



# WATER RESOURCES MANAGEMENT

ECONOMIC ANALYSIS MATHEMATICS

# DEFINING ALTERNATIVES

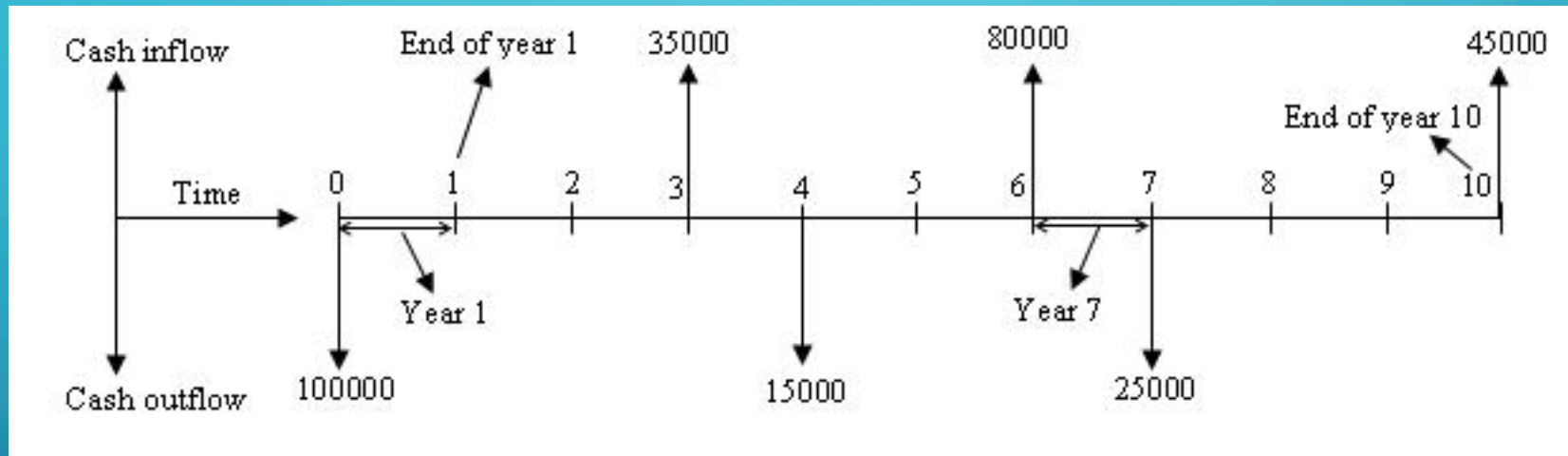
- Application of economics principles to assign value to an alternative
- Decision principles used to select the best alternative based on value

# PHYSICAL CONSEQUENCES

- Economic value – a basis for comparisons
- Definitions/Concepts
- Economic Analysis
  - Principles of engineering economics
  - Cash flows
  - Discounting methods

# CASH FLOW DIAGRAMS

- Graphic representation of cash outflow (costs) and inflow (revenue).



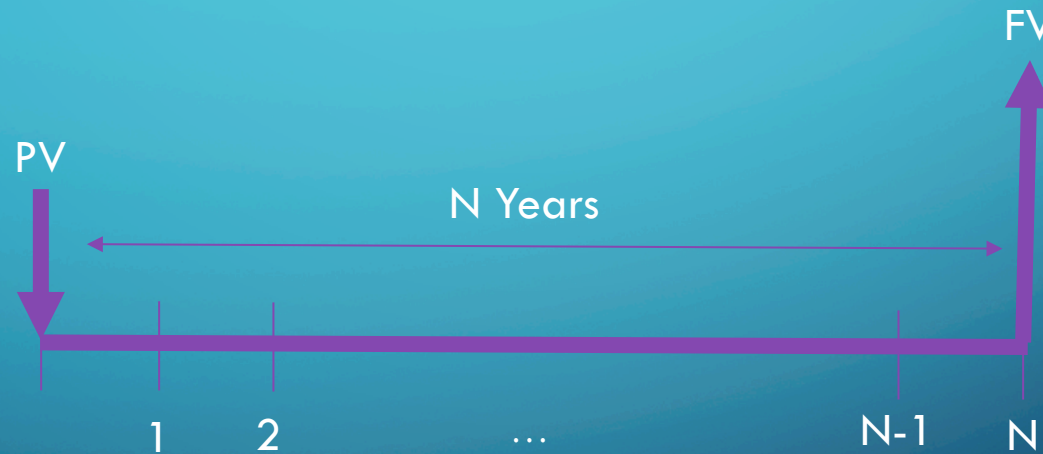
- Convention is revenue (benefit) is plotted upward, and expenses (costs) is plotted downward.
- All cash flows during a year are usually lumped into sums occurring at the end of the year

# DISCOUNTING FACTORS

- Discounting factor is a tool to convert a value at one date to an economically equivalent value at another date
- All discounting problems can be expressed as combinations of two fundamental factors
  - Single-payment Compound-amount Factor
  - Single-payment Present-worth Factor

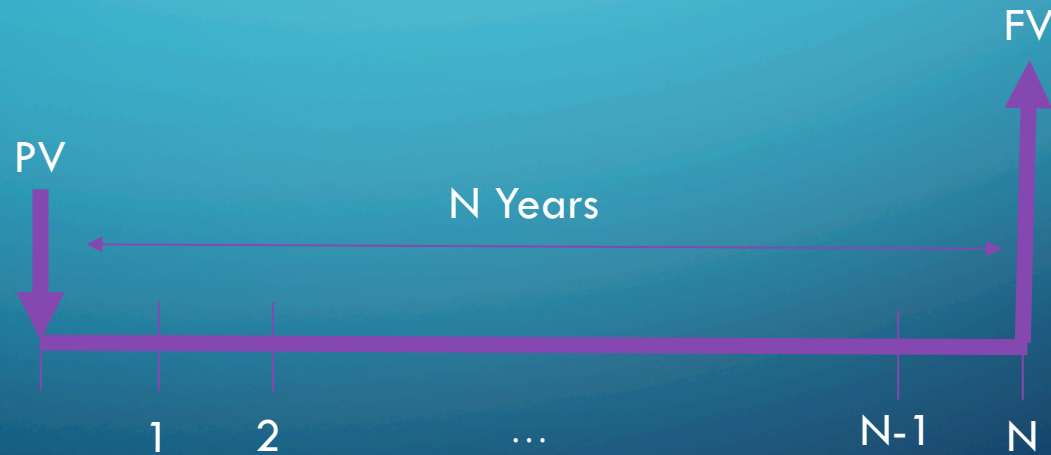
# CONVERT PRESENT VALUE TO FUTURE VALUE

- The factor computes the number of monetary units (FV) that will accumulate in  $N$  years for every initial unit (PV) invested at a rate of return of  $i$ -percent.
- $FV = PV(1 + i)^N$



# CONVERT FUTURE VALUE TO PRESENT VALUE

- The factor computes the number of current monetary units (PV) that a future value (FV) is worth if the monetary units are invested at a rate of return of  $i$ -percent.
- $PV = FV(1 + i)^{-N}$

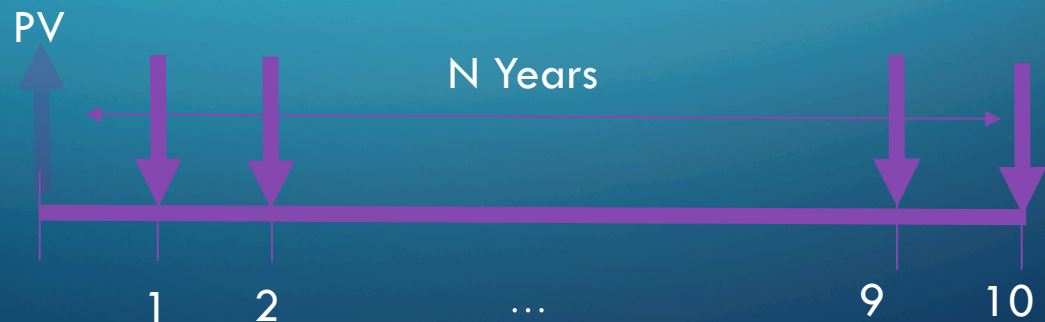


# OTHER FACTORS

- Notice the cash flow diagram looks the same – it is a tool to move values along the time axis. All other diagrams can be determined by combinations of these two concepts.
- Can either apply algebra to make the combinations, or just use simple computer programming (e.g. R)

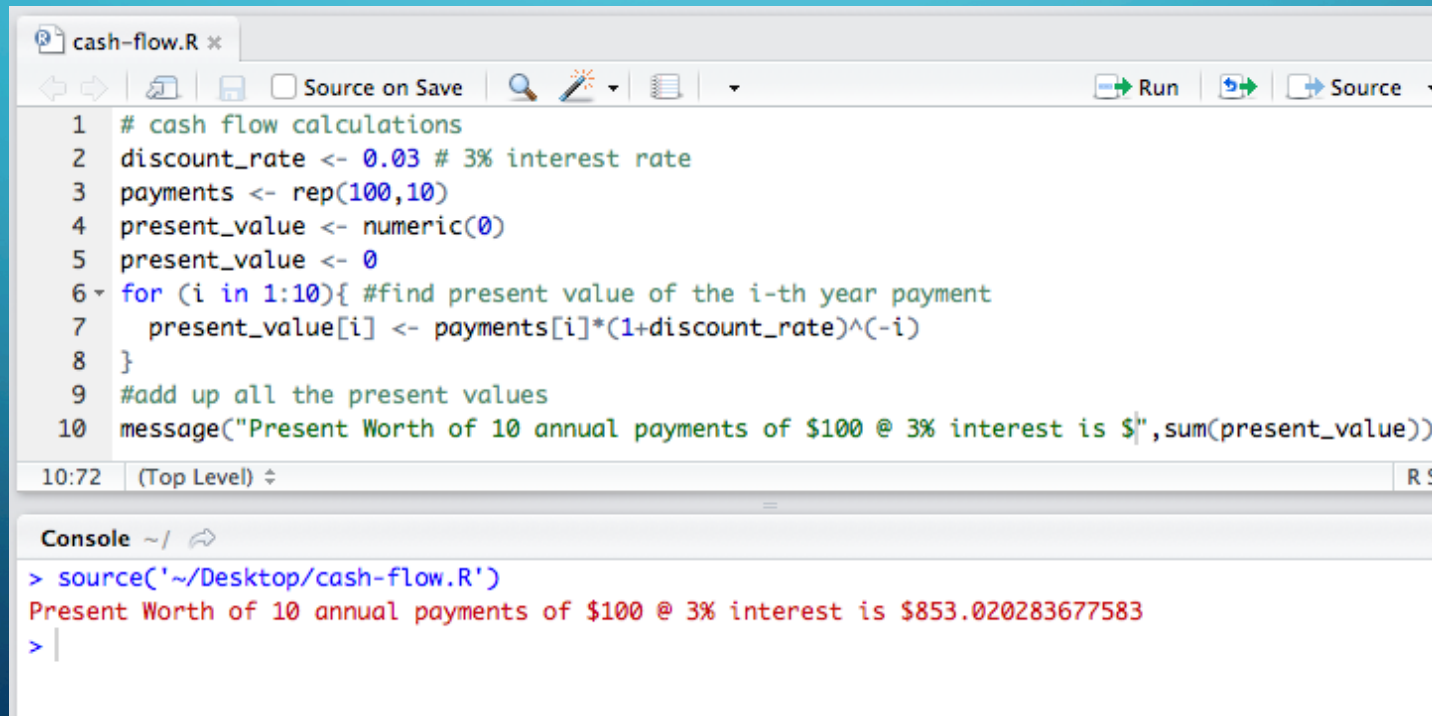
## EXAMPLE

- Determine the present worth (PV) of a stream of annual payments, each payment is \$100 for a period of 10 years. The discount rate is 3-percent per year.
- Sketch a cash flow diagram



# EXAMPLE

- Determine the present worth of each future payment (  $PV = FV(1 + i)^{-N}$  )
- Sum these present worth(s) to determine the present worth of the entire series of payments



```
cash-flow.R
1 # cash flow calculations
2 discount_rate <- 0.03 # 3% interest rate
3 payments <- rep(100,10)
4 present_value <- numeric(0)
5 present_value <- 0
6 for (i in 1:10){ #find present value of the i-th year payment
7   present_value[i] <- payments[i]*(1+discount_rate)^(-i)
8 }
9 #add up all the present values
10 message("Present Worth of 10 annual payments of $100 @ 3% interest is $",sum(present_value))

10:72 (Top Level)
R Sc

Console ~/
> source('~\\Desktop\\cash-flow.R')
Present Worth of 10 annual payments of $100 @ 3% interest is $853.020283677583
> |
```

# DISCOUNTING TECHNIQUES

- Refers to systematic application of discounting factors to compare alternatives
- The 4 accepted techniques are:
  - Present worth method
  - Rate-of-return method
  - Benefit-cost ratio method
  - Annual-cost method
- Each method produces the same evaluation of relative value

# PRESENT WORTH METHOD

- Move all present worths to the same time base
- Compute all present worths using the same discount rate
- Base all present worths on the same period of analysis
- Calculate the present worth of each alternative, retain only alternatives with positive present worth.
- Choose the alternative in a set of mutually exclusive alternatives having the greatest present worth
- If alternatives have intangibles that are approximately equal (policy), choose the alternative having the least cost.

# RATE OF RETURN METHOD (PART 1)

- Compare all alternatives over the same period of analysis
- Calculate the rate of return for each alternative. Retain alternatives with a rate of return exceeding the minimum acceptable value.
- If sets of mutually exclusive alternatives are involved, proceed to the next part.

## RATE OF RETURN METHOD (PART 2)

- Rank the alternatives in the set of mutually exclusive alternatives in order of increasing cost. Calculate the rate of return on the incremental cost and incremental benefits of the next alternative above the least costly alternative. Choose the more costly alternative if the incremental rate of return exceeds the minimum acceptable discount rate. Otherwise choose the less costly alternative. Continue the analysis by considering the alternatives in order of increased costliness, the alternative on the less costly side of each increment being the most costly project chosen this far.

# BENEFIT/COST RATIO METHOD (PART 1)

- Compute all benefit/cost ratios using the same discount rate
- Compare all alternatives over the same period of analysis
- Calculate the benefit/cost ratio for each alternative. Retain alternatives with ratios exceeding unity
- If sets of mutually exclusive alternatives are retained, proceed to next part

## BENEFIT/COST RATIO METHOD (PART 2)

- Rank the alternatives in the set of mutually exclusive alternatives in order of increasing cost. Calculate the benefit/cost ratio by using the incremental cost and incremental benefit of the next alternative above the least costly alternatives. Choose the more costly alternative if the incremental benefit/cost ratio exceeds unity. Otherwise choose the less costly alternative. Continue the analysis by considering the alternatives in order of increasing costliness, the alternative on the less costly side of each increment being the most costly project chosen thus far

# ANNUAL COST METHOD

- Compute all annual costs by using the same discount rate
- Base all annual costs on the same period of analysis
- Calculate the net annual benefit of each alternative. Retain alternatives with positive net annual benefit
- Choose the alternative in a set of mutually exclusive alternatives having the greatest net annual benefit.

# UNCERTAINTY

- Comparing alternatives usually has to look into the future – inherent uncertainty
  - Uncertain objectives
  - Uncertain constraints
  - Uncertain public response
  - Uncertain technological change
  - Uncertainty in recurring events (e.g. flooding magnitude and times)

# PLANNING HORIZON

- Design life (economic)
  - Typically 50 to 100 years
- Service life (physical life)
  - Variable – technology can change, need can change, the thing can break
- Compare things over the same period of analysis – using negative (salvage) cash flow if necessary to use comparable planning horizons

# ALTERNATIVE STRUCTURE (FOR COMPARISONS)

- Alternatives capable of achieving the design objective should be defined
- Identify consequences of each alternative and express in monetary units
- Comparisons should be on cost-to-go (to eliminate consideration of sunk costs)
- Intangibles should be identified – search policy to see if economic values are already assigned (or surrogates are available)
- Compare on uniform basis over common analysis periods