Name:______________________________________

Instructions:

(1) Be sure you have five (5) different problems.
(2) Read each problem carefully.
(3) Indicate the fundamental principles that you are using for each problem.

Solutions

Name:____

1.
(a) What volume of solid material is present in one cubic meter of sandstone if the porosity is 20%?
Show calculations:

$$\omega = \frac{V_{voids}}{V_{total}} \qquad V_{solids} = (1-\omega)V_{total}$$

$$V_{solids} = (1-0.2)(1 m^3) = 0.8 m^3$$

(b) What volume of water is contained in one cubic meter of material if the porosity is 20% and the saturation expressed as a percentage of the pore space is 30%?
Show calculations:

$$S_\omega = \frac{V_{water}}{V_{voids}} \qquad \omega = \frac{V_{voids}}{V_{total}} \qquad \therefore V_{voids} = \omega V_{total}$$

$$S_\omega = (0.3)(0.2)(1 m^3) = 0.06 m^3$$

(c) Write a Darcy's law. Define each term in your equation.

$$Q = -KA\frac{\Delta h}{\Delta x}$$

K - hydraulic conductivity
A - Flow area
$\frac{\Delta h}{\Delta x}$ - gradient of head

(d) Write the water budget equation.

$$\mathring{I} - \mathring{O} = \mathring{S}$$

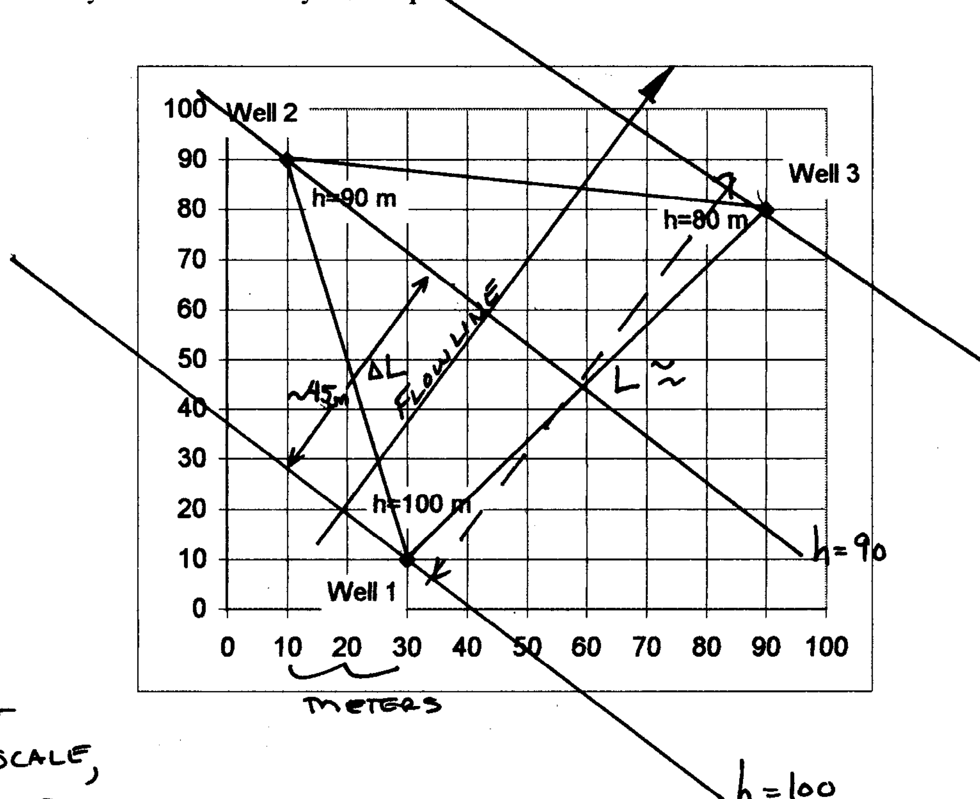
$\mathring{I}$ - Rate of mass inflow
$\mathring{O}$ - Rate of mass outflow
$\mathring{S}$ - Rate of change of mass stored

Name: Solution

2. An aquifer is monitored by three wells located as shown. The head in each well is indicated on the map.

a) Find the magnitude and direction of the hydraulic gradient in this aquifer.

b) A tracer released near Well 1 arrives near Well 3 in 25 days. The aquifer has a porosity of 30%. Estimate the hydraulic conductivity of the aquifer.



DRAWING NOT TO SCALE, BUT CLOSE

$$\frac{\Delta h}{\Delta L} \sim \frac{10\,m}{45\,m} = 0.222$$

DIRECTION ~60°

$L \approx 92\,m$

$$L = v t = \frac{U}{\omega} t$$

$t = 25$ day

$$\therefore \; v = \frac{L}{t} = \frac{92\,m}{25\,day} = 3.68\,m/d$$

$$\omega v = U = (0.30)(3.68\,m/d) = 1.104\,m/d$$

$$U = K \frac{\Delta h}{\Delta L} = K\,(0.222)$$

$$K = \frac{1.104\,m/d}{(0.222)} = 4.97\,m/d$$

OVER

ALTERNATIVE

$$h = ax + by + c$$

$$100 = a\,30 + b\,10 + c$$
$$90 = a\,10 + b\,90 + c$$
$$80 = a\,90 + b\,80 + c$$

SOLVE

$$\begin{array}{l}\text{i}\\ \text{ii}\\ \text{iii}\end{array}\begin{pmatrix} 30 & 10 & 1 \\ 10 & 90 & 1 \\ 90 & 80 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 100 \\ 90 \\ 80 \end{pmatrix}$$

$$\begin{array}{lrrrr} \text{i} & 30 & 10 & 1 & 100 \\ \text{ii*30} \rightarrow \text{ii} & 30 & 270 & 3 & 270 \\ \text{iii} & 90 & 80 & 1 & 80 \end{array} \rightarrow \begin{array}{lrrrr} \text{i} & 30 & 10 & 1 & 100 \\ \text{ii}-\text{i} & 0 & 260 & 2 & 170 \\ \text{iii}-3*\text{i} & 0 & 50 & -2 & -220 \end{array}$$

$$\begin{array}{lrrrr} & 30 & 10 & 1 & 100 \\ \rightarrow \frac{\text{ii}}{260} & 0 & 1 & 0.00769 & 0.6538 \\ \rightarrow \frac{\text{iii}}{50} & 0 & 1 & -0.04 & -4.4 \end{array} \rightarrow \begin{array}{lrrrr} \frac{\text{i}}{30} & 1 & 0.333 & 0.0333 & 3.333 \\ & 0 & 1 & 0.00769 & 0.6538 \\ \text{iii}-\text{ii} & 0 & 0 & -0.04769 & -5.0538 \end{array}$$

back substitute

$$c = \frac{-5.0538}{-0.04769} = 105.97$$

$$b = 0.6538 - 0.00769\,(105.97) = -0.161$$

$$a = 3.33 - 0.333(-0.161) - 0.0333(105.97)$$
$$= -0.145$$

$$-\text{grad}(h) = 0.145\,\underline{i} + 0.161\,\underline{j}$$

$$|-\text{grad}(h)| = \sqrt{0.145^2 + 0.161^2} = 0.216 \quad \text{(close to graphical)}$$

UNIT VECTOR ALONG FLOWLINE

$$\frac{0.145}{0.216}\,\underline{i} + \frac{0.161}{0.216}\,\underline{j} = 0.671\,\underline{i} + 0.745\,\underline{j}$$

2 CONTINUED

DISTANCE WELL 1 TO WELL 3

$$D = \sqrt{60^2 + 70^2} = 92.19$$

DIRECTION COSINES

$$\frac{60i}{92.19} + \frac{70j}{92.19} = 0.65\underline{i} + 0.759\underline{j}$$

VECTOR WELL 1 – WELL 3

$$\underline{D} = 92.19\,(0.65\underline{i} + 0.759\underline{j})$$

PROJECTION ALONG FLOWLINE

$$\underline{D} \cdot \underline{d}_h = 92.19\,(0.65\underline{i} + 0.759\underline{j}) \cdot (0.671\underline{i} + 0.745\underline{j})$$

$$= 92.19\,(0.436 + 0.565) = 92.32$$

FLOWLINE IS NEARLY PARALLEL WITH $\overline{D}$.

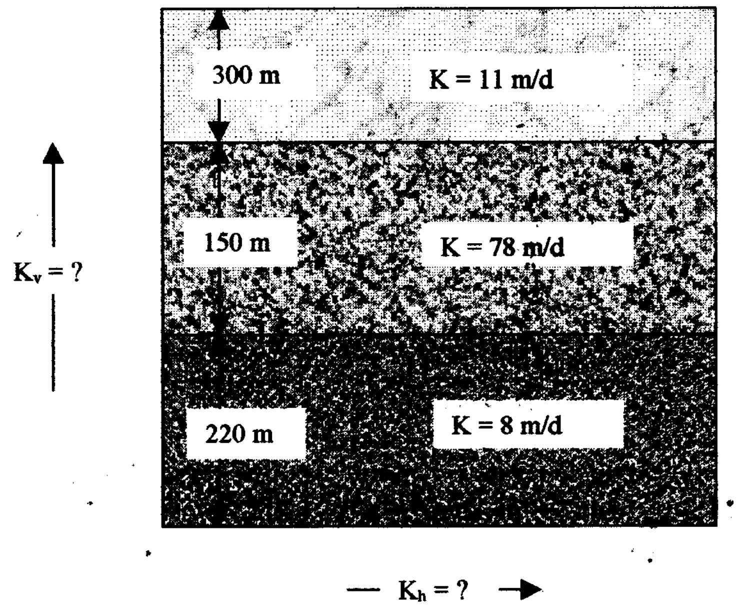
$$U = \frac{L}{t} = \frac{92.32\,m}{25\,day} = 3.69\ m/d$$

$$\omega U = U = (0.30)(3.69) = 1.107\ m/d$$

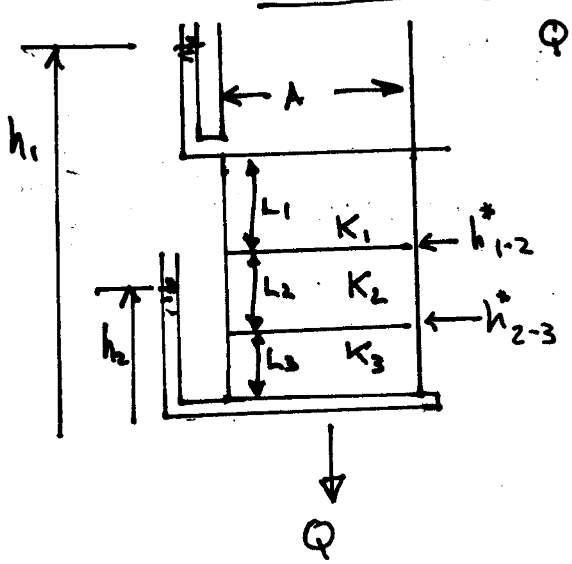
$$K = \frac{1.107\ m/d}{0.216} = 5.125\ m/d$$

Name:____________________________________

3. An aquifer consists of the three formations shown. Each individual formation is homogeneous and isotropic. Derive formulas to determine the overall apparent hydraulic conductivity in the horizontal and vertical directions.



VERTICAL



$$Q = \bar{K}A\frac{h_1-h_2}{L_1+L_2+L_3} \quad \therefore \quad h_1-h_2 = \frac{Q(L_1+L_2+L_3)}{\bar{K}A}$$

but continuity of mass and head loss principles require

$$h_1-h_2 = h_1-h^*_{1-2} + h^*_{1-2}-h^*_{2-3} + h^*_{2-3}-h_2$$

So:

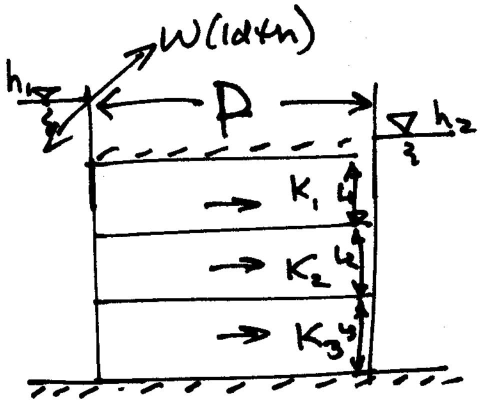
$$\frac{\cancel{Q}(L_1+L_2+L_3)}{\bar{K}\cancel{A}} = \frac{\cancel{Q}L_1}{K_1\cancel{A}} + \frac{\cancel{Q}L_2}{K_2\cancel{A}} + \frac{\cancel{Q}L_3}{K_3\cancel{A}}$$

$$\therefore \bar{K} = \frac{L_1+L_2+L_3}{\frac{L_1}{K_1}+\frac{L_2}{K_2}+\frac{L_3}{K_3}}$$

$$\text{So} \quad K_V = \frac{300+150+220}{\frac{300}{11}+\frac{150}{78}+\frac{220}{8}} = \frac{670}{56.69} = \underline{\underline{11.8\ m/d}}$$

over

Horizontal



$$Q = \bar{K} A \frac{h_1 - h_2}{D} = \bar{K}(L_1 + L_2 + L_3) W \frac{h_1 - h_2}{D}$$

$$= Q_1 + Q_2 + Q_3$$

$$= K_1 L_1 W \frac{h_1 - h_2}{D} + K_2 L_2 W \frac{h_1 - h_2}{D} + K_3 L_3 W \frac{h_1 - h_2}{D}$$

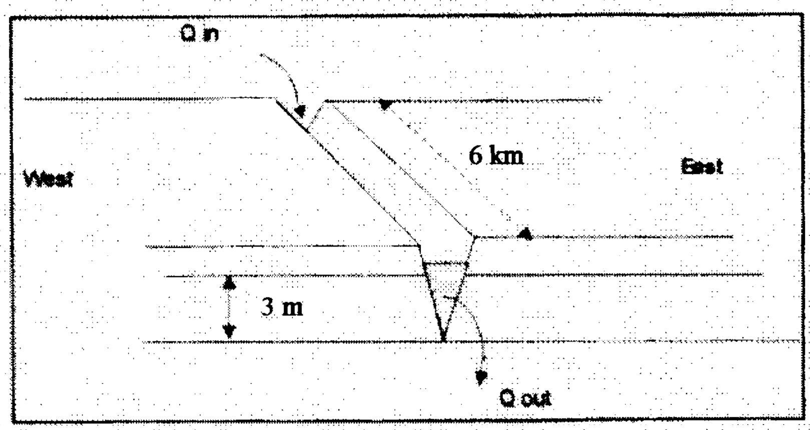
$$= (K_1 L_1 + K_2 L_2 + K_3 L_3) \frac{W}{D} (h_1 - h_2)$$

$\therefore$

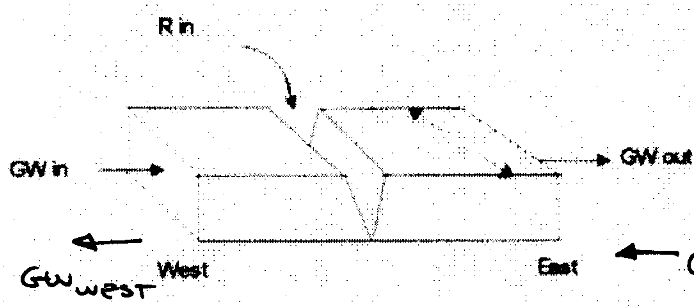
$$\bar{K}(L_1 + L_2 + L_3) \cancel{\frac{W}{D}} \cancel{(h_1 - h_2)} = (K_1 L_1 + K_2 L_2 + K_3 L_3) \cancel{\frac{W}{D}} \cancel{(h_1 - h_2)}$$

$$\bar{K} = \frac{K_1 L_1 + K_2 L_2 + K_3 L_3}{L_1 + L_2 + L_3}$$

4. Dog Run Creek penetrates a confined aquifer 3 meters thick (Figure 3.1). During a long drought the flow in the creek decreases by 1.1 cubic meters per second between two gaging (flow measurement) stations along the creek located 6 kilometers apart. On the west side of the creek the hydraulic head contours run parallel to the bank of the creek and the contour levels decrease as one moves away from the creek at a rate of 0.0007 m/m. The head contours on the east side of the creek are also parallel to the creek and the levels decrease as one moves towards the creek at a rate of 0.0003 m/m.



(a) Write a water balance for the aquifer in the 6 km section near the creek.



OK TO HAVE $-GW_{OUT}$ SAME AS GW_{EAST}

$$GW_{IN} + R_{IN} = GW_{OUT}$$

$$GW_{IN} = -K(6000)(3)(0.0007)$$

$$GW_{OUT} = -K(6000)(3)(0.0003)$$

$$R_{IN} = 1.1\ m^3/sec$$

(b) Use Darcy's Law and the water balance to estimate the hydraulic conductivity of the aquifer.

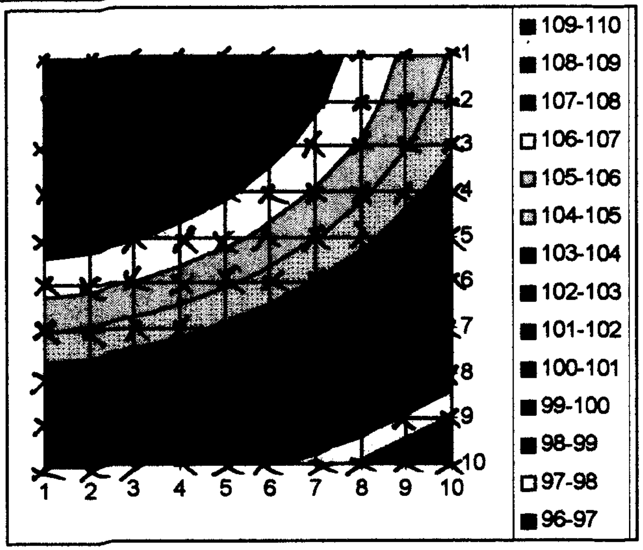
$$-K(6000)(3)(0.0007) + 1.1\ m^3/s = -K(6000)(3)(0.0003)$$

$$1.1\ m^3/s = -K(6000)(3)(0.0003) + K(6000)(3)(0.0007)$$

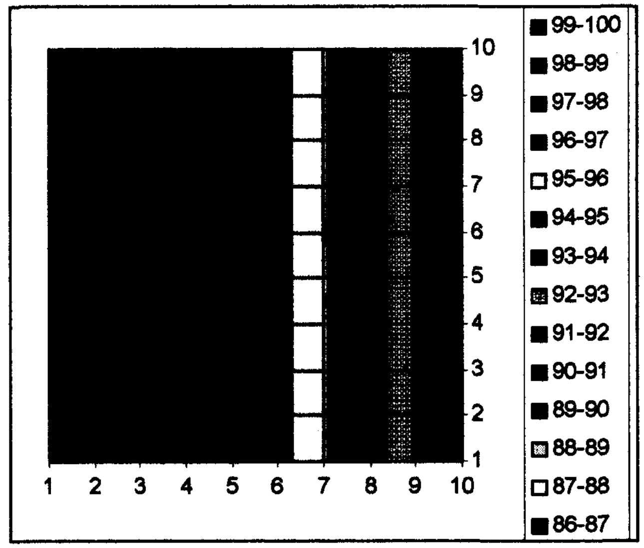
$$K = \frac{1.1}{(6000)(3)(0.0004)} = 0.153\ m/sec$$

Name:______________________________

5. The two maps below are shaded contour maps of an aquifer before and after 2.5 million cubic meters were pumped from the aquifer. The grid cells are 100 meters x 100 meters. The head values are in meters. The contour intervals are shown by the different color panels. Estimate the storativity of the aquifer.



Before Pumping



After Pumping

A worksheet grid is provided to help calculate Δh for the aquifer.

Name:________________________________

Head in Aquifer - Before:

	1	2	3	4	5	6	7	8	9	10
1	110	110	110	109	109	108	107	107	106	105
2	110	110	110	109	109	108	107	107	106	105
3	110	109	109	109	108	108	107	106	105	105
4	109	109	109	108	109	107	106	106	105	104
5	108	108	108	107	107	106	106	105	104	103
6	107	107	107	106	106	105	104	104	103	102
7	106	105	104	103	102	102	103	102	102	100
8	104	104	104	103	102	102	102	101	100	99
9	102	102	102	102	101	101	100	99	98	97
10	100	100	100	100	99	99	98	97	97	96

Head in Aquifer - After:

	1	2	3	4	5	6	7	8	9	10
1	100	100	100	99	98	97	96	94	92	91
2	100	100	100	99	98	97	96	94	92	91
3	100	100	100	99	98	97	96	94	92	91
4	100	100	100	99	98	97	96	94	92	91
5	100	100	100	99	98	97	96	94	92	91
6	100	100	100	99	98	97	96	94	92	91
7	100	100	100	99	98	97	96	94	92	91
8	100	100	100	99	98	97	96	94	92	91
9	100	100	100	99	98	97	96	94	92	91
10	100	100	100	99	98	97	96	94	92	91

Name:________________________________

Change in head (Δh) decline

	1	2	3	4	5	6	7	8	9	10
1	10	10	10	10	11	11	11	13	14	14
2	10	10	10	10	11	11	11	13	14	14
3	10	9	9	10	10	11	11	12	13	14
4	9	9	9	9	10	10	10	12	13	13
5	8	8	8	8	9	9	10	11	12	12
6	7	7	7	7	8	8	8	10	11	11
7	6x	5x	4x	4x	4x	5x	7	8	10	10
8	4x	4x	4x	4x	4x	5x	6x	7	8	8
9	2x	2x	2x	3x	3x	4x	4x	5x	x6	6x
10	0	0	0	1	1	2	2	3x	5x	5x

Δh

$14 \times 5 = 70$
$13 \times 5 = 65$
$12 \times 4 = 48$
$11 \times 11 = 121$
$10 \times 18 = 180$
$9 \times 8 = 72$
$8 \times 10 = 80$
$7 \times 6 = 42$
$6 \times 4 = 24$
$5 \times 6 = 30$
$4 \times 10 = 40$
$3 \times 3 = 9$
$2 \times 5 = 10$

791

AREA = 100×100

TOTAL VOLUME DEWATERED

$791 * 100 * 100$

7,91,0000

$7.91 \cdot 10^6\ m^3$

$$S_y = \frac{V_{pump}}{V_{dewatered}} = \frac{2.5 \cdot 10^6\ m^3}{7.91 \cdot 10^6\ m^3}$$

$= 0.316$

$= 0.32$