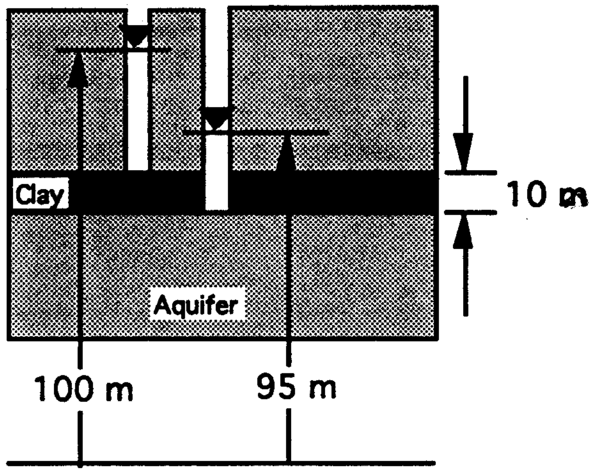
solutions

Solutions
(Please post in Library)

CE6361 Groundwater Hydrology , HW#2, Fall 1996 Due: ________________

1) Calculate the specific discharge across the clay layer in cm/sec where the vertical conductivity of the clay layer is 1.0E-7 cm/sec. Which direction is the flow?



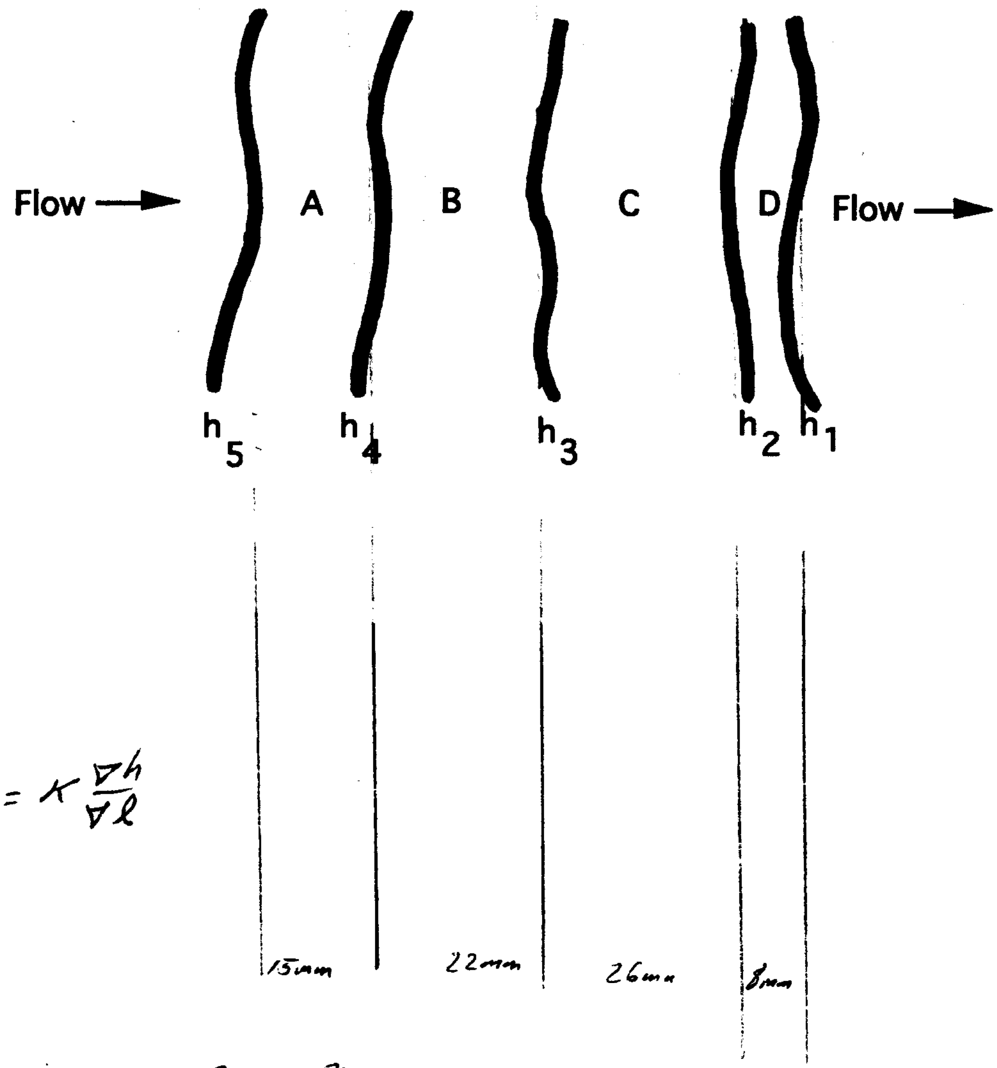
2) Given the following observations of the piezometric heads in three observation wells:

Well	LJ-65-21-226	LJ-65-21-227	LJ-65-21-228
x-cood.	100.0m	400.0m	100.0m
y-cood.	110.0m	100.0m	310.0m
head(m)	12.0m	13.5m	10.4m

Assume the wells all penetrate the same homogeneous, isotropic, confined aquifer of constant thickness, B=20 meters, effective porosity, n = 20%, hydraulic conductivity, K = 15m/day, and that the piezometric surface between the wells can be approximated as a plane. Determine the hydraulic gradient, the flow direction, the total discharge in the aquifer per unit width, and the average pore velocity at at point P (200m,200m). ~~Use the graphical method and the computer program "GRADIENT" and compare results~~.

3) Repeat the exercise for the same aquifer above if the tensor of hydraulic conductivity is Kxx=10.0m/d, Kxy=-5m/d,Kyy=20.0m/d. Plot the flow paths for both aquifers on the same plot. Determine the position of a particle of water that starts at P=200m,200m and moves in each flow field for 10 days, 100 days, and 1000 days. Comment on the implications of any differences you observe.

4) If the hydraulic conductivity in area A (see attached figure) is 1.0E-6 meters/sec, determine the hydraulic conductivity in the other areas. Assume the medium is isotropic, and that inflow equals outflow.



$$q = K \frac{\nabla h}{\nabla l}$$

$$q_A = q_B = q_c = q_d$$

$$K_A \frac{\nabla h}{\nabla l} = K_b \frac{\nabla h}{\nabla l} = K_c \frac{\nabla h}{\nabla l} = K_d \frac{\nabla h}{\nabla l}$$

$$\nabla h = 1$$

$$\therefore \quad \frac{K_A}{\Delta l} = \frac{K_B}{\Delta l} = \frac{K_c}{\Delta l} = \frac{K_d}{\Delta l}$$

So

$$K_B = \frac{\Delta l_B}{\Delta l_A} K_a \qquad K_d = \frac{\Delta l_d}{\Delta l_A} K_a$$

$$K_c = \frac{\Delta l_c}{\Delta l_A} K_a$$

Solution(s)

$K_a = 1.0 \cdot 10^{-6}$ m/s (given)

$K_b = \left(\frac{22}{15}\right)(1.0 \cdot 10^{-6}) = 1.46 \cdot 10^{-6}$ m/s

$K_c = \left(\frac{26}{15}\right)(1.0 \cdot 10^{-6}) = 1.73 \cdot 10^{-6}$ m/s

$K_d = \left(\frac{8}{15}\right)(1.0 \cdot 10^{-6}) = 5.33 \cdot 10^{-7}$ m/s

#1

$$q = -K \frac{\partial h}{\partial l}$$

$K = 1 \cdot 10^{-7}$ cm/sec

$h_2 = 95m$
$h_1 = 100m$
$dl = 10m$

$$q = -(1 \cdot 10^{-7} \text{cm/sec}) \frac{95m - 100m}{10m} = 5.0 \cdot 10^{-8} \text{cm/sec}$$

~~flow is~~ downward

#2. Using GRAD.WKS (output attached)

$-\frac{\nabla h}{\nabla l} = -0.00473\hat{i} + 0.008\hat{j}$ $\left|\frac{\nabla h}{\nabla l}\right| = 0.00929$

Using Graphical Method (Method attached)

$\left|\frac{-\nabla h}{\nabla l}\right| = 0.0092$ (similar results)

Flow/unit width

$$Q = -KA \frac{\nabla h}{\nabla l}$$
$$Q = -KB(\text{width}) \frac{\nabla h}{\nabla l}$$

Flow/unit width means: $f = \frac{Q}{(\text{width})}$

so $f = -KB \frac{\nabla h}{\nabla l}$

Groundwater Hydrology Gradient Spreadsheet

	x-coord	y-coord	head
Well #1	100	110	12
Well #2	400	100	13.5
Well #3	100	310	10.4

Hydraulic	-0.004733	<-x-comp.
Gradient	0.008	<-y-comp.

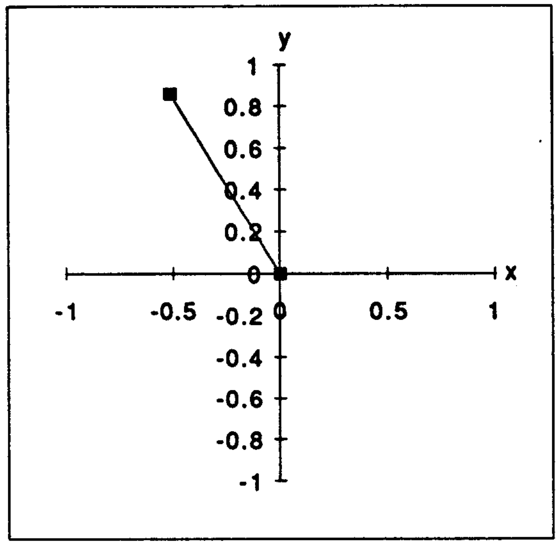
59°

A-Matrix			b-vector	x-vector
100	110	1	12	0.00473333
400	100	1	13.5	-0.008
100	310	1	10.4	12.4066667

A-Inverse		
-0.0035	0.0033333	0.0001667
-0.005	-1.95E-19	0.005
1.9	-0.333333	-0.566667

Head(x,y)	+	0.1088667	23	<-x value
	+	-0.192	24	<-y value
	+	12.406667		
	=	12.323533		

This spreadsheet prepared by
Theodore G. Cleveland, Ph.D.



Instructions:

Enter data for three wells in shaded area above left.

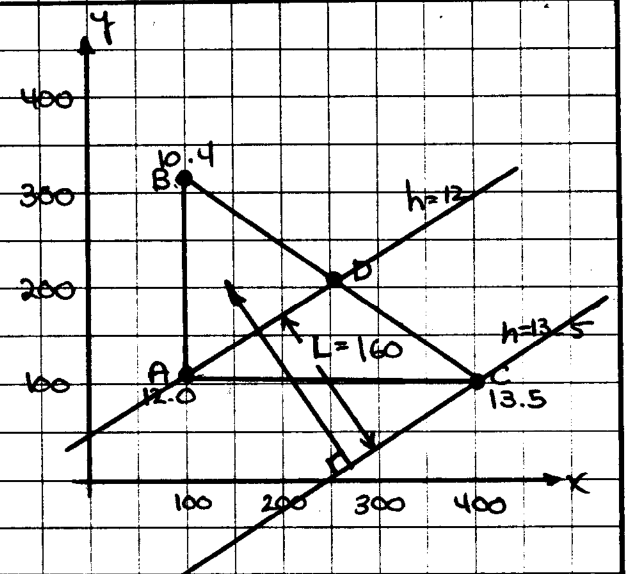
Spreadsheet solves linear system and computes gradient.

Plot shows gradient direction.

Name: Date:

Experiment:

#2 graphical method for gradient



$\underline{AB} = 200$

$\underline{AC} = 300$

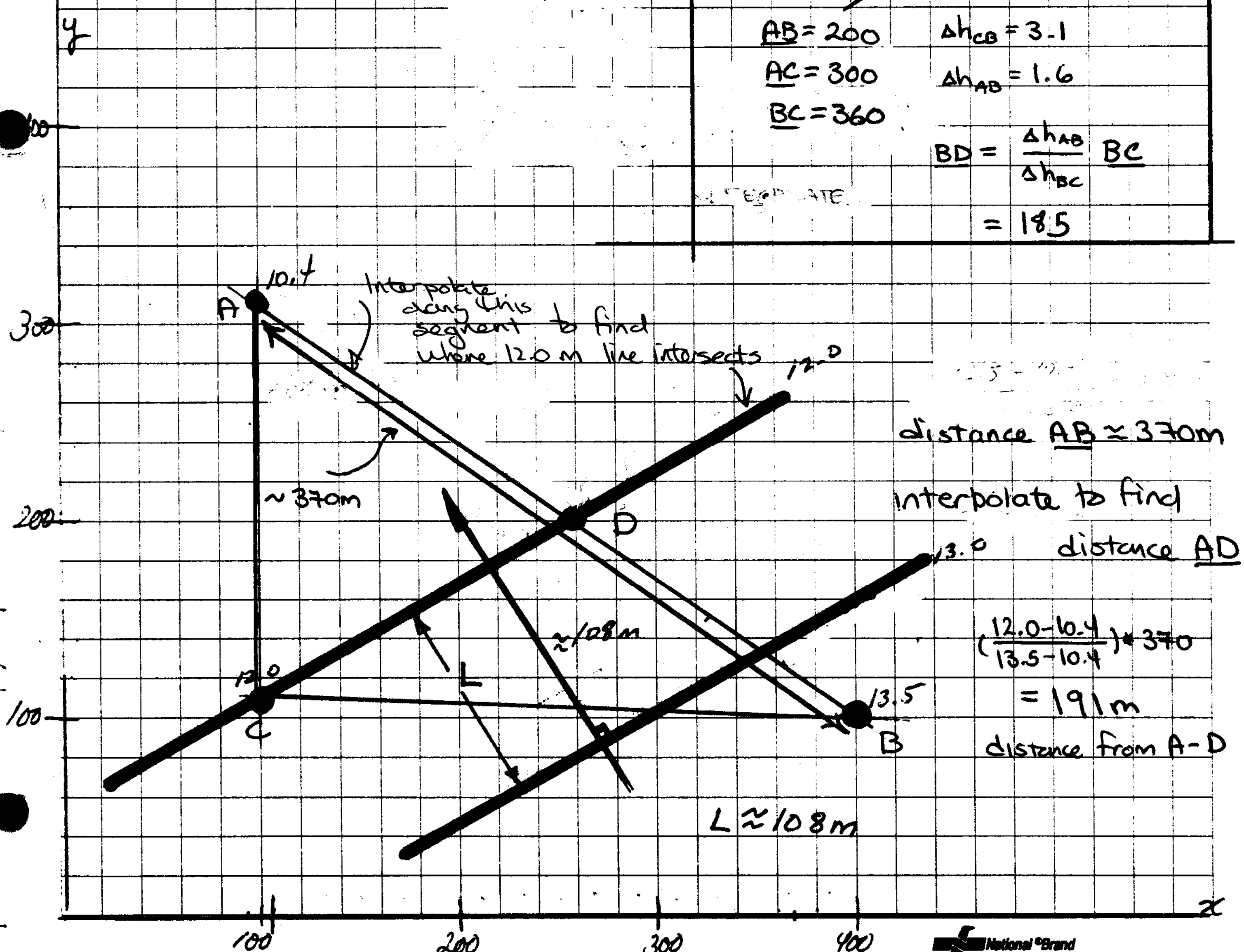
$\underline{BC} = 360$

$\Delta h_{CB} = 3.1$

$\Delta h_{AB} = 1.6$

$$\underline{BD} = \frac{\Delta h_{AB}}{\Delta h_{BC}} \underline{BC}$$

$$= 185$$



distance $\underline{AB} \approx 370m$

interpolate to find distance $\underline{AD}$

$$\left(\frac{12.0-10.4}{13.5-10.4}\right) * 370$$

$= 191m$

distance from A-D

#2 (continued)

$$f = (15\,m/day)(20\,meters)(0.00929)$$

$$= 2.788\,m^2/day \text{ (from high head to low head)}$$

Average pore velocity $\quad v = \frac{q}{n} \quad q_z = -K \operatorname{grad} h = K\left(-\frac{\partial h}{\partial z}\right)$

$$v = \frac{q}{n} = \frac{(15\,m/day)(0.00929)}{0.2} = \frac{0.139}{0.2} = 0.696\,m/d$$

Solution(s)

hydraulic gradient = 0.00929  ← flow direction

$$\frac{Q}{W} = 2.788\,\frac{m^2}{m \cdot day}$$

$$v = 0.696\,m/d$$

Note for problem #3

$q_x = -0.0709\,m/d$; $v_x = -0.354\,m/d$

$q_y = 0.120\,m/d$, $v_y = 0.60\,m/d$

#3) ~~Gradient same~~

$q = -K \, grad(h)$ hydraulic gradient $= -grad(h)$

$\therefore \quad q = K \frac{\nabla h}{\nabla l}$

but K is a tensor

$q_x = K_{xx} \frac{\nabla h}{\nabla x} + K_{xy} \frac{\nabla h}{\nabla y}$

$q_y = K_{xy} \frac{\nabla h}{\nabla x} + K_{yy} \frac{\nabla h}{\nabla y}$

$q_x = (10)(-0.00473) + (-5)(0.008) = -0.0873$
$q_y = (-5)(-0.00473) + (20)(0.008) = 0.1836$

$v_x = q_x/n = \frac{-0.0873}{0.2} = -0.4365$
$v_y = q_y/n = \frac{0.1836}{0.2} = 0.9182$

$\frac{\text{Total discharge}}{\text{unit width}} = |q| B = (0.2032)(20) = 4.065 \; m^2/day$

see plot attached

Implications – the distance between positions of particles after 1000 days is significant $d \approx 328$ meters or about 2/10th of a mile off.

Name: Date:

Experiment: