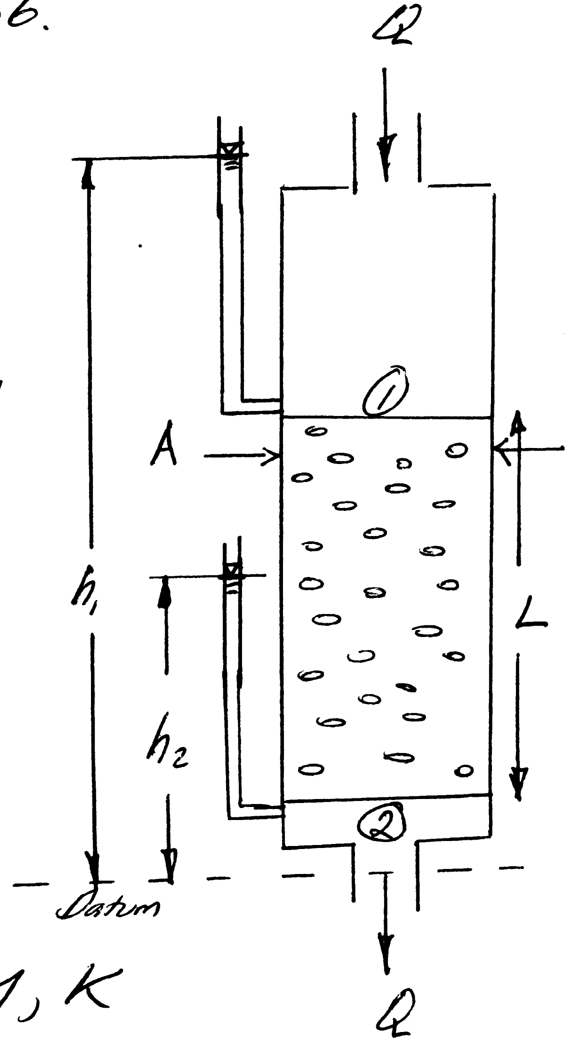
# CIVE 6361 Groundwater Hydrology

## Darcy's Law and Continunity

- Darcy's Experiments (pp 24-27).
- Gradients from field data (pp 28-29).
- Groundwater systems (pp 30-31).
- Measurement techniques (pp 47).

# Darcy's Law

Darcy's law was established experimentally by Henri Darcy in 1856.




Darcy observed that total discharge, Q was proportional to A, $\Delta h = h_1 - h_2$, and inversely proportional to L.

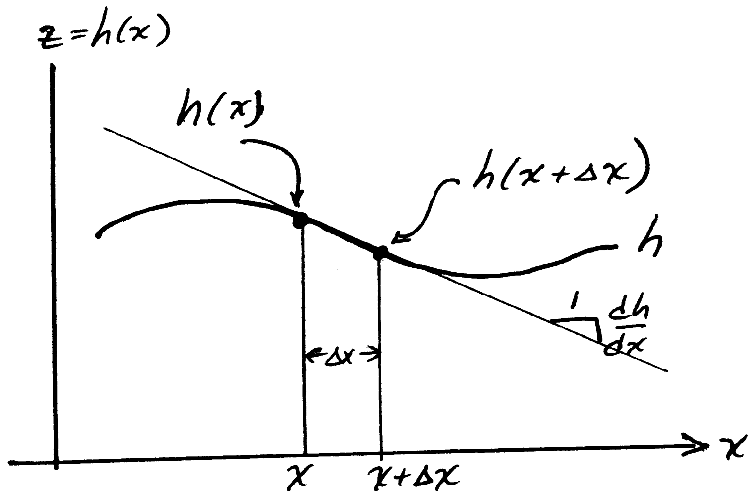
The constant of proportionality is known as the hydraulic conductivity, K

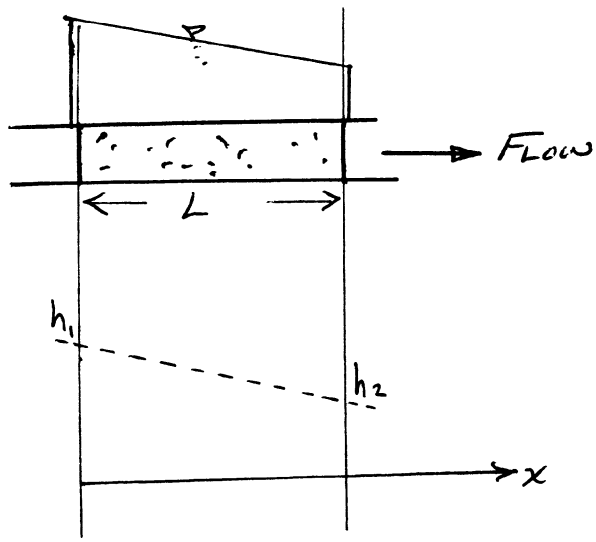
$$Q = KA\frac{\Delta h}{L}$$

The group of terms $\frac{\Delta h}{L}$ is called the hydraulic gradient. It represents the energy loss per unit length in the direction of flow.

A related concept is the gradient of the head distribution. It is defined as

$$\frac{dh}{dx} = \lim_{\Delta x \to 0} \frac{h(x+\Delta x) - h(x)}{\Delta x}$$






$$\frac{dh}{dx} = \frac{h_2 - h_1}{L} \quad \text{(Gradient of } h\text{)}$$

$$\frac{\Delta h}{L} = \frac{h_1 - h_2}{L} \quad \text{(hydraulic gradient)}$$

Observe that $\frac{\Delta h}{L} = -\frac{dh}{dx}$

And generally (in higher dimensions)

$$\frac{\Delta h}{L} = -\text{grad}(h)$$

Later experimental and theoritical work has established Darcy's Law as an expression of momentum in an irrotational flow field.

The general expression is

$$\vec{U} = -\frac{k}{\nu}\left(\mathrm{grad}(p) + \rho g\, \mathrm{grad}(z)\right)$$

k is called the intrinsic permeability and is a function of porous structure.

The hydraulic conductivity is related to K through specific fluid properties as

$$K = \frac{k \rho g}{\nu}$$

In many media the hydraulic conductivity depends on the direction of the applied pressure gradient and is expressed as a tensor quantity.

The most general form of Darcy's law is

$$U_x = -\frac{k_{xx}}{\nu}\frac{\partial p}{\partial x} - \frac{k_{xy}}{\nu}\frac{\partial p}{\partial y} - \frac{k_{xz}}{\nu}\left(\frac{\partial p}{\partial z} + \rho g\right)$$

$$U_y = -\frac{k_{xy}}{\nu}\frac{\partial p}{\partial x} - \frac{k_{yy}}{\nu}\frac{\partial p}{\partial y} - \frac{k_{yz}}{\nu}\left(\frac{\partial p}{\partial z} + \rho g\right)$$

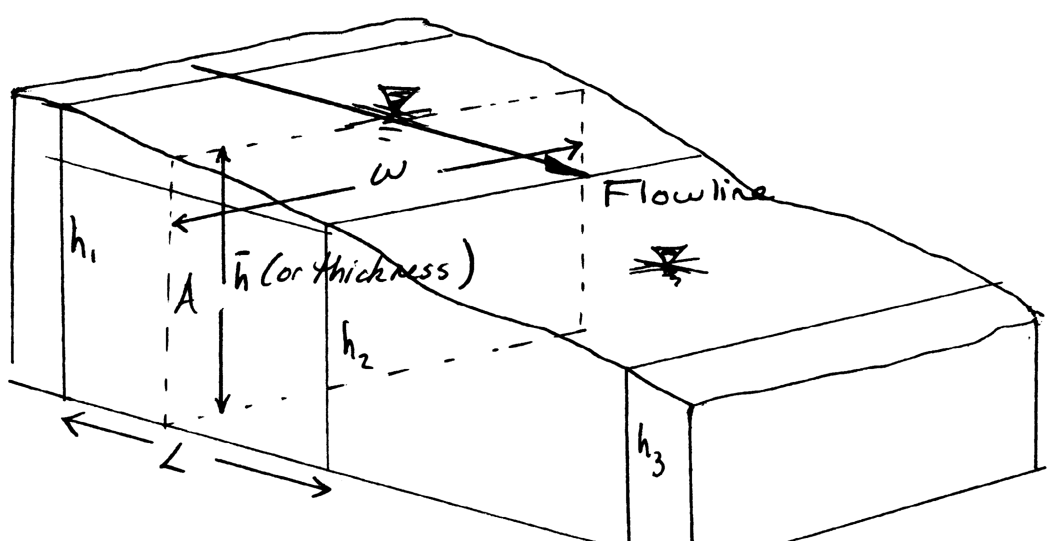
$$U_z = -\frac{k_{xz}}{\nu}\frac{\partial p}{\partial x} - \frac{k_{yz}}{\nu}\frac{\partial p}{\partial y} - \frac{k_{zz}}{\nu}\left(\frac{\partial p}{\partial z} + \rho g\right)$$

Usually the simpler forms suffice for problems of engineering significance

Qualitative & Quantitative Analysis Using Darcy's Law

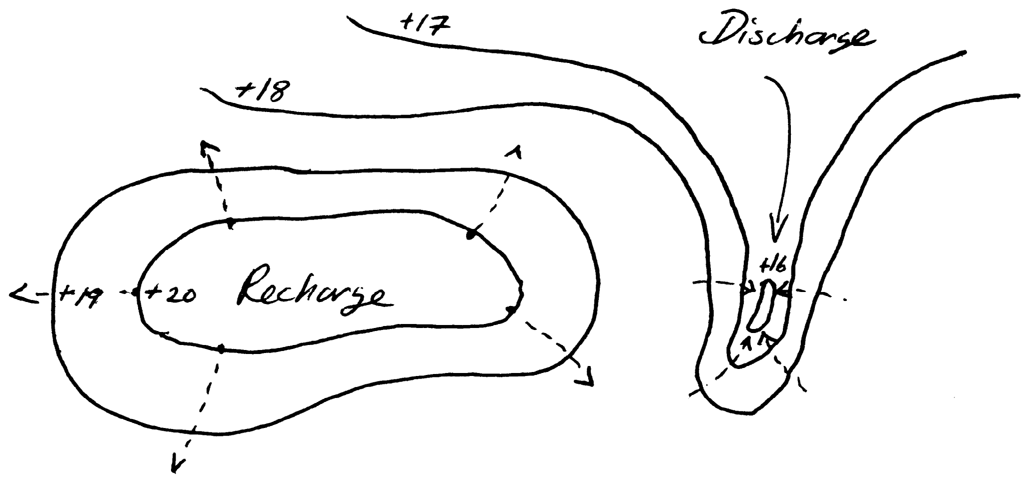
① Flow is from high head to low head

② In an isotropic system flowlines are perpindicular to lines of constant head



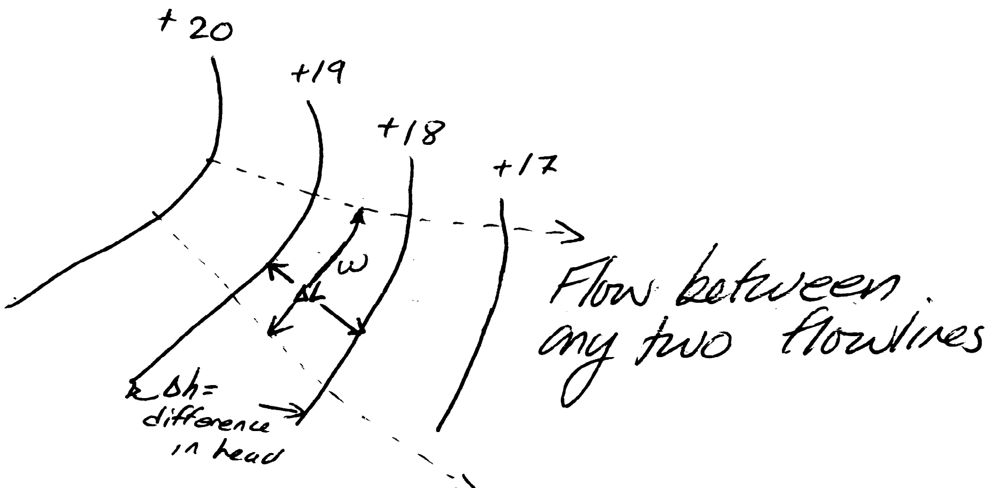

$$Q = K w \bar{h} \frac{\Delta h}{L}$$

# Head Maps



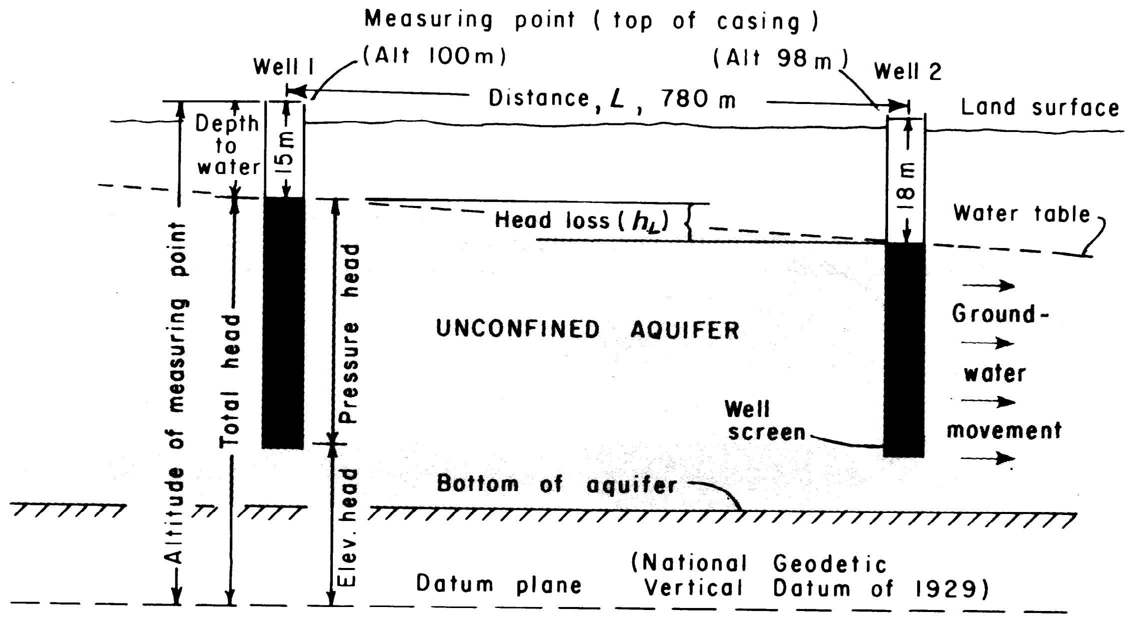

Flowlines diverge ⇒ recharge area

Flowlines converge ⇒ discharge area




$$Q = K w \bar{h} \frac{\Delta h}{\Delta L}$$

# HEADS AND GRADIENTS




( 1 )

The depth to the water table has an important effect on use of the land surface and on the development of water supplies from unconfined aquifers (1). Where the water table is at a shallow depth, the land may become "waterlogged" during wet weather and unsuitable for residential and many other uses. Where the water table is at great depth, the cost of constructing wells and pumping water for domestic needs may be prohibitively expensive.

The direction of the slope of the water table is also important because it indicates the direction of ground-water movement (1). The position and the slope of the water table (or of the potentiometric surface of a confined aquifer) is determined by measuring the position of the water level in wells from a fixed point (a measuring point) (1). (See "Measurements of Water levels and Pumping Rates.") To utilize these measurements to determine the slope of the water table, the position of the water table at each well must be determined relative to a *datum plane* that is common to all the wells. The datum plane most widely used is the National Geodetic Vertical Datum of 1929 (also commonly referred to as "sea level") (1).

If the depth to water in a nonflowing well is subtracted from the altitude of the measuring point, the result is the *total head* at the well. Total head, as defined in fluid mechanics, is composed of *elevation head*, *pressure head*, and *velocity head*. Because ground water moves relatively slowly, velocity head can be ignored. Therefore, the total head at an observation well involves only two components: elevation head and pressure head (1). Ground water moves in the direction of decreasing total head, which may or may not be in the direction of decreasing pressure head.

The equation for total head ($h_t$) is

$$h_t = z + h_p$$

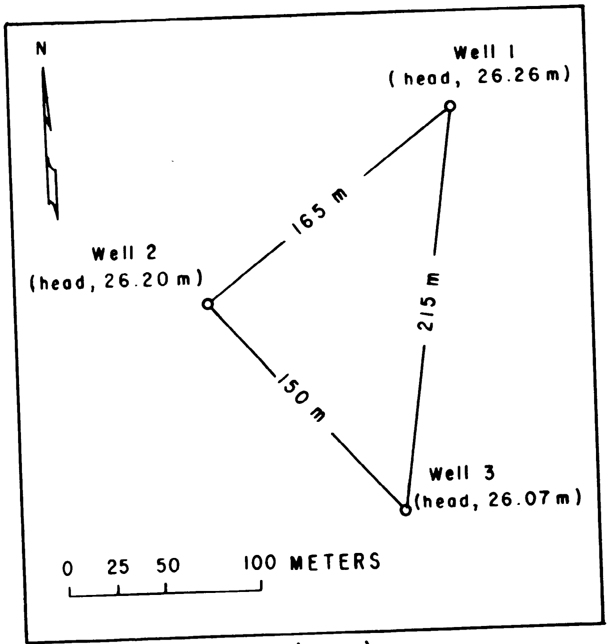
where $z$ is elevation head and is the distance from the datum plane to the point where the pressure head $h_p$ is determined.

All other factors being constant, the rate of ground-water movement depends on the *hydraulic gradient*. The hydraulic gradient is the change in head per unit of distance in a given direction. If the direction is not specified, it is understood to be in the direction in which the maximum rate of decrease in head occurs.

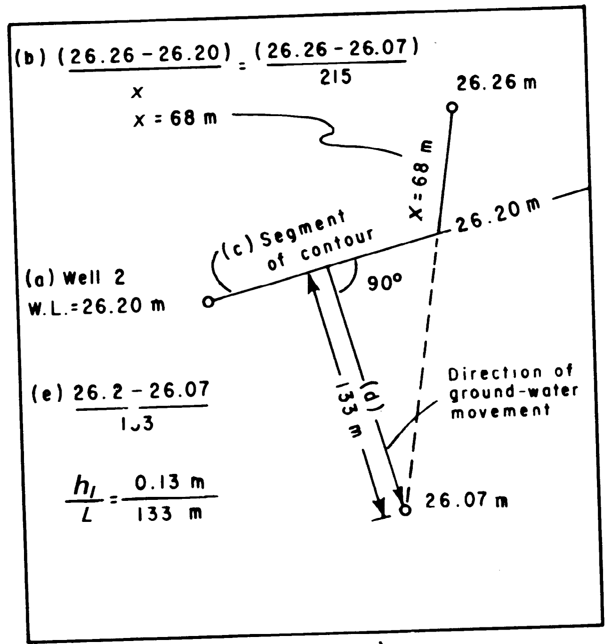
If the movement of ground water is assumed to be in the plane of sketch 1—in other words, if it moves from well 1 to well 2—the hydraulic gradient can be calculated from the information given on the drawing. The hydraulic gradient is $h_L/L$, where $h_L$ is the head loss between wells 1 and 2 and $L$ is the horizontal distance between them, or

$$\frac{h_L}{L} = \frac{(100\text{ m} - 15\text{ m}) - (98\text{ m} - 18\text{ m})}{780\text{ m}} = \frac{85\text{ m} - 80\text{ m}}{780\text{ m}} = \frac{5\text{ m}}{780\text{ m}}$$

When the hydraulic gradient is expressed in consistent units, as it is in the above example in which both the numerator and the denominator are in meters, any other consistent units of length can be substituted without changing the value of the gradient. Thus, a gradient of 5 ft/780 ft is the same as a gradient of 5 m/780 m. It is also relatively common to express hydraulic gradients in inconsistent units such as meters per




( 2 )




( 3 )

kilometer or feet per mile. A gradient of 5 m/780 m can be converted to meters per kilometer as follows:

$$\left(\frac{5\text{ m}}{780\text{ m}}\right) \times \left(\frac{1{,}000\text{ m}}{\text{km}}\right) = 6.4\text{ m km}^{-1}$$

Both the direction of ground-water movement and the hydraulic gradient can be determined if the following data are available for three wells located in any triangular arrangement such as that shown on sketch 2:

1. The relative geographic position of the wells.
2. The distance between the wells.
3. The total head at each well.

Steps in the solution are outlined below and illustrated in sketch 3:

a. Identify the well that has the intermediate water level (that is, neither the highest head nor the lowest head).

b. Calculate the position between the well having the highest head and the well having the lowest head at which the head is the same as that in the intermediate well.

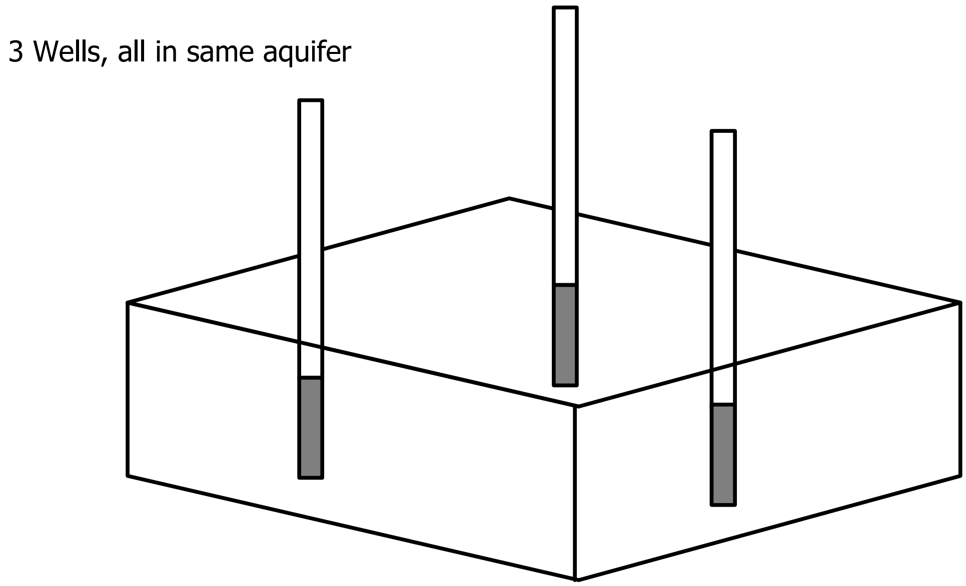
c. Draw a straight line between the intermediate well and the point identified in step *b* as being between the well having the highest head and that having the lowest head. This line represents a segment of the water-level contour along which the total head is the same as that in the intermediate well.

d. Draw a line perpendicular to the water-level contour and through either the well with the highest head or the well with the lowest head. This line parallels the direction of ground-water movement.

e. Divide the difference between the head of the well and that of the contour by the distance between the well and the contour. The answer is the hydraulic gradient.

<u>Determining Gradients from Field Data (Three Well Triangulation)</u>

Computing gradients is important in ground water hydrology. A simple method that can be used with three wells is triangulation. Suppose three wells are completed in an aquifer as depicted below:




From head measurements in each well, the groundwater hydraulic gradient can be computed and flow directions predicted. This triangulation approach has some special assumptions built into it:

(1) Piezometric surface in the vicinity of the three wells can be approximated by a plane.
(2) All three wells sample the same aquifer unit
(3) The wells all measure vertically averaged head, or head at identical elevations in the aquifer.
(4) Head measurements are taken at essentially the same time

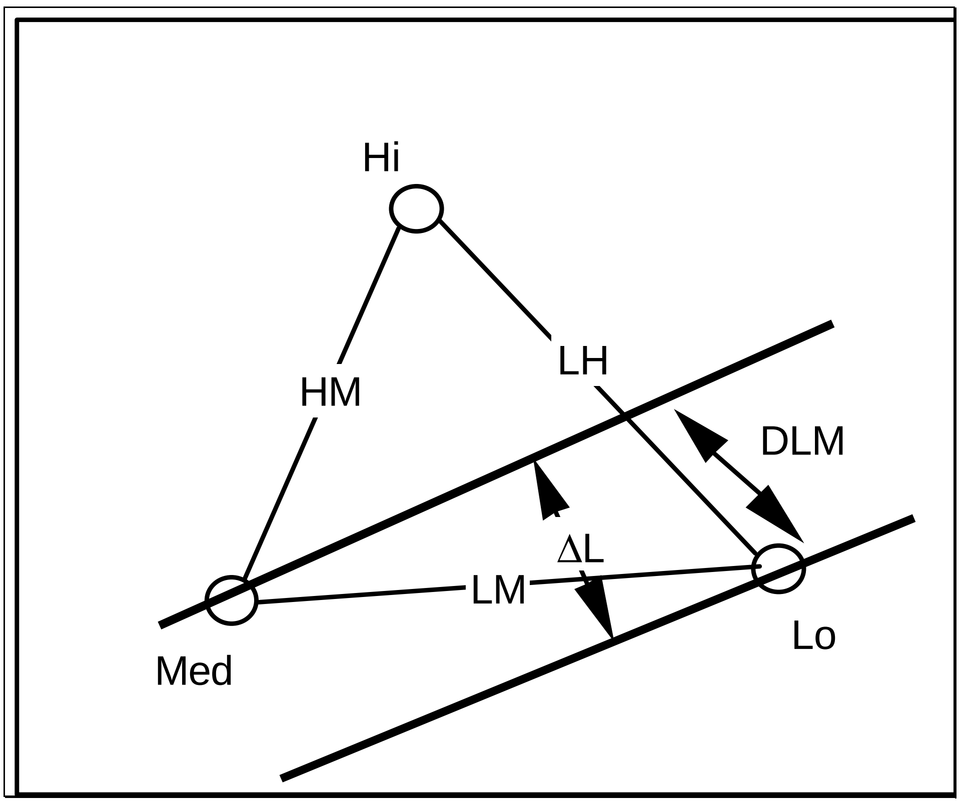
Objective is to determine the direction and magnitude of the hydraulic gradient.

Methods:

(1) Graphical method
(2) Mathematical method (solve the equation of the plane)

Graphical method:

a) Plot locations and static water level (SWL) elevations of each well.




b) Identify High, Medium, and Low SWL elevations. (Shown as Hi, Med, and Lo in picture)

c) Calculate point between High and Low wells where SWL = Medium Value from :

$$DLM = \frac{Med-Lo}{Hi-Lo} * LH.$$

Hi = head value at high well
Med = head value at medium well
Lo = head value at low well
LH = distance from High well to Low well.

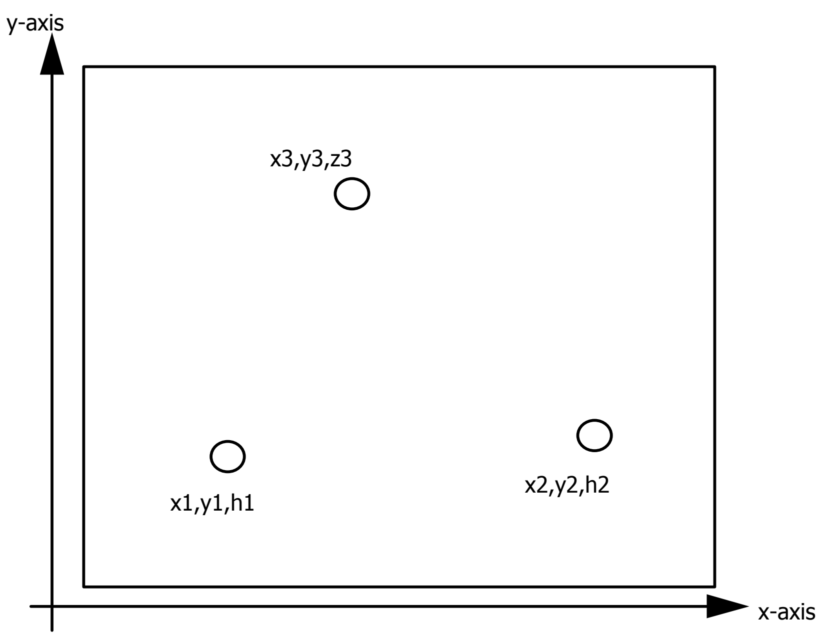
d) Draw equipotential line connecting points of equal head from Medium well to DLM along segment LH.

e) Draw gradient orientation line perpendicular to this equipotential line.

f) Compute hydraulic gradient as: $\Delta h/\Delta L$ where $\Delta h$ is the change in head between Medium well and Low Well, and $\Delta L$ is the distance between the medium well equipotential line and the low well equipotential line along the gradient orientation line.

<u>Mathematical Method:</u>

a) Plot locations and SWL elevations of each well.




b) Choose any arbitrary origin - measure x-component and ycomponent distances from this origin to each of the wells.

c) Solve the equation of the plane for the piezometric surface (h(x,y) = ax + by + c) by constructing a linear system of equations:

$$h_1 = a\,x_1 + b\,y_1 + c$$

$$h_2 = a\,x_2 + b\,y_2 + c$$

$$h_3 = a\,x_3 + b\,y_3 + c$$

This system can be written in vector matrix form as:

$$\mathbf{A}\,(\mathbf{x}) = \mathbf{b}$$

$$\mathbf{A} = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

Now solve this system of linear equations ( 3 eqn. 3 unk.) for the unknowns (a,b,c).

Recall from linear algebra, $\mathbf{x} = \mathbf{A}^{\mathbf{-1}}\,\mathbf{b}$ Lotus 1-2-3, Excel, and Quattro spreadsheets all have matrix inversion procedures built in so that solving the system should be trivial. The spreadsheet named **MODEL1.XLS** implements this approach.

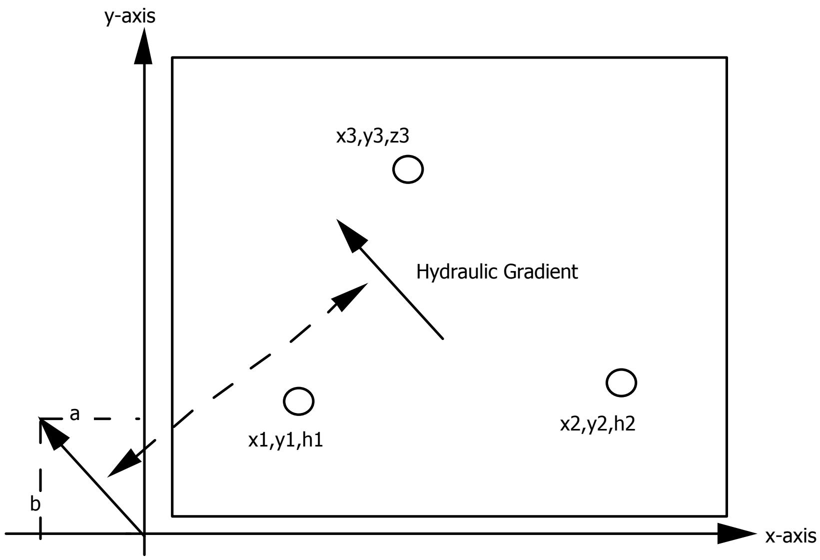
The results give coefficients of the plane equation, that when differentiated give the hydraulic gradient.

$$h(x,y) = a\,x + b\,y + c$$

-grad(h) = - a$\mathbf{i}$ -b$\mathbf{j}$ where **i** and **j** are the unit directions of the arbitrary coordinate system.

The sign of the result of differentiation indicates which quadrant the vector components reside.

Simply slide this vector to any useful place on your map for subsequent flow calculations - be careful not to change the direction or length of the vector!

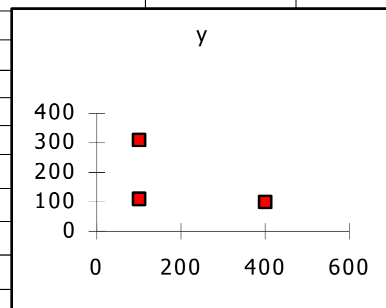


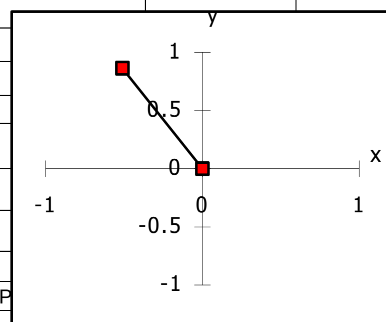

To find the magnitude of the gradient (slope along the orientation direction) just compute the length of the vector:

$$|-\text{grad(head)}| = \sqrt{a^2 + b^2}$$

This gradient method also approximates the **flow direction** <u>if the aquifer is homogeneous and isotropic with respect to the hydraulic conductivity field.</u> Otherwise, the generalized (2 or 3D matrix) form of Darcy's law must be used.

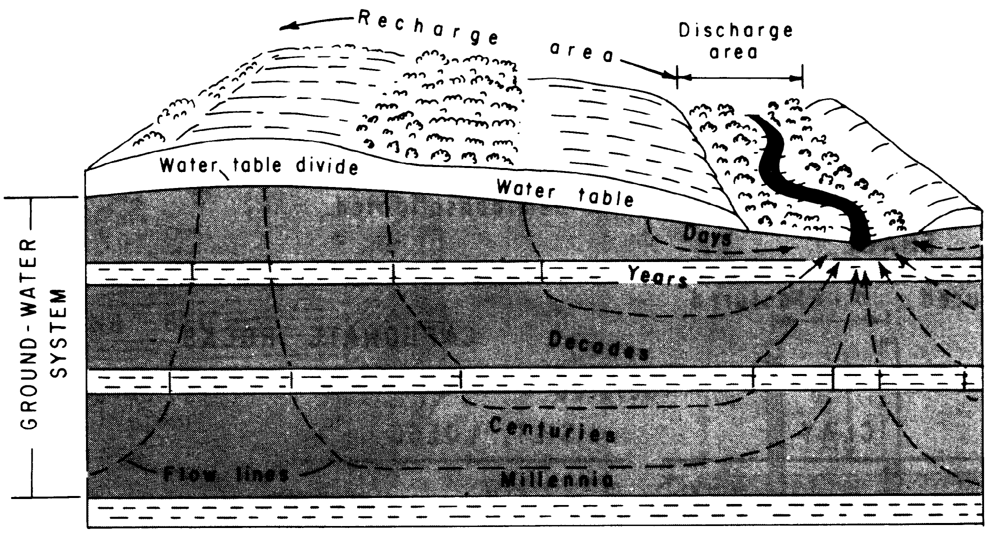
| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | Instructions: Enter data for three wells in shaded area above left. Spreadsheet solves linear system and computes gradient. Plot shows gradient direction. | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |
| 6 | | Groundwater Hydrology Gradient Spreadsheet | | | | | | | |
| 7 | | Field Data from Three Wells | | | | | | | |
| 8 | | | x | y | head | | | | |
| 9 | | Well 1 | **100** | **110** | **12** | | | | |
| 10 | | Well 2 | **400** | **100** | **13.5** | | | | |
| 11 | | Well 3 | **100** | **310** | **10.4** | | | | |
| 12 | | | | | | | | | |
| 13 | | Hydraulic | **-0.0047333** | <-i. | | | | | |
| 14 | | Gradient | **0.008** | <-j. | | | | | |
| 15 | | | | | | | | | |
| 16 | | A-Matrix | | | b-vector | x-vector | | | |
| 17 | | 100 | 110 | 1 | 12 | 0.00473333 | | | |
| 18 | | 400 | 100 | 1 | 13.5 | -0.008 | | | |
| 19 | | 100 | 310 | 1 | 10.4 | 12.4066667 | | | |
| 20 | | A-Inverse | | | | | | | |
| 21 | | -0.0035 | 0.0033333 | 0.00016667 | | | | | |
| 22 | | -0.005 | -1.948E-19 | 0.005 | | | | | |
| 23 | | 1.9 | -0.3333333 | -0.5666667 | | | | | |
| 24 | | | | | | | | | |
| 25 | | Head | + | 0 | **0** | <-x | P | | |
| 26 | | Function | + | 0 | **0** | <-y | | | |
| 27 | | | + | 12.4066667 | | | | | |
| 28 | | | | **12.4066667** | <= h(x,y) | | | | |
| 29 | | | | | | | | | |
| 30 | | This spreadsheet prepared by | | | | | | | |
| 31 | | Theodore G. Cleveland, Ph.D. | | | | | | | |
| 32 | | All rights reserved | | | | | | | |
| 33 | | | | | | | | | |
| 34 | | Hydraulic Gradient Vector Magnitude | | | | | | | |
| 35 | | Magnitude= | 0.0092954 | | | | | | |
| 36 | | i= | j= | | | | | | |
| 37 | | 0 | 0 | Hydraulic Gradient | | | | | |
| 38 | | -0.509212491 | 0.8606408 | Direction Cosines | | | | | |
| 39 | | | | x-dir.=> | -0.509212 | | | | |
| 40 | | | | y-dir.=> | 0.860641 | | | | |
| 41 | | | | | | | | | |




MODEL1.XLS Spreadsheet program for 3-well gradient calculations.

# FUNCTIONS OF GROUND-WATER SYSTEMS




(1)

The aquifers and confining beds that underlie any area comprise the *ground-water system* of the area (1). Hydraulically, this system serves two functions: it stores water to the extent of its porosity, and it transmits water from recharge areas to discharge areas. Thus, a ground-water system serves as both a reservoir and a conduit. With the exception of cavernous limestones, lava flows, and coarse gravels, ground-water systems are more effective as reservoirs than as conduits.

Water enters ground-water systems in *recharge areas* and moves through them, as dictated by hydraulic gradients and hydraulic conductivities, to *discharge areas* (1).

The identification of recharge areas is becoming increasingly important because of the expanding use of the land surface for waste disposal. In the humid part of the country, recharge occurs in all interstream areas—that is, in all areas except along streams and their adjoining flood plains (1). The streams and flood plains are, under most conditions, discharge areas.

In the drier part (western half) of the conterminous United States, recharge conditions are more complex. Most recharge occurs in the mountain ranges, on alluvial fans that border the mountain ranges, and along the channels of major streams where they are underlain by thick and permeable alluvial deposits.

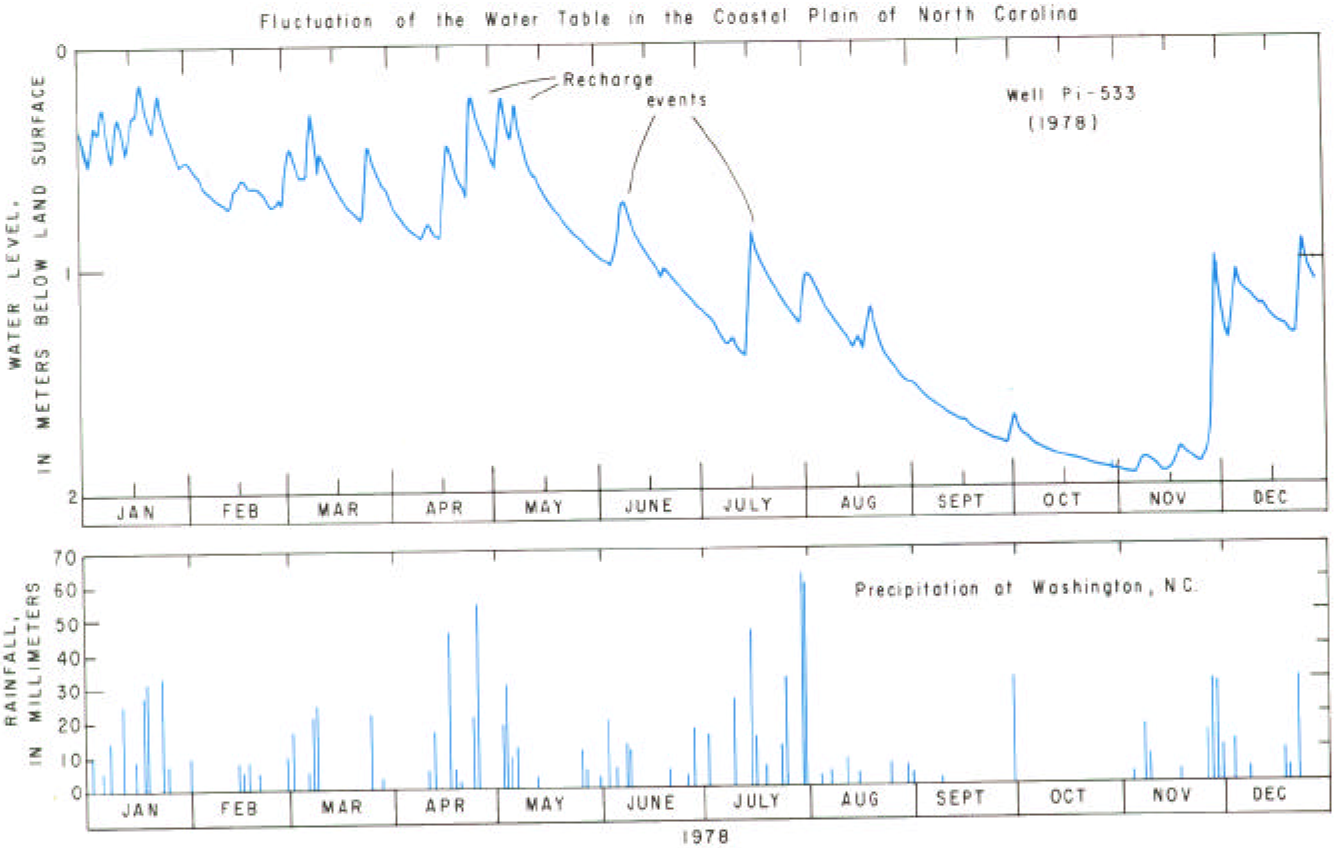
Recharge rates are generally expressed in terms of volume (such as cubic meters or gallons) per unit of time (such as a day or a year) per unit of area (such as a square kilometer, a square mile, or an acre). When these units are reduced to their simplest forms, the result is recharge expressed as a depth of water on the land surface per unit of time. Recharge varies from year to year, depending on the amount of precipitation, its seasonal distribution, air temperature, land use, and other factors. Relative to land use, recharge rates in forests are much higher than those in cities.

Annual recharge rates range, in different parts of the country, from essentially zero in desert areas to about 600 mm $yr^{-1}$ (1,600 $m^3\ km^{-2}\ d^{-1}$ or $1.1 \times 10^6$ gal $mi^{-2}\ d^{-1}$) in the rural areas on Long Island and in other rural areas in the East that are underlain by very permeable soils.

The rate of movement of ground water from recharge areas to discharge areas depends on the hydraulic conductivities of the aquifers and confining beds, if water moves downward into other aquifers, and on the hydraulic gradients. (See "Ground-Water Velocity.") A convenient way of showing the rate is in terms of the time required for ground water to move from different parts of a recharge area to the nearest discharge area. The time ranges from a few days in the zone adjacent to the discharge area to thousands of years (millennia) for water that moves from the central part of some recharge areas through the deeper parts of the ground-water system (1).

Natural discharge from ground-water systems includes not only the flow of springs and the seepage of water into stream channels or wetlands but also evaporation from the upper part of the capillary fringe, where it occurs within a meter or so of the land surface. Large amounts of water are also withdrawn from the capillary fringe and the zone of saturation by plants during the growing season. Thus, discharge areas include not only the channels of perennial streams but also the adjoining flood plains and other low-lying areas.

One of the most significant differences between recharge areas and discharge areas is that the areal extent of discharge areas is invariably much smaller than that of recharge areas. This size difference shows, as we would expect, that discharge areas are more "efficient" than recharge areas. Recharge involves unsaturated movement of water in the vertical direction; in other words, movement is in the direction in which the hydraulic conductivity is generally the lowest. Discharge, on the other hand, involves saturated movement, much of it in the horizontal direction—that is, in the direction of the largest hydraulic conductivity.




(2)

Another important aspect of recharge and discharge involves timing. Recharge occurs during and immediately following periods of precipitation and thus is intermittent (2). Discharge, on the other hand, is a continuous process as long as ground-water heads are above the level at which discharge occurs. However, between periods of recharge, ground-water heads decline, and the rate of discharge also declines. Most recharge of ground-water systems occurs during late fall, winter, and early spring, when plants are dormant and evaporation rates are small. These aspects of recharge and discharge are apparent from graphs showing the fluctuation of the water level in observation wells, such as the one shown in sketch 2. The occasional lack of correlation, especially in the summer, between the precipitation and the rise in water level is due partly to the distance of 20 km between the weather station and the well.

# GROUND-WATER MOVEMENT AND TOPOGRAPHY

It is desirable, wherever possible, to determine the position of the water table and the direction of ground-water movement. To do so, it is necessary to determine the altitude, or the height above a datum plane, of the water level in wells. However, in most areas, general but very valuable conclusions about the direction of ground-water movement can be derived from observations of land-surface topography.

Gravity is the dominant driving force in ground-water movement. Under natural conditions, ground water moves "downhill" until, in the course of its movement, it reaches the land surface at a spring or through a seep along the side or bottom of a stream channel or an estuary.

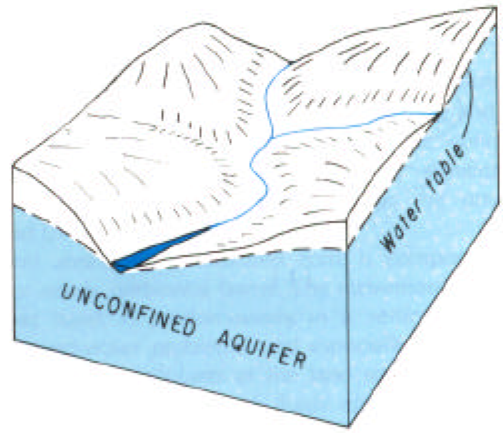
Thus, ground water in the shallowest part of the saturated zone moves from interstream areas toward streams or the coast. If we ignore minor surface irregularities, we find that the slope of the land surface is also toward streams or the coast. The depth to the water table is greater along the divide between streams than it is beneath the flood plain. In effect, the water table usually is a subdued replica of the land surface.

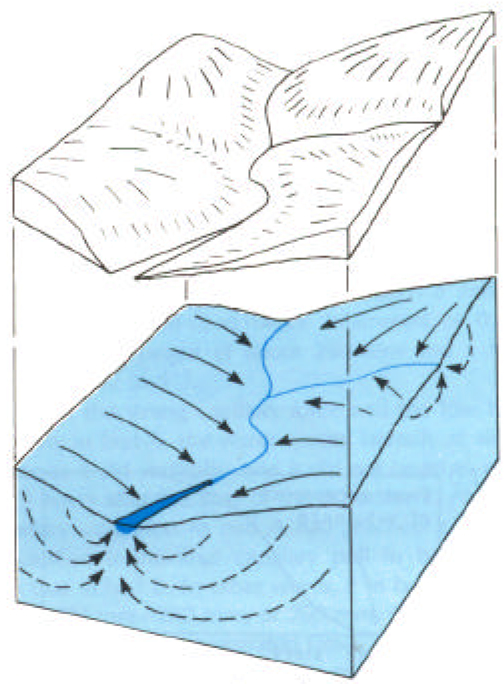
In areas where ground water is used for domestic and other needs requiring good-quality water, septic tanks, sanitary landfills, waste ponds, and other waste-disposal sites should not be located uphill from supply wells.

The potentiometric surface of confined aquifers, like the water table, also slopes from recharge areas to discharge areas. Shallow confined aquifers, which are relatively common along the Atlantic Coastal Plain, share both recharge and discharge areas with the surficial unconfined aquifers. This sharing may not be the case with the deeper confined aquifers. The principal recharge areas for these are probably in their outcrop areas near the western border of the Coastal Plain, and their discharge areas are probably near the heads of the estuaries along the major streams. Thus, movement of water through these aquifers is in a general west to east direction, where it has not been modified by withdrawals.

In the western part of the conterminous United States, and especially in the alluvial basins region, conditions are more variable than those described above. In this area, streams flowing from mountain ranges onto alluvial plains lose water to the alluvial deposits; thus, ground water in the upper part of the saturated zone flows down the valleys and at an angle away from the streams.

Ground water is normally hidden from view; as a consequence, many people have difficulty visualizing its occurrence and movement. This difficulty adversely affects their ability to understand and to deal effectively with ground-water-related problems. This problem can be partly solved through the use of flow nets, which are one of the most effective means yet devised for illustrating conditions in ground-water systems.



Arrows show direction of ground-water movement

# GROUND-WATER FLOW NETS

*Flow nets* consist of two sets of lines. One set, referred to as *equipotential lines*, connects points of equal head and thus represents the height of the water table, or the potentiometric surface of a confined aquifer, above a datum plane. The second set, referred to as *flow lines*, depicts the idealized paths followed by particles of water as they move through the aquifer. Because ground water moves in the direction of the steepest hydraulic gradient, flow lines in isotropic aquifers are perpendicular to equipotential lines—that is, flow lines cross equipotential lines at right angles.

There are an infinite number of equipotential lines and flow lines in an aquifer. However, for purposes of flow-net analysis, only a few of each set need be drawn. Equipotential lines are drawn so that the drop in head is the same between adjacent pairs of lines. Flow lines are drawn so that the flow is equally divided between adjacent pairs of lines and so that, together with the equipotential lines, they form a series of "squares."

Flow nets not only show the direction of ground-water movement but can also, if they are drawn with care, be used to estimate the quantity of water in transit through an aquifer. According to Darcy's law, the flow through any "square" is

$$q = Kbw \left(\frac{dh}{dl}\right) \qquad (1)$$

and the total flow through any set or group of "squares" is

$$Q = nq \qquad (2)$$

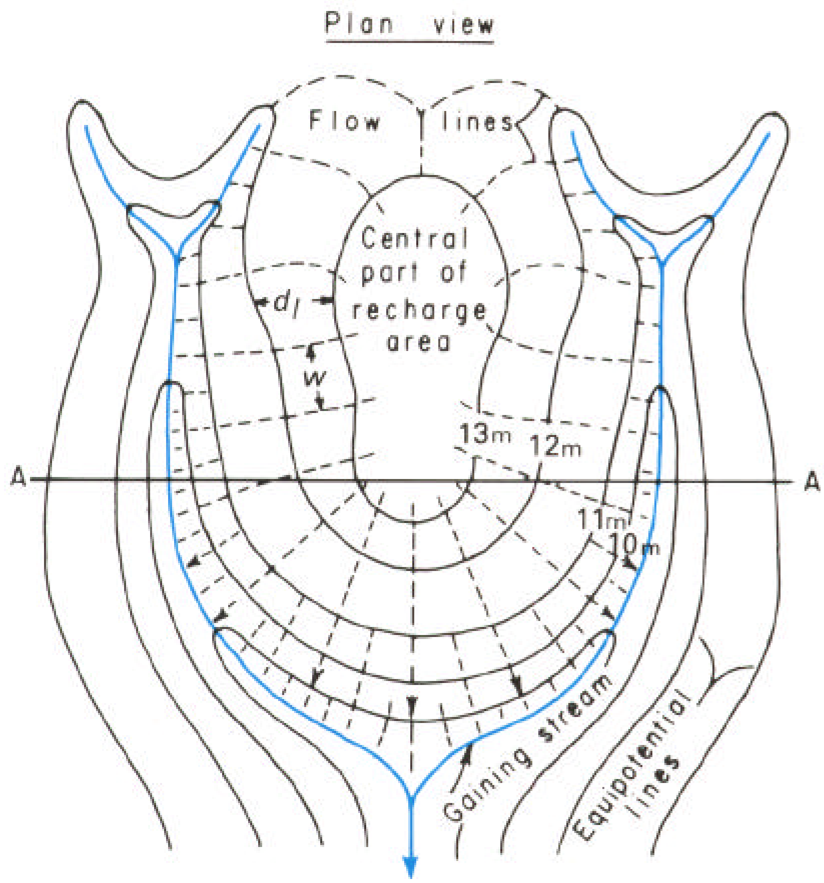
where $K$ is hydraulic conductivity, $b$ is aquifer thickness at the midpoint between equipotential lines, $w$ is the distance between flow lines, $dh$ is the difference in head between equipotential lines, $dl$ is the distance between equipotential lines, and $n$ is the number of squares through which the flow occurs.

Drawings 1 and 2 show a flow net in both plan view and cross section for an area underlain by an unconfined aquifer composed of sand. The sand overlies a horizontal confining bed, the top of which occurs at an elevation 3 m above the datum plane. The fact that some flow lines originate in the area in which heads exceed 13 m indicates the presence of recharge to the aquifer in this area. The relative positions of the land surface and the water table in sketch 2 suggest that recharge occurs throughout the area, except along the stream valleys. This suggestion is confirmed by the fact that flow lines also originate in areas where heads are less than 13 m.

As sketches 1 and 2 show, flow lines originate in recharge areas and terminate in discharge areas. Closed contours (equipotential lines) indicate the central parts of recharge areas but do not normally indicate the limits of the areas.

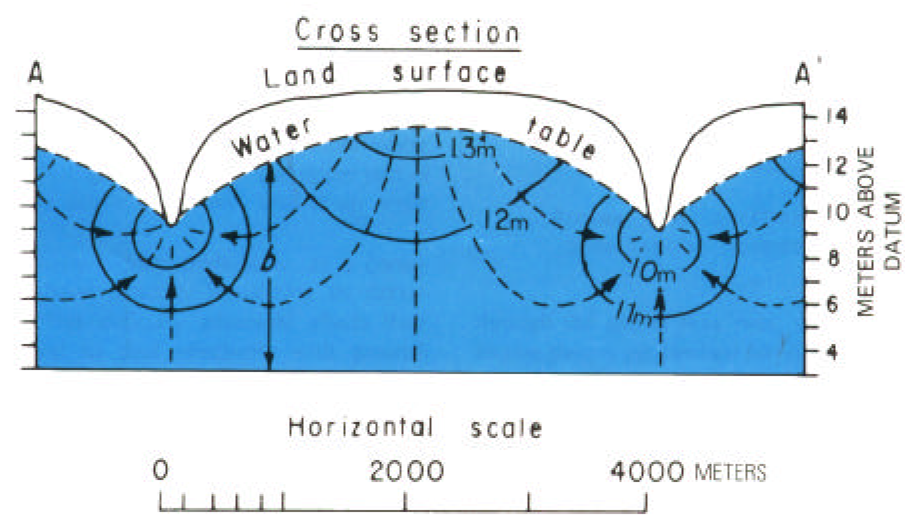
In the cross-sectional view in sketch 2, heads decrease downward in the recharge area and decrease upward in the discharge area. Consequently, the deeper a well is drilled in a recharge area, the lower the water level in the well stands below land surface. The reverse is true in discharge areas. Thus, in a discharge area, if a well is drilled deeply enough in an unconfined aquifer, the well may flow above land surface. Consequently, a flowing well does not necessarily indicate artesian conditions.

Drawings 3 and 4 show equipotential lines and flow lines in the vicinity of a stream that gains water in its headwaters and loses water as it flows downstream. In the gaining reaches, the equipotential lines form a V pointing upstream; in the losing reach, they form a V pointing downstream.



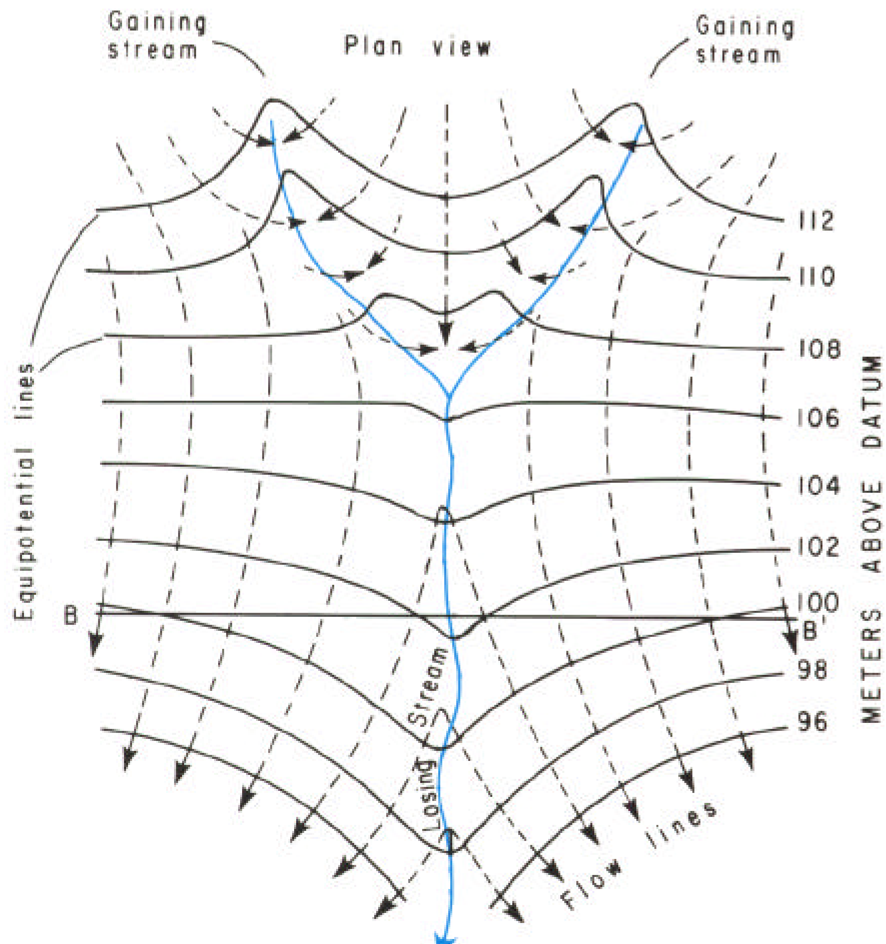
Plan view
Flow lines
Central part of recharge area
$d_l$
w
13m
12m
11m
10m
A
A'
Gaining stream
Equipotential lines


(1)

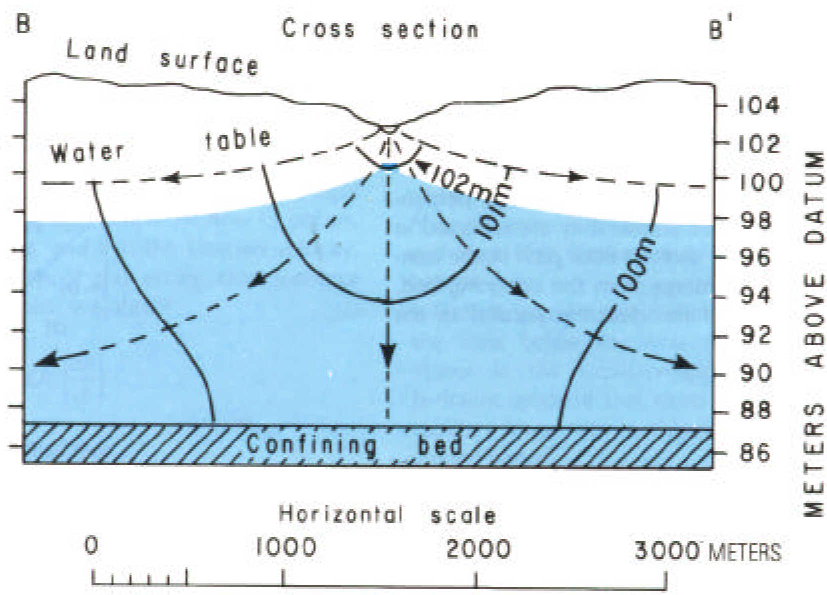


Cross section
A
A'
Land surface
Water table
13m
12m
10m
11m
b
14
12
10
8
6
4
METERS ABOVE DATUM
Horizontal scale
0
2000
4000 METERS


(2)


Gaining stream
Plan view
Gaining stream
112
110
108
106
104
102
100
98
96
METERS ABOVE DATUM
Equipotential lines
B
B'
Losing Stream
Flow lines

(3)


B
Cross section
B'
Land surface
Water table
102m
101m
100m
104
102
100
98
96
94
92
90
88
86
METERS ABOVE DATUM
Confining bed
Horizontal scale
0
1000
2000
3000 METERS

(4)

$$\Delta\varphi = (\varphi_{\max} - \varphi_{\min})/n; \qquad \Delta Q = Q_{\text{total}}/m.$$

$$Q_{\text{total}} = m\,\Delta Q = mK\,\Delta n\,\frac{\Delta\varphi}{\Delta s} = mK\,\Delta n\,\frac{\varphi_{\max} - \varphi_{\min}}{n\,\Delta s} = \frac{m}{n}\frac{\Delta n}{\Delta s}K(\varphi_{\max} - \varphi_{\min}) \tag{6.6.4}$$

or:

$$\frac{\Delta n}{\Delta s} = \frac{n}{m}\frac{Q_{\text{total}}}{K(\varphi_{\max} - \varphi_{\min})}. \tag{6.6.5}$$

Again, the ratio $\Delta n/\Delta s$ (in general $\neq 1$) remains constant throughout the flow domain. Sometimes $n$ and $m$ (not necessarily integers) are chosen such that squares with $\Delta s = \Delta n$ are obtained. Then:

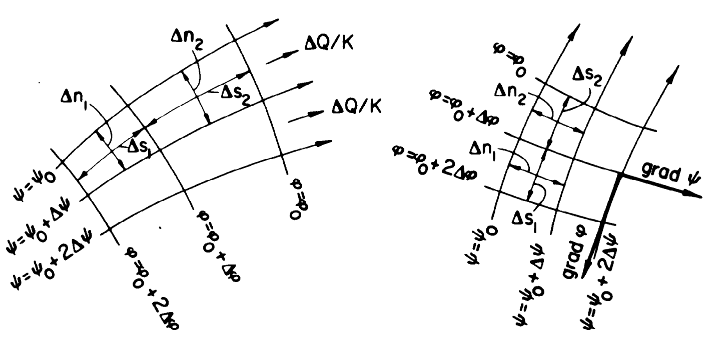
$$Q_{\text{total}} = \frac{m}{n}K(\varphi_{\max} - \varphi_{\min}). \tag{6.6.6}$$

In a zoned porous medium, where $K$ varies abruptly from $K_1$ to $K_2$ along a specified boundary, we have refraction of both equipotentials and streamlines across this boundary (par. 7.1.10). In plane flow through a porous medium of constant thickness, $K$ should be replaced by the transmissivity $T = Kb$, with $Q$ referring to the discharge through the entire thickness.

It is important to emphasize that when a homogeneous isotropic flow domain has boundaries on which the boundary conditions are given in terms of $\varphi$, and the flow is steady, described by the Laplace equation, the flow net is independent of the hydraulic conductivity. It depends only on the geometry of the flow domain. This is also true when one of the boundaries is a phreatic surface.

### 6.6.2 The Ground Water Contour Map

We shall use here the term ground water flow (or aquifer flow) in the sense discussed in section 6.4, i.e., a flow that is essentially horizontal. In general, the only information that we have about what happens in the aquifer is the piezometric heads observed and recorded at observation wells. In a phreatic aquifer these heads give the elevation




(a) $\Delta n \neq \Delta s$ (b) $\Delta n = \Delta s$

FIG. 6.6.2. A portion of a flow-net.

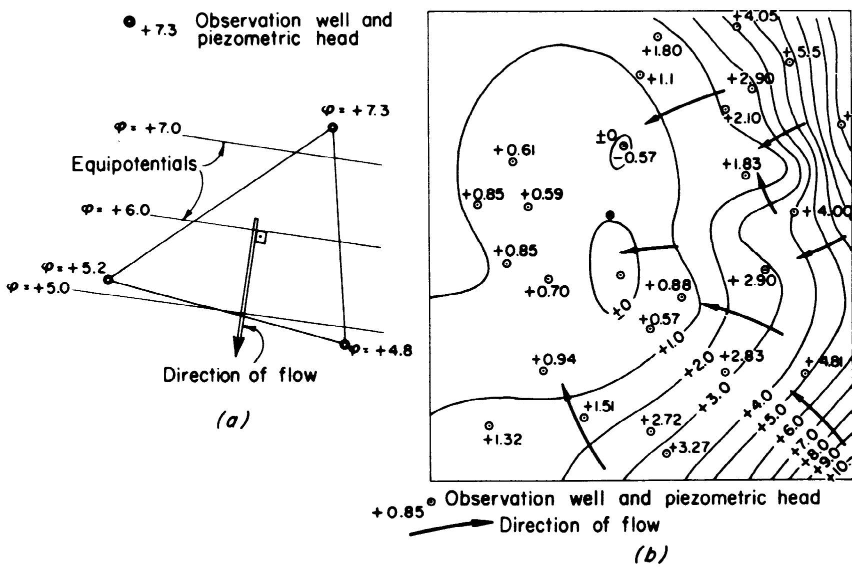



FIG. 6.6.3. Ground water contour map with arrows indicating directions of flow.

of the water table at the observation wells (neglecting the capillary fringe and employing the Dupuit assumption (chap. 8)). Except for special cases, such as when impervious faults are known to be present in the aquifer, we may assume that the piezometric head is a continuous function of the plane coordinates. Hence, using the information on values of $\varphi$ at discrete points (observation wells), smooth equipotential curves $\varphi = \text{const}$ are drawn, usually by linear interpolation, over the flow domain enclosed between the observation wells. Care must be taken to use only the information from wells tapping the aquifer being considered. An example of drawing equipotential curves, also called *ground water contours*, is given in figures 6.6.3a and 6.6.3b. The contours give the shape of the piezometric surface (or phreatic surface in a water table aquifer).

Once the equipotentials are drawn, a family of streamlines that are everywhere orthogonal to the ground water contours (assuming that the aquifer is isotropic) can be drawn. In certain cases the information on piezometric heads is insufficient to draw closely spaced contours that permit the exact drawing of streamlines. In such cases we draw on the contour map only arrows that indicate direction of flow (fig. 6.6.3b).

In unsteady flow the contour map gives an instantaneous picture of what happens in the aquifer. When interpreting a contour map (as to regions of recharge and discharge, etc.), care must be taken to distinguish between steady and unsteady situations. For example, figure 6.6.4 shows a mound in the water table of a phreatic aquifer. If this map describes steady flow, there *must be* a source of water inside the area enclosed by the + 5 contour curve. If, however, the flow is unsteady and this is only an instantaneous picture that changes with time, the water leaving the area



FIG.

along the streamlines is ta
ubiquitous drop in the wa

In reservoir engineering
piezometric head.

**6.7 The Partial Differen**
**Incompressible Fluid i**

This section is a continua
the present section we co
density varies as a result
in it. From the discussion
the concept of the stream
inhomogeneous fluids. It
partial differential equatic
inhomogeneous fluid is di

6.7.1 Two-Dimensional Fl

Consider flow in a two-d
homogeneous and isotropi
the specific discharge is gi
$\Psi$, and the relationship (

By cross-differentiation

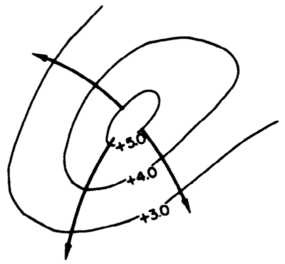



FIG. 6.6.4. A mound in the phreatic surface.

along the streamlines is taken from storage within the aquifer itself, producing a ubiquitous drop in the water table.

In reservoir engineering contours are usually drawn of pressure rather than of piezometric head.

## 6.7 The Partial Differential Equations Describing Flow of an Inhomogeneous Incompressible Fluid in Terms of $\Psi$

This section is a continuation of the discussion presented in paragraph 6.5.4. In the present section we consider isothermal flow of an incompressible fluid whose density varies as a result of changes in the concentration of some solute dissolved in it. From the discussion presented at the beginning of section 6.5, it follows that the concept of the stream function $\Psi$ is a general one, applicable also to flow of inhomogeneous fluids. It would therefore be advantageous to express the required partial differential equations in terms of this function. The problem of flow of an inhomogeneous fluid is discussed in detail in chapter 10.

### 6.7.1 Two-Dimensional Flow

Consider flow in a two-dimensional vertical $xz$ plane. The medium is assumed homogeneous and isotropic ($k$ = constant). Since we allow both $\rho$ and $\mu$ to vary, the specific discharge is given by (4.7.17). With the definition of the stream function $\Psi$, and the relationship (6.5.43), we obtain:

$$q_x = -\frac{k}{\mu}\frac{\partial p}{\partial x} = -\frac{\partial \Psi}{\partial z}$$

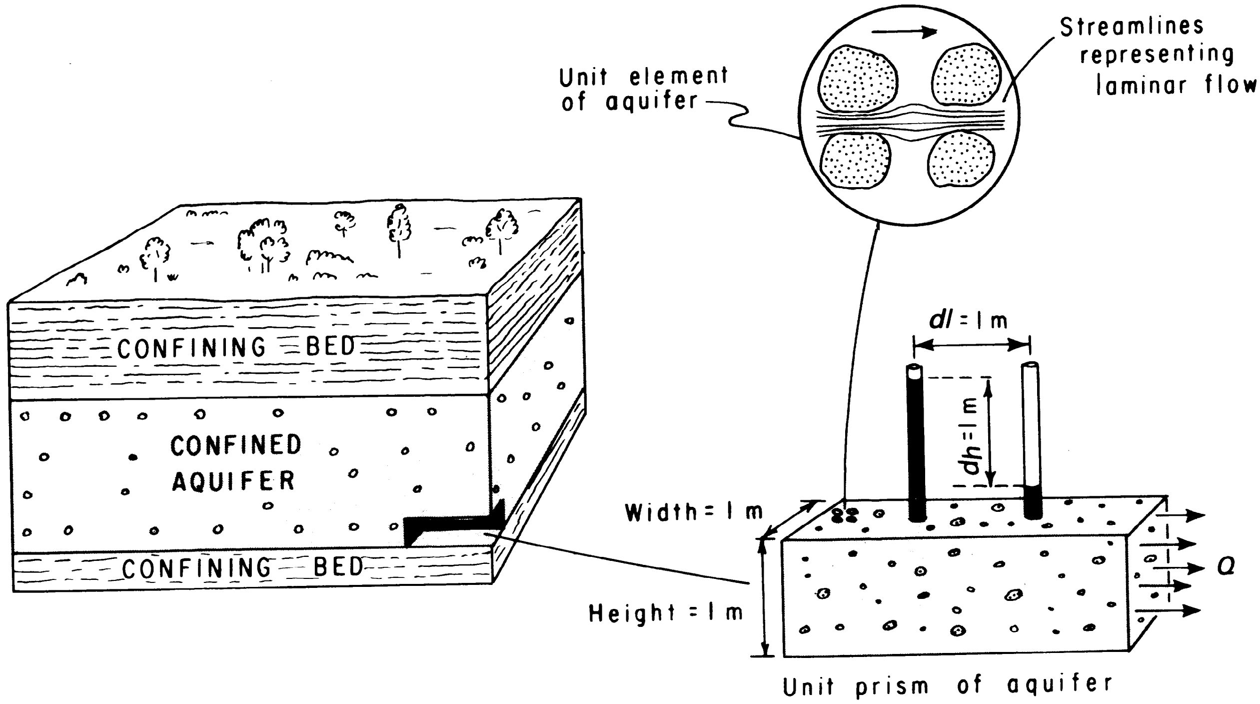
$$q_z = -\frac{k}{\mu}\frac{\partial p}{\partial z} - \frac{k\rho g}{\mu} = +\frac{\partial \Psi}{\partial x}. \tag{6.7.1}$$

By cross-differentiation of (6.7.1) we obtain:

$$\frac{\partial q_x}{\partial z} - \frac{\partial q_z}{\partial x} = -\frac{\partial(k/\mu)}{\partial z}\frac{\partial p}{\partial x} - \frac{k}{\mu}\frac{\partial^2 p}{\partial x\,\partial z} + \frac{\partial(k/\mu)}{\partial x}\frac{\partial p}{\partial z} + \frac{k}{\mu}\frac{\partial^2 p}{\partial x\,\partial z} + \frac{\partial(k\rho g/\mu)}{\partial x}$$

$$= -\left(\frac{\partial^2 \Psi}{\partial z^2} + \frac{\partial^2 \Psi}{\partial x^2}\right) \tag{6.7.2}$$

# HYDRAULIC CONDUCTIVITY




(1)

Aquifers transmit water from recharge areas to discharge areas and thus function as porous conduits (or pipelines filled with sand or other water-bearing material). The factors controlling ground-water movement were first expressed in the form of an equation by Henry Darcy, a French engineer, in 1856. Darcy's law is

$$Q = KA\left(\frac{dh}{dl}\right) \tag{1}$$

where $Q$ is the quantity of water per unit of time; $K$ is the hydraulic conductivity and depends on the size and arrangement of the water-transmitting openings (pores and fractures) and on the dynamic characteristics of the fluid (water) such as kinematic viscosity, density, and the strength of the gravitational field; $A$ is the cross-sectional area, at a right angle to the flow direction, through which the flow occurs; and $dh/dl$ is the hydraulic gradient.[1]

Because the quantity of water ($Q$) is directly proportional to the hydraulic gradient ($dh/dl$), we say that ground-water flow is *laminar*—that is, water particles tend to follow discrete streamlines and not to mix with particles in adjacent streamlines (1). (See "Ground-Water Flow Nets.")

[1]Where hydraulic gradient is discussed as an independent entity, as it is in "Heads and Gradients," it is shown symbolically as $h_L/L$ and is referred to as head loss per unit of distance. Where hydraulic gradient appears as one of the factors in an equation, as it does in equation 1, it is shown symbolically as $dh/dl$ to be consistent with other ground-water literature. The gradient $dh/dl$ indicates that the unit distance is reduced to as small a value as one can imagine, in accordance with the concepts of differential calculus.

If we rearrange equation 1 to solve for $K$, we obtain

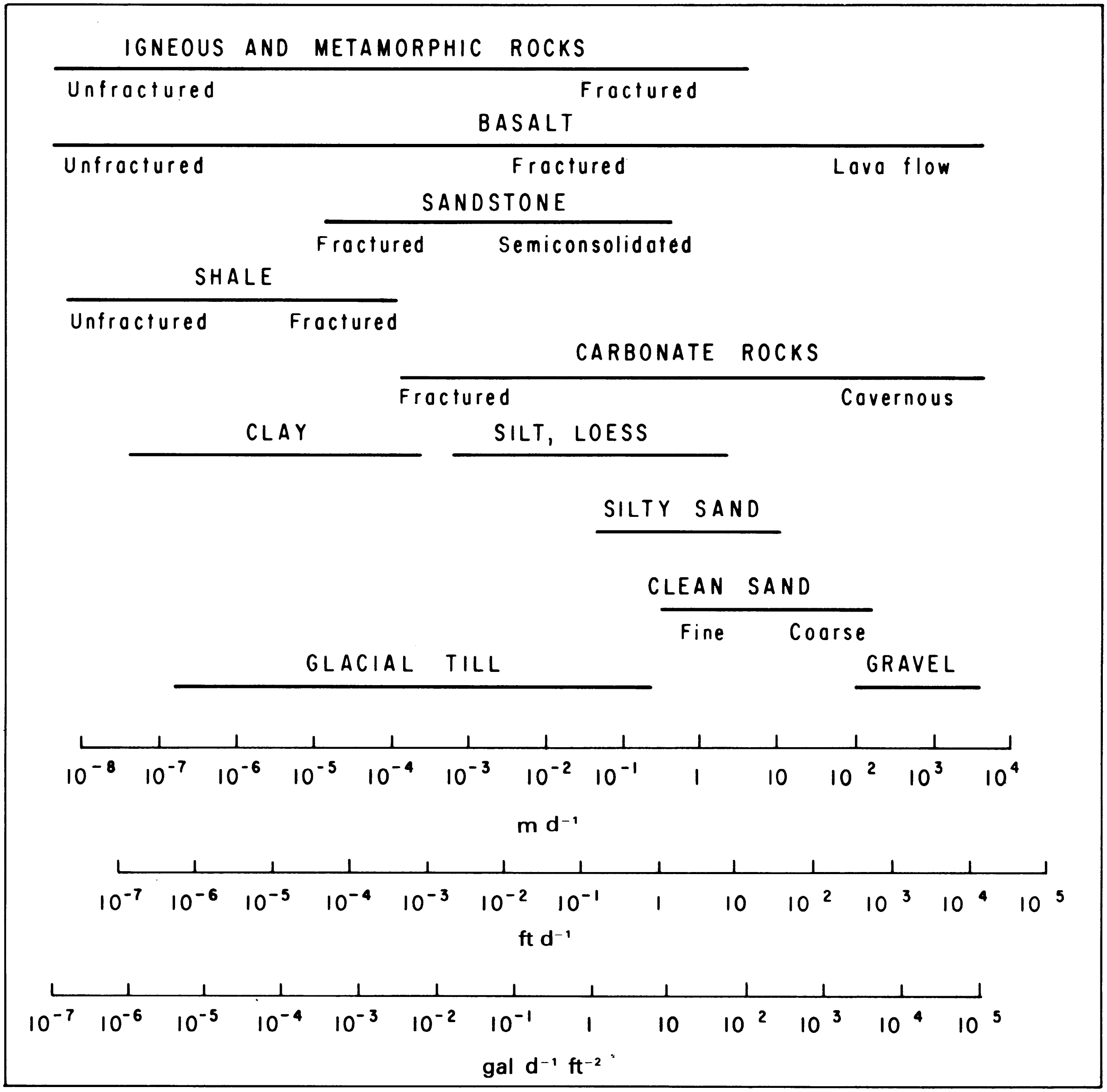
$$K = \frac{Qdl}{Adh} = \frac{(\mathrm{m^3\,d^{-1}})(\mathrm{m})}{(\mathrm{m^2})(\mathrm{m})} = \frac{\mathrm{m}}{\mathrm{d}} \tag{2}$$

Thus, the units of hydraulic conductivity are those of velocity (or distance divided by time). It is important to note from equation 2, however, that the factors involved in the definition of hydraulic conductivity include the volume of water ($Q$) that will move in a unit of time (commonly, a day) under a unit hydraulic gradient (such as a meter per meter) through a unit area (such as a square meter). These factors are illustrated in sketch 1. Expressing hydraulic conductivity in terms of a unit gradient, rather than of an actual gradient at some place in an aquifer, permits ready comparison of values of hydraulic conductivity for different rocks.

Hydraulic conductivity replaces the term "field coefficient of permeability" and should be used in referring to the water-transmitting characteristic of material in quantitative terms. It is still common practice to refer in qualitative terms to "permeable" and "impermeable" material.

The hydraulic conductivity of rocks ranges through 12 orders of magnitude (2). There are few physical parameters whose values range so widely. Hydraulic conductivity is not only different in different types of rocks but may also be different from place to place in the same rock. If the hydraulic conductivity is essentially the same in any area, the aquifer in

Hydraulic Conductivity of Selected Rocks




(2)

that area is said to be *homogeneous*. If, on the other hand, the hydraulic conductivity differs from one part of the area to another, the aquifer is said to be *heterogeneous*.

Hydraulic conductivity may also be different in different directions at any place in an aquifer. If the hydraulic conductivity is essentially the same in all directions, the aquifer is said to be *isotropic*. If it is different in different directions, the aquifer is said to be *anisotropic*.

Although it is convenient in many mathematical analyses of ground-water flow to assume that aquifers are both homogeneous and isotropic, such aquifers are rare, if they exist at all. The condition most commonly encountered is for hydraulic conductivity in most rocks and especially in unconsolidated deposits and in flat-lying consolidated sedimentary rocks to be larger in the horizontal direction than it is in the vertical direction.

Constant Head Permeameter

The constant head permeameter is simply a device that exactly duplicates Darcy's original experiment. A sample is placed in the permeameter and constant head gradient is maintained across the sample. The hydraulic conductivity is inferred by direct application of Darcy's Law.

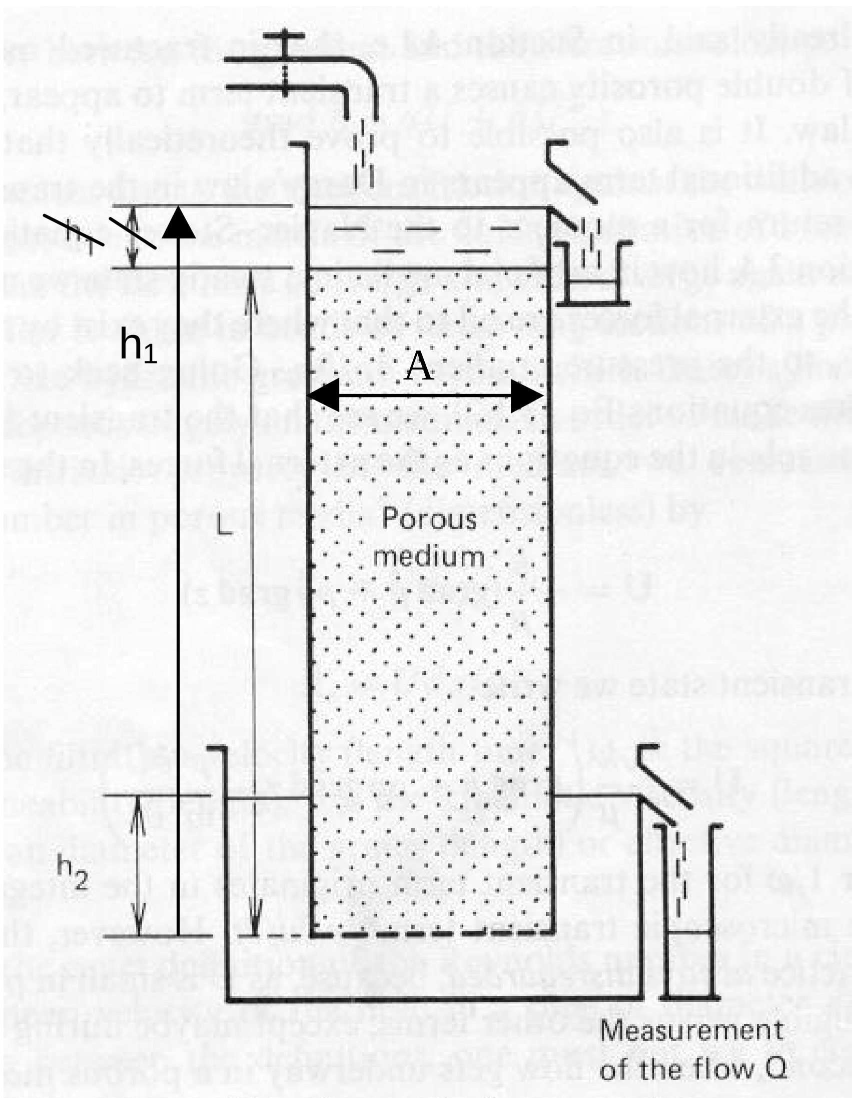



Figure 6.1 Schematic of Constant Head Permeameter

Figure 6.1 is a schematic diagram of a constant head permeameter. A sample of porous medium is placed the device. The length of the sample is *L*, the cross sectional area of the sample is *A*. The head is measured at the inlet and outlet of the sample. $h_1$ is the head at the inlet of the sample as measured from the outlet of the sample. $h_2$ is the head at the outlet of the sample. *Q* is the rate of flow through the sample measured by recording the time required for a known volume of water to pass through the sample.

Darcy's law for this experiment is

$$Q = KA\frac{h_1 - h_2}{L} \qquad (6.1).$$

Rearranging this equation gives the following formula for hydraulic conductivity;

$$K = \frac{Q}{A}\frac{L}{h_1 - h_2} \qquad (6.2).$$

These computations can be conviently entered into a spreadsheet program. MODEL2_CHPerm.xls is a spreadsheet program that implements these equations for a constant head permeameter.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **Constant Head Permeameter** | | | | | | | |
| 2 | **Temperature (oC)** | | 20 | | | | | |
| 3 | | | | | | | | |
| 4 | **Data** | | | Cell Formulas | | **Diagram** | | |
| 5 | Area (A) | 225 | cm^2 | | | | | |
| 6 | Length (L) | 25 | cm | | | | | |
| 7 | Volume(V) | 50 | cm^3 | | | | | |
| 8 | Time (t) | 456 | sec | | | | | |
| 9 | Head Loss (dh) | 15 | cm | | | | | |
| 10 | | | | | | | | |
| 11 | density | 0.998203 | g/cm^3 | | | | | |
| 12 | viscosity | 0.01005 | g/(sec cm) | | | | | |
| 13 | gravitational | 980 | cm/sec^2 | | | | | |
| 14 | acceleration | | | | | | | |
| 15 | | | | | | | | |
| 16 | **Computed Values** | | | B5/B6 | | | | |
| 17 | Flow(Q) | 0.109649123 | =V/t | | | | | |
| 18 | | | | (B15/B3)*(B4/B7) | | | | |
| 19 | Hydraulic (K) | 8.1E-04 | cm/sec | | | | | |
| 20 | Conductivity | | | | | | | |
| 21 | | | | B17*B10/(B9*B11) | | | | |
| 22 | Intrinsic (k) | 8.34435E-09 | cm^2 | | | | | |
| 23 | Permeability | | | | | | | |

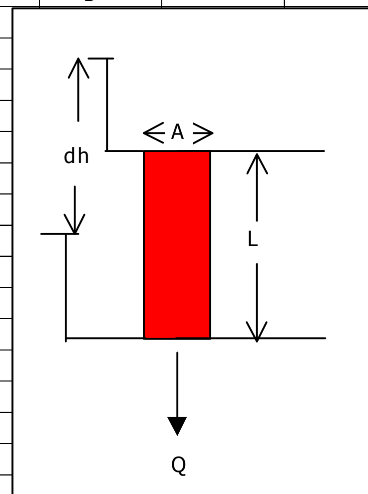



Figure 6.2 Example of MODEL2_CHPerm.xls

To use the program the data is entered for a particular analysis. If intrinsic permeability is to be calculated then the density and viscosity of water should be determined from a tabulation and entered into the spreadsheet.

Falling Head Permeameter

The falling head permeameter is used when the hydraulic conductivity of the porous material is small. In this case the constant head permeameter cannot easily generate a high enough gradient to produce measurable flow in a reasonable amount of time.

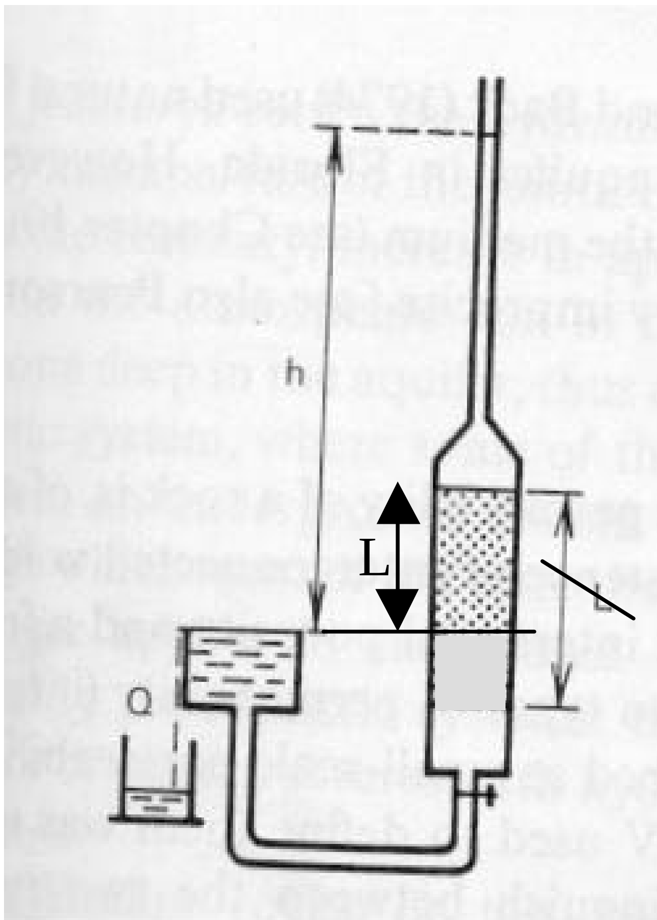


Figure 6.2 Falling-head permeameter

Figure 6.2 is a schematic diagram of a falling head permeameter. A sample of porous medium is placed the device. The length of the sample is *L*, the cross sectional area of the sample is *A*. A smaller area tube rises above the sample. This tube provides the driving force required to move water through the porous sample with reasonably small water volume. The area of this tube is *a*. The head is measured at the inlet of the sample as the height of water in the tube above the sample, *h(t)*. *Q(t)* is the varying rate of flow through the sample. Observe that in a falling head permeameter, both the head and the flow rate vary with time.

Darcy's law for this situation is

$$Q(t) = KA\frac{h(t)}{L} \qquad (6.3).$$

A volume balance for the device leads to the following additional expression for discharge rate;

$$Q(t) = -a\frac{dh(t)}{dt} \qquad (6.4).$$

Equating these two expressions gives a differential equation for flow in the porous medium;

$$KA\frac{h(t)}{L} = -a\frac{dh(t)}{dt} \qquad (6.5)$$

Separation of variables produces the following expression that can be integrated to relate head and time;

$$\frac{dh(t)}{h(t)} = -\frac{KA}{aL}dt \qquad (6.6).$$

Integration of this equation will provide a formula that relates *h(t)* and *t* in terms of hydraulic conductivity, tube and sample areas and the length of the sample.

$$\int\frac{dh(t)}{h(t)} = -\frac{KA}{aL}\int dt \qquad (6.7).$$

The result of the integration is

$$\ln(h(t)) + C = -\frac{KA}{aL}t \qquad (6.8).$$

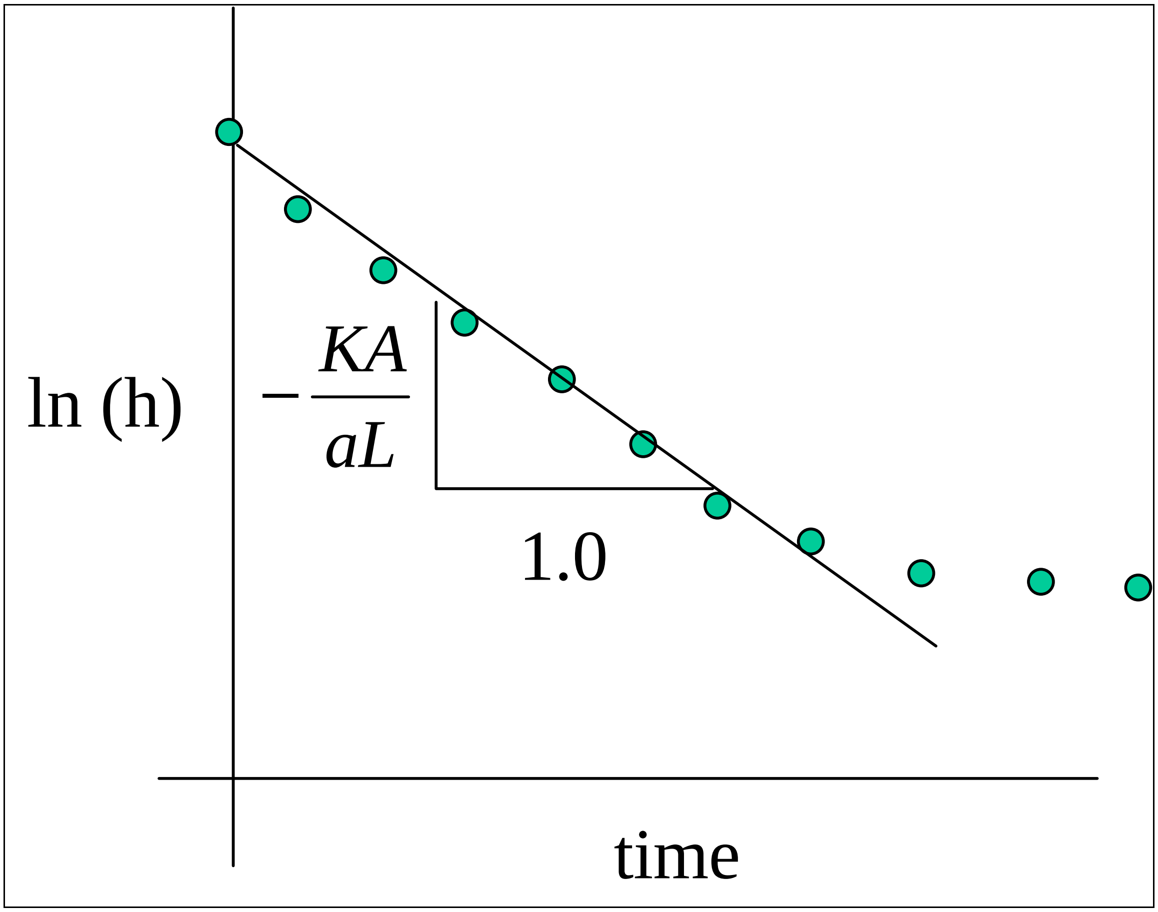
To evaluate the constant of integration one considers at $t=0$, $h=h_0$.

$$\ln(h(t)) + C = 0 \Rightarrow C = -\ln(h_0) \quad (6.9).$$

So the equation that describes the falling head permeameter is

$$\ln(h(t)) = \ln(h_0) - \frac{KA}{aL}t \quad (6.10).$$

This equation is linear in time, so that a plot of the logarithm of *h* versus time should be a straight line, the slope of the line is proportional to the hydraulic conductivity, *K*.




MODEL3_FHPerm.xls is a spreadsheet program that implements the falling head equations.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **Falling Head Permeameter** | | | | | | | |
| 2 | **Data** | | | | | | | |
| 3 | Tube Diameter (d) | 2 | cm | | | | | |
| 4 | Sample Diameter (D) | 10 | cm | | | | | |
| 5 | Sample Length (L) | 15 | cm | | | | | |
| 6 | Initial head ( h(t=0) ) | 5 | cm | | | | | |
| 7 | Final Head ( h(t) ) | 0.5 | cm | | | | | |
| 8 | Elapsed time ( t ) | 528 | sec | | | | | |
| 9 | | | | | | | | |
| 10 | density | 0.999099 | g/cm^3 | | | | | |
| 11 | viscosity | 0.011404 | g/(sec cm) | | | | | |
| 12 | gravitational | 980 | cm/sec^2 | | | | | |
| 13 | acceleration | | | | | | | |
| 14 | | | | | | | | |
| 15 | **Computed Values** | | | | | | | |
| 16 | Volume(Q) | 14.13717 | | | | | | |
| 17 | | | | | | | | |
| 18 | Hydraulic (K) | 0.002617 | cm/sec | | | | | |
| 19 | Conductivity | | | | | | | |
| 20 | | | | | | | | |
| 21 | Intrinsic (k) | 3.05E-08 | cm^2 | | | | | |
| 22 | Permeability | | | | | | | |
| 23 | | | | | | | | |
| 24 | | | | | | | | |
| 25 | | | | | | | | |

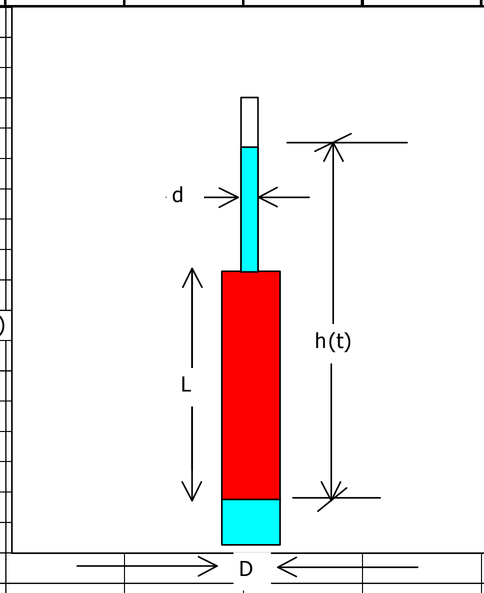



Figure 6.3 Example of MODEL3_FHPerm.xls

Field Methods

In principle the measurement of hydraulic conductivity in the field should simply extend concepts of the permeameter devices, however the measurement of flow rates in field applications is not trivial.

Wells and pumps are used in aquifer pumping tests, a subject of later articles. If the flow rate is known in the field, and two wells are known to be located on the same flow line, then Darcy's law can be applied directly to infer the hydraulic conductivity from head measurements at two wells, using concepts identical to the constant head permeameter.

Infiltration tests, and auger hole tests can be used to infer hydraulic conductivity in a manner analogous to the falling head permeameter test.

Tracers

A tracer is a chemical introduced into an aquifer that is easy to detect in very low concentrations and is acceptable to intentionally introduce into an aquifer. Existing pollutants already in the aquifer can be used as tracers, and certain types of isotope ratios can be used as natural tracers.

A tracer introduced between two wells that lie on the same flow line can be used to estimate the hydraulic conductivity of the aquifer between the two wells..

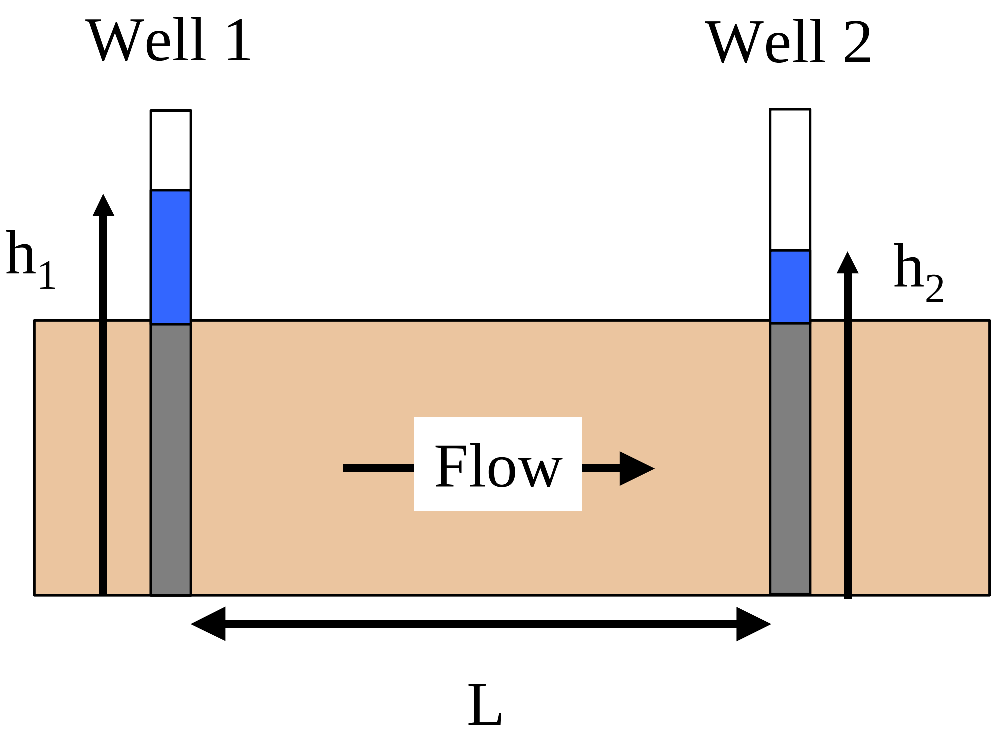



Figure 6.4 Schematic of Confined Aquifer System for Tracer Test.

A tracer is introduced at well 1, for example a kilogram of rock salt is added to the well suddenly. The concentration of chloride will increase immediately in well 1 proportional to the amount of salt added.

As time elapses, the concentrated salt water moves downstream towards well 2. The water mixes with the aquifer water so that the tracer parcel (labeled water) spreads out, so that at well 2 we do not expect to recover all the salt mass, nor see the same peak concentration. We do expect to observe a peak concentration whose arrival time is proportional to the average velocity of the pore water between the two wells.

Figure 6.5 is a plot of how the concentration histories might appear. The spreading provides useful information of mixing, but that is a later subject. The time of travel of the tracer peak provides the information required to infer hydraulic conductivity.

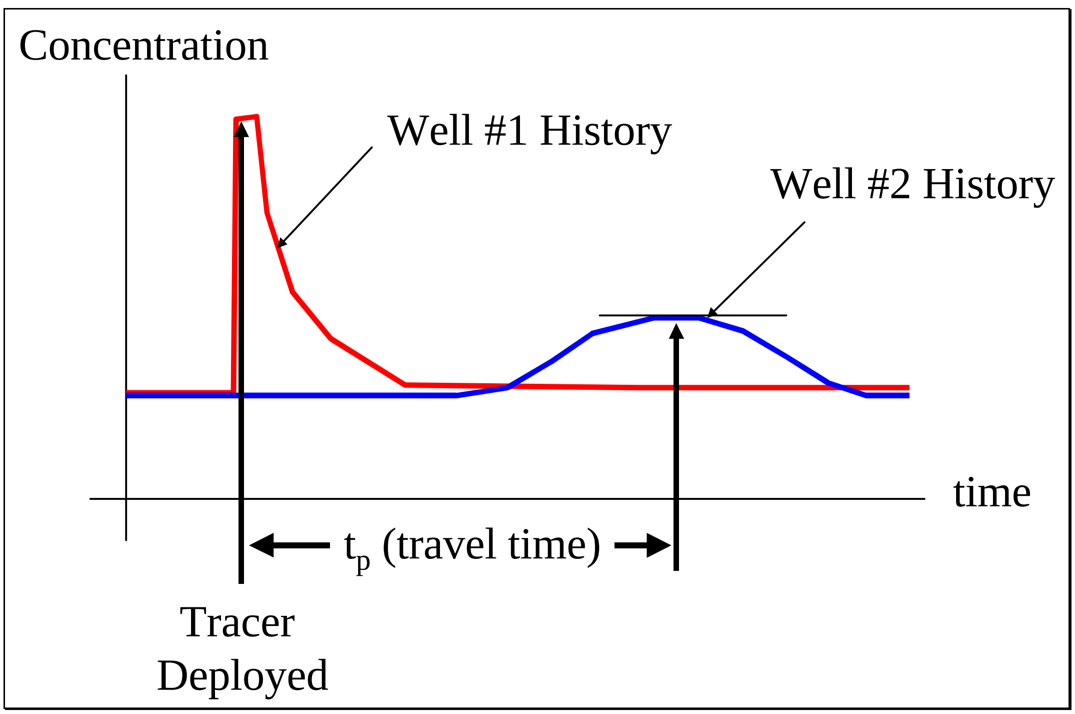



Figure 6.5 Tracer Concentration Histories

Darcy's Law in terms of specific discharge is

$$Q = UA \text{ where } U = K\frac{h_1 - h_2}{L} \quad (6.11).$$

The pore water velocity that is the same as the tracer velocity is the specific discharge divided by the formation porosity, *n*;

$$u = \frac{U}{n} = \frac{K}{n}\frac{h_1 - h_2}{L} \qquad (6.12).$$

The tracer travels a distance *L* in time period $t_p$. So the tracer velocity is $L/t_p$. This velocity should be the same as the pore water velocity, so by equating these two velocities and solving for *K* we arrive at a formula for estimating *K* from a tracer study.

$$u = \frac{K}{n}\frac{h_1 - h_2}{L} = \frac{L}{t_p}$$
$$\Rightarrow K = \frac{nL^2}{t_p(h_1 - h_2)} \qquad (6.13).$$