Measuring Soil Properties

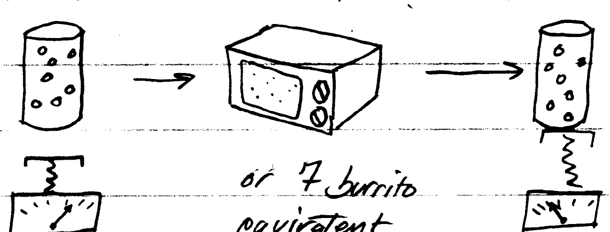
Using the Richards equation requires knowledge of
$k_{rw}(\theta_w)$ or $k_{rw}(\psi)$

Additionally
$\theta_w(\psi)$ is required (Soil characteristic curve)

Water Content

gravimetric method:

105°C / 24hrs



m_1 = mass soil + water

or 7 burrito equivalent times in a microwave

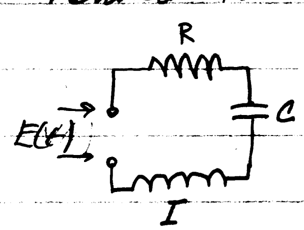
m_2 = mass soil

$$\theta_{wg} = \frac{m_1 - m_2}{m_2}$$

neutron & gamma absorbtion:
already discussed in geophysics lecture. Need to calibrate tools

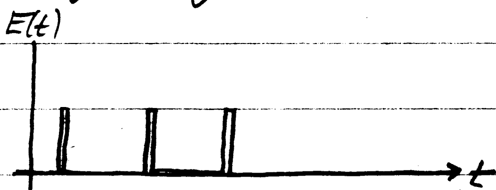
time domain reflectometry (TDR)
similar to sending acoustic pulses, time of travel of EM pulse is related to water content

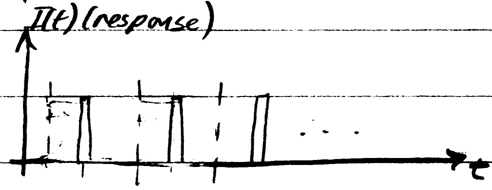
Circuit is:



$$L\dot{I} + RI + \frac{1}{C}\int I\,dt = E(t)$$

$E(t)$ is typically a set of pulses





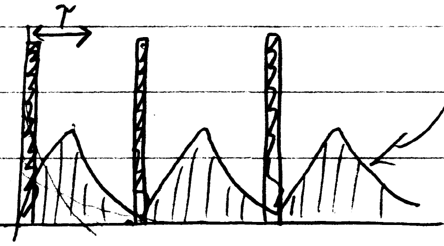
ratio of spacing & dispersion of current waveform provides information about soil properties

For a given soil we would have

$$\lambda_1 = -\frac{R}{2L} + \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

$$\lambda_2 = -\frac{R}{2L} - \frac{1}{2L}\sqrt{R^2 - \frac{4L}{C}}$$

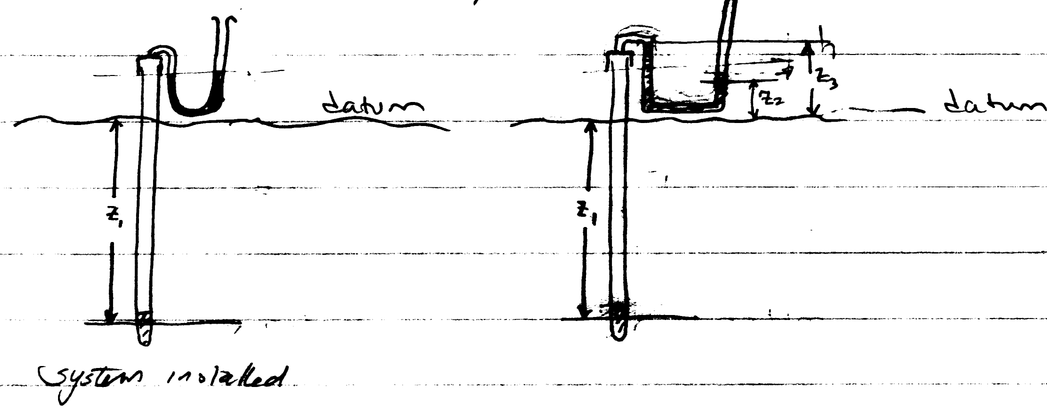
$$I(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \quad \text{(for each impulse)}$$



τ is related to R,C,L which change depending on water content

Capillary Pressure

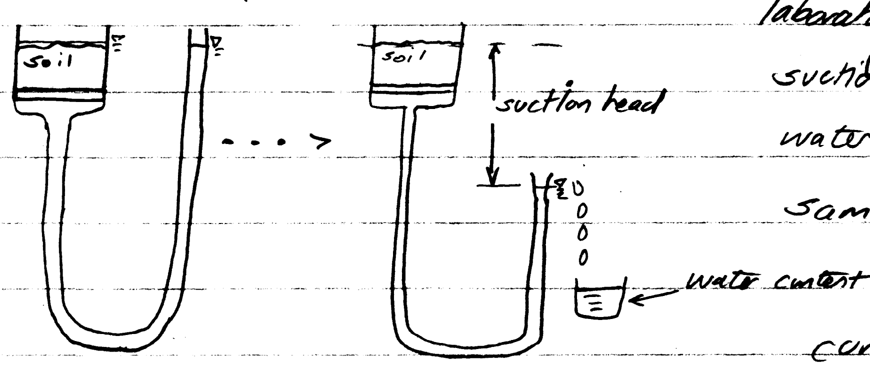
Tensiometer - tensiometer is very similar to a piezometer except "screen" end is a porous material



$$p_{soil} = p_{Atm} - (z_3 - z_2)\gamma \qquad h_{soil} = \frac{p_{Atm}}{\gamma} - (z_3 - z_2) - z_1$$

At high suctions (90 kPa) water cavitates in device and psychrometric methods (thermodynamic techniques) are required.

Soil-Water characteristic Curve



laboratory method involves suction head measurements as water is drawn from sample. Typically drainage and imbibition (wetting) curves are different.

Other approaches

- Estimate properties for similar soils
- Estimate properties from grain size distribution
- Assume analytic functional form for $\psi(\theta_w)$, obtain fitting parameters from soil textural descriptions

Common analytic models

Brooks & Corey (power-law model)
van Genuchten (3-parameter power law model)

Brooks and Corey model

$$\Theta = \begin{cases} 1 & \psi \le \psi_b \\ \left(\frac{\psi_b}{\psi}\right)^{\lambda} & \psi > \psi_b \end{cases}$$

ψ_b is bubbling pressure head
λ is pore size distribution index

$$\Theta = \frac{\theta_w - \theta_{wr}}{n - \theta_{wr}}$$

θ_{wr} - irreducible water content (can use wilting point as an approximation)

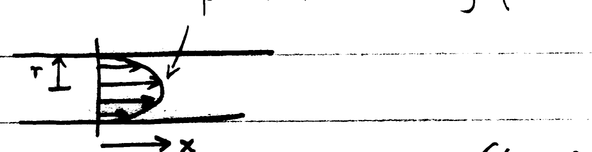
van Genuchten model

$$\Theta = \left(\frac{1}{1 + (\alpha\psi)^n}\right)^m$$

Hydraulic Conductivity/Relative Permeability

Usually determine K_{ws} by permeameter methods.
Determine k_{rw} from pore-size distribution correlations and application of the capillarity formula to an idealized fluid mechanics geometry (Poiseuille formula)

$$\frac{dp}{dx} = \mu \frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr}$$



Integration of above PDE using parabolic velocity profile (laminar flow) produces

$$Q_{pore} = \frac{\pi r^4}{8\mu}\frac{p_1 - p_2}{L} = \underbrace{\frac{\pi r^4}{8\mu}}_{\text{"flow resistance" (like K)}} \cdot \frac{dp}{dx}$$

Observe that

$$\underbrace{\frac{\pi r^2}{4}}_{\text{pore area}} \underbrace{\left(\frac{r^2}{2\mu}\right)}_{\text{"viscous resistance"}}$$

∴ The resistance is proportional to r^2

Combined with a tortuosity equation, the Burdine equation relates pore size and k_{rw} as

$$k_{rw} = \Theta^2 \left(\frac{\int_0^{\Theta} \frac{d\Theta}{\psi^2}}{\int_0^1 \frac{d\Theta}{\psi^2}} \right)$$

a similar relationship for air permeability results

Relative Permeability Measurements

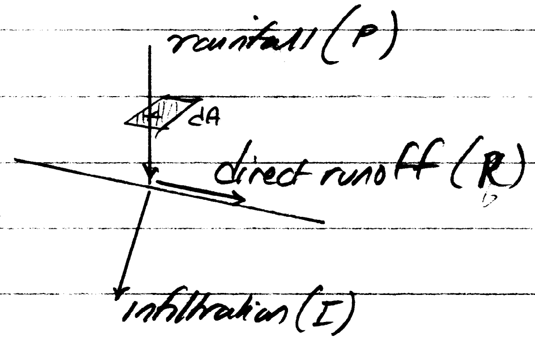
Measuring K_{rw} is a non-trivial soil science exercise
Methods include

- pressure plate (Tempe cell)
- depth profiling under constant infiltration
- Guelph permeameter

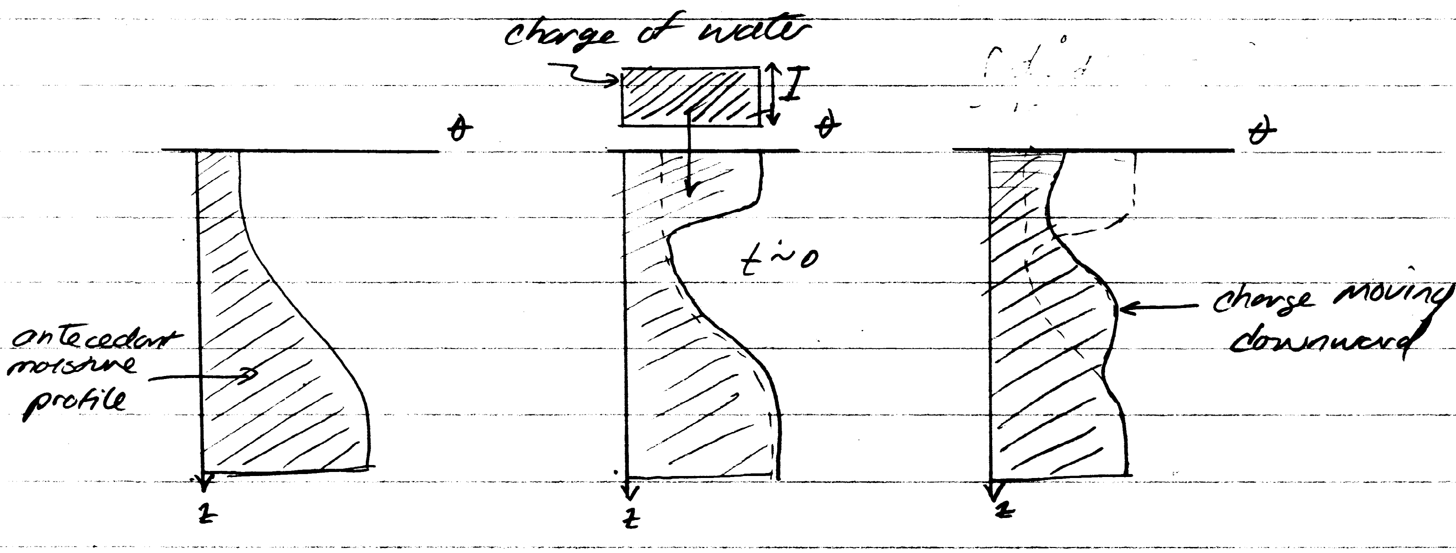
tempe cell in papers
refs.?

Infiltration models

Infiltration is an important hydrologic process for both surface water behavior and subsurface behavior



$PA = RA + IA$



$I(t)$ cumulative infiltration depth

$$I(t) = \int_0^\infty \left[\theta(z,t) - \theta_A(z,t)\right] dz$$

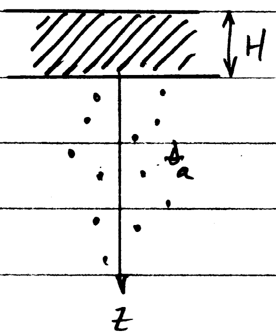
$I(t)$ has dimensions of L

$$i(t) = \frac{dI}{dt} = \frac{d}{dt}\int_0^\infty \theta(z,t) - \theta_A(z,t)\, dz$$

$i(t)$ is infiltration rate

$i(t)$ has dim. $\frac{L}{t}$

Green-Ampt Model



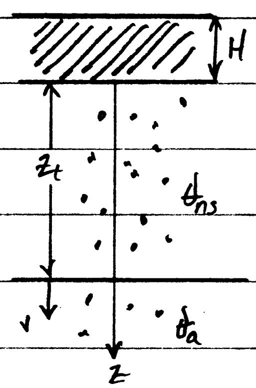
K_{ns} - saturated hydraulic conductivity

θ_{ns} - water content behind wetting front

θ_a - water content ahead of wetting front

$$\underbrace{(\theta_{ns}-\theta_a)\frac{dz}{dt}}_{\text{volume infiltrated}} = \underbrace{K_{ns}\,\frac{H+\psi+z}{z}}_{\text{Darcy's Law}}$$

Separate & integrate



$$\frac{z}{H+\psi+z}\,dz = \frac{K_{ns}}{(\theta_{ns}-\theta_a)}\,dt$$

$$\frac{z}{H+\psi+z} = \frac{H+\psi+z}{H+\psi+z} - \frac{H+\psi}{H+\psi+z} = 1 - \frac{H+\psi}{H+\psi+z}$$

$$\int\left[1-\frac{H+\psi}{H+\psi+z}\right]dz = \int\frac{K_{ns}}{(\theta_{ns}-\theta_a)}\,dt$$

$$z + (H+\psi)\ln\left(\frac{H+\psi}{H+\psi+z}\right) = \frac{K_{ns}\,t}{(\theta_{ns}-\theta_a)}$$

Cumulative infiltration is

$$I(t) = (\theta_{ns}-\theta_a)\,z(t)$$

$$\therefore I(t) = K_{ns}t - (\theta_{ns}-\theta_a)(H+\psi)\ln\left(\frac{H+\psi}{H+\psi+\frac{I}{(\theta_{ns}-\theta_a)}}\right)$$

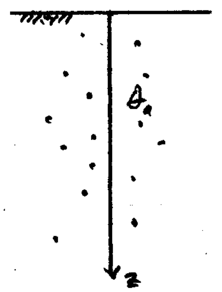
$$= K_{ns}t + (\theta_{ns}-\theta_a)(H+\psi)\ln\left(\frac{(H+\psi)(\theta_{ns}-\theta_a)+I}{(\theta_{ns}-\theta_a)\,H+\psi}\right)$$

$$= K_{ns}t + (\theta_{ns}-\theta_a)(H+\psi)\ln\left(1+\frac{I}{(\theta_{ns}-\theta_a)(H+\psi)}\right)$$

Green-Ampt Model

K_{ns} – Saturated hydraulic conductivity
θ_{ns} – water content behind wetting front
θ_a – water content ahead of wetting front

t<0



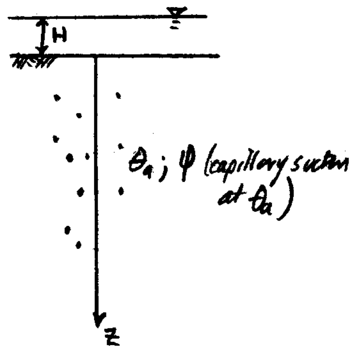
$$I(t) = z_t\,(\theta_{ns} - \theta_a)$$

$$\frac{dI}{dt} = i = \frac{d}{dt}\left(z_t\,(\theta_{ns} - \theta_a)\right)$$

Green-Ampt model assumes θ_{ns} & θ_a are constants with respect to time so that

$$i = (\theta_{ns} - \theta_a)\frac{dz_t}{dt}$$

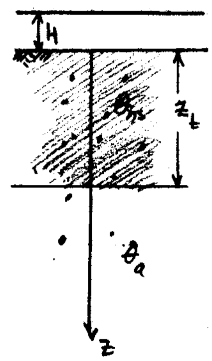
t=0



At the soil-water interface the infiltration rate is governed by Darcy's Law

$$i = K_{ns}\left(\frac{(H+\psi)-(-z_t)}{z_t}\right) = K_{ns}\left(\frac{H+\psi+z_t}{z_t}\right)$$

t>0



← Volume of moisture infiltrated into shaded area $z_t\,(\theta_{ns} - \theta_a)$

Equate these two expressions for i and solve for z_t

$$(\theta_{ns} - \theta_a)\frac{dz}{dt} = K_{ns}\left(\frac{H+\psi+z_t}{z_t}\right)$$

$$\frac{z_t}{H+\psi+z_t}\,dz = \frac{K_{ns}}{(\theta_{ns} - \theta_a)}\,dt$$

$$\frac{z_t}{H+\psi+z_t} = \frac{H+\psi+z_t}{H+\psi+z_t} - \frac{H+\psi}{H+\psi+z_t} \Rightarrow \int 1 - \frac{H+\psi}{H+\psi+z_t}\,dz = \int \frac{K_{ns}}{(\theta_{ns} - \theta_a)}\,dt$$

$$= \int dz - H+\psi \int \frac{1}{H+\psi+z}\,dz = \frac{K_{ns}\,t}{(\theta_{ns} - \theta_a)} + c$$

$$= z - (H+\psi)\ln(H+\psi+z) = \frac{K_{ns}\,t}{(\theta_{ns} - \theta_a)} + C$$

evaluate C from initial conditions at $t=0,\ z=0$

$$\therefore\ c = -(H+\psi)\ln(H+\psi)$$

Thus the equation that relates z and t is

$$z - (H+\psi)\ln(H+\psi+z) = \frac{K_{ns}\,t}{(\theta_{ns}-\theta_a)} - (H+\psi)\ln(H+\psi)$$

$$z + (H+\psi)\left[\ln(H+\psi) - \ln(H+\psi+z)\right] = \frac{K_{ns}\,t}{(\theta_{ns}-\theta_a)}$$

$$z + (H+\psi)\ln\left[\frac{H+\psi}{H+\psi+z}\right] = \frac{K_{ns}\,t}{(\theta_{ns}-\theta_a)}$$

Now substitute $I(t) = z_t(\theta_{ns}-\theta_a)$ to obtain the infiltration equation

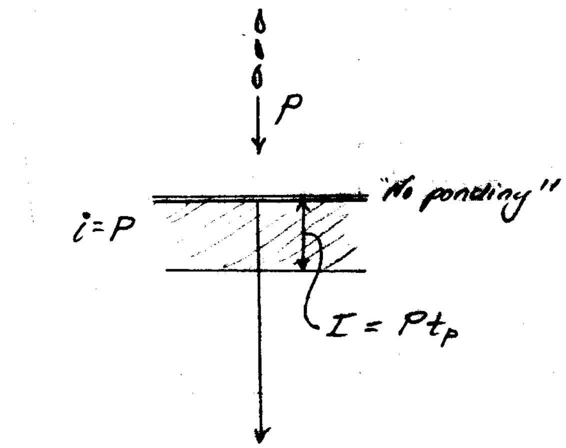
$$I(t) = z_t(\theta_{ns}-\theta_a) = (\theta_{ns}-\theta_a)\left[\frac{K_{ns}\,t}{(\theta_{ns}-\theta_a)} - (H+\psi)\ln\left(\frac{H+\psi}{H+\psi+\frac{I}{(\theta_{ns}-\theta_a)}}\right)\right]$$

$$= K_{ns}t - (H+\psi)\ln\left(\frac{H+\psi}{H+\psi+\frac{I}{\theta_{ns}-\theta_a}}\right)$$

$$= K_{ns}t + (H+\psi)\ln\left(\frac{H+\psi+\frac{I}{\theta_{ns}-\theta_a}}{H+\psi}\right)$$

$$\underline{\underline{I(t) = K_{ns}t + (H+\psi)\ln\left(1+\frac{I(t)}{(H+\psi)(\theta_{ns}-\theta_a)}\right)}} \longleftarrow$$ cumulative infiltration (Green-Ampt Model)

Application to Rainfall-Runoff modeling



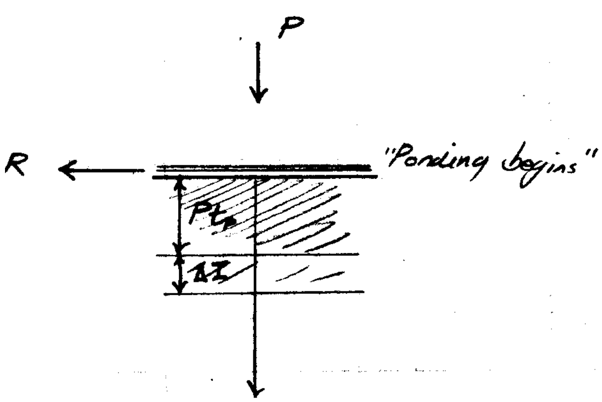
Time to ponding (runoff)

Substitute into rate equation

$$i = K_{ns}\left(\frac{H+\psi+z}{z}\right)$$

recall $z = \dfrac{I(t)}{\theta_{ns}-\theta_a}$

So $$i = K_{ns}\left(\frac{H+\psi+\dfrac{I}{\theta_{ns}-\theta_a}}{\dfrac{I}{\theta_{ns}-\theta_a}}\right)$$



Thus

$$i t_p = P t_p = K_{ns}\, t_p\left(\frac{(H+\psi)(\theta_{ns}-\theta_a)}{P t_p} + 1\right)$$

Solve for t_p (time to ponding)

$$\frac{P t_p}{K t_p} = \left(\frac{(H+\psi)(\theta_{ns}-\theta_a)}{P t_p} + \frac{P t_p}{P t_p}\right)$$

$$P t_p P \cancel{t_p} = (H+\psi)(\theta_{ns}-\theta_a) K \cancel{t_p} + P t_p K \cancel{t_p}$$

$$t_p(P P) - P K t_p = (H+\psi)(\theta_{ns}-\theta_a) K$$

$$t_p\, P(P-K) = (H+\psi)(\theta_{ns}-\theta_a) K$$

$$t_p = \frac{(H+\psi)(\theta_{ns}-\theta_a) K}{P(P-K)}$$

Since ponding depth is $H=0$

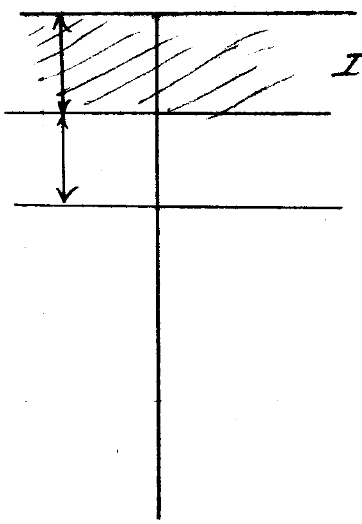
$$t_p = \frac{\psi(\theta_{ns}-\theta_a) K}{P(P-K)}$$ (Compare to 4.5.24 in text)

Cumulative infiltration after ponding

t_p has occured

$$I(t+\Delta t) = K(t+\Delta t) + (\theta_{ns}-\theta_a)(H+\psi)\ln\left(1+\frac{I(t+\Delta t)}{(H+\psi)(\theta_{ns}-\theta_a)}\right)$$

$$I(t) = Kt + (\theta_{ns}-\theta_a)(H+\psi)\ln\left(1+\frac{I(t)}{(H+\psi)(\theta_{ns}-\theta_a)}\right)$$



$I=(\theta_{ns}-\theta_a)z_{t_p}$

$+\Delta t$

$$I(t+\Delta t) - I(t) =$$

$$\cancel{Kt} + K\Delta t - \cancel{Kt} + (\theta_{ns}-\theta_a)(H+\psi)\ln\left[\frac{1+\frac{I(t+\Delta t)}{(H+\psi)(\theta_{ns}-\theta_a)}}{1+\frac{I(t)}{(H+\psi)(\theta_{ns}-\theta_a)}}\right]$$

$$I(t+\Delta t) - I(t) = K\Delta t + (\theta_{ns}-\theta_a)(H+\psi)\ln\left[\frac{(H+\psi)(\theta_{ns}-\theta_a)+I(t+\Delta t)}{(H+\psi)(\theta_{ns}-\theta_a)+I(t)}\right]$$

$$\therefore\ I(t+\Delta t) = I(t) + K\Delta t + (\theta_{ns}-\theta_a)(H+\psi)\ln\left[\frac{(H+\psi)(\theta_{ns}-\theta_a)+I(t+\Delta t)}{(H+\psi)(\theta_{ns}-\theta_a)+I(t)}\right]$$

As ~~before~~, the ponding depth is assumed to be zero, and all Precip. not infiltrated is runoff so

$$** \longrightarrow I(t+\Delta t) = I(t) + K\Delta t + (\theta_{ns}-\theta_a)\psi\ln\left[\frac{\psi(\theta_{ns}-\theta_a)+I(t+\Delta t)}{\psi(\theta_{ns}-\theta_a)+I(t)}\right]$$

cumulative runoff is determined from a mass balance

$$R_t = Pt - I_t$$

** is easier to write in terms of Δt

$$\frac{I(t+\Delta t) - I(t) - (\theta_{ns}-\theta_a)\psi\ln\left[\frac{\psi(\theta_{ns}-\theta_a)+I(t+\Delta t)}{\psi(\theta_{ns}-\theta_a)+I(t)}\right]}{K} = \Delta t$$

To use Green-Ampt one needs to know $(\theta_{ns} - \theta_a)$: (Change in water content across front)

ψ : capillary suction at the front.

Using Brooks & Corey power law model is typical where

$$k_r = \begin{cases} |\Theta|^{\varepsilon} & \psi > \psi_b^* \\ 1 & \psi < \psi_b^* \end{cases}$$

Solving for $\psi_f = \int_0^{\infty} k_r \, d\psi$ (from Darcy's law at front)

$$\psi_f = \frac{2 + 3\lambda}{1 + 3\lambda} \psi_b^*$$

Typically $\psi_b^* = \dfrac{\psi_b}{2}$ (to account for differences between wetting & drainage curves)

$$\underset{\uparrow}{K} = \frac{K_{ws}}{2}$$

*K in cumulative infiltration model

$\varepsilon = 3 + \dfrac{2}{\lambda}$ (from Burdine eqn.)

$$\theta_{ns} = \theta_{wr} + (n - \underbrace{\theta_{wr}}_{\text{determined experimentally}}) \left(\frac{1}{2}\right)^{1/\varepsilon}$$

Lastly $\theta_a \approx \theta_{fc}$ (field capacity)

<u>Using the model</u>

$P = 11\,cm/2\,hrs. \quad = \quad 5.5\,cm/hr.$

$\lambda = 0.60$

$\theta_{wr} = 0.06 \qquad n = 0.44 \qquad \psi_b = 30\,cm \qquad K_{ws} = 0.8\,m/d \qquad \theta_a = \theta_{fc}$

① $\theta_a = \theta_{fc} = \theta_{wr} + (n - \theta_{wr})\,\Theta^{\lambda}$ (Brooks & Corey model, 4.4.2)

$$= 0.06 + (0.44 - 0.06)\left(\frac{30\,cm}{344\,cm}\right)^{0.60}$$

$$= 0.148$$

1 bar ≈ 10 meters head $= \dfrac{101.325\,kPa}{9.8\ m/s^2}$ $= 10.33\,m$

② $\theta_{ns} = \theta_{wr} + (n - \theta_{wr})\left(\frac{1}{2}\right)^{1/\varepsilon}$

$$= 0.06 + (0.44 - 0.06)\left(\frac{1}{2}\right)^{\frac{1}{(3 + 2/0.6)}} = 0.401$$

$\therefore\ \theta_{ns} - \theta_a = 0.401 - 0.148 = 0.253$

$$\psi_f = \frac{2 + 3\lambda}{1 + 3\lambda}\,\frac{\psi_b}{2} = \frac{2 + 3(0.6)}{1 + 3(0.6)}\left(\frac{30}{2}\right) = 20.4\,cm = 0.204\,m$$

$K_{ns} = K_{ws}/2 = 0.4\,m/d$

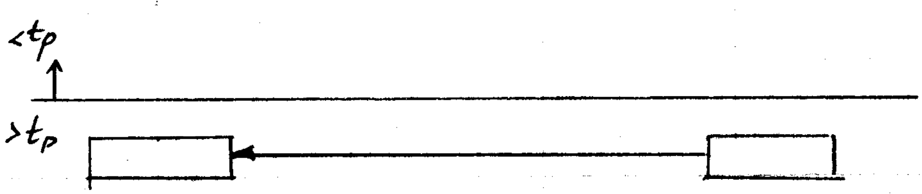
$i = 5.5\,cm/hr = 1.32\,m/d$

$$t_p = \frac{(0.204\,m)(0.253)(0.4\,m/d)}{(1.32\,m/d)(1.32\,m/d - 0.4\,m/d)} = 0.017\,d = 0.408\,hr$$

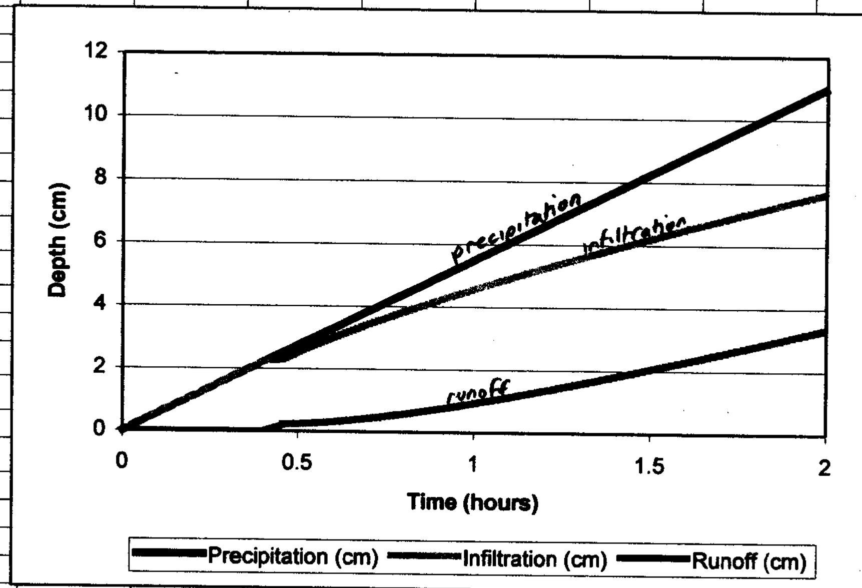
$I(t_p) = (1.32\,m/d)(0.017\,d) = 0.0224\,m$

Next construct a spreadsheet

Δt	t	P	I



	A	B	C	D	E	F	G
1	Green-Ampt Infiltration Model						
2							
3	Data						
4	P	132.00	cm/d	precipitation rate			
5	λ	0.600		Brooks and Corey parameter			
6	θ_{wr}	0.060		irreducible water content			
7	n	0.440		porosity			
8	Ψ_b	30.000	cm	bubbling pressure			
9	K_{ws}	80	cm/d	saturated hydraulic conductivity			
10	Ψ_{fc}	345.332143	cm	suction at field capacity (1/3 atmosphere)			
11	Θ	0.231		reduced water content (Brooks and Corey)			
12							
13	θ_a	0.148		water content ahead of front			
14	θ_{ns}	0.401		water content behind front			
15	$\Delta\theta$	0.253		change in water content at front			
16	Ψ_{front}	20.357	cm	suction at front			
17	K	40.000	cm/d	effective hydraulic conductivity			
18	t_p	0.01695644	d	time to pond			
19							
20	dl				0.05		
21	dt	time (days)	Time (hrs)	Precipitati	Infiltration	Runoff (c	z (cm)
22		0	0	0	0	0	0
23	0.001695644	0.00169564	0.040695	0.223825	0.223825	0	0.885093
24	0.001695644	0.00339129	0.081391	0.44765	0.44765	0	1.770186
25	0.001695644	0.00508693	0.122086	0.671475	0.671475	0	2.65528
26	0.001695644	0.00678258	0.162782	0.8953	0.8953	0	3.540373
27	0.001695644	0.00847822	0.203477	1.119125	1.119125	0	4.425466
28	0.001695644	0.01017387	0.244173	1.34295	1.34295	0	5.310559
29	0.001695644	0.01186951	0.284868	1.566775	1.566775	0	6.195652
30	0.001695644	0.01356516	0.325564	1.790601	1.790601	0	7.080745
31	0.001695644	0.0152608	0.366259	2.014426	2.014426	0	7.965839
32	0.001695644	0.01695644	0.406955	2.238251	2.238251	0	8.850932
33	0.002118277	0.01907472	0.457793	2.517863	2.288251	0.229613	9.048652
34	0.000387542	0.01946226	0.467094	2.569019	2.338251	0.230768	9.246371
35	0.000393283	0.01985555	0.476533	2.620932	2.388251	0.232681	9.444091
36	0.000398948	0.02025449	0.486108	2.673593	2.438251	0.235343	9.641811



BROOKS COREY. XLS

Summarize:

① Darcy's law from pond to wetting front
② Volume infiltrated from "porosity" concept

Result: $$I(t) = K_{ns}t + (\theta_{ns}-\theta_a)(H+\psi)\ln\left[1+\frac{I}{(\theta_{ns}-\theta_a)(H+\psi)}\right]$$

$$i(t) = K_{ns}\left[1+\frac{H+\psi}{\frac{I}{(\theta_{ns}-\theta_a)}}\right] = K_{ns}\left[1+\frac{(\theta_{ns}-\theta_a)(H+\psi)}{I}\right]$$

at $t\to\infty$ $I\to\infty$
$i(t)\to K_{ns}$ which agrees with intuition

Horton's Equation: $$i(t) = i_\infty + i_0 e^{-\alpha t}$$

i_0 = initial rate, i_∞ = equilibrium rate
α = decay rate

$i_\infty = K_{ns}$; $$\int_0^t i_\infty + i_0 e^{-\alpha\tau}\,d\tau = i_\infty\tau - \frac{i_0 e^{-\alpha\tau}}{\alpha}\Big|_0^t$$

$$= i_\infty t - \frac{i_0}{\alpha}e^{-\alpha t} + \frac{i_0 e^{-\alpha 0}}{\alpha} = i_\infty t + \frac{i_0}{\alpha}\left[1-e^{-\alpha t}\right]$$

at $t=0$, $i(t)\neq\infty$ so does not satisfy initial "physics" used in Green-Ampt model, but Horton's equation is a useful model of infiltration

Philip's model

$$I(t) = K_a t + \sum_{m=1}^{M} S_m t^{m/2} \quad \text{(Series of non-linear reservoirs)}$$

$$i(t) = \frac{S}{\sqrt{4t}} + A$$

Kostiakov's model

$$i(t) = \alpha_1 t^{-\alpha_2}$$

$$I(t) = \frac{\alpha_1}{1-\alpha_2} t^{1-\alpha_2}$$

Only Green-Ampt model fits most "physics" of process.
Horton's model also fits observations

All models agree well with sand and clay data (see pg 210-211)

Use of models

Infiltration models are crucial in rainfall-runoff modeling and in contaminant transport models.

Rainfall-Runoff

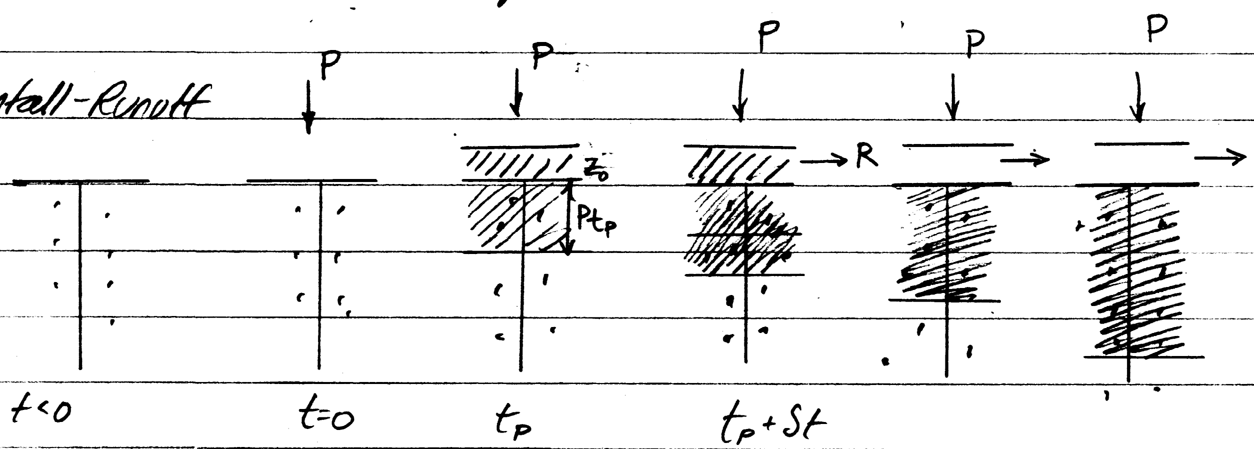


Figure 5–5 Solution of runoff equation