Groundwater

Use of Groundwater Level Fluctuations near an Operating Water Supply Well to Estimate Aquifer Transmissivity

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Abstract

We developed a method to estimate aquifer transmissivity from the hydraulic-head data associated with the normal cyclic operation of a water supply well thus avoiding the need for interrupting the water supply associated with a traditional aquifer test. The method is based on an analytical solution that relates the aquifer's transmissivity to the standard deviation of the hydraulic-head fluctuations in one or more observation wells that are due to the periodic pumping of the production well. We analyzed the resulting analytical solution and demonstrated that when the observation wells are located near the pumping well, the solution has a simple, Dupuit like form. Numerical analysis demonstrates that the analytical solution can also be used for a quasi-periodic pumping of the supply well. Simulation of cyclic pumping in a statistically heterogeneous medium confirms that the method is suitable for analyzing the transmissivity of weakly or moderately heterogeneous aquifers. If only one observation well is available, and the shift in the phase of hydraulic-head oscillations between the pumping well and the observation well is not identifiable. Prior knowledge of aquifer's hydraulic diffusivity is required to obtain the value of the aquifer transmissivity.

Introduction

The use of a water supply well for an aquifer test entails shutting off the supply well, allowing for the hydraulic head in and around the supply well to stabilize, followed by restarting the pumping well and monitoring the resulting hydraulic-head declines, then again shutting off the supply well, allowing for the recovery of hydraulic heads, and then returning the supply well to its pretest pumping schedule. However, turning off an existing supply well can be very difficult and inconvenient. This paper presents a method of obtaining transmissivity from observation well data associated with the normal uninterrupted cyclic operation of a water supply well.

The typical pumping cycle associated with a water-supply well is characterized by a period when the well pumps and fills a water tank. When the tank is full, the pump in the water supply well turns off. Water use then drains the tank until the pump in the water supply well automatically turns on to refill the tank. The water tank typically has a capacity of at least one day's consumption resulting in the pump being switched on approximately once a day. Cycling the pump in this manner creates hydraulic-head oscillations in the pumped aquifer. These oscillations are controlled by the a combination of the pumping of the water supply well and the aquifer's hydraulic properties.

Herein we define a periodic mode as the cycling on and off of the well. In other words, the periodic mode includes the well discharge rate's stepwise change from zero to its maximum value and back to zero. In this formulation, the overall duration of the cycle and the time when the pump is on can vary from pumping cycle to pumping cycle.

Changes in hydraulic heads caused by cyclic well discharge have been studied by a number of previous investigators. An analytical solution for the hydraulic-head oscillations caused by a pulse-test was derived by Kamal and Brigham (1975, 1989, 1990, 1991) using a superposition technique for transient flow toward a well with a constant pumping rate. Noaman and King (1990) and Noaman and King (1991) applied this solution to estimate hydraulic parameters from pulse testing.

Another well-known way of conducting tests in a well with a variable flow rate is to use harmonic fluctuations

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in the flow rate of the production well along with the associated water-level fluctuations in observation wells. Black and Kipp (1981) related oscillating hydraulic heads to harmonic pumping from and injection into a well completed in a confined, homogeneous aquifer. Black and Kipp (1981) also investigated the connection between aquifer parameters and the amplitude of oscillations in quasi-steady-state flow. Rasmussen et al. (2003) developed a more general solution for sinusoidal pumping by considering the case of a leaky aquifer and the effect of different initial conditions on the amplitudes of the hydraulic-head oscillations. Rasmussen et al. (2003) successfully applied their method to aquifer parameter estimation. Renner and Messar (2006) developed a similar solution for radial flow considering specified head-boundary conditions. For a confined, unbounded aquifer the amplitude of oscillations in quasi-steady-state flow equations employed by all these researchers have a form that coincides with Black and Kipp's (1981) results.

In recent decades the methodology and applications of aquifer testing using harmonic oscillations have been widely used in the application of hydraulic tomography to determine hydraulic heterogeneity. For example, Cardiff and Barrash (2015) and Cardiff and Sayler (2016) used Black and Kipp's (1981) results to develop a process for aquifer test analysis that involved periodic discharge rates. Bakhos et al. (2014) noted the major benefit of harmonic oscillation tests is the extraction of the oscillation signal from different types of noise. Using Rasmussen et al. (2003) results, Bakhos et al. (2014) developed a numerical inversion method for the estimation of transmissivity and storage using a synthetic heterogeneous aquifer. Fischer et al. (2018) used numerical inversion of harmonic aquifer tests to identify the structure of hydraulic heterogeneity in a karst aquifer.

In this paper, we present a method of obtaining estimates of aquifer transmissivity from the uninterrupted operation of a water supply well. This method of aquifer testing only requires continuous data from one or more observation wells, no new or special use pumping well, no interruption or change in water production, and a simple Dupuit-like analytical solution to obtain an estimate of the aquifer transmissivity. For the test analysis, we developed an analytical solution for a pumping well operating in a periodic or quasi-periodic mode using Fourier series and Black and Kipp's (1981) results for harmonic oscillations. We evaluate our approach for estimating transmissivity from observation well data for homogeneous and heterogeneous aquifers using field data and synthetic examples. In applying our method, we assume that head fluctuations in observation wells occur only due to the operation of the pumping well, that is, the extraneous changes of the head at observation wells resulting from barometric pressure changes and earth tides have been removed by preprocessing of the raw monitoring data using available procedures (Toll and Rasmussen 2007) and software (Halford 2006; Halford et al. 2012).

Theoretical Background

Consider a pumping well that operates with a constant flow rate (Q) in a homogeneous, confined aquifer. If there are no hydraulic boundaries near this well, the relationship between the drawdown $S_c(t)$ at a given distance from pumping well (r) and the flow rate can be described by the well-known Theis solution:

$$S_c(t) = \frac{Q}{4\pi T} W(u); \quad u = \frac{r^2}{4at}; a = \frac{T}{S_s} \tag{1}$$

where T is the aquifer transmissivity, a is the hydraulic diffusivity, S_s is the aquifer storativity, and $W(u)$ is the well function.

Next, consider a well that pumps in a periodic or cyclic mode with a flow rate $Q_{\max}$ and a mean pumping duration of t_Q within an overall period (pump on and off) of t_p. The fraction of the time when the pump is on is denoted as $\chi = t_Q / T_P$.

The relationship between the actual drawdown $S_p(t)$ and the time of periodic pumping for pumping cycle number N can be found with using superposition of Equation 1:

$$S_p(t) = \frac{Q_{\max}}{4\pi T} \sum_{i=1}^{N} [W(u_i) - W(u_i^*)]; \quad u_i = \frac{r^2}{4at_i}; u_i^* = \frac{r^2}{4a(t_i - t_Q)} \tag{2}$$

where t_i is the time from the beginning of the ith well cycle, that is, for a constant period length, $t_i = t-(i - 1)$ T_p where t is the total time from the start of the first cycle.

An example of the drawdown at an observation well in a confined aquifer with elastic storage due to cyclic pumping with constant values of t_Q and T_p is shown in Figure 1. Also shown is the hypothetical drawdown from an equivalent constant-rate pumping well. The solid line in Figure 1 is the drawdown, S_p, calculated with Equation 2. The dashed line in Figure 1 shows the hypothetical drawdown, S_c, calculated with Equation 1 for a well with an effective constant-pumping rate of $Q_{av} = \chi Q_{\max}$. The actual volume of water pumped is the same as the volume calculated by using the effective constant rate.

Consider the drawdown $S(t)$ with zero mean to be defined as:

$$S(t) = S_p(t) - S_s(t) \tag{3}$$

Because Equation 3 is the superposition of two processes described by Equations 1 and 2, it represents the drawdown caused by a stepwise flow rate within the ith well cycle:

$$Q(t) = Q_{\max}[H(t_Q - t_i) - \chi] \tag{4}$$

where $H(t_Q - t_i)$ is Heaviside unit step function whose value is zero for negative arguments and one for positive arguments.

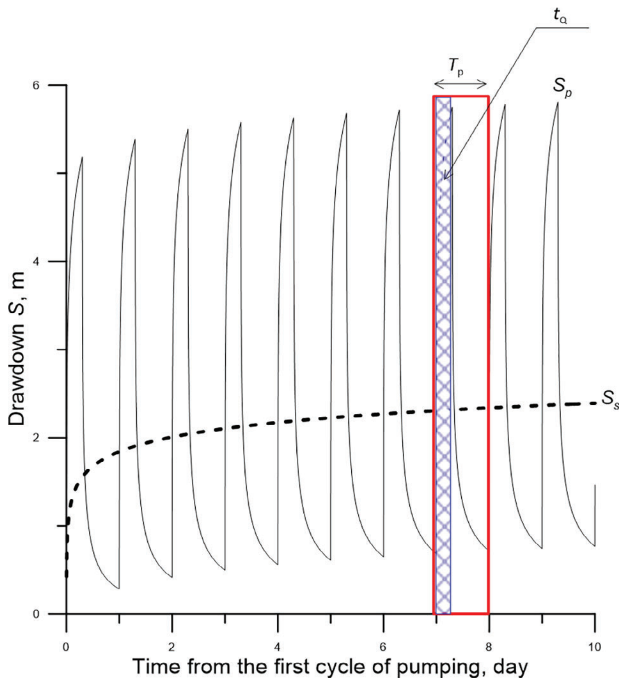

Figure 1. Drawdown at the observation well due to cyclic pumping along with the hypothetical drawdown from an equivalent constant-rate pumping well.

One can see from Figure 1 that for large times, more than 5-10 times the period T_p, the amplitude of actual drawdown, S_p, within a cycle is much larger than the maximum hypothetical change of drawdown of S_c during one cycle. Thus, for large times, the periodic $S(t)$ process can be considered quasi-steady-state: its moving average and variance stop changing in time. Next, we expand Equation 4 into a Fourier series so that the stepwise flow rate, $Q(t)$, is represented as an infinite sum of harmonic oscillations (Abramowitz and Stegun 1964):

$$Q(t) = \sum_{n=1}^{\infty} Q_n^{\max} \cos(\omega_n t) \tag{5}$$

with Fourier coefficients $Q_n^{\max} = \frac{2Q_{\max}}{\pi n} \sin(n\pi\chi)$ and frequencies $\omega_n = 2\pi n / T_p$.

The aquifer response ΔS_n at the distance r to a given harmonic, $Q(t) = Q_n^{\max}\cos(\omega_n t)$, can be written in terms of an amplitude solution (Black and Kipp 1981; Depner and Rasmussen 2016) for harmonic flow oscillation:

$$\Delta S_n = \frac{Q_n^{\max}}{2\pi T} N_0\left(\frac{r}{R_n}\right) = \frac{Q_{\max}}{\pi T n} N_0\left(\frac{r}{R_n}\right) \tag{6}$$

where $R_n = \sqrt{a/\omega_n}$; $N_0(x) = \sqrt{\mathrm{Ker}^2(x) + \mathrm{Kei}^2(x)}$; ΔS_n is the oscillation amplitude of the head change due to component $Q_n^{\max}$, N_0 is the amplitude of the Kelvin function of the second kind and is the sum of squares of its real part Ker and its imaginary part Kei (Abramowitz and Stegun 1964).

Because Equation 5 is a harmonic process, the variance of the resulting harmonic process $S(t)$ in Equation 3 can be formally written taking into account Equation 6 as the sum of variances of all harmonics in Equation 5:

$$\sigma_s^2 = \sum_{n=1}^{\infty} \sigma_n^2 = \left(\frac{Q_{\max}}{\pi T}\right)^2 \sum_{n=1}^{\infty} n^{-1} N_0^2\left(\frac{r}{R_n}\right); \tag{7}$$

Equation 7 can be rewritten in the following form:

$$\sigma_s^2 = \left(\frac{Q_{\max}}{2\pi T}\right)^2 F^2\left(\frac{r}{R_T}\right)$$

$$R_T = \sqrt{a/\omega};\ \omega = 2\pi/T_P;\quad v = 4\chi(1-\chi);\quad F(x, v) = \frac{2}{\pi}\sqrt{\sum_{n=1}^{\infty} n^{-2}\sin^2\left[n\frac{\pi}{2}(1-\sqrt{1-v})\right] N_0^2(x\sqrt{n})}. \tag{8}$$

Thus, introducing the characteristic amplitude of the drawdown fluctuation $\Delta S_{\max}$ we obtain:

$$\Delta S_{\max} = \sqrt{2}\sigma_s = \frac{Q_{\max}}{2\pi T} F\left(\frac{r}{R_T}, v\right) \tag{9}$$

where σ_s is the standard deviation of the drawdown fluctuation.

Numerical analysis of the function $F(r/R_T, v)$ demonstrates that when $r/R_T < 1$ and $v > 0.5$ it can be approximated with an error of less than 3% using the following logarithmic equation:

$$F\left(\frac{r}{R_T}, v\right) \approx C\cdot\left(\ln\frac{R_T}{0.907r} + 0.45\frac{r}{R_T}\right); \quad C = 0.697v^{0.58}. \tag{10}$$

The logarithmic character of the function $F(r/R_T, v)$ in Equation 10 allows us to use a Dupuit-like equation for estimating transmissivity with the use of two or more observation wells at the distances $r < 0.2R_T$ from the pumping well as follows:

$$\Delta S_{\max}(r) = \frac{Q_{ef}}{2\pi T}\ln\frac{R_{Dp}}{r};\quad R_{Dp} = 1.11R_T;\quad Q_{ef} = C\cdot Q_{\max} \tag{11}$$

where R_{Dp} is the Dupuit radius, that is, the distance from pumping well where the logarithmic approximation of $\Delta S_{\max}(R_{Dp}) = 0$. Note that the form of Equation 11 coincides with the approximation of the Theis equation (Equation 1) by Cooper and Jacob (1946), if we assume: $R_{Dp} = 1.5\sqrt{at}$.

Theoretical analysis of variable cyclic pumping reveals that the shift in phase between the drawdown curve at an observation well and the flow rate are controlled by the hydraulic diffusivity and the distance to the observation well (Kamal and Brigham 1975; Black and Kipp 1981; Rasmussen et al. 2003). Therefore, measurements of this phase shift in a well located at

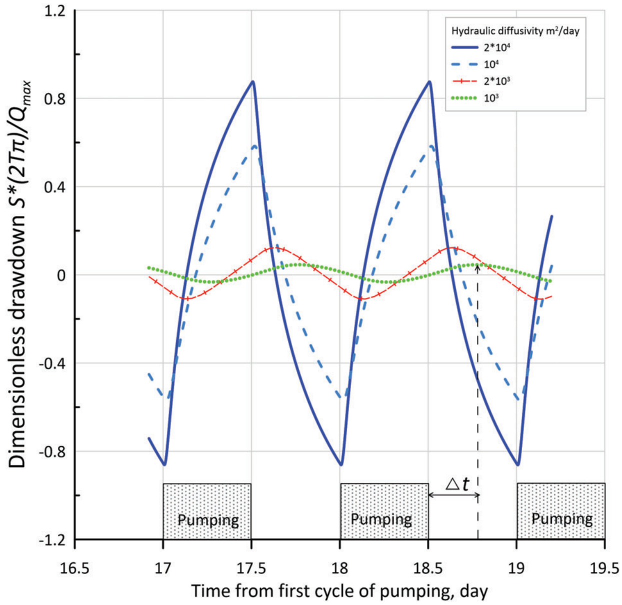

Figure 2. Dimensionless drawdowns versus time at the observation well at distance 53 m from the pumping well due to cyclic pumping with period 1 day.

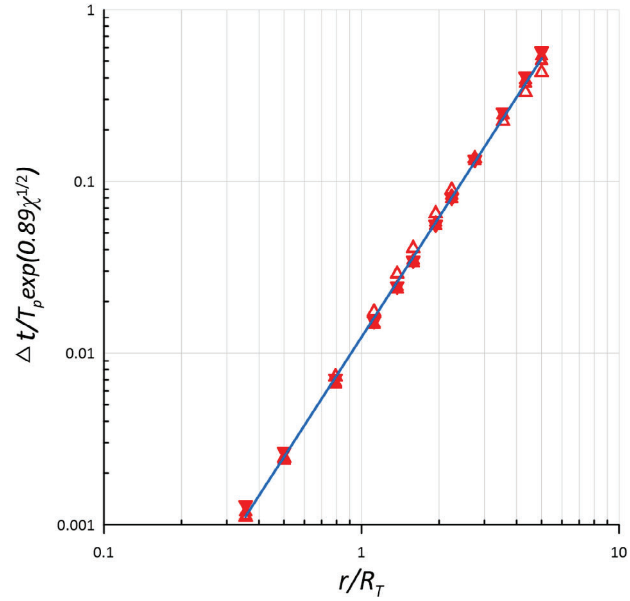

Figure 3. Dimensionless phase shift versus dimensionless distance. Different symbols correspond to the various fraction χ of the time when the pump is on, changed in the range 0.1–0.7.

a known distance can be used to estimate hydraulic diffusivity (Rasmussen et al. 2003). However, this shift in phase may be so small that it is difficult to identify in a confined aquifer were the pumping well is close to the observation well As an example, shown in Figure 2 are drawdown curves calculated with Equation 2 for an observation well located 48 m from the pumping well for different values of hydraulic diffusivity and a typical pumping cycle of 1 day. From this figure, we can see that noticeable phase shifts, Δt, occur at characteristic values of hydraulic diffusivity of the order 10^3 m^2/d. These values are typical of unconfined aquifers.

For confined aquifers, dimensionless analysis was used to relate the phase shift to the distance to an observation well and the aquifer hydraulic diffusivity. The phase shift is identified by finding local minimums of drawdown, $S(t)$, that is, we will find points at which $\mathrm{d}S/\mathrm{d}t = 0$. To do this, we take the time derivative of Equation 2 for a large time after the beginning of pumping and get the following expression which can be used to calculate the phase shift Δt (which is shown in Figure 2):

$$\sum_{i=1}^{N}(i\cdot T_p - t_Q + \Delta t)^{-1}\exp\left(-\frac{r^2}{4a(i\cdot T_p - t_Q + \Delta t)}\right) = \sum_{i=1}^{N}(i\cdot T_p + \Delta t)^{-1}\exp\left(-\frac{r^2}{4a\cdot(i\cdot T_p + \Delta t)}\right) \tag{12}$$

The results of the numerical solution of Equation 12 using five members of the series ($N = 5$) in a dimensionless form are presented in Figure 3. It follows from this figure that for typical confined aquifers of dimensionless distances of $r/R_T < 1$ to an observation well, the

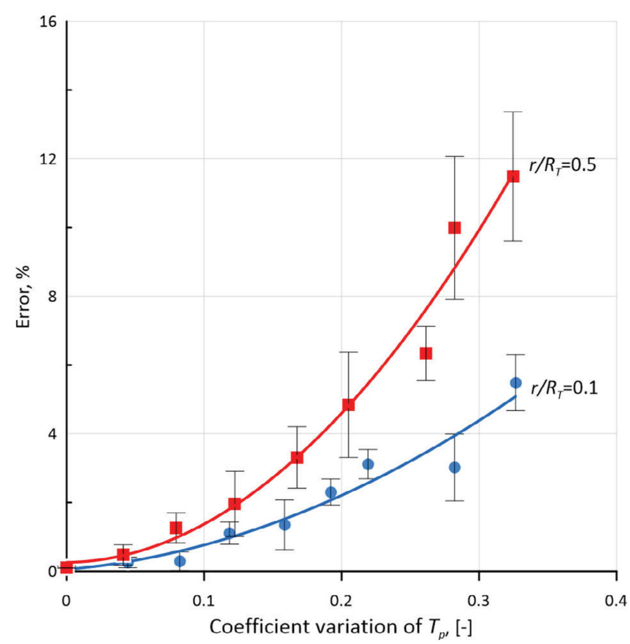

Figure 4. Errors of transmissivity estimations versus coefficient variation of T_p for $M\{\chi\} = 1/2$ and $M\{T_p\} = 1$, where T_p and χ are uniformly distributed random variables. Vertical bars are the standard deviation of error averaged over 5 runs.

phase shift is only a fraction of a percent and within the first percent of the length of the pumping cycle, T_p. This inability to identify the phase shift makes it impossible to estimate hydraulic diffusivity.

To solve this problem two or more observation wells are required. Using two or more observation wells, the hydraulic diffusivity can be estimated by matching observed data $\Delta S_{\max}(r) - \lg(r)$ with theoretical curves (Equations 9-11).

The Sensitivity of the Estimated Transmissivity to Quasi-Periodic Pumping

Equations 7-11 are valid for periodic pumping and constant values of the fraction of the time the pump is on, χ, and a constant pumping cycle, T_p. In the real world, pumping from supply wells may be quasi-periodic—not exactly periodic. In this section, the sensitivity of the estimated transmissivity to deviations from strictly periodic pumping is investigated.

To evaluate the error associated with the estimated transmissivity when the pumping is not exactly periodic, a synthetic indicator time-series of quasi-periodic pumping $I(t)$ ($I = 1$ if the pump is cycled on and $I(t) = -1$ if the pump is cycled off) were created using uniformly distributed random series of $\chi(t)$ and $T_p(t)$ with the given mean values: $\mathbf{M}\{\chi\} = 1/2$, $\mathbf{M}\{T_p\} = 1$. Here $\mathbf{M}\{\}$means expected mean value. Then, for each generated random time series of the pumping indicator, Equation 13 is used to calculate a time series of dimensionless drawdown $\overline{S}(t)$:

$$\overline{S}(t) = \frac{4\pi[S_p(t) - S_c(t)]T}{Q_{\max}} = \sum_{i=1}^{N}[I(t_i)W(u_i) + I(t_i - t_{Qi})W(u_i^*)] - \mathbf{M}\{\chi\} \cdot W(u);$$

$$u = \frac{r^2}{4at}; \quad u_i = \frac{r^2}{4at_i}; u_i^* = \frac{r^2}{4a(t_i - t_{Qi})} \tag{13}$$

where t_i is the time from the beginning of the ith well cycle and t_{Qi} is the length of pumping time in this cycle.

The error, $\varepsilon\%$, associated with the estimated transmissivity when the pumping is not exactly periodic depends on a dimensionless distance and is defined as:

$$\varepsilon = \left(1 - \frac{2F\left(\frac{r}{R_T}, \nu\right)}{\sqrt{2}\sigma_{\overline{S}}}\right) \cdot 100 \tag{14}$$

where $\sigma_{\overline{S}}$ is the standard deviation of processes defined by Equation 13 and $\nu = 4\mathbf{M}\{\chi\} \cdot (1 - \mathbf{M}\{\chi\})$.

Shown in Figure 4 is the dependence of the error $\varepsilon\%$ on the dimensionless distance and the coefficient of variation of the period length T_p. Each point on this graph results from averaging five simulations each with a length of 40 periods. It follows from this figure that the error associated with a quasi-periodic pumping at small coefficients of variation (<0.2) is quite acceptable. Interestingly, this error turnes out to be positive. That is, a quasi-periodic series of pumping leads to an increase in the standard deviation of drawdowns in comparison with a strictly periodic series due to the increasing random component in the fluctuation of the well operation time.

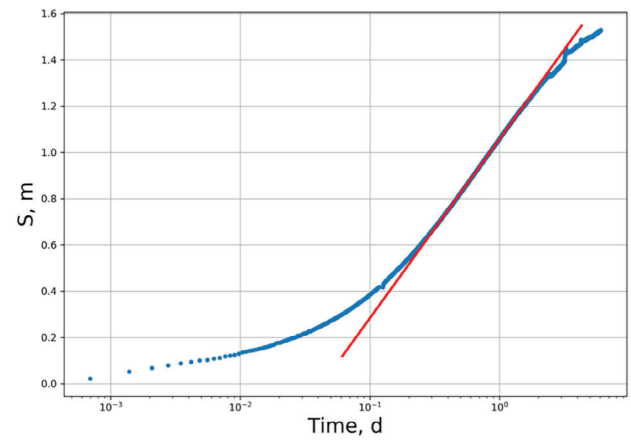

Figure 5. Interpretation results of the 6-day constant-flow rate aquifer test. Blue points are measured with data logger drawdown in the observation well. Redline is the semilog approximation of drawdown for processing test results by Cooper-Jacob method.

Comparison to a Conventional Constant-Rate Aquifer Test

In this section, the transmissivity obtained with this new method is compared to the transmissivity obtained from a constant-flow rate aquifer test that was analyzed using the Theis equation with the Cooper-Jacob (Cooper and Jacob 1946) approximation. The test involved one water supply well and one monitoring well completed in a confined aquifer. Under normal conditions, the water supply well operates in a quasi-periodic mode, automatically turning on and off for several hours at a time.

The studied aquifer is composed of carboniferous limestone and dolomite and has a constant thickness of 20 m. Below the aquifer, is a carboniferous clay and above the aquifer, there is quaternary clay. We assumed that the aquifer is unlimited in areal extension, homogenous, isotropic, and has a constant thickness.

Two wells were used for the constant-rate pumping test, one pumping and one observation well. The distance between wells is 53 m. The water supply well is fully penetrating while the observation well penetrates more than half of the thickness. The pumping rate of 322 m^3/d was measured with an in-line flow meter and was adjusted once every hour. Water levels were measured with pressure transducers that had errors of less than 1 cm. Data were recorded on a schedule that was close to logarithmic. The test lasted more than 6 days (the pumping well was a production well and it started to work in normal operation mode soon after the test).

The interpretation of the test was performed by curve matching with the straight-line method on a semilog time-drawdown plot, based on the Cooper-Jacob solution for a pumping test in a confined aquifer. The resulting transmissivity is 77 m^2/d and hydraulic diffusivity is 2700 m^2/d. (Figure 5).

Upon completion of the constant-flow rate test and drawdown recovery, the pumping well continued usual

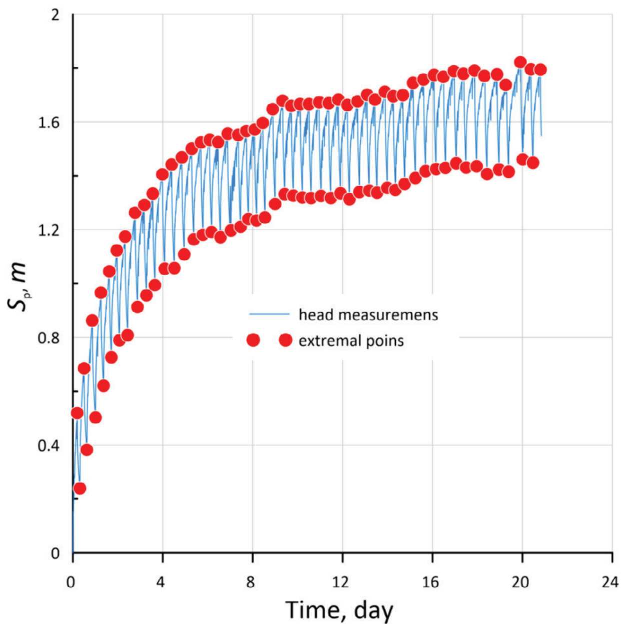

Figure 6. Drawdown data from cyclic pumping.

quasi-periodic operation as a production well while hydraulic-head recording in the monitoring well continued (Figure 6).

In analyzing the data of quasi-periodic operation, first the average $T_{pav.}$ and $\chi_{av.}$ were calculated from pressure head monitoring data with the use of a VBA script in Excel which found the extreme points in the hydraulic-head monitoring data used in this analysis (Figure 6).

Using average values for $T_{pav.}$ and $\chi_{av.}$ and the known aquifer hydraulic diffusivity, the transmissivity was estimated using Equation 9. Shown in Table 1 is the transmissivity estimated using our cyclic pumping method along with the transmissivity obtained from the constant-flow rate aquifer test. The difference between these values is 6.4%, very reasonable for aquifer test analyses.

Applicability of the Analysis of Cyclic Pumping for a Heterogeneous Aquifer

Numerical simulation was used to assess the impact of the spatial heterogeneity on the estimate of transmissivity obtained from the analysis of cycling pumping. The sandy-gravel alluvium of the Neogene paleo-valley of the Don River (Muromets et al. 2018) were used as an example of a heterogeneous aquifer. This paleo-alluvial aquifer is currently used for water supply. Nine cluster-well aquifer tests and indicator semilog graphs of drawdowns demonstrated that the Theis model could be used to interpret these tests. The resulting transmissivity values varied between 900 and 2200 m^2/d and the estimated specific storage was about 0.001.

Analysis of the transmissivity and grain-size distribution data from drilling cores supported the development of a stochastic model of spatial heterogeneity of the alluvial aquifer. In this model, the spatial variability of transmissivity is described by a lognormal two-dimensional Gaussian field with an isotropic exponential variogram with the spatial scale $\lambda = 300$ m and the following parameters of $\mathbf{Y} = \ln T$ distribution: mean $\mathbf{M}\{\mathbf{Y}\} = 7.48$ and variation $\sigma_Y{}^2 = 0.25$. Such parameters are typical of field heterogeneity of weakly and moderately heterogeneous aquifers—potential sources for groundwater pumping.

A numerical model containing 600×600 grid cells was chosen for simulating periodic pumping using MODFLOW 2005 (Harbaugh 2005). The model grid had 566*566 internal blocks with a constant block size with $\Delta x = \Delta y = 3$ m. The external 17 blocks had a variable spacing that gradually increased from 5 to 350 m away from the center of the grid making the total simulated area equal to 5800*5800 m.

The pumping well was placed in the center of the simulated area. During the first 3 days (i.e., the first stress-period) pumping was constant and equal to 1500 m^3/d. This period corresponded to the standard 72-h aquifer test. Then, during the next 20 stress periods of 0.5 days each, the cyclic well operation was simulated: 0.5 days with a flow rate of 3000 m^3/d and the next 0.5 days with a zero flow rate.

From the Theis solution, the radius of influence, R_{inf}, of the pumping well is proportional to the square root of test time and can be estimated as $3 \div 4\sqrt{at}$. This means that even for the first stress period, at characteristic hydraulic diffusivity values a, the pumping effect reaches the external boundaries of the simulated domain. To simulate an unbounded aquifer using a bounded simulated domain, the MODFLOW 2005 General Head Boundary (GHB) package (Harbaugh 2005) was used. For each stress period, the hydraulic head at the assumed external source and the associated GHB hydraulic conductance were set at the external boundaries of the model. This external hydraulic head and hydraulic conductance required by the GHB package were calculated using the Theis equation by requiring the distance to the GHB

Table 1
Transmissivity from Cyclic Pumping Compared to the Transmissivity From a Constant-Flow Rate Aquifer Test

Period Duration T_{pav}, Days	$\chi_{av.}$	Q, m^3/d	r, m	Constant Flow-Rate a, m^2/d	Constant Flow-Rate T, m^2/d	Amplitude ΔS_{max}, m	Cyclic Pumping $T_{est.}$, m^2/d	Error, %
0.41	0.76	322	53	27,000	77	0.15	72	−6.4

Table 2
Transmissivity and Specific Storage Values of the Entire Model Domain Obtained by Processing the Simulation Results at Individual Virtual Observation Wells for Four Homogeneous Aquifers

	Estimated Hydraulic Parameters					
	S-lg(t/r^2)		S-lg(r) for $t = 3$ days		ΔS_{max}-lg(r)	
Simulation Parameters	T	S_s*10^{-3}	T	S_s*10^{-3}	T	S_s*10^{-3}
$T = 1000\,m^2/d\ S_s = 10^{-3}$	1033	0.92	1004	1.08	1000	0.93
$T = 1500\,m^2/d\ S_s = 10^{-3}$	1530	0.97	1505	1.11	1498	0.95
$T = 2000\,m^2/d\ S_s = 10^{-3}$	2025	0.99	2006	1.11	1995	0.95
$T = 2500\,m^2/d\ S_s = 10^{-3}$	2515	1.02	2508	1.10	2492	0.95

boundary be equal to the radius of influence $R_{\text{inf}}(t)$, where there is zero drawdown.

The Theis equation gives the following relationship between the radial flow $Q(r)$ and pumping rate Q_{well}:

$$Q(r) = Q_{well} \exp\left(-\frac{r^2}{4at}\right). \qquad (15)$$

Assuming that the groundwater flow remains close to radial at a large distance from the pumping well and using Equation 15, the following expression was used in calculating the GHB hydraulic conductivity assigned for each stress period in the external grid blocks:

$$T_{i,j}^{GHB}(t_{sp\,k}) = \Delta L_{i,j} T_{grd} / \left(r_{i,j} \ln \frac{3\sqrt{at_{sp\,k}}}{r_{i,j}} \exp(-u_{i,j}) \right);$$
$$u_{i,j} = \frac{r_{i.j}^2}{4at_{sp\,k}} \qquad (16)$$

where $t_{sp\,k}$ is time from the beginning of the simulation to the kth stress period, $T_{i,j}^{GHB}(t_{sp\,k})$ is the GHB hydraulic conductivity for the boundary block node with coordinates ith column and jth row, $r_{i,j}$ the distance from the node to the center of grid, T_{grd} is grid effective transmissivity which is equal to the geometric mean of the transmissivity of the simulated domain $\Delta L_{i,j} = \Delta x_j$ Δor Δy_i, depending on the position of the boundary segment where the boundary condition is specified.

The first stress-period consisted of 30 increasing time steps with a multiplication factor equal to 1.2 and the remaining stress-periods used constant time steps equal to 1 of 48 of the stress period. Ten locations around the pumping well were selected to analyze the model results. These virtual observation wells were randomly located at distances of 30 to 300 m from the pumping well. The choice of a minimum distance of 30 m from the pumping wells is to avoid the influence of numerical effects caused by radially converging flow being simulated with a square grid.

As a first step in assessing the adequacy of the design grid for evaluating the ability of the cyclic pumping method for estimating transmissivities in heterogeneous media, four numerical experiments with constant transmissivities of 1000, 1500, 2000, and 2500 m^2/d were simulated. Using the drawdowns from each of these simulations in the virtual observation wells, three methods were applied in determining the hydraulic parameters of the simulated aquifer. The first method, the Cooper-Jacob method, analyzes the drawdown over time at a fixed location using a semi-logarithmic straight-line method of S-lg(t/r^2) for all observations during the first constant pumping stress-period. The second method looks at the drawdown as a function of distance at a fixed time of the end of 3 days of constant pumping and uses the semi-logarithmic method of a straight line in $S(r) - \lg(r)$ to estimate aquifer parameters. The third method uses the approach presented in this paper to estimate hydraulic parameters from the amplitudes of hydraulic-head oscillations data $\Delta S_{\max}(r) - \lg(r)$associated with the cyclic pumping after the initial 3 days of constant pumping.

The results for all three methods are in agreement and agree well with the prescribed model parameters (Table 2) indicating that the numerical grid and associated boundary conditions are not negatively affecting the evaluation of hydraulic parameters based on the use of virtual observation wells across the model domain.

Next, 25 variants of transmissivity heterogeneity provided the basis for simulating the hydraulic response of a heterogeneous aquifer to cycling pumping. SGeMS (Remy et al. 2009) was used to generate the transmissivity realizations for the internal part of the grid (566 × 566 blocks). For the 17 external grid blocks, a simplified approach was used for setting their transmissivity. First, the external blocks were divided into square superblocks, each 600 m on a side and each had its own transmissivity. The logarithm of the transmissivity of each superblock was chosen randomly from a normal distribution with a mean value equal to the mean of the simulated field and a variance of $\varsigma^2 {*} \sigma_Y{}^2$. The parameter ς was calculated based on the ratio of the superblock side length to the correlation field length $x = (L/\lambda)$ according to (Pozdniakov and Tsang 1999) as $\zeta(x) = 2x^{-2}[\exp(-x) + x - 1]$. The geometric average of the transmissivity, T_{grd}, was used in calculating the GHB hydraulic conductivity for each realization of the transmissivity field.

All 25 model simulations were processed as well as the simulations with constant transmissivities using

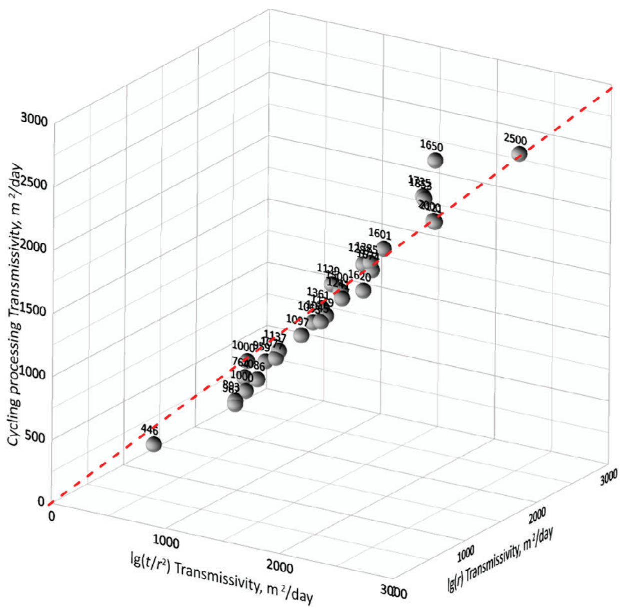

Figure 7. Relationship of the transmissivity values obtained by three different methods. The red dashed line is for X = Y = Z—representing a perfect between the different methods. The numbers above the 3D symbols are the geometric mean transmissivities calculated in the 300 m radius surrounding the pumping well.

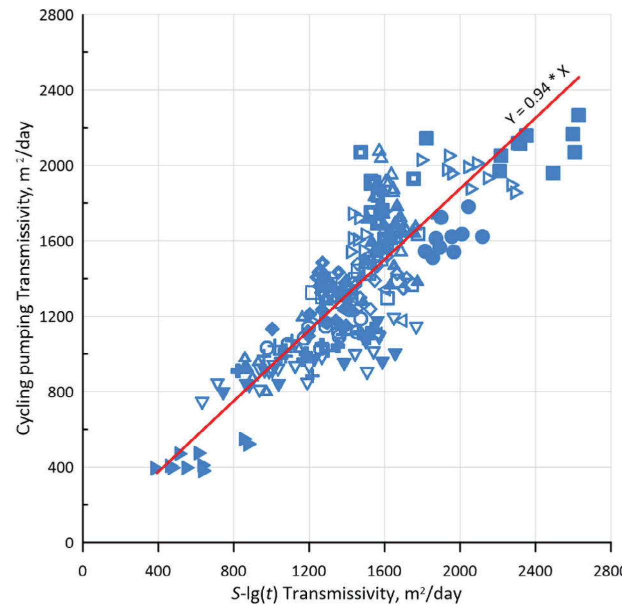

Figure 8. Correlation of the transmissivity values obtained by Cooper-Jacob semilogarithmic straight-line method S-lg(t) and the cyclic pumping method for each observation point for all 25 simulations. The different symbols indicate different simulation runs. Red straight-line is the equation of regression of transmissivity values obtained by two methods.

the three aquifer test analysis methods described above. The results of these simulations are shown in Figure 7 and listed in Table 3. All three methods give similar, well-correlated values. The averaged results in Table 3 show that all methods give transmissivity values close to the geometric mean in the area of the virtual observation wells. This result is consistent with the theoretical study (Indelman 2003) on the relationship between the transmissivity obtained by the Cooper-Jacob method and the effective conductivity of a weakly heterogeneous aquifer and results of stochastic analysis of oscillatory pumping in heterogeneous aquifers (Cheng et al. 2019).

The next test of the cyclic pumping analysis is to find out how well it performs at individual locations in a heterogeneous aquifer. First, a straight line fit to S-lg(t) data was used to estimate the transmissivity for each of the 25 simulations of the transmissivity field. Then, for each well, the transmissivity was also calculated using the cyclic pumping analysis presented in this paper (Equations 9-10). The hydraulic diffusivity was taken from the previous analysis using the average of 10 virtual wells and it is constant for the given realization of transmissivity. The correlation between the results of these two methods is shown in Figure 8 for 25 run and 10 locations. The results of the semilog and cyclic methods show similar transmissivity values as well as similar variations of transmissivity for the 25 realizations.

In general, the results of constant and cycling rate pumping tests show that the proposed method is as reliable as the classical methods based on Cooper-Jacob approximation of Theis solution in analyzing aquifer tests in heterogeneous aquifers.

Table 3

Transmissivity and Specific Storage Estimations Obtained by Processing the Simulation Results With Variable Transmissivity Values—Results Averaged over 25 Simulations

	Method Used in Processing the Simulation Results				
				Average Value Within 300 m of the Pumping Well	
Parameter	**S-lg(t/r^2)**	**S-lg(r)**	**Cycle Processing**	**Geometric Mean**	**Harmonic Mean**
Mean transmissivity, m^2/d	1489	1426	1492	1452	1322
Coefficient variation of transmissivity	0.26	0.35	0.36	0.32	0.34
Mean storage coefficient*10^3	0.98	2.5	0.99	1	1

Conclusion

We developed a method to estimate aquifer transmissivity from the hydraulic-head fluctuations associated with the normal cyclic operation of a water supply well thus avoiding the need for a traditional aquifer test and the associated interruption of the water supply.

The method relates the aquifer's transmissivity to the amplitude of hydraulic-head oscillations associated with periodic pumping. Our method produces transmissivities similar to those obtained from constant-rate aquifer tests and is suitable for analyzing the transmissivity of weakly or moderately statistically heterogeneous aquifers. Sensitivity analysis demonstrates that the error in the estimated transmissivity is generally less than 5% for quasi-periodic pumping where the coefficient variation of the period length is less than 0.2.

The major limitation of our method arises when only one observation well is available and the shift in the phase of hydraulic-head oscillations between the pumping well and the observation well is not identifiable. In this case, prior knowledge of the aquifer's hydraulic diffusivity is required to obtain the aquifer's transmissivity. The phase shift typically is identifiable in observation wells in unconfined aquifers with relatively low hydraulic diffusivities located within a few hundreds of meters of the pumping well. This phase shift together with the amplitudes of the hydraulic-head oscillations can be used to estimate both the aquifer's hydraulic diffusivity and transmissivity.

Prior knowledge of the aquifer's diffusivity is not required with two or more observation wells.

In summary, we are providing a means for estimating or confirming aquifer parameters from an uninterrupted water supply well with one or more observation wells.

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Authors' Note

The author(s) does not have any conflicts of interest

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