

- 1) Construct a multiple-cell balance model to compute heads in the aquifer depicted in Figure 1. The transmissivity is spatially variable, so an averaging scheme must be incorporated into your balance equations. Determine the steady-flow head distribution for Figure 1 and Figure 2.
- 2) Sketch flowlines for the flow system of Figure 2. Estimate what fraction of recharge water is captured by the discharge zone, and what fraction of induced river flow is captured by the discharge zone.

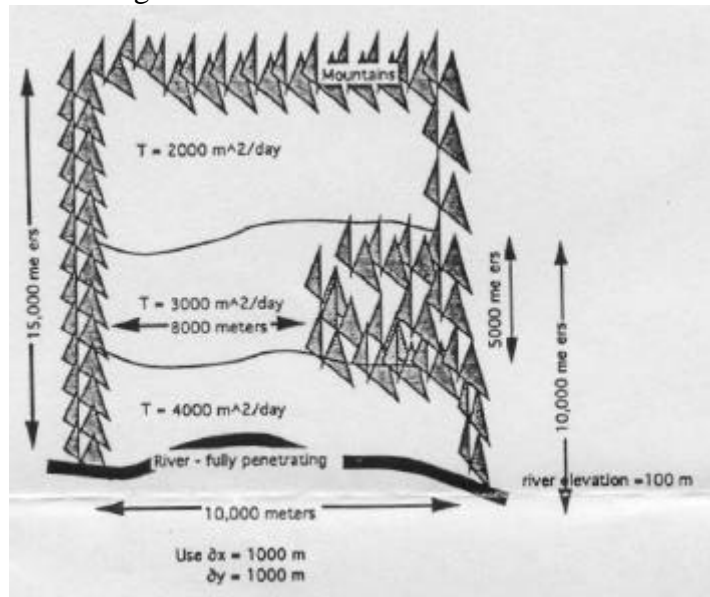


Figure 1. Pre-development case

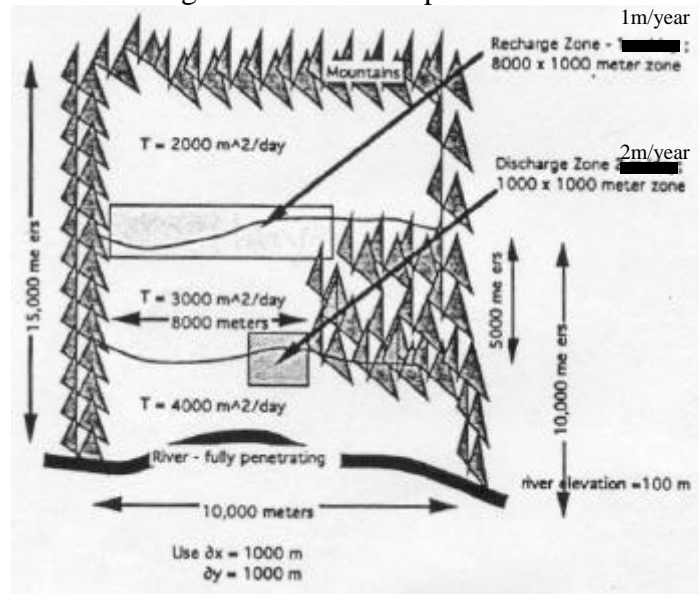
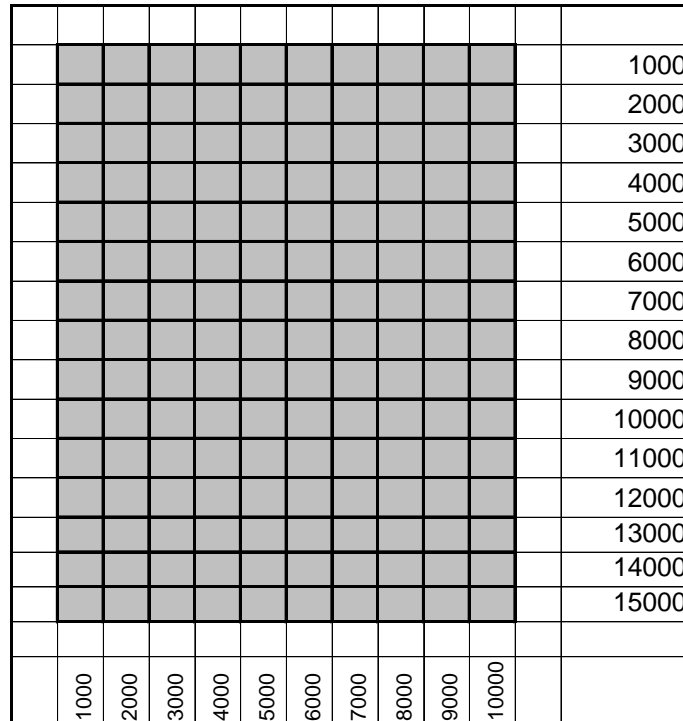


Figure 2. Post-development case

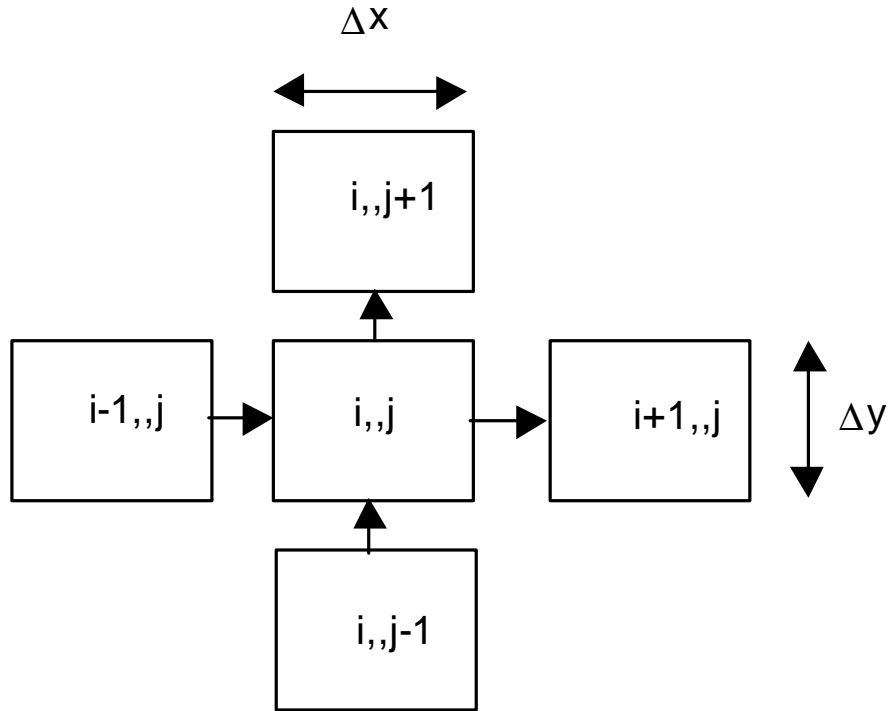
Solution

Sketch the geometry of the model to help write governing equations and establish a computational grid.



The figure above depicts the computational domain. Observe that there two columns and two rows “extra”. These rows and columns will be used to map boundary conditions into the solution domain. The solution domain is indicated by the shaded area.

Now write balance equations for a generic cell somewhere in the interior.
Schematic of Multiple Cells (Two-Dimensional Flow)



Balance Expressions for i,j-th Cell

Mass Flow Into Cell

$$\frac{1}{2}(T_{x;i-1,j} + T_{x;i,j}) \Delta y [h_{i-1,j} - h_{i,j}]/\Delta x + \frac{1}{2}(T_{y;i,j-1} + T_{y;i,j}) \Delta x [h_{i,j-1} - h_{i,j}]/\Delta y + R\Delta x\Delta y$$

Mass Flow Out of Cell

$$\frac{1}{2}(T_{x;i,j} + T_{x;i+1,j}) \Delta y [h_{i,j} - h_{i+1,j}]/\Delta x + \frac{1}{2}(T_{y;i,j} + T_{y;i,j+1}) \Delta x [h_{i,j} - h_{i,j+1}]/\Delta y - D\Delta x\Delta y$$

Complete Balance Equation for i,j-th Cell

$$0 = \frac{1}{2}(T_{x;i-1,j} + T_{x;i,j}) \Delta y [h_{i-1,j} - h_{i,j}]/\Delta x + \frac{1}{2}(T_{y;i,j-1} + T_{y;i,j}) \Delta x [h_{i,j-1} - h_{i,j}]/\Delta y + R\Delta x\Delta y - \frac{1}{2}(T_{x;i,j} + T_{x;i+1,j}) \Delta y [h_{i,j} - h_{i+1,j}]/\Delta x - \frac{1}{2}(T_{y;i,j} + T_{y;i,j+1}) \Delta x [h_{i,j} - h_{i,j+1}]/\Delta y - D\Delta x\Delta y$$

Let

$$A_{ij} = \frac{1}{2}(T_{x;i-1,j} + T_{x;i,j}) / \Delta x^2$$

$$B_{ij} = \frac{1}{2}(T_{y;i,j-1} + T_{y;i,j}) / \Delta y^2$$

$$C_{ij} = \frac{1}{2}(T_{x;i,j} + T_{x;i+1,j}) / \Delta x^2$$

$$D_{ij} = \frac{1}{2}(T_{y;i,j} + T_{y;i,j+1}) / \Delta y^2$$

Divide the balance equation by the cell area $\Delta x\Delta y$, and make the indicated substitutions to obtain,

$$0 = A_{ij} [h_{i-1,j} - h_{i,j}] + B_{ij} [h_{i,j-1} - h_{i,j}] - C_{ij} [h_{i,j} - h_{i+1,j}] - D_{ij} [h_{i,j} - h_{i,j+1}] + R - D$$

Rearrange the equation to isolate the i,j cell.

$$(A_{i,j} + B_{i,j} + C_{i,j} + D_{i,j})h_{i,j} = A_{i,j} [h_{i-1,j}] + B_{i,j} [h_{i,j-1}] + C_{i,j} [h_{i+1,j}] + D_{i,j} [h_{i,j+1}] + R - D$$

Finally, divide by the coefficient $A+B+C+D$ to obtain a formula for estimating h in cell i,j .

$$h_{i,j} = \{A_{i,j} [h_{i-1,j}] + B_{i,j} [h_{i,j-1}] + C_{i,j} [h_{i+1,j}] + D_{i,j} [h_{i,j+1}] + R - D\} / (A_{i,j} + B_{i,j} + C_{i,j} + D_{i,j})$$

An algorithm (recipe) to compute h from unknown values is to guess values for the entire aquifer (iteration k). One then uses the formula to update the guess (iteration $k+1$). The updates are then put into the formula and the process repeated until the guesses stop changing. (This algorithm is called the Jacobi iteration method).

Now program the routine into a spreadsheet.

Case 1: Pre-development

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Aquifer Model												
2													
3		1000	Delta x										
4		1000	Delta y										
5													
6	Tx Array		1	2	3	4	5	6	7	8	9	10	
7			2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	
8	1	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
9	2	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
10	3	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
11	4	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
12	5	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
13	6	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
14	7	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
15	8	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
16	9	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
17	10	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
18	11	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
19	12	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
20	13	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
21	14	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
22	15	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
23			4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	
24	Ty Array		1	2	3	4	5	6	7	8	9	10	
25			2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	
26	1	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
27	2	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
28	3	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
29	4	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
30	5	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000	2000
31	6	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
32	7	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
33	8	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
34	9	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
35	10	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000	3000
36	11	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
37	12	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
38	13	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
39	14	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
40	15	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	4000
41			4000	4000	4000	4000	4000	4000	4000	4000	4000	4000	

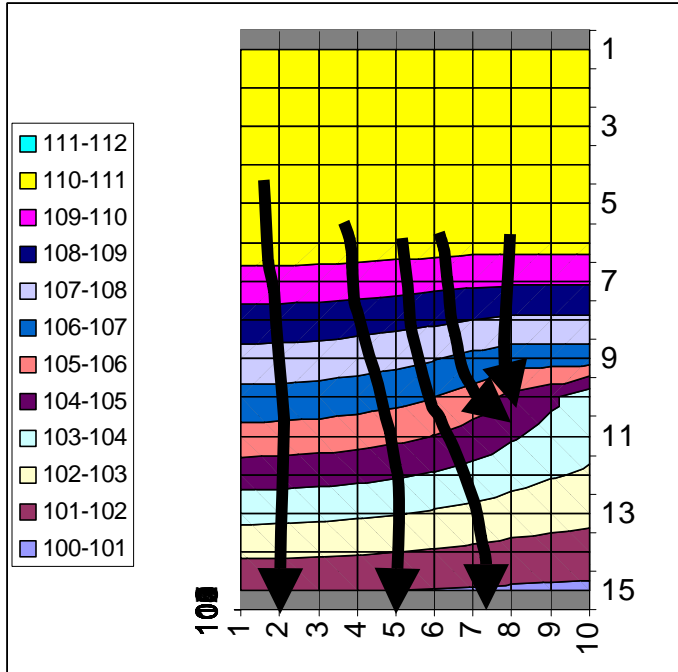
This image is the input data for this problem. The next two windows are the A,B,C,D arrays that are the result of calculations based on this first window.

	A	B	C	D	E	F	G	H	I	J	K	L	M	
150	Rese	1												
151	Iter	46724	=IF(B150=0,0,B151+1)											
152														
153	head (k)		1	2	3	4	5	6	7	8	9	10		
154		0	100	100	100	100	100	100	100	100	100	100	0	
155	1	100	100	100	100	100	100	100	100	100	100	100	100	
156	2	100	100	100	100	100	100	100	100	100	100	100	100	
157	3	100	100	100	100	100	100	100	100	100	100	100	100	
158	4	100	100	100	=IF(\$B\$150=0,0,D174)						100	100	100	
159	5	100	100	100								100	100	
160	6	100	100	100								100	100	
161	7	100	100	100	100	100	100	100	100	100	100	100	100	
162	8	100	100	100	100	100	100	100	100	100	100	100	100	
163	9	100	100	100	100	100	100	100	100	100	100	100	100	
164	10	100	100	100	100	100	100	100	100	100	100	100	100	
165	11	100	100	100	100	100	100	100	100	100	100	100	100	
166	12	100	100	100	100	100	100	100	100	100	100	100	100	
167	13	100	100	100	100	100	100	100	100	100	100	100	100	
168	14	100	100	100	100	100	100	100	100	100	100	100	100	
169	15	100	100	100	100	100	100	100	100	100	100	100	100	
170		0	100	100	100	100	100	100	100	100	100	100	0	
171	head (k+1)		1	2	3	4	5	6	7	8	9	10		
172			100	100	100	100	100	100	100	100	100	100		
173	1	100	100	100	100	100	100	100	100	100	100	100	100	
174	2	100	100	100	100	100	100	100	100	100	100	100	100	
175	3	100	100	100	100	100	100	100	100	100	100	100	100	
176	4	100	=(D45*C156+D63*D157+D81*E156+D99*D155+D117-										00	
177	5	100	D135)/(D99+D81+D63+D45)										00	
178	6	100	100	100	100	100	100	100	100	100	100	100	100	
179	7	100	100	100	100	100	100	100	100	100	100	100	100	
180	8	100	100	100	100	100	100	100	100	100	100	100	100	
181	9	100	100	100	100	100	100	100	100	100	100	100	100	
182	10	100	100	100	100	100	100	100	100	100	100	100	100	
183	11	100	100	100	100	100	100	100	100	100	100	100	100	
184	12	100	100	100	100	100	100	100	100	100	100	100	100	
185	13	100	100	100	100	100	100	100	100	100	100	100	100	
186	14	100	100	100	100	100	100	100	100	100	100	100	100	
187	15	100	100	100	100	100	100	100	100	100	100	100	100	
188			100	100	100	100	100	100	100	100	100	100		

Case 2: Recharge and Discharge

	A	B	C	D	E	F	G	H	I	J	K	L	M
150	Rese	1											
151	Iter	46733											
152													
153	head (k)		1	2	3	4	5	6	7	8	9	10	
154		0	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	0
155	1	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4
156	2	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4
157	3	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4	110.4
158	4	110.6	110.6	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4	110.4
159	5	110.6	110.6	110.6	110.5	110.5	110.5	110.4	110.4	110.4	110.4	110.4	110.4
160	6	110.6	110.6	110.6	110.6	110.5	110.5	110.4	110.4	110.3	110.4	110.4	110.4
161	7	109.6	109.6	109.6	109.5	109.5	109.4	109.3	109.2	109.1	109.1	109.1	109.1
162	8	108.6	108.6	108.6	108.5	108.4	108.3	108.2	108	107.9	107.9	107.9	107.9
163	9	107.6	107.6	107.6	107.5	107.4	107.3	107	106.8	106.4	106.4	106.4	106.4
164	10	106.7	106.7	106.7	106.6	106.5	106.3	106	105.6	104.7	104.2	103.3	103.3
165	11	105.6	105.6	105.6	105.5	105.4	105.2	105	104.6	104.1	103.7	103.3	103.3
166	12	104.5	104.5	104.4	104.4	104.3	104.1	103.9	103.6	103.3	103.1	102.9	102.9
167	13	103.3	103.3	103.3	103.2	103.2	103	102.9	102.7	102.5	102.4	102.3	102.3
168	14	102.2	102.2	102.2	102.1	102.1	102	101.9	101.8	101.7	101.6	101.6	101.6
169	15	101.1	101.1	101.1	101.1	101	101	101	100.9	100.9	100.8	100.8	100.8
170		0	100	100	100	100	100	100	100	100	100	100	0
171	head (k+1)		1	2	3	4	5	6	7	8	9	10	
172			110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	
173	1	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4
174	2	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4
175	3	110.5	110.5	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4	110.4
176	4	110.6	110.6	110.5	110.5	110.5	110.5	110.5	110.4	110.4	110.4	110.4	110.4
177	5	110.6	110.6	110.6	110.5	110.5	110.5	110.4	110.4	110.4	110.4	110.4	110.4
178	6	110.6	110.6	110.6	110.6	110.5	110.5	110.4	110.4	110.3	110.4	110.4	110.4
179	7	109.6	109.6	109.6	109.5	109.5	109.4	109.3	109.2	109.1	109.1	109.1	109.1
180	8	108.6	108.6	108.6	108.5	108.4	108.3	108.2	108	107.9	107.9	107.9	107.9
181	9	107.6	107.6	107.6	107.5	107.4	107.3	107	106.8	106.4	106.4	106.4	106.4
182	10	106.7	106.7	106.7	106.6	106.5	106.3	106	105.6	104.7	104.2	103.3	103.3
183	11	105.6	105.6	105.6	105.5	105.4	105.2	105	104.6	104.1	103.7	103.3	103.3
184	12	104.5	104.5	104.4	104.4	104.3	104.1	103.9	103.6	103.3	103.1	102.9	102.9
185	13	103.3	103.3	103.3	103.2	103.2	103	102.9	102.7	102.5	102.4	102.3	102.3
186	14	102.2	102.2	102.2	102.1	102.1	102	101.9	101.8	101.7	101.6	101.6	101.6
187	15	101.1	101.1	101.1	101.1	101	101	101	100.9	100.9	100.8	100.8	100.8
188			100	100	100	100	100	100	100	100	100	100	

Contour Plot



All the water in the discharge area comes from the recharge source. There is no water from the river flowing to the well. We conclude this remark because all the flowlines near the river flow toward the river.