

CIVE 6361 Groundwater Hydrology Fall 1994

Exam 2

Instructions:

Exam is closed book. Work each problem as completely as you can. Please show work as well as indicate any numerical answer. If a value is missing or otherwise required, assume a value, indicate clearly that assumption and continue with the analysis. Use the back of the exam as needed for calculations, etc.

- 1) Verify that you have seven (7) problems.
- 2) Verify that you have a table of error function values
- ~~3) Verify that you have a Walton Method Type Curve.~~
- 4) Read the exam one time quickly, then begin working.
- 5) The exam is 80 minutes long.
- 6) When the instructor tells you to stop, close the exam.
- 7) Fill out your name below, put your name on each sheet of the exam.

Your Name: _____

Your Student Number: _____

Your Signature: _____

Useful Formulas:

Well Function Approximations

$$W(u) \approx -0.57721566 - \ln(u)$$

$$W(u, v) \approx \left(\sqrt{\frac{\pi}{2v}}\right) \exp(-v) \operatorname{erfc}\left(-\frac{v-2u}{2\sqrt{u}}\right)$$

A table of error function values is included with the exam.

~~A Walton Method Type Curve is included with the exam.~~

Darcy's Law: Discharge = Hydraulic Conductivity * Area * Hydraulic Gradient.

Continuity: Inflow Rate - Outflow Rate + Internal Sources = Rate of Change of Storage.

1 acre-foot = 43,560 cubic-feet

2.54 cm = 1.0 inches.

1 ppm ~ 1 mg/L (for dilute systems)

Problem 1 (5 pts) Identify the five contaminant transport models below with the proper descriptions of the scenarios for which they might be appropriate. Indicate appropriate values of hydraulic and transport parameters as applicable:

Model	Description
	One-dimensional flow, instantaneous input of contamination, no decay, no retardation.
	One-dimensional flow, constant source of contaminated water upstream of origin, no decay, no retardation.
	One dimensional flow, continuous injection of contamination, decay and retardation included.
	Two-Dimensional flow, continuous injection of contamination, no decay, no retardation.
	Two-dimensional flow, instantaneous input of contamination, no decay, retardation included.

$$\text{Model 1: } C(x,t) = \frac{C_o V}{n} \frac{1}{wb} \frac{1}{2\sqrt{\pi D t}} \exp\left[-\frac{(x-vt)^2}{4Dt}\right]$$

$$\text{Model 2: } C(x,t) = \frac{C_o V}{n} \frac{1}{wb} \frac{1}{2\sqrt{\pi \frac{D}{R} t}} \exp\left[-\frac{(x-\frac{v}{R}t)^2}{4\frac{D}{R}t}\right]$$

$$\text{Model 3: } C(x,t) = \frac{C_o V}{n} \frac{1}{wb} \frac{1}{2\sqrt{\pi \frac{D}{R} t}} \exp\left[-\frac{(x-\frac{v}{R}t)^2}{4\frac{D}{R}t} - \lambda t\right]$$

$$\text{Model 4: } C(x,t) = \frac{C_o}{2} \left\{ \text{erfc}\left[\frac{x-vt}{2\sqrt{Dt}}\right] + \exp\left[\frac{xv}{D}\right] \text{erfc}\left[\frac{x+vt}{2\sqrt{Dt}}\right] \right\}$$

$$\text{Model 5: } C(x,t) = \frac{C_o}{2} \left\{ \text{erfc}\left[\frac{x-\frac{v}{R}t}{2\sqrt{\frac{D}{R}t}}\right] + \exp\left[\frac{xv}{D}\right] \text{erfc}\left[\frac{x+\frac{v}{R}t}{2\sqrt{\frac{D}{R}t}}\right] \right\}$$

Model 6:

$$C(x,t) = \frac{C_o}{2} \exp\left[\frac{xv}{2D}\right] \left\{ \exp\left[-\frac{xv^2}{4D^2} - \frac{x\lambda R}{D}\right] \text{erfc}\left[\frac{x - \sqrt{\left(\frac{v}{R}\right)^2 - 4\lambda \frac{D}{R}t}}{2\sqrt{\frac{D}{R}t}}\right] \right. \\ \left. + \exp\left[\frac{xv^2}{4D^2} + \frac{x\lambda R}{D}\right] \text{erfc}\left[\frac{x + \sqrt{\left(\frac{v}{R}\right)^2 - 4\lambda \frac{D}{R}t}}{2\sqrt{\frac{D}{R}t}}\right] \right\}$$

$$\text{Model 7: } C(x,t) = \frac{C_o}{2} \left\{ \text{erfc}\left[\frac{x-vt}{2\sqrt{Dt}}\right] - \exp\left[\frac{xv}{D}\right] \text{erfc}\left[\frac{x+vt}{2\sqrt{Dt}}\right] \right\}$$

$$\text{Model 8: } C(x,t) = \frac{C_o}{2} \left\{ \operatorname{erfc}\left[\frac{x - \frac{v}{R}t}{2\sqrt{\frac{D}{R}t}}\right] - \exp\left[\frac{xv}{D}\right] \operatorname{erfc}\left[\frac{x + \frac{v}{R}t}{2\sqrt{\frac{D}{R}t}}\right] \right\}$$

Model 9:

$$C(x,t) = \frac{C_o}{2} \exp\left[\frac{xv}{2D}\right] \left\{ \exp\left[-\frac{xv^2}{4D^2} - \frac{x\lambda R}{D}\right] \operatorname{erfc}\left[\frac{x - \sqrt{\left(\frac{v}{R}\right)^2 - 4\lambda \frac{D}{R}t}}{2\sqrt{\frac{D}{R}t}}\right] \right. \\ \left. - \exp\left[\frac{xv^2}{4D^2} + \frac{x\lambda R}{D}\right] \operatorname{erfc}\left[\frac{x + \sqrt{\left(\frac{v}{R}\right)^2 - 4\lambda \frac{D}{R}t}}{2\sqrt{\frac{D}{R}t}}\right] \right\}$$

$$\text{Model 10: } C(x,y,t) = \frac{C_o V}{4\pi nb} \frac{1}{\sqrt{D_L D_T}} \frac{1}{t} \exp\left[-\frac{(x-vt)^2}{4D_L t} - \frac{y^2}{4D_T t}\right]$$

$$\text{Model 11: } C(x,y,t) = \frac{C_o V}{4\pi nb} \frac{1}{\sqrt{D_L D_T}} \frac{R}{t} \exp\left[-\frac{(x - \frac{v}{R}t)^2}{4\frac{D_L}{R}t} - \frac{y^2}{4\frac{D_T}{R}t}\right]$$

$$\text{Model 12: } C(x,y,t) = \frac{C_o V}{4\pi nb} \frac{1}{\sqrt{D_L D_T}} \frac{R}{t} \exp\left[-\frac{(x - \frac{v}{R}t)^2}{4\frac{D_L}{R}t} - \frac{y^2}{4\frac{D_T}{R}t} - \lambda t\right]$$

$$\text{Model 13: } C(x,y,t) = \frac{C_o Q}{bn} \frac{1}{4\pi\sqrt{D_L D_T}} \exp\left[\frac{xv}{2D_L}\right] W\left[\frac{x^2 + y^2 \left(\frac{D_L}{D_T}\right)}{4D_L t}, \sqrt{\frac{x^2 + y^2 \left(\frac{D_L}{D_T}\right)}{2D_L}}\right]$$

$$W(a,b) = \sqrt{\frac{\pi}{2b}} \exp[-b] \operatorname{erfc}\left[\frac{-(b-2a)}{2\sqrt{a}}\right]$$

$$\text{Model 14: } C(x,y,t) = \frac{C_o}{2} \operatorname{erfc}\left[\frac{(x-vt)\sin\alpha - y\cos\alpha}{\sqrt{4(D_L \sin^2\alpha + D_T \cos^2\alpha)t}}\right]$$

Problem 2 (5 pts) Define the following terms using the variable list provided. Define new variables as needed.

D - Coefficient of dispersion

a - dispersivity

v - average linear velocity

n - porosity

q - specific discharge (scalar)

K - hydraulic conductivity (scalar)

dh/dl - hydraulic gradient.

K_{xx} - hydraulic conductivity in x-direction due to a gradient in the x-direction.

K_{xy} - hydraulic conductivity in the x-direction due to a gradient in the y-direction.

K_{yy} - hydraulic conductivity in the y-direction due to a gradient in the y-direction.

K_{yx} - hydraulic conductivity in the y-direction due to a gradient in the x-direction.

dh/dx - gradient of head in the x-direction.

dh/dy - gradient of head in the y-direction.

q_x - specific discharge in x-direction.

q_y - specific discharge in the y-direction.

a) The coefficient of dispersion in terms of dispersivity and average linear velocity.

b) The average linear velocity in terms of specific discharge and porosity.

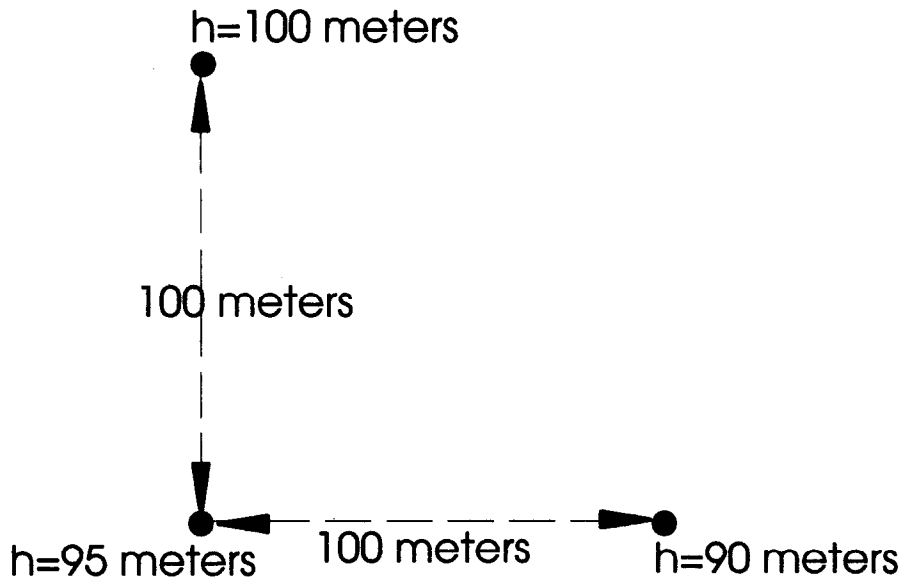
c) Specific discharge in terms of hydraulic conductivity, and hydraulic gradient (scalar form).

d) Specific discharge in the x-direction in terms of hydraulic conductivities and gradients of head.

e) Specific discharge in the y-direction in terms of hydraulic conductivities and gradients of head.

Problem 3 (10 pts.) Three wells monitor an anisotropic aquifer as indicated below. Determine the gradient of head in the x-direction and the y-direction. Determine the path of a water particle starting at (0,0) and travelling under advection for 100 days. The aquifer porosity is $n=0.35$, and the hydraulic conductivity matrix is:

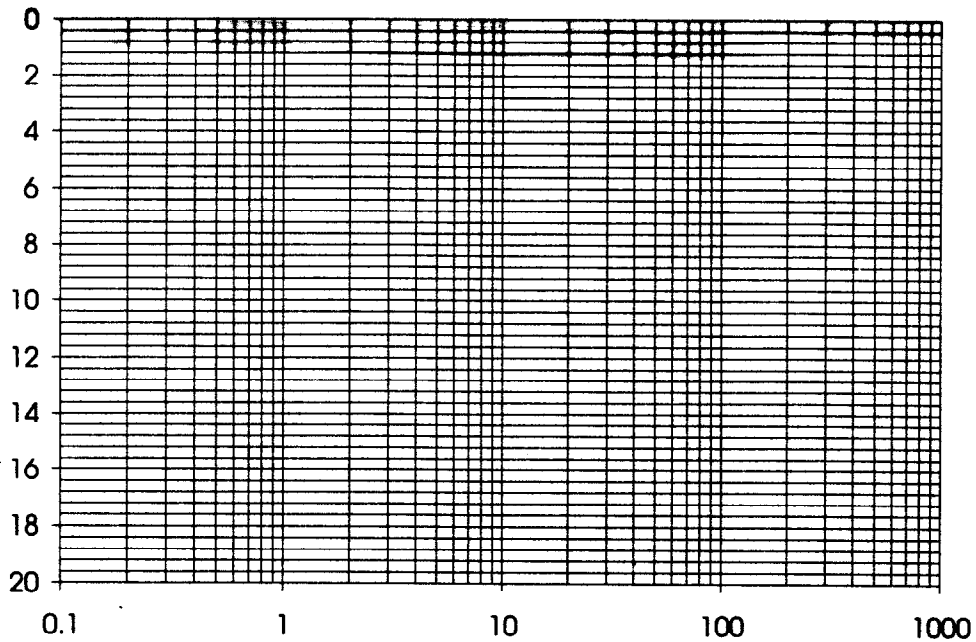
$$\begin{pmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{pmatrix} = \begin{pmatrix} 1.0 & -0.01 \\ -0.01 & 0.6 \end{pmatrix} \text{ m/day}$$



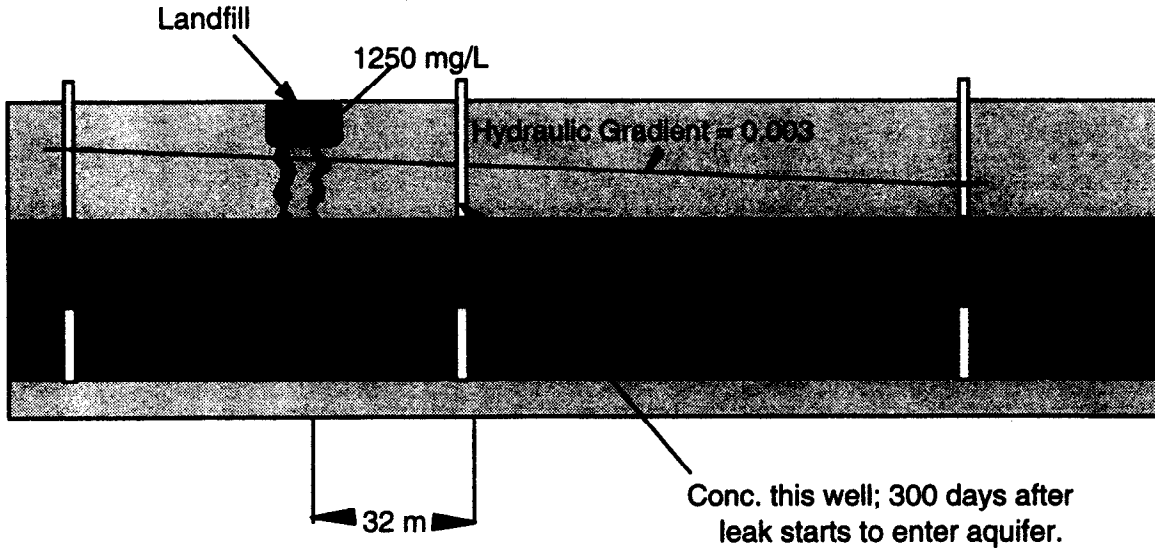
Problem 4 (20 pts.) A well in a confined aquifer is pumped at a rate of 125,000 cubic feet/day. The drawdown at different distances is shown below. Plot the time-drawdown data on the semi-logarithmic chart below. Determine the aquifer transmissivity and storativity using the Jacob straight-line method.

Pumping Rate 125,000 (ft³/day)

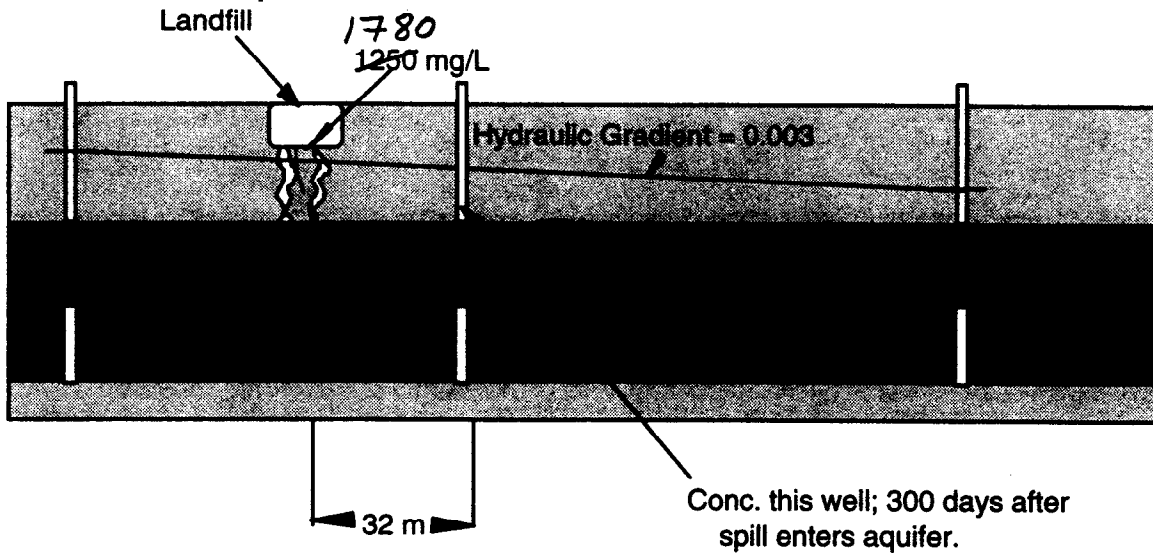
Time (days)	Radius (feet)	Drawdown (feet)
30	50	21.56
30	150	16.89
30	250	14.72
30	500	11.77
30	1000	8.83
30	3000	4.31
30	5000	2.42
30	10000	0.56



Problem 6 (20 pts.) A landfill is leaking an effluent with a concentration of iodine of 1250 mg/L. It seeps into an aquifer. The aquifer has a hydraulic conductivity of 7.3 meters/day, a gradient of 0.003, and an effective porosity of 25%. A down-gradient monitoring well is located 32 meters from the landfill. The aquifer is 3 meters thick. Use an appropriate analytical model to determine what the iodine concentration will be in this monitoring well 300 days after the leak first enters the aquifer?



Problem 7 (20 pts.) The same landfill also suddenly releases chromium that is strongly adsorbed in the aquifer. The solids density in the aquifer is 2.65 g/L. The linear distribution coefficient of chromium for the aquifer solids is 23.2 L/mg. The aquifer is 3 meters thick. The 1000 liters of chromium is released at a concentration of 1780 mg/L. Determine the chromium concentration in the monitoring well 300 days after the release enters the aquifer. The well's diameter is 0.1 meters.



$$K_d = 23.2$$

$$R = 1 + \frac{1-w}{w} \rho_s K_d$$

$$= 1 + \frac{1-0.25}{0.25} (2.65)(23.2) = 185$$

$$t^* = \frac{t}{R} = \frac{300}{185} = 1.6 \text{ day}$$