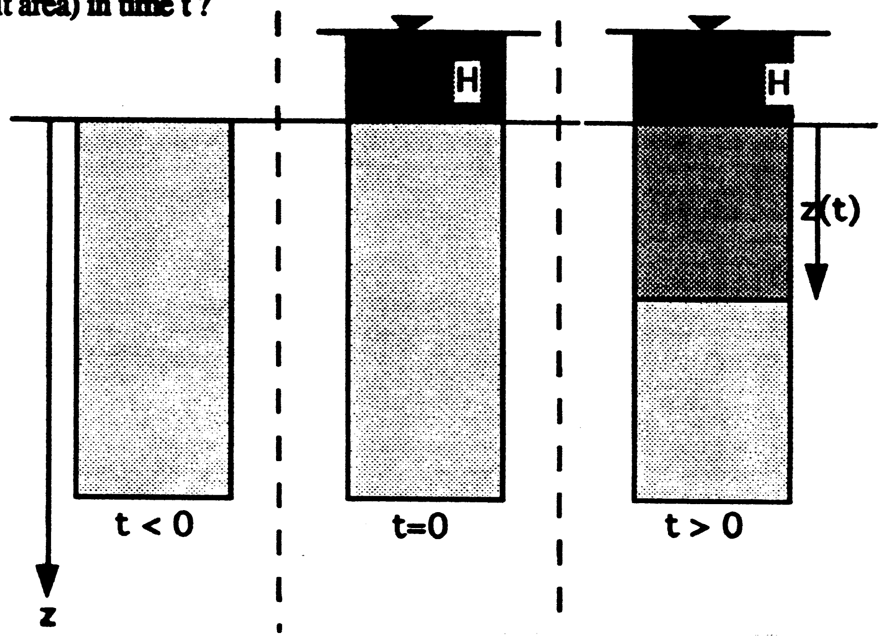
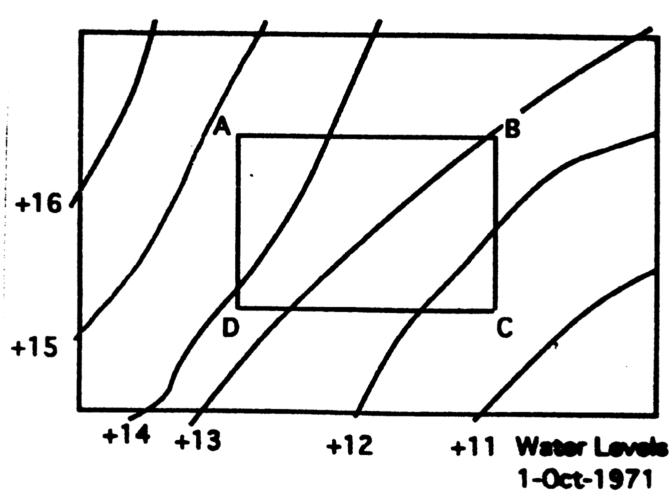
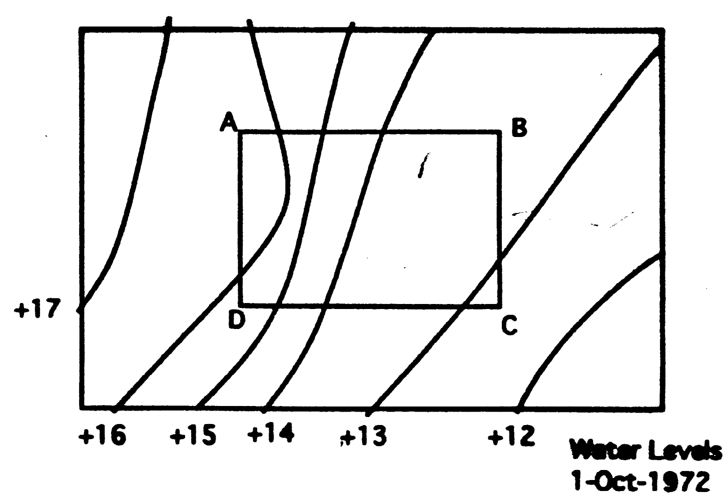
CE6361 Groundwater Hydrology , HW#3, Fall 1997 Due: ______________

1) Suppose at time t=0, the head in figure 1 below is raised instantly to a value H above the ground surface. The hydraulic conductivity is K and the effective porosity is n. Assume the ground is initially dry and capillary effects are negligible. Assume the porous medium is completely saturated above the wetting front z(t) and completely dry below. What is the position of the wetting fromt at any time t ? How much water infiltrates (per unit area) in time t ?



2)The attached figures are two contour maps (scale 1:25000) of a portion of an unconfined aquifer with storativity 10% and transmissivity (assumed constant even though the water elevations vary) of 1000 sq.meters/day. Determine the net change in water storage during the indicated time interval.





3) Prepare a groundwater balance for the subdomain ABCD and use it to estimate the net volume of water pumped from or recharged into the area over the indicated time interval.

that significant differences in the velocity are possible under influence of waviness. In Fig. 366 are given the lines of equal head and equal (reduced) flow rate.

§3. <u>One-dimensional Vertical Flow for Constant Operating Head</u>. We examine the flow of water along a vertical line in the soil, considering the seepage coefficient k and the porosity m to be constant quantities.

We take the general equation of motion with the inertia terms (the y-axis is directed downward)

$$\frac{\partial v_y}{\partial t} + \frac{\partial v_y}{\partial y} v_y = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g - \frac{mg}{k} v_y . \tag{3.1}$$

We drop the rest of the equation, since we consider one-dimensional flow, parallel to the y-axis. Therefore, the equation of continuity takes on the form:

$$\frac{\partial v_y}{\partial y} = 0 .$$

This shows that v_y only depends upon time (which was noticed by N. N. Pavlovsky). Therefore (3.1) may be rewritten, introducing instead of v_y the seepage velocity v through

$$v_y = \frac{1}{m} v ,$$

in the following form

$$\frac{1}{m}\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g - \frac{g}{k} v . \tag{3.2}$$

But we have shown in Chapter I (and also in Chapter II), that the term $\frac{1}{m}\frac{\partial v}{\partial t}$ may be neglected in all practically interesting cases. If we introduce the quantity

$$h = \frac{p}{\rho g} - y , \tag{3.3}$$

we come back to Darcy's law

$$v = -k\frac{dh}{dy} , \tag{3.4}$$

valid, consequently, even in the case of unsteady flows. We assume now that water percolates in the soil under a constant head H (Fig. 368), and that at the moment t it has seeped to the depth y_0 from the boundary of the reservoir. The y-axis will be oriented vertically downward. Let $y_0 = 0$ at the origin of time $t = 0$.

Since $v = v(t)$ depends only upon time and not upon y, h is a linear function of y:

$$h = a(t)y + b(t) .$$

For $y = 0$, the head is equal to H.

$$h(0) = b(t) = H . \tag{3.5}$$

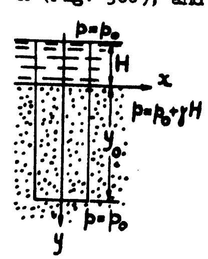


Fig. 368

For $y = y_0$, considering the atmospheric pressure to be zero, we have after (3.3):

$$h(y_0) = -h_k - y_0 = ay_0 + b , \tag{3.6}$$

where by h_k the height of capillary rise is designated.

$$h_k = -\frac{p_k}{\rho g} .$$

Therefore for "a" we may write this expression [after (3.5) and (3.6)]:

$$a = \frac{dh}{dy} = -\frac{h_k + y_0 + H}{y_0} . \tag{3.7}$$

Seepage velocity v and derivative $\frac{dy_0}{dt}$ are related as

$$v = m\frac{dy_0}{dt} . \tag{3.8}$$

Comparison of (3.7) and (3.8) leads to the equation for y_0:

$$m\frac{dy_0}{dt} = k\frac{H + h_k + y_0}{y_0} . \tag{3.9}$$

We notice that according to the obtained equation, the capillary height is added to the acting head, as if instead of the head H we had the head $H + h_k$.

To integrate equation (3.9) it suffices to write it in the form

$$\frac{y_0 dy_0}{y_0 + H + h_k} = \frac{k}{m} dt$$

or

$$dy_0 - \frac{H + h_k}{y_0 + H + h_k} dy_0 = \frac{k}{m} dt ,$$

after which we find, considering that $y_0 = 0$ for $t = 0$:

$$\frac{y_0}{H + h_k} - \ln\left(1 + \frac{y_0}{H + h_k}\right) = \frac{kt}{m(H + h_k)} . \tag{3.10}$$

Introducing the dimensionless quantities:

$$\left\{\begin{aligned} \frac{y_0}{H + h_k} &= \eta , \\ \frac{kt}{m(H + h_k)} &= \tau , \end{aligned}\right. \tag{3.11}$$

we rewrite (3.10) in the following form:

$$\tau = \eta - \ln(1 + \eta) . \tag{3.12}$$

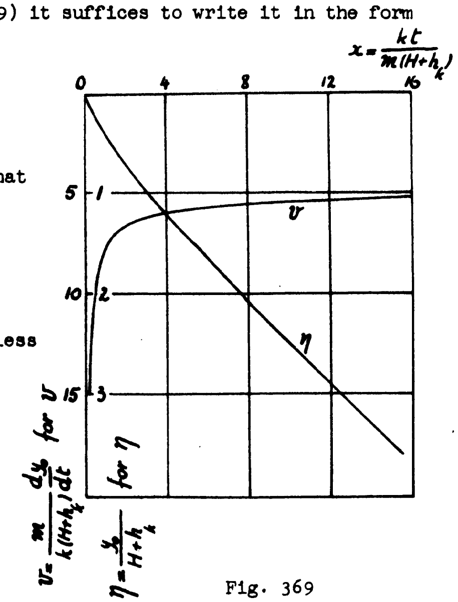


Fig. 369

The graph of the dependence of η on τ is given in Fig. 369. Also, the dependence of v upon τ is given there.

For small values of η it is possible to carry out calculation of

#1 a) $h = 0$ @ $t < 0$ Position of wetting front

@ $t=0$ $h = H$ @ $t > 0$ $h = H + z$

Given: K & n (note: H remains constant)

from Darcy's $q = k \dfrac{H+z}{z}$

also $q = n \dfrac{dz}{dt}$

$\therefore$ $k\left(\dfrac{H+z}{z}\right) = n \dfrac{dz}{dt}$

rewriting & integrate

$$\int \frac{z\,dz}{H+z} = \int \frac{k}{n}\,dt$$

$$z - H\ln|H+z| = \frac{k}{n}t + C$$

at $t=0$ $z=0$ $\therefore$ $C = -H\ln|H|$

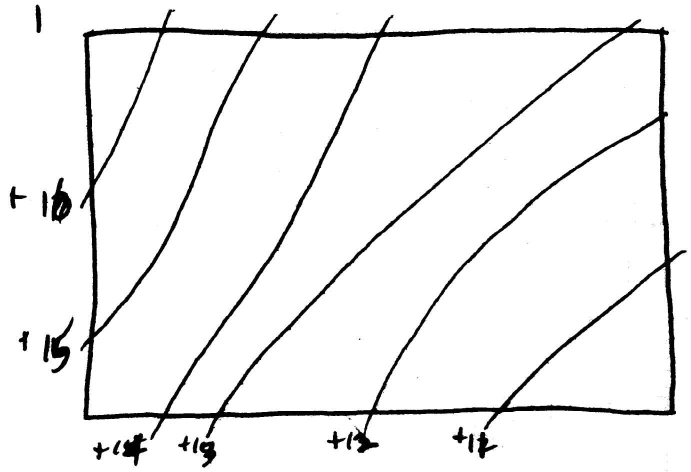
$$\boxed{z - H\ln|H+z| + H\ln|H| = \frac{k}{n}t}$$

b) Vol. of water infiltrating

$\boxed{\dfrac{V}{a} = nz}$ from $V = nza$

#2

t_1 = 1-oct-1971



Head (m)	# of Squares	% of Area	Weighted Head (m)
16.5	4	2.67	0.44
15.5	16	10.67	1.65
14.5	30	20.0	2.90
13.5	30	20.0	2.70
12.5	32	21.33	2.67
11.5	28	18.67	2.15
10.5	10	6.66	0.70
	150	100%	$\Sigma = 13.2\,m = H_1$

t_2 = 1-Oct-'72

Head (m)	# of Squares	% of Area	Weighted Head (m)
17.5	11	7.33	1.28
16.5	30	20.0	3.30
15.5	13	8.67	1.34
14.5	16	10.67	1.55
13.5	46	30.67	4.14
12.5	25	16.67	2.08
11.5	9	6.00	0.69
	150		$\Sigma = 14.38\,m = H_2$

<cont'd>

#2/cont'd

$$\Delta V = S A \Delta h$$

Given: $S = 0.10$

Area $= 1250\ m \times 1900 m$

$= 2.375 \times 10^6\ m^2$

$\Delta h = 14.4 - 13.2 = 1.2 m$

$$\Delta V = (0.10)(2.375 \times 10^6\ m^2)(1.2 m)$$

$$\boxed{\Delta V = 285{,}000\ m^3}$$

Note: You have to compute a change in head & then calculate ΔV using the above equation.

You cannot compute $V_{t=1}$ & $V_{t=2}$ & then $\Delta V = V_2 - V_1$. You know the head values ~~[illegible]~~ (top elev. of aquifer), but you do not know the bottom elev. Therefore, you don't really know the actual volume of water being stored. Although you will get the same absolute value for ΔV by calculating "volumes", the method is wrong. (Just get lucky numbers are the same).

#3

Given: Head contours @ t_1 & t_2 for Area ABCD
$T = 1000\ m^2/day$
$S = 0.10$

from maps: $A = 575\,m \times 875\,m$
$\approx 500{,}000\ m^3$

Find: Groundwater Balance for subdomain & estimate net. volume of water pumped or recharged into area over time indicated.

Groundwater Balance:

Avg Flow In − Avg Flow Out + Net Recharge = Δ Storage

where: Net Recharge = Recharge − Pumping

& + Net Recharge = Recharge

− Net Recharge = Pumping

∴ GW Balance:

$$-\frac{(Q_{in_1} + Q_{in_2})}{2} + \frac{Q_{out_1} + Q_{out_2}}{2} + \Delta\,\text{Storage} = \text{Net Recharge}$$

@ t_1

14.5	14.2	14	13.5	13.2	13	12.6
14.4	14	13.8	13.5	13	12.6	12.3
14.2	14	13.5	13	12.6	12.3	12
14	13.5	13	12.6	12.3	12	11.8
13.5	13	12.6	12.3	12	11.8	11.5

@ t_2

16.2	15.5	14.8	14.2	13.8	13.6	13.4
16.2	15.5	14.5	14	13.8	13.6	13.4
16.2	15.5	14.5	13.8	13.6	13.4	13.2
16	15	14	13.8	13.6	13.3	13
15.5	14.6	14	13.7	13.4	13.2	13

ΔH:

1.7	1.3	0.8	0.7	0.6	0.6	0.8
1.8	1.5	0.7	0.5	0	1.0	1.1
2.0	1.5	1.0	0.8	1.0	1.1	1.2
2	1.5	1.0	1.2	1.3	1.3	1.2
2	1.5	1.4	1.4	1.4	1.4	1.5

Divide area with grid
Determine ΔH for each cell
Average over area to find
ΔH for subdomain

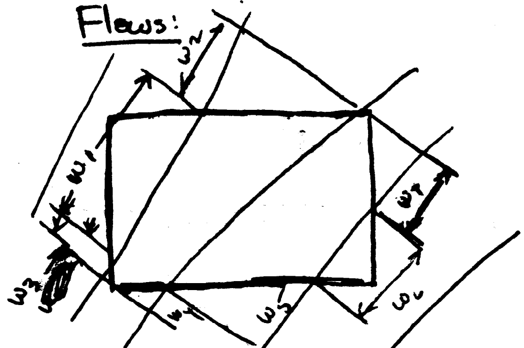
Σ Heads = 36.5 m
over 31.5 cells (bottom row counted as 1/2 each)

Ave ΔH = 1.16 m

$$\Delta V = S A \Delta H = 0.1 (500{,}000 m^2)(1.16 m)$$

$$\underline{\Delta V = 58{,}000 m^3}$$

Flows:



$$Q_i = T w \frac{\Delta h}{\Delta L}$$

$$Q_{IN} = T \left(w_1 \frac{\Delta h}{\Delta L} + w_2 \frac{\Delta h}{\Delta L} + w_3 \frac{\Delta h}{\Delta L} \right)$$

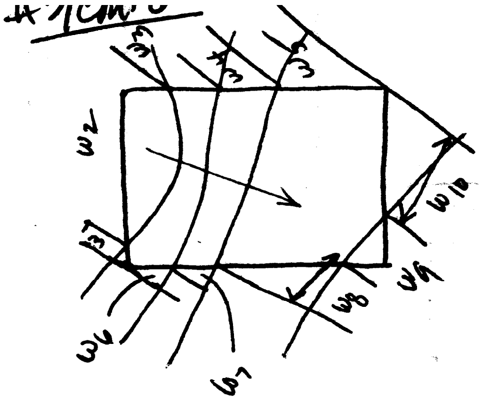
$$= T \left(575 m \frac{1 m}{350 m} + 300 m \frac{1 m}{350 m} + 75 \frac{1 m}{300 m} \right)$$

$$= 1000 m^2/day \, (1.64 + 0.86 + 0.25) m$$

$$\underline{Q_{IN_1} = 2750 m^3/day}$$

$$Q_{OUT} = T \left(w_4 \frac{\Delta h}{\Delta L} + w_5 \frac{\Delta h}{\Delta L} + w_6 \frac{\Delta h}{\Delta L} + w_7 \frac{\Delta h}{\Delta L} \right)$$

$$= 1000 \, (0.35 + 0.71 + 1.04 + 0.74)$$



$$Q_{IN_2} = T\left(\frac{w_1 \Delta h}{\Delta L} + \frac{w_2 \Delta h}{\Delta L} + \frac{w_3 \Delta h}{\Delta L} + \frac{w_4 \Delta h}{\Delta L} + \frac{w_5 \Delta h}{\Delta L}\right)$$

$$= 1000\ m^2/day\,(0.29 + 1.58 + 0.33 + 0.50 + 0.89)$$

w/o cm → m conversion

$$= 1000\left(\frac{0.35(1)}{1.2} + \frac{1.9(1)}{1.2} + \frac{0.4(1)}{1.2} + \frac{0.4(1)}{0.8} + \frac{0.8(1)}{0.9}\right)$$

$$\underline{Q_{IN_2} = 3597\ m^3/day}$$

$$Q_{OUT_2} = 1000\left[\frac{0.2(1)}{0.65} + \frac{0.3(1)}{1.0} + \frac{0.8(1)}{1.5} + \frac{0.8(1)}{1.7} + \frac{1.4(1)}{2.0}\right)$$

$$= 1000\left(\frac{w_6 \Delta h}{\Delta L} + w_7\frac{\Delta h}{\Delta L} + w_8\frac{\Delta h}{\Delta L} + w_9\frac{\Delta h}{\Delta L} + w_{10}\frac{\Delta h}{\Delta L}\right)$$

$$\underline{Q_{OUT_2} = 2312\ m^3/day}$$

Balance

$$\text{Net } Q = \Delta Vol - \text{Ave Flow In} + \text{Ave Flow Out}$$

$$= 58{,}000\ m^3 - \left(\frac{2750 + 3600}{2}\right)\times 365 + \left(\frac{2850 + 2300}{2}\right)\times 365$$

$$= 58{,}000 - 1{,}160{,}000 + 940{,}000 \quad m^3$$

$$\boxed{\text{Net } Q = -162{,}000\ m^3 \text{ Pumped Out}}$$