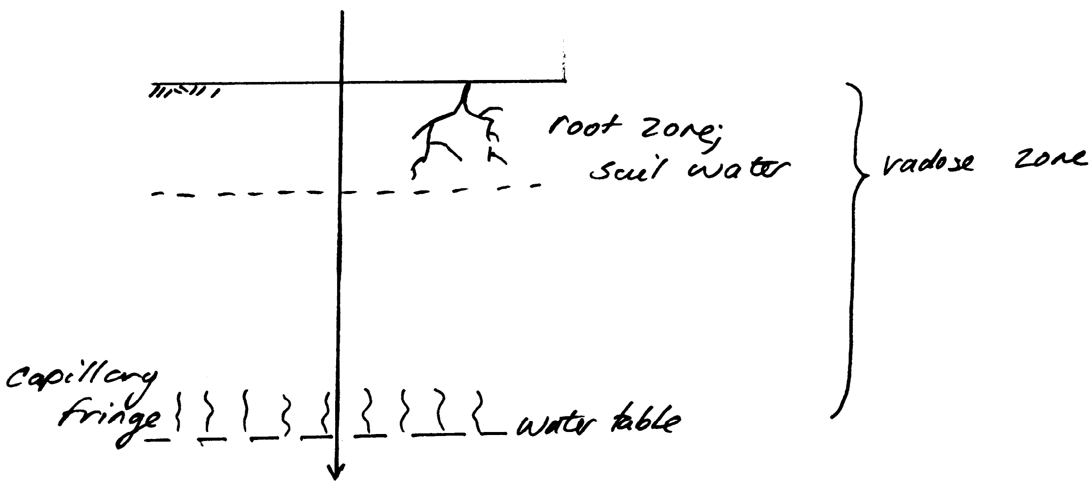
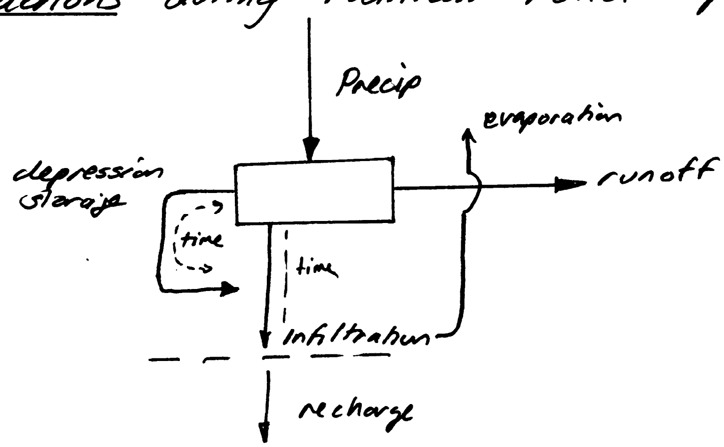
Unsaturated Zone & Recharge

Portion of groundwater system where liquids and gasses share pore space. Discontinuous liquid phase.



Abstractions during rainfall-runoff process



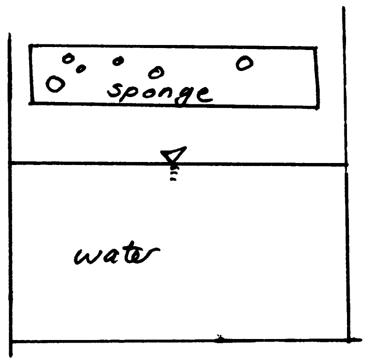
Water Balance during recharge; contaminant migration

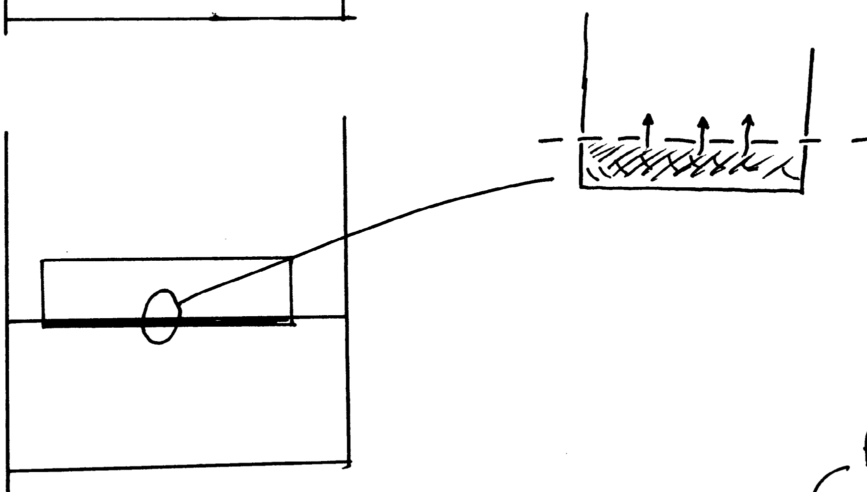
infiltration

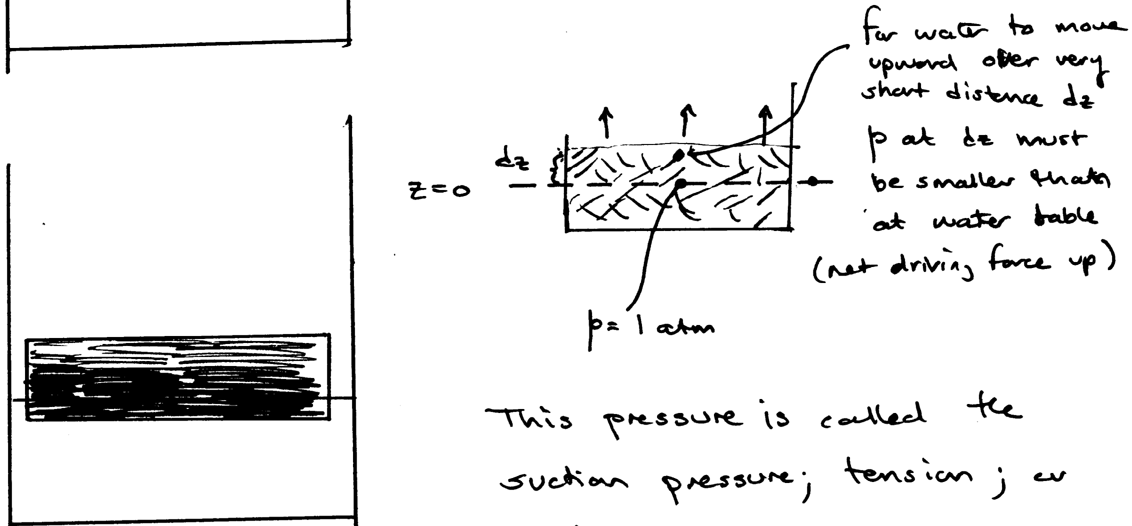
percolation

recharge

Tension (suction); capillary pressure

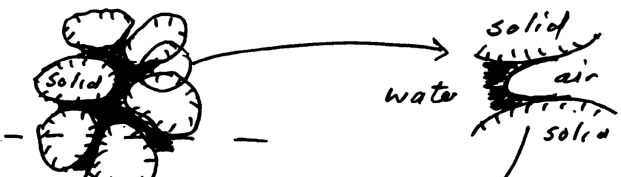






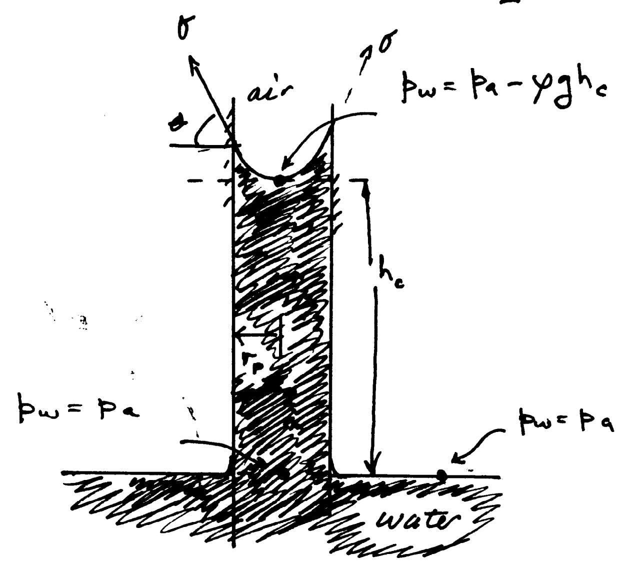
This pressure is called the suction pressure; tension; or capillary pressure.

Capillary pressure



Typically soils are water wet.

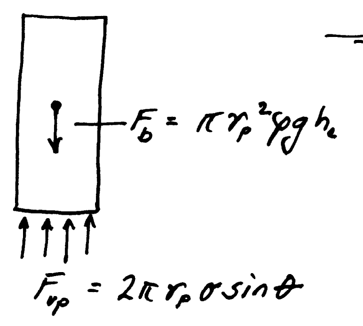
Surface tension forces in small diameter pores creates a pressure differential at air water interfaces — similar to capillary tube behavior



Laplace capillary equation (text)

$$p_a - p_w = p_c = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{2\sigma}{r_p}$$

In a capillary tube using a force balance one arrives a similar relation



⇒ at equilibrium

$$2\pi r_p \sigma \sin\theta = \pi r_p^2 \rho g h_c$$

$$h_c = \frac{2\sigma \sin\theta}{\rho g r_p}$$

θ is usually large so $\sin\theta \approx 1$

$$\therefore h_c \approx \frac{2\sigma}{\rho g r_p}$$

or $p_c = \rho g h_c = \dfrac{2\sigma}{r_p}$

r_p - radius of pore throat

Now $p_c = p_a - p_w$

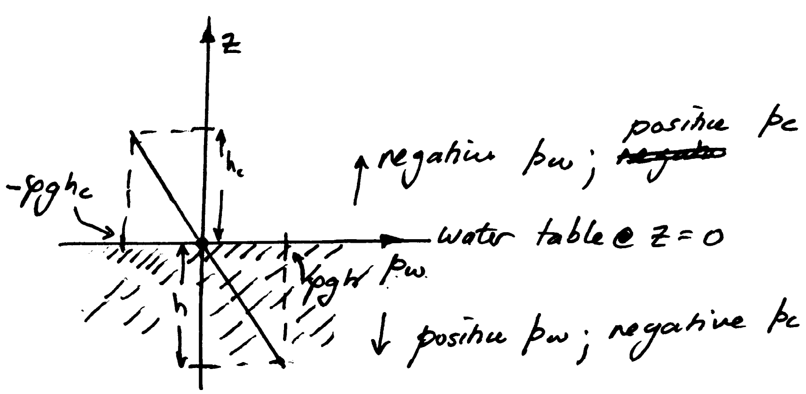
$\therefore$ with $p_a \approx 1$ atm

$$p_w = -p_c = -\frac{2\sigma}{r}$$

As $r \to$ small, $p_w \to$ "− big"

$\therefore$ Large negative pressures are associated with small pore dimensions

Typically soil scientists talk about tension or suction "p_c". Suction head ψ is defined as $\psi = \frac{p_c}{\rho g}$



Summary: $p_w = \frac{-2\sigma}{r}$ (static equilibrium)

$\psi = \frac{p_c}{\rho g}$ (suction head)

$p_w > 0$ below water table

$p_w < 0$ above water table

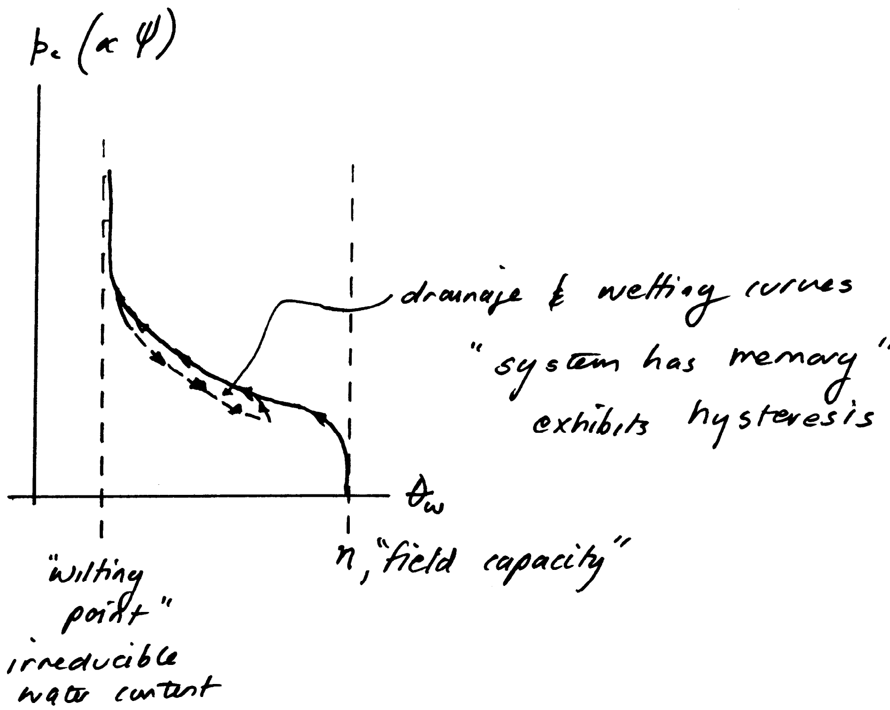
$\theta_w + \theta_a = n$ (air volume + water volume = porosity)

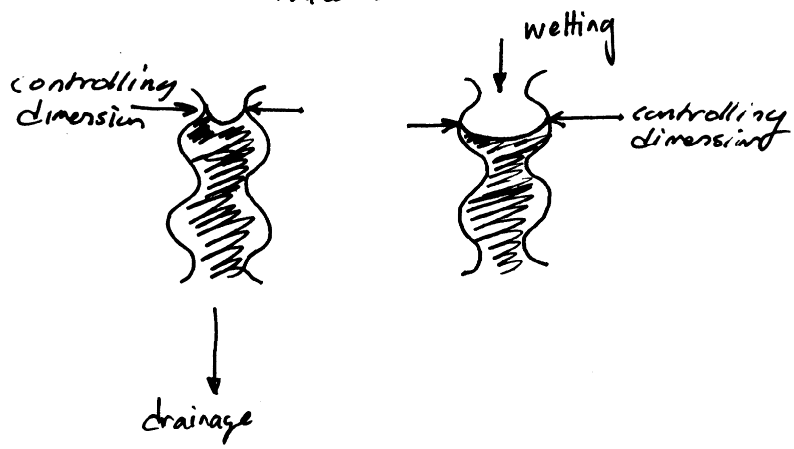
$S_w + S_a = 1$

Soil characteristic curve

Relationship between p_c and water content.

Related to pore size distribution in soil





Soil characteristic curve relates ψ to θ_w. Slope of curve at any value θ_w is $\frac{d\psi}{d\theta_w}$

Darcy's Law in Unsaturated Flow.

① Flow only in continuous phase of water.

② K_w is a function of θ_w

③ $h_w = z - \psi$

$\theta_{w①} < \theta_{w②}$

$K_{w①} < K_{w②}$

$$q_w = -K_w(\theta_w)\frac{dh_w}{dl}$$

K is a function of θ_w

Typically interested in vertical flow

$$q_z = -K_w(\theta_w)\frac{d}{dz}(z - \psi) = -K_w(\theta_w)\left[1 - \frac{d\psi}{dz}\right]$$

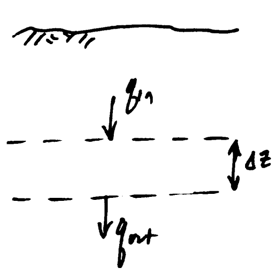
Relative permeability – ratio of $K_w(\theta_w)$ to fully saturated K

$$k_{rw}(\theta_w) = \frac{K_w(\theta_w)}{K}$$

$$\therefore \quad q_z = -Kk_{rw}(\theta_w) + Kk_{rw}(\theta_w)\frac{d\psi}{dz}$$

Continuity

mechanism of storage is increase/decrease θ_w.



$$\frac{d\theta_w}{dt} = \lim_{\Delta z \to 0} \frac{q(z-\Delta z) - q(z)}{\Delta z} = -\frac{dq}{dz}$$

$$\frac{d\theta_w}{dt} = -\frac{d}{dz}\left[-K k_{rw}(\theta_w) + K k_{rw}(\theta_w)\frac{d\psi}{dz}\right]$$

From the soil-characteristic curve we have

$$\frac{d\psi}{d\theta_w} \propto f(\theta_w) \propto F(\psi)$$

$$\therefore \quad F(\psi)\cdot\frac{d\theta_w}{dz} = \frac{d\psi}{d\theta_w}\cdot\frac{d\theta_w}{dz} = \frac{d\psi}{dz}$$

So

$$\frac{d\theta_w}{dt} = -\frac{d}{dz}\left[-K k_{rw}(\theta_w) + \underbrace{K k_{rw}(\theta_w) F(\psi)}\frac{d\theta_w}{dz}\right]$$

(negative) soil water diffusivity, $-D(\theta_w)$

$$\underbrace{\frac{d\theta_w}{dt} = \frac{d\,K k_{rw}(\theta_w)}{dz} + \frac{d}{dz}\left[D(\theta_w)\frac{d\theta_w}{dz}\right]}$$

"Non-linear diffusion equation"

Called "Richards'" equation

There are various forms of the equation. All are non-linear. Solutions exist for relatively simplistic representations of $D(\theta_w)$ or $\frac{d\psi}{d\theta_w}$. Otherwise numerical methods are required. Despite being non-linear; it is a diffusion equation and relatively well behaved.