FE Stats Review

Thursday, October 2, 2025 8:02 PM

Dispersion, mean, median & mode ← pg 63

Statistic	Sample symbol	population symbol	Formula
Mean	$\bar{X}$	μ	$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$
Variance	s^2	σ^2	$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(X_i - \mu)^2$
			$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$
standard deviation	s	σ	

median ($\neq$ mean)

mode : values repeats most often.

Statistics

Dispersion, mean, median, mode…

1. What is the sample variance, median, and mode of the following numbers?

1,3,5,7,9,11,3,3

re-order: 1, 3, 3, 3, 5, 7, 9, 11

median: $(3+5)/2 = 4$

mode: 3

find mean: $\bar{x} = (1+3+3+3+5+7+9+11)/8 = 5.25$

$$s^2 = \frac{1}{8-1}\left[(1-5.25)^2 + (3-5.25)^2 + \cdots + (11-5.25)^2\right]$$

$$= 11.929$$

Permutation ← pg 64

Permutation ← pg 64

$P(n,r) = \frac{n!}{(n-r)!}$ n - distinct object
r - # taken at a time

Example: numbers of different 3-digit pin can be formed using numbers 0-9

□ □ □ $10 \times 9 \times 8 = \boxed{720}$

# of choice: 10 9 8

math expression: $\frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7 \cdots}{\cancel{7 \cdots}}$

Combination : order doesn't matter

$C(n,r) = \frac{P(n,r)}{r!} = \left[\frac{n!}{r!(n-r)!} \right]$

Example: if we choose pin 1-2-3

order matter	order doesn't matter
1 2 3	1-2-3
1 3 2	
2 1 3	
2 3 1	
3 2 1	
3 1 2	

Law of total Probability

$P(A+B) = P(A) + P(B) - P(A,B)$

either A or B or both occur

A & B occur simultaneously

probability

What is the probability that either two heads or three heads will be thrown if six fair coins are tossed at once?

(A) 0.35
(B) 0.55
(C) 0.59
(D) 0.63

$P(2 \text{ heads}) = \frac{\text{total \# of ways 2 heads occur}}{\text{total \# of possible outcomes}}$

$= \frac{C(6,2)}{} = \frac{6!}{} \quad /64 = \frac{15}{}$

(C) 0.59
(D) 0.63

Total # of possible outcomes

$$= \frac{C(6,2)}{2^6} = \frac{6!}{2!(6-2)!}/64 = \frac{15}{64}$$

$$P(3\,heads) = \frac{C(6,3)}{2^6} = \frac{20}{64}$$

here, these are mutally exclusive events

$$P(2\,heads,\ 3\,heads) = 0$$

$$\therefore P(2\,heads + 3\,heads) = P(2\,heads) + P(3\,heads) = \frac{15}{64} + \frac{20}{64} = 0.547$$

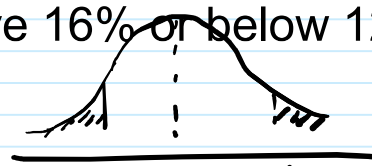
Normal distribution ← pg 67

Given normal distributed observations: μ, σ
Convert to standard normal distribution function.
$\leftarrow \mu = 0, \sigma = 1$ $\qquad z = \frac{x-\mu}{\sigma}$
then use unit normal distribution table ← pg 76

Normal Distribution

1. The water content of soil from a borrow site is normally distributed with a mean of 14.2% and a standard deviation of 2.3%. What is the probability that sample taken from the site will have a water content above 16% or below 12%?

(A) 0.13
(B) 0.25
(C) 0.37
(D) 0.42



$$z_{16\%} = \frac{16\% - 14.2\%}{2.3\%} = 0.78 \rightarrow \text{use } 0.8$$

$$z_{12\%} = \frac{12\% - 14.2\%}{2.3\%} = -0.96 \rightarrow \text{use } -1.0$$

from table $R(0.80) = 0.2119$, $R(1.0) = 0.1587$

$$\therefore P(z < 12\% \text{ or } z > 16\%) = 0.2119 + 0.1587 = 0.3706$$

Binomial Distribution

Binomial Distribution

$$P_n(x) = \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$P_n(x)$ ← P(x success occurs in n trials)

$\frac{n!}{x!(n-x)!}$ ← C(n,x)

p ← P(success)

q ← P(failure)

Binomial distribution

1. What is the approximate probability of exactly two people in a group of seven having a birthday on April 15?

(A) 1.2×10^{-18}
(B) 2.4×10^{-17}
(C) 7.4×10^{-6}
(D) 1.6×10^{-4}

sample size $n = 7$

$x = 2$

$p = \frac{1}{365}$, $q = 1 - p = \frac{364}{365}$

$$P = \frac{7!}{2!(7-2)!} \left(\frac{1}{365}\right)^2 \left(\frac{364}{365}\right)^{7-2} = 1.555 \times 10^{-4}$$

Confidence interval ← pg 74

(A) σ is known (population, $n > 30$), use table on pg 75

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

(B) σ is not known (sample), use table on pg 77

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}}$$

α = significance level = 1 - confidence interval

$t_{\alpha/2}$ corresponds to $n-1$ degrees of freedom

Confidence Intervals

1. You collect 10 observations from an experiment. The sample average is 14.0, and the standard deviation is 5.8. The 90% confidence interval on the mean is:

(A) 11.57 < μ <16.43
(B) 10.64 < μ < 17.36
(C) 8.2 < μ < 19.8
(D) 8.78 < μ < 19.22

$n < 30$, σ unknown, $s = 5.8$, $\bar{x} = 14.0$

$\alpha = 1 - 0.9 = 0.1$, $\alpha/2 = 0.05$

conf. int.

dof = 10 - 1 = 9

$t_{0.05,9} = 1.833$ ← table on pg 77

$t_{0.05,9} \frac{s}{\sqrt{n}} = 1.833 \times \frac{5.8}{\sqrt{10}} = 3.3619$

$14 - 3.3619 \leq \mu \leq 14 + 3.3619$

$10.638 \leq \mu \leq 17.3619$

Expected Value (Discrete) ← pg 65

$X = x_1, x_2, \ldots, x_n$

Probability mass function

$f(x_k) = P(X = x_k)$, $k = 1, 2, \ldots, n$

Expected value of X:

$E[X] = \sum_{k=1}^{n} x_k f(x_k)$

Example: civil practice exam #16

Linear Regression ← manual pg 69