CIVE 6361 Groundwater Hydrology

Basic Concepts

Porosity

- Soils and aquifers are composed of both solids and voids.
- A porous medium is a medium composed of solids and voids, where the dimensions of the void space are such that capillary forces are significant.

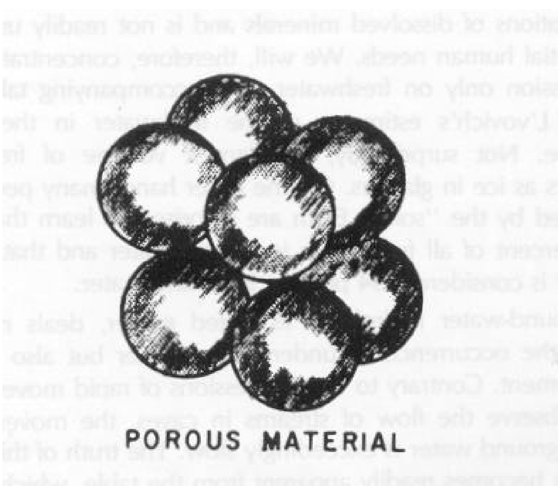


Figure 2.1 Porous Material of Packed Spheres

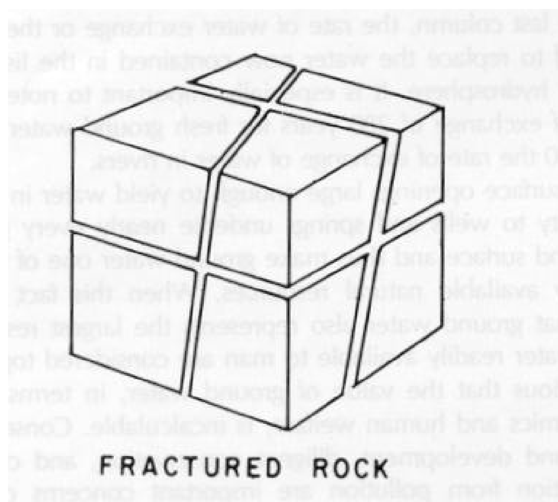


Figure 2.2 Porous Material of Fractured Solid

- The portion of porous material not consisting of solids is called the void space.

- The void space contains air, water, and other fluids.
- Connected voids serve as conduits for fluids to flow in a porous medium.

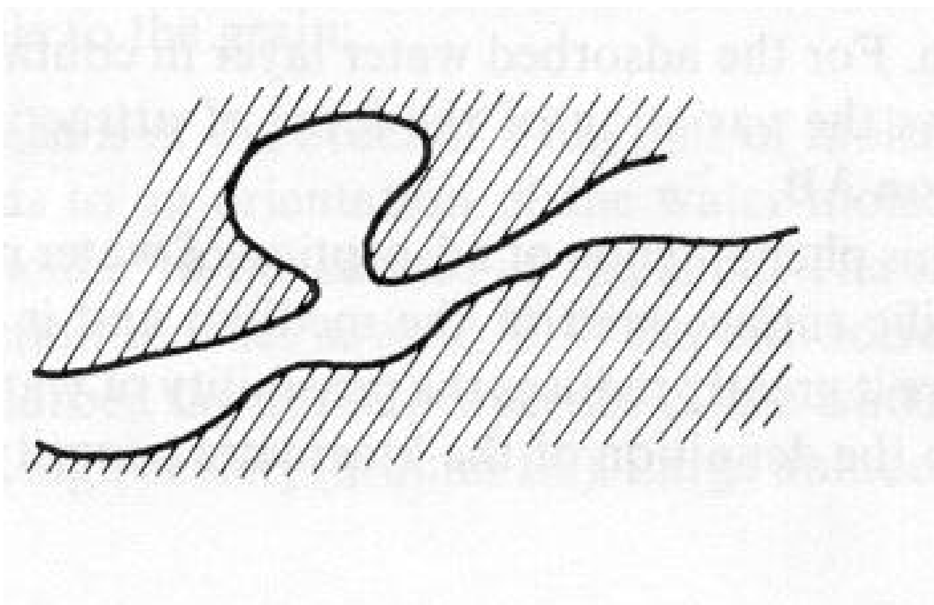
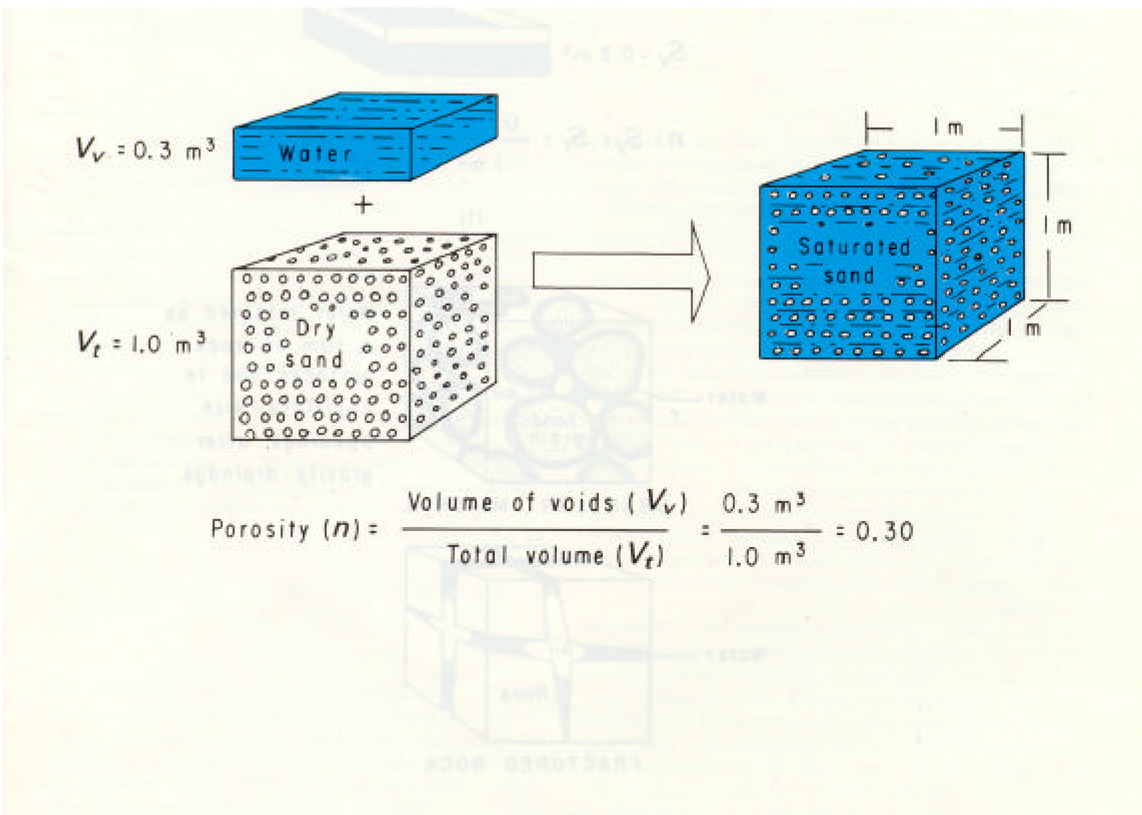


Figure 2.3 Dead-end pore attached to interconnected pore space.

- Soils are among the most porous of natural materials because soil particles tend to form loose clumps and because of the presence of root holes and animal burrows.

CIVE 6361 Groundwater Hydrology

- Porosity of unconsolidated deposits depends on the range in grain size (sorting) and on the shape of the rock particles but not on their size.
- Fine-grained materials tend to be better sorted and, thus, tend to have the largest porosity.



- The porosity of a specimen of porous material is the ratio of open pore space volume in the specimen to the bulk volume of the specimen.

$$n = \frac{V_{void}}{V_{total}}$$

- In groundwater studies one is normally interested in the interconnected, or effective, porosity, which is the ratio of the volume of interconnected pore space - excluding completely isolated pores - to the bulk volume.
- In most work, the term “porosity” usually refers to the interconnected or effective porosity.

Void Ratio

- The ratio of void volume to solid volume is called the void ratio.

$$e = \frac{V_{void}}{V_{solids}}$$

- The pore space or void space can fill with many different types of fluids.

- Groundwater is said to occur under saturated conditions when all the interconnected pore space is completely filled with water, and it occurs under unsaturated conditions when a part of the pores contain air (or other gas).

CIVE 6361 Groundwater Hydrology

- In unsaturated flow problems, the degree of saturation is often expressed as a percentage of the interconnected pore space.

Moisture Content

- The volume fraction of water in a sample is called the moisture content or volumetric water content.

$$q_{water} = \frac{V_{water}}{V_{total}}$$

- The fraction of pore space occupied by water in a sample is called the volumetric water saturation or the saturation ratio.

$$S_{water} = \frac{V_{water}}{V_{voids}}$$

The moisture content plays a crucial role in the flow of water in partially saturated media.

Discharge (Fluid mechanics definition)

- Discharge is the volume of flow per unit time in a fluid system. It has dimensions of length^3/time.

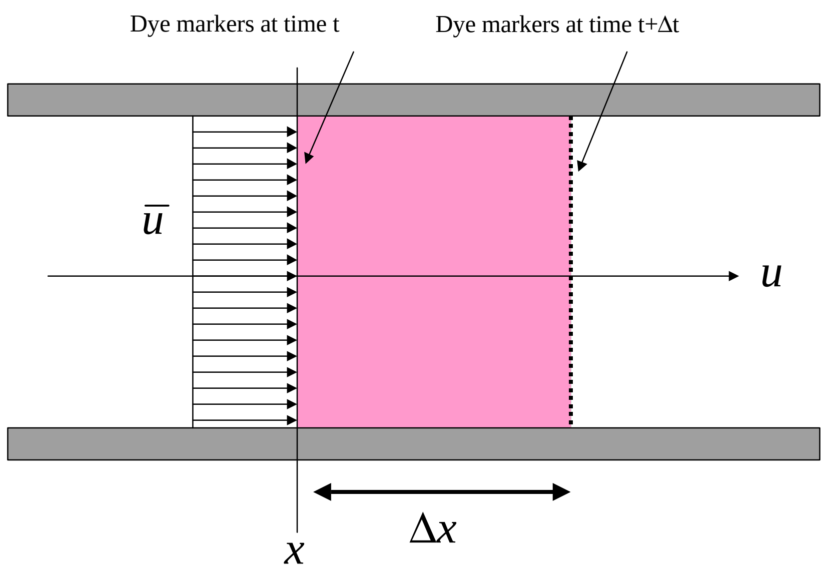


Figure 2.5. Uniform flow in a conduit

- Figure 2.5 is a schematic of a conduit completely filled with fluid. Dye markers are placed at location *x* at some time *t*. A short period later, the position of the dye markers has moved to the location shown on the diagram.
- The product of the area of the conduit and the distance swept by the dye markers is a volume.
- The ratio of this volume and time it takes for this volume to be defined is called the volumetric flow rate.

In mathematical terms, the area of the conduit is A. The volume of fluid that passed x in the time interval Δt is $\Delta x A$. The volumetric flow rate is then

$$Q = \frac{V}{\Delta t} = \frac{\Delta x}{\Delta t} A$$

- In the limit this flow rate is defined in terms of the mean section velocity

$$Q = \bar{u}A$$

.

- If the velocity varies across the section, as in Figure 2.6, the mean sectional velocity is found by integration.

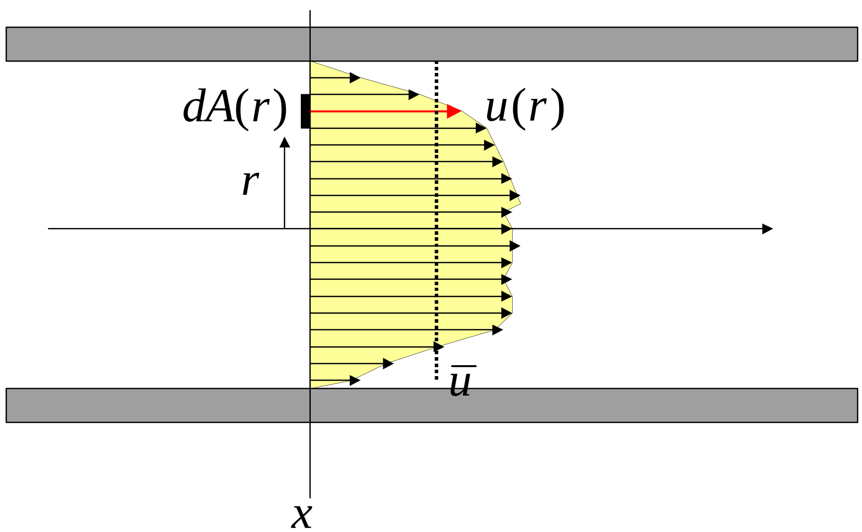


Figure 2.6. Non-uniform flow in a conduit

- From calculus we define the differential increment of discharge as

$$dQ = udA \, .$$

- Integration of all the differential elements is expressed as,

$$\int dQ = \int udA \, .$$

- From the conceptual definition of average section velocity we can compute its value as the ratio of these two integrals,

$$\bar{u} = \frac{\int udA}{\int dA}$$

- Observe that $Q = \bar{u}A$ is perpendicular to u .

- For an arbitrary orientation one must compute the scalar product of the velocity vector and the area vector, as depicted in Figure 2.7.

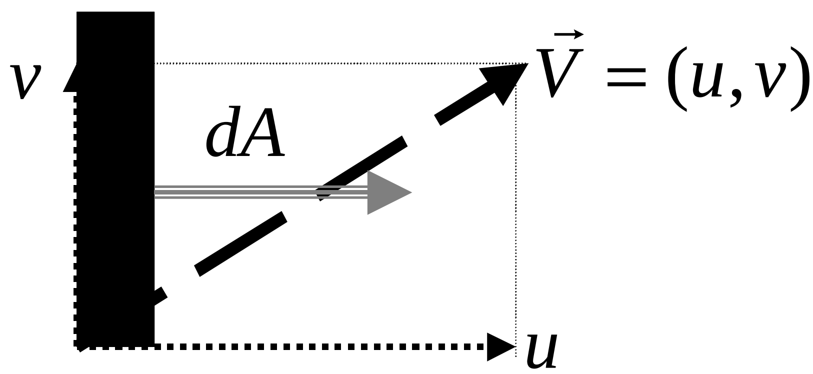


Figure 2.7. Diagram of non-collinear velocity and area vector(s).

$$Q = \int \vec{V} \bullet dA = \int u \bullet dA_x + \int v \bullet dA_y$$

Specific Discharge

- Variation of flow velocity of an individual fluid particle is inherent in the nature of flow through porous media.

- Within an individual pore, boundary resistance causes the velocity to decrease from a maximum along the centerline to essentially zero at the pore wall.

- Another form of variation is caused by the tortuous character of the flow - that is, the repeated branching and reconnecting of flow paths as the particles of fluid make their way around the individual grains of solid.

- This braided pattern causes the velocity of a fluid particle to vary from point to point in both magnitude and direction, even if its motion occurs along the centerline of the pore space.

- If one considers a small segment of porous material, but large enough to contain many pores, we will observe that these small-scale variations in flow tend to cancel in all except one average direction of flow.

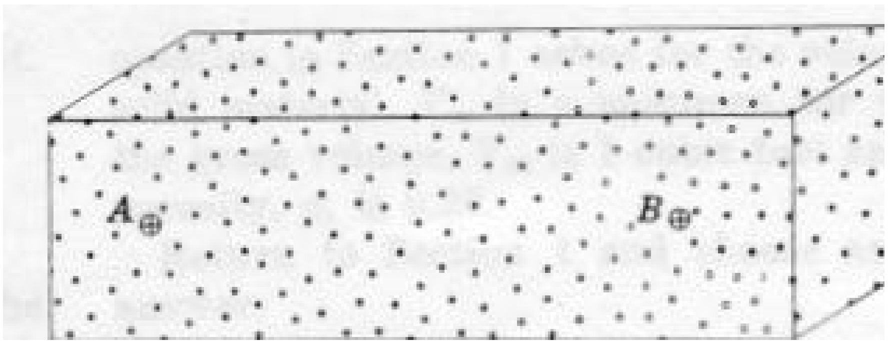


Figure 2.8 Porous medium

- Thus in the figure a particle of fluid moving from A to B would travel a distance greater than the straight-line distance between the two points.
- It is generally impossible to know the actual distance, but it should be related to porosity and the pore structure.
- Pores containing the fluid will occupy only a part of any cross-sectional area of a porous medium. The remainder of the area will be solid.

CIVE 6361 Groundwater Hydrology

- The traditional definition of average flow velocity from fluid mechanics has to be modified to reflect this fact.

- The average velocity of flow in pipe flow is defined as the ratio of discharge (volumetric flow rate) and flow area.

- In most groundwater studies the ratio of actual area of flow to gross area (the surface porosity) is assumed to be equal to the porosity. Thus the average flow velocity in terms of traditional hydraulics principles is unchanged, but the fluid particle velocity is increased in an amount inversely proportional to the porosity.

- In groundwater, the ratio of discharge to area is called the specific discharge or specific flux.

- The velocity of individual fluid particles is called the pore velocity or average linear velocity.

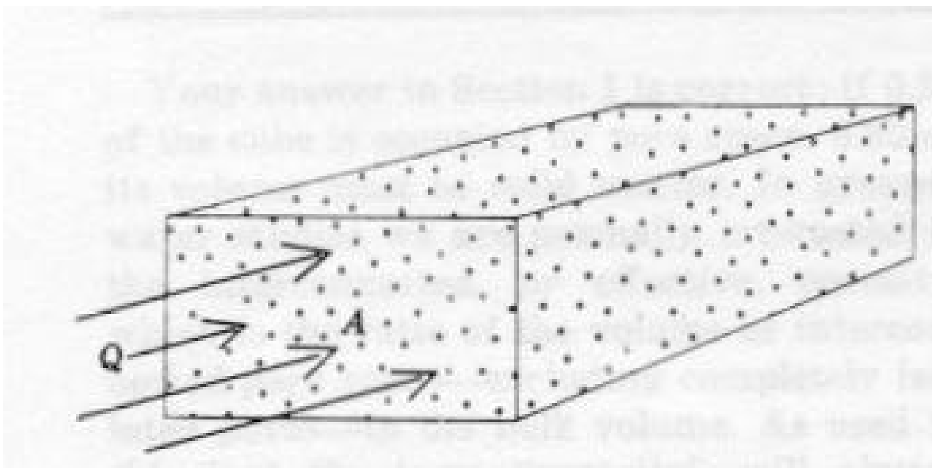


Figure 2.9 Flow across an area

- In the figure the ratio of discharge to area is less than the average linear velocity of fluid in the pore space because the gross cross sectional area is greater than the actual cross sectional area of the pore space.

- Symbolically we use *U* (or *q*) for specific discharge, and *u* for average linear velocity.

$$U = q = \frac{Q}{A}; \quad u = \frac{U}{n}$$

where *Q, A, n* are the total discharge, the bulk cross-sectional area, and the porosity, respectively.

Potential Energy and Head

- The static or piezometric head at a point in a groundwater system is the elevation of the top of a column of water that can be supported above that point.
- The density of the water in the measuring column is assumed to be equal to that of the groundwater, and the density of the groundwater is assumed to be uniform.

CIVE 6361 Groundwater Hydrology

- Head consists of two terms in a groundwater system: the elevation of the point of interest, and the height of a column of water that can be supported above that point.

- The height of the column of water above the point is a measure of the pressure at the point and is called the pressure head, while the elevation of the point of interest is called the elevation head.

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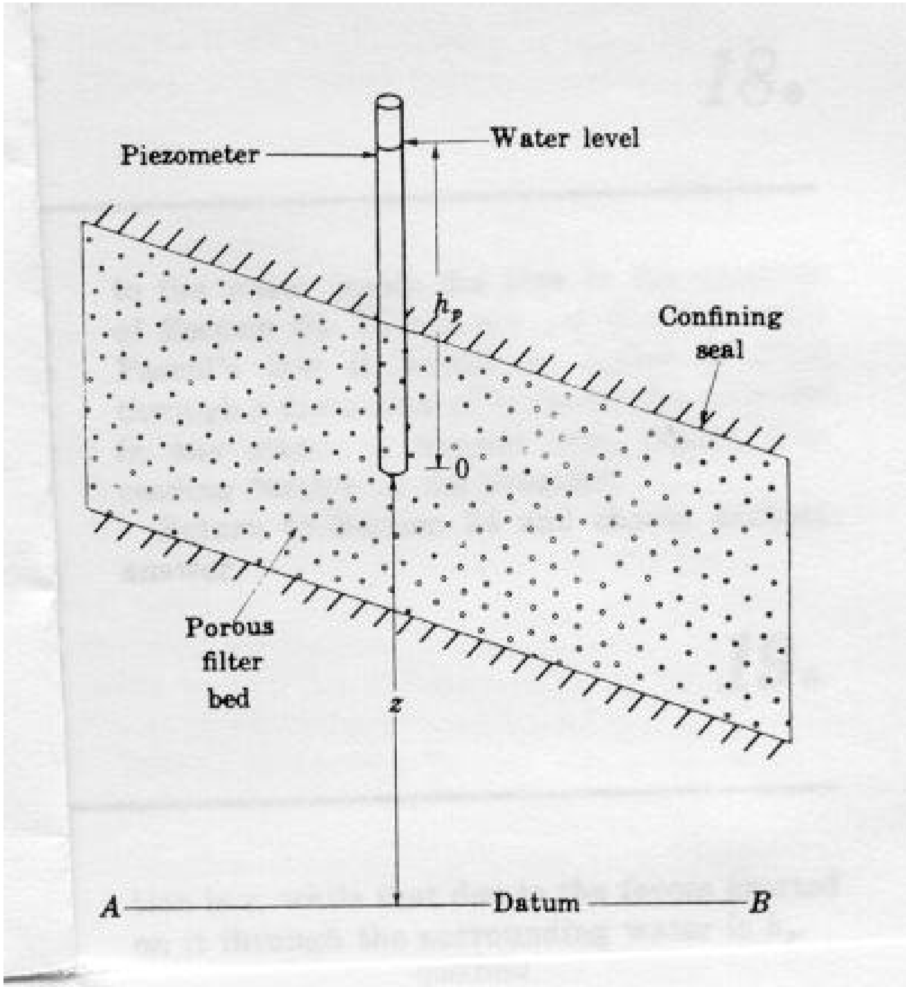


Figure 2.10 Illustration of Static Head

- Figure 2.10 represents an enclosed porous bed; the plane *AB* is the datum and a piezometer is inserted to the point *O*. In this figure the head at point *O* is the distance h_p+z; the sum of elevation and pressure head.

Consider that a pipe is driven into the ground. The bottom of the pipe comes to rest at a point below the water table where the fluid pressure is p. Water rises in the pipe to a height h_p above the lower end of the pipe. The pressure at the top of the water column in the pipe is zero (gage). The height of the column of water in the pipe expressed in terms of fluid density, pressure, and the gravitational constant is the piezometric (pressure) head at the bottom of the pipe.

Because the water in the pipe is not in motion, it must obey the laws of fluid statics. The pressure at the bottom of the pipe is related to the height of water in the pipe by Pascal's law;

$$p = \rho g h_p$$

In this fashion, h_p is actually a measure of the pressure in the fluid at the lower end of the pipe. The sum of this pressure head and the elevation head (height of the bottom of the pipe above a datum) is the static head in an aquifer.

- The static head is also a measure of the potential energy per unit weight of fluid. This concept is important, as the difference in static head between two locations is the driving force for flow.

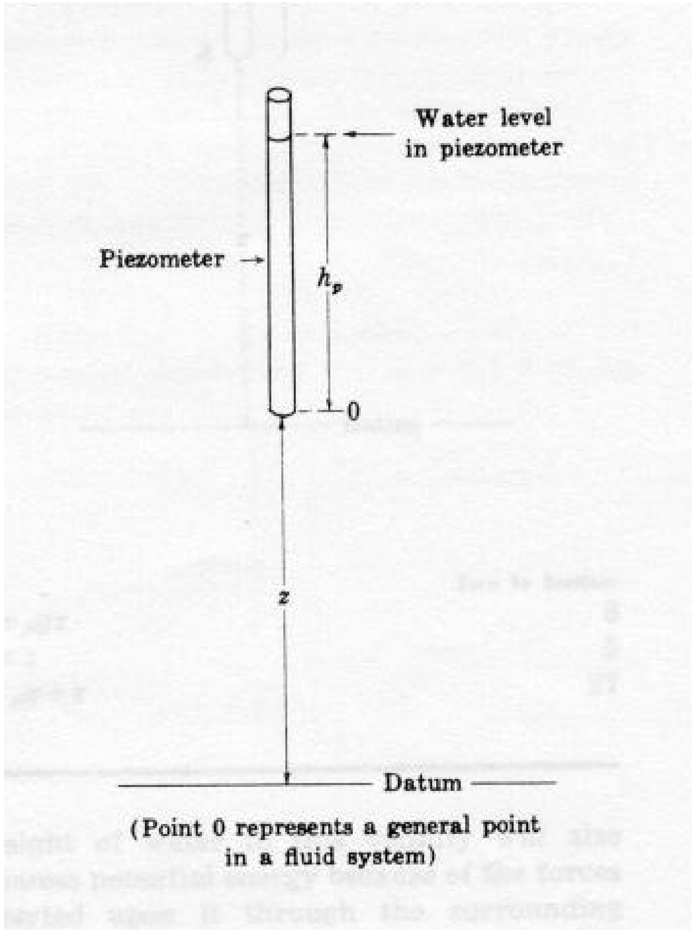


Figure 2.11 Piezometer in fluid system

- Figure 2.11 is a schematic of a piezometer (pipe) sampling a location in a fluid system.

- The elevation term z is the potential energy of a unit weight of water at point *O* that exists because point *O* is above the datum.

- This potential is equal to the amount of work required to raise one unit weight of water from the datum to point *O*. For instance, if z is 10 meters, and 10 Newtons of water is raised from the datum elevation to point O, then 100 Newton-meters (N-m) of work would be required. Conversely, if 10 Newtons of water were allowed to fall from point O back to the datum, this weight of water could perform 100 N-m of work.
- The pressure term also represents a capacity to do work.
- Pressure is usually expressed as force per unit area. However pressure can also be expressed as work per unit volume.

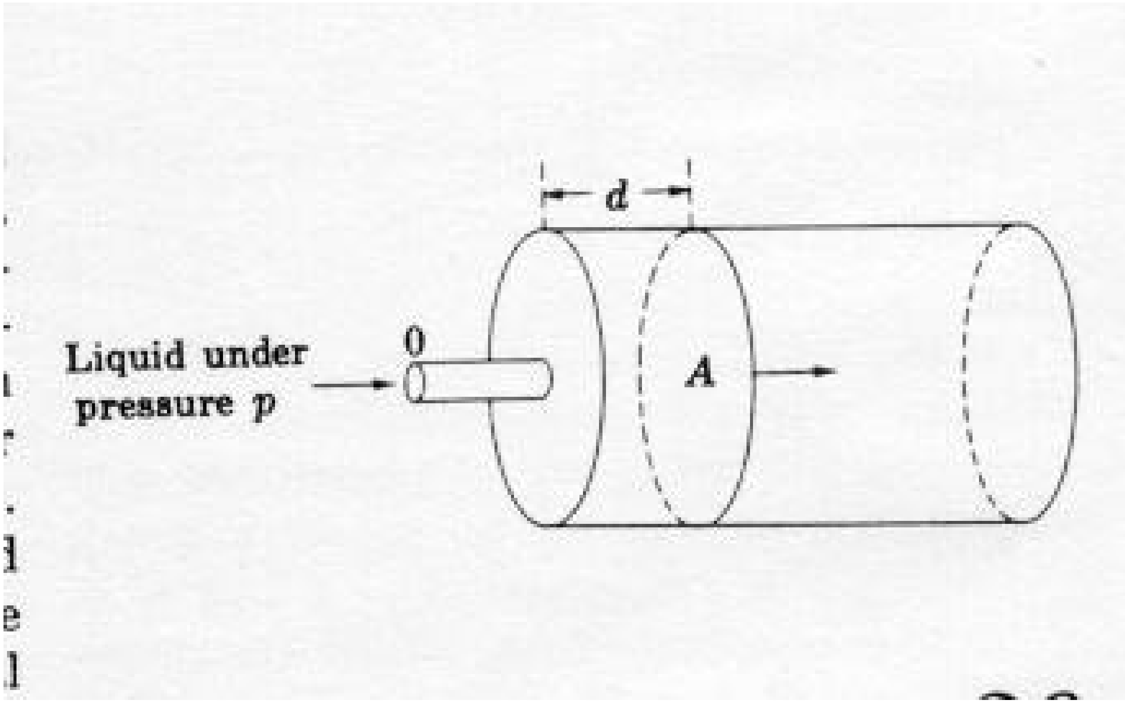


Figure 2.12 Hydraulic cylinder

- One way of illustrating the alternate expression of pressure is to consider a hydraulic cylinder. Figure 2.12 is a schematic of a hydraulic press.
- Liquid under pressure p is fed into the cylinder through the port at O. As the liquid enters, the piston is displaced to the right. Pressure is force per unit area, so the total force on the piston is the product of pressure p and piston face area A.

- The work required to move the piston through the distance *d* is the product of force and distance.

$$W = Fd = p \times A \times d$$

- The product of piston area *A* and distance *d* is a volume; thus the ability of the volume of liquid *Ad* to perform work *W* is given by the fluid pressure *p*.

- Potential energy is the ability to do work. In the hydraulic piston example, the potential energy per unit volume of liquid that is allowed to enter the piston is the pressure.

- This concept of potential energy can be extended to many kinds of flow systems provided that one understands that the potential is only from forces exerted on a fluid element by the surrounding fluid. To obtain total potential, one needs to add potential energy from the force of gravity, from chemical forces, etc.

- In many groundwater problems, only the pressure potential and the gravity potential are important, and the other potentials can be neglected.

- If the pressure, representing potential energy per unit volume, is divided by the specific weight of the fluid, ρg, then one obtains the term

$$h_p = \frac{p}{\rho g}$$

Which is called the pressure head. It represents the energy per unit weight of the fluid.

- In figure 2.11 the potential energy of a unit weight of water at point O can be expressed as h_p+z, while the potential energy per unit volume would be $p+\rho gz$.

Kinetic Energy

The last important term is kinetic energy, or the energy from motion.

- In particle mechanics, the kinetic energy is the product of mass and the square of velocity divided by 2.

$$K.E. = \frac{mv^2}{2}$$

- In a fluid, we represent kinetic energy as the mass per unit volume (*V*) of fluid moving at the specified velocity (*v*).

$$K.E. = \frac{\rho V v^2}{2}$$

- In terms of energy per unit volume of fluid, the kinetic energy potential is

$$\frac{rv^2}{2}$$

- In terms of unit weight (same dimensions as head) the energy per unit weight is

$$\frac{v^2}{2g}$$

- In a groundwater system the flow velocities are usually small and the square of these velocities is even smaller. Thus the kinetic energy terms are usually negligible when compared to the pressure and elevation potentials.

- Normally in groundwater problems, on generally neglects the kinetic energy terms and only considers the pressure and elevation potentials, and loss in potential energy from friction.

- When the other potentials are important (chemical, osmotic, thermal, etc.) the flow velocities are extremely small, and can also be neglected.

References

Bennett, G.D. (1989) Introduction to Ground-Water Hudraulics. TWRI Book 3, Chapter B2. U.S. Geological Survey, Denver, CO.

Darcy's Law

In engineering mechanics one equates forces producing motion with forces opposing motion. The same concept is used in considering the steady motion of fluid through a porous medium.

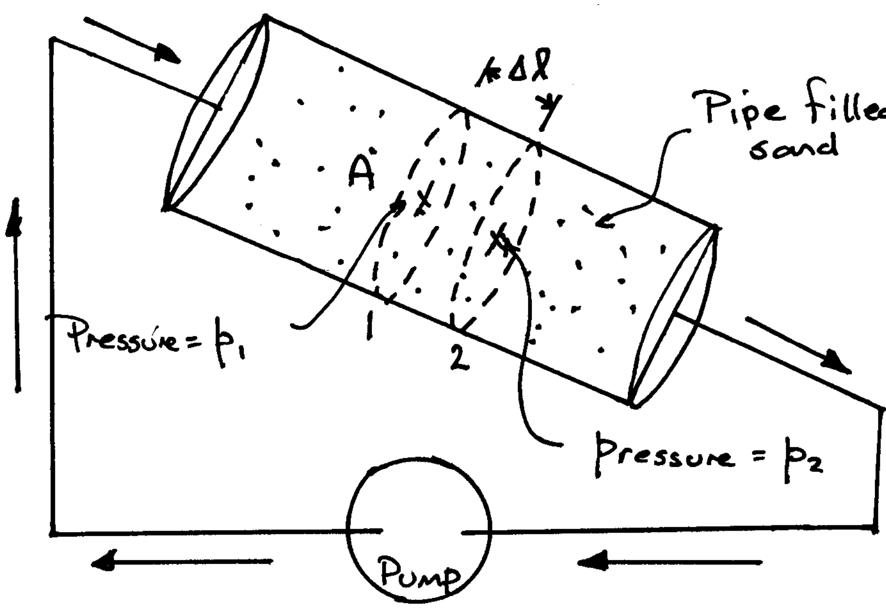
In the motion of a particle falling in a fluid we find that the force of friction opposing motion is proportional to the velocity of the particle – at least in laminar flow (Stokes Law).

Likewise in the flow of fluid through a porous medium the forces opposing flow are proportional to the fluid velocity – at least in laminar flow.

To develop an equation of motion one equates the forces that produce flow with the forces that oppose flow. The opposing forces are assumed to be proportional to the fluid velocity.

In steady flow, no acceleration is observed therefore the vector sum of all forces on the fluid must be zero. The resulting expression will be a form of Darcy's Law.

First we start with driving forces.



Consider a pipe packed with sand with porosity n. Liquid of density ρ is circulated through the pipe with a pump.

A small cylindrical segment of the pipe of length Δl and cross sectional area A is indicated by the dotted lines.

A small element (volume) of fluid occupies this segment. The fluid pressure on the upstream face is p_1.

The pressure force exherted on the upstream face of the fluid element is

$$F_{P_{UP}} = p_1 n A.$$

The area of fluid actually exposed is assumed to be the product of porosity and total cross sectional area.

Similarly the pressure on the downstream face is

$$F_{P_{down}} = p_2 n A$$

Assuming that $p_1 > p_2$, the net pressure force on the fluid element is

$$p_1 n A - p_2 n A$$

This expression can also be written as

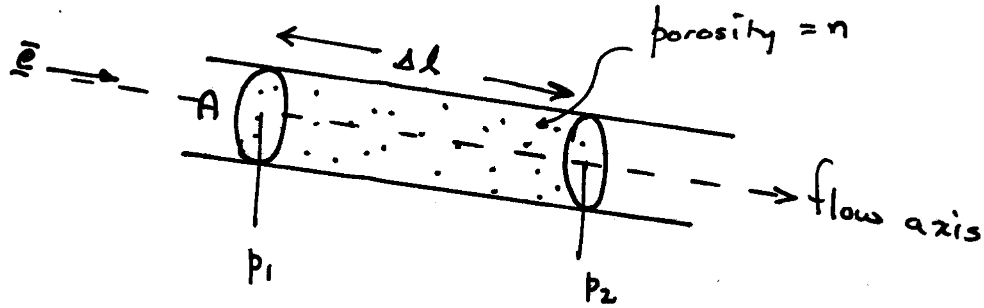
$$-(p_2 - p_1) n A$$

Because pressure varies from point to point in this system, we refer to the change of pressure with distance l along the flow line as a pressure gradient.

The gradient is expressed as $\frac{dp}{dl}$ and

is referred to as the derivative of pressure with respect to distance in the direction l. (directional derivative)

A plot of pressure versus distance in the l direction would have slope $\frac{dp}{dl}$



$\Delta p = p_2 - p_1$

Let $\bar{e}$ be a unit vector along the flow axis.

The net force on the fluid element is

$$-\Delta p\, n A = -\frac{\Delta p}{\Delta \ell} n A \cdot \Delta \ell$$

but $\frac{\Delta p}{\Delta \ell}$ is the pressure gradient so the net pressure force (magnitude) is

$$-\frac{dp}{d\ell} n A \Delta \ell$$

Expressed as a force vector the net pressure force is

$$\underline{F}_p = -\frac{dp}{d\ell} n A \Delta \ell \, \underline{e}$$

Next we consider the gravitational force acting on the fluid element.

Total body force is the product of fluid mass in the element multiplied by the gravitational acceleration constant, g.

The mass of the element is the product of the fluid volume and its density.

For this discussion the fluid mass in the element is

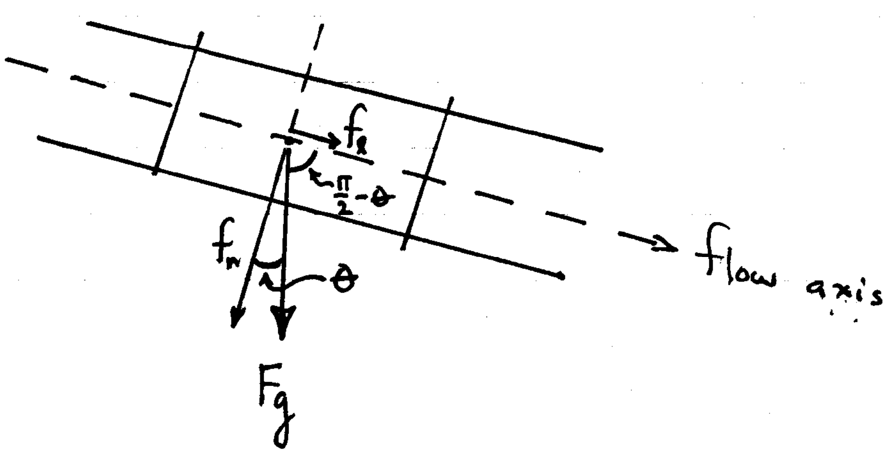
$$m = \varphi n A \Delta l$$

The magnitude of the gravitational force is therefore

$$F_g = \varphi n \Delta \Delta l g A$$

This force is always directed downward.

In the original flow diagram, the pipe has slope, so some part of the gravitational force acts along the flow axis and some part acts normal to the axis.



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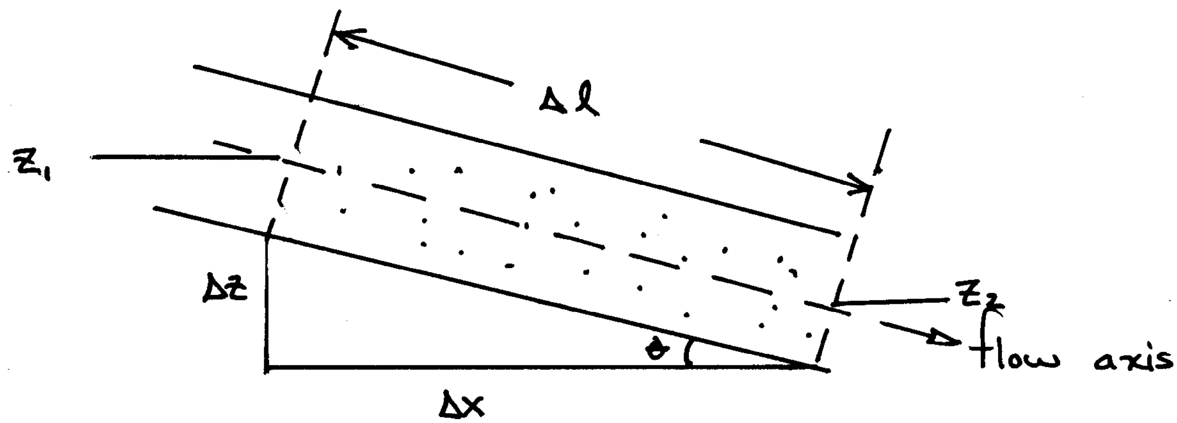
Therefore the magnitude of the gravity force along the flow axis is

$$f_l = F_g \sin\theta = \rho n \Delta l g \sin\theta A$$



Now consider the geometry in a cartesian sense

$$\sin\theta = \frac{\Delta z}{\Delta l}$$

Therefore

$$f_l = \rho n \Delta l A g \frac{\Delta z}{\Delta l}$$

This expression states that the gravitational force is proportional to the change in elevation <u>along the flow path</u>

As with pressure $\Delta z = -(z_2 - z_1)$.

Also when expressed as a gradient we will obtain

$$\underline{f}_l = -\varphi n \Delta l A g \frac{dz}{dl} \underline{e}$$

Now if we combine these two forces we obtain the net driving force vector

$$\left(-\frac{dp}{dl} - \varphi g \frac{dz}{dl}\right) \Delta l \, n A \, \underline{e} .$$

This vector expresses the net driving force ~~per~~ in the flow direction for the fluid element.

The net force <u>per unit volume of fluid</u> from pressure and gravity is

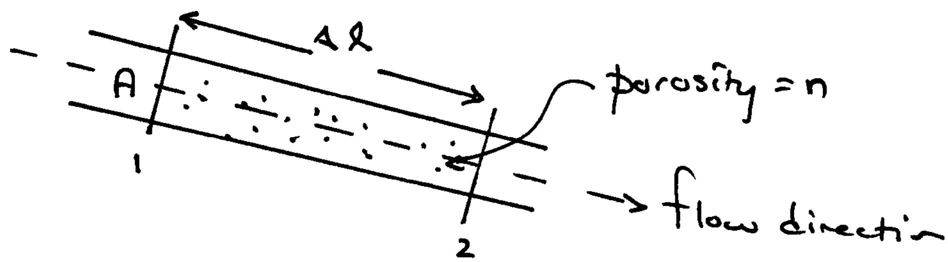
$$-\left(\frac{dp}{dl} + \varphi g \frac{dz}{dl}\right) \underline{e}$$

Now consider forces opposing motion (frictional drag force).

Typically the frictional force is assumed to be proportional to the specific discharge. The specific discharge (an apparent velocity) is determined by the distribution of pore velocities and one, in effect, is assuming that frictional drag is determined by this distribution of pore velocities.

The drag force is also assumed to be proportional the volume of fluid in the element on the theory that the total area of fluid-solid contact within the element increases in proportion to the volume of the element.

Finally the drag force is assumed to be proportional to the dynamic viscosity of the fluid. - one expects a low viscosity fluid to move more easily than a high viscosity fluid through a porous medium



Using these assumptions we have the following relationships.

$$F_{FR} \propto \frac{Q}{A} \;;\quad F_{FR} \propto \nu \;;\quad F_{FR} \propto n A \Delta \ell$$

proportional to sp. discharge; proportional to viscosity; proportional volume in fluid element

Friction always opposes motion so its direction will be $-\underline{e}$.

Now if we express the friction term as a product of the factors we have

$$F_{FR} \propto \nu (n A \Delta \ell) \frac{Q}{A}$$

Let the constant of proportionality be $\frac{1}{k}$ where k is called the intrinsic permeability. Then the frictional drag force is

$$F_{FR} = \frac{\mu}{k} \cdot \frac{Q}{A} (n A \Delta l)$$

And expressed as a force vector we obtain

$$\underline{F}_{FR} = -\frac{\mu}{k} \frac{Q}{A} (n A \Delta l)$$

Now we write the static (equilibrium) force balance for the fluid element.

$$\underline{F}_{NET} + \underline{F}_{FR} = 0 \qquad (\Sigma F = 0)$$

↖ Eqn. of static equilibrium

$$\therefore \left\{ -\left(\frac{dp}{dl} + \rho g \frac{dz}{dl}\right) n A \Delta l - \frac{\mu}{k} \frac{Q}{A} n A \Delta l \right\} \underline{e} = \underline{0}$$

After simplification we obtain the vector expression

$$-\left(\frac{dp}{dl} + \rho g \frac{dz}{dl}\right) \underline{e} = \frac{\mu}{k} \frac{Q}{A} \underline{e}$$

or

$$-\frac{k}{\mu}\left(\frac{dp}{dl} + \rho g \frac{dz}{dl}\right) \underline{e} = \frac{Q}{A} \underline{e}$$

One more rearrangement leads to

$$-\frac{k\rho g}{\nu}\left(\frac{1}{\rho g}\frac{dp}{dl}+\frac{dz}{dl}\right)\underline{e} = \frac{Q}{A}\underline{e}$$

For many engineering problems the groundwater is isothermal and density is nearly constant. In this instance the term $\frac{k\rho g}{\nu}$ is replaced by the hydraulic conductivity K.

That is $K = \frac{k\rho g}{\nu}$

When this substitution can be made the vector equation becomes

$$-K\left(\frac{1}{\rho g}\frac{dp}{dl}+\frac{dz}{dl}\right)\underline{e} = \frac{Q}{A}\underline{e}$$

Recall that hydraulic head is given by

$$h = \frac{p}{\rho g} + z$$

Therefore

$$\frac{dh}{dl} = \frac{1}{\rho g}\frac{dp}{dl} + \frac{dz}{dl} \qquad (\rho g = \text{constant})$$

So when ρ, μ are constants the vector equation becomes

$$-K\frac{dh}{dl}\underline{e} = \frac{Q}{A}\underline{e}$$

This last expression is often expressed as

$$-KA\frac{dh}{dl}\underline{e} = Q\,\underline{e}$$

And is called Darcy's law. This relationship between specific discharge and the gradient of head was obtained experimentally by Henri Darcy in 1856.

The hydraulic conductivity K incorporated fluid properties (ρ & μ) and geometric properties of the porous medium (k).

The relationship states that groundwater flows in the direction of decreasing head.

Darcy's law is a differential equation. The specific discharge is a vector quantity. The vector equation relates flow per unit area, or flux, to the energy consumed per unit distance by friction.

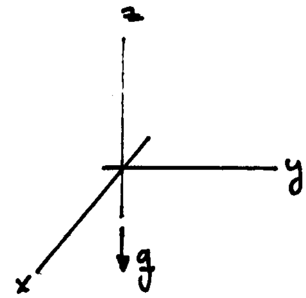
Darcy's law is a linear flux law. So is Fourier's law (heat flow), Ohms law (electron flow), Ficks law (chemical flow), etc.

Darcy's law has three dimensional forms that are best expressed using principles of vector algebra.

The most generalized form considers that the permeability at some point has a different value depending on direction. This property is called anisotropy.

Anisotropy means, in a practical sense, that a gradient perpindicular to a direction of interest, can produce flow in that direction of interest.

$$\left(\frac{Q}{\underline{A}}\right) = U\underline{i} + V\underline{j} + W\underline{k} = q$$



$$U = -\frac{k_{xx}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial x}\right) - \frac{k_{xy}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial y}\right) - \frac{k_{xz}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial z}\right)$$

$$V = -\frac{k_{yx}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial x}\right) - \frac{k_{yy}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial y}\right) - \frac{k_{yz}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial z}\right)$$

$$W = -\frac{k_{zx}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial x}\right) - \frac{k_{zy}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial y}\right) - \frac{k_{zz}\rho g}{\nu}\left(\frac{1}{\rho g}\frac{\partial p}{\partial z} + 1\right)$$

This form is used when ν, ρ are not constants because of thermal changes & density changes.

The most general form in common use is

$$U = -K_{xx}\frac{dh}{dx} - K_{xy}\frac{dh}{dy} - K_{xz}\frac{dh}{dz}$$
$$V = -K_{yx}\frac{dh}{dx} - K_{yy}\frac{dh}{dy} - K_{yz}\frac{dh}{dz}$$
$$W = -K_{zx}\frac{dh}{dx} - K_{zy}\frac{dh}{dy} - K_{zz}\frac{dh}{dz}$$

The form sufficient for this course is

$$U = -K_x\frac{dh}{dx}, \quad V = -K_y\frac{dh}{dy}, \quad W = -K_z\frac{dh}{dz}$$

This general form of darcy's law can be written in vector shorthand as

$$\underset{\sim}{q} = -\underline{K} \cdot \nabla h$$

Where

$$\underline{K} = \begin{pmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{pmatrix}$$

$$\nabla h = \frac{\partial h}{\partial x}\underline{i} + \frac{\partial h}{\partial y}\underline{j} + \frac{\partial h}{\partial z}\underline{k} = \begin{pmatrix} \frac{\partial h}{\partial x}\underline{i} \\ \frac{\partial h}{\partial y}\underline{j} \\ \frac{\partial h}{\partial z}\underline{k} \end{pmatrix}$$

$\therefore$

$$\underset{\sim}{q} = -K_x \frac{dh}{dx}\underline{i} - K_y \frac{dh}{dy}\underline{j} - K_z \frac{dh}{dz}\underline{k}$$

Also we could write

$$\underset{\sim}{q} = -K \cdot \mathrm{grad}(h)$$