Problem 1.

Figure 1.1 is a diagram of an unconfined aquifer near a creek. Assume that the average saturated thickness of the aquifer is 49 feet. During a long drought, the flow in the creek decreases by 16 cubic feet per second between the two gaging stations located 4 miles apart. The observation wells on the west side of the creek show that the slope of the water table is 0.0004 ft/ft. The observation wells on the east side of the creek (wells are not shown on the figure – the gaging stations are on the east side of the creek in the picture) show that the water table slopes away from the creek with slope 0.0006 ft/ft. Estimate the transmissivity and hydraulic conductivity of the aquifer.

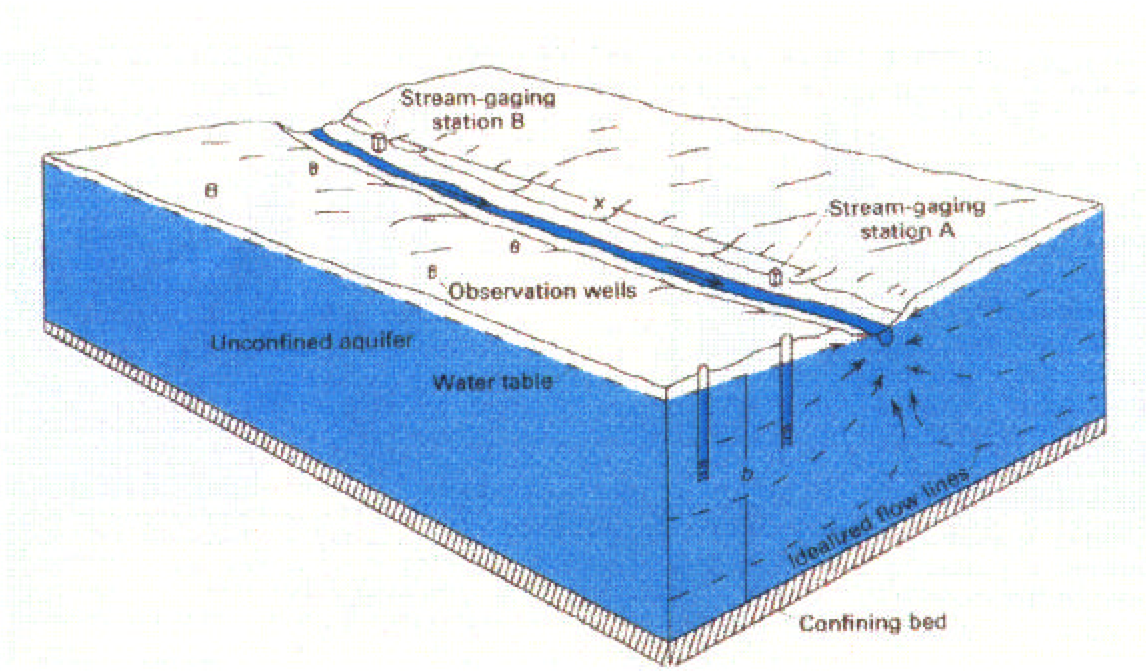
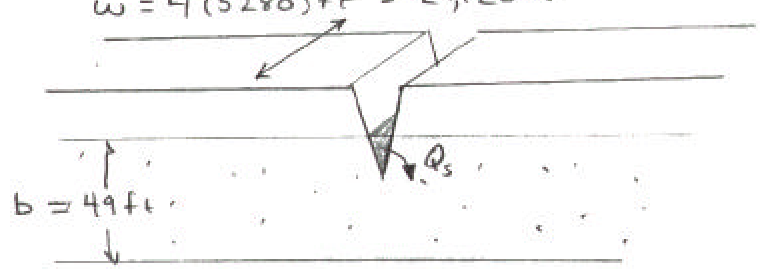


Figure 1.1 Schematic of aquifer-stream system.

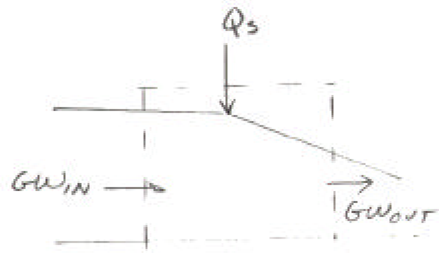
$w = 4(5280)\,ft = 21{,}120\,ft$



$$GW_{IN} = K b w \frac{\Delta h}{\Delta L}$$

$$GW_{OUT} = \underbrace{Kb}_{=T} w \frac{\Delta h}{\Delta L}$$

$$Q_s = 16\ cfs$$



BALANCE

$$GW_{OUT} - GW_{IN} - Q_s = 0$$

$$T(21{,}120\,ft)(0.0006) - T(21{,}120\,ft)(0.0004) - 16 = 0$$

SOLVE FOR T

$$T = \frac{16}{21{,}120(0.0002)} = 3.78\ ft^2/sec$$

$$K = \frac{3.78}{49} = 0.07$$

Problem 2.

Figure 2.1 is a diagram of a portion of a confined aquifer. Determine the groundwater discharge (cubic meters per day) flowing out the right face of the section of aquifer.

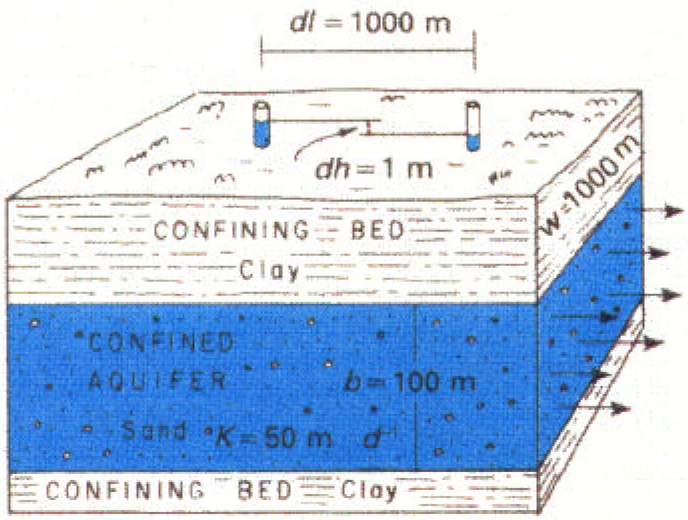


Figure 2.1 Schematic of Aquifer

$$Q = KA\frac{dh}{dl}$$

$$= (50\,m/d)(100\,m)(1000\,m)\left(\frac{1\,m}{1000\,m}\right)$$

$$= 5000\,m^3/d$$

Problem 3

Figure 3.1 below shows a grid with numbers. The numbers represent the piezometric head in an aquifer located at the center of each grid cell. Figure 3.2 is the same aquifer 90 days later than the earlier figure. Each grid cell is 100 meters x 100 meters. Water was withdrawn from the aquifer at a rate of 5,000 cubic meters per day. Draw a contour map of the water levels at the two times and estimate the storativity of the aquifer.

110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180
110	120	130	140	150	160	170	180

Flow

110 120 130 140 150 160 170 180

Figure 3.1 Head at t=0 days

125 Flow 160 150 140 120 120 125 130

119	123	130	137	145	155	163	165
123	125	129	134	140	146	151	154
124	125	127	130	135	140	144	146
124	124	123	123	129	135	139	141
123	122	119	109	124	131	135	137
124	123	122	120	125	130	133	135
125	124	124	124	127	130	132	134
125	125	125	126	128	130	132	133

Figure 3.2 Head at t=90 days

$\forall_{WATER} = 5000 \frac{m^3}{d} \times 90\ d$

$= 450,000\ m^3$

$\forall_{AQUIFER\ DEWATER}$

$\forall_{WATER} = \underbrace{S A \Delta h}_{\forall_{AQUIFER}}$

$S = \frac{\forall_{water}}{A \Delta h}$

$= \frac{450000}{(13.8)(640000)}$

$\simeq 0.05$

+9	+3	0	-3	-5	-7	-10	-15
+13	+5	-1	-6	-10	-14	-19	-26
+14	+5	-3	-10	-15	-20	-26	-34
+14	+4	-7	-17	-21	-25	-31	-39
+13	+2	-11	-31	-26	-29	-35	-43
+14	+3	-8	-20	-25	-30	-37	-45
+15	+4	-6	-16	-23	-30	-36	-46
+15	+5	-5	-14	-22	-30	-36	-47

Area $8 \times 8 \times 100 \times 100$

$640,000\ m^2$

$\overline{\Delta h} = -\frac{877}{64}$

$= -13.8$ over entire area

OR

-877 over 100×100

Problem 4.

Three wells monitor an aquifer as shown in Figure 4.1. The head in each well is listed in the table below. Determine the magnitude and direction of the hydraulic gradient in this aquifer. If a tracer released near well 1 arrives near well 2 in 23 days, and the aquifer has porosity of 30%, what is the hydraulic conductivity of this aquifer?

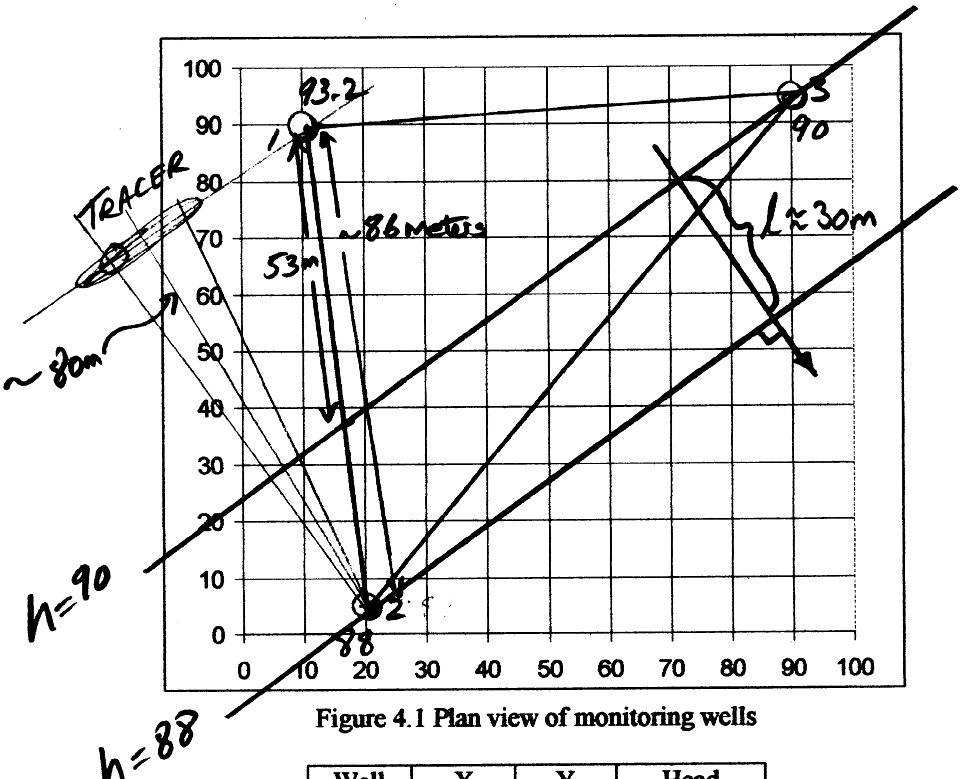


Figure 4.1 Plan view of monitoring wells

Well	X	Y	Head
#1	10	90	93.2
#2	20	5	88
#3	90	95	90

$$\frac{x}{93.2-90} = \frac{86}{93.2-88}$$

$$x = \frac{86(3.2)}{5.2} = 52.9$$

$$\frac{\Delta h}{\Delta l} = \frac{2m}{30m} = 0.067$$

x_{1-2} tracer ~ 80 m

$x = vt$

$$v = \frac{x}{t} = \frac{80m}{23d} = 3.47 m/d$$

$$U = nv = (0.3)(3.47 m/d)$$
$$= 1.043 m/d$$

$U = K\frac{\Delta h}{\Delta l}$ Solve for K

$$K = \frac{U}{\frac{\Delta h}{\Delta l}} = \frac{1.043 m/d}{0.067} = 15.5 m/d$$

Problem 5

Two boreholes are located at points A and B along the same flowline. The soil between them has a hydraulic conductivity of 3.0×10^{-5} ft/sec and a porosity of 40%. Water flows from B to A in the water table. A 500 foot x 100 foot waste cell is planned to be built between points A and B as shown on Figure 5.1. The bottom of the waste cell must be at least 5 feet above the water table at all points. Find the hydraulic gradient between points A and B and the minimum elevation of the waste cell. Estimate the travel time for waste to reach point A if the protective liner of the cell fails and the waste material reaches the groundwater.

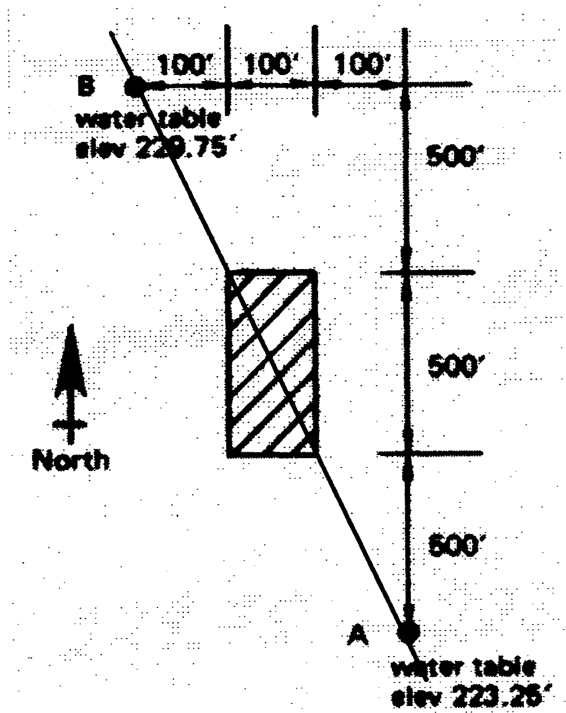
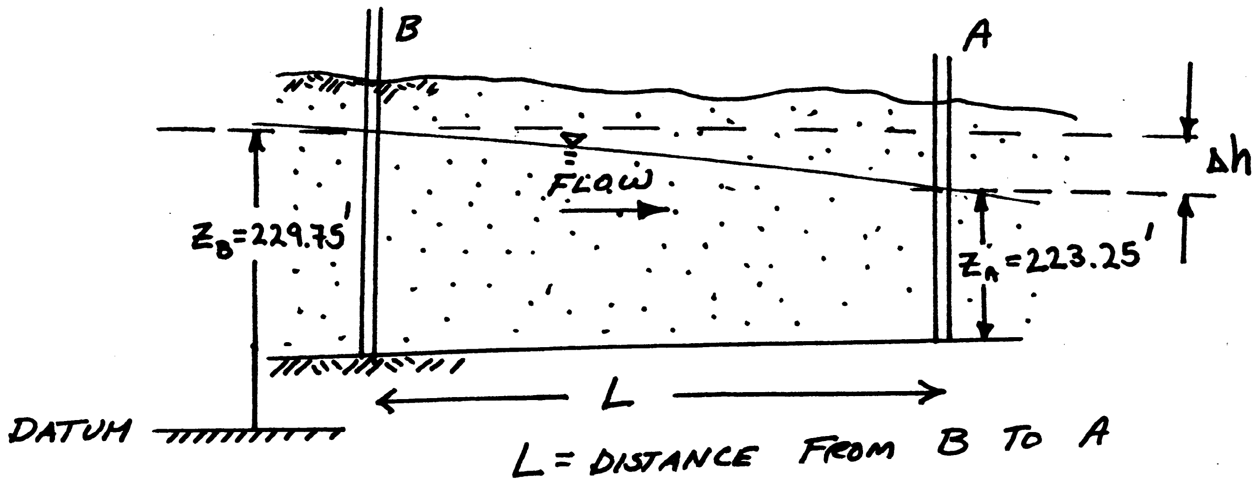


Figure 5.1 Plan view of proposed waste cell.

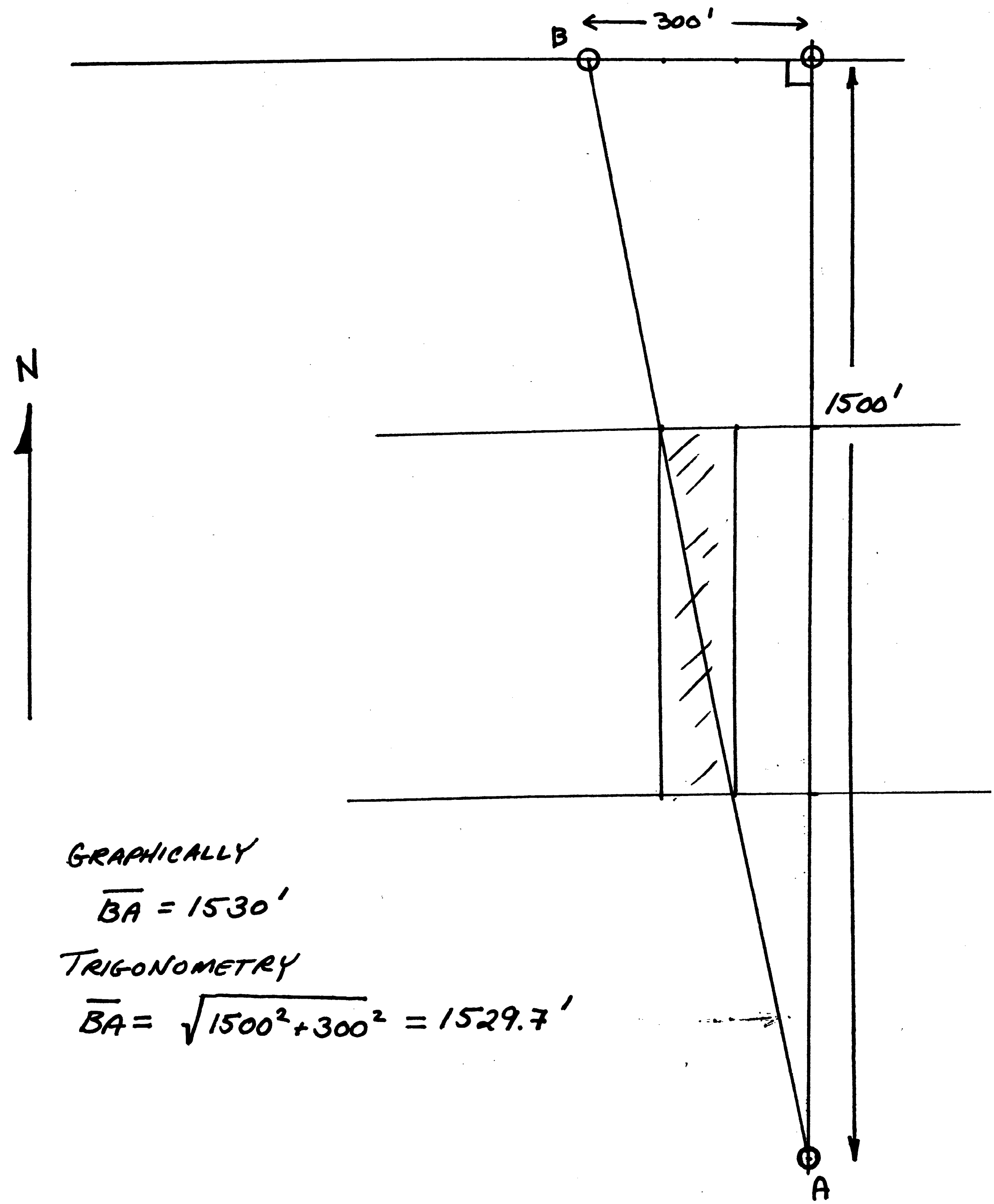
(a) Hydraulic gradient between points A and B.



Hydraulic gradient is $\dfrac{\Delta h}{L}$

$$\Delta h = 229.75 - 223.25$$
$$= 6.5'$$

Determine L by trigonometry or graphically.



GRAPHICALLY

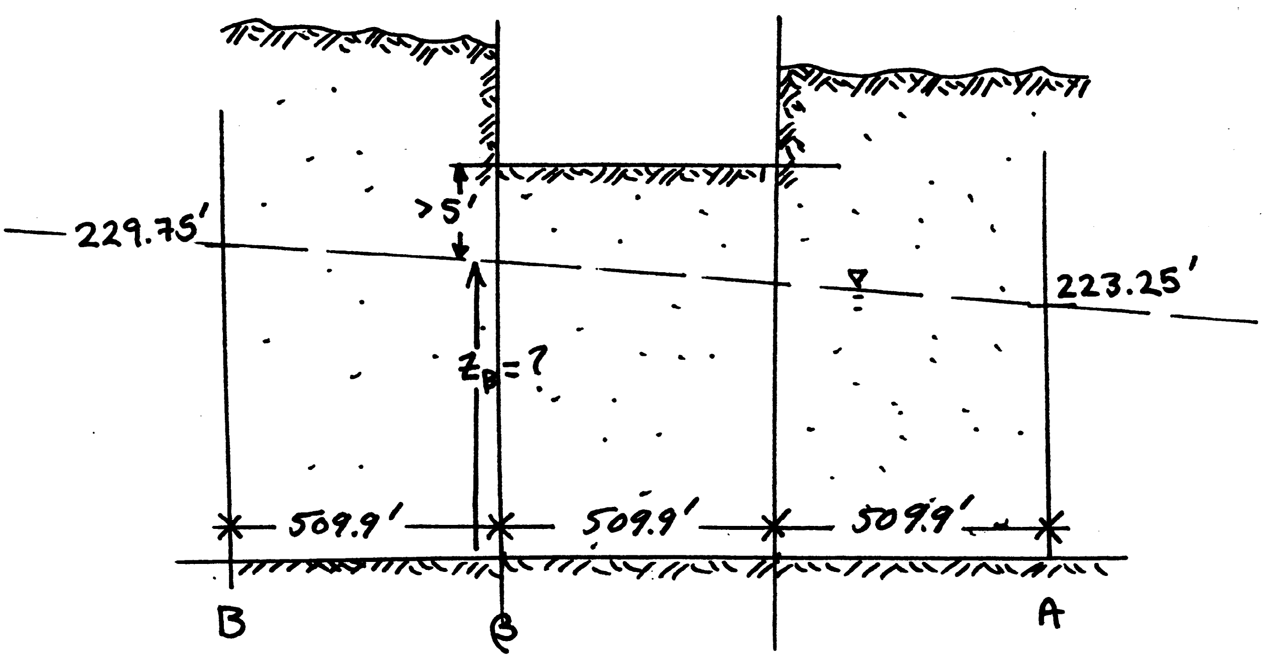
$\overline{BA} = 1530'$

TRIGONOMETRY

$\overline{BA} = \sqrt{1500^2 + 300^2} = 1529.7'$

$$\frac{\Delta h}{L} = \frac{6.5'}{1529.7'} = \underline{\underline{0.00425}}$$

(b) Minimum acceptable elevation of containment cell.



Find water table elevation at β.

Add 5' minimum clearance required.

Result is minimum elevation.

$$Z_\beta = Z_B - \frac{\Delta h}{\Delta L}(509.9')$$

$$= 229.75' - (0.00425)(509.9')$$

$$= 227.58'$$

Add 5' minimum clearance

$$\underline{\underline{Z_{CELL} = 232.58'}} \longleftarrow$$

(c) Assume the liner fails and the contaminant reaches groundwater. Estimate the arrival time of contaminant at point A.

Darcy's Law

$$U = K\frac{\Delta h}{L}$$

U - Seepage velocity (Specific discharge)

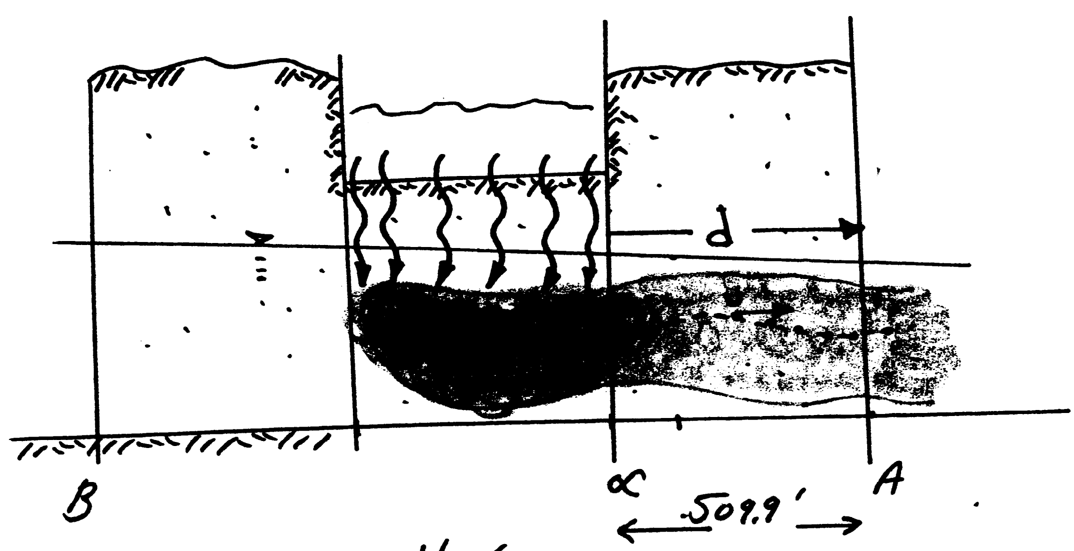
$$u = \frac{U}{n}$$

K - Hydraulic conductivity (Permeability)

n - porosity

$\frac{\Delta h}{L}$ - Hydraulic gradient

u - Pore water velocity



Velocity of a contaminated water particle is the pore water velocity v.

$$v = \frac{K}{n}\frac{\Delta h}{L}$$

$$= \left(\frac{3 \cdot 10^{-5}\ \text{ft/sec}}{0.40}\right)(0.00425)$$

$$= 3.1875 \cdot 10^{-7}\ \text{ft/sec.}$$

Kinematics

$d = vt$ $\quad$ t - travel time

d - distance

v - velocity

$$t = \frac{d}{v}$$

$$t = \frac{509.9'}{3.1875 \cdot 10^{-7}\ ft/sec}$$

$$= 1.59 \cdot 10^{9} sec$$

Convert into meaningful units

$$1.59 \cdot 10^{9} sec \frac{day}{86400\ sec} = 1.8515 \cdot 10^{4}\ days$$

$$1.8515 \cdot 10^{4}\ days \frac{1\ yr}{365\ days} = \underline{\underline{50.73\ years}}$$

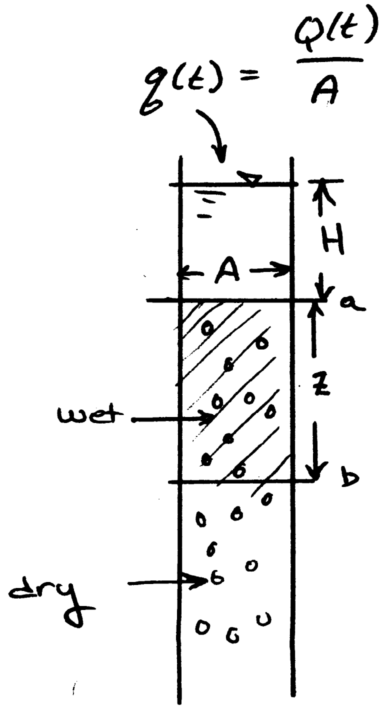
Problem 6 (Hard!)

An artificial recharge pond is located in a sandy area (K=10 m/day; n=0.30). The pond covers a large area. A flood wave fills the pond within a very short time up to a water level of 5.5 m above the sandy bottom. By controlling the inflow the water level is maintained at this elevation. During previous operations, the bottom of the pond was covered by a layer of fines of thickness 0.5 meters, K=0.1 m/day; n=0.50.

Assuming infiltration is only vertically downward, and under practically saturated flow conditions, show the rate of advance of the wetting front from the recharge pond into an assumed dry soil as a function of time. Assume a very deep water table. What will be the rate of infiltration after a very long time?

Case 1: Homogeneous Soil

Per unit area



Volume infiltrated $= nz$.

$$\frac{dnz}{dt} = n\frac{dz}{dt} = q(t) \quad (1^{**})$$

Pressure at a = H

Elevation at a = 0

Pressure at b = 0

Elevation at b = $-z$

$\therefore$ Head at a = H

Head at b = $-z$

Darcy's law from a to b

$$q(t) = K\frac{H-(-z)}{z} = K\frac{H+z}{z} \quad (2^{**})$$

Equate (1^{**}) and (2^{**})

$$n\frac{dz}{dt} = K\frac{H+z}{z}$$

Separate & Integrate

$$\frac{K}{n}\,dt = \frac{z}{H+z}\,dz$$

$$= \frac{H\left(\frac{z}{H}\right)}{H\left(1+\frac{z}{H}\right)}\,dz$$

$$\frac{K}{n}\,dt = \frac{\frac{z}{H}}{1+\frac{z}{H}}\,dz$$

$$\int \frac{K}{n}\,dt = \int \frac{\frac{z}{H}}{1+\frac{z}{H}}\,dz \qquad \text{let } u = \frac{z}{H}$$

$$du = \frac{1}{H}\,dz$$

$$\frac{K}{n}t = H\int \frac{\overbrace{\tfrac{z}{H}}^{u}}{\underbrace{1+\tfrac{z}{H}}_{u}}\;\underbrace{\frac{1}{H}\,dz}_{du}$$

$$\frac{K}{n}t = H\int \frac{u}{1+u}\,du$$

$$\frac{K}{n}t = H\left[1+u-\ln|1+u|\right]+C$$

Substitute original variables

$$\frac{K}{n}t = H\left[1+\frac{z}{H}-\ln\left|1+\frac{z}{H}\right|\right]+C$$

8

at $t=0$ $z=0$

$\therefore 0 = H\left[1 - \ln|1|\right] + c$

$\ln|1| = 0 \quad \therefore \quad c = -H$

So

$$\frac{K}{n}t = \cancel{H} + z - H\ln\left|1 + \frac{z}{H}\right| - \cancel{H}$$

$$\frac{K}{n}t = z - H\ln\left|1 + \frac{z}{H}\right|$$

for the specific problem

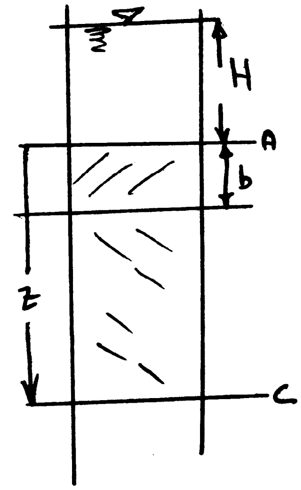
$$\frac{0.1\,m/d}{0.50}\,t = z - 5.5m\ln\left(1 + \frac{z}{5.5}\right)$$

$$t = \frac{0.5}{0.1}\left(z - 5.5\ln\left(1 + \frac{z}{5.5}\right)\right)$$

Stratified Soil

Wetting front passed b.

$$V = n_1 b + n_2 z - n_2 b$$



$$\frac{dv}{dt} = q(t) = n_2 \frac{dz}{dt} \qquad (1)$$

Darcy's law from ~~A~~ to c

$$q(t) = \bar{K} \, \frac{H + z}{z}$$

$$\frac{z}{\bar{K}} = \frac{b}{K_1} + \frac{z - b}{K_2}$$

$$\therefore \quad \frac{\bar{K}}{z} = \frac{1}{\frac{b_1}{K_1} + \frac{z-b}{K_2}}$$

So

$$q(t) = \frac{H + z}{\frac{b}{K_1} + \frac{z-b}{K_2}} \qquad (2)$$

Equate (1) and (2)

$$n_2 \frac{dz}{dt} = \frac{H + z}{\frac{b}{K_1} + \frac{z-b}{K_2}}$$

Separate & Integrate. (A lot more complicated)

let $u = \frac{z}{H}$

$du = \frac{1}{H} dz$

$$n_2 \frac{dz}{dt} = \frac{H\left(1+\frac{z}{H}\right)}{\frac{b}{K_1} + \frac{H\frac{z}{H}}{K_2} - \frac{b}{K_2}} = \frac{H}{H} \frac{\left(1+\frac{z}{H}\right)}{\frac{b}{K_1 H} + \frac{z}{K_2 H} - \frac{b}{K_2 H}}$$

$$n_2 \frac{dz}{dt} = \frac{\left(1+\frac{z}{H}\right)}{\frac{b}{K_1 H} - \frac{b}{K_2 H} + \frac{1}{K_2}\frac{z}{H}} = \frac{\left(1+\frac{z}{H}\right)}{\frac{1}{K_2}\left(\frac{bK_2}{K_1 H} - \frac{b\cancel{K_2}}{\cancel{K_2} H} + \frac{z}{H}\right)}$$

$$n_2 \frac{dz}{dt} = \frac{1+\frac{z}{H}}{\frac{1}{K_2}\left(\frac{bK_2}{K_1 H} - \frac{b}{H} + \frac{z}{H}\right)}$$

$$\frac{\cancel{K_2} n_2}{K_2} \frac{dz}{dt} = \frac{1+\frac{z}{H}}{\underbrace{\left(\frac{bK_2}{K_1 H} - \frac{b}{H}\right)}_{\text{Constant} = A} + \frac{z}{H}} = \frac{1+\frac{z}{H}}{A+\frac{z}{H}}$$

$$\frac{A+\frac{z}{H}}{1+\frac{z}{H}} dz = \frac{K_2\, dt}{\cancel{K_2} n_2}$$

$$\int \frac{A + \frac{z}{H}}{1 + \frac{z}{H}}\, dz = \int \frac{K_2}{\cancel{K_2} n_2}\, dt$$

Substitute $u = \frac{z}{H}$

$$H \int \frac{A + u}{1 + u}\, du = \int \frac{K_2}{\cancel{K_2} n_2}\, dt$$

$$H \int \frac{A}{1+u}\, du + H \int \frac{u}{1+u}\, du = \int \frac{K_2}{\cancel{K_2} n_2}\, dt$$

$$HA \int \frac{1}{1+u}\, du + H \int \frac{u}{1+u}\, du = \int \frac{K_2}{\cancel{K_2} n_2}\, dt$$

$$HA \ln|1+u| + H\{1 + u - \ln|1+u|\} + c = \frac{K_2 t}{\cancel{K_2} n_2}$$

$$HA \ln\left|1 + \frac{z}{H}\right| + H\left\{1 + \frac{z}{H} - \ln\left|1 + \frac{z}{H}\right|\right\} + c = \frac{K_2 t}{\cancel{K_2} n_2}$$

$$H\left(\frac{bK_2}{K_1 H} - \frac{b}{H}\right) \ln\left|1 + \frac{z}{H}\right| + H\left\{1 + \frac{z}{H} - \ln\left(1 + \frac{z}{H}\right)\right\} + c = \frac{K_2}{n_2} t$$

At $z = b$, $t = 0$ Solve for c

$$H\left(\frac{bK_2}{K_1 H} - \frac{b}{H}\right) \ln\left(1 + \frac{b}{H}\right) + H + b - H\ln\left(1 + \frac{b}{H}\right) + c = 0$$

$$\frac{bK_2}{K_1} - (b + H)\ln\left(1 + \frac{b}{H}\right) + H + b + c = 0$$

Substitution and simplification leads to.

$$\frac{z}{H} - \left(1 + \frac{b}{H} - \frac{K_2 b}{K_1 H}\right)\ln\left(\frac{z+b+H}{b+H}\right) = \frac{K_2 t}{n_2(H)}$$

Where $t=0$ when wetting front first passes interface

for given variables

$\frac{z}{5.5}$

z	time	
0	0	
		$\frac{1m}{0.34d}$
0.5m	0.10718 days	
1.0m	0.23 + 0.107 = 0.34 days	
		$\frac{4m}{0.72d}$
5.0m	0.95 + 0.107 = 1.05 days	
		$\frac{5m}{0.64d}$
10.0m	1.59 + 0.107 = 1.70 days	
		$\frac{10m}{0.94d}$
20.0m	2.64 day	

t	v(wetting front)	nv (flux)
0.3d	2.94 m/d	0.8 m/d
1.0d	5.5 m/d	1.6 m/d
1.7d	7.8 m/d	2.34 m/d
2.64d	10.6 m/d	3.18 m/d

At large time what is infiltration?

At large time $z \to$ big

$\therefore \quad H + z \approx z$

So

$$n_2 \frac{dz}{dt} \approx \frac{z}{\frac{b}{K_1} + \frac{z-b}{K_2}}$$

but $\dfrac{1}{\frac{z-b}{K_2} + \frac{b}{K_1}} = \dfrac{\bar{K}}{z}$

Thus

$$n_2 \frac{dz}{dt} \approx \bar{K} \frac{z}{z} = \bar{K}$$

$$n_2 \frac{dz}{dt} = q(t).$$

$\therefore \quad q(t) = \bar{K}$ at large time.

In present case at large z

$$\bar{K} \approx \frac{z}{\frac{b}{K_1} + \frac{z}{K_2}} = \frac{z}{\frac{0.5}{0.1} + \frac{z}{10}}$$

z	$\bar{K}$
1	0.196
10	1.6
20	2.8
50	5
100	6.6
1000	9.5

$$\lim_{z \to \infty} \bar{K} = 10 \text{ m/d}$$