

CE 5366 – Water Resources Management Exercise Set 4

Purpose: Engineering economic mathematics

Exercise

- Two mutually exclusive project alternatives that provide identical service are described below:

Project ID	Initial Cost	Annual O&M	Salvage Value	Lifespan
A	\$10,000	\$2,000	\$1,000	10
B	\$25,000	\$1,500	\$5,000	20

Assuming a discount rate of 5% and using the **R** script in Listing 1 (or write your own)¹, determine:

- Which alternative has the lower annual cost?
- What is the incremental annual cost of going from the less to the more expensive alternative?
- Select the best alternative by the present-worth method.
- What is the rate of return on the incremental investment of B?
- What initial cost of replacing A after 10 years would make the two alternatives equivalent, assuming none of the other costs change?

Listing 1. R code for Trial-Error to find equivalent annual cost.

```
# cash flow calculations for ES4
rm(list=ls())
discount_rate <- 0.03 # 3% interest rate
# option A
# compute present values of expenses first 10 years
initial_costA <- 10000
operationsA <- rep(2000,10)
salvageA <- 1000
### compute the PV of the operations cost
present_value <- numeric(0)
present_value <- 0
for (i in 1:10){ #find present value of the i-th year payment
  present_value[i] <- operationsA[i]*(1+discount_rate)^(-i)
}
operationsAPV <- sum(present_value)
### compute the PV of the salvage payment
salvageAPV <- salvageA*(1+discount_rate)^(-10)
#print(cbind(initial_costA,operationsAPV,salvageAPV))
# now compute the PV for the second 10 years
presentValueA1 <- initial_costA+operationsAPV+salvageAPV
presentValueA2 <- presentValueA1*(1+discount_rate)^(-10)
presentValueA <- presentValueA1+presentValueA2
message("Present Value A = $",presentValueA)
##### Alternative B #####
initial_costB <- 25000
operationsB <- rep(1500,20)
salvageB <- 5000
```

¹The script here is pretty crude. User supplies a guess of annual costs, and by repeated application changes the guess until the computed present value of the annual costs is equal to the pre-determined present value based on the supplied components. A vast improvement would be to make the guess-and-check automatic; Newton's method (finite-difference approximations to the derivative) or bisection would work well.

```

### compute the PV of the operations cost
present_value <- numeric(0)
present_value <- 0
for (i in 1:20){ #find present value of the i-th year payment
  present_value[i] <- operationsB[i]*(1+discount_rate)^(-i)
}
operationsBPV <- sum(present_value)
### compute the PV of the salvage payment
salvageBPV <- salvageB*(1+discount_rate)^(-20)
#print(cbind(initial_costB,operationsBPV,salvageBPV))
presentValueB <- initial_costB+operationsBPV+salvageBPV
message("Present Value B = $",presentValueB)
# now convert both to annual costs
### compute the PV of the operations cost
present_valueA <- numeric(0)
present_valueA <- 0
### compute the PV of the operations cost
present_valueB <- numeric(0)
present_valueB <- 0
## read in guess for annual cost
avA <- readline(prompt="Enter annual cost for alternative A: ")
avA <- as.numeric(avA)
avB <- readline(prompt="Enter annual cost for alternative B: ")
avB <- as.numeric(avB)
annualA <- rep(avA,20)
annualB <- rep(avB,20)
for (i in 1:20){ #find present value of the i-th year payment
  present_valueA[i] <- annualA[i]*(1+discount_rate)^(-i)
  present_valueB[i] <- annualB[i]*(1+discount_rate)^(-i)
}
message("annual cost A = $",avA," PVA = $",sum(present_valueA)," PValue A = $",
       presentValueA)
message("annual cost B = $",avB," PVB = $",sum(present_valueB)," PValue B = $",
       presentValueB)

### CONSOLE INTERACTION ###
> source('~/Dropbox/1-Teaching/ce-5366/2-Exercises/ES-3o/cash-flow-ES3.R')
Present Value A = $48493.6585389613
Present Value B = $50084.5910616149
Enter annual cost for alternative A: 3259.5
Enter annual cost for alternative B: 3366.5
annual cost A = $3259.5 PVA = $48493.1293076547 PValue A = $48493.6585389613
annual cost B = $3366.5 PVB = $50085.0191177235 PValue B = $50084.5910616149
>

```