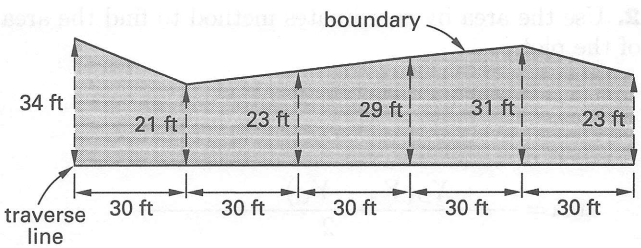
48 Plane Surveying

PRACTICE PROBLEMS

1. Boundary and traverse lines bounding an irregular area are shown.



The total area between the irregular boundary and the traverse line is most nearly

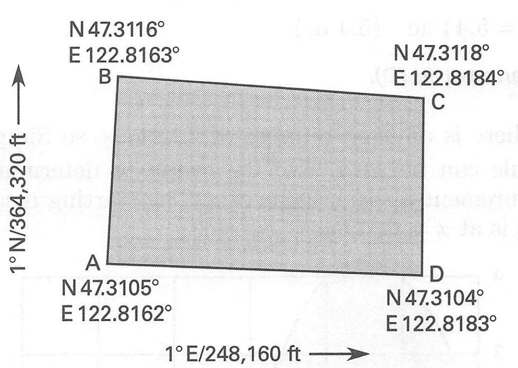
(A) 3600 ft^2

(B) 3800 ft^2

(C) 4000 ft^2

(D) 4200 ft^2

2. Global positioning system (GPS) latitudes and longitudes were taken of a plot of land. In the region where the plot is located, the length of a degree of latitude is 364,320 ft, and the length of a degree of longitude is 248,160 ft.



What is most nearly the area of the plot?

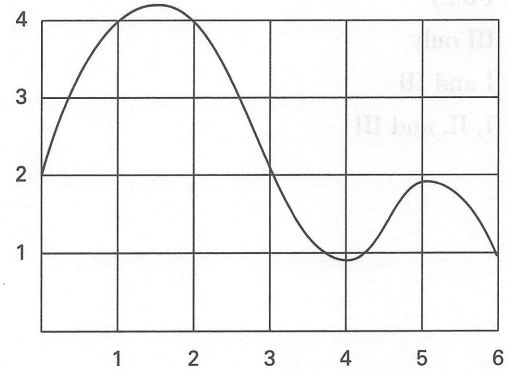
(A) 5.0 ac

(B) 5.1 ac

(C) 5.3 ac

(D) 5.4 ac

3. The illustration shows a curve from $x = 0$ to $x = 6$.



Using intervals of 1, what is most nearly the area under the curve predicted by Simpson's $1/3$ rule?

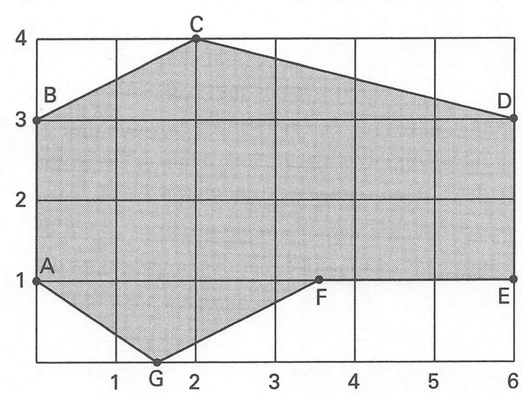
(A) 14

(B) 15

(C) 16

(D) 17

4. A polygon is created by enclosing lines as shown.



Using an interval of 1, what is most nearly the area of the polygon predicted by the trapezoidal rule?

(A) 15

(B) 16

(C) 17

(D) 18

5. Which statement(s) concerning methods used to determine areas under a curve or line is/are true?

I. The trapezoidal rule applies to areas where the irregular sides are curved.

II. Simpson's 1/3 rule only applies to an odd number of segments.

III. The area by coordinates method can be used if the coordinates of the traverse leg end points are known.

(A) I only

(B) III only

(C) I and III

(D) I, II, and III

SOLUTIONS

1. The trapezoidal rule is

$$\begin{aligned}\text{area} &= w\left(\frac{h_1+h_6}{2}+h_2+h_3+h_4+h_5\right)\\ &= (30\text{ ft})\left(\begin{array}{l}\dfrac{34\text{ ft}+23\text{ ft}}{2}+21\text{ ft}\\ \qquad +23\text{ ft}+29\text{ ft}+31\text{ ft}\end{array}\right)\\ &= 3975\text{ ft}^2 \quad (4000\text{ ft}^2)\end{aligned}$$

The answer is (C).

2. Use the area by coordinates method to find the area of the plot.

$$\text{area} = \frac{\begin{array}{l}X_A(Y_B-Y_D)+X_B(Y_C-Y_A)\\ \quad +X_C(Y_D-Y_B)\\ \quad +X_D(Y_A-Y_C)\end{array}}{2}$$

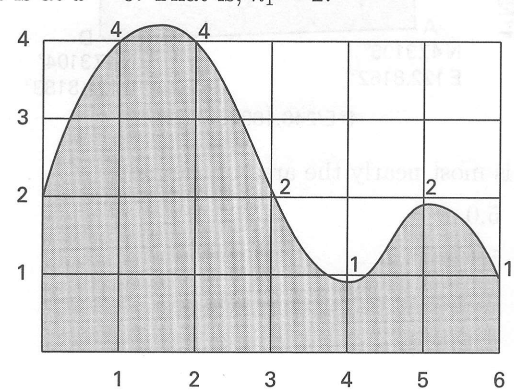
$$\begin{aligned}&= \frac{\begin{array}{l}(122.8162^\circ)(47.3116^\circ-47.3104^\circ)\\ \quad +(122.8163^\circ)(47.3118^\circ-47.3105^\circ)\\ \quad +(122.8184^\circ)(47.3104^\circ-47.3116^\circ)\\ \quad +(122.8183^\circ)(47.3105^\circ-47.3118^\circ)\end{array}}{2}\\ &= -2.62\times10^{-6}\text{ deg}^2\end{aligned}$$

Use the lengths per degree of latitude and longitude to convert the angles to distance.

$$\begin{aligned}\text{area} &= \frac{(-2.62\times10^{-6}\text{ deg}^2)\left(248{,}160\ \frac{\text{ft}}{^\circ\text{E}}\right)\left(364{,}320\ \frac{\text{ft}}{^\circ\text{N}}\right)}{43{,}560\ \frac{\text{ft}^2}{\text{ac}}}\\ &= 5.44\text{ ac} \quad (5.4\text{ ac})\end{aligned}$$

The answer is (D).

3. There is an even number of intervals, so Simpson's 1/3 rule can be used. Use the graph to determine the measurement at each value of x. The starting measurement is at $x = 0$. That is, $h_1 = 2$.



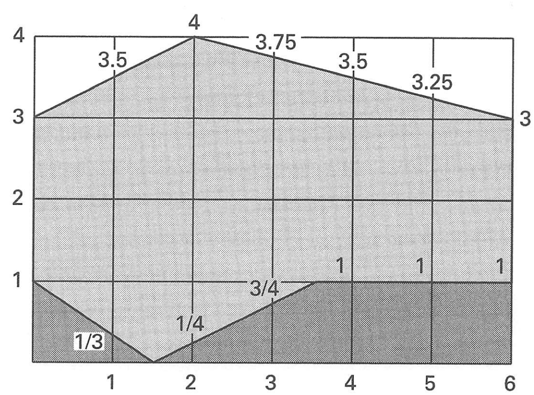
The area under the curve using Simpson's 1/3 rule is

$$\text{area} = \frac{w\left(h_1 + 2\left(\sum\limits_{k=3,5,\ldots}^{n-2} h_k\right) + 4\left(\sum\limits_{k=2,4,\ldots}^{n-1} h_k\right) + h_n\right)}{3}$$
$$= \frac{(1)\left(2 + (2)(4+1) + (4)(4+2+2) + 1\right)}{3}$$
$$= 15$$

The answer is (B).

4. The area of the polygon can be determined by subtracting the area under the bottom line from the area under the top line.

Since the common interval is 1, there will be seven height measurements (i.e., $n = 7$). The height of each trapezoid can be determined as shown in the figure.



The area under the top line is

$$\text{area}_{\text{top}} = w\left(\frac{h_1 + h_7}{2} + h_2 + h_3 + h_4 + h_5 + h_6\right)$$
$$= (1)\left(\frac{3+3}{2} + 3.5 + 4 + 3.75 + 3.5 + 3.25\right)$$
$$= 21$$

The area under the bottom line is

$$\text{area}_{\text{bottom}} = w\left(\frac{h_1 + h_7}{2} + h_2 + h_3 + h_4 + h_5 + h_6\right)$$
$$= (1)\left(\frac{1+1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{3}{4} + 1 + 1\right)$$
$$= 4.333$$

The area of the polygon is

$$\text{area} = \text{area}_{\text{top}} - \text{area}_{\text{bottom}}$$
$$= 21 - 4.333$$
$$= 16.667 \quad (17)$$

The answer is (C).

5. The trapezoidal rule is applicable to areas where the irregular sides are straight or nearly straight. Option I is false. Simpson's 1/3 rule is applicable to areas whose irregular sides are curved, but applies only to an even number of segments (i.e., an odd number of segment end points). Option II is false. The area by coordinates method calculates the traverse area using the coordinates of the leg end points. Option III is true.

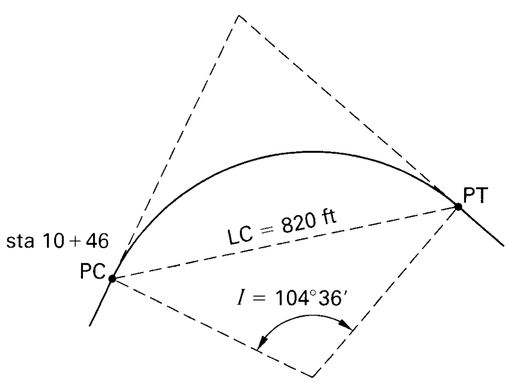
The answer is (B).

Transportation/ Surveying

49 Geometric Design

PRACTICE PROBLEMS

1. A horizontal curve is laid out with the point of curve, PC, station and the length of long chord, LC, as shown.



The radius of the curve is most nearly

(A) 520 ft

(B) 560 ft

(C) 620 ft

(D) 670 ft

2. A freeway route has a horizontal curve with a PI at sta 11+01.86, an intersection angle, I, of 12°24′00″ right, and a radius of 1760 ft. The PC is located at

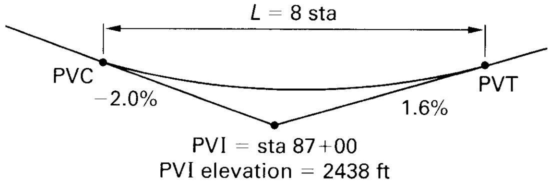
(A) sta 9+10

(B) sta 9+22

(C) sta 10+11

(D) sta 10+24

3. A vertical sag curve has a length of 8 sta and connects a −2.0% grade to a 1.6% vertical grade. The PVI is located at sta 87+00 and has an elevation of 2438 ft.



The elevation of the lowest point on the vertical curve is most nearly

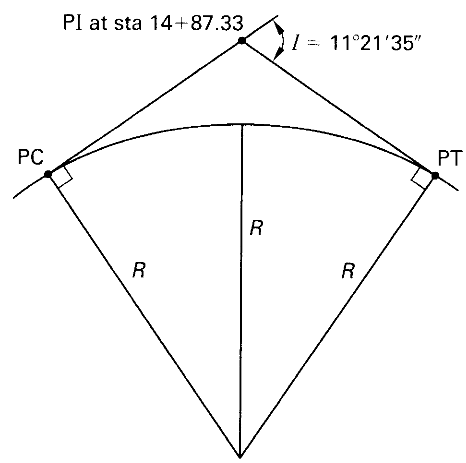
(A) 2420 ft

(B) 2430 ft

(C) 2440 ft

(D) 2450 ft

4. A 6° curve has forward and back tangents that intersect at sta 14+87.33.



The station of the point of curve, PC, is most nearly

(A) sta 5+32

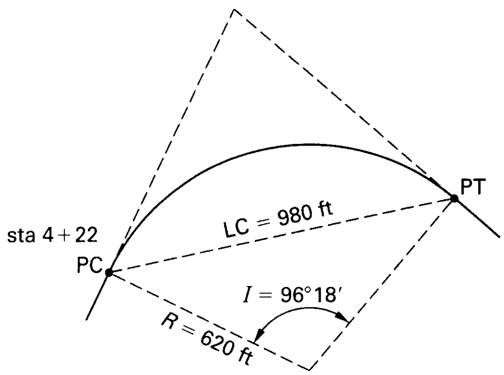
(B) sta 9+93

(C) sta 11+28

(D) sta 13+92

Transportation/ Surveying

5. A horizontal curve is laid out with the point of curve, PC, station and the length of long chord, LC, as shown.



The curve radius, R, is 620 ft. With stationing around the curve, the stationing of the point of tangent, PT, is most near to

(A) sta 6+20

(B) sta 10+42

(C) sta 14+02

(D) sta 14+64

SOLUTIONS

1. The intersection angle, I, in decimal degrees is

$$I = 104^\circ + \frac{36 \text{ min}}{60 \frac{\text{min}}{\text{deg}}} = 104.6^\circ$$

The radius of the curve is

$$\begin{aligned} R &= \frac{\text{LC}}{2 \sin \frac{I}{2}} = \frac{820 \text{ ft}}{2 \sin \frac{104.6^\circ}{2}} \\ &= 518.18 \text{ ft} \quad (520 \text{ ft}) \end{aligned}$$

The answer is (A).

2. Convert the intersection angle to a decimal value.

$$I = 12^\circ + \frac{24 \text{ min}}{60 \frac{\text{min}}{\text{deg}}} = 12.4^\circ$$

The tangent length is

$$\begin{aligned} T &= R \tan \frac{I}{2} \\ &= (1760 \text{ ft}) \tan\left(\frac{12.4^\circ}{2}\right) \\ &= 191.20 \text{ ft} \end{aligned}$$

The PC is located at

$$\begin{aligned} \text{sta PC} &= \text{sta PI} - T \\ &= 1101.86 \text{ ft} - 191.20 \text{ ft} \\ &= 910 \text{ ft} \quad (\text{sta } 9+10) \end{aligned}$$

The answer is (A).

3. The PVI is located at the curve's midpoint. The elevation of the PVC is

$$\begin{aligned} Y_{\text{PVC}} &= Y_{\text{PVI}} + |g_1| \frac{L}{2} \\ &= 2438 \text{ ft} + (0.02)\left(\frac{8 \text{ sta}}{2}\right)\left(100 \ \frac{\text{ft}}{\text{sta}}\right) \\ &= 2446 \text{ ft} \end{aligned}$$

The distance from the PVC to the lowest point on the curve is

$$\begin{aligned} x_m &= \frac{g_1 L}{g_1 - g_2} \\ &= \frac{(-0.02)(8 \text{ sta})\left(100 \ \frac{\text{ft}}{\text{sta}}\right)}{-0.02 - 0.016} \\ &= 444.44 \text{ ft} \end{aligned}$$

Determine the elevation at the lowest point.

$$\begin{aligned} Y &= Y_{\text{PVC}} + g_1 x + \left(\frac{g_2 - g_1}{2L}\right)x^2 \\ &= 2446 \text{ ft} + (-0.02)(444.44 \text{ ft}) \\ &\quad + \left(\frac{0.016 - (-0.02)}{(2)(8 \text{ sta})\left(100 \ \frac{\text{ft}}{\text{sta}}\right)}\right)(444.44 \text{ ft})^2 \\ &= 2441.55 \text{ ft} \quad (2440 \text{ ft}) \end{aligned}$$

The answer is (C).

4. The radius of the curve is

$$\begin{aligned} R &= \frac{5729.58}{D} = \frac{5729.58 \text{ ft-deg}}{6^\circ} \\ &= 954.93 \text{ ft} \end{aligned}$$

Convert the intersection angle to a decimal value.

$$\begin{aligned} I &= 11^\circ + \frac{21 \text{ min}}{60 \ \frac{\text{min}}{\text{deg}}} + \frac{35 \text{ sec}}{\left(60 \ \frac{\text{sec}}{\text{min}}\right)\left(60 \ \frac{\text{min}}{\text{deg}}\right)} \\ &= 11.36^\circ \end{aligned}$$

The tangent length is

$$\begin{aligned} T &= R\tan\frac{I}{2} = (954.93 \text{ ft})\tan\left(\frac{11.36^\circ}{2}\right) \\ &= 94.98 \text{ ft} \end{aligned}$$

The station of the point of curve, PC, is

$$\begin{aligned} \text{sta PC} &= \text{sta PI} - T \\ &= 1487.33 \text{ ft} - 94.98 \text{ ft} \\ &= 1392 \text{ ft} \quad (\text{sta } 13{+}92) \end{aligned}$$

The answer is (D).

5. The intersection angle, I, in decimal degrees is

$$I = 96^\circ + \frac{18 \text{ min}}{60 \ \frac{\text{min}}{\text{deg}}} = 96.3^\circ$$

The length of curve from PC to PT is

$$\begin{aligned} L &= RI\left(\frac{\pi}{180^\circ}\right) \\ &= (620 \text{ ft})(96.3^\circ)\left(\frac{\pi \text{ rad}}{180^\circ}\right) \\ &= 1042.07 \text{ ft} \end{aligned}$$

The stationing of the point of tangent is

$$\begin{aligned} \text{sta PT} &= \text{sta PC} + L \\ &= 422 \text{ ft} + 1042.07 \text{ ft} \\ &= 1464 \text{ ft} \quad (\text{sta } 14{+}64) \end{aligned}$$

The answer is (D).

50 Earthwork

PRACTICE PROBLEMS

1. Earthwork quantities for a section of roadway indicate a transition from fill to cut. The following areas are scaled from the print cross sections. Where there are transitions between cut and fill, the cut and fill roadway cross sections are both triangular in shape.

station	cut (ft^2)	fill (ft^2)
20+00	–	1864.42
20+10.50	–	468.88
20+21.50	154.14	103.66
20+28.45	696.75	–
20+40	2017.37	–

The total volume of fill required for this section of road is most nearly

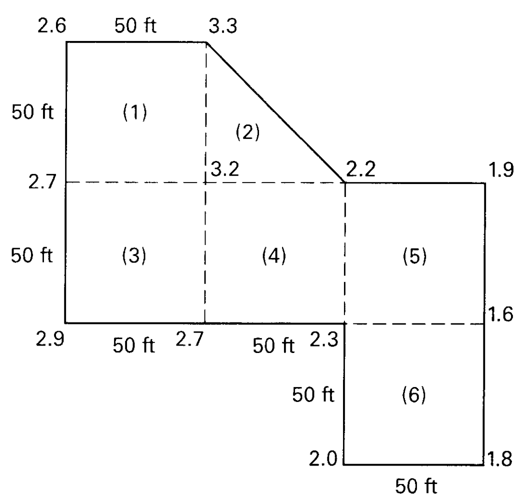
(A) 11,000 ft^3

(B) 16,000 ft^3

(C) 19,000 ft^3

(D) 21,000 ft^3

2. Consider the borrow pit grid shown. Existing excavation depths (in feet) are shown for each corner.



The total undercut volume of this borrow pit is most nearly

(A) 1300 yd^3

(B) 1600 yd^3

(C) 1900 yd^3

(D) 2100 yd^3

3. The table represents the areas of cut and fill at three consecutive roadway points (survey stations) along a rural road project.

point	cut (ft^2)	fill (ft^2)
10+00	1600	270
20+00	0	810
30+00	1100	270

The amount of borrow or waste between the points is most nearly

(A) 27,000 ft^3 borrow

(B) 240,000 ft^3 waste

(C) 250,000 ft^3 borrow

(D) 270,000 ft^3 waste

4. Earthwork quantities for a section of roadway indicate a transition from fill to cut. The following areas are scaled from the print cross sections. Where there are transitions between cut and fill, the cut and fill roadway cross sections are both triangular in shape.

station	cut (ft^2)	fill (ft^2)
10+30	–	126.5
10+60	160.7	50.6
10+82	505.0	–

The total volume of fill required for this section of road is most nearly

(A) 3000 ft^3

(B) 4900 ft^3

(C) 5900 ft^3

(D) 7200 ft^3

SOLUTIONS

1. Earthwork volumes for fill areas and cut areas can be calculated using the average end area formula. Since the cut and fill areas are triangular in shape, earthwork volumes in the transition region from fill to cut can be calculated from the formula that gives the volume of a pyramid.

For sta 20+00 to sta 20+10.50,

$$L = 10.50 \text{ ft} - 0 \text{ ft} = 10.50 \text{ ft}$$

The fill volume is

$$\begin{aligned} V_{\text{fill}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (10.50 \text{ ft})\left(\frac{1864.42 \text{ ft}^2 + 468.88 \text{ ft}^2}{2}\right) \\ &= 12{,}249.83 \text{ ft}^3 \end{aligned}$$

For sta 20+10.50 to sta 20+21.50,

$$L = 21.50 \text{ ft} - 10.50 \text{ ft} = 11.00 \text{ ft}$$

The fill volume is

$$\begin{aligned} V_{\text{fill}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (11.00 \text{ ft})\left(\frac{468.88 \text{ ft}^2 + 103.66 \text{ ft}^2}{2}\right) \\ &= 3148.97 \text{ ft}^3 \end{aligned}$$

Cut is required at sta 20+21.50, but no cut is required at sta 20+10.50, so a cut-to-fill transition occurs between these two points. (The cut area could conceivably decrease to zero at any point before sta 20+21.50, but a reasonable assumption is that data was given at sta 20+10.50 for a reason: that is the point where the cut becomes zero.)

The cut area cross section is triangular at sta 20+21.50, tapering to zero at sta 20+10.50, so the soil mass is essentially a triangular-base pyramid on its side, with the apex at sta 20+10.50. The "height" of this pyramid is 11.00 ft. Calculate the volume of a pyramid of cut.

$$\begin{aligned} V_{\text{cut}} &= h\left(\frac{\text{area of base}}{3}\right) \\ &= (11.00 \text{ ft})\left(\frac{154.14 \text{ ft}^2}{3}\right) \\ &= 565.18 \text{ ft}^3 \end{aligned}$$

For sta 20+21.50 to sta 20+28.45,

$$\begin{aligned} L &= 28.45 \text{ ft} - 21.50 \text{ ft} \\ &= 6.95 \text{ ft} \end{aligned}$$

Fill is required at sta 20+21.50, but no fill is required at sta 20+28.45, so a fill-to-cut transition occurs between these two points. The fill area cross section is triangular at sta 20+21.50, tapering to zero at sta 20+28.45. The "height" of the pyramid is 6.95 ft. Calculate the volume of a pyramid of fill.

$$\begin{aligned} V_{\text{fill}} &= h\left(\frac{\text{area of base}}{3}\right) \\ &= (6.95 \text{ ft})\left(\frac{103.66 \text{ ft}^2}{3}\right) \\ &= 240.15 \text{ ft}^3 \end{aligned}$$

The cut volume is

$$\begin{aligned} V_{\text{cut}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (6.95 \text{ ft})\left(\frac{154.14 \text{ ft}^2 + 696.75 \text{ ft}^2}{2}\right) \\ &= 2956.84 \text{ ft}^3 \end{aligned}$$

For sta 20+28.45 to sta 20+40,

$$\begin{aligned} L &= 40 \text{ ft} - 28.45 \text{ ft} \\ &= 11.55 \text{ ft} \end{aligned}$$

The cut volume is

$$\begin{aligned} V_{\text{cut}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (11.55 \text{ ft})\left(\frac{696.75 \text{ ft}^2 + 2017.37 \text{ ft}^2}{2}\right) \\ &= 15{,}674.04 \text{ ft}^3 \end{aligned}$$

A table that summarizes earthwork volumes is now made.

station	cut area (ft^2)	fill area (ft^2)	cut volume (ft^3)	fill volume (ft^3)
20+00	–	1864.42		
			–	12,249.83
20+10.50	–	468.88		
			565.18	3148.97
20+21.50	154.14	103.66		
			2956.84	240.15
20+28.45	696.75	–		
			15,674.04	–
20+40	2017.37	–		
		total	19,196.06	15,638.95

Therefore, the total volume of fill required for this section of road is 15,638.95 ft^3 (16,000 ft^3).

The answer is (B).

Transportation/ Surveying

2. Calculate the average depth of undercut by summing the undercut depths at each of the corners and dividing the total by the number of corners. Calculate the undercut volume by multiplying the area by the average depth of undercut.

The area of each full cell is

$$(50\text{ ft})(50\text{ ft}) = 2500\text{ ft}^2$$

The area of each triangular half-cell is

$$\frac{2500\text{ ft}^2}{2} = 1250\text{ ft}^2$$

cell number	area (ft^2)	average depth of undercut (ft)	volume (ft^3)
1	2500	2.95	7375
2	1250	2.90	3625
3	2500	2.88	7200
4	2500	2.60	6500
5	2500	2.00	5000
6	2500	1.93	4825
		total	34,525

The total volume is

$$V_{\text{total}} = \frac{34{,}525\text{ ft}^3}{\left(3\ \frac{\text{ft}}{\text{yd}}\right)^3} = 1279\text{ yd}^3 \quad (1300\text{ yd}^3)$$

The answer is (A).

3. Use the average end area method. The points are 10 stations apart. 100 ft stations are used. The cut volumes are

$$\begin{aligned} V_{\text{cut,sta 1 to 2}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (10\text{ sta})\left(100\ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{1600\text{ ft}^2 + 0\text{ ft}^2}{2}\right) \\ &= 800{,}000\text{ ft}^3 \\ V_{\text{cut,sta 2 to 3}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (10\text{ sta})\left(100\ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{0\text{ ft}^2 + 1100\text{ ft}^2}{2}\right) \\ &= 550{,}000\text{ ft}^3 \end{aligned}$$

The fill volumes are

$$\begin{aligned} V_{\text{fill,sta 1 to 2}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (10\text{ sta})\left(100\ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{270\text{ ft}^2 + 810\text{ ft}^2}{2}\right) \\ &= 540{,}000\text{ ft}^3 \\ V_{\text{fill,sta 2 to 3}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (10\text{ sta})\left(100\ \frac{\text{ft}}{\text{sta}}\right)\left(\frac{810\text{ ft}^2 + 270\text{ ft}^2}{2}\right) \\ &= 540{,}000\text{ ft}^3 \end{aligned}$$

Determine the total cut and fill volumes.

$$\begin{aligned} \text{cut:}\quad V &= 800{,}000\text{ ft}^3 + 550{,}000\text{ ft}^3 \\ &= 1{,}350{,}000\text{ ft}^3 \\ \text{fill:}\quad V &= 540{,}000\text{ ft}^3 + 540{,}000\text{ ft}^3 \\ &= 1{,}080{,}000\text{ ft}^3 \end{aligned}$$

There is $1{,}350{,}000\text{ ft}^3 - 1{,}080{,}000\text{ ft}^3 = 270{,}000\text{ ft}^3$ more waste than borrow.

The answer is (D).

4. Earthwork volumes for fill areas and cut areas can be calculated using the average end area formula. Since the cut and fill areas are triangular in shape, earthwork volumes in the transition region from fill to cut can be calculated from the formula that gives the volume of a pyramid.

For sta 10+30 to sta 10+60,

$$L = 60\text{ ft} - 30\text{ ft} = 30\text{ ft}$$

The fill volume is

$$\begin{aligned} V_{\text{fill}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (30\text{ ft})\left(\frac{126.5\text{ ft}^2 + 50.6\text{ ft}^2}{2}\right) \\ &= 2656.5\text{ ft}^3 \end{aligned}$$

Cut is required at sta 10+60, but no cut is required at sta 10+30, so a cut-to-fill transition occurs between these two points. (The cut area could conceivably decrease to zero at a point before sta 10+60, but a reasonable assumption is that data was given at sta 10+30 for a reason: that is the point where the cut becomes zero.)

Transportation/ Surveying

The cut area cross section is triangular at sta 10+60, tapering to zero at sta 10+30, so the soil mass is essentially a triangular-base pyramid on its side, with the apex at sta 10+30. The height of this pyramid is 30 ft. Calculate the volume of a pyramid of cut.

$$\begin{aligned} V_{\text{cut}} &= h\left(\frac{\text{area of base}}{3}\right) \\ &= (30 \text{ ft})\left(\frac{160.7 \text{ ft}^2}{3}\right) \\ &= 1607.0 \text{ ft}^3 \end{aligned}$$

For sta 10+60 to sta 10+82,

$$L = 82 \text{ ft} - 60 \text{ ft} = 22 \text{ ft}$$

Fill is required at sta 10+60, but is not required at sta 10+82, so a fill-to-cut transition occurs between these two points. The fill area cross section is triangular at sta 10+60, tapering to zero at sta 10+82. The height of the pyramid is 22 ft. Calculate the volume of a pyramid of fill.

$$\begin{aligned} V_{\text{fill}} &= h\left(\frac{\text{area of base}}{3}\right) \\ &= (22 \text{ ft})\left(\frac{50.6 \text{ ft}^2}{3}\right) \\ &= 371.1 \text{ ft}^3 \end{aligned}$$

The cut volume is

$$\begin{aligned} V_{\text{cut}} &= L\left(\frac{A_1 + A_2}{2}\right) \\ &= (22 \text{ ft})\left(\frac{160.7 \text{ ft}^2 + 505.0 \text{ ft}^2}{2}\right) \\ &= 7322.7 \text{ ft}^3 \end{aligned}$$

A table that summarizes earthwork volumes is now made.

station	cut area (ft^2)	fill area (ft^2)	cut volume (ft^2)	fill volume (ft^3)
10+30	–	126.5	–	2656.5
10+60	160.7	50.6	1607.0	371.1
10+82	505.0	–	7322.7	–
		total	8929.7	3027.6

Therefore, the total volume of fill required for this section of road is 3027.6 ft^3 (3000 ft^3).

The answer is (A).

Transportation/ Surveying