(1) Given: Observations of the piezometric heads in three observation wells:

Well	A	B	C
x-coordinate	0	300 meters	0
y-coordinate	0	0	200 meters
Piezometric head (m)	+10.0 m	+11.5 m	+8.4 m

The wells penetrate a homogeneous, isotropic, confined aquifer of constant thickness B=20 meters, porosity = 0.2, and hydraulic conductivity K = 15 m/day. Assume that the piezometric surface can be approximated as a plane.

Transmissivity

20m $K = 15\,m/d$

$T = (15)(20) = 300\,m^2/d$

Determine:

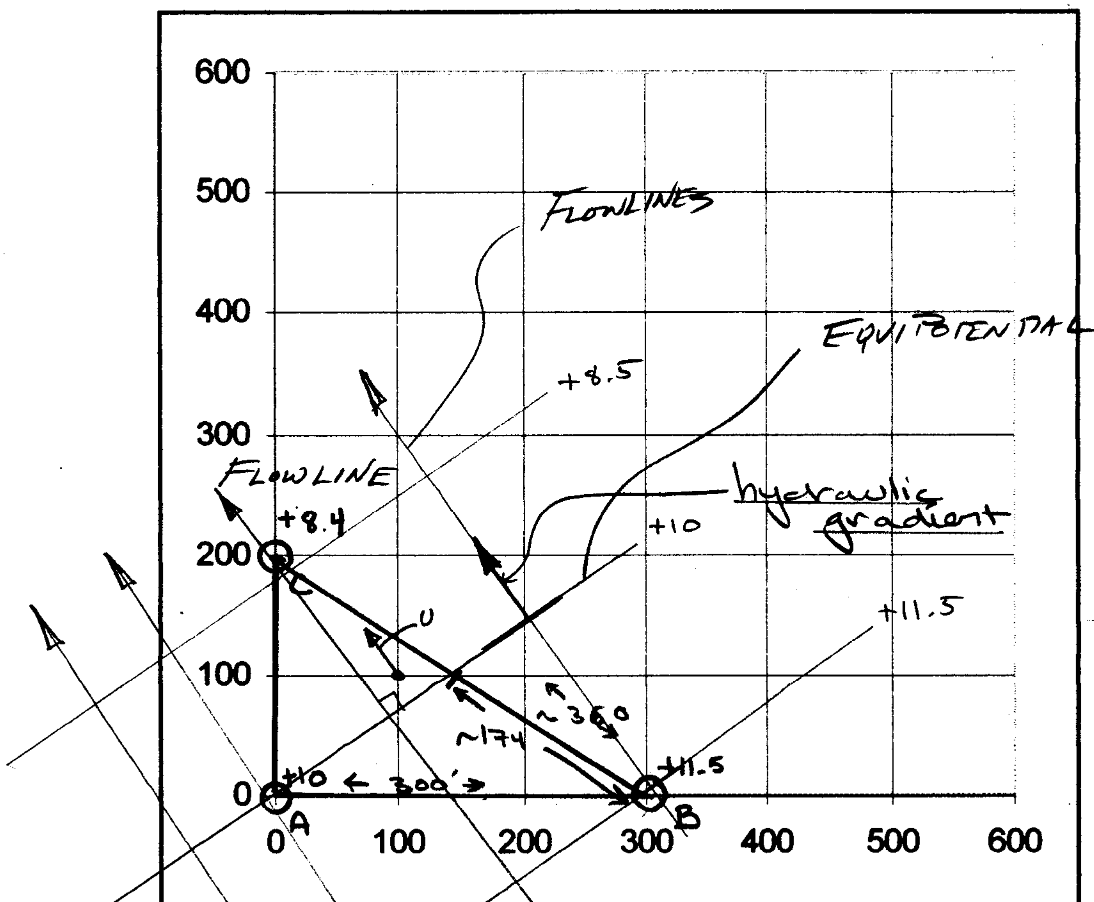
a) The hydraulic gradient (magnitude and direction)

b) The aquifer transmissivity $T = 300\,m^2/d$

c) The total discharge per unit width in the aquifer along a flow line

d) The pore velocity at point P(100,100).

e) Sketch the flowlines and equipotential lines for this aquifer.



hyd. gradient

$$\frac{\Delta h}{\Delta l} = \frac{1.5\,m}{\sim 170} = 0.008$$

$$\frac{11.5 - 8.4}{360} = \frac{11.5 - 10}{x}$$

solve for x

$$x = \frac{(1.5)\,360}{(3.1)} = 174$$

Problem 1 Worksheet.

$$Q = KA\frac{\Delta h}{\Delta l} = K \cdot B \cdot w \frac{\Delta h}{\Delta l}$$

$$\frac{Q}{w} = \text{flow/width} = K \cdot B \frac{\Delta h}{\Delta l}$$

$$= 300\,m^2/d \; (0.008)$$

$$= 2.64\,m^3/d/m$$

$$v_{pore} = \frac{K}{w}\frac{\Delta h}{\Delta l} = \frac{15\,m/d}{0.20}\;0.008 = 0.6\,m/d$$

(2) Given: Same aquifer as problem 1. Aquifer dispersivities are $\alpha_L = 2.0$ m, α_T=0.2 m.

Determine:

a) Longitudinal dispersion coefficient along a flowline.
b) Transverse dispersion coefficient along a flowline.
c) Distance along a flowline where the concentration from a reservoir-type source is reduced to 10% of its input concentration.

Table of Functions						
x	ERF(x)	ERFC(x)		x	ERF(x)	ERFC(x)
-3	-1.0000	2.0000		0	0.0000	1.0000
-2.9	-1.0000	2.0000		0.1	0.1125	0.8875
-2.8	-0.9999	1.9999		0.2	0.2227	0.7773
-2.7	-0.9999	1.9999		0.3	0.3286	0.6714
-2.6	-0.9998	1.9998		0.4	0.4284	0.5716
-2.5	-0.9996	1.9996		0.5	0.5205	0.4795
-2.4	-0.9993	1.9993		0.6	0.6039	0.3961
-2.3	-0.9989	1.9989		0.7	0.6778	0.3222
-2.2	-0.9981	1.9981		0.8	0.7421	0.2579
-2.1	-0.9970	1.9970		0.9	0.7969	0.2031
-2	-0.9953	1.9953		1	0.8427	0.1573
-1.9	-0.9928	1.9928		1.1	0.8802	0.1198
-1.8	-0.9891	1.9891		1.2	0.9103	0.0897
-1.7	-0.9838	1.9838		1.3	0.9340	0.0660
-1.6	-0.9763	1.9763		1.4	0.9523	0.0477
-1.5	-0.9661	1.9661		1.5	0.9661	0.0339
-1.4	-0.9523	1.9523		1.6	0.9763	0.0237
-1.3	-0.9340	1.9340		1.7	0.9838	0.0162
-1.2	-0.9103	1.9103		1.8	0.9891	0.0109
-1.1	-0.8802	1.8802		1.9	0.9928	0.0072
-1	-0.8427	1.8427		2	0.9953	0.0047
-0.9	-0.7969	1.7969		2.1	0.9970	0.0030
-0.8	-0.7421	1.7421		2.2	0.9981	0.0019
-0.7	-0.6778	1.6778		2.3	0.9989	0.0011
-0.6	-0.6039	1.6039		2.4	0.9993	0.0007
-0.5	-0.5205	1.5205		2.5	0.9996	0.0004
-0.4	-0.4284	1.4284		2.6	0.9998	0.0002
-0.3	-0.3286	1.3286		2.7	0.9999	0.0001
-0.2	-0.2227	1.2227		2.8	0.9999	0.0001
-0.1	-0.1125	1.1125		2.9	1.0000	0.0000
0	0.0000	1.0000		3	1.0000	0.0000

Problem 2 Worksheet

$$D_L = \alpha_L v$$
$$= (2.0m)(0.6m/d)$$
$$= 1.2m^2/d$$

$$D_T = \alpha_T v$$
$$= (0.2m)(0.6m/d)$$
$$= 0.12m^2/d$$

Constant reservoir

$$C(x,t) = \frac{C_0}{2}\left[\operatorname{erfc}\left(\frac{x-vt}{2\sqrt{D_L t}}\right) - \underbrace{\exp\left(\frac{xv}{D}\right)\operatorname{erfc}\left(\frac{x+vt}{2\sqrt{D_L t}}\right)}_{\text{negligible at } x > 10vt}\right]$$

Use

$$C(x,t) \approx \frac{C_0}{2}\operatorname{erfc}\left(\frac{x-vt}{2\sqrt{D_L t}}\right)$$

Find x for $C(x,t) = 0.1\,C_0$

$$0.1\,C_0 = \frac{C_0}{2}\operatorname{erfc}\left(\frac{x-vt}{2\sqrt{Dt}}\right)$$
$$0.2\,C_0 = C_0\operatorname{erfc}\left(\frac{x-vt}{2\sqrt{Dt}}\right)$$
$$0.2 = \operatorname{erfc}\left(\frac{x-vt}{2\sqrt{Dt}}\right)$$

From table

$$\frac{x-vt}{2\sqrt{Dt}} = 0.9$$

$$\boxed{x = 1.8\sqrt{Dt} + vt}$$

(3) Given: Confined aquifer system depicted in figure P-4. Water enters the confined aquifer in the recharge area and leaves in the form of a spring.

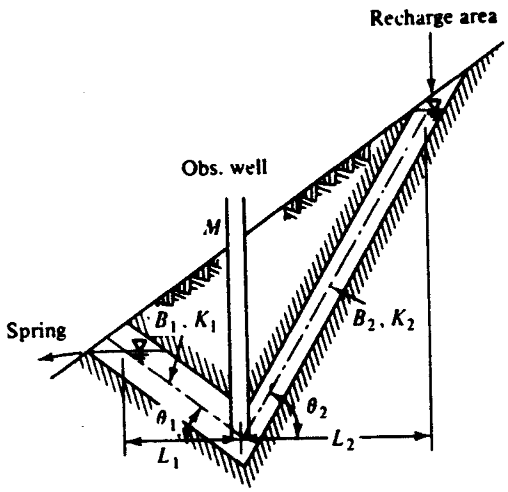


Figure P-4.

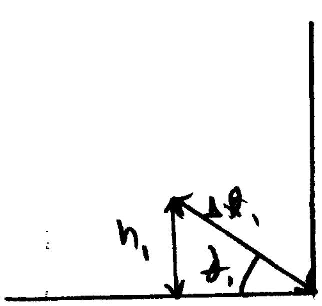
Determine:

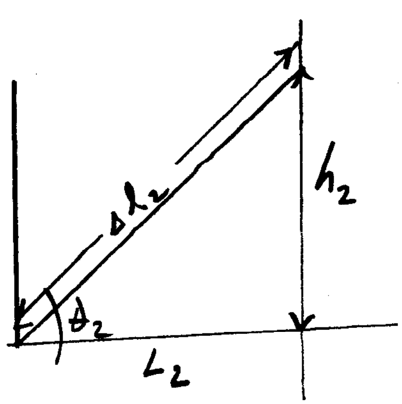
a) Piezometric head at a well located at M (in terms of the other variables)

b) Specify the conditions for the well to become a flowing well (water flows from well without pumping)

$$\Delta l_2 = \sqrt{h_2^2 + L_2^2}$$

$$h_2 = L_2 \tan\theta_2$$





$$\Delta l_2 = \sqrt{L_2^2 \tan^2\theta_2 + L_2^2}$$

$$= L_2\sqrt{\tan^2\theta_2 + 1}$$

$$\Delta l_1 = L_1\sqrt{\tan^2\theta_1 + 1}$$

$$h_1 = L_1 \tan\theta_1$$

Problem 3 Worksheet

$$Q_2 = K_2 B_2 \frac{\Delta h}{\Delta l} = K_2 B_2 \frac{L_2 \tan\theta_2 - h_m}{L_2 \sqrt{\tan^2\theta_2 + 1}}$$

$$Q_1 = K_1 B_1 \frac{\Delta h}{\Delta l} = K_1 B_1 \frac{h_m - L_1 \tan\theta_1}{L_1 \sqrt{\tan^2\theta_1 + 1}}$$

By continuity

$$Q = Q_1 = Q_2$$

$$\therefore K_1 B_1 \frac{h_m - L_1 \tan\theta_1}{L_1 \sqrt{\tan^2\theta_1 + 1}} = K_2 B_2 \frac{L_2 \tan\theta_2 - h_m}{L_2 \sqrt{\tan^2\theta_2 + 1}}$$

Solve for h_m

$$\frac{h_m - L_1 \tan\theta_1}{L_1 \sqrt{\tan^2\theta_1 + 1}} = \frac{K_2 B_2}{K_1 B_1} \frac{L_2 \tan\theta_2 - h_m}{L_2 \sqrt{\tan^2\theta_2 + 1}}$$

$$\frac{h_m}{L_1 \sqrt{\tan^2\theta_1 + 1}} = \frac{K_2 B_2}{K_1 B_1} \frac{L_2 \tan\theta_2}{L_2 \sqrt{\tan^2\theta_2 + 1}} - \frac{K_2 B_2 h_m}{K_1 B_1 L_2 \sqrt{\tan^2\theta_2 + 1}} + \frac{L_1 \tan\theta_1}{L_1 \sqrt{\tan^2\theta_1 + 1}}$$

$$\frac{h_m}{L_1 \sqrt{\tan^2\theta_1 + 1}} + \frac{K_2 B_2 h_m}{K_1 B_1 L_2 \sqrt{\tan^2\theta_2 + 1}} = \underbrace{\frac{K_2 B_2}{K_1 B_1} \frac{L_2 \tan\theta_2}{L_2 \sqrt{\tan^2\theta_2 + 1}} + \frac{L_1 \tan\theta_1}{L_1 \sqrt{\tan^2\theta_1 + 1}}}_{R}$$

$$\frac{h_m K_1 B_1 L_2 \sqrt{\tan^2\theta_2 + 1} + h_m K_2 B_2 L_1 \sqrt{\tan^2\theta_1 + 1}}{K_1 B_1 L_1 L_2 \sqrt{\tan^2\theta_2 + 1} \sqrt{\tan^2\theta_1 + 1}} = R$$

$$h_m = \left(\frac{K_1 B_1 L_1 L_2 \sqrt{\tan^2\theta_2 + 1} \sqrt{\tan^2\theta_1 + 1}}{K_1 B_1 L_2 \sqrt{\tan^2\theta_2 + 1} + K_2 B_2 L_1 \sqrt{\tan^2\theta_1 + 1}} \right) \left(\frac{K_2 B_2 L_2 \tan\theta_2}{K_1 B_1 L_2 \sqrt{\tan^2\theta_2 + 1}} + \frac{L_1 \tan\theta_1}{L_1 \sqrt{\tan^2\theta_1}} \right)$$

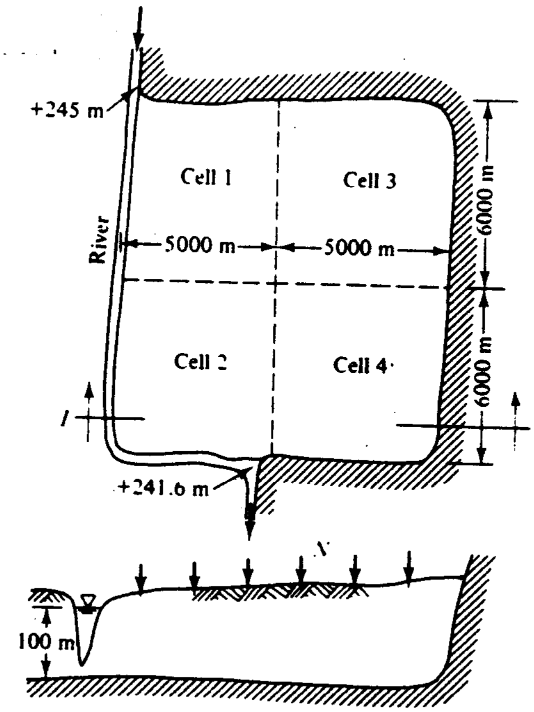
$$h = \frac{K_2 B_2 L_1 L_2 \sqrt{\tan^2\theta_1 + 1} \tan\theta_2 + K_1 B_1 L_1 L_2 \sqrt{\tan^2\theta_2 + 1} \tan\theta_1}{\ldots}$$ (a)

$$\frac{h_2 - h_1}{L_2 + L_1} = \text{slope of land.}$$

$$\left(\frac{L_2 \tan\theta_2 - L_1 \tan\theta_1}{L_2 + L_1}\right) \cdot L_1 + L_1 \tan\theta_1 < h_m$$

then well will flow!

(4) Given: A four-cell conceptual model of an aquifer with the following cell-by-cell data summary.



Cross section I-I

R = mm/yr * 1000 * 5000 * 6000

	Pumping (P) ($10^6\ m^3/yr$)	Nat. replenishment (mm/yr)	R ($10^6\ m^3/yr$)	K (m/day)
Cell 1	17.8	200	6.0	40
Cell 2	0	420	12.6	40
Cell 3	6.3	300	9.0	30
Cell 4	2.7	300	9.0	30
Σ	26.8		36.6	

Determine:

a) Write the governing steady-state balance equations that must be solved to calculate the steady average water levels in the four cells.

b) Without actually solving these equations determine if the aquifer is a net supplier of water to the river or if the aquifer draws water from the river. (Show how you arrived at your answer).

Problem 4 Worksheet

a) Cell #1

$$0 = P_1 - R_1 + \frac{K_1h_1 + K_3h_3}{2}\left[\frac{h_3 - 2h_1 + h_R}{\Delta x^2}\right] + \frac{K_1h_1 + K_2h_2}{2}\left[\frac{h_2 - 2h_1 + h_1}{\Delta y^2}\right]$$

Cell #2

$$0 = P_2 - R_2 + \frac{K_2h_2 + K_4h_4}{2}\left[\frac{h_4 - 2h_2 + h_R}{\Delta x^2}\right] + \frac{K_2h_2 + K_1h_1}{2}\left[\frac{h_R - 2h_2 + h_2}{\Delta y^2}\right]$$

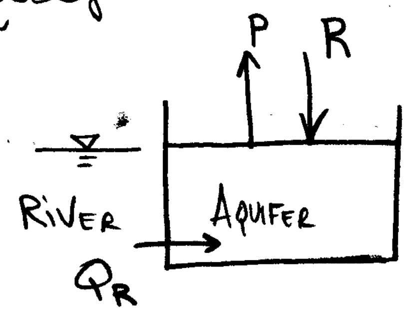
Cell #3

$$0 = P_3 - R_3 + \frac{K_3h_3 + K_1h_1}{2}\left[\frac{h_3 - 2h_3 + h_1}{\Delta x^2}\right] + \frac{K_3h_3 + K_4h_4}{2}\left[\frac{h_3 - 2h_3 + h_4}{\Delta y^2}\right]$$

Cell #4

$$0 = P_4 - R_4 + \frac{K_4h_4 + K_2h_2}{2}\left[\frac{h_4 - 2h_4 + h_2}{\Delta x^2}\right] + \frac{K_4h_4 + K_3h_3}{2}\left[\frac{h_3 - 2h_4 + h_4}{\Delta y^2}\right]$$

b) Water Budget entire aquifer



$$0 = R - P + Q_R$$

$$Q_R = P - R$$
$$= 26.8 - 36.6$$
$$= -9.8$$

$\therefore$ AQUIFER IS NET <u>SUPPLIER</u> TO RIVER

(5) Given: Corrected time-drawdown data for a pumping test in a confined aquifer pumped at 42,400 cubic feet per day. The aquifer is 48 feet thick. The observation well is 824 feet from the pumping well.

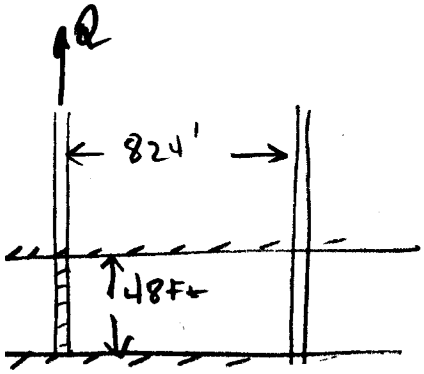
Determine:
a) Sketch the pumping well and observation well conceptual model.
b) Find the aquifer transmissivity and storage coefficient using Jacob's method.

– Convert Q to ft^3/min

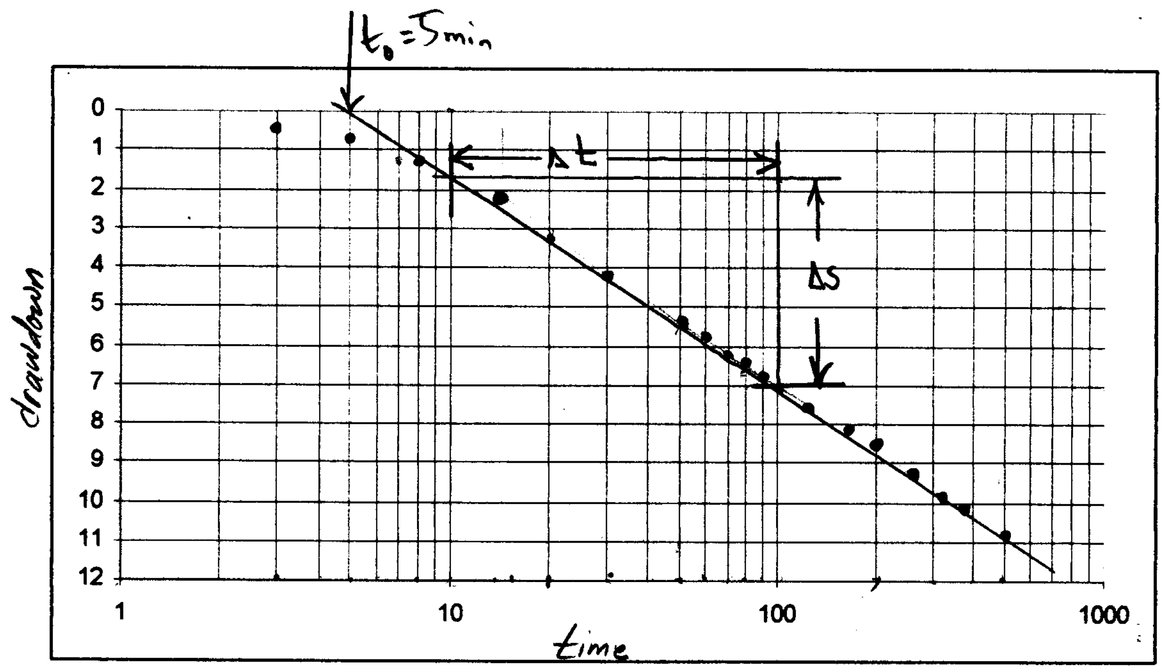
$r_0 = 824\ ft$

$$42400\ ft^3/day \cdot \frac{1\ day}{1440\ min}$$

$29.4\ ft^3/min$



Time after pumping started (min)	Drawdown (feet)
3	0.3
5	0.7
8	1.3
12	2.1
20	3.2
24	3.6
30	4.1
38	4.7
47	5.1
50	5.3
60	5.7
70	6.1
80	6.3
90	6.7
100	7.0
130	7.5
160	8.3
200	8.5
260	9.2
320	9.7
380	10.2
500	10.9



$t_2 = 100$ $\Delta s = 5.2\ ft$

$t_1 = 10$

Problem 5 Worksheet

$$T = \frac{Q}{4\pi \Delta s} \ln\left(\frac{t_2}{t_1}\right)$$

$$S = \frac{2.25\, T\, t_0}{r^2}$$

$$T = \frac{29.4\ ft^3/min}{4\pi\,(5.2\,ft)} \ln(10) = 1.03\ ft^2/min$$

$$S = \frac{2.25\,(1.03\,\tfrac{ft^2}{m})(5\,min)}{(824\,ft)^2} = 0.0000017$$