INTRODUCTION

- PUMPING TEST IS A TEST WHERE AN AQUIFER IS STRESSED (PUMPED) AND THE RESPONSE TO THE STRESS (DRAWDOWN) IS OBSERVED.
- OBSERVATIONS ARE INTERPRETED TO INFER CHARACTERISTICS ABOUT THE AQUIFER
- CHARACTERISTICS (FORMATION CONSTANTS) ARE USED TO DESIGN WATER SUPPLY WELLS, PREDICT RATES AND DIRECTIONS OF GROUNDWATER FLOW, AND DESIGN EFFECTIVE REMEDIATION SYSTEMS

TYPES OF TESTS

- SPECIFIC CAPACITY
- PUMPING TEST
- SLUG TEST

SPECIFIC CAPACITY

- USUALLY AVAILABLE FOR EXISTING PRODUCTION WELLS
- SPECIFIC CAPACITY = $\frac{\text{PUMPING RATE}}{\text{DRAWDOWN}}$
- SPECIFIC CAPACITY IS STRONGLY CORRELATED WITH TRANSMISSIVITY
- USUALLY INTERPRETED USING THEIS SOLUTION

$$s(r,t) = \frac{Q_w}{4\pi T} Ei\left(\frac{r^2 S}{4Tt}\right)$$

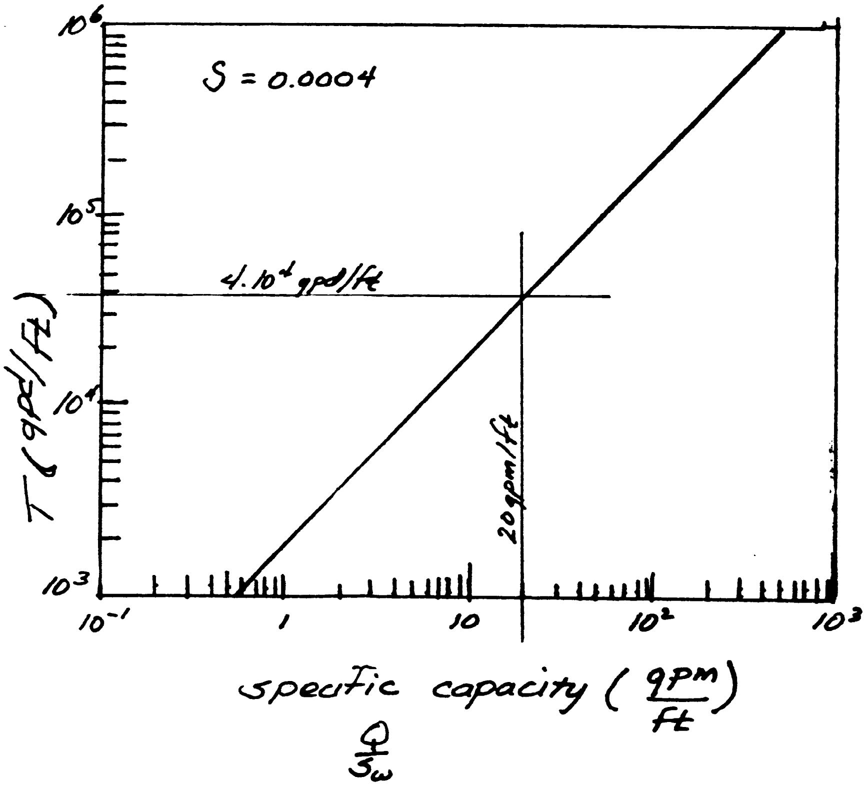
- INTERPRETING SPECIFIC CAPACITY DATA (GRAPHICAL)
  - -- LENGTH OF TEST (1 HR OR 1 DAY)
  - -- VARIABLE PUMP RATE
    - (a) USE AVERAGE RATE
    - (b) TIME CONVOLUTION
  - -- REARRANGE THEIS SOLUTION

$$\frac{Q}{S_w} = \frac{4\pi T}{Ei\left(\frac{r_w^2 S}{4Tt}\right)}$$

  - -- USE A STORAGE COEFFICIENT APPLICABLE TO TEST SITE. - OR CHOOSE REASONABLE RANGES
  - -- PREPARE SET OF GRAPHS OF $\frac{Q}{S_w}$ VS. T FOR DIFFERENT VALUES OF S.
  - -- READ T FROM GRAPH.

EXAMPLE: SUPPOSE $S = 0.0004$, $\frac{Q}{S_w} = 20.67 \frac{gpm}{ft}$, $t = 60\,min$

$$T \approx 4.0 \cdot 10^{-4}\,gpd/ft \quad \text{(FIGURE 1)}$$



LOGARITHMIC PLOT OF TRANSMISSIVITY VS SPECIFIC CAPACITY FOR 1-HOUR TEST

PUMPING TESTS

CONFINED AQUIFERS

* THEIS METHOD
* COOPER-JACOB METHOD

THEIS RECOVERY
PAPADOPULOS-COOPER
COOPER-BREDEHOEFT-PAPADOPULOS (SLUG)

UNCONFINED AQUIFERS

THEIS METHOD
COOPER-JACOB METHOD
NEUMAN METHOD
* BOUWER-RICE METHOD (SLUG)

LEAKY AQUIFERS

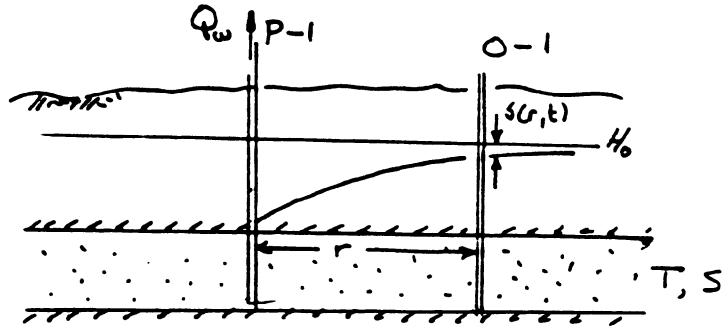
* HANTUSH

MOENCH

FRACTURED AQUIFERS

MOENCH

TYPE CURVE METHOD



DATA: r, t, $s(r,t)$

USUALLY WILL HAVE A COUPLE OF OBSERVATION WELLS.

DATA ARE REPORTED AS:

WELL: O-1

TIME	DRAWDOWN
15 sec	0.0
30 sec	0.0
⋮	⋮
2 min	1.0 ft
5 min	2.3 ft
⋮	⋮

STEP ① REDUCE DATA:

COMPUTE: $\frac{r^2}{t}$, s IF USING A u vs. $W(u)$ TYPE CURVE

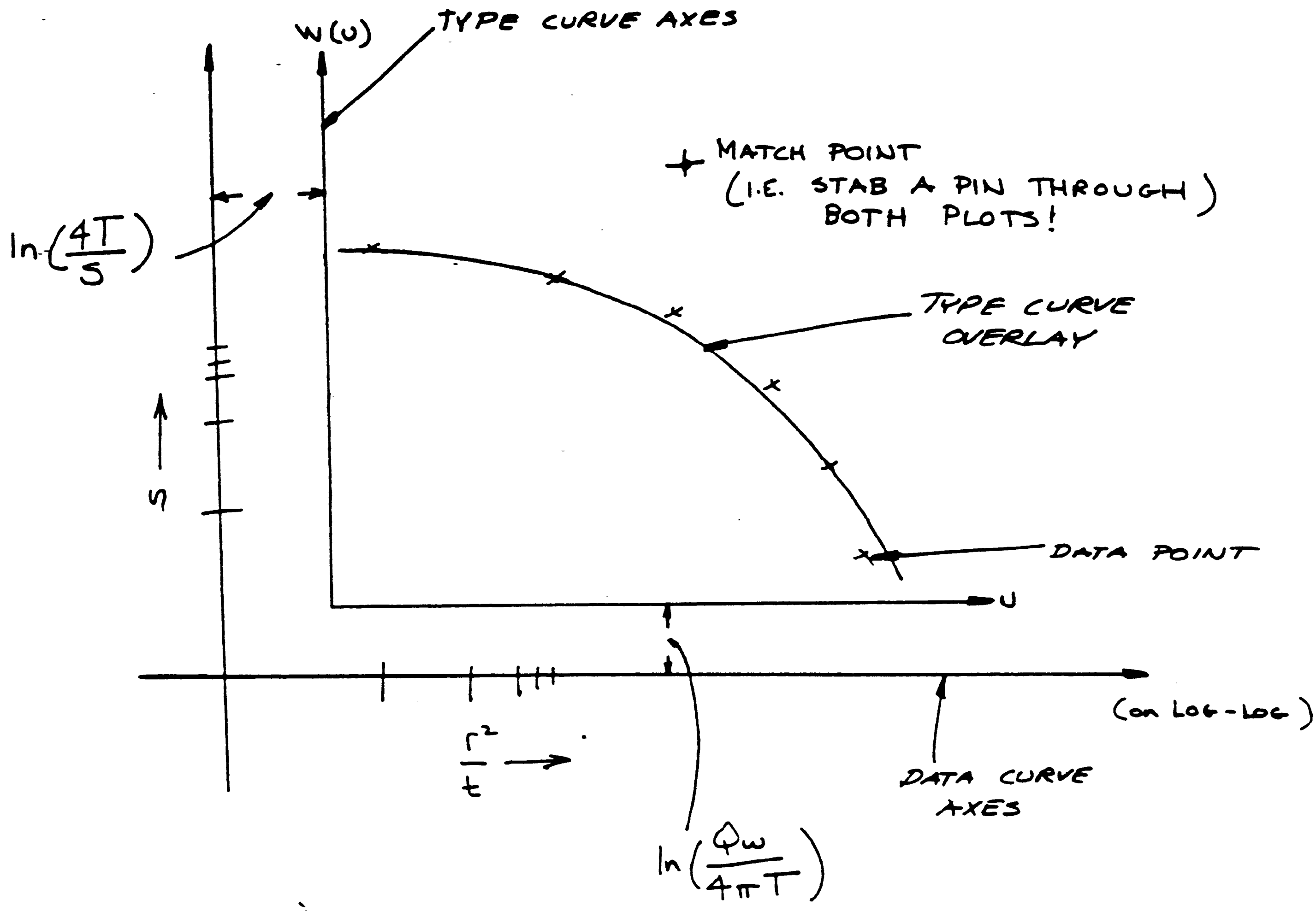
OR $\frac{t}{r^2}$, s IF USING A $\frac{1}{u}$ vs. $W(u)$ TYPE CURVE.

(OR DO BOTH, THEN YOU ARE COVERED!)

STEP ② PLOT ON LOG-LOG PAPER THAT HAS SAME SCALES AS TYPE CURVE

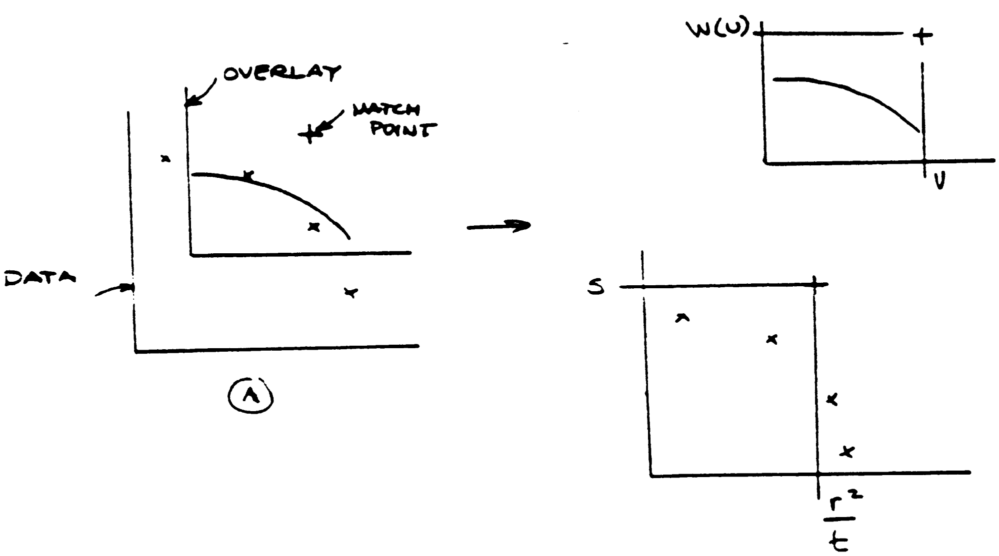
STEP ③ OVERLAY TYPE CURVE

STEP ④ CHOOSE A CONVIENENT MATCH POINT, READ s, u FROM TYPE CURVE.



AT MATCH POINT (HOLE IN <u>BOTH</u> PLOTS)

READ S FROM DATA AXES, U FROM TYPE CURVE (AND W(U) TOO!).



STEP (5) COMPUTE T FROM:

$$s = \frac{Q_w}{4\pi T} W(u) \longrightarrow T = \frac{Q_w W(u)}{4\pi s}$$

STEP (6) COMPUTE S FROM:

$$u = \frac{r^2 S}{4Tt} \longrightarrow S = \frac{u\,4Tt}{r^2}$$

HOW THIS METHOD WORKS:

ASSUME $s(r,t) = \frac{Q_w}{4\pi T} W\left(\frac{r^2 S}{4Tt}\right)$ IS APPROPRIATE MODEL.

OBSERVE:

$$u = \frac{r^2 S}{4Tt} \longrightarrow \frac{r^2}{t} = \frac{4Tu}{S}$$

$$\ln(s) = \ln\left[\frac{Q_w}{4\pi T} W(u)\right] = \underbrace{\ln\left(\frac{Q_w}{4\pi T}\right)}_{\text{some constant}} + \ln W(u)$$

$$\ln\left(\frac{r^2}{t}\right) = \ln \frac{4Tu}{S} = \underbrace{\ln\left(\frac{4T}{S}\right)}_{\text{some constant}} + \ln(u)$$

$$\Rightarrow \quad \ln(s) \propto \ln W(u)$$

and

$$\ln\left(\frac{r^2}{t}\right) \propto \ln(u)$$

SO THE TYPE CURVE, AND THE DATA CURVE ARE THE SAME EXCEPT FOR A PARALLEL SHIFT IN AXES!! THE SHIFT IS PROPORTIONAL TO T AND S.

IF YOU KNOW THE SHIFT — YOU KNOW T & S.

EXAMPLE

A WELL IN A CONFINED AQUIFER IS PUMPED AT 220 gpm FOR 8 HOURS. AN OBSERVATION WELL 824 FT. AWAY WAS MONITORED. THE TABLE BELOW SHOWS THE DATA.

DETERMINE T & S FOR THE AQUIFER USING THE THEIS SOLUTION:

STEP ①

Time After Pumping Started (min)	t/r^2	Drawdown (ft)
3	4.46×10^{-6}	0.3 ✓
5	7.46×10^{-6}	0.7 ✓
8	1.18×10^{-5}	1.3 ✓
12	1.77×10^{-5}	2.1 ✓
20	2.95×10^{-5}	3.2 ✓
24	3.53×10^{-5}	3.6 ✓
30	4.42×10^{-5}	4.1 ✓
38	5.57×10^{-5}	4.7 ✓
47	6.94×10^{-5}	5.1 ✓
50	7.41×10^{-5}	5.3 ✓
60	8.85×10^{-5}	5.7 ✓
70	1.03×10^{-4}	6.1 ✓
80	1.18×10^{-4}	6.3 ✓
90	1.33×10^{-4}	6.7 ✓
100	1.47×10^{-4}	7.0 ✓
130	1.92×10^{-4}	7.5 ✓
160	2.36×10^{-4}	8.3 ✓
200	2.95×10^{-4}	8.5 ✓
260	3.83×10^{-4}	9.2 ✓
320	4.72×10^{-4}	9.7 ✓
380	5.62×10^{-4}	10.2 ✓
500	7.35×10^{-4}	10.9 ✓

STEP (2)

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GEOLOGICAL SURVEY

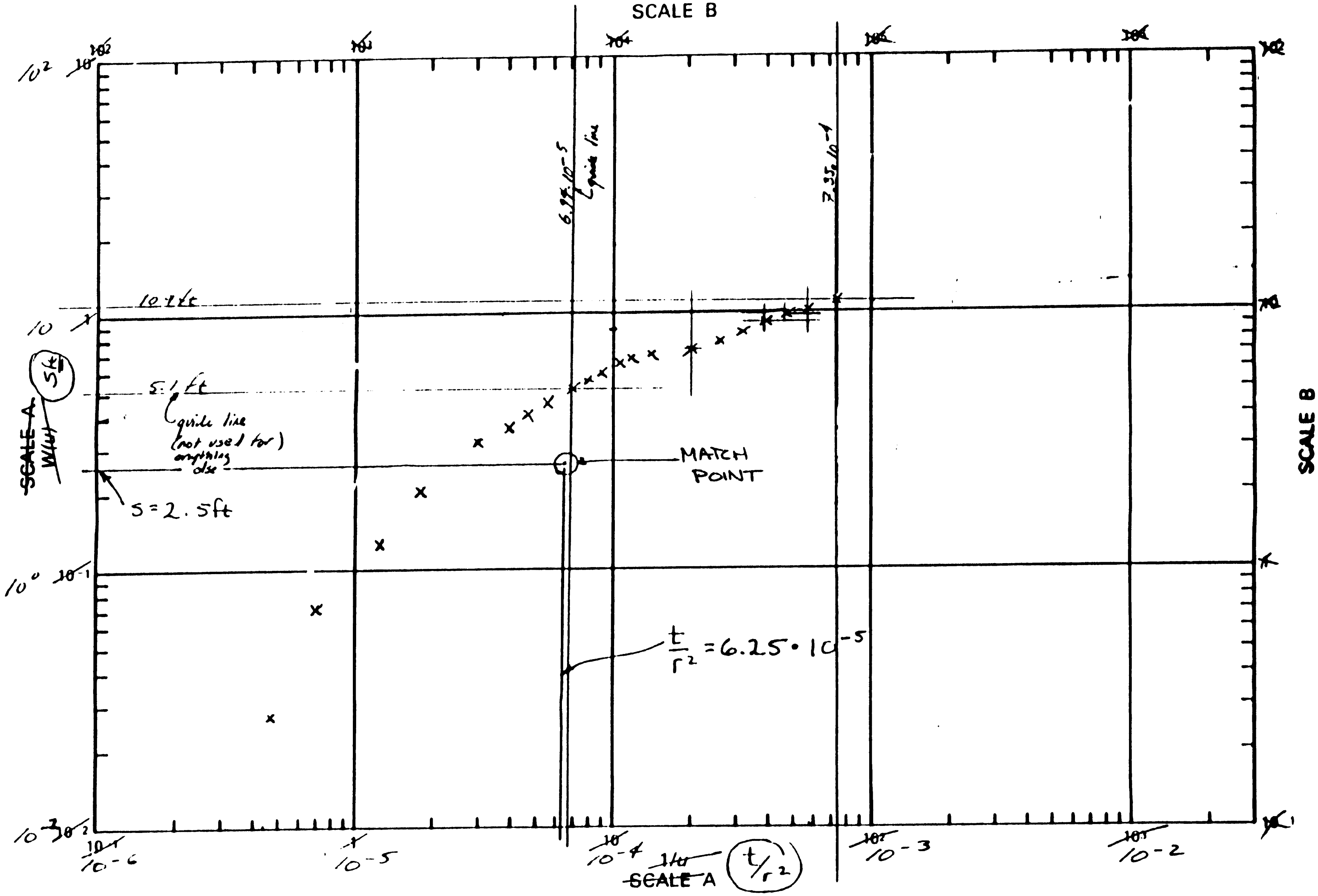

FIGURE 1.2 Type curve of dimensionless drawdown (W(u)) versus dimensionless time (1/u) for constant discharge from an artesian

STEP ③ & ④

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GEOLOGICAL SURVEY

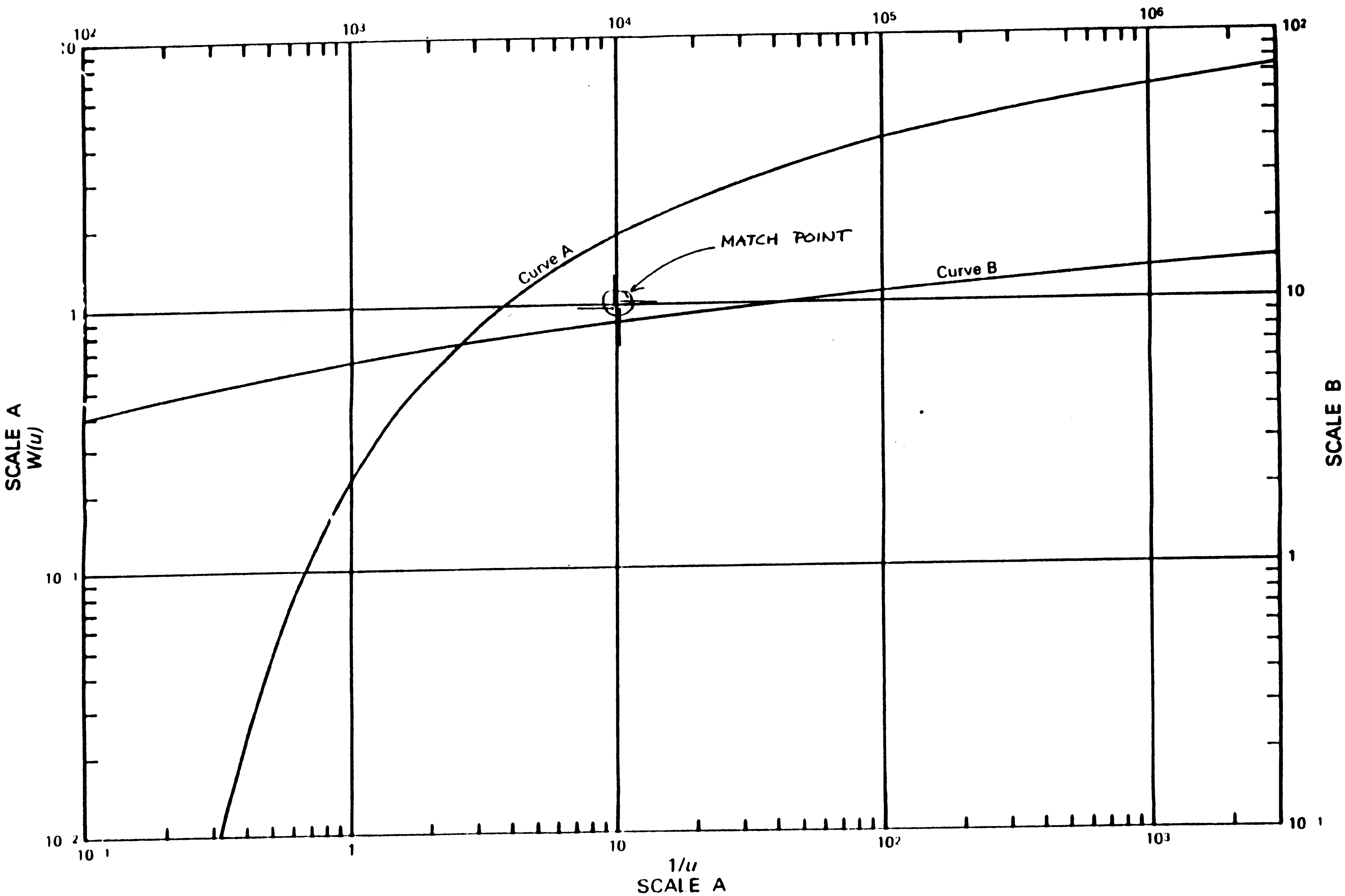

FIGURE 1.2 Type curve of dimensionless drawdown ($W(u)$) versus dimensionless time ($1/u$) for constant discharge from an artesian

STEP (5) MATCH POINT:

$\frac{1}{U} = 10$; $W(U) = 1$ (USED CURVE A)

$\frac{t}{r^2} = 6.25 \cdot 10^{-5}$; $s = 2.5 \text{ ft}$

$$T = \frac{220 \text{ gpm} \cdot 1}{4\pi (2.5 \text{ ft})} = 7.0028 \text{ gpm/ft}$$

$$\frac{7.0028 \text{ gal}}{\text{min} \cdot \text{ft}} \cdot \frac{\text{ft}^3}{7.48 \text{ gal}} \cdot \frac{60 \text{ min}}{\text{hr}} \cdot \frac{24 \text{ hr}}{\text{day}} = 1348 \text{ ft}^2/\text{day}$$

STEP (6)

$$S = \frac{u 4 T t}{r^2}$$

$$= \frac{1}{10} \cdot 4 \cdot (0.93611 \text{ ft}^2/\text{min}) \left(6.25 \cdot 10^{-5} \frac{\text{min}}{\text{ft}^2}\right)$$

$$= 0.000023$$

∴ DONE!!

TYPICALLY YOU WOULD REPORT:

$T = 1350 \text{ ft}^2/\text{day}$
$S = 2.3 \cdot 10^{-5}$

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GEOLOGICAL SURVEY

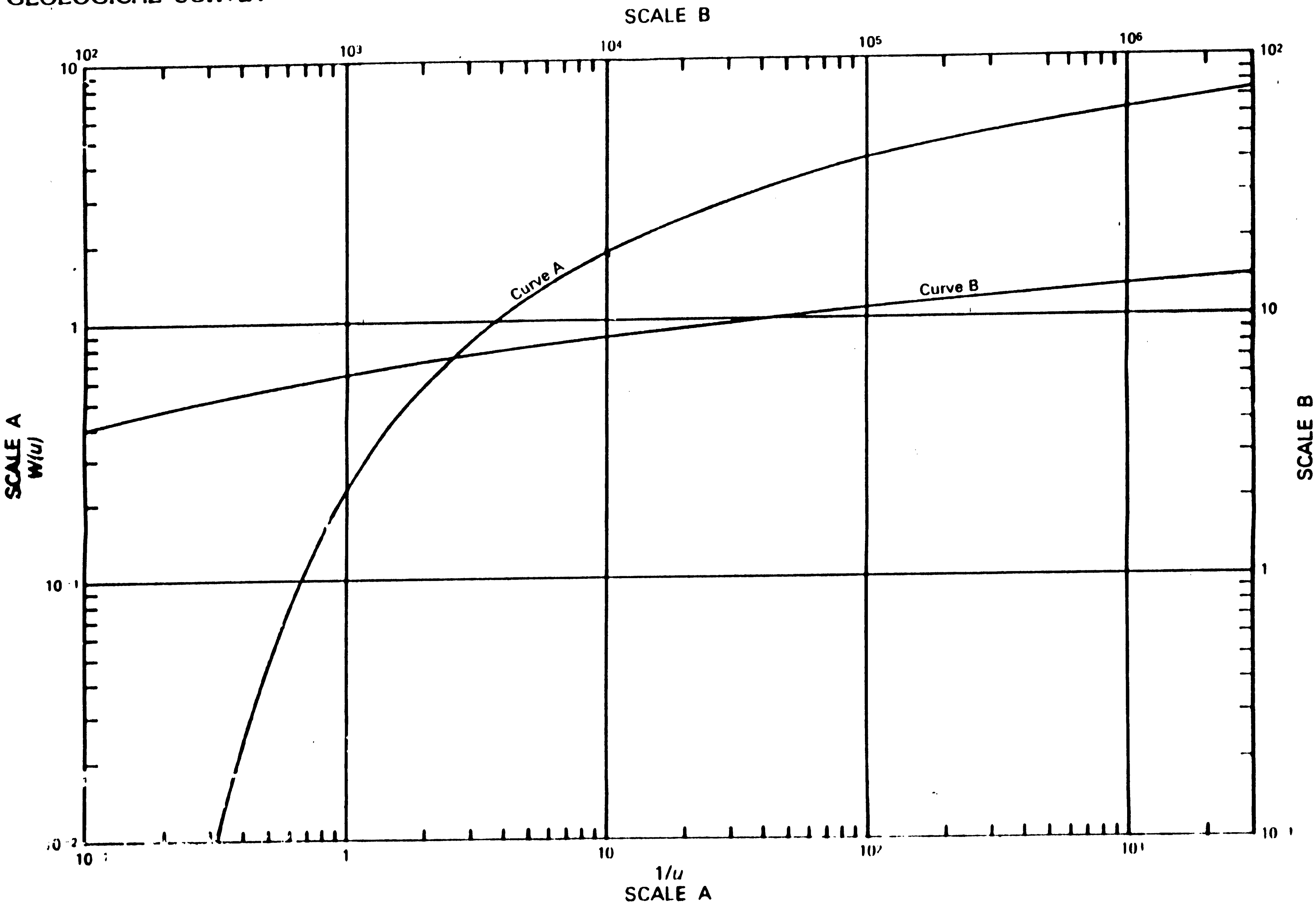


FIGURE 1 2   Type curve of dimensionless drawdown ($W(u)$) versus dimensionless time ($1/u$) for constant discharge from an artesian

COOPER-JACOB'S METHOD FOR INTERPRETING PUMPING TEST DATA

ASSUMPTIONS

SAME AS FOR THEIS SOLUTION, PLUS EITHER r IS SMALL OR TIME t IS LARGE SO THAT

$Ei(u)$ CAN BE APPROXIMATED AS

$$Ei(u) \approx -0.577216 - \ln(u).$$

USING THIS APPROXIMATION, THE DRAWDOWN s IS GIVEN BY

$$s = \frac{Q}{4\pi T} \ln\left(\frac{4Tt}{1.78 r^2 S} \right)$$

$$= \frac{Q}{4\pi T} \ln\left(\frac{2.25 Tt}{r^2 S} \right)$$

WHEN THIS APPROXIMATION IS JUSTIFIIED,

$s \propto \ln(t)$; $s \propto \ln(t/r^2)$; $s \propto \ln(r)$

FURTHERMORE PLOTS OF s vs. $\ln(t)$

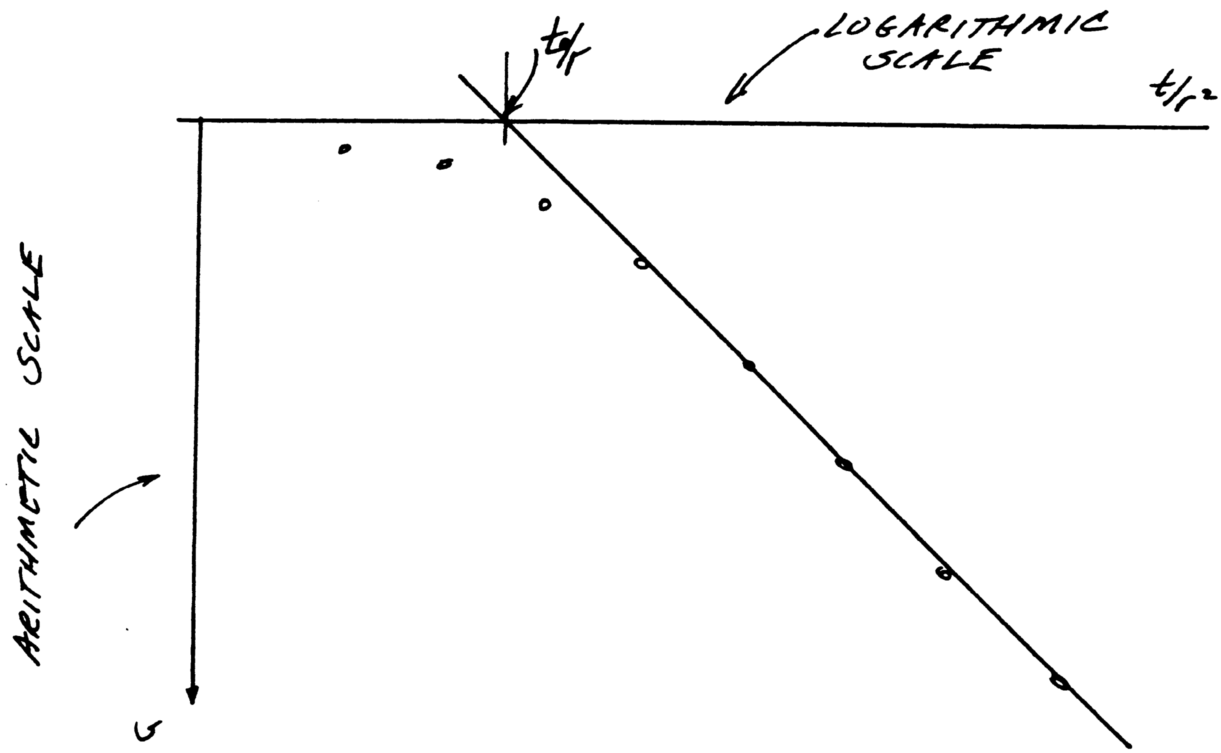
s vs. $\ln\left(\frac{t}{r^2}\right)$

s vs. $\ln(r)$

TO USE JACOB'S METHOD

① REDUCE DATA TO s vs t/r^2
(OR OTHER DIMENSIONLESS TIME/DISTANCE)
(SEE MARSILY pg. 168 or FETTER pg. 170)

② PLOT DRAWDOWN (USUALLY ON A ⊕DOWN AXIS)
VS. DIMENSIONLESS TIME



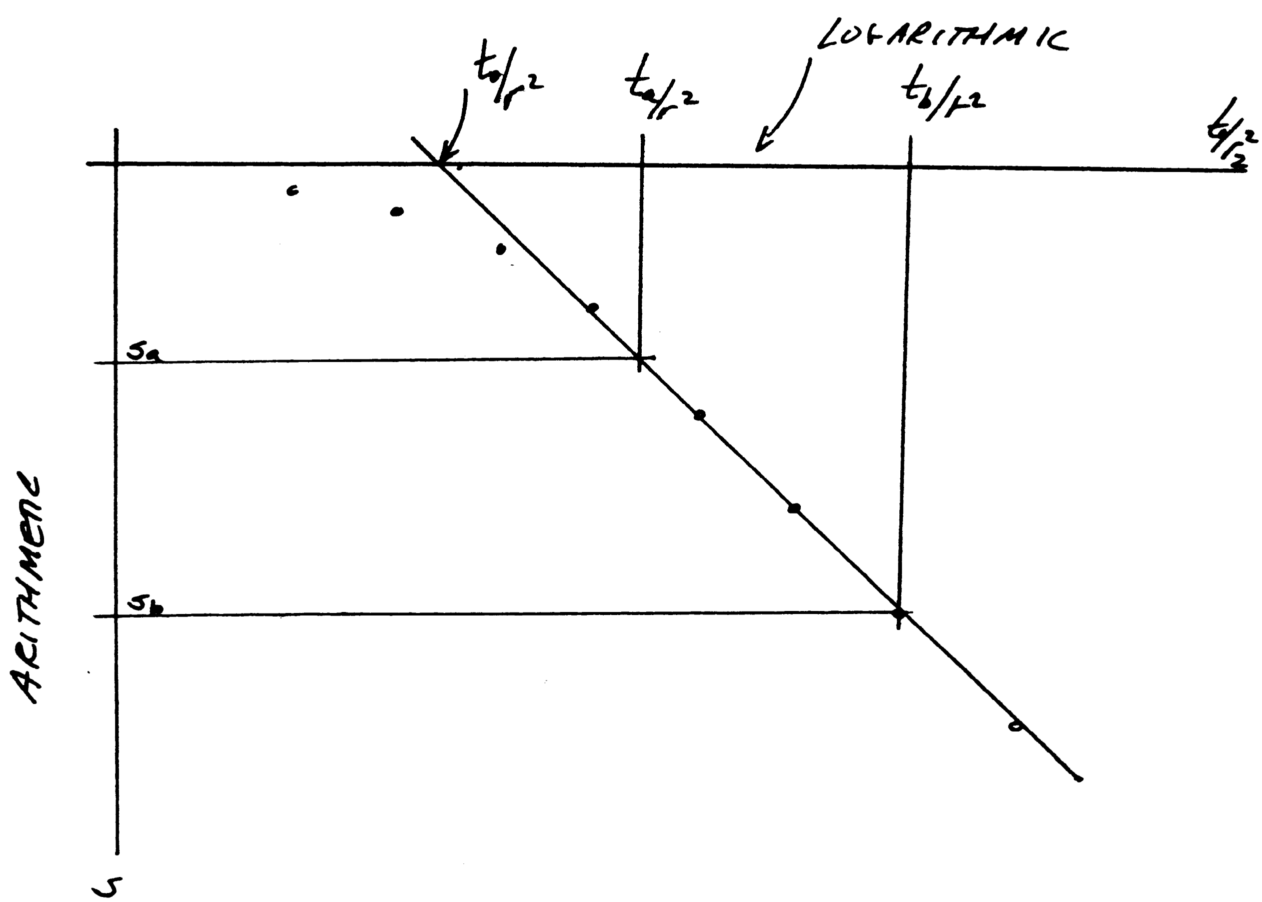
③ USING STRAIGHT LINE PORTION OF DATA, LOCATE t_0 $(t/r^2)_0$ WHERE STRAIGHT PORTION INTERCEPTS AXIS

④ SLOPE OF LINE IS PROPORTIONAL TO TRANSMISSIVITY

③ STORATIVITY IS PROPORTIONAL TO t_0

$$T = \frac{Q}{4\pi(s_b - s_a)} \ln\left(\frac{t_b}{t_a}\right)$$

$$S = \frac{2.25\, T\, t_0}{r^2}$$



PUMPING TESTS FOR LEAKY CONFINED AQUIFER WITH NEGLIGIBLE STORAGE IN CONFINING LAYER

HANTUSH SOLUTION; WALTON'S METHOD

① FIELD DATA ARE PLOTTED AS $\frac{t}{r^2}$ VS. S.

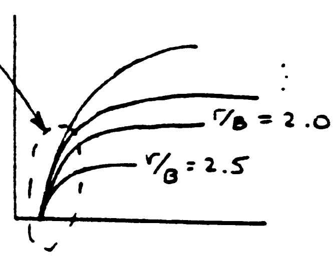
② OVERLAY A $W(u, r/B)$ VS $\frac{1}{u}$ TYPE CURVE (WALTON'S CURVE)

OR A $L(u,v)$ VS $\frac{1}{u}$ (LOHMAN'S CURVE)

③ MATCH DATA TO ONE OF THE r/B CURVES ON THE TYPE CURVE OVERLAY

- EARLY DATA WILL TEND TO FOLLOW NON-EQUILIBRIUM PORTION

  AS LEAKAGE STARTS TO CONTRIBUTE TO FLOW TO WELL,



  THE DRAWDOWN WILL FOLLOW AN r/B CURVE

④ SELECT A CONVIENENT MATCH POINT

- $r/B = 0 \triangleq$ THEIS SOLUTION

(5) READ $W(u, r/B)$, $\frac{1}{u}$, $\frac{t}{r^2}$, s FROM MATCH POINT.

(6) READ r/B FROM TYPE CURVE

(7) FIND FORMATION CONSTANTS FROM

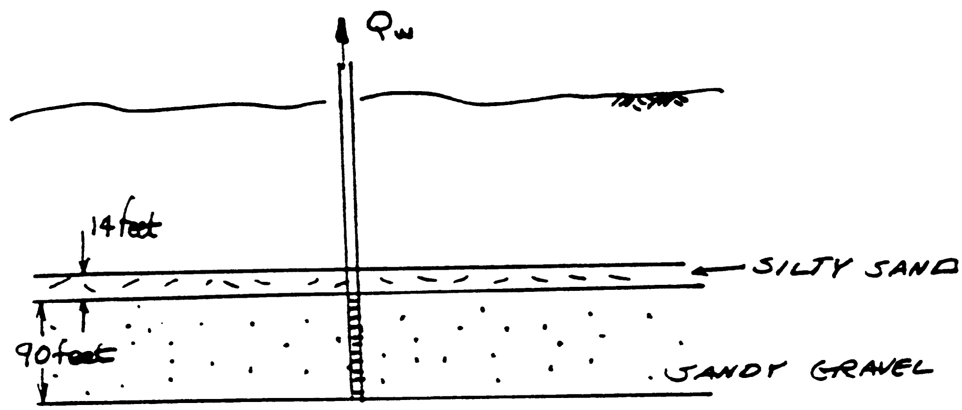
(a) $$T = \frac{Q_w}{4\pi (s)} W(u, r/B)$$

(b) $$S = \frac{u 4 T t}{r^2}$$

(c) $$B = \left(\frac{r}{B}\right)^{-1} r$$

(d) $$K' = \frac{T b' (r/B)^2}{r^2}$$

EXAMPLE (WALTON'S CURVE)



WELL CONFINED BY 14' THICK SILTY FINE SAND IS PUMPED AT 25gpm. DRAWDOWN IS OBSERVED IN ANOTHER WELL 96' AWAY. DETERMINE THE AQUIFER TRANSMISSIVITY, STORAGE COEFFICIENT, AND VERTICAL HYDRAULIC CONDUCTIVITY OF THE AQUITARD.

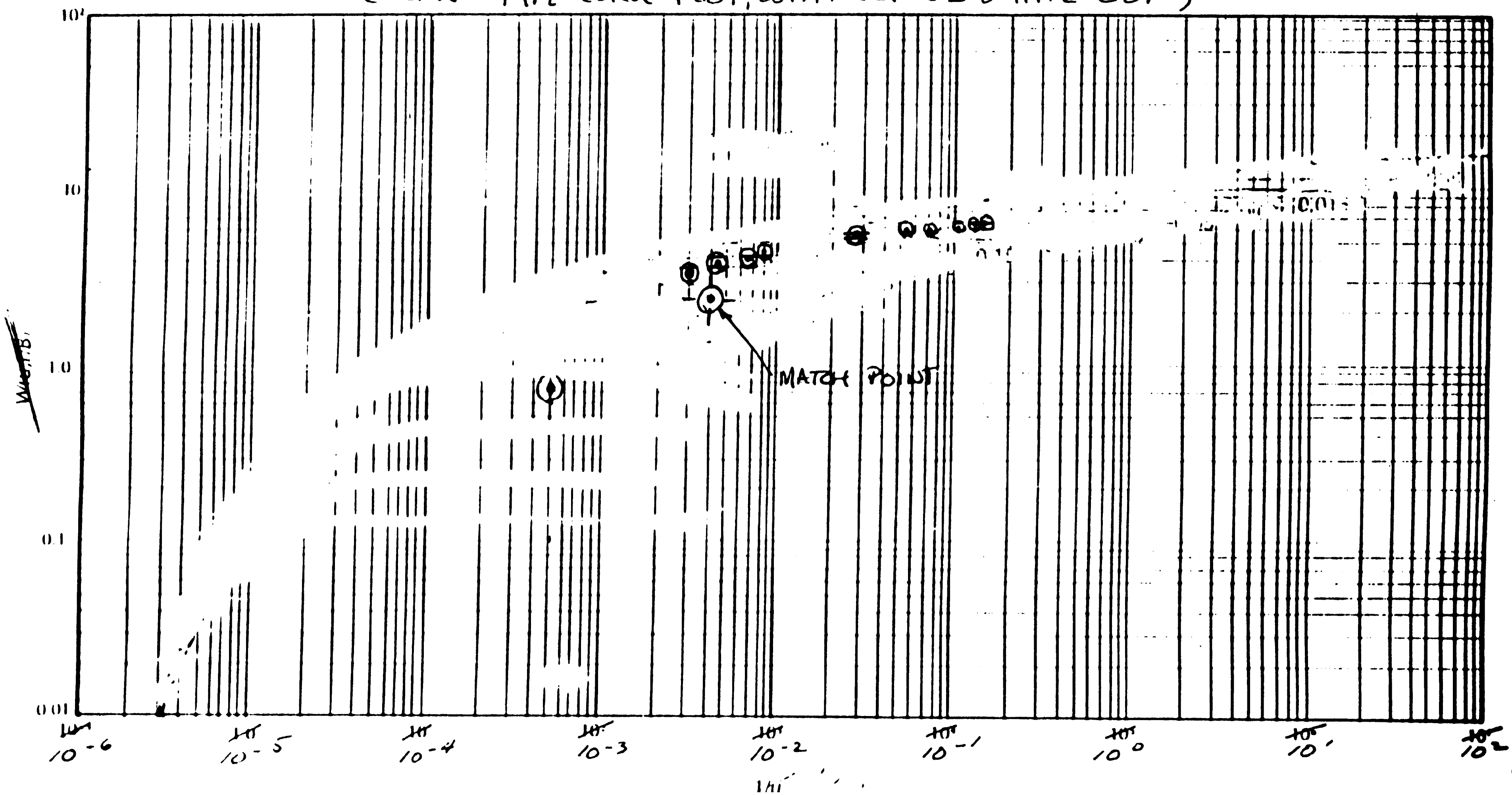
(USE WALTON'S CURVE)

DATA		REDUCE DATA →
TIME	DRAWDOWN	t/r^2
5 (min.)	0.76 (ft.)	$5.425 \cdot 10^{-4}$
28	3.30	$3.038 \cdot 10^{-3}$
41	3.59	$4.449 \cdot 10^{-3}$
60	4.08	$6.510 \cdot 10^{-3}$
75	4.39	$8.138 \cdot 10^{-3}$
244	5.47	$2.648 \cdot 10^{-2}$
493	5.96	$5.349 \cdot 10^{-2}$
669	6.11	$7.259 \cdot 10^{-2}$
958	6.27	$1.039 \cdot 10^{-1}$
1129	6.40	$1.225 \cdot 10^{-1}$
1185	6.42	$1.286 \cdot 10^{-1}$

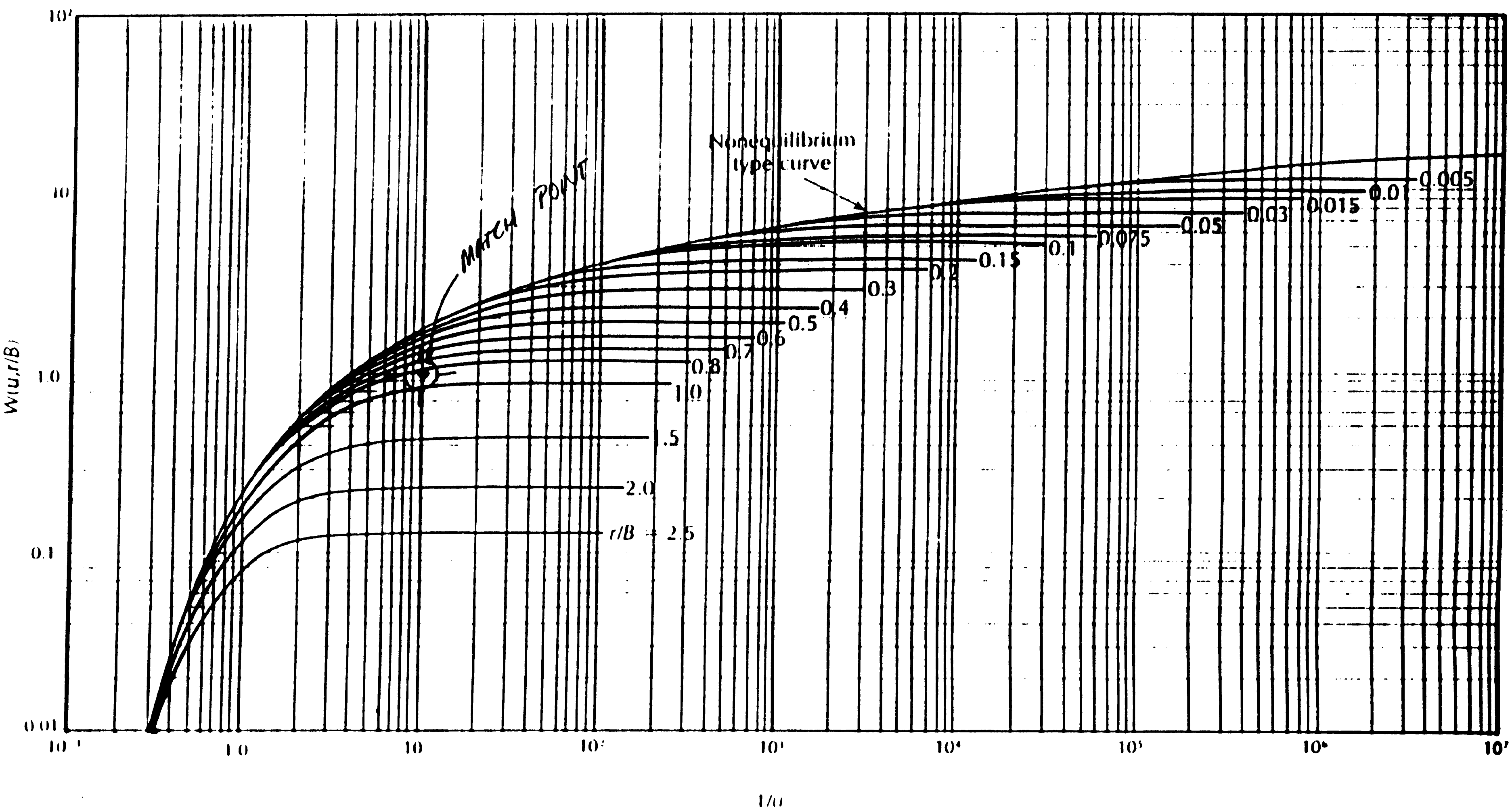
1. PLOT $\frac{t}{r^2}$ vs s ON LOG-LOG PAPER
2. OVERLAY TYPE CURVE
3. MATCH DATA
4. SELECT MATCH POINT
5. $W(u, r/B)$, $\frac{1}{u}$, $\frac{t}{r^2}$, s FROM MATCH POINT; r/B FROM CURVE
6. FORMATION CONSTANTS

DATA PLOT

(USING TYPE CURVE PLOT, WITH CURVES WHITE-OUT)



WALTON'S CURVE



FROM MATCH POINT

$W(u) = 1.0 \qquad \frac{1}{u} = 10 \qquad u = 0.10$

$\frac{t}{r^2} = 5.0 \cdot 10^{-4} \frac{min}{ft^2}, \qquad s = 0.72\ ft$

FROM TYPE CURVE

$\frac{r}{B} = 0.40$

FORMATION CONSTANTS

$$T = \frac{25gpm}{4\pi(0.72ft)} 1.0 = 2.763 gpm/ft = 3.694 \cdot 10^{-1} ft^2/min$$

$$S = (0.10)(4)(3.694 \cdot 10^{-1} ft^2/min)\left(5.0 \cdot 10^{-4} \frac{min}{ft^2}\right) = 7.388 \cdot 10^{-5}$$

$$K' = \frac{\left(3.694 \cdot 10^{-1} \frac{ft^2}{min}\right)(14ft)(0.40)^2}{(96ft)^2}$$

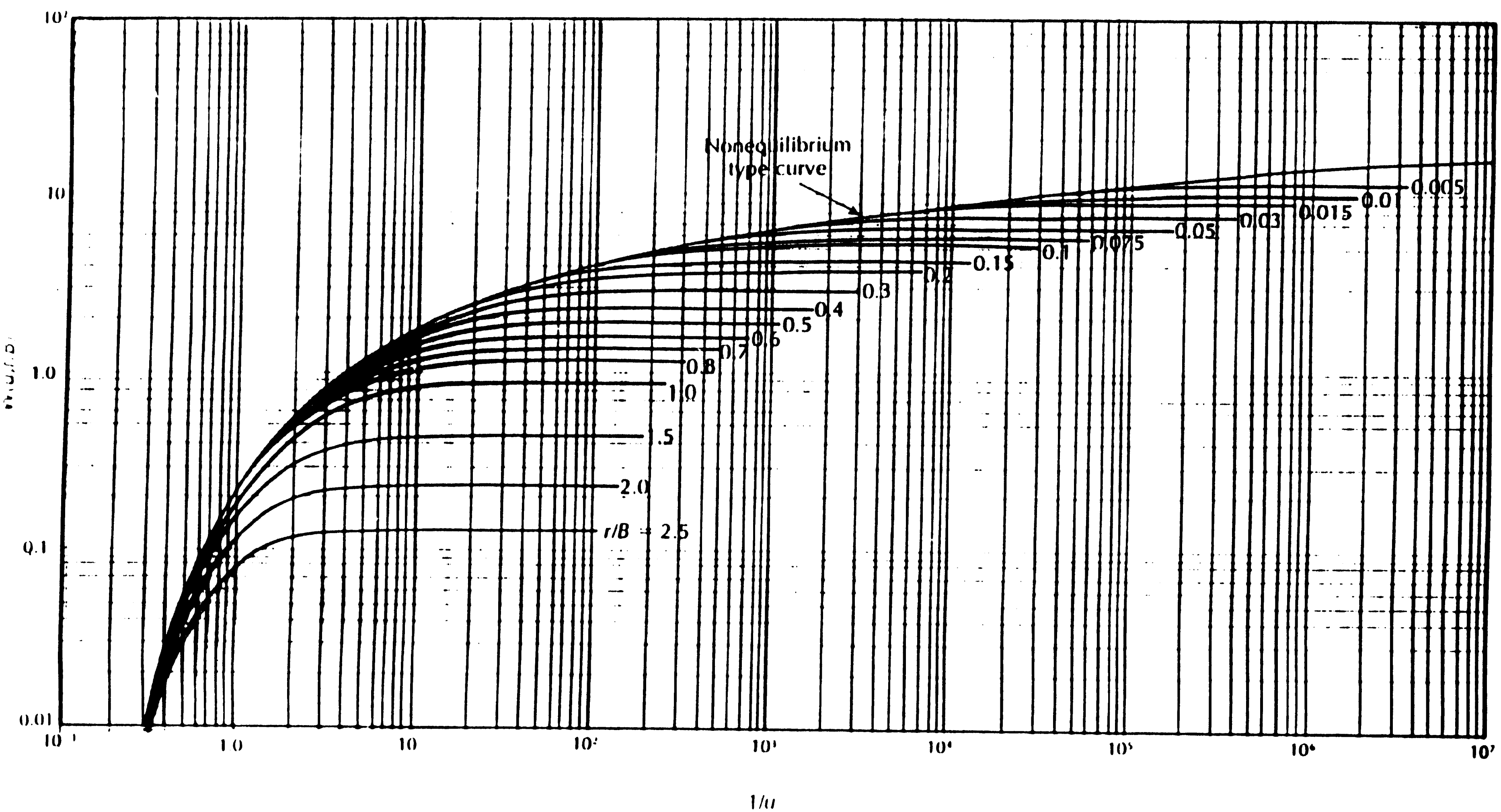
$$= 8.978 \cdot 10^{-5} ft/min$$

REPORT ~~RESULTS AS~~

$T = 570\ ft^2/day$

$S = 7.4 \cdot 10^{-5}$

$K' = 1.3 \cdot 10^{-1} ft^2/day$



Nonequilibrium type curve
0.005
0.01
0.015
0.03
0.05
0.075
0.1
0.15
0.2
0.3
0.4
0.5
0.6
0.7
0.8
1.0
1.5
2.0
r/B = 2.5
1.0
10
0.1
0.01
1/u

Slug Tests

Slug tests involve the use of a single borehole for determining aquifer characteristics. A volume of water is suddenly added or removed and observations of recovery or drawdown are noted through time. Evaluation of the recovery curve and knowledge of borehole geometry allows one to estimate the hydraulic conductivity of the formation near the borehole.

Typical Procedure

A displacement rod (the slug) slightly smaller than the borehole is lowered into the borehole and the water level is allowed to come to equilibrium (Figures 1 and 2).

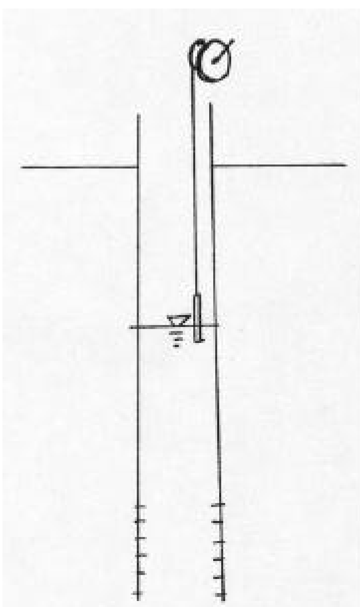


Figure 1. Measure equilibrium water level.

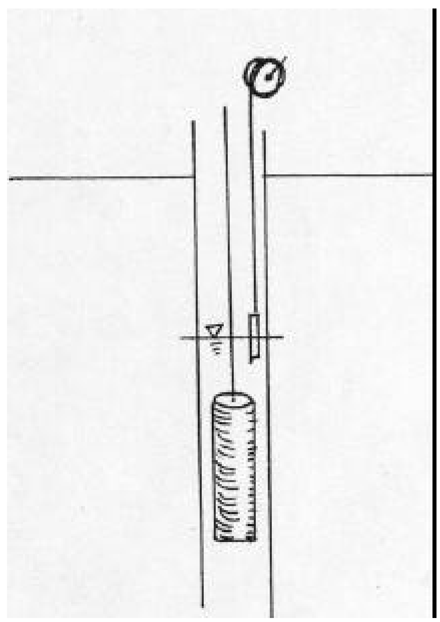


Figure 2. Lower slug allow water to come to equilibrium.

The rod is then quickly removed, its volume equivalent to removing the same volume of water from the borehole (Figure 3). Water level measurements are then collected (Figure 4) and analyzed to infer the aquifer characteristics.

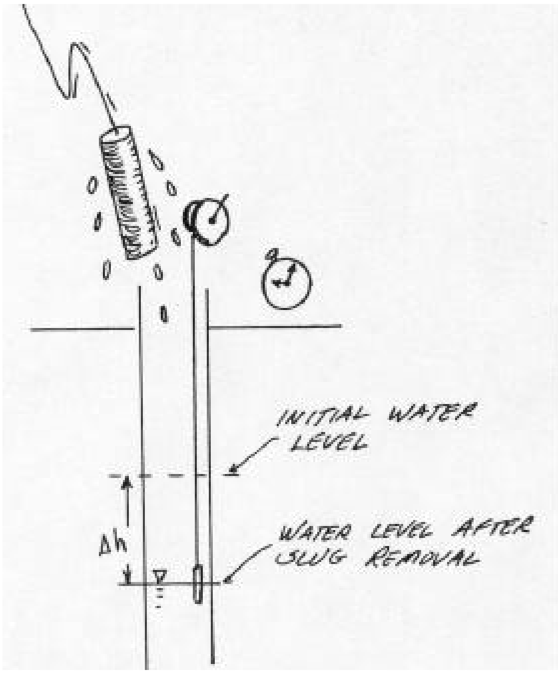


Figure 3. Rapidly pull slug from well, start timer, measure depth to water at frequent intervals.

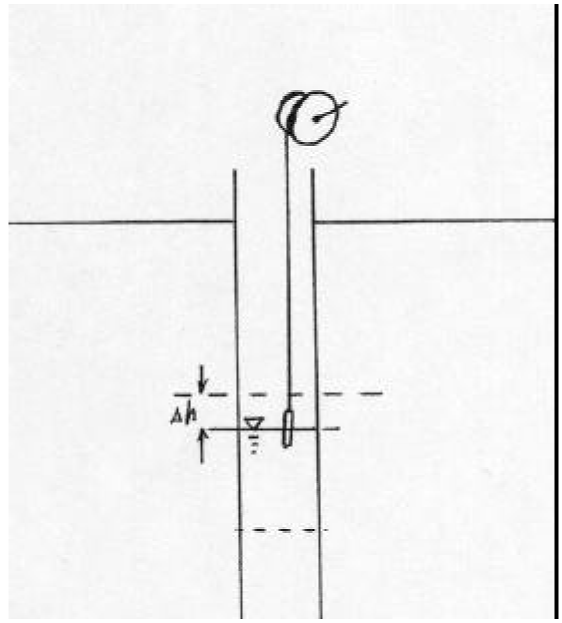


Figure 4. Continue to monitor until water level has returned at least 90% of distance back

Typical measuring intervals are every fifteen seconds for the first two minutes, then every thirty seconds from two to five minutes, and every minute afterwards until 90% recovery is observed.

Analysis

Slug tests are analyzed using a variety of conceptual models. One common model is the Hvorslev (1951) approach. Figure 5 is a diagram of the variables used in slug test data analysis. The assumptions used in this analysis are that the aquifer is bounded by aquicludes, Darcy's law is valid, the aquifer is horizontal, the aquifer is incompressible, flow is essentially horizontal, and there is negligible head loss through the well screen and filter pack.

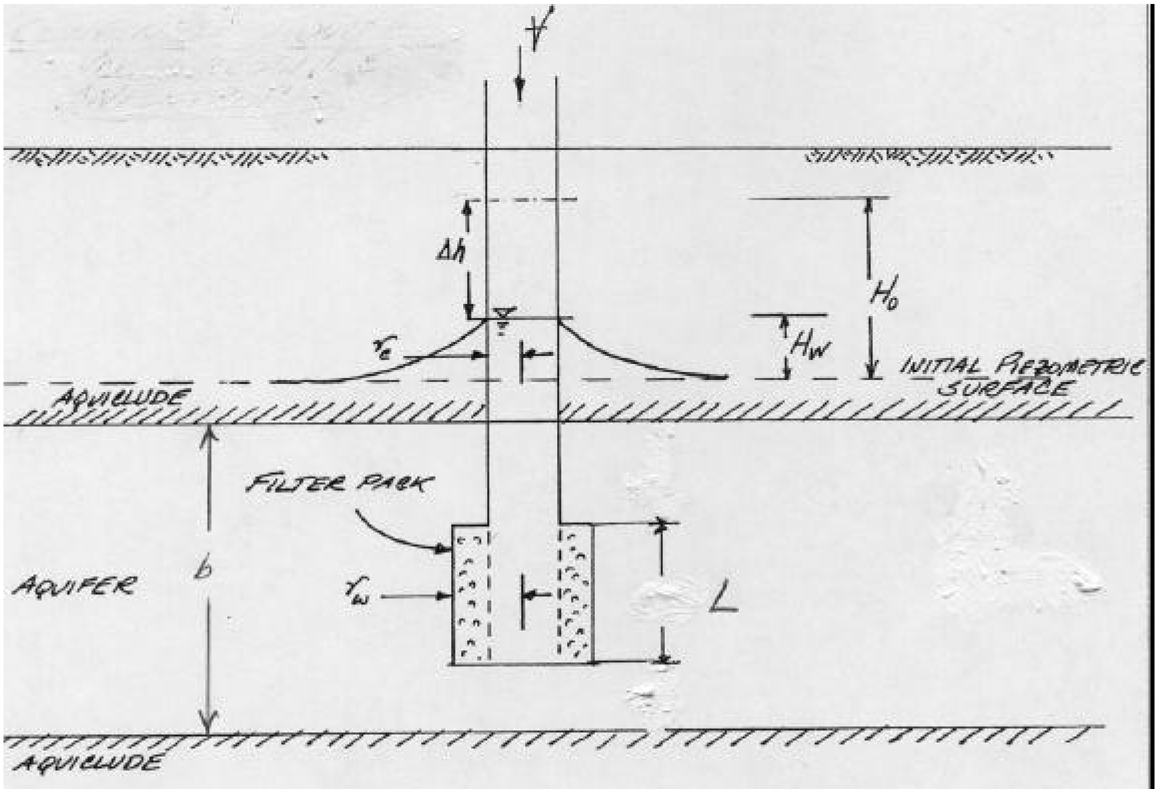


Figure 5. Sketch of relevant variables for slug tests.

The analysis assumed that a volume of water is added instantly. After the addition of this volume the flow rate of water into the formation is given by Darcy's law as

$$Q = KAH_w \tag{1}$$

where A is a flow area (shape factor) based on borehole and flow geometry. The flow rate out of the borehole is

$$Q = -pr_c^2 \frac{dH_w}{dt} \tag{2}$$

From continuity the flow into the aquifer must equal the flow out of the borehole and this relationship allows one to relate aquifer flow to borehole flow.

$$-pr_c^2 \frac{dH_w}{dt} = KAH_w \tag{3}$$

Integration of this equation will provide a formula to estimate the hydraulic conductivity.

Separating variables produces

$$\frac{dH}{H} = -\frac{KA}{pr_c^2}dt \tag{4}$$

Integration of this equation produces

$$\begin{aligned} \int \frac{dH}{H} &= -\int \frac{KA}{pr_c^2}dt \\ \ln|H| + C &= -\frac{KA}{pr_c^2}t \end{aligned} \tag{5}$$

The constant of integration is evaluated from the boundary conditions $(H,0)=H_o$

$$\ln|H| - \ln|H_o| = -\frac{KA}{pr_c^2}t \tag{6}$$

This last expression can be written as

$$\ln\left|\frac{H}{H_o}\right| = -\frac{KA}{pr_c^2}t \tag{7}$$

A plot of $\ln|\frac{H}{H_o}|$ versus t should be a straight line with slope $-\frac{KA}{pr_c^2}$. Thus from a plot of the data we can determine the slope and then the value of K. The slope is determined from the data as

$$\frac{-\ln|\frac{H_1}{H_2}|}{t_2 - t_1} = -\frac{KA}{pr_c^2} \tag{8}$$

And solving this equation for the hydraulic conductivity produces

$$\frac{pr_c^2 \ln|\frac{H_1}{H_2}|}{A(t_2 - t_1)} = K \tag{9}$$

Typical shape factors are chosen based on geometry of the test.

Cylindrical (very thin screen length, in middle of aquifer)

$$A = 2pr_w L; K = \frac{r_c^2 \ln|{}^{H_1}/_{H_2}|}{2Lr_w(t_2 - t_1)} \quad (10)$$

Elongated (Screen about 80% of entire thickness)

$$A = \frac{2pL}{\ln({}^{L}/_{r_w})}; K = \frac{r_c^2 \ln({}^{L}/_{r_w}) \ln|{}^{H_1}/_{H_2}|}{2L(t_2 - t_1)} \quad (11)$$

Various other shape factors are available from the original reference. Equation 11 is the same shape factor and solution used on pg. 249 in the textbook by Fetter.

References

Fetter, C.W. 1994 Applied Hydrogeology, Third Ed., Macmillan Pub. New York.

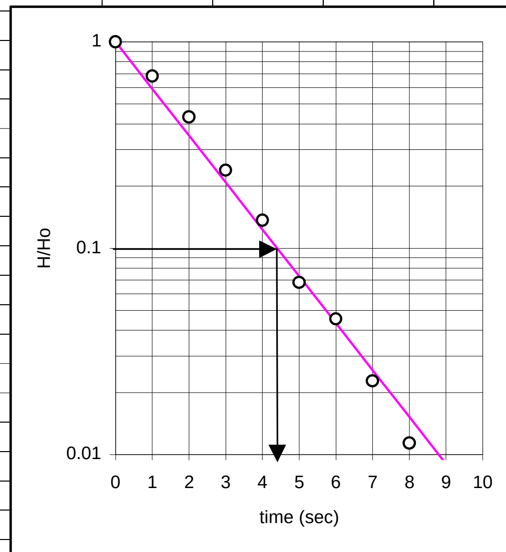
Hvorslev, M.J., 1951. Time lag and soil permeability in ground water observations. U.S. Army Corps of Engineerg, Waterways Experiment Station, Bulletin 36. (Also a Naval Facilities Manual by the same name and author exists).

Example

A slug test is performed by lowering a metal slug into a piezometer that is screened in a coarse sand. The inside diameter of the well screen and well casing is 2 inches. The well screen is 10 feet long. A pressure transducer was used to record the water level every second. The attached spreadsheet lists the data obtained and the calculations used to find the hydraulic conductivity.

The spreadsheet also plots the values using equation 7 to illustrate how the model represents the data. In this particular example, the fit is quite good.

	A	B	C	D	E	F	G	H	I	J
1	Purpose: Hvorslev Slug Test									
2	Author: T.G. Cleveland									
3	Date:	10/14/99								
4										
5	r	0.083	ft							
6	R	0.083	ft							
7	L	10	ft							
8	K	8.64E-04	ft/sec	7.46E+01	ft/day					
9	A	1.31E+01	shape factor							
10										
11				Observed	Modeled					
12	Time (sec)	Depth	Hw	Hw/Ho	Hw/Ho					
13	<0	13.99								
14	0	14.87	0.88	1	1					
15	1	14.59	0.6	0.681818	0.592444					
16	2	14.37	0.38	0.431818	0.35099					
17	3	14.2	0.21	0.238636	0.207942					
18	4	14.11	0.12	0.136364	0.123194					
19	5	14.05	0.06	0.068182	0.072985					
20	6	14.03	0.04	0.045455	0.04324					
21	7	14.01	0.02	0.022727	0.025617					
22	8	14	0.01	0.011364	0.015177					
23	9	13.99	0	0	0.008991					
24										
25	t1	0								
26	t2	4.4								
27	H2/Ho	0.1								
28	H1/Ho	1								
29	Slope	0.523315								
30	K	8.64E-04								
31										



SLUG TESTS

-- SLUG TESTS INVOLVE THE USE OF A <u>SINGLE</u> <u>BOREHOLE</u> (OR WELL) FOR DETERMINING AQUIFER FORMATION CHARACTERISTICS.

-- A VOLUME OF WATER IS <u>SUDDENLY</u> REMOVED OR ADDED AND OBSERVATIONS OF RECOVERY OR DRAWDOWN ARE NOTED THROUGH TIME.

-- BY CAREFUL EVALUATION OF THE RECOVERY CURVE AND KNOWLEDGE OF BOREHOLE GEOMETRY, IT IS POSSIBLE TO DETERMINE ESTIMATES OF HYDRAULIC CONDUCTIVITY

TYPICAL PROCEDURE

-- A DISPLACEMENT ROD (THE "SLUG"), SLIGHTLY SMALLER THAN THE BOREHOLE DIAMETER IS LOWERED INTO THE BOREHOLE, AND THE WATER LEVEL IS ALLOWED TO COME TO EQUILIBRIUM

-- ROD IS QUICKLY REMOVED, ITS VOLUME EQUIVALENT TO REMOVING THE SAME VOLUME OF WATER FROM THE HOLE.

-- WATER LEVEL MEASUREMENTS ARE THEN COLLECTED AND ANALYZED TO DETERMINE AQUIFER CHARACTERISTICS

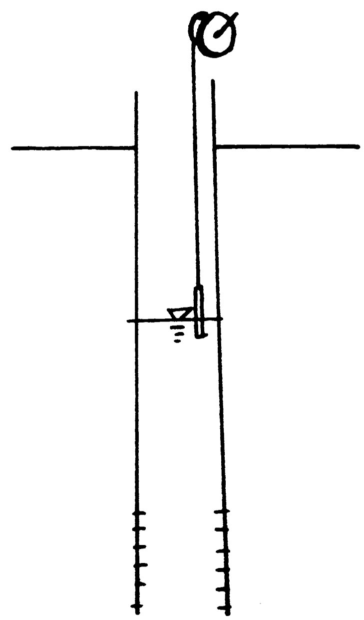
TYPICAL MONITORING INTERVALS

-- FROM TIME ZERO TO 2 MINUTES, MONITOR EVERY 15 SECONDS.

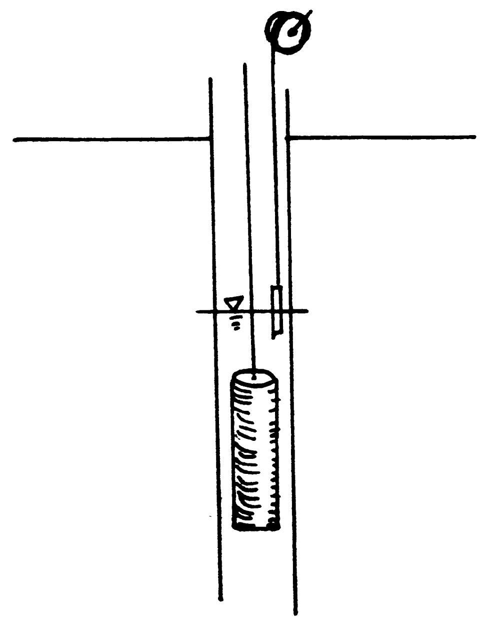
-- FROM 2 MINUTES TO 5 MINUTES, MONITOR EVERY 30 SECONDS

-- FROM 5 MINUTES TO 10 MINUTES, MONITOR EVERY 60 SECONDS

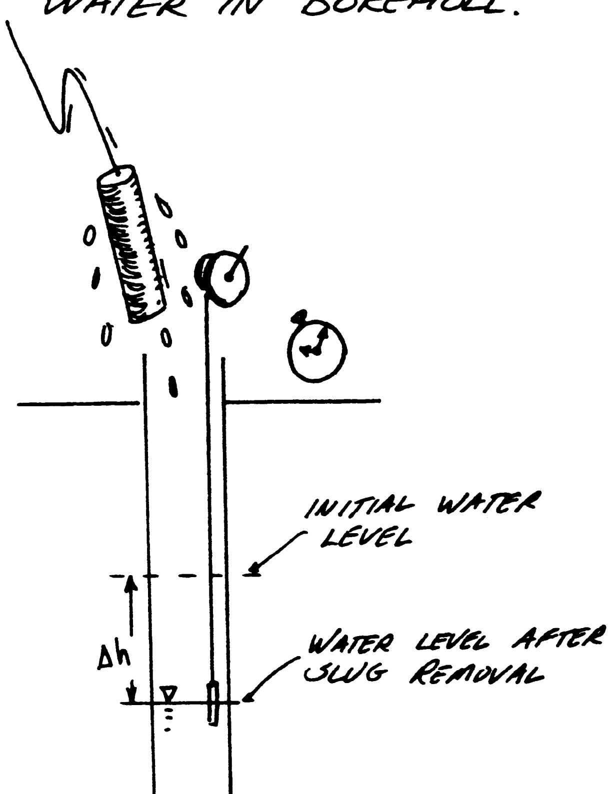
TYPICAL SLUG TEST PROCEDURE



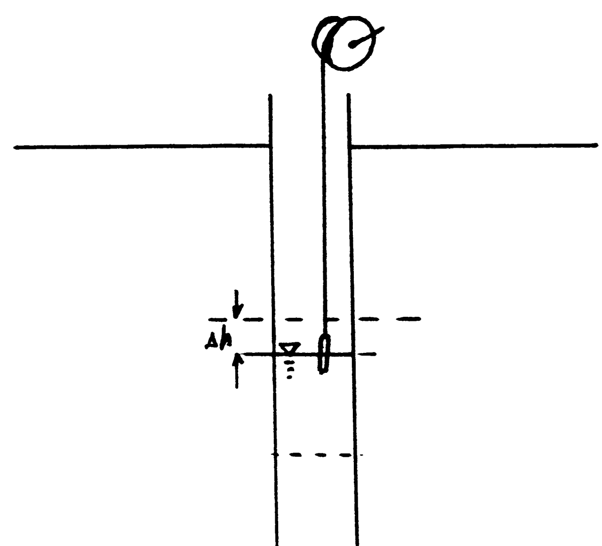
① MEASURE DEPTH TO WATER IN BOREHOLE.



② LOWER SLUG BELOW WATER SURFACE. MONITOR DEPTH TO WATER UNTIL WATER LEVEL RETURNS TO INITIAL READING.



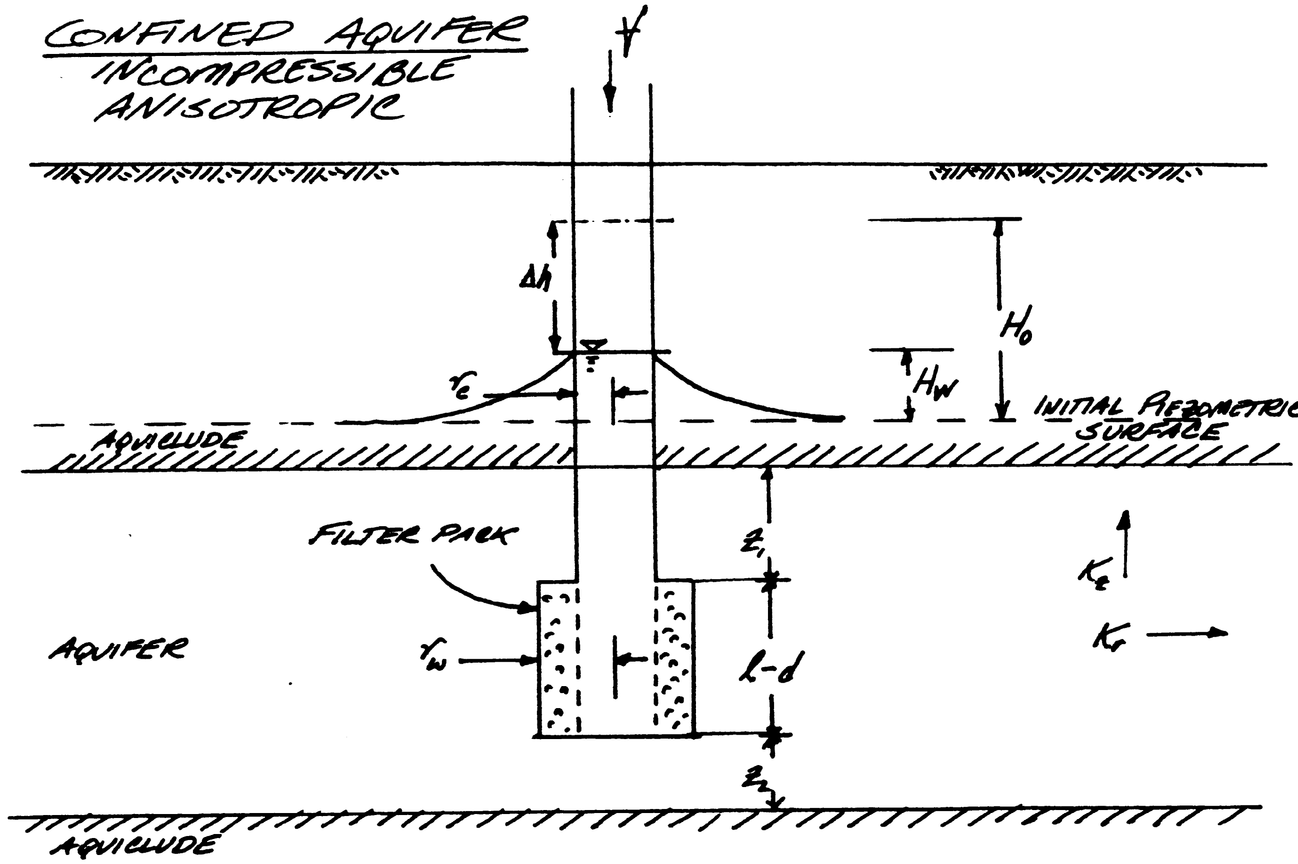
③ RAPIDLY PULL SLUG FROM WELL, START WATCH, MEASURE DEPTH TO WATER AT FREQUENT INTERVALS



④ CONTINUE TO MONITOR UNTIL WATER LEVEL HAS RISEN AT LEAST 90% OF DISTANCE BACK TO INITIAL LEVEL.

HVORSLEV (1951) METHOD

CONFINED AQUIFER
INCOMPRESSIBLE
ANISOTROPIC



ASSUMPTIONS

BOUNDED ABOVE & BELOW BY AQUICLUDES

ALL LAYERS HORIZONTAL OF INFINITE AREAL EXTENT

AQUIFER HOMOGENEOUS, VERTICALLY ANISOTROPIC

DARCY'S LAW IS VALID

WATER DENSITY & VISCOSITY CONSTANT

AQUIFER INCOMPRESSIBLE

FLOW IS HORIZONTAL (ESSENTIALLY)

NEGLIGIBLE HEAD LOSS THROUGH FILTER PACK

FLOW AWAY FROM WELL (DARCY'S LAW)

$$Q = F K_r H_w = -\pi r_c^2 \frac{dH_w}{dt}$$

F IS A SHAPE FACTOR THAT DEPENDS ON BOREHOLE GEOMETRY

HVORSLEV DEFINED LAG TIME AS THE TIME THE BOREHOLE WOULD EMPTY IF FLOW RATE IS MAINTAINED AT THE INITIAL ($t=0$) RATE

$$t_L = \frac{V}{Q} = \frac{\pi r^2 H_0}{F K_r H_0} = \frac{\pi r^2}{F K_r}$$

GOVERNING EQUATION IS NOW:

$$F K_r H_w = -\pi r_c^2 \frac{dH_w}{dt}$$

REARRANGE

$$\frac{dH_w}{dt} = -\frac{F K_r}{\pi r_c^2} H_w = -\frac{1}{t_L} H_w$$

SEPARATE & INTEGRATE

$$\int \frac{dH_w}{H_w} = -\frac{1}{t_L}\int dt$$

$$\ln H_w - \ln H_0 = -\frac{t}{t_L} \iff \ln\left(\frac{H_w}{H_0}\right) = -\frac{t}{t_L}$$

SOLUTION:

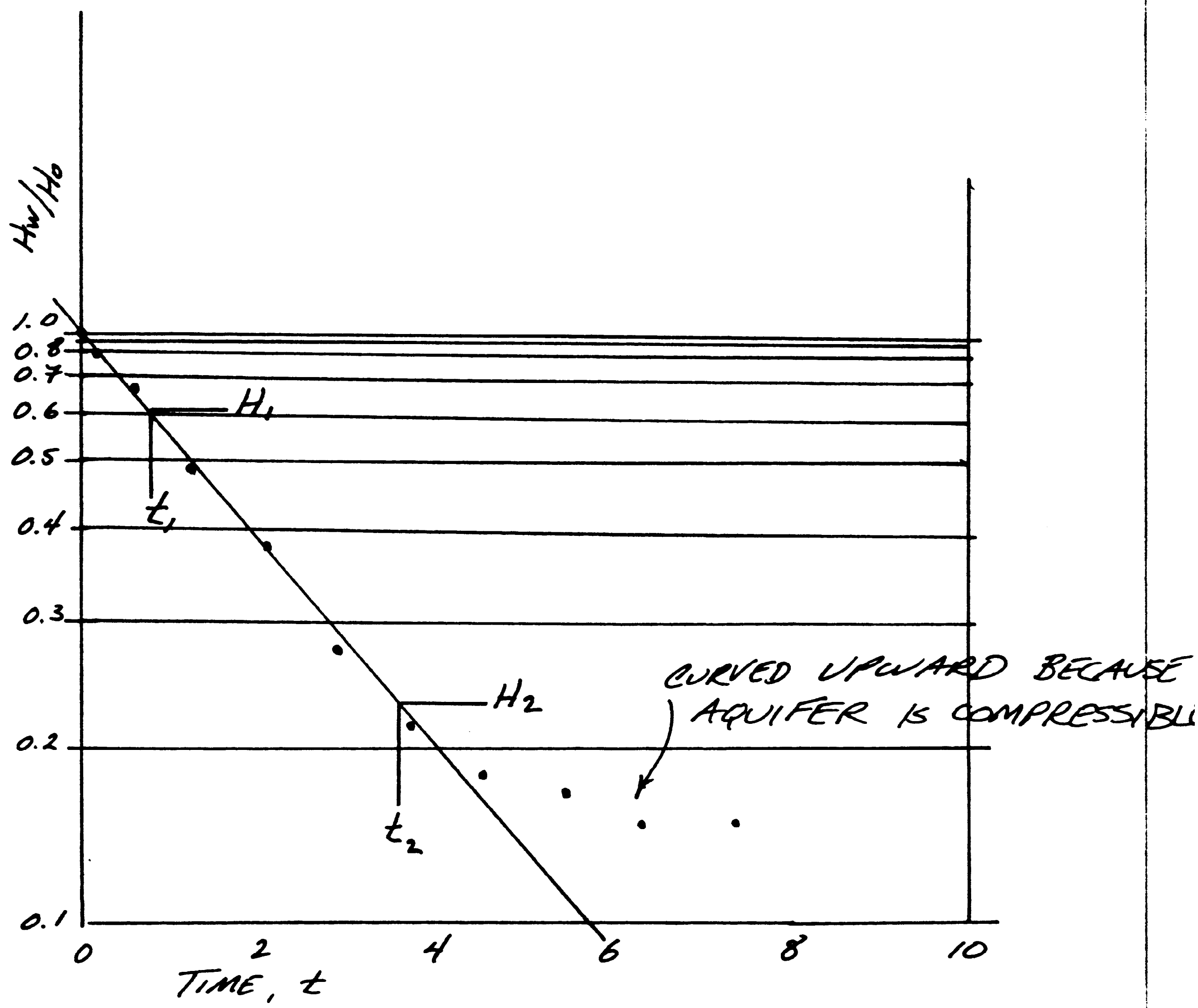
$$t_L = \frac{t}{\ln\left(\frac{H_w}{H_0}\right)}$$

$$K_r = \frac{\pi r_c^2}{F t_L}$$

DATA ANALYSIS HVORSLEV'S "ORIGINAL" METHOD

FROM TYPICAL TEST PROCEDURE DATA

① PLOT $\left(\frac{H_w}{H_o}\right)$ VERSUS TIME, t ON SEMI-LOG PAPER



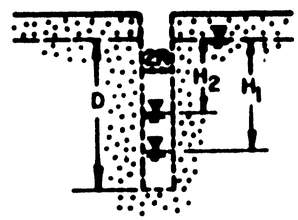
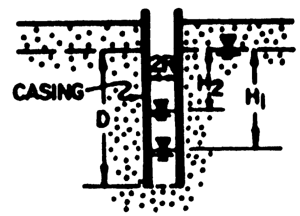
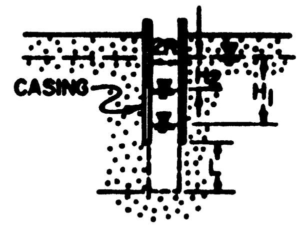
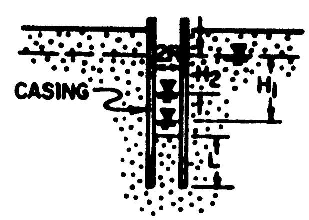
② USE EARLY TIME DATA TO ESTIMATE H_1, H_2 AS SHOWN ON PLOT ABOVE

③ INTERPRET $$K = \frac{\pi r_c^2}{F(t_2 - t_1)} \ln\left(\frac{H_1}{H_2}\right)$$

④ SEE ATTACHED SHEETS FOR SHAPE FACTORS "F"

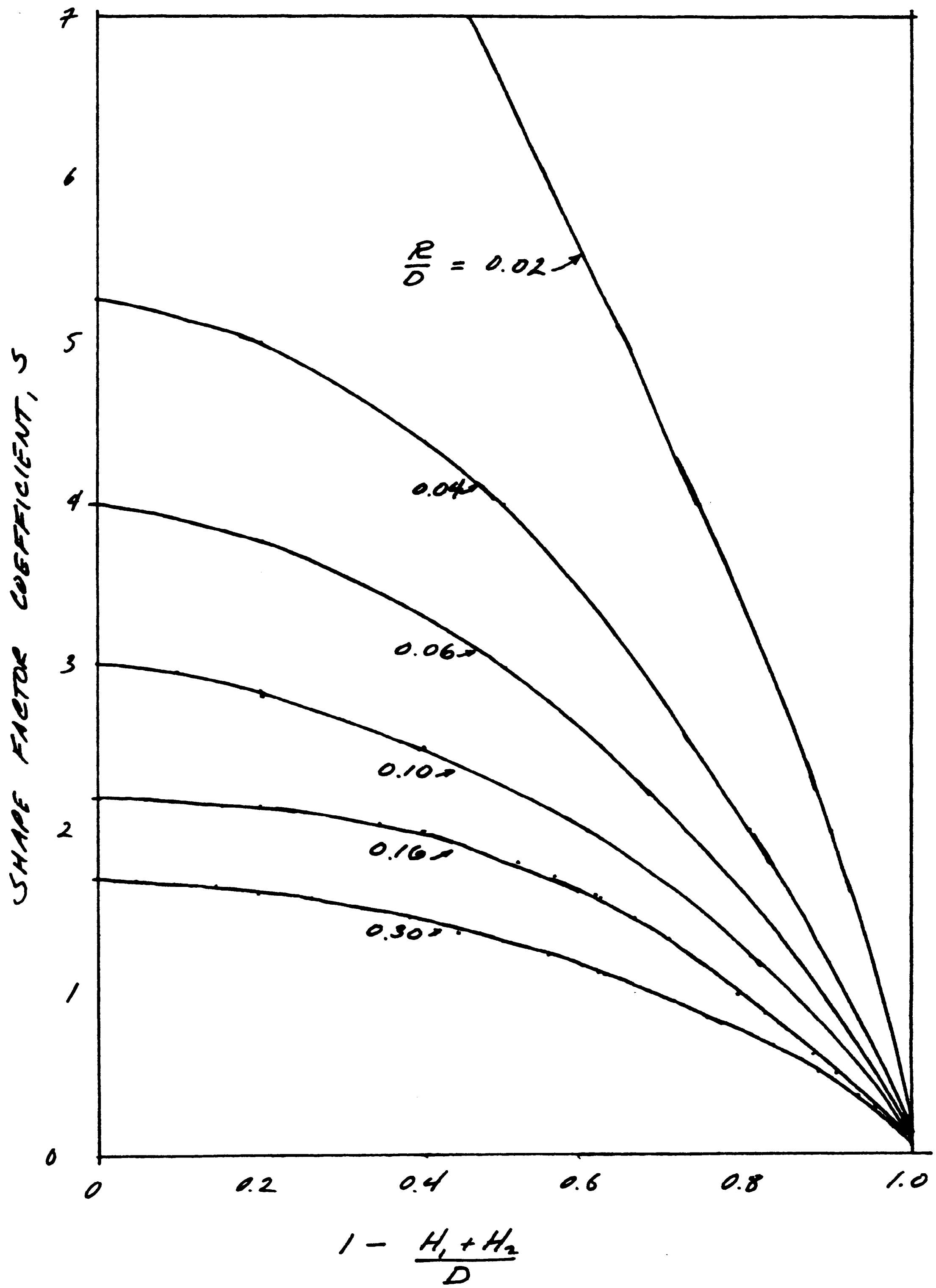
OBSERVATION WELL OR PIEZOMETER IN SATURATED ISOTROPIC STRATUM OF LARGE DEPTH

SHAPE FACTORS FOR COMPUTATION OF PERMEABILITY FROM FALLING HEAD BOREHOLE TESTS

CONDITION	DIAGRAM	SHAPE FACTOR F	CONDUCTIVITY K	APPLICATION
UNCASED HOLE		$F=16\pi D S R_w$	$K=\frac{R_c^2}{16DSR_w}\cdot\frac{H_2-H_1}{t_2-t_1}$ FOR $\frac{D}{R_w}<50$	SIMPLEST METHOD. NOT APPLICABLE IN STRATIFIED SOILS FOR S, SEE ATTACHED CHART
CASED HOLE, FLUSH WITH BOTTOM		$F=\frac{11}{2}R_w$	$K=\frac{2\pi R_c^2}{11(t_2-t_1)R_w}\ln\left(\frac{H_1}{H_2}\right)$ FOR $6'' \le D \le 60''$	USE FOR DETERMINATI[ON] OF K AT SHALLOW DEPTH BELOW WATER TABLE. UNRELIABLE IN FALLIN[G] HEAD WITH SILTING OF HOLE
CASED HOLE, UNCASED OR PERFORATED EXTENSION OF LENGTH L		$F=\frac{2\pi L}{\ln\left(\frac{L}{R_w}\right)}$	$K=\frac{R_c^2}{2L(t_2-t_1)}\ln\left(\frac{L}{R_w}\right)\ln\left(\frac{H_1}{H_2}\right)$ FOR $\frac{L}{R}>8$	USE FOR DETERMINA[TION] OF K AT GREATER DEPTHS BELOW WAT[ER] TABLE UNRELIABLE IF HOLE SILTS
CASED HOLE, COLUMN OF SOIL INSIDE CASING TO HEIGHT L		$F=\frac{11\pi R_c^2}{2\pi R_c+11L}$	$K=\frac{2\pi R_c+11L}{11(t_2-t_1)}\ln\left(\frac{H_1}{H_2}\right)$	USE FOR VERTICAL PERMEABILITY IN ANISOTROPIC SOILS

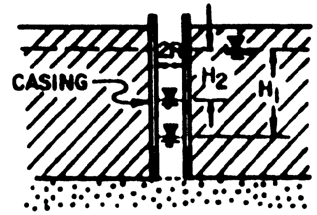
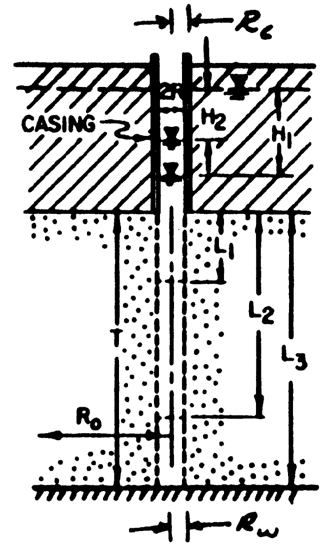
NOTE: FORMULAS MODIFIED TO BE CONSISTENT

SHAPE FACTOR COEFFICIENT S
FOR UNCASED HOLE TESTS



SHAPE FACTORS FOR COMPUTATION OF PERMEABILITY FROM FALLING HEAD BOREHOLE TESTS

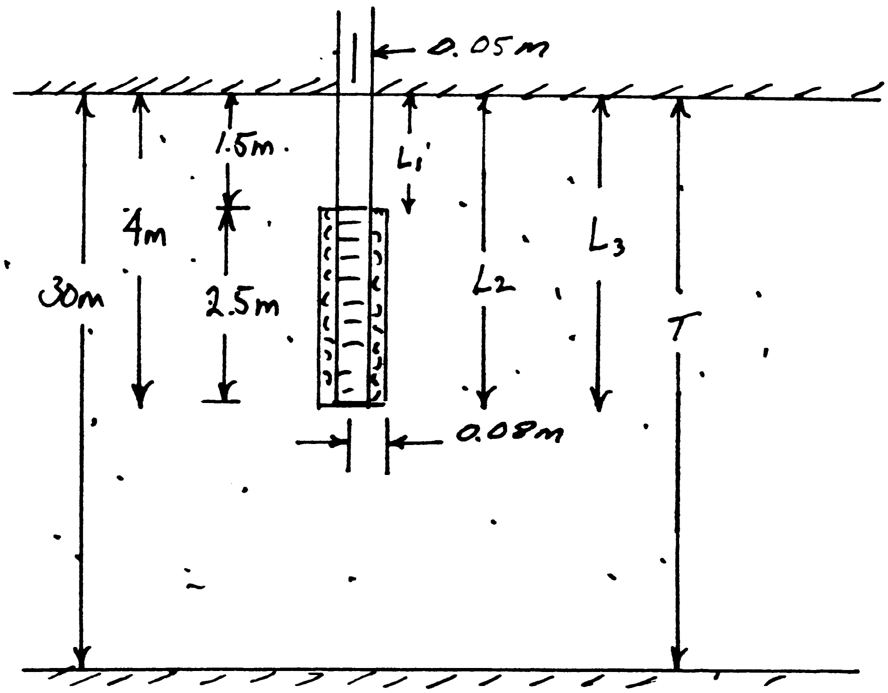
OBSERVATION WELL OR PIEZOMETER IN AQUIFER WITH IMPERVIOUS UPPER LAYER

CONDITION	DIAGRAM	SHAPE FACTOR F	CONDUCTIVITY K	APPLICATION
CASED HOLE, OPENING FLUSH WITH UPPER CONFINING UNIT OF AQUIFER OF LARGE THICKNESS		$F = 4R_w$	$K = \frac{\pi R}{4(t_2 - t_1)} \ln\left(\frac{H_1}{H_2}\right)$	FOR K WHEN SURFACE IMPERVIOUS LAYER IS THIN. UNRELIABLE IN FALLING HEAD WITH SILTING OF BOTTOM OF HOLE
CASED HOLE, UNCASED OR PERFORATED EXTENSION INTO AQUIFER OF FINITE THICKNESS. R_o IS EFFECTIVE RADIUS TO SOURCE OF CONSTANT HEAD		$F = C_s R_w$ FOR $\frac{L_1}{T} \leq 0.2$ AND $L_2/T \leq 0.2$	$K = \frac{\pi R_c^2}{C_s(t_2 - t_1)R_w} \ln\left(\frac{H_1}{H_2}\right)$	FOR K AT DEPTHS GREATER THAN 5 FT. FOR C_s SEE BELOW
		$F = \frac{2\pi L_2}{\ln\left(\frac{L_2}{R_w}\right)}$ FOR $0.2 < \frac{L_2}{T} \leq 0.85$	$K = \frac{R_c^2 \ln\left(\frac{L_2}{R_w}\right)}{2L_2(t_2 - t_1)} \ln\left(\frac{H_1}{H_2}\right)$ FOR $\frac{L}{R} \geq 8$	FOR K AT GREATER DEPTHS AND FOR FINE GRAINED SOILS USING POROUS INTAKE POINT OF PIEZOMETER
		$F = \frac{2\pi L_3}{\ln\left(\frac{R_o}{R_w}\right)}$ FOR $\frac{L_3}{T} = 1.00$ (FULLY PENETRATING)	$K = \frac{R_c^2 \ln\left(\frac{R_o}{R_w}\right)}{2L_3(t_2 - t_1)} \ln\left(\frac{H_1}{H_2}\right)$	FOR FULLY PENETRATING CONDITIONS. USE $\frac{R_o}{R} = 200$ UNLESS ACTUAL VALUES OF R_o ARE KNOWN
			$\log C_s \approx 0.5489 \log(L/R) + 0.8739 \quad \frac{L}{R} < 30$	
			$\log C_s \approx 0.42202 \log(L/R - 29) + 1.6846 \quad \frac{L}{R} \geq 30$	

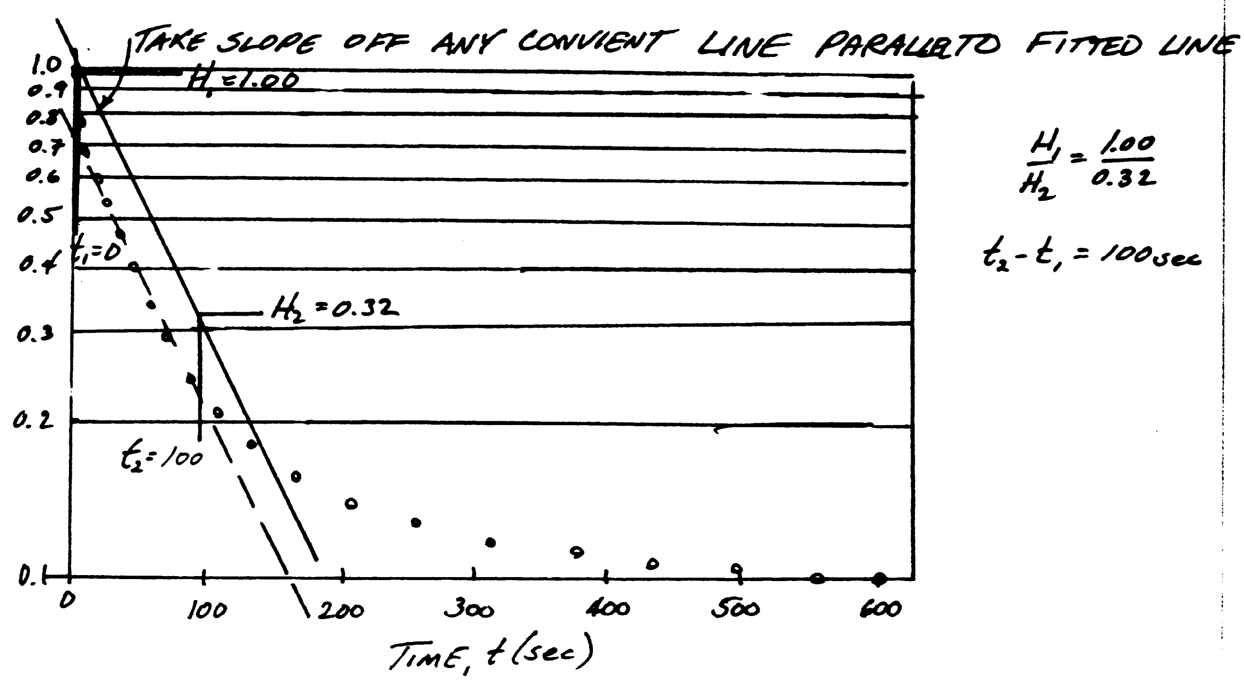
NOTE: FORMULAS MODIFIED TO BE CONSISTENT

EXAMPLE

A SLUG TEST IN A CONFINED SAND AQUIFER 30 METERS THICK WITH A PIEZOMETER EXTENDING 4 METERS BELOW CONFINING LAYER, WITH A 2.5 METER LONG SCREENED INTERVAL. CASING DIAMETER IS $r_c = 5.0$ cm. WHAT IS HYDRAULIC CONDUCTIVITY, K ?



STEP ① PLOT DATA (ALREADY DONE BELOW), USE EARLY TIME DATA TO FIT STRAIGHT LINE.



$$\frac{H_1}{H_2} = \frac{1.00}{0.32}$$

$$t_2 - t_1 = 100 \text{ sec}$$

STEP ② DETERMINE SHAPE FACTOR

ⓐ $l-d = 2.5m$

$r_w = 0.08m$

$r_c = 0.05m$

$z_1 = 1.5m$

$z_2 = 26m$

ⓑ LOOK ON ATTACHED SHEETS FOR SHAPE FACTOR — NOTE CHANGE IN GEOMETRY TERMINOLOGY!

CASED HOLE WITH EXTENSION, OBSERVATION WELL IN CONFINED AQUIFER.

$$\frac{L_1}{T} = \frac{z_1}{z_1+z_2+(l-d)} = \frac{1.5m}{30m} \leq 0.05$$

$$\frac{L_2}{T} = \frac{z_1+(l-d)}{z_1+z_2+(l-d)} = \frac{4.0m}{30m} \leq 0.02$$

∴ USE SHAPE FACTOR:

$$F = C_s R_w$$

$$\frac{L}{R} = \frac{l-d}{r_c} = \frac{2.5m}{0.08m} = 31.25$$

$$\log C_s = 0.42202 \log(31.25 - 29) + 1.6846$$

$$\log C_s = 1.8332$$

$$C_s = 10^{1.8332} = 68.1126$$

STEP ③ $$K = \frac{\pi(0.05m)^2}{(68)(0.08m)} \cdot \frac{\ln\left(\frac{1.00}{0.33}\right)}{100sec} = 0.000016\ m/sec$$

$1.6 \cdot 10^{-3}$ cm/sec

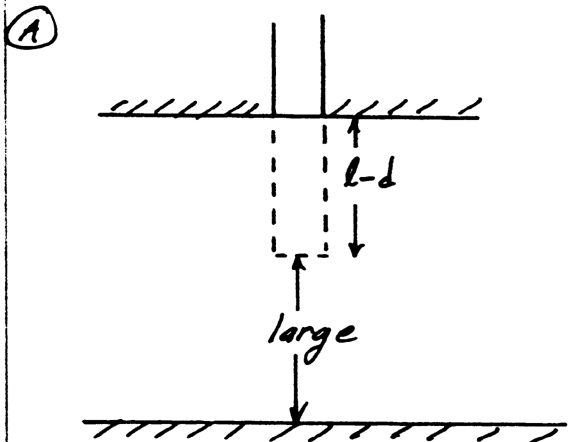
FOR ANISOTROPIC SOILS: USE

$$a_k = \sqrt{\frac{K_r}{K_z}}$$

, MUST BE ESTIMATED FROM SITE GEOLOGY — OR USE AN ADVANCING CASED HOLE TEST.

SHAPE FACTORS:

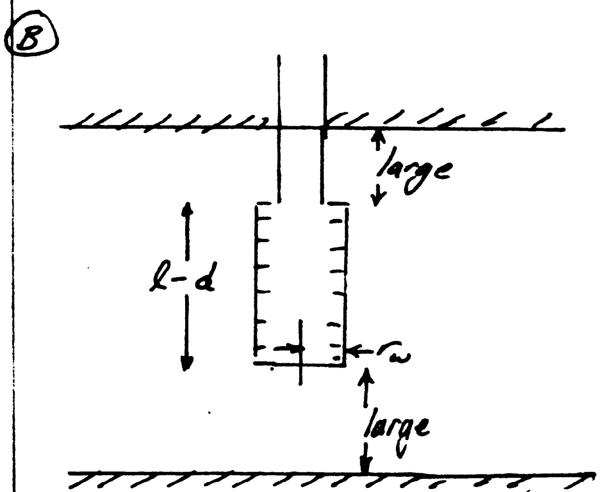
(A)



$$F = \frac{2\pi(l-d)}{\ln\left[\frac{a_k(l-d)}{r_w} + \sqrt{1+\left(\frac{a_k(l-d)}{r_w}\right)^2}\right]}$$

$$F = \frac{2\pi(l-d)}{\ln\left[\frac{2a_k(l-d)}{r_w}\right]}, \text{ for } \frac{a_k(l-d)}{r_w} > 4$$

(B)



$$F = \frac{2\pi(l-d)}{\ln\left[\frac{a_k(l-d)}{2r_w} + \sqrt{1+\left(\frac{a_k(l-d)}{2r_w}\right)^2}\right]}$$

$$F = \frac{2\pi(l-d)}{\ln\left(\frac{a_k(l-d)}{r_w}\right)}, \text{ for } \frac{a_k(l-d)}{2r_w} > 4$$

REFERENCE: DAWSON, K, AND J.D. ISTOK, AQUIFER TESTING: DESIGN AND ANALYSIS OF PUMPING AND SLUG TESTS, LEWIS PUBLISHERS, CHELSEA, MICHIGAN, 1991

REFERENCE: HVORSLEV, M.J. TIME LAG AND SOIL PERMEABILITY IN GROUNDWATER OBSERVATIONS, U.S. ARMY CORPS OF ENGINEERS, WATERWAYS EXPERIMENTAL STATION, BULLETIN 36, 1951

Hvorslev method:

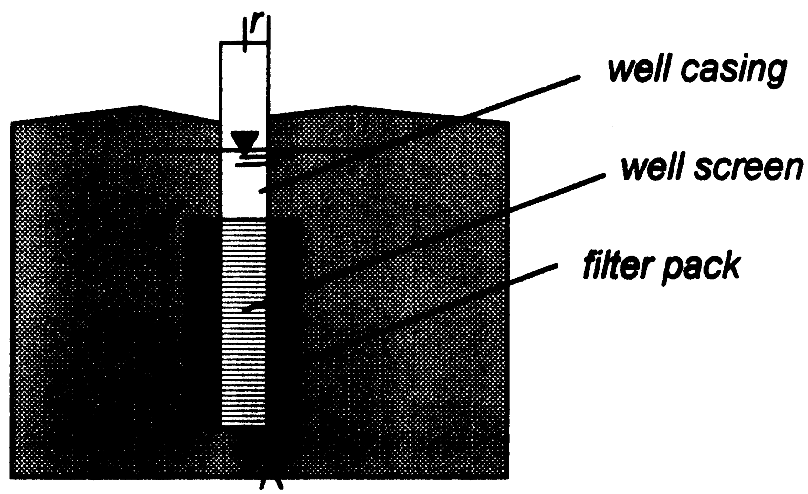
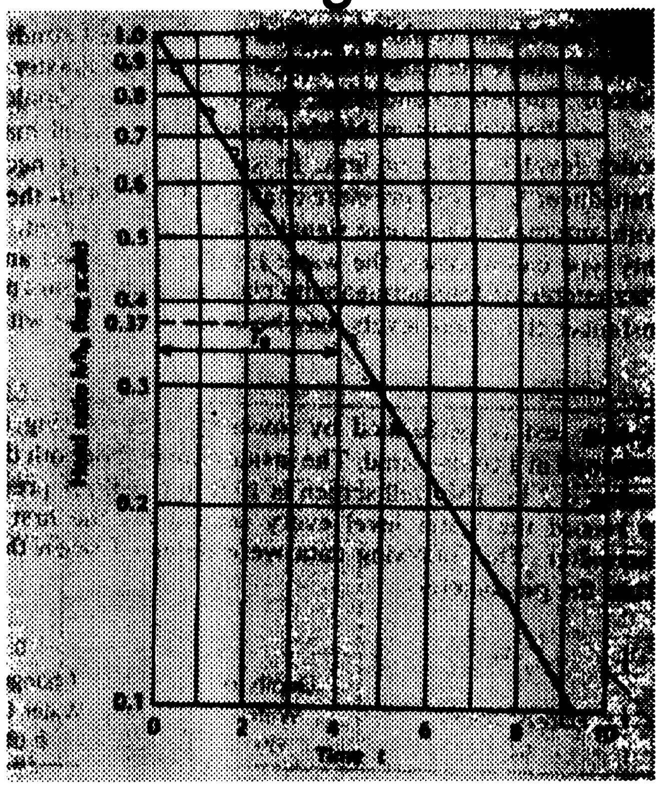


Diagram for Hvorslev analysis of slug test

Analysis; Plot h(t)/h(0) versus time for slug test.



Determine time required for 37% recovery.

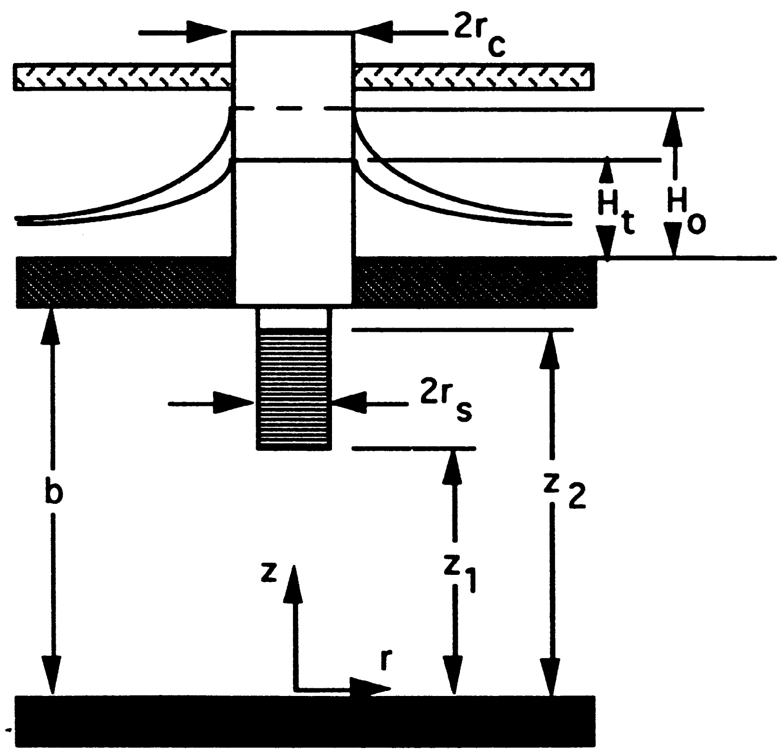
Estimate hydraulic conductivity as:

$$K = \frac{r^2 \ln(L / R)}{2 L_e T_o}$$

Slug Test Analysis (Nguyen and Pinder Method)

Nguyen,V. and G.F. Pinder, 1984. "Direct Calculation of Aquifer Parameters in Slug Test Analysis." in Groundwater Hydraulics, American Geophysical Union, Water Resources Monograph No. 9., pp 222-239.

Definition Sketch



Governing Equation(s)

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial z^2} = \frac{S}{K}\frac{\partial s}{\partial t}$$

where

s = drawdown or buildup
S = specific storage
K = hydraulic conductivity/

Subject to following auxiliary conditions:

$$s(r,z,0) = 0$$ *initial condition*

$$\frac{\partial s}{\partial z} = 0 \; at \; z = 0; z = b$$ *top,bottom; no-flow boundary conditions.*

$$s(\infty,z,t) = 0$$ *infinite radius boundary condition*

$$H_t = \frac{1}{(z_2 - z_1)} \int_{z_1}^{z_2} \frac{\partial}{\partial r} s(r_s,z,t)dz$$ *average drawdown in well.*

$$2\pi r_s K \int_{z_1}^{z_2} \frac{\partial}{\partial r} s(r_s,z,t)dz = \pi r_c \frac{\partial H_t}{\partial t}$$ *change in borehole storage equals flux into (out) of aquifer.*

Solution, by LaPlace Transform is

$$S = \frac{r_c^2}{r_s^2}\frac{C_3}{(z_2 - z_1)}; K = \frac{r_c^2}{4C_4}\frac{C_3}{(z_2 - z_1)}$$

where

C_3 and C_4 are obtained from the procedure that follows.

- Plot ln(H_t) versus ln(t) from test data.
- Compute slope of best fit line that passes through plot.
- The negative of this slope is C_3.

- Plot ln (- DHt/Dt) versus 1/t.
- Compute slope of best fit line that passes through plot.
- This slope is C_4.