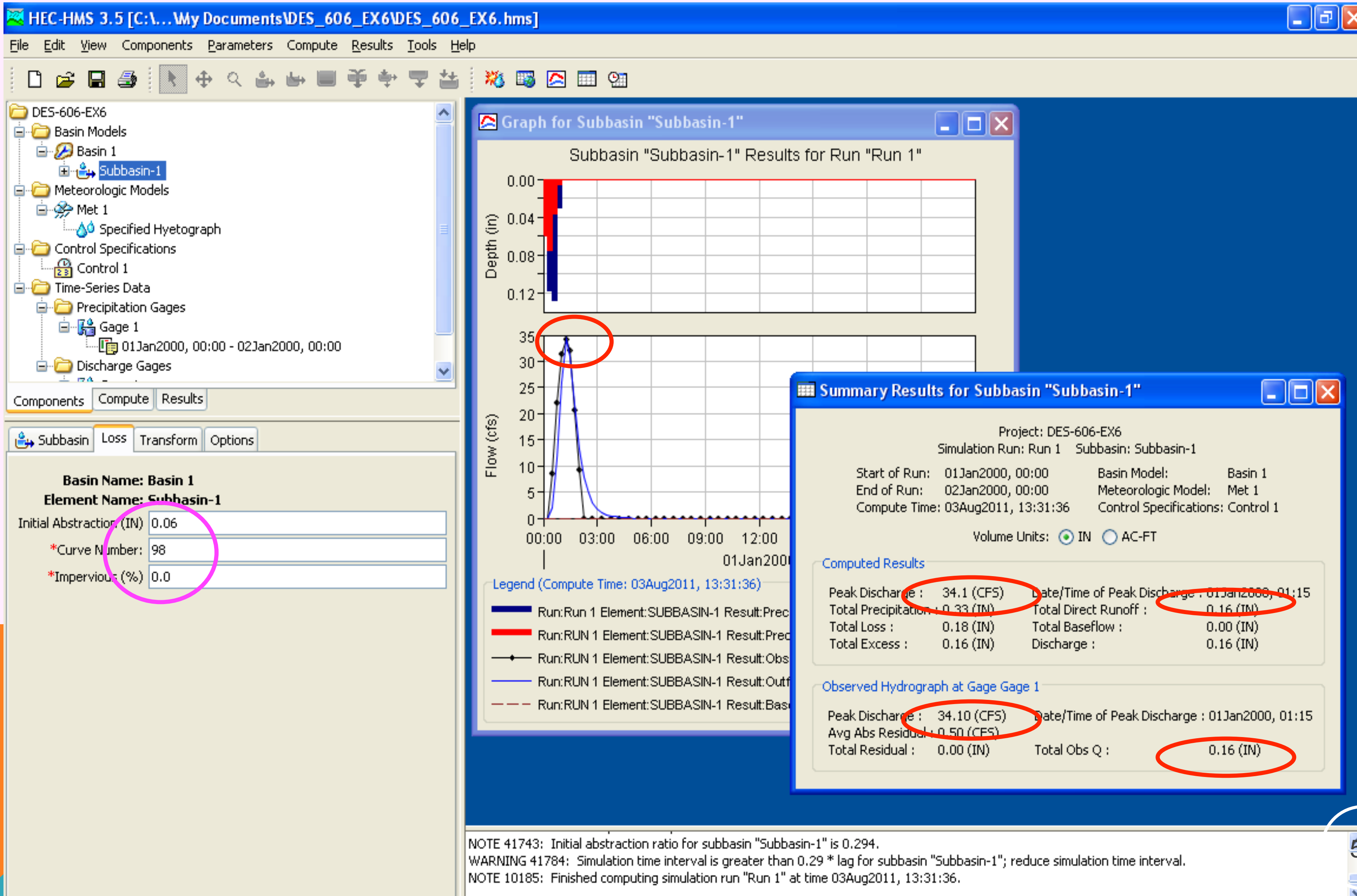
We have just "fit" the data to an NRCS DUH with T_{lag} = 45.3 minutes



DIFFERENT LOSS MODEL

We could try a different loss model

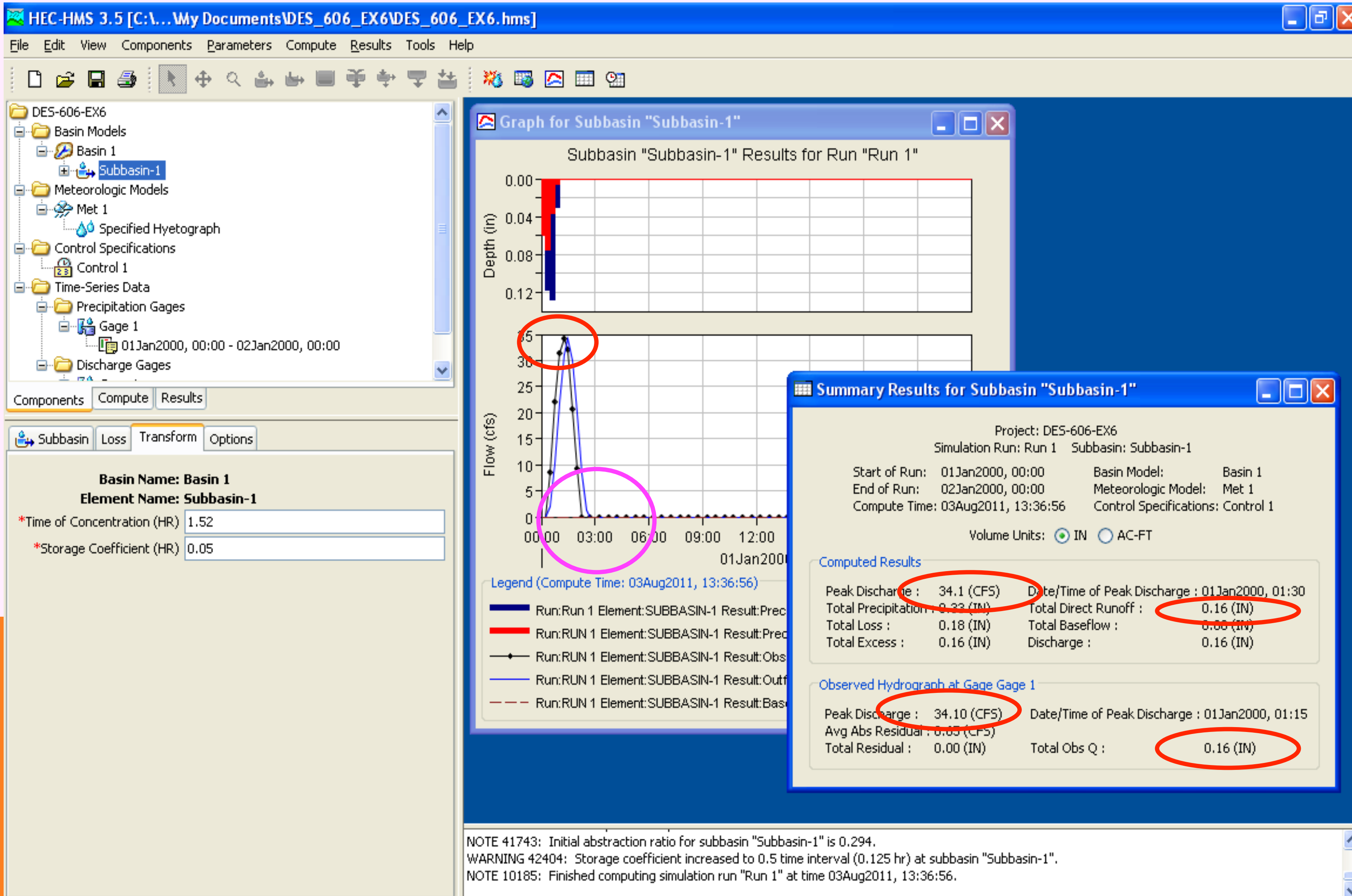
- NRCS CN model; Initial Loss = 0.06, initial guess CN=98



DIFFERENT UH MODEL

Clark UH

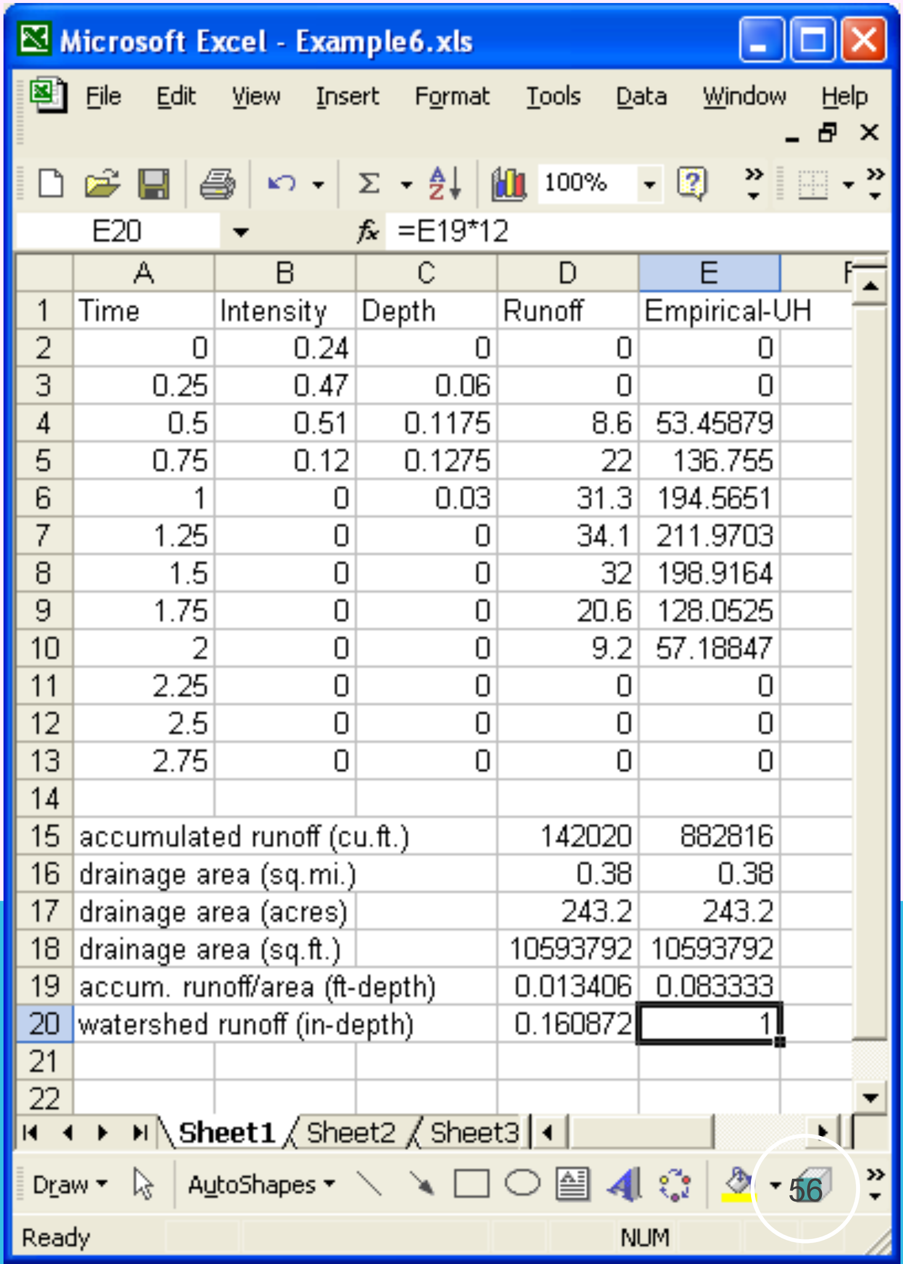
- Two parameters, Tc and R



USER SUPPLIED UH MODEL

Empirical UH

- Prepared externally
- Import using PAIRED data manager

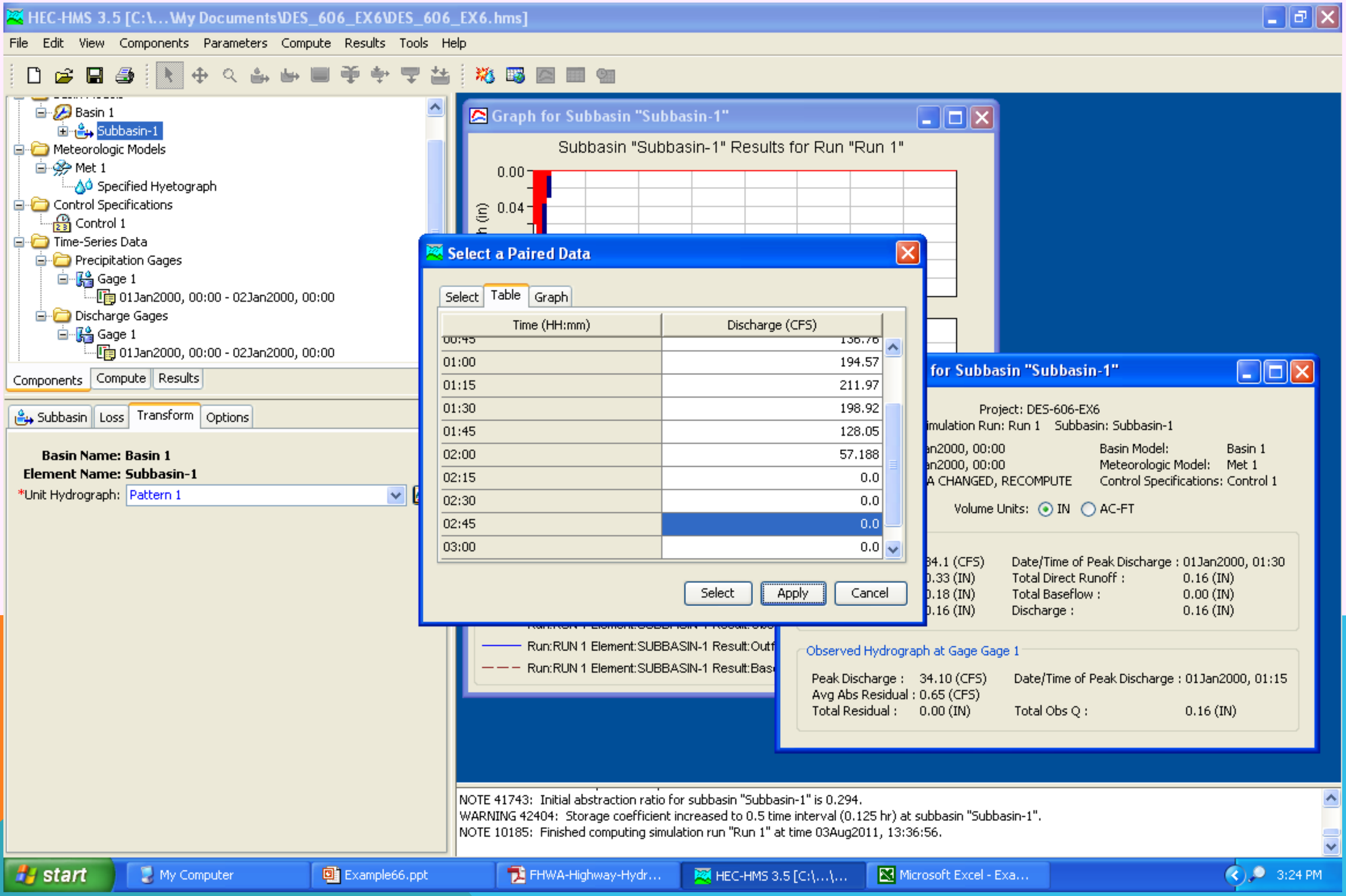


	A	B	C	D	E
1	Time	Intensity	Depth	Runoff	Empirical-UH
2	0	0.24	0	0	0
3	0.25	0.47	0.06	0	0
4	0.5	0.51	0.1175	8.6	53.45879
5	0.75	0.12	0.1275	22	136.755
6	1	0	0.03	31.3	194.5651
7	1.25	0	0	34.1	211.9703
8	1.5	0	0	32	198.9164
9	1.75	0	0	20.6	128.0525
10	2	0	0	9.2	57.18847
11	2.25	0	0	0	0
12	2.5	0	0	0	0
13	2.75	0	0	0	0
14					
15	accumulated runoff (cu.ft.)			142020	882816
16	drainage area (sq.mi.)			0.38	0.38
17	drainage area (acres)			243.2	243.2
18	drainage area (sq.ft.)			10593792	10593792
19	accum. runoff/area (ft-depth)			0.013406	0.083333
20	watershed runoff (in-depth)			0.160872	1
21					
22					

USER SUPPLIED UH MODEL

Empirical UH

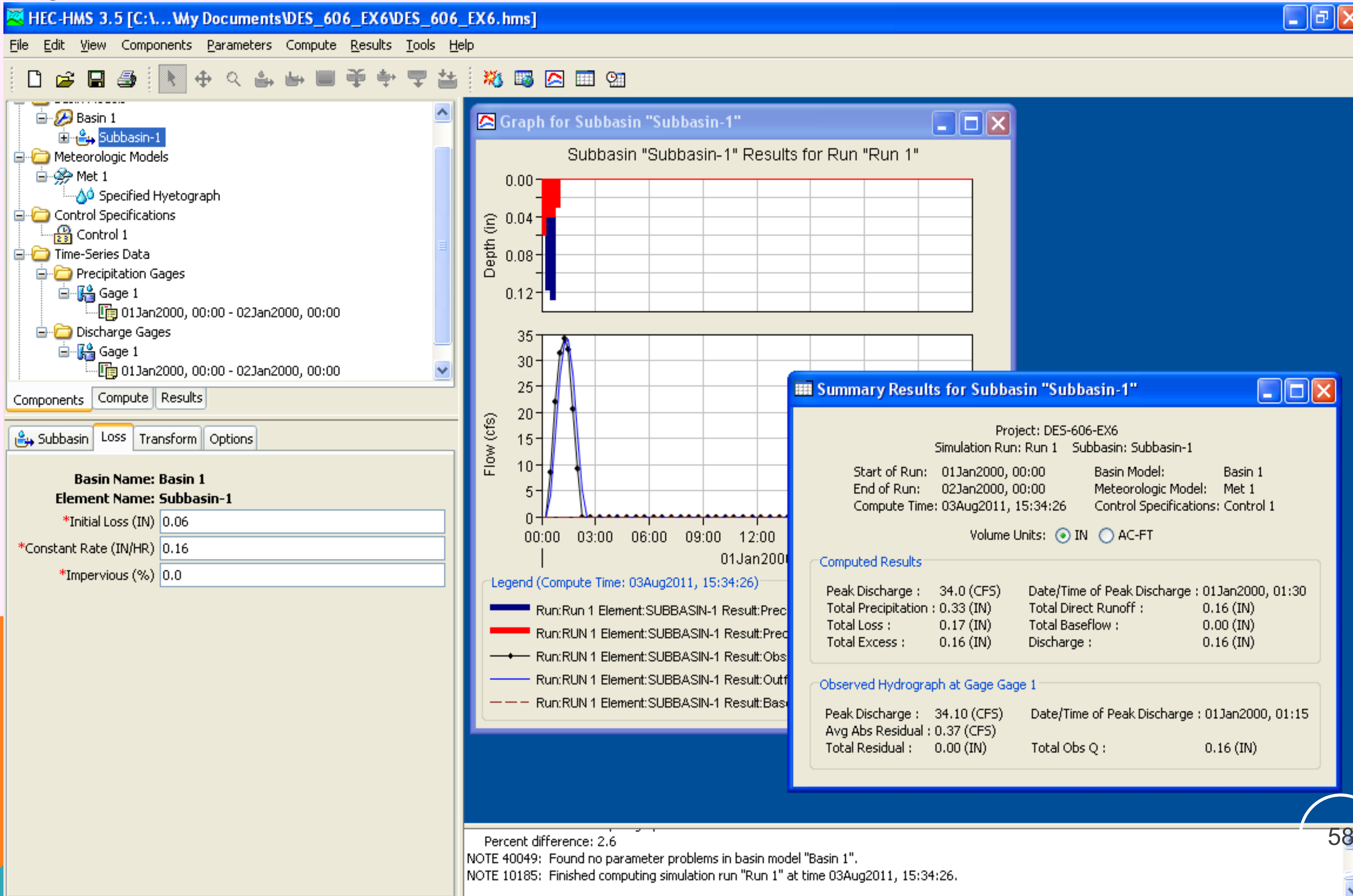
- Normalized to produce 1-unit (inch) runoff.
- 1 unit excess input will produce the hydrograph.



USER SUPPLIED UH MODEL

Empirical UH

- Original loss model.



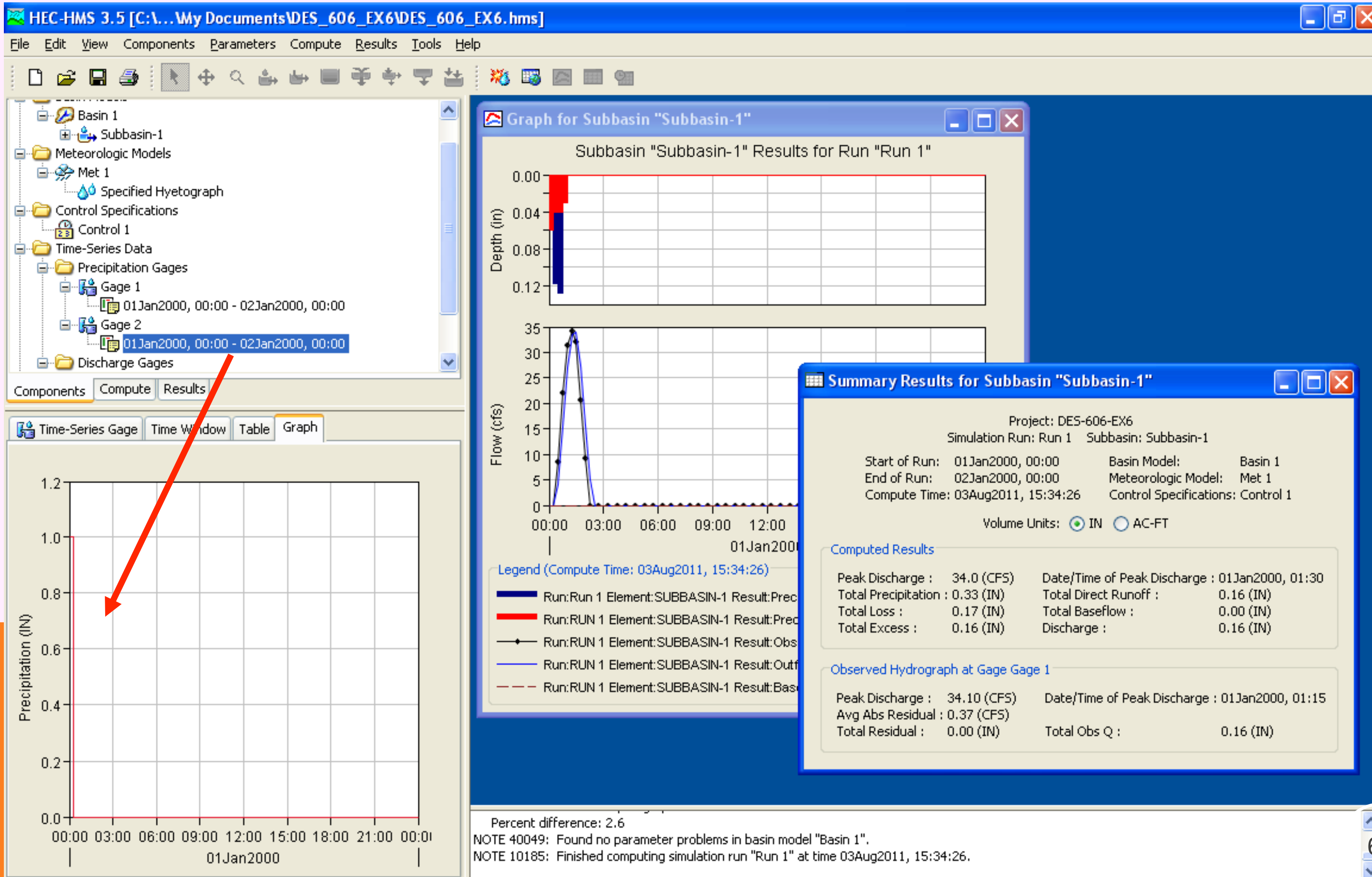
USER SUPPLIED UH MODEL

How to check that indeed a unit hydrograph?

- Add a “design storm” that has total excess of 1-unit (inch).
- Verify returns 1-unit (inch) of runoff.

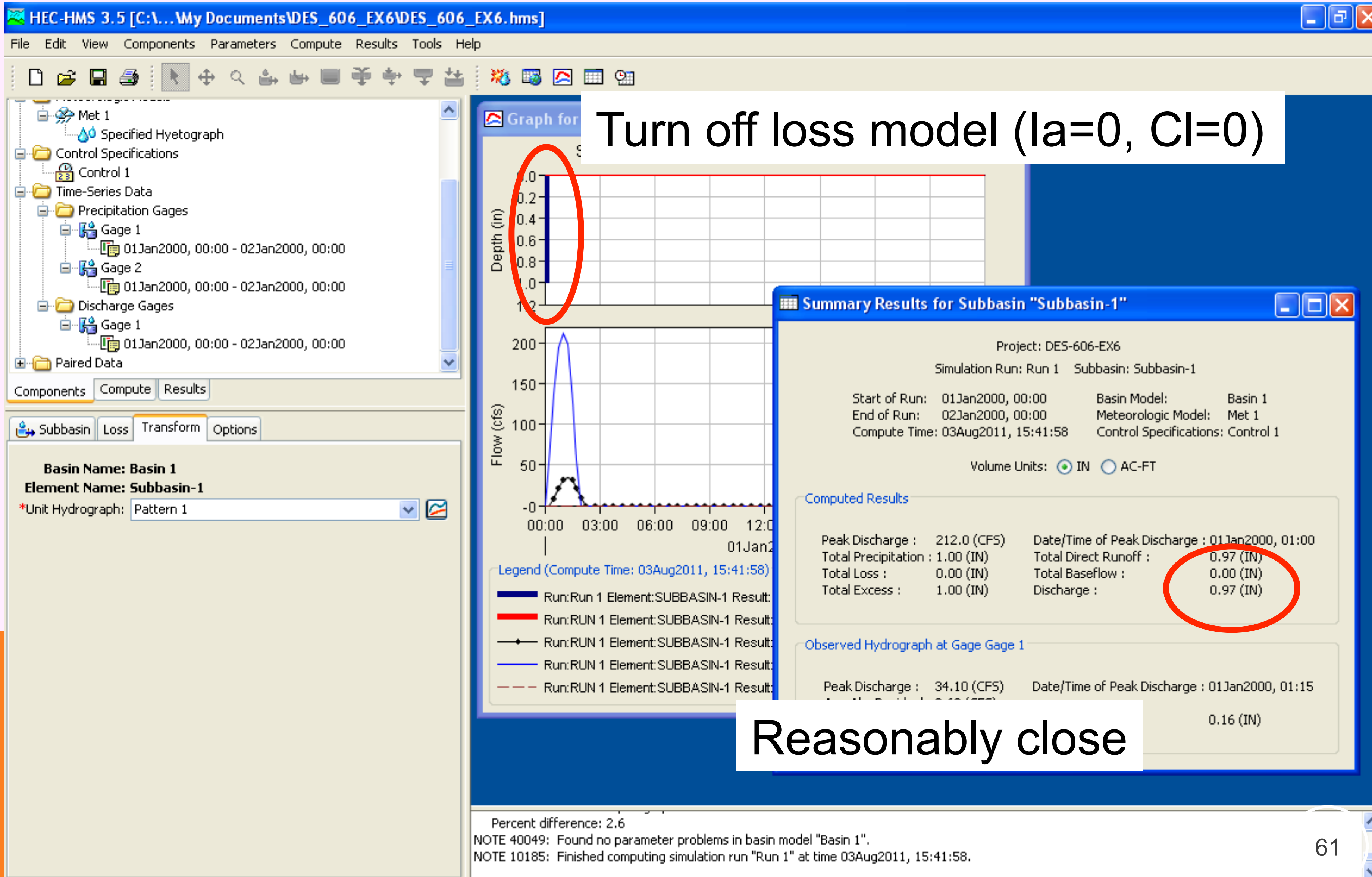
USER SUPPLIED UH MODEL

Add a "design storm" that has total excess of 1-unit (inch).



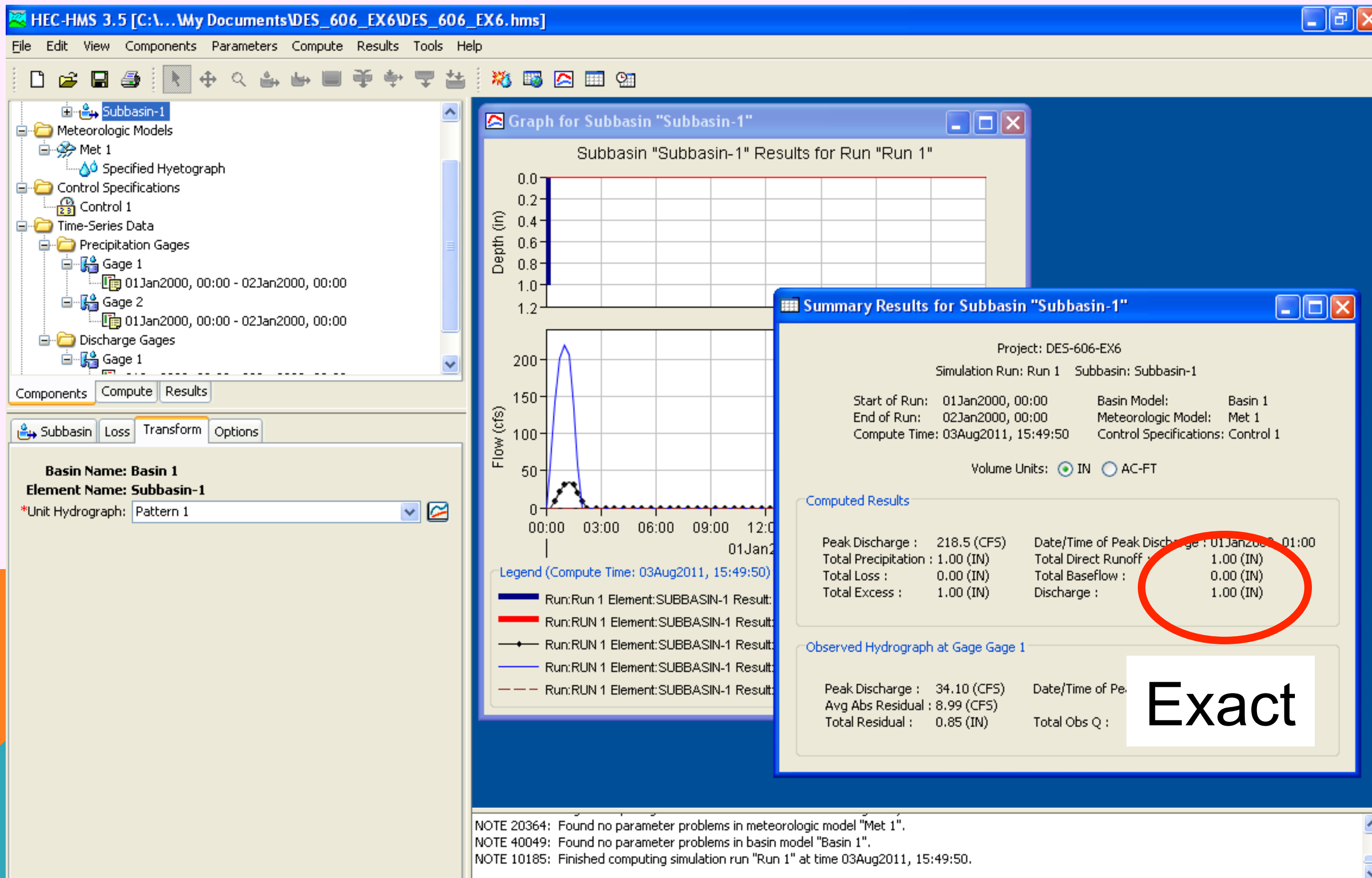
USER SUPPLIED UH MODEL

Verify returns 1-unit (inch) of runoff.



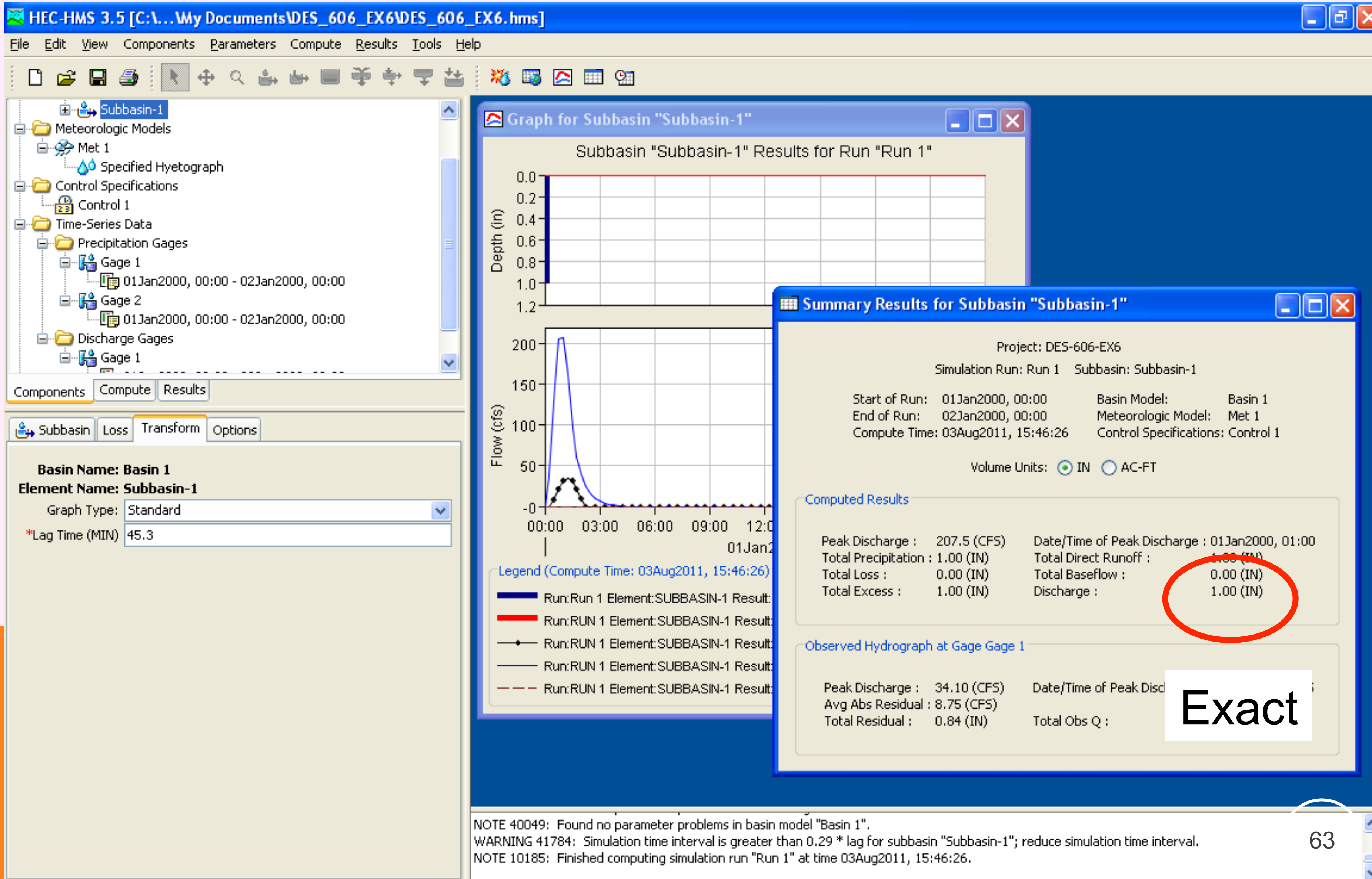
USER SUPPLIED UH MODEL

Rescale the ordinates and repeat.



PARAMETRIC UH MODEL

Verify returns in same fashion.



COMPARISONS

- The empirical UH and parametric UH are both unit hydrographs.
- Parametric has flexibility if changing time bases considerably, and easier to communicate to other analysts
  - Only need send the parameter set, and not a paired series of unit weights.

HEC-HMS EXAMPLE

Learning Summary

- Configured a single sub-basin model
- Used parametric UH models to simulate responses
  - Adjusted parameters until fit was “good”
- Used an empirical UH model based on the supplied runoff hydrograph
  - Normalized and time-shifted into a UH

HEC-HMS EXAMPLE

Learning Summary

- Verified both models were UH by
  - Apply single 1-inch depth pulse
  - Disable loss model to guarantee 1-inch excess input.
  - Observed that computed watershed runoff depth was 1-inch

HEC-HMS EXAMPLE

Learning Summary

- The empirical UH rescaling is how durations could be changed to produce a UH with a different duration.
- Empirical UH are entered as PAIRED data.

HEC-HMS EXAMPLE

Closing Remarks

- Example shows how to enter a user-specified hydrograph, thus the Gamma Unit Hydrograph is now a useable tool for HEC-HMS

HEC-HMS EXAMPLE

Learn more

- HEC HMS user manual
- FHWA-NHI-02-001 Highway Hydrology

SUMMARY

- Unit hydrographs map the excess precipitation signal to the outlet
- Base-flow separation isolates the total discharge from the storm-induced discharge
- Loss models are implicit; the unit hydrograph maps excess to the outlet
    - HEC-HMS Example inferred a UH from data

NEXT TIME

Unit Hydrographs/HMS Workshop

- CMM pp. 201-223
- Bring a laptop if you want to play along!