UNIVERSITY OF HOUSTON
Department of Civil and Environmental Engineering

CIVE 2330 Engineering Mechanics I (Statics)

Fall 1999

1999 Catalog Data:	Composition and resolution of forces, free-body diagrams, analysis of forces acting on structures and machines, friction, centroids, and moments of inertia.
Instructor:	Theodore G. Cleveland, Ph.D. Associate Professor, Civil and Environmental Engineering
Required Text:	Engineering Mechanica – Statics. Fourth Edition. J.L. Meriam and L.G. Kraige. J. Wiley & Sons, New York 1997

Prerequisites by Topic:

1. Algebra
2. Calculus (Differentiation, Integration)
3. Physics

Course Objectives[1]:

Objective 1: Teach students the principles of equilibrium mechanics (1 and 5).

Objective 2: Teach the practice of mechanical analysis, including effective communication of ideas by legible sketching and the application of mathematics (2).

Topics:

1. Forces in a Plane; Particle Equilibrium; Forces in Space; Rigid Bodies (5 classes).
2. Moments; Couples; Reduction of Force Systems; 2D Equilibrium (4 classes).
3. 3D Equilibrium; Centroids: Lines, Areas, Volumes; Fluid Statics (5 classes).
4. Structures; Method of Joints;, Method of Sections; Frames and Machines (5 classes).
5. Beams: Internal Forces; Shear and Bending Moment; Cables (4 classes)
6. Friction; Wedges and Screws; Bearings (3 classes)
7. Moments of Inertia; Area Moment, Mass Moment (3 classes)

---

[1] Numbers in parenthesis refers to the Department of Civil and Environmental Engineering educational objectives.

Assessment Tools:

1. Exercises.
2. Examinations.
3. Portfolio.

Performance Criteria[2]:

Objective 1.

1.1 Students will demonstrate knowledge of the principles of equilibrium mechanics (a)

Objective 2.

2.1 Students will demonstrate an ability to formulate and solve an equilibrium mechanics problem (e,k)

2.2 Students will demonstrate an ability to effectively communicate and engineering problem and solution to a technical audience (g).

---

[2] Letters in parenthesis refer to ABET EC 2000 outcomes and assessment items (Criterion 3).

CIVE 2330
Engineering Mechanics I
Statics

Fall 1999

1999 Catalog Data: Composition and resolution of forces, free-body diagrams, analysis of forces acting on structures and machines, friction, centroids, and moments of inertia.

Required Text: Engineering Mechanics – Volume 1: Statics. Fourth Edition. J.L. Meriam and L.G. Kraige. Wiley and Sons, Inc. New York, 1997. 524p.

The minimum prerequisites for this course are college algebra, calculus through analytical geometry (concurrent OK) and physics up to electricity and magnetism.

Course Objectives:

1. Teach students the principles of equilibrium mechanics.
2. Teach the practice of mechanical analysis, including effective communication of ideas by legible sketching and application of mathematical formulas.

Schedule:

	Date	Subject	Exercises	Reference
1	8-23	Introduction & Review	1.1,1.4,1.10	M&K pp 1-16
2	8-25	Computational Methods		M&K pp 493-508
3	8-30	Vectors and Forces	2.5,2.14,2.19,2.27	M&K pp 19-34;495-498
4	9-1	Vectors and Moments	2.31,2.33,2.37,2.38	M&K pp 34-45;496-498
5	9-8	Couples	2.40,2.41,2.47,2.52	M&K pp 46-53
6	9-13	Resultants	2.54,2.58,2.61,2.66	M&K pp 54-70
7	9-15	3D Force Systems	2.72,2.75,2.76,2.83	M&K pp 71-83
8	9-20	3D Moments and Couples	2.92,2.94,2.98,2.103	M&K pp 84-101
9	9-22	Review Chapters 1-2	2.113,2.125,2.138,2.142	
10	9-27	Exam 1		
11	9-29	Free Body Diagrams	3.1,3.3,3.8,3.11	M&K pp 103-115
12	10-4	2D Equilibrium	3.24,3.25,3.40,3.54,3.55	M&K pp 116-141
13	10-6	3D Equilibrium	3.61,3.73,3.104,3.109	M&K pp 142-173
14	10-11	Method of Joints	4.4,4.6,4.9,4.15,4.26	M&K pp 175-190
15	10-13	Method of Sections	4.30,4.32,4.35,4.37,4.46	M&K pp 191-200
16	10-18	Space Trusses	4.53,4.54,4.59,4.63	M&K pp 201-207
17	10-20	Frames and Machines	4.67,4.74,4.77,4.94,4.95	M&K pp 208-238
18	10-25	Review	4.104,4.108,4.117,4.118	
19	10-27	Exam 2		
20	11-1	Centroids	5.1,5.5,5.7,5.8,5.13,5.16	M&K pp 241-262
21	11-3	Composite Bodies	5.46,5.51,5.74,5.92	M&K pp 262-279
22	11-8	Beams-Reactions	5.93,5.96,5.102,5.107	M&K pp 280-286
23	11-10	Beams-Shear;Bending	5.113,5.116,5.121,5.131	M&K pp 287-298
24	11-15	Cables	5.141,5.143,5.144,5.148	M&K pp 299-313
25	11-17	Fluid Statics	5.161,5.162,5.165	
26	11-22	Fluid Statics	5.167,5.171,5.177,5.190	M&K pp 314-333
27	11-24	Friction	6.1,6.4,6.5,6.8,6.23	M&K pp 345-368
28	11-29	Wedges	6.50,6.53,6.55,6.64	M&K pp 369-379
29	12-1	Bearings	6.68,6.86,6.93,6.101	M&K pp 380-406
30	12-15	Area Moment of Inertia	A.1;A.14;A.29;A.36	M&K pp 455-490
		Final Exam **2:00-5:00pm**		

Assessment Tools:

Exercises:

Exercises to reinforce lecture material will be assigned from the problem sets in the textbook. Solutions to the exercises are located in a binder at the department. Exercises are self-assessment tools. Exercises will be collected at the beginning of each class meeting and returned the next meeting. The exercises will be spot-checked. The teaching assistant will hold office hours during which the students can ask questions about the exercises.

Examinations:

Three in-class examinations will be assigned. The examinations are shown on the course schedule. The examinations will be comprised of material from exercise problems. Examination grades comprise a significant portion of your grade.

Portfolio:

The student will maintain a portfolio of their self-assessment exercise papers and graded exams. A copy of the portfolio will be handed in at the final examination period. It will not be returned. Failure to hand in the portfolio will result in a reduced course grade. The portfolio should be bound with a cover sheet showing the student's name, semester, and course number. The preferred binding is spiral bound, but a three-hole folio is acceptable.

Performance Criteria:

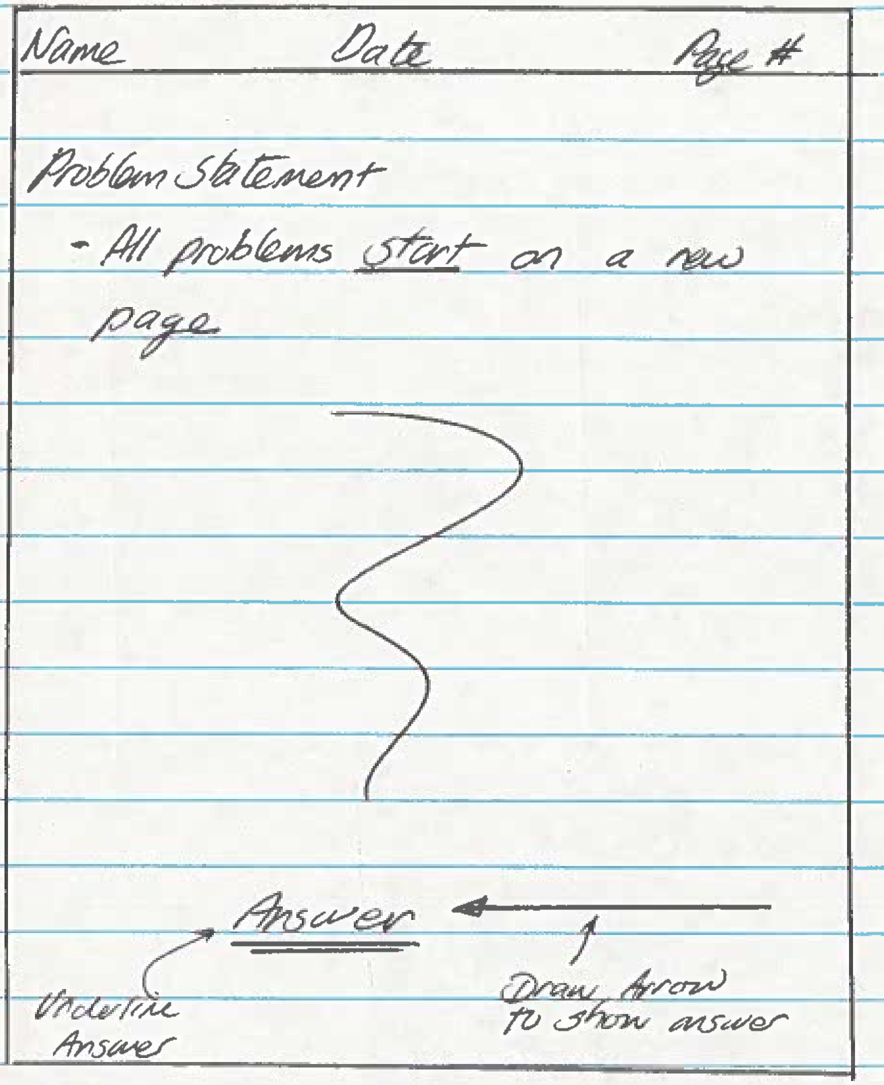
Students will be evaluated using three in-class examinations. Students must completely solve at least 4 examination problems to receive a passing grade. The examinations will be comprised of exercise problems and material similar to the exercise problems.

A complete solution must have the following attributes:

- Problem statement listing the known and required quantities.
- Clear sketch and free-body diagram showing all required forces, directions, and coordinate systems.
- Correct selection of the fundamental principles of mechanics for the problem.
- Correct algebraic and arithmetic reduction of the problem to the required quantity.
- Correct numerical or algebraic answer with correct units.

Partial credit will be awarded for correct components of an incorrect problem.

Exercise Format



- All problems start on a new page

Introduction

Mechanics – The study of the state of rest or (pg 2)
motion of bodies under the action
of forces.

Rigid body, deformable body, fluid mechanics

Rigid body: Statics; bodies at zero acceleration;
force equilibrium
Dynamics; bodies at non-zero acceleration;
unbalanced force systems.

Basic Concepts (pg 3)

Space; geometric region occupied by the
bodies of interest
Time; Measure of succession of events
Mass; Amount of matter contained in a body;
measure of resistance of a body to a
change in velocity.
Force; Action of one body upon another; defined
by magnitude, direction, and point of
application
Moment; Measure of tendency of a force to
rotate a body about an axis.

✓

Idealizations

Particle: point mass; negligible dimension

Rigid body: a body occupying finite space, whose relative movements between its parts are negligible for the problem at hand.

Scalar: magnitude alone

Vector: magnitude and direction

free vector: action not confined to a unique point in space

fixed vector: line of action and point of application is unique.

Newton's Laws of Motion (pg 4)

1. A body remains at rest or continues to move in a straight line with uniform velocity if all forces on the body are balanced.

2. The acceleration of a particle is proportional to the resultant force acting on it and takes place in the direction of that force

$$\underline{R} = \Sigma \underline{F} = m\underline{a}$$

3. The forces of action and reaction between two bodies are equal in magnitude, opposite in direction, and collinear.

Gravitation : The mutual attraction between any two bodies in a gravitational field has magnitude

$$F = \frac{G\, m_1 m_2}{r^2} \quad ; \quad G = 6.673 \cdot 10^{-11} \frac{m^3}{kg\, sec^2}$$

The gravitational attraction of earth on a body is called weight

When a body is near the surface of the earth

$$W = mg$$

where m is the mass of the body and g is the gravitational acceleration constant.

Weight has dimensions of a force

Fundamental Quantities

The three fundamental quantities in mechanics are mass (m), length (L), and time (T).

All other quantities may be expressed in terms of these fundamentall quantities

eg: force = mass . acceleration = $m \frac{L}{T^2}$

Two systems of units are in common use; SI units and U.S. Units.

Use consistent units in a problem

A dimension is a particular quantity

e.g. length, force, time etc.

The amount or magnitude of that quantity is a unit.

eg. meters, Newtons, seconds.

Unit conversions can be made by remembering two base conversions

1 meter = 3.28 ft. 1 kg = 2.204 lbm

Problem Solving (pg 12)

- Problem statement - clear and precise statement. It should state known information and required information
- Free body diagram - legible sketch showing all quantities involved, indicating forces and a coordinate system.
- Fundamental principles of mechanics are used to write relevant equations
- Algebra is used to solve the equations for the unknown quantity

- Accuracy and precision: Numerical answers must be expressed only to the precision justified by the given quantities.

Force Systems

Force is a vector quantity. Its effect on an object depends on its magnitude, direction, and point of application

The action of a force can be separated into two effects: external and internal

External effects of a force on a body can be applied or reactive forces.

Internal effects are internal stresses and strains in the material

Force Classification

Body forces – applied by remote action (gravitational, electromagnetic)

Contact forces – generated through physical contact

Forces may be concentrated or distributed.

Weight of a body is a body force distributed over its volume, it may be replaced by a concentrated force acting through the center of gravity.

Newton's third law states that the action of a force is always accompanied by an equal and opposite reaction

An object has a mass of 1.00 kg. What is its weight?

$$W = mg = (1.00\,kg)(9.81\,m/s^2)$$

$$= 9.81\,kg\,m/s^2 = 9.8\,N$$

What is the weight of this object expressed in pounds-mass?

$$1\,kg = 2.204\,lbm$$

$$\therefore \quad (1.00\,kg)\underbrace{\left(\frac{2.204\,lbm}{1.00\,kg}\right)}_{A} = 2.2\,lbm$$

The quantity "A" is called a unit conversion factor.

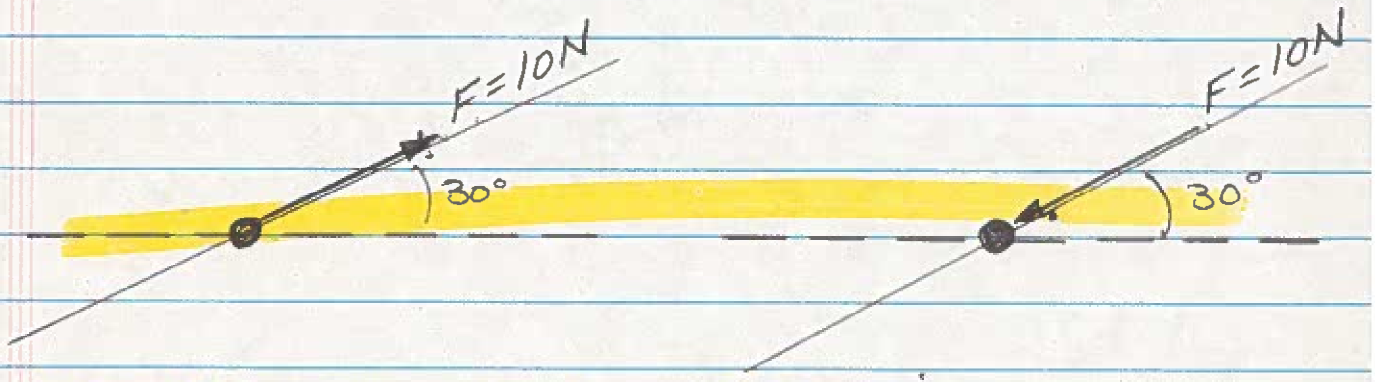
Forces in a Plane (p16)

A force represents the action of one body on another, and is characterized by point of application, magnitude, and direction

Point of application is the location on the body where the force is applied.

Magnitude is the amount, or number of units of force. eg: 1000N

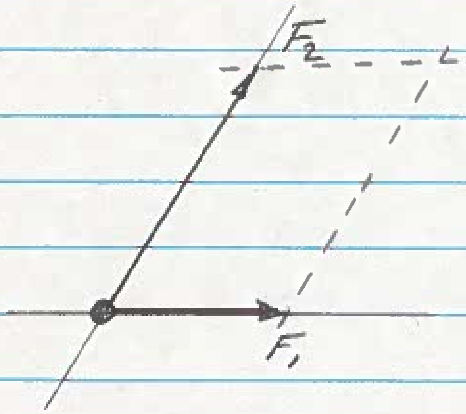
Direction is the line of action and sense of the force

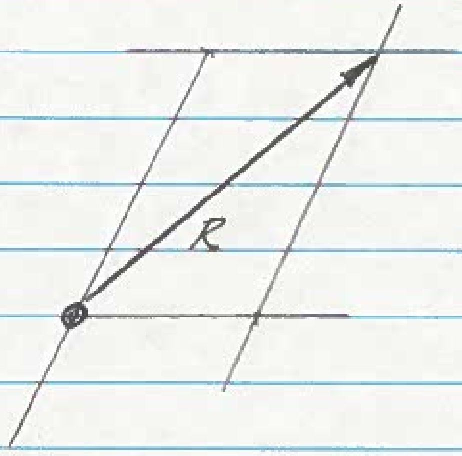


Two forces; same point of application; same line of action; opposite sense.

Resultant (p17)

Experience shows that two forces F_1 and F_2 acting on the same body, can be replaced by a single force, R that has same effect on the body.



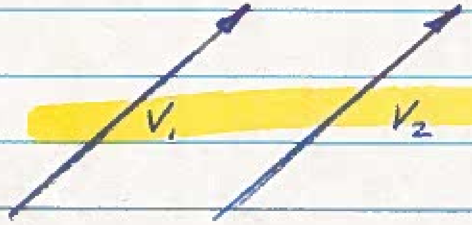


Vector is a mathematical quantity that possesses magnitude and direction, and obeys the parallelogram law of addition

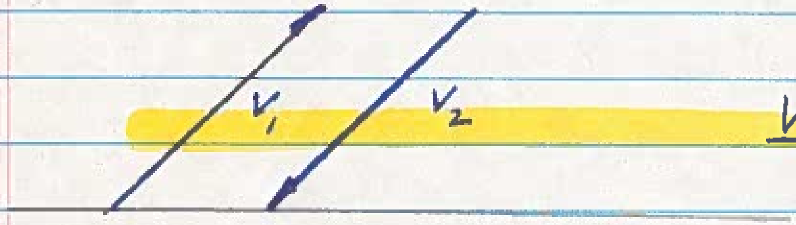
Vectors

Two vectors with same magnitude and direction are equal

Two vectors with same magnitude and line of action and opposite sense are additive (inverses) of each other



$\underline{V}_1 = \underline{V}_2$



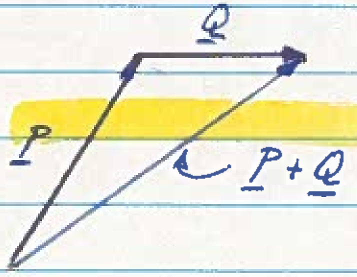
$\underline{V}_1 = -(\underline{V}_2)$

$\therefore \underline{V}_2 = -\underline{V}_1$

Vector Addition

$\underline{P} + \underline{Q} = \underline{Q} + \underline{P}$

but $P + Q \neq \text{magnitude}\,(\underline{P} + \underline{Q})$



Vector addition is

commutative $\underline{P}+\underline{Q} = \underline{Q}+\underline{P}$

associative $(\underline{P}+\underline{Q})+\underline{S} = \underline{P}+(\underline{Q}+\underline{S})$

Product of scalar & Vector

A scalar "scales" the magnitude of a vector.

∴ $2\underline{P}$ has magnitude twice as large as $\underline{P}$ but same direction and point of application.

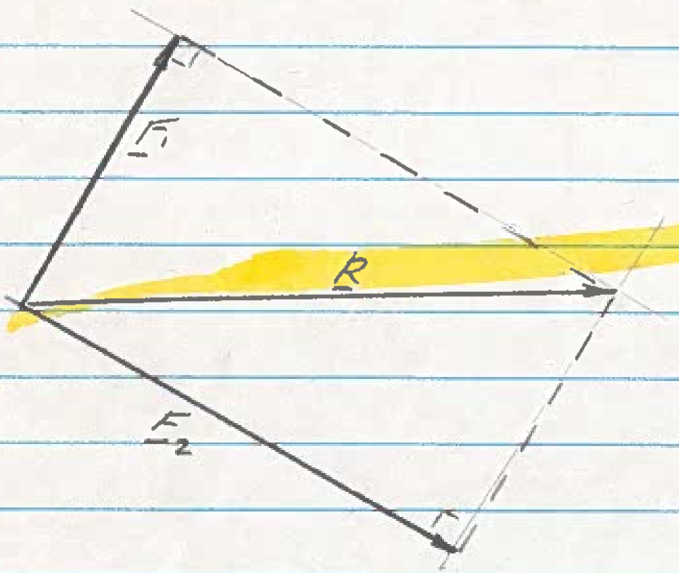
$(-1)\underline{P}$ changes sense of the vector.

Vector components

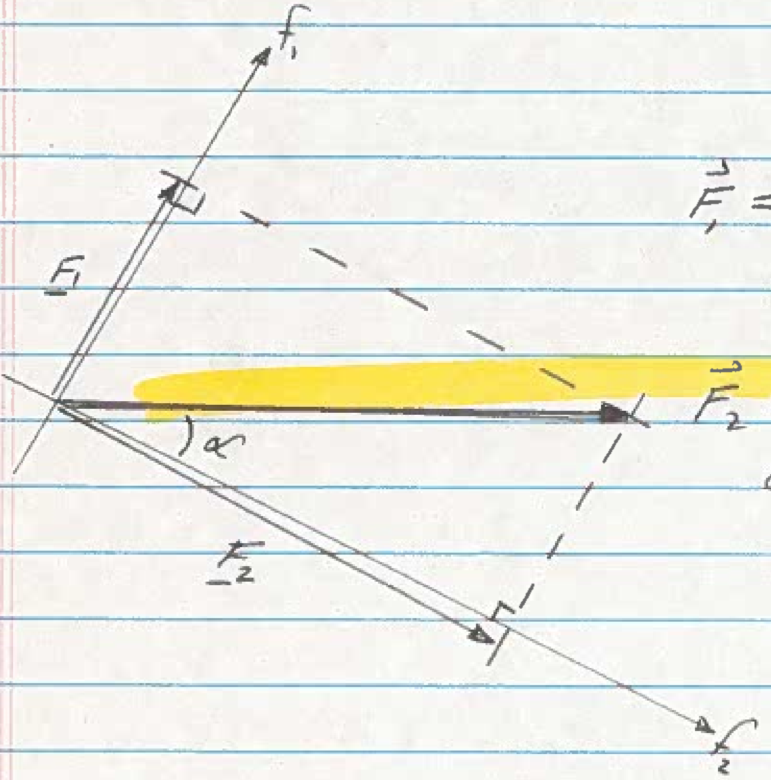
Vectors (and forces) are much easier to manipulate when expressed as components in certain directions.

This process is called resolving a force into its vector components in the principal directions.

p21.



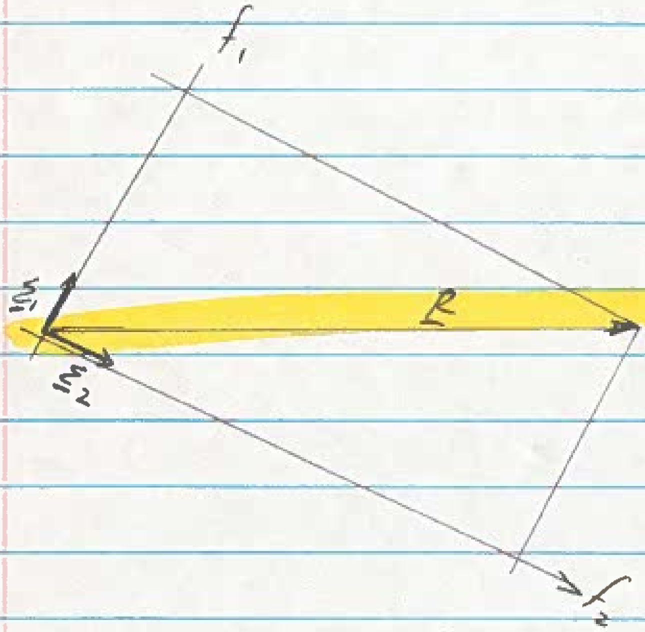
$\underline{R} = \underline{F}_1 + \underline{F}_2$



$\vec{F}_1$ = component of $\vec{R}$ along f_1 direction

$\vec{F}_2$ = component of $\vec{R}$ along f_2 direction

Unit vectors are vectors in a specified direction with magnitude one (unity).



$\underline{F}_1 = F_1 \underline{\varepsilon}_1$

$\underline{F}_2 = F_2 \underline{\varepsilon}_2$

$$\underline{R} = \underline{F}_1 + \underline{F}_2 = F_1 \underline{\varepsilon}_1 + F_2 \underline{\varepsilon}_2$$

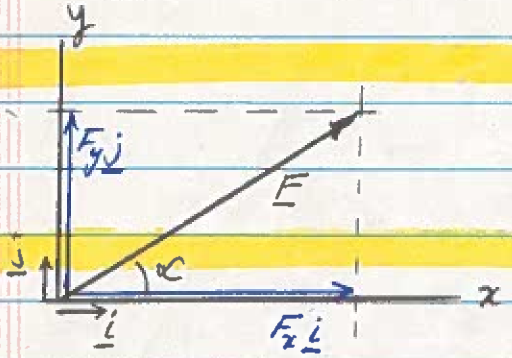
Unit vectors are useful for resolving forces into arbitrary directions:

$$F_1 = \frac{\underline{R} \cdot \underline{e}_1 - (\underline{\varepsilon}_1 \cdot \underline{\varepsilon}_2)\, \underline{R} \cdot \underline{e}_2}{1 - (\underline{\varepsilon}_1 \cdot \underline{\varepsilon}_2)^2}$$

The unit vectors in the x, y, z directions in a cartesian coordinate system are named

$\underline{i}$, $\underline{j}$, and $\underline{k}$

Rectangular Components (p27)



F_x = magnitude of $\underline{F}$ along the x-direction

F_y = magnitude of $\underline{F}$ along the y-direction

By vector addition laws:

$$\underline{F} = F_x \underline{i} + F_y \underline{j}$$

Suppose the magnitude F is known.

Then $F_x = F\cos\alpha$

$F_y = F\sin\alpha$

So $\underline{F} = F\cos\alpha\, \underline{i} + F\sin\alpha\, \underline{j}$

cosα and sinα are called "direction cosines"

By the Pythagorean theorem we can find the magnitude of any vector expressed in rectangular components

$$F = (F_x^2 + F_y^2)^{1/2}$$

(p 30)

Forces in rectangular components are summed by addition of individual components

$$\underline{R} = \underline{F}_1 + \underline{F}_2$$

$$= F_{1x}\underline{i} + F_{1y}\underline{j} + F_{2x}\underline{i} + F_{2y}j = R_x\underline{i} + R_y\underline{j}$$

$\therefore$

$$R_x = F_{1x} + F_{2x}$$

$$R_y = F_{1y} + F_{2y}$$

Particle Equilibrium (p35)

When the resultant of all the forces on a particle is zero, the particle is in equilibrium and will not accelerate

$$\underline{R} = \boxed{\Sigma \underline{F} = 0}$$ Fundamental Equation #1 of statics.

In rectangular components:

$$\Sigma F_x = 0 \;;\; \Sigma F_y = 0 \;;\; \Sigma F_z = 0$$

In terms of Newton's First Law

$$\Sigma \underline{F} = m\cancelto{=0}{\underline{a}}$$

Statics is a special case of Newton's first law:

If the resultant force acting on a particle is zero, the particle will remain at rest or will move with constant speed in a straight line

Free Body Diagrams

A sketch showing physical conditions of a problem is called a space diagram

When the problem can be reduced to a problem concerning a single particle, a separate diagram (showing this particle and all forces acting on it is called the free body diagram

To analyze a mechanical system under equilibrium conditions each component of the system must be defined and then isolated from its surroundings, showing all forces acting on the component.

A component may be a particle, single rigid body, single deformable body, or a combination of connected bodies.

The forces acting on each system component are shown using a free-body diagrams. All forces from mechanical contact and those from magnetic or gravitational attraction are to be shown.

Steps for FBDs

- determine which system components to isolate
- draw a clear diagram that represents its complete external boundary
- all forces acting <u>on</u> the isolated body from contact or attraction are represented as vectors in their proper positions on the diagram.
- coordinate axes are shown on the diagram.

The most difficult part of a FBD is determining <u>support reactions</u>. All forces <u>on</u> the body act in a direction opposite to that in which the body would move if the support were removed.

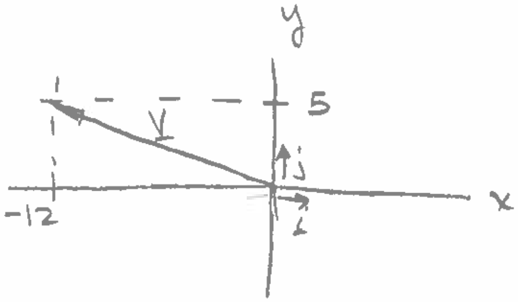
Problem Solving

1. Show all work except basic arithmetic
2. State all assumptions
3. Draw clearly all appropriate figures
4. Isolate the body to be considered and draw a complete FBD. Label on the diagram all external forces, both known and unknown
5. Identify and state applicable force principles for the problem
6. Show all equations in algebraic form prior to substituting numerically.
7. Indicate on FBD which directions and angles are positive. Always use a right-hand coordinate system whenever vector cross products are used
8. Match the number of independent equations with the number of unknowns
9. Carry-out the numerical solution of the equations. Show proper number of significant figures. Include units.

1.1

GIVEN: $\underline{V} = -12\underline{i} + 5\underline{j}$

FIND: ANGLE θ $\underline{V}$ FORMS WITH $+x$ AXIS; $\underline{n}$ UNIT VECTOR ALONG $\underline{V}$



FROM VECTOR ALGEBRA

$$\underline{P}\cdot\underline{Q} = |P||Q|\cos\theta$$

$$\therefore \frac{\underline{V}\cdot\underline{i}}{(\underline{V}\cdot\underline{V})^{1/2}} = \cos\theta$$

$$\underline{V}\cdot\underline{i} = -12$$

$$\underline{V}\cdot\underline{V} = 12^2 + 5^2 = 144 + 25 = 169$$

$$\underline{V}\cdot\underline{V}^{1/2} = \sqrt{169} = 13$$

$$\therefore \cos\theta = \frac{-12}{13}$$

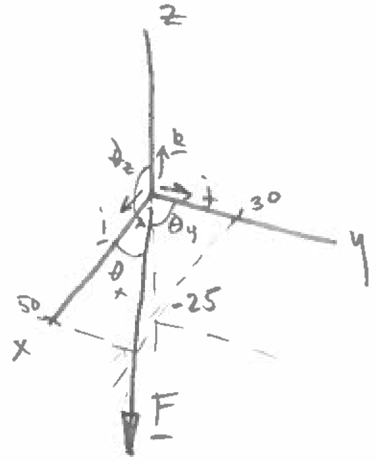
$$\theta = \cos^{-1}\left(-\frac{12}{13}\right) = \underline{157.4^\circ} \qquad \longleftarrow \theta$$

$$\underline{n} = \frac{\underline{V}}{(\underline{V}\cdot\underline{V})^{1/2}} = \frac{-12\underline{i} + 5\underline{j}}{13} = -\frac{12}{13}\underline{i} + \frac{5}{13}\underline{j}$$

$$= \underline{-0.92\underline{i} + 0.38\underline{j}} \qquad \longleftarrow \underline{n}$$

1.4 GIVEN: $\underline{F} = 50\underline{i} + 30\underline{j} - 25\underline{k}$

FIND: $\theta_x, \theta_y, \theta_z$



$F_x = F\cos\theta_x$

$F_y = F\cos\theta_y$

$F_z = F\cos\theta_z$

$$\cos\theta_x = \frac{F_x}{F} = \frac{50}{\sqrt{50^2+30^2+25^2}} = \frac{50\,N}{63.4\,N} = 0.778$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{30}{63.4N} = 0.473$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-25}{63.4} = -0.394$$

$\theta_x = \cos^{-1}(0.778) = \underline{38°}$ $\longleftarrow$ θ_x

$\theta_y = \cos^{-1}(0.473) = \underline{61.8°}$ $\longleftarrow$ θ_y

$\theta_z = \cos^{-1}(-0.394) = \underline{113.2°}$ $\longleftarrow$ θ_z

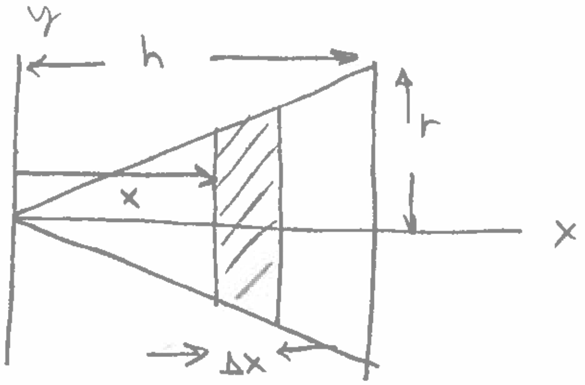
1.10

GIVEN: CONE

FIND: EXPRESSION FOR ΔV

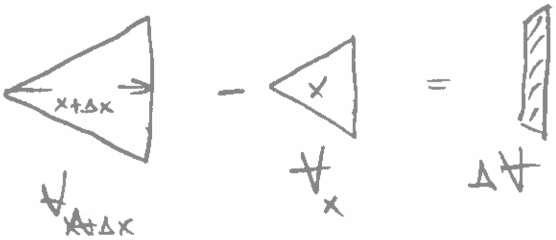
$$y(x) = \frac{r}{h}x$$

$$y(x+\Delta x) = \frac{r}{h}(x+\Delta x)$$



$$\forall = \frac{1}{3}\pi r^2 h$$

$\Delta \forall =$



$$= \frac{1}{3}\pi \frac{r^2}{h^2}(x+\Delta x)^2(x+\Delta x) - \frac{1}{3}\pi\frac{r^2}{h^2}x^3$$

$$= \frac{1}{\cancel{3}}\frac{\pi r^2}{h^2}\left[(\cancel{x^3} + \cancel{3}x^2\Delta x + \cancel{3}x\Delta x^2 + \frac{\Delta x^3}{3}) - (\cancel{x^3})\right.$$

$$\Delta V = \frac{\pi r^2}{h^2}\left(x^2\Delta x + \underbrace{x\Delta x^2 + \frac{\Delta x^3}{3}}_{\text{negligible when } \Delta x \to 0}\right)$$

$$\underset{\Delta x \to \delta x}{\Delta V \longrightarrow} \frac{\pi r^2}{h^2}x^2\Delta x$$

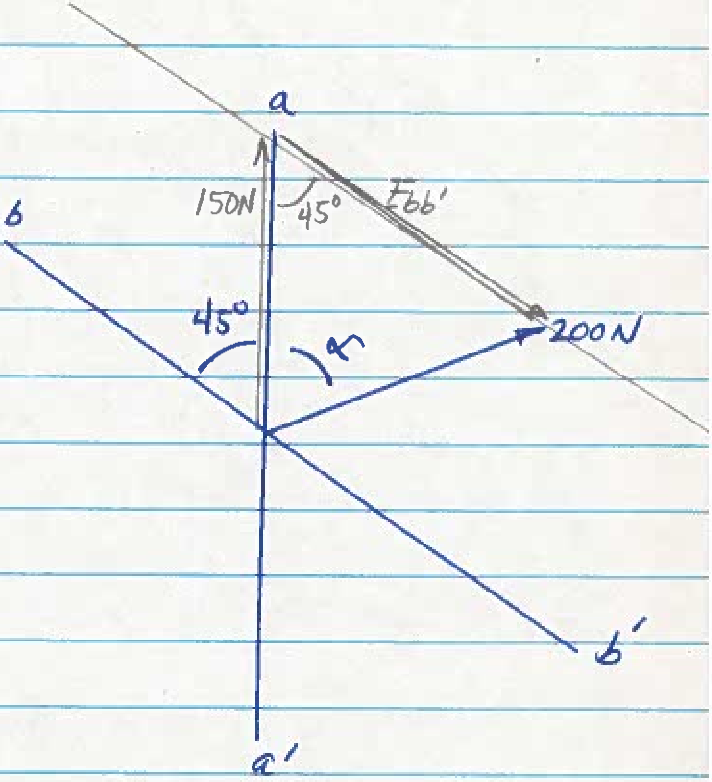
2.5 <u>Given</u>

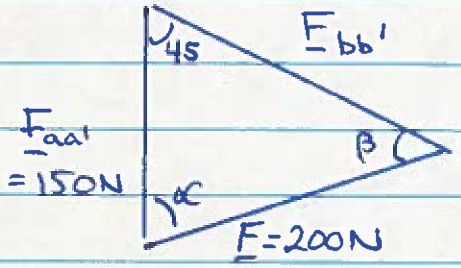
Component of 200N force along a-a' is 150N

<u>Find</u>

a) α

b) Component along b-b'





Law of Sines

$$\frac{\sin 45}{200N} = \frac{\sin\beta}{150N} = \frac{\sin\alpha}{F_{bb'}}$$

Solve for β.

$$\frac{\sin 45 \cdot 150N}{200N} = 0.53 = \sin\beta$$

$\therefore$

$$\beta = \sin^{-1}(0.53) = 32.1^\circ$$

Angles of a triangle sum to 180°

$\therefore \alpha = 180 - 45 - 32.1 = 102.9 = 103^\circ$

$$\underline{F_{bb'}} = \frac{\sin(103)}{\sin(45)} 200N = 275.6 = 276N$$

$\underline{\alpha = 103^\circ}$ ←

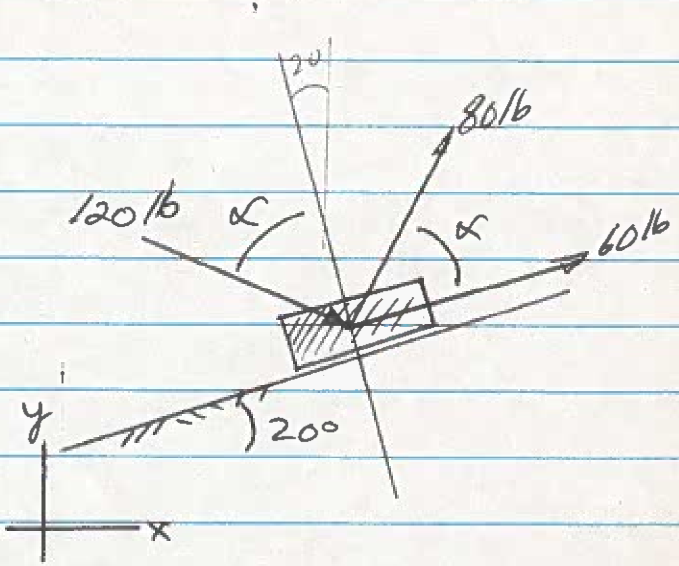
$\underline{F_{bb'} = 276N}$ ←

2.38 Given

$\alpha = 75°$; Forces as shown

Find

Resultant of the forces



Ⓐ Express all forces in rectangular coordinates

60 lb

$F_x = 60 \cos 20 = 56.4 \text{ lb}$

$F_y = 60 \sin 20 = 20.5 \text{ lb}$

80 lb

$F_x = 80 \cos(20+\alpha) = 80 \cos 95 = -6.97 \text{ lb}$

$F_y = 80 \sin(20+\alpha) = 80 \sin 95 = 79.69 \text{ lb}$

120 lb

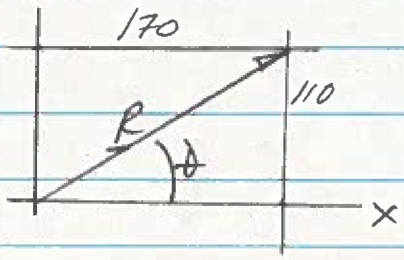
$F_x = 120 \sin(20+\alpha) = 120 \sin 95 = 119.54 \text{ lb}$

$F_y = -120 \cos(20+\alpha) = -120 \cos 95 = 10.46 \text{ lb}$

Ⓑ Resultant is sum of all forces

$R_x = 56.4 - 6.97 + 119.54 = 168.9 \text{ lb}$

$R_y = 20.5 + 79.69 + 10.46 = 110.6 \text{ lb}$



$\underline{R} = 168.9 \text{ lb}\, \underline{i} + 110.6 \text{ lb}\, \underline{j}$

$R = (168.9^2 + 110.6^2)^{1/2} = 201.9 \text{ lb}$

$\theta = \tan^{-1}\left(\frac{110.6}{169}\right) = 33.2°$

2.58 <u>Given</u>

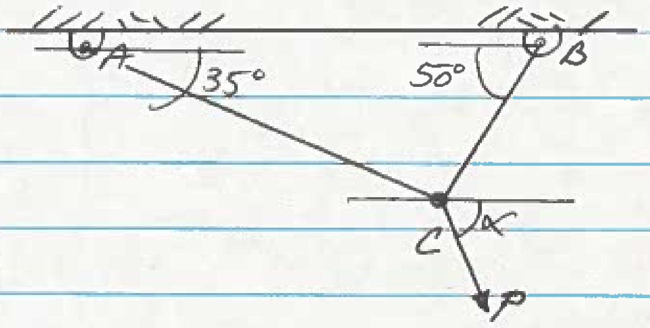
Max allowable $T = 1200N$ in AC

<u>Max</u> allowalbe $T = 600N$ in BC

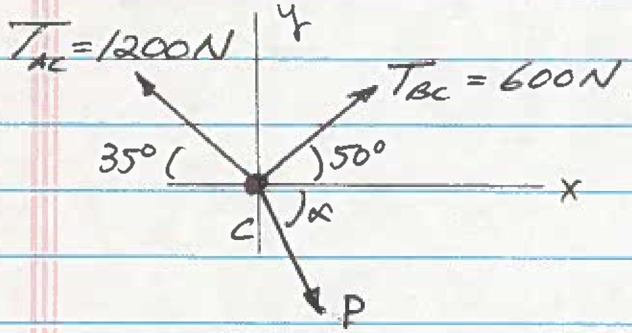
<u>Find</u>

(a) Max allowable value of P.

(b) α



FBD of C



$\Sigma F_x = 0$ $\quad P\cos\alpha + T_{BC}\cos 50 - T_{AC}\cos 35 = 0$

$\Sigma F_y = 0$ $\quad -P\sin\alpha + T_{BC}\sin 50 + T_{AC}\sin 35 = 0$

$$P\cos\alpha = \overset{982}{1200\cos 35} - \overset{385}{600\cos 50} = 597.3$$

$$P\sin\alpha = 1200\sin 35 + 600\sin 50 = 1147.9$$

$$\frac{P\sin\alpha}{P\cos\alpha} = \frac{1147.9}{597.3} = \tan\alpha = 1.92$$

$$\alpha = \tan^{-1}(1.92) = \underline{62.5°} \longleftarrow \alpha$$

$$P = \frac{597.3}{\cos(62.5)} = \underline{1294N} \longleftarrow P$$

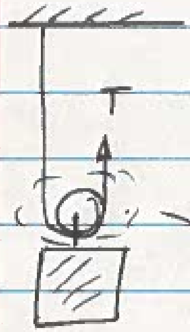
2-67 Given

600 lb crate supported by string-pulley system

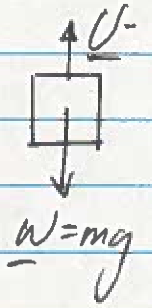
Find

Tension in string

a)



FBD - crate (all cases)

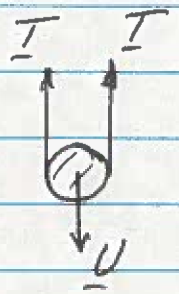


$\Sigma F_x = 0$

$\Sigma F_y = U - W = 0$

$\therefore U = mg$

FBD Pulley



$\Sigma F_y = 0 = 2T - U$

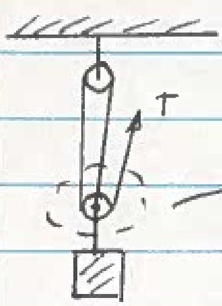
$\therefore U = 2T$

$\frac{U}{2} = T$

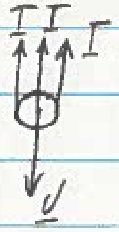
$\therefore T = \frac{mg}{2} = \frac{600}{2} = \underline{\underline{300 lb}}$

b) Same analysis as (a)
Same answer

c)



FBD Pulley

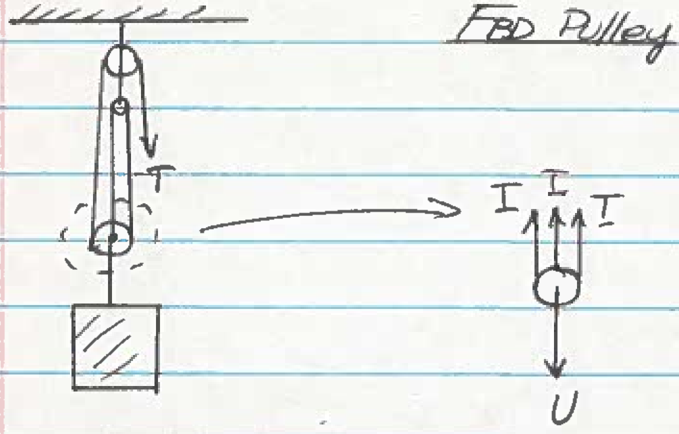


$\Sigma F_y = 3T - U = 0$

$U = 3T$

$T = \frac{U}{3} = \frac{mg}{3} = \frac{600}{3} = \underline{\underline{200 lb}}$

d)

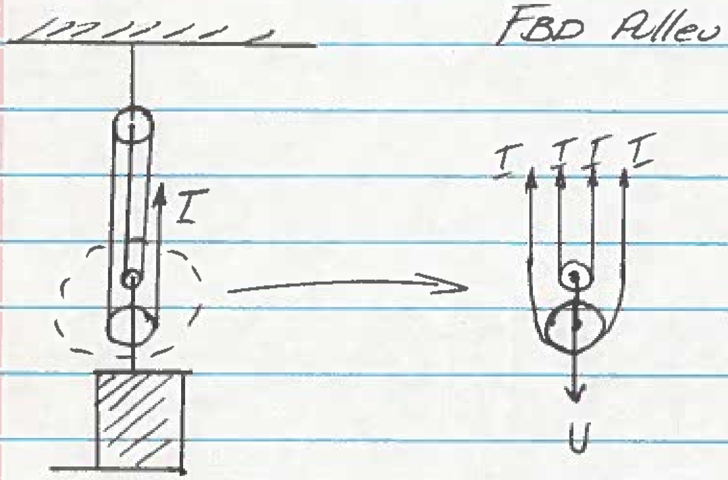


$$\Sigma F_y = 0 = 3T - U$$

$$T = \frac{U}{3}$$

$$T = \frac{600}{3} = \underline{\underline{200\ lb}} \leftarrow$$

e)



$$\Sigma F_y = 0 = 4T - U$$

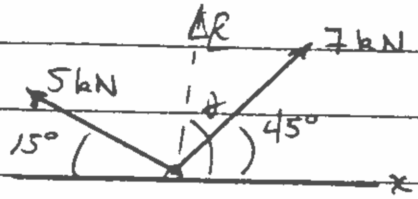
$$T = \frac{U}{4} = \frac{600}{4} = \underline{\underline{150\ lb}} \leftarrow$$

2/6 Given

Forces as shown

Find

Resultant, and θ wrt x axis



$R = \Sigma F$ $\quad R_x = \Sigma F_x = 7kN\cos 45 - 5kN\cos 15$

$= 0.1201\,kN$

$R_y = \Sigma F_y = 7kN\sin 45 + 5kN\sin 15$

$= 6.24\,kN$

$\tan\theta = \frac{R_y}{R_x} = \frac{6.24}{0.1201}$ $\quad \theta = \tan^{-1}\left(\frac{6.24}{0.1201}\right) = \underline{88.9°} \leftarrow$

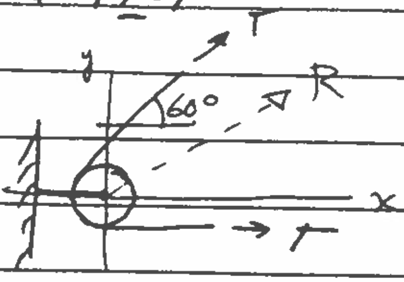
$|R| = (R_x^2 + R_y^2)^{1/2} = (0.1201^2 + 6.24^2)^{1/2} = \underline{6.24\,kN} \leftarrow$

2.15 Given

Pulley & Forces Shown $T = 400N$

Find

$\underline{R}$ & $|\underline{R}|$



$\underline{R} = \Sigma \underline{F}$ $R_x = \Sigma F_x = T\cos 60 + T =$

$400\cos 60 + 400 = 600N$

$R_y = \Sigma F_y = T\sin 60 + 0 =$

$T\sin 60 = 346N$

$\underline{R} = 600N\underline{i} + 346N\underline{j}$ ←

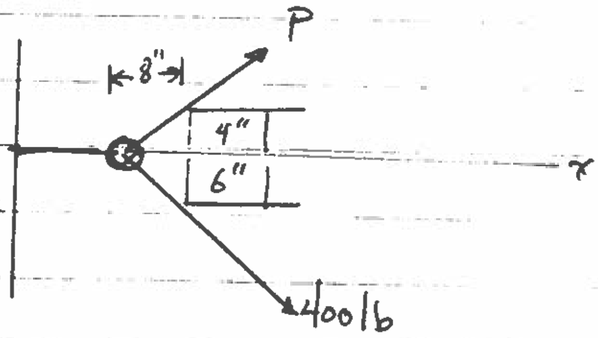
$|\underline{R}| = (600^2 + 346^2)^{1/2} = 693N$ ←

2/20 Given

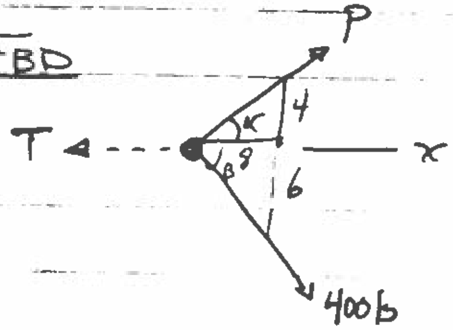
Forces and Obstructions as shown

Find

P required to generate T horizontal to spike and magnitude of T.



FBD



$\alpha = \tan^{-1}\left(\frac{4}{8}\right) = 26.56°$

$\beta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87°$

$\Sigma F = 0$ $\quad$ $\Sigma F_x = 0$

(a) $0 = P\cos 26.56 + 400\cos 36.87° - T$

$\Sigma F_y = 0$

(b) $0 = P\sin 26.56 - 400\sin 36.87°$

Solve ~~either~~ (b) for P

$P = \frac{400\sin 36.87}{\sin 26.56} = \underline{537\,lb}$ ←

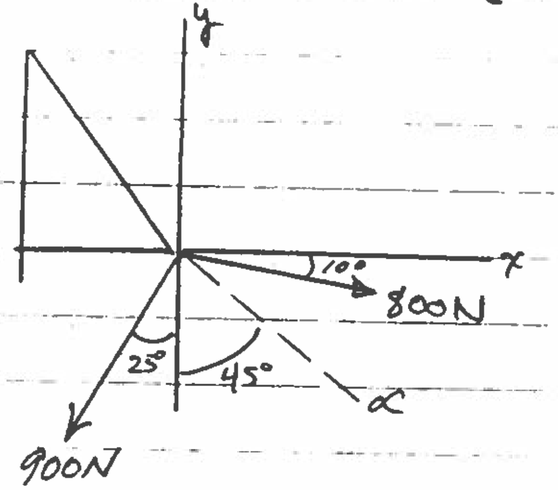
Substitute into (a), solve for T

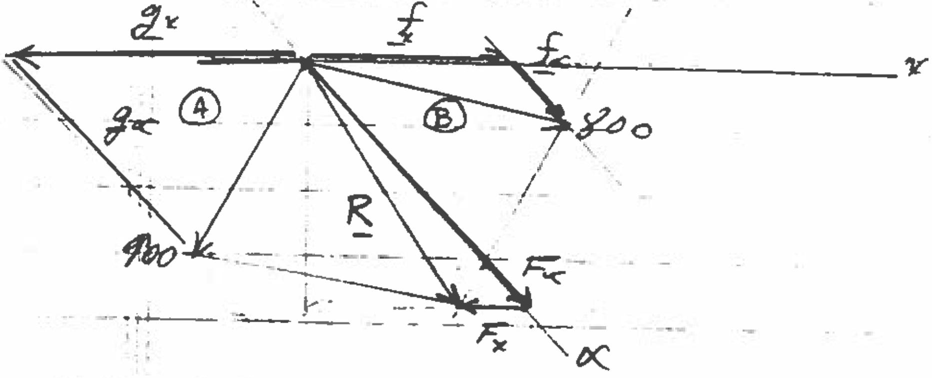
$T = 537\cos 26.56 + 400\cos 36.87 = 800.3$

$\underline{800\ lbs}$ ←

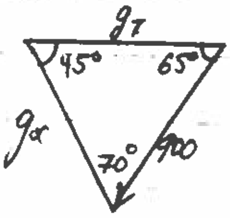
2/28 Given Forces as shown

Find F_x and F_α. Solve geometrically





Ⓐ



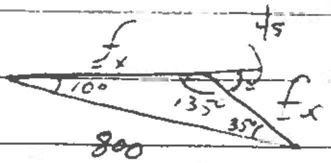
$$\frac{900}{\sin 45} = \frac{g_x}{\sin 70} = \frac{g_\alpha}{\sin 65}$$

$$g_x = \frac{900 \sin 70}{\sin 45} = 1196,03$$

$$g_\alpha = \frac{900 \sin 65}{\sin 45} = 1153.54$$

2/28 continued

Ⓑ



$$\frac{800}{\sin 135} = \frac{f_\alpha}{\sin 10} = \frac{f_x}{\sin 35}$$

$$f_\alpha = \frac{800 \sin 10}{\sin 135} = 196.46$$

$$f_x = \frac{800 \sin 35}{\sin 135} = 648.9$$

$$\underline{R} = \Sigma \underline{F} = \underline{F}_{Ⓐ} + \underline{F}_{Ⓑ}$$

$$R_x = -1196.03 + 648.9 = -547.1$$

$$R_\alpha = +1153.54 + 196.46 = +1350.$$

Observe α is + ↘

x is + →

2.9 Given

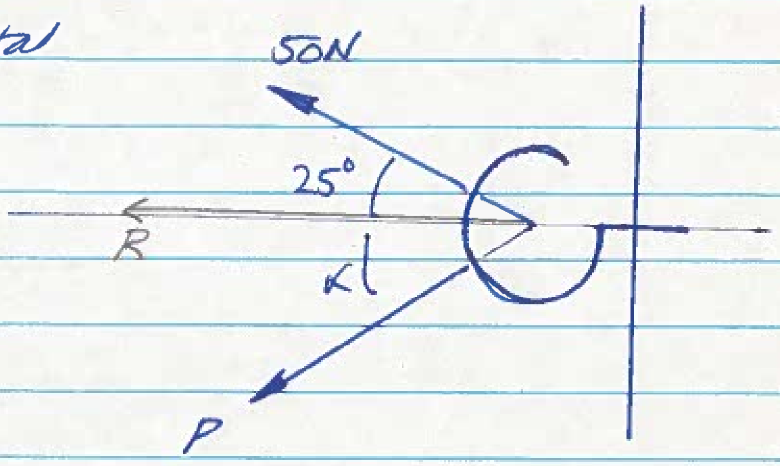
R must be horizontal

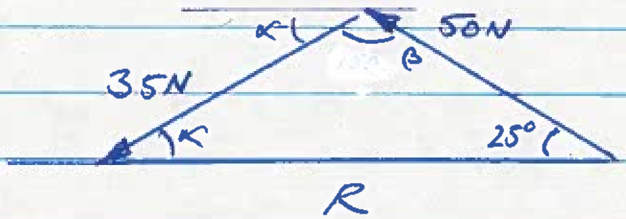
$P = 35N$

Find

a) α

b) R





$180 - \alpha - 25 = \beta$

$180 - 37 - 25 = 118$

Law of sines: $\frac{\sin\alpha}{50N} = \frac{\sin\beta}{R} = \frac{\sin 25}{35N}$

Solve for α

$$\alpha = \sin^{-1}\left(\frac{50}{35}\sin 25\right) = 37.1°$$

$\underline{\underline{\alpha = 37°}}$ ←

Solve for R

$$R = \frac{\sin(118)}{\sin 25} \cdot 35N = 73.1\ N$$

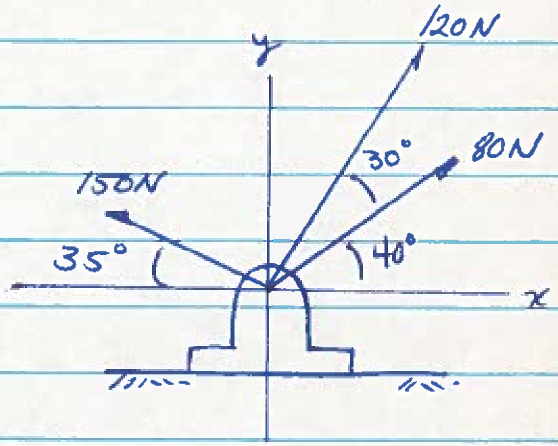
$R = 73N$ ←

2.21 Given

Magnitudes & Directions of forces

Find

x & y components



80N

$F_x = 80 \cos 40°$

$= \underline{61.3 N}$

$F_y = 80 \sin 40°$

$= \underline{51.4 N}$

120N

$F_x = 120 \cos 70°$

$= \underline{41. \quad N}$

$F_y = 120 \sin 70°$

$= \underline{112.8 N}$

150N

$F_x = - 150 \cos 35$

$= \underline{-112.9 N}$

$F_y = 150 \sin 35$

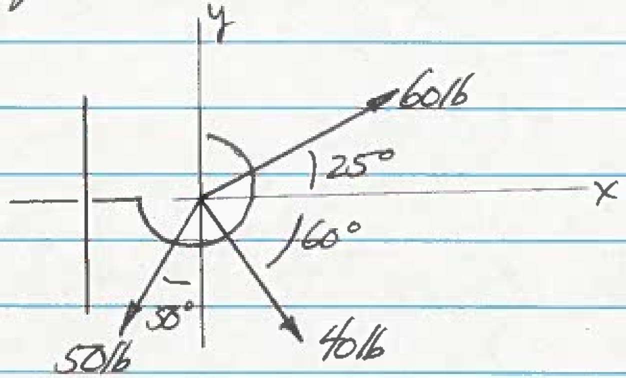
$= \underline{86. \quad N}$

2.22 Given

Magnitudes & directions of forces

Find

x and y components of each force



60 lb

$F_x = 60 \cos 25 = 54.4 \text{ lb}$ ←

$F_y = 60 \sin 25 = 25.4 \text{ lb}$ ←

40 lb

$F_x = 40 \cos 60 = 20 \text{ lb}$ ←

$F_y = -40 \sin 60 = 34.6 \text{ lb}$ ←

50 lb

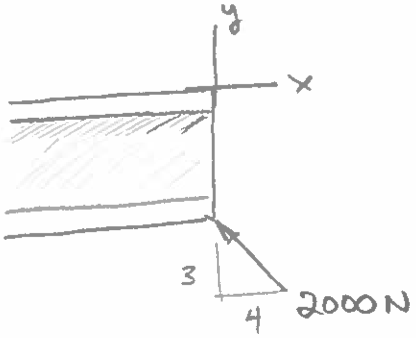
$F_x = -50 \sin 50 = -38.3 \text{ lb}$ ←

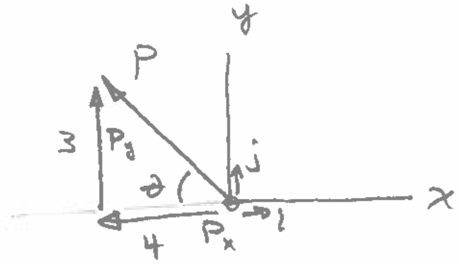
$F_y = -50 \cos 50 = -32.1 \text{ lb}$ ←

2.5

GIVEN: 2000N FORCE AS SHOWN

F.: $\underline{P}$ AS VECTOR EXPRESSED IN COMPONENTS





$P_x = -P\cos\theta$

$P_y = P\sin\theta$

$\tan\theta = \frac{3}{4}$ $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.869°$

$\cos\theta = \frac{4}{5} = \cos 36.869 = 0.8$

$\sin\theta = \frac{3}{5} = \sin 36.869 = 0.6$

$P_x = -2000\cos\theta = 2000(0.8) = -1600\text{ N}$

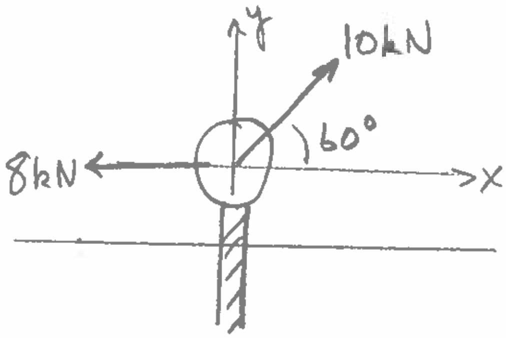
$P_y = 2000\sin\theta = 2000(0.6) = 1200\text{ N}$

$\underline{P} = -1600\underline{i} + 1200\underline{j}\text{ N}$ ←

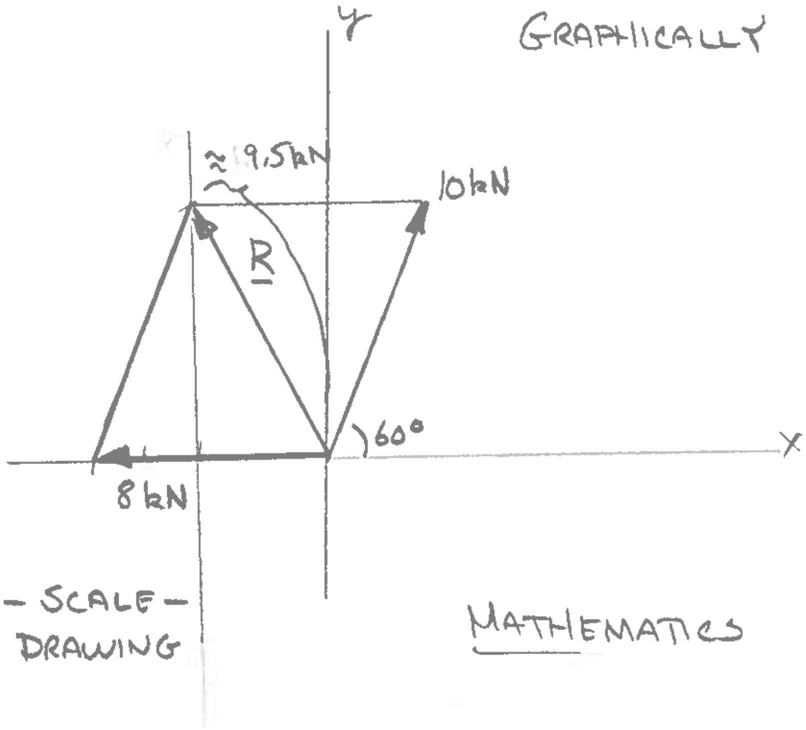
2.14

GIVEN: FORCES AND DIAGRAM AS SHOWN

FIND: RESULTANT, GRAPHICALLY & MATHEMATICALLY



GRAPHICALLY



MATHEMATICS

$$\underline{R} = -8\,\underline{i}\ \text{kN} + 10(\cos 60\,\underline{i} + \sin 60\,\underline{j})\ \text{kN}$$

$$= -8\,\underline{i} \quad 5\underline{i} + 8.66\,\underline{j}\ \text{kN}$$

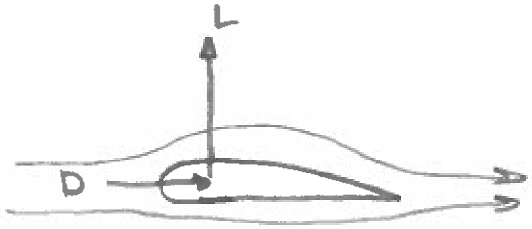
$$\underline{R} = -3\underline{i} + 8.66\underline{j}\ \text{kN} \leftarrow$$

$$R = \sqrt{3^2 + 8.66^2} = \underline{9.165\ \text{kN}} \leftarrow$$

2.19 GIVEN: AIRFOIL WITH LIFT TO DRAG RATIO 10:1

LIFT IS 50 lb.

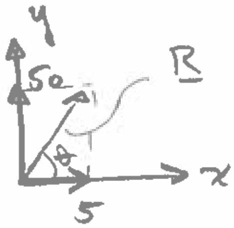
FIND: RESULTANT $\underline{R}$ & θ WITH HORIZONTAL



$\frac{L}{D} = 10$

$L = 50\,lbs$

$\therefore D = 5\,lbs$



$\underline{R} = 5\underline{i} + 50\underline{j}$ $\underline{\underline{lbs}}$

$R = \sqrt{5^2 + 50^2} = 50.249\,lbs$

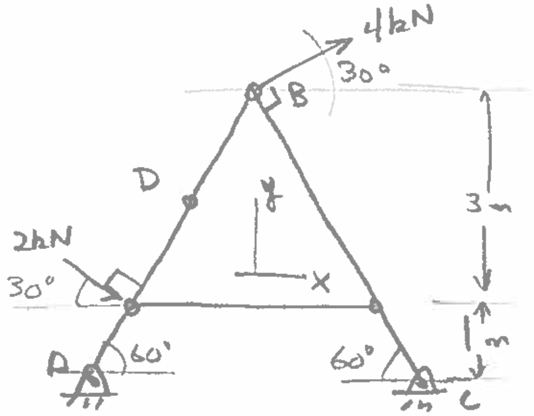
$\cos\theta = \frac{5}{50.249} = 0.0995$

$\theta = \cos^{-1}(0.0995) = \underline{\underline{84.2°}}$

2.27 GIVEN: 2 FORCES SHOWN ACTING ON A-FRAME TRUSS

FIND: $\underline{R}$, R, θ with x-axis

IF $\underline{R}$ applied at D, find distance AD



$\underline{R} = 4\cos 30\underline{i} + 2\cos 30\underline{i} + 4\sin 30\underline{j} - 2\sin 30\underline{j}$ kN

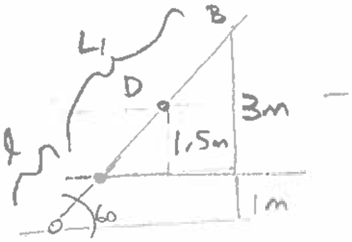
$= 6\cos 30\underline{i} + 2\sin 30\underline{j}$ kN

$= 5.196\underline{i} + 1\underline{j}$ kN ←

$R = \sqrt{5.196^2 + 1^2} = 5.291$ kN ←

$\theta = \tan^{-1}\left(\frac{1}{5.196}\right) = 10.9°$ ←

D appears to be between joint B and cross member



$L_1\sin 60 = 3$

$L_1 = \frac{3}{\sin 60} = 3.464$

$L = L_1 + l$

$L\sin 60 = 4$

$L = \frac{4}{\sin 60} = 4.618$

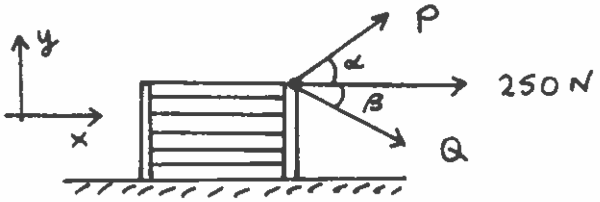
$\Rightarrow s = 1.154 + 1.732$

$= 2.886$

$= 2.89$m ←

EXAMPLE 1 : Rectangular Components - 2D (Forces)

Two forces $\underline{P}$ and $\underline{Q}$ with magnitudes 100 N and 200 N are applied to the upper corner of the crate shown. The sum of the two forces is a horizontal force of 250N. Find α and β.



$$\underline{P} + \underline{Q} = 250\,\underline{i}$$

and

$$\underline{P} = 100\cos\alpha\,\underline{i} + 100\sin\alpha\,\underline{j}$$
$$\underline{Q} = 200\cos\beta\,\underline{i} - 200\sin\beta\,\underline{j}$$

Substituting

$$(100\cos\alpha + 200\cos\beta)\,\underline{i} + (100\sin\alpha - 200\sin\beta)\,\underline{j} = 250\,\underline{i}$$

Scalar equations

$\underline{i}$: $100\cos\alpha + 200\cos\beta = 250$ ①

$\underline{j}$: $100\sin\alpha - 200\sin\beta = 0$ ②

Equation ② $\Rightarrow$ $\sin\alpha = 2\sin\beta$ ③

$\Rightarrow$ $\cos\alpha = (1 - 4\sin^2\beta)^{1/2}$ ④

Substitute from ④ into ① $\Rightarrow$

$$100\,(1 - 4\sin^2\beta)^{1/2} = -200\cos\beta + 250$$

square both sides

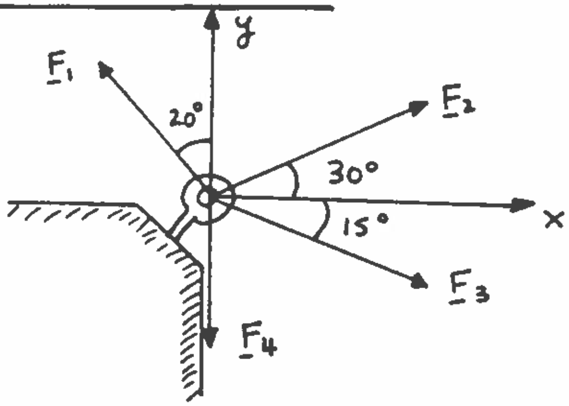
$$1 - 4\sin^2\beta = 4\cos^2\beta - 10\cos\beta + 6.25$$
$$1 - 4(\sin^2\beta + \cos^2\beta) = 6.25 - 10\cos\beta$$
$$10\cos\beta = 9.25$$
$$\underline{\underline{\beta = 22.3^\circ}}$$

Substitute into ③ $\rightarrow$ $\sin\alpha = 2\sin 22.3^\circ = .759$

$$\underline{\underline{\alpha = 49.4^\circ}}$$

EXAMPLE 2 : Rectangular Components - 2D (Forces)

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt if $F_1 = 80$ N, $F_2 = 150$ N, $F_3 = 100$ N, $F_4 = 110$ N.



Express $\underline{F_1} - \underline{F_4}$ in rectangular components

$$\underline{F_1} = 80\,(-\sin 20\,\underline{i} + \cos 20\,\underline{j})$$
$$= -27.4\,\underline{i} + 75.2\,\underline{j}$$
$$\underline{F_2} = 150\,(\cos 30\,\underline{i} + \sin 30\,\underline{j})$$
$$= 129.9\,\underline{i} + 75.0\,\underline{j}$$
$$\underline{F_3} = 100\,(\cos 15\,\underline{i} - \sin 15\,\underline{j})$$
$$= 96.6\,\underline{i} - 25.9\,\underline{j}$$
$$\underline{F_4} = -110\,\underline{j}$$

The resultant force $\underline{R} = (\Sigma F_x)\,\underline{i} + (\Sigma F_y)\,\underline{j}$

$$= (199.1\,\underline{i} + 14.3\,\underline{j}) \quad N$$

Resultant magnitude $R = \{(199.1)^2 + (14.3)^2\}^{1/2}$

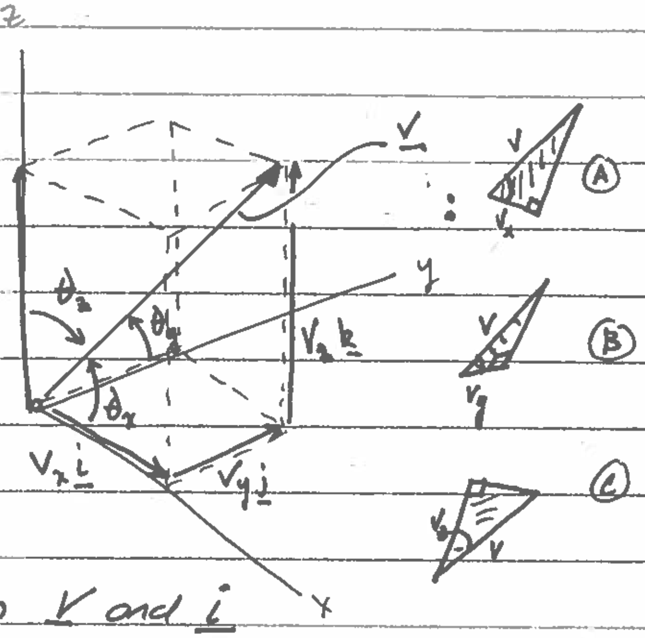
$$\underline{\underline{= 199.6 \quad N}}$$

Resultant direction (line of action) $\theta = \tan^{-1}(14.3/199.1)$

$$\underline{\underline{= 4.1°}}$$

Vector Arithmetic

Consider $\underline{V}$ in rectangular co-ordinates



$\underline{V} = V_x\underline{i} + V_y\underline{j} + V_z\underline{k}$

$|\underline{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$

θ_x = angle between $\underline{V}$ and $\underline{i}$
θ_y = angle between $\underline{V}$ and $\underline{j}$
θ_z = angle between $\underline{V}$ and $\underline{k}$

From trigonometry

$V_x = V\cos\theta_x$, $V_y = V\cos\theta_y$, $V_z = V\cos\theta_z$

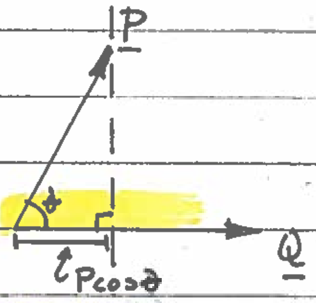
$\therefore$

$\frac{V_x}{V} = \cos\theta_x$, $\frac{V_y}{V} = \cos\theta_y$, $\frac{V_z}{V} = \cos\theta_z$

These ratios are called the direction cosines of the vector $\underline{V}$.

Dot or Scalar Product

$\underline{P} \cdot \underline{Q} = |\underline{P}||\underline{Q}|\cos\theta$



"Projection of the Vector $\underline{P}$ on $\underline{Q}$ multiplied by $|\underline{Q}|$

Operation: Dot product is sum of component by component multiplication

$$\underline{P}\cdot\underline{Q} = (P_x\underline{i} + P_y\underline{j} + P_z\underline{k})\cdot(Q_x\underline{i} + Q_y\underline{j} + Q_z\underline{k})$$

$$= P_xQ_x + P_yQ_y + P_zQ_z$$

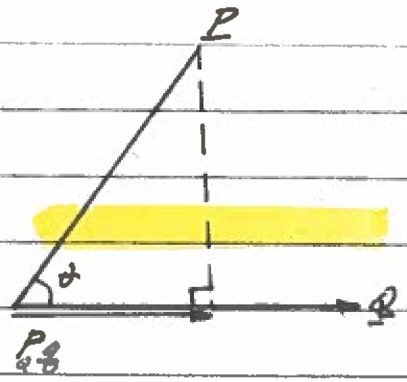
$\underline{i}\cdot\underline{i} = \underline{j}\cdot\underline{j} = \underline{k}\cdot\underline{k} = 1$ Why?

$\underline{i}\cdot\underline{j} = \underline{i}\cdot\underline{k} = \underline{j}\cdot\underline{k} = 0$ Why?

Scalar product is commutative

$$\underline{P}\cdot\underline{Q} = \underline{Q}\cdot\underline{P}$$

If we want to determine the component of P along the direction of Q we write



$$P_Q = P\cos\theta = \frac{\underline{P}\cdot\underline{Q}}{(\underline{Q}\cdot\underline{Q})^{1/2}} = \frac{PQ\cos\theta}{(QQ\underset{\downarrow\,=1}{\cos 0})^{1/2}} = P\cos\theta \checkmark$$

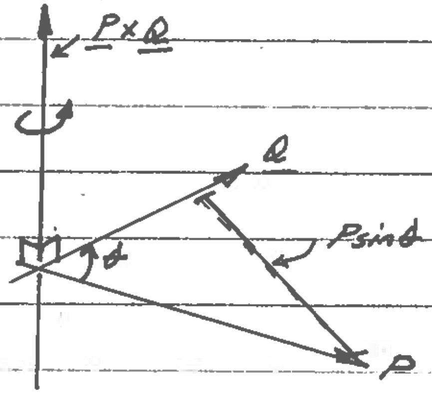
This rule is valuable for projecting force components along particular lines of action

Vector or Cross Product

$\underline{P} \times \underline{Q}$ is a vector with magnitude

$$\left|\underline{P} \times \underline{Q}\right| = PQ\sin\theta$$

that is mutually orthogonal to $\underline{P}$ and $\underline{Q}$ and oriented according to a right-hand-rule



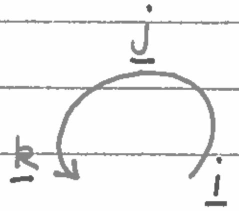
$\underline{P} \times \underline{Q}$ is not commutative

$$\underline{P} \times \underline{Q} \neq \underline{Q} \times \underline{P}$$

but instead

$$\underline{P} \times \underline{Q} = -\underline{Q} \times \underline{P}$$

$$\underline{i} \times \underline{i} = \underline{j} \times \underline{j} = \underline{k} \times \underline{k} = 0$$



$\underline{i} \times \underline{j} = \underline{k}$ $\quad \underline{j} \times \underline{k} = \underline{i}$ $\quad \underline{k} \times \underline{i} = \underline{j}$

$\underline{j} \times \underline{i} = -\underline{k}$ $\quad \underline{k} \times \underline{j} = -\underline{i}$ $\quad \underline{i} \times \underline{k} = -\underline{j}$

Vector product is distributive

$$\underline{P} \times (\underline{Q} + \underline{R}) = \underline{P} \times \underline{Q} + \underline{P} \times \underline{R}$$

Operation:

$$\underline{P}\times\underline{Q} = (P_x\underline{i} + P_y\underline{j} + P_z\underline{k})\times(Q_x\underline{i} + Q_y\underline{j} + Q_z\underline{k})$$

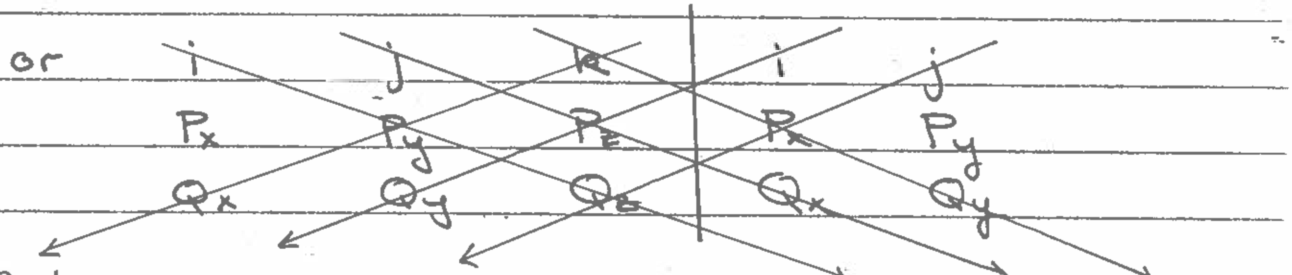
$$= P_xQ_x\cancelto{=0}{\underline{i}\times\underline{i}} + P_xQ_y\cancelto{=k}{\underline{i}\times\underline{j}} + P_xQ_z\cancelto{=-j}{\underline{i}\times\underline{k}}$$
$$+ P_yQ_x\cancelto{=-k}{\underline{j}\times\underline{i}} + P_yQ_y\cancelto{=0}{\underline{j}\times\underline{j}} + P_yQ_z\cancelto{=i}{\underline{j}\times\underline{k}}$$
$$+ P_zQ_x\cancelto{=j}{\underline{k}\times\underline{i}} + P_zQ_y\cancelto{=-i}{\underline{k}\times\underline{j}} + P_zQ_z\cancelto{=0}{\underline{k}\times\underline{k}}$$

$$= (P_yQ_z - P_zQ_y)\underline{i} + (P_zQ_x - P_xQ_z)\underline{j}$$

The operation may also be expressed as a determinant

$$\underline{P}\times\underline{Q} = \begin{vmatrix} i & j & k \\ R_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

or



$-P_yQ_x\underline{k}$ $\quad -P_zQ_y\underline{i}$ $\quad -P_xQ_z\underline{j}$ $\quad P_yQ_z\underline{i}$ $\quad P_zQ_x\underline{j}$ $\quad P_xQ_y\underline{k}$

assemble like terms

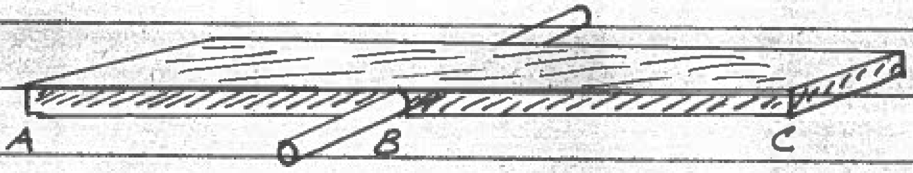
$$\underline{P}\times\underline{Q} = (P_yQ_z - P_zQ_y)\underline{i} + (P_zQ_x - P_xQ_z)\underline{j}$$
$$+ (P_xQ_y - P_yQ_x)\underline{k}$$

Vector products are important for definition of moments

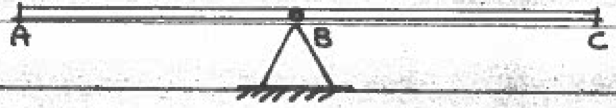
Moment

The moment of a force is a measure of the force's tendency to rotate a body about a point

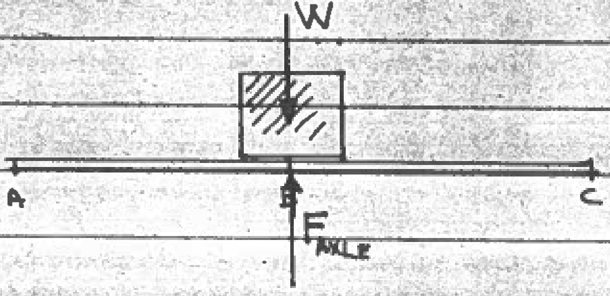
Consider a massless beam on an axle



Assume the beam is supported at the axle

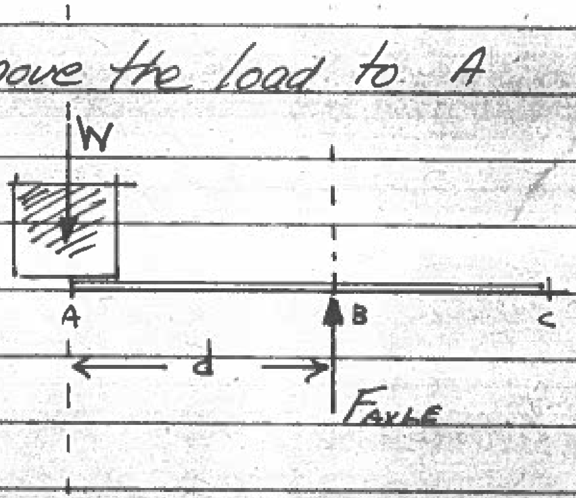


Apply a load at B



Load of weight W is restrained by the upward support force Faxle

Now move the load to A

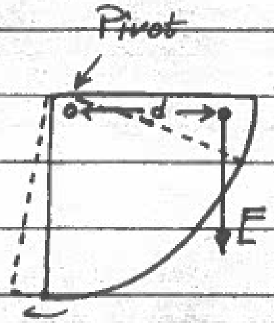


The force system is still balanced in terms of displacement, but the beam wants to rotate (and will if not restrained).

This tendency to cause rotation is called the moment of force W about the point B

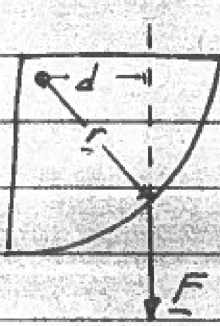
The magnitude of the moment is proportional to the magnitude of the force and the distance from the point of application to the axis of rotation.

consider a flat plate



Wants to rotate this way

The distance "d" is called the moment arm.



$\underline{r}$ is the position vector connecting the pivot location and the point of application of the force.

d is the moment arm from the pivot to the line of application of the force, that is perpindicular to the line of application.

Any force $\underline{F}$ along this lice of action causes the same moment about the pivot

The "direction" of the moment is positive if the sense of rotation is counterclockwise (CCW⇔+), and negative if the sense of rotation is clockwise (CW⇔−).

Because a moment has magnitude and direction, it is a vector quantity.

The vector relationship that preserves the direction convention is

$$\underline{M} = \underline{r} \times \underline{F}$$

Where $\underline{r}$ is the position vector that connects the point of rotation to the point of the force application.

$\underline{M}$ is the moment vector.

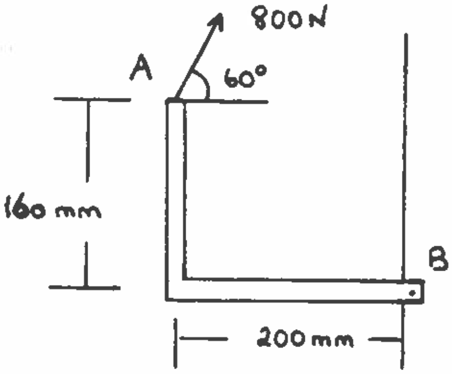
Varignon's Theorem

The moment of a force about a point is equal to the sum of the moments of the components of the force

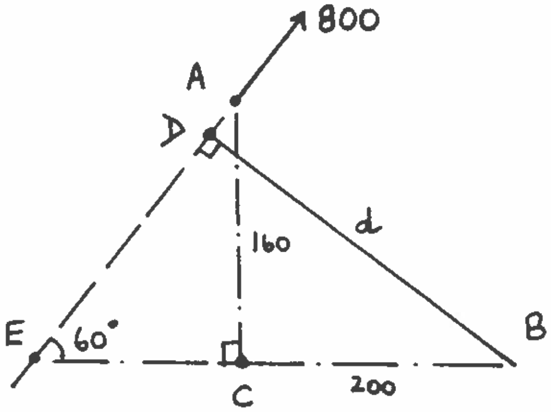
$$\underline{M} = \underline{r} \times \underline{F} = \underline{r} \times F_x \underline{i} + \underline{r} \times F_y \underline{j} + \underline{r} \times F_z \underline{k}$$

EXAMPLE 1 : Rectangular Components - 2D (Moments)

A force of 800 N acts on the bracket shown. Determine the moment of this force about B.



Scalar Approach:



Need to calculate distance d.

In Δ ACE : tan 60 = 160 / EC

EC = 92.34 mm

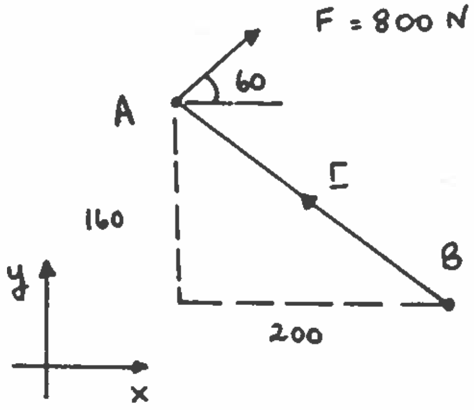
So BE = EC + BC = 292.34 mm

In Δ EBD : sin 60 = d / BE

d = 253.17 mm

Moment M = Fd = $\underline{\underline{202.54 \text{ Nm} \quad (\text{cw})}}$

Vector Approach:



Express $\underline{r}$ and $\underline{F}$ in rectangular components :

$$\underline{r} = -200\,\underline{i} + 160\,\underline{j} \quad \text{mm}$$

$$= -.2\,\underline{i} + .16\,\underline{j} \quad \text{m}$$

$$\underline{F} = 800\cos 60\,\underline{i} + 800\sin 60\,\underline{j}$$

$$= 400\,\underline{i} + 692.82\,\underline{j} \quad \text{N}$$

$$\underline{M} = \underline{r} \times \underline{F} = (-.2\underline{i} + .16\underline{j}) \times (400\underline{i} + 692.82\underline{j}) \ \text{Nm}$$

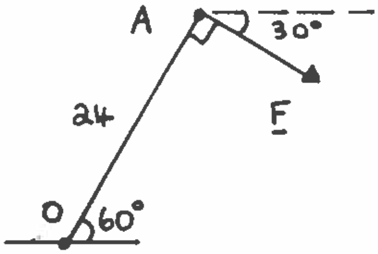
$$= -138.56\,\underline{k} - 64\,\underline{k} \quad \text{Nm}$$

$$\underline{\underline{= -202.56\,\underline{k} \quad \text{Nm}}}$$

(negative sign means cw)

(c) Smallest Force to cause same moment about O:

F has smallest magnitude when d is maximum. So choose F perpendicular to OA, then d = 24 in.

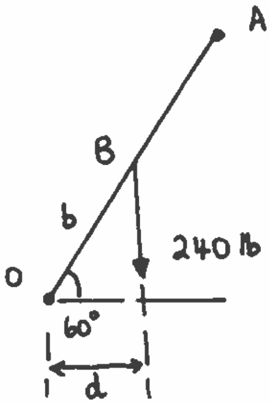


$M_o = F d$

$1200 = 24 F$

$F = 50$ lbs acting at -30° to horizontal.

(d) Distance from O a 240 lb vertical force must act:



$M_o = F d$

$1200 = 240\ d$

$d = 5$ inches

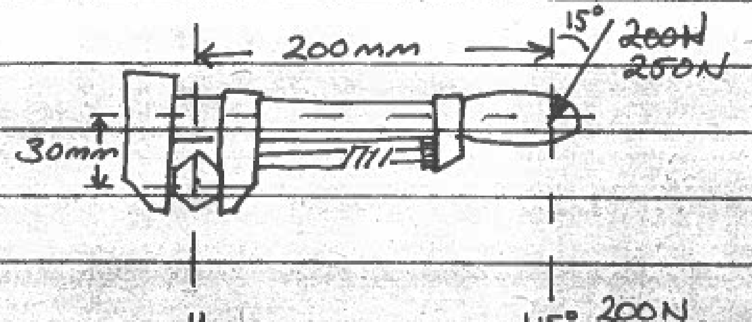
Now $\cos 60 = d / b$

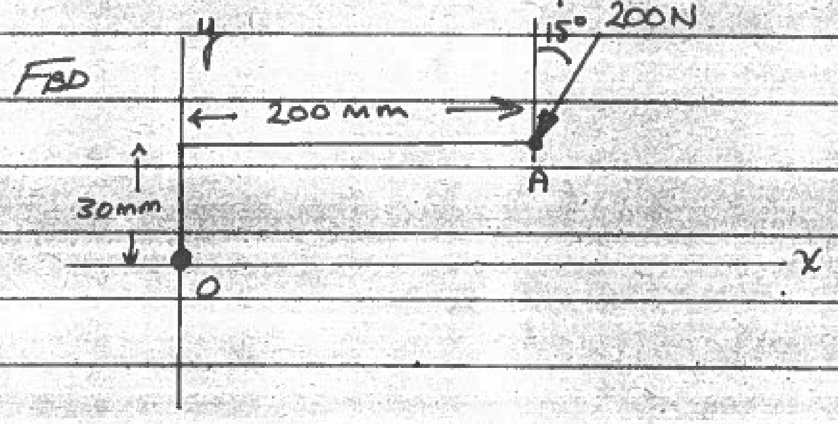
$\Rightarrow$ $b = 10$ inches

2.34

Given: Wrench with 250N force as shown

Find: Moment of force about center of bolt





$$\underline{M}_o = \underline{r} \times \underline{F} \qquad \underline{r} = 200\,\text{mm}\,\underline{i} + 30\,\text{mm}\,\underline{j}$$

$$= 0.2\,\text{m}\,\underline{i} + 0.03\,\text{mm}\,\underline{j}$$

$$\underline{F} = -\overset{250}{\cancel{200}}\,\text{N}\sin 15^\circ\,\underline{i} - \overset{250}{\cancel{200}}\,\text{N}\cos 15^\circ\,\underline{j}$$

$$= -\underset{64.70}{\cancel{51.76}}\,\text{N}\,\underline{i} - \underset{241.48}{\cancel{193.18}}\,\text{N}\,\underline{j}$$

$$\underline{r}\times\underline{F} = \begin{array}{ccc|cc} \underline{i} & \underline{j} & \underline{k} & \underline{i} & \underline{j} \\ 0.2 & 0.03 & 0 & 0.2 & 0.03 \\ -51.76 & -193.18 & 0 & -51.76 & -193.8 \end{array}$$

$$(-\underset{64.70}{\cancel{51.76}})(0.03)\,\underline{k} \quad 0 \quad 0 \qquad 0 \quad 0 \quad (-\underset{241.48}{\cancel{193.18}})(0.2)\,\underline{k}$$

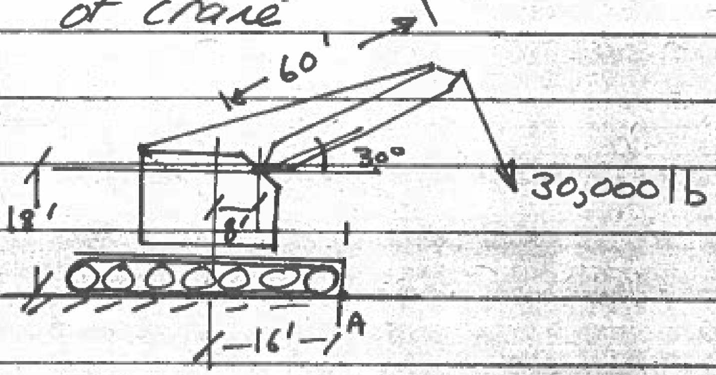
$$\underline{M}_o = (-\underset{241.48}{\cancel{193.18}})(0.2) - (-\underset{64.70}{\cancel{51.76}})(0.03)\ \underline{k}$$

$$= \cancel{-37.08\,\underline{k}}\ \ \underline{-46.4\,\text{N}\cdot\text{m}\ \underline{k}} \longleftarrow$$

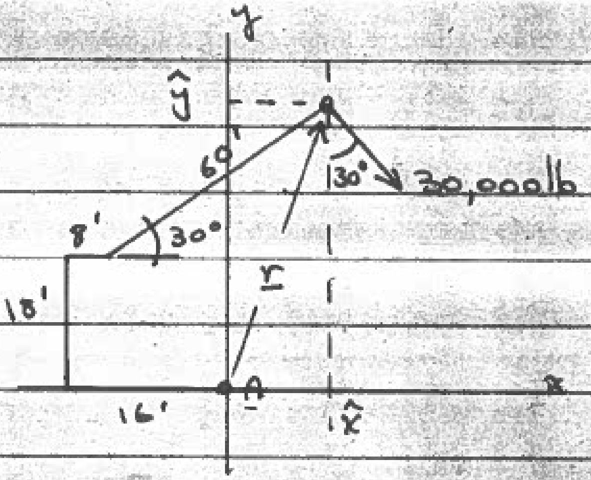
2.38

Given: Crane as shown

Find: overturning moment at front edge (A) of crane



FBD



$\hat{x} = -16 + 8 + 60\cos 30 = 43.96$

$\hat{y} = 18 + 60\sin 30 = 48$

$\underline{M}_a = \underline{r} \times \underline{F}$

$\underline{r} = 43.96\underline{i} + 48\underline{j}$

$\underline{F} = 30{,}000\sin 30\,\underline{i} - 30{,}000\cos 30\,\underline{j}$

$= 15{,}000\underline{i} - 25{,}980\underline{j}$

$$\underline{r}\times\underline{F} = \begin{array}{ccc|cc} i & j & k & i & j \\ 43.96 & 48 & 0 & 43.96 & 48 \\ 15000 & -25980 & 0 & 15000 & -25980 \end{array}$$

$-(15{,}000)(48)\underline{k}$ 0 0 0 0 $(-25980)(4396)\underline{k}$

$\underline{M}_a$ ~~=~~ $(-25980)(43.96) - (+15000)(48)\ \underline{k}$

$= -1862114$ ft·lbs $\underline{k}$

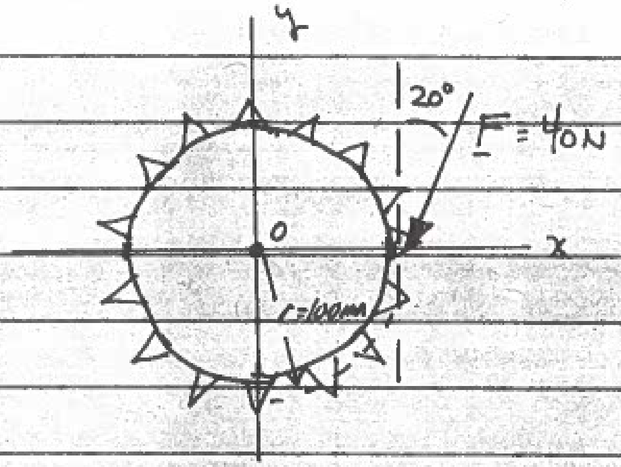
$= -1.86 \cdot 10^6$ ft·lbs $\underline{k}$

Keaton

2.32

Given: Force on gear as shown, $F = 40N$

Find: moment of F about point 0



$\underline{M} = \underline{r} \times \underline{F}$ $\qquad$ $\underline{r} = 100mm\ \underline{i} + 0\ \underline{j}$

$\underline{F} = -40N \sin 20°\ \underline{i} - 40N \overset{cos}{\cancel{sin}}\ 20°\ \underline{j}$

$= -13.68\ \underline{i} - 37.59\ \underline{j}$

$$\underline{r} \times \underline{F} = \begin{array}{ccc|cc} i & j & k & i & j \\ 100mm & 0 & 0 & 100mm & 0 \\ -13.68 & -37.59 & 0 & -13.68 & -37.59 \end{array}$$

$0 \quad 0 \quad 0 \quad 0 \quad 0 \quad (-37.59)(100)\ \underline{k}$

$= (.100\ meters)(-37.59N)\ \underline{k}$

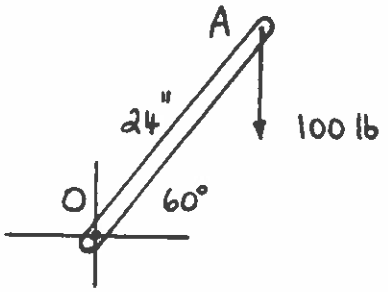
$= -3.76 N\cdot m\ \underline{k}$

Note: 2D Force systems will always have a geometric dimension with zero components.

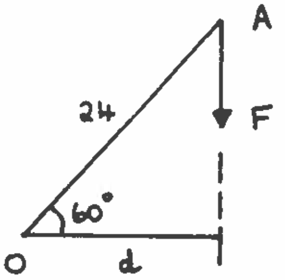
3/12

EXAMPLE 2 : Rectangular Components - 2D (Moments)

A 100 lb vertical force is applied to the end of a lever attached to a shaft at O. Find (a) the moment of the 100 lb force about O ; (b) the magnitude of the horizontal force about O which causes the same moment ; (c) the smallest force applied at A which creates the same moment about O ; (d) How far from the shaft a 240 lb vertical force must act to create the same moment about O.



(a) Moment about O :

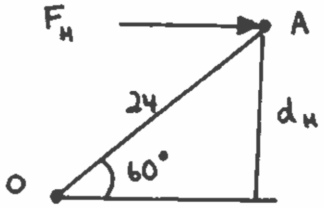


$d = 24 \cos 60 = 12$ inches

$M_o = 100\,(12) = \underline{\underline{1200 \text{ lb. in}}}$

(CW)

(b) Horizontal force causing same moment about O :



$M_o = F_H \, d_H$

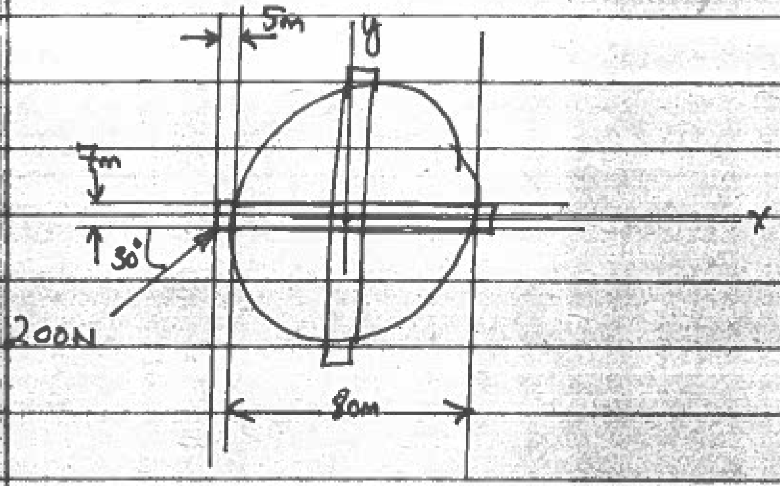
but $d_H = 24 \sin 60 = 20.8$ inches

Using $M_o = 1200$ lb.in $\Rightarrow$ $\underline{\underline{F_H = 57.7 \text{ lb} \quad (\rightarrow)}}$

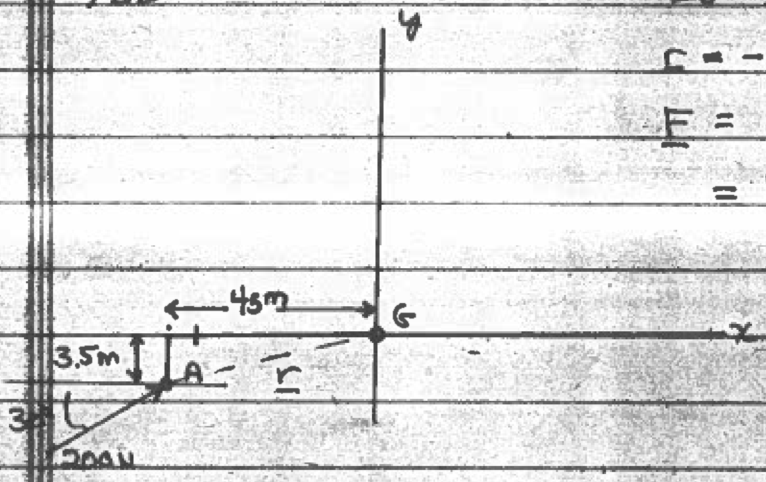
2.39

Given: Space Station as shown fires 200N thruster

Find: Moment generated about station center of mass



FBD

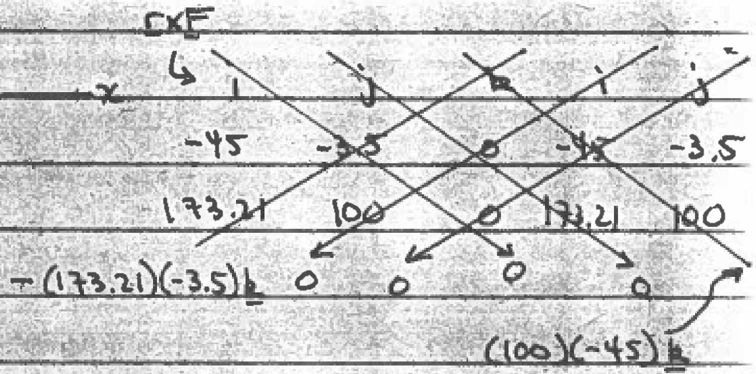


$\underline{M}_G = \underline{r} \times \underline{F}$

$\underline{r} = -45\text{m}\,\underline{i} - 3.5\text{m}\,\underline{j}$

$\underline{F} = 200\text{N}\cos 30\,\underline{i} + 200\text{N}\sin 30\,\underline{j}$

$= 173.21\text{N}\,\underline{i} + 100\text{N}\,\underline{j}$



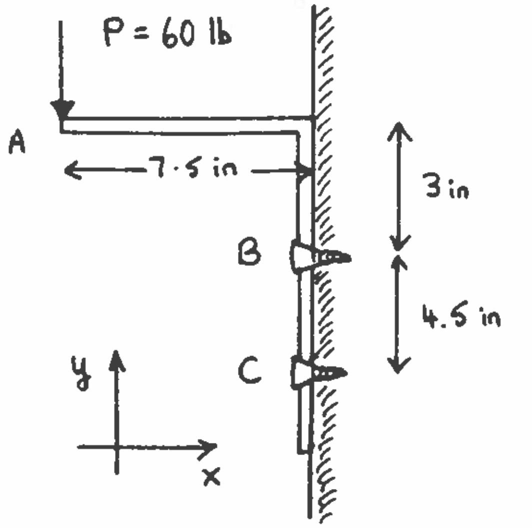
$\underline{M}_G = (100)(-45) - (173.21)(-3.5)$

$= -3893.76\,\text{N}\cdot\text{m}\,\underline{k}$

$= -3890\,\text{N}\cdot\text{m}\,\underline{k}$

EXAMPLE 1 : Force - Couple Systems

A 60 lb vertical force is applied at A to the bracket shown, which is held by screws at B, C (a) Replace P by an equivalent force - couple system at B; (b) Find the two horizontal forces at B, C which are equivalent to the couple obtained in (a).



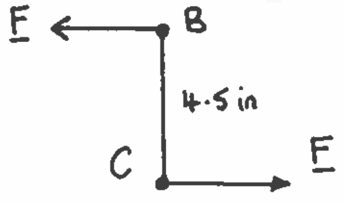
Above system causes moment at B given by

$$\underline{M}_B = \underline{r}_{A/B} \times \underline{F}$$
$$= (3\underline{j} - 7.5\underline{i}) \times (-60\underline{j}) \quad \text{lb.in}$$
$$= 450\,\underline{k} \quad \text{lb.in}$$

Equivalent system at B : $\underline{F}_B = -60\,\underline{j}$ lb force

$\underline{M}_B = 450\,\underline{k}$ lb.in couple

Replace couple with horizontal forces as shown



Moment of this couple = F (4.5) lb.in

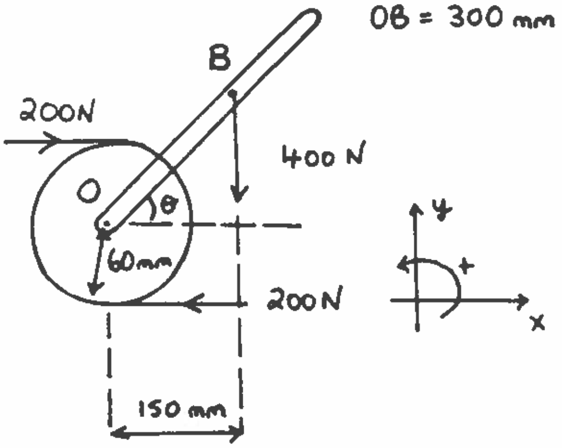
So $4.5\,F = 450$

$$\underline{\underline{F = 100 \text{ lb}}}$$

So $\underline{F}_B = 100\,\underline{i}$ lb, $\underline{F}_C = -100\,\underline{i}$ lb, to produce same couple as in (a).

EXAMPLE 2 : Force - Couple Systems

Replace the couple and force shown by an equivalent single force applied to the lever. Determine the distance from the shaft to the point of application of this equivalent force.



In Cartesian coordinates :

$$\underline{F} = -400\,\underline{j} \quad N$$

$$\underline{M}_c = -200(120)\,\underline{k} \quad Nmm \quad = -24\,\underline{k} \quad Nm$$

Total moment about O :

$$\underline{M}_o = \underline{M}_c + \underline{r}_{B/O} \times \underline{F}$$

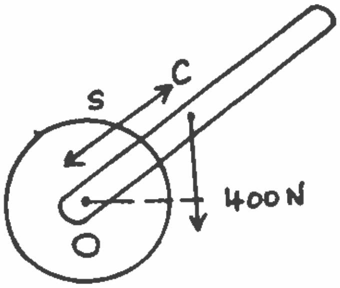
$$= -24\underline{k} + (.3\cos\theta\,\underline{i} + .3\sin\theta\,\underline{j}) \times (-400\,\underline{j})$$

From geometry, $\cos\theta = 150/300 = .5 \Rightarrow \theta = 60°$, so

$$\Sigma\underline{M}_o = -24\underline{k} + .15(-400)\,\underline{k}$$

$$= -84\,\underline{k} \quad Nm$$

Let equivalent force act at point C, a distance s along lever, then $\underline{M}_o$ caused by this force must equal $-84\underline{k}$ Nm, so



$$\underline{r}_{C/O} \times \underline{F} = -84\underline{k}$$

$$(s\cos 60\,\underline{i} + s\sin 60\,\underline{j}) \times (-400\underline{j}) = -84$$

$$-200\,s\,\underline{k} = -84\,\underline{k}$$

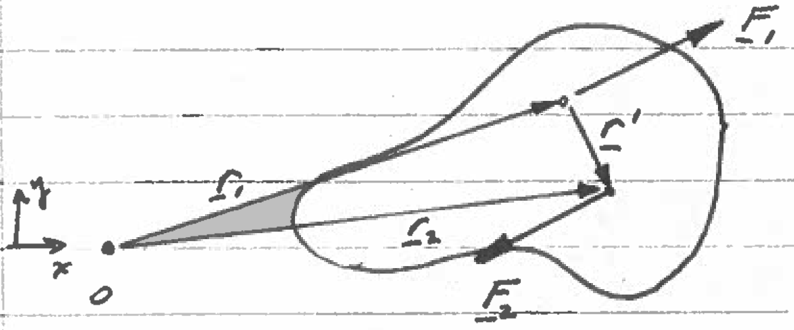
$$s = 0.42 \text{ m}$$

$$= 420 \text{ mm}$$

Couple

A moment produced by two equal, opposite, and non-collinear forces is known as a couple

Consider moment about O produced by $\underline{F}_1$ and $\underline{F}_2$ as shown



$$\Sigma \underline{F} = \underline{F}_1 + \underline{F}_2 = 0 \quad \therefore \text{ No translation}$$

$$\underline{M}_o = \Sigma \underline{r} \times \underline{F} = \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2$$

From Vector addition $\underline{r}_2 = \underline{r}_1 + \underline{r}'$

$$\therefore \underline{M}_o = \underline{r}_1 \times \underline{F}_1 + (\underline{r}_1 + \underline{r}') \times \underline{F}_2$$
$$= \underline{r}_1 \times \underline{F}_1 + \underline{r}_1 \times \underline{F}_2 + \underline{r}' \times F_2$$

but $\underline{F}_1 = \underline{F}_2 \quad \therefore \quad \underline{M}_o = \underline{r}' \times \underline{F}_2$

where $\underline{r}' = \underline{r}_2 - \underline{r}_1$

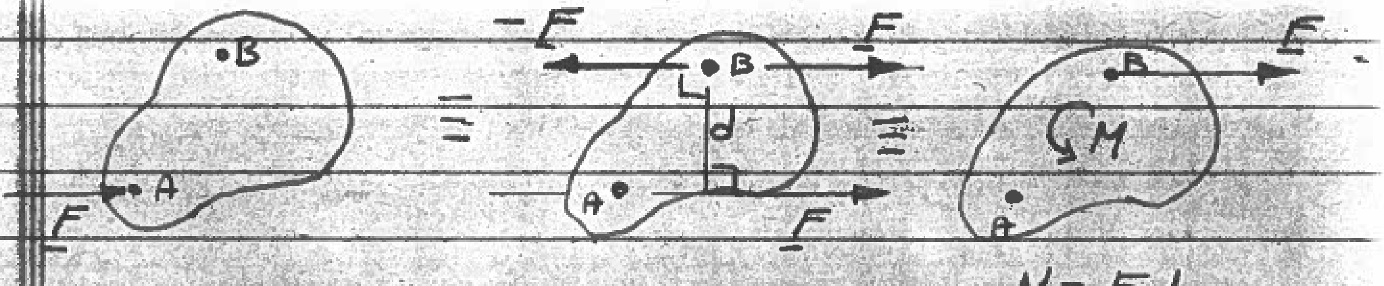
Because this moment does not depend on "O", but only on the distance between the forces a couple is a free vector.

The moment caused by a couple has the same value for all moment centers.

Force - Couple systems

The effect of a force on a body is to cause translation and rotation.

Often it convienent to represent the effect of a force by its equivalent force-couple system.



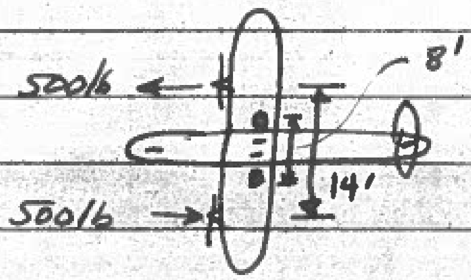
M = Fd

Force at A is replaced by same force at B plus a couple

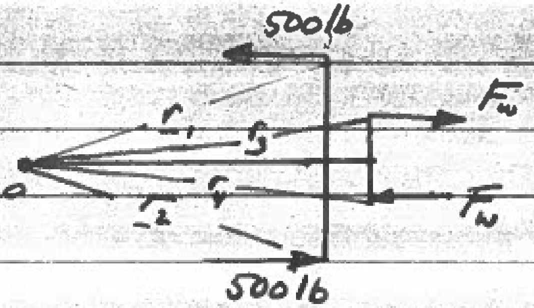
~~2.55~~ 2.60

Given: Aircraft tested as shown

Find: F required to counteract propeller thrusts (supplied by brakes on wheels)



FBD (We can pick any moment center!)



$r_1 = 100\underline{i} + 7\underline{j}$

$r_2 = 100\underline{i} - 7\underline{j}$

$r_3 = 110\underline{i} + 4\underline{j}$

$r_4 = 110\underline{i} + 4j$

$$\Sigma M_o = 0 = \underline{r}_1 \times F_1 + \underline{r}_2 \times \underline{F}_2 + \underline{r}_3 \times \underline{F}_3 + \underline{r}_4 \times \underline{F}_4$$

$$= (100\underline{i} + 7\underline{j}) \times (-500\underline{i}) + (100\underline{i} - 7\underline{j}) \times (500\underline{i})$$

$$+ (110\underline{i} + 4\underline{j}) \times (F_w\underline{i}) + (110\underline{i} - 4\underline{j}) \times (-F_w\underline{i}) \qquad = 0$$

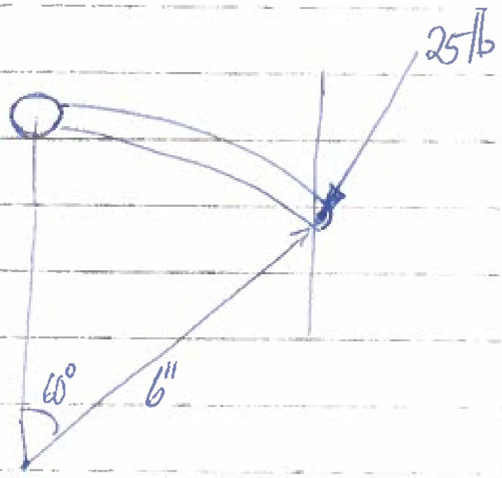
$$= +3500\underline{k} + 3500\underline{k} - 4F_w\underline{k} - 4F_w\underline{k} \qquad = 0$$

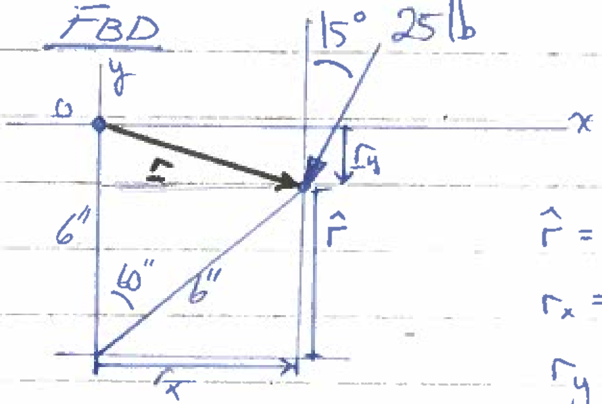
$$= 7000\underline{k} - 8F_w\underline{k} = 0$$

$$F_w = \frac{\underline{7000}\,\text{ft.lbs}}{8\ \text{ft}} = \underline{\underline{875\,\text{lbs}}} \longleftarrow$$

2.41 Given: Force on semi-circular bracket as shown

Find: Moment of force about axle "O".





$\hat{r} = 6'' \cos 60° = 3''$

$r_x = 6'' \sin 60° = 5.196''$

$r_y = 6'' - \hat{r} = 3''$

$\underline{M}_o = \underline{r} \times \underline{F}$

$F_x = -25 \sin 15° = -6.47 \text{ lbs}$

$F_y = -25 \cos 15° = -24.148 \text{ lbs}$

$$\underline{r} \times \underline{F} = \begin{array}{ccc|cc} i & j & \underline{k} & i & j \\ 5.196 & -3 & 0 & 5.196 & -3 \\ -6.47 & -24.148 & 0 & -6.47 & -24.148 \end{array}$$

$-(-6.47)(-3)\underline{k}$ $\quad 0 \quad 0 \quad 0 \quad 0 \quad$ $(-24.148)(5.196)\underline{k}$

$$\underline{M}_o = [(-24.148)(5.196) - (-6.47)(-3)]\,\underline{k} = -144.88\,\underline{k}$$

$= \underline{-145 \text{ in-lbs}}\ \underline{k}$ ←

$= 145$ in-lbs)

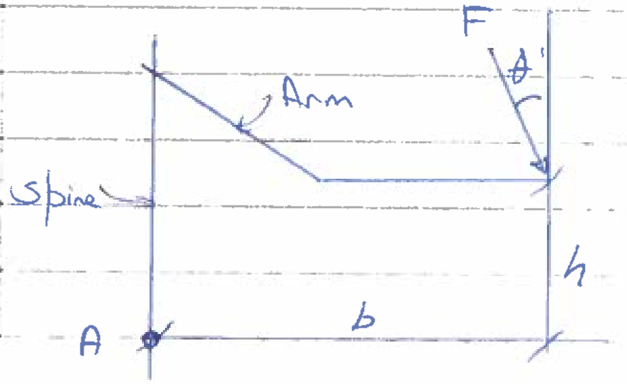
$$b\cancel{F}\sin\theta = h\cancel{F}\cos\theta$$

$$\frac{b}{h} = \frac{\cos\theta}{\sin\theta} \quad \text{or} \quad \frac{h}{b} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

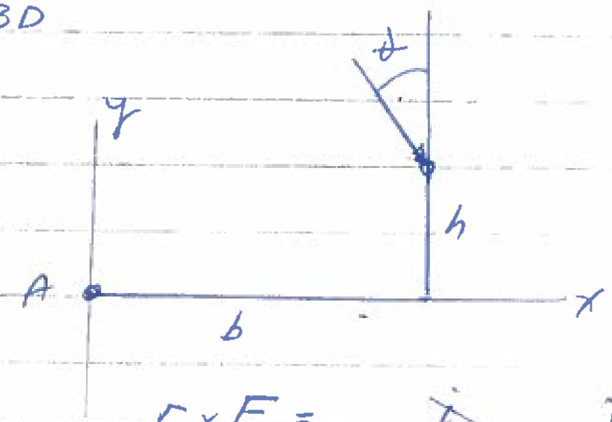
$$\therefore \; \theta_{max} = \tan^{-1}\left(\frac{h}{b}\right)$$

2.42 Given: Body as shown

Find: Find θ that produces most severe bending strain (moment) about A.



FBD



$$\underline{M}_A = \underline{r} \times \underline{F}$$

$$\underline{F} = F\sin\theta\,\underline{i} - F\cos\theta\,\underline{j}$$

$$\underline{r} = b\,\underline{i} + h\,\underline{j}$$

$$\underline{r} \times \underline{F} = \begin{array}{ccc|cc} i & j & k & i & j \\ b & h & 0 & b & h \\ F\sin\theta & -F\cos\theta & 0 & F\sin\theta & -F\cos\theta \end{array}$$

$$-hF\sin\theta \qquad 0 \qquad 0 \qquad -bF\cos\theta\,\underline{k}$$

$$\underline{M}_A = (-bF\cos\theta - hF\sin\theta)\,\underline{k}$$

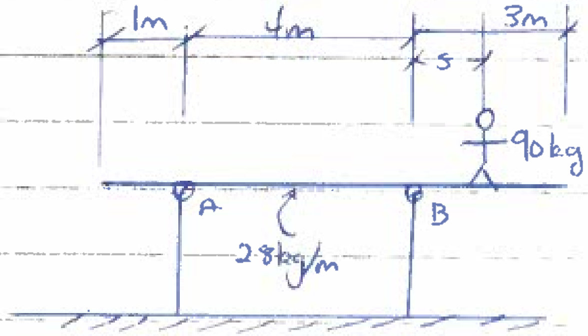
Max $\underline{M}_A$: $\dfrac{dM_A}{d\theta} = bF\sin\theta - hF\cos\theta = 0$

solve for θ

2.48 Given: Work platform shown, $\frac{M}{L} = \frac{28kg}{m}$

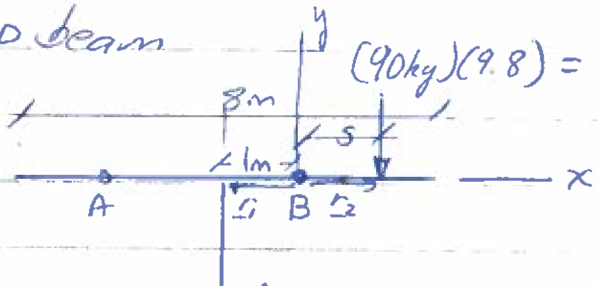
90kg worker walks out platform

Find: Distance s when moment at B is zero.



Weight acts through GC. of beam — occurs at ½ length of beam

FBD beam



$(90kg)(9.8) = 882N$

$W = mg = (\frac{28kg}{m})(8m)(9.8m/s^2) = 2195.2N$

$\Sigma M_B = 0$ $\quad r_1 = -1m\,\underline{i}$ $\quad r_2 = s\,m\,\underline{i}$

$F_1 = -2195.2N\,\underline{j}$ $\quad F_2 = -882N\,\underline{j}$

$r_1 \times F_1 = 2195.2 N\cdot m\,\underline{k}$ $\quad r_2 \times F_2 = -882s\,N\cdot m\,\underline{k}$

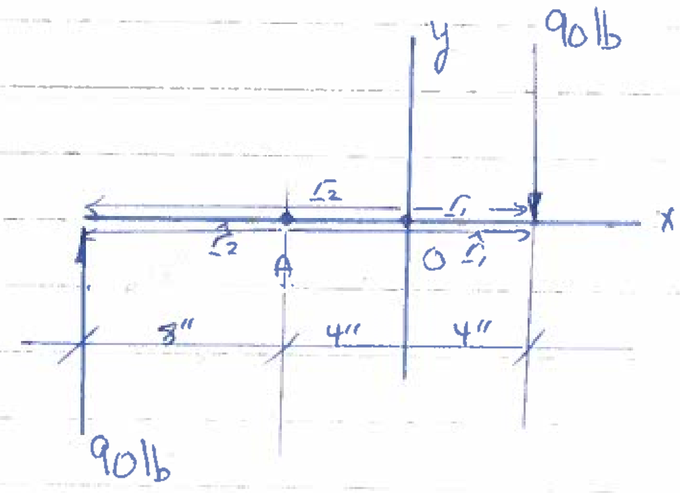
$\Sigma M_B = 2195.2\,N\cdot m - 882s\,N\cdot m\,\underline{k} = 0$

Solve for s

$$\frac{2195.2N\cdot m}{882N} = s\,m \quad = 2.48m$$

2.53 Given: Geometry and Forces Shown

Find: Moment of two forces about point O and point A



$\underline{r}_1 = 4'' \underline{i}$

$\underline{r}_2 = -12'' \underline{i}$

$\hat{\underline{r}}_1 = 8'' \underline{i}$

$\hat{\underline{r}}_2 = -8'' \underline{i}$

$\underline{F}_1 = -90 \, lb \, \underline{j}$

$\underline{F}_2 = 90 \, lb \, \underline{j}$

$\underline{M}_O = \underline{r} \times \underline{F} = \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2$

$\underline{r}_1 \times \underline{F}_1 = 4\underline{i} \times -90 \, lb \, \underline{j} = -90(4) \, \underline{k}$

$\underline{r}_2 \times \underline{F}_2 = -12\underline{i} \times 90 \, \underline{j} = -90(12) \, \underline{k}$

$\underline{M}_O = (-90(4) - 90(12))\underline{k} = \underline{\underline{-1440 \, lb \cdot in}} \, \underline{k} \longleftarrow$

$\underline{M}_A = \underline{r} \times \underline{F} = \hat{\underline{r}}_1 \times \underline{F}_1 + \hat{\underline{r}}_2 \times \underline{F}_2$

$\hat{\underline{r}}_1 \times \underline{F}_1 = 8\underline{i} \times -90 \, lb \, \underline{j} = -90(8) \, \underline{k}$

$\hat{\underline{r}}_2 \times \underline{F}_2 = -8\underline{i} \times 90 \, lb \, \underline{j} = -90(8) \, \underline{k}$

$\underline{M}_A = (-90(8) - 90(8))\underline{k} = \underline{\underline{-1440 \, lb \cdot in}} \, \underline{k} \longleftarrow$

Moments the same because force system is a couple.

Resultants

The <u>resultant</u> of a system of forces is the <u>simplest</u> force combination that can replace all the original forces without altering the external effect of the force system on the rigid body.

<u>Equilibrium</u> occurs when all forces that ~~acton~~ act on the body have a resultant that vanishes. ($\underline{R} = \underline{0}$)

If all forces act in a single plane then for equilibrium to occur
- (a) the resultant must cause the same tendency for <u>translation</u>
- (b) the resultant must cause the same tendency for <u>rotation</u>

Mathematically (a) is expressed as

$$\underline{R} = \Sigma \underline{F}$$

The line of action of $\underline{R}$ is determined by requiring that $\underline{R}$ cause the same moment about any moment center as the original system.

Thus (6) is expressed as

$$\underline{M}_o = \Sigma \underline{M}_o = \Sigma \underline{r} \times \underline{F} = \underline{r}' \times \underline{R}$$

This last equation is called the principle of moments

Special cases

1) A concurrent system of forces all passing through a common point, O will cause zero moment through that point. The line of action of the resultant $\underline{R} = \Sigma F$ will also pass through O.

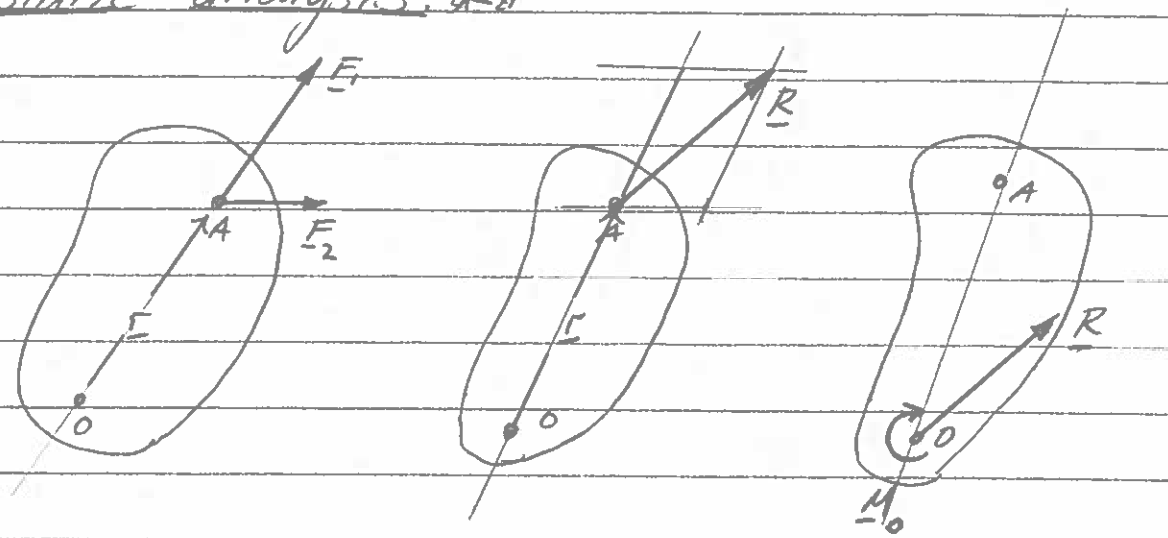
2) If $\Sigma \underline{F} = \underline{0}$ for a given force system, the resultant of the system may not be zero, it could be a couple.

At equilibrium the force balance is zero and the moment about any moment center is zero.

$$\boxed{\Sigma F = \underline{0}}$$

$$\boxed{\Sigma \underline{M}_o = \underline{0}}$$

*** These are the fundamental equations of static analysis. **

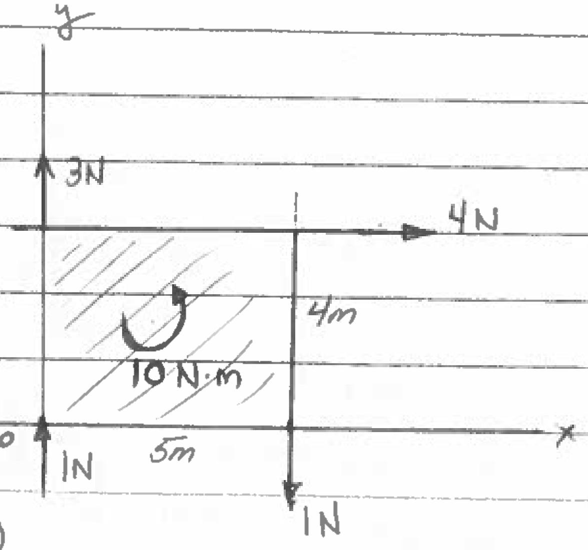


Original
Force System

Resultant
Force System

Given: Plate as shown

Find: Resultant of the system



$\underline{R} = \Sigma F$

$= 1\underline{j} + 3\underline{j} + 4\underline{i} - 1\underline{j}$

$= 4\underline{i} + 3\underline{j}$

$\Sigma M_o = \Sigma r \times F + \Sigma M$

$= (\underline{0} \times 1\underline{j}) + (4j \times 3\underline{j})$

$+ (5i + 4j) \times (4i) + (5\underline{i} \times -1\underline{j})$

$+ 10\underline{k}$

$= 10\underline{k} - 5\underline{k} - 16\underline{k} = -11\underline{k}$

But $\Sigma M_o = \hat{\underline{r}} \times \underline{R}$ let $\hat{\underline{r}} = x\underline{i} + y\underline{j}$

$(x\underline{i} + y\underline{j}) \times (4\underline{i} + 3\underline{j}) = -11\underline{k}$

$3x\underline{k} - 4y\underline{k} = -11\underline{k}$

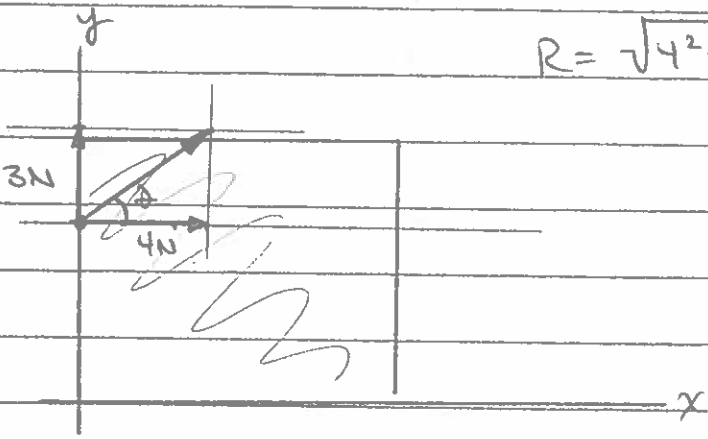
$$y\underline{k} = \frac{11\underline{k}}{4} + \frac{3x\underline{k}}{4} = \underline{\underline{2.75\underline{k} + 0.75x\underline{k}}}$$

giving line of action of $\underline{R}$

Choose point of application at $x = 0$ then $y = 2.75$

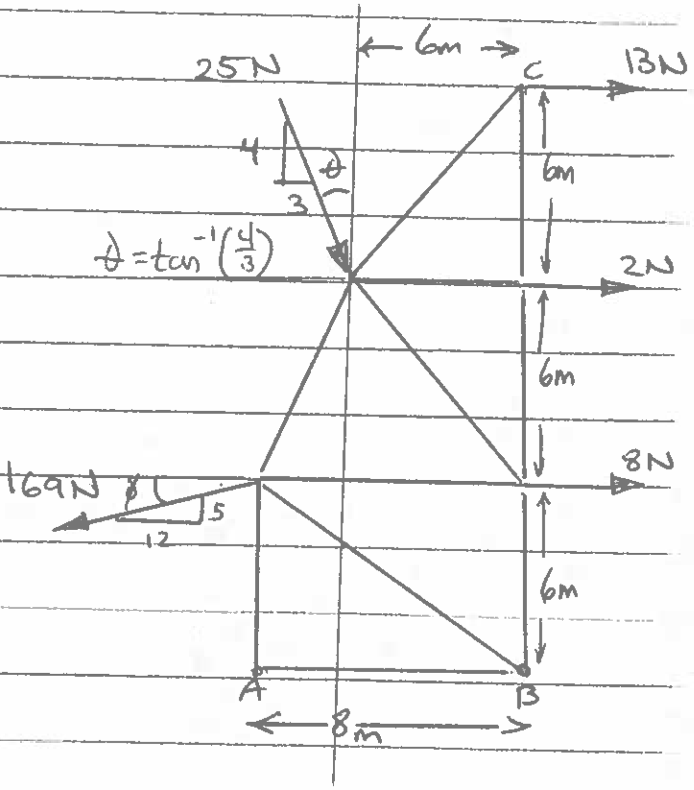
$\theta = \tan^{-1}(3/4) = 37°$

$R = \sqrt{4^2+3^2} = 5$

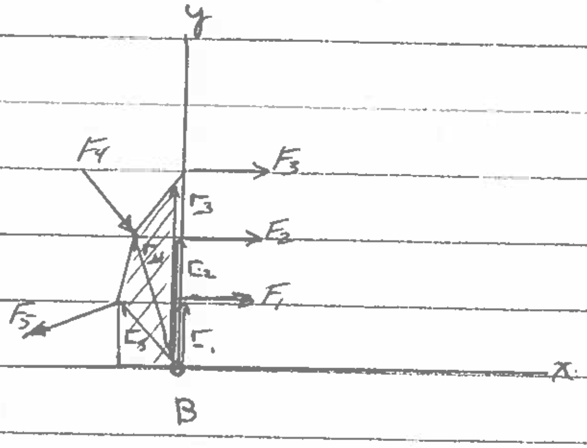


Find the resultant for the system shown.

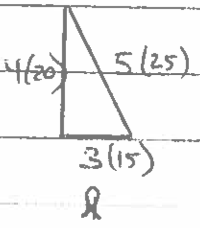
Locate the intersection of the line of action of the resultant with the segment $\overline{BC}$

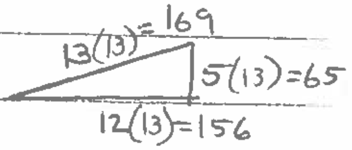


$$\underline{R} = \Sigma \underline{F}$$
$$= 8\underline{i} + 2\underline{i} + 13\underline{i}$$
$$+ 15\underline{i} - 20\underline{j}$$
$$- 156\underline{i} - 65\underline{j}$$
$$= -118\underline{i} - 85\underline{j}$$



$$\Sigma M_B = \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2 + \underline{r}_3 \times \underline{F}_3$$
$$+ \underline{F}_4 \times \underline{F}_4 + \underline{r}_5 \times \underline{F}_5$$
$$= 6\underline{j} \times 8\underline{i} + 12\underline{j} \times 2\underline{i}$$
$$+ 18\underline{j} \times 13\underline{i}$$
$$+ (-6\underline{i} + 12\underline{j}) \times (15\underline{i} - 20\underline{j})$$
$$+ (-8\underline{i} + 6\underline{j}) \times (-156\underline{i} - 65\underline{j})$$
$$= -48\underline{k} - 24\underline{k} - 234\underline{k}$$
$$+ 120\underline{k} - 180\underline{k} + 520\underline{k}$$
$$+ 936\underline{k}$$
$$= 1090\underline{k}$$





$\underline{\Sigma M_o} = \underline{\hat{r}} \times R$

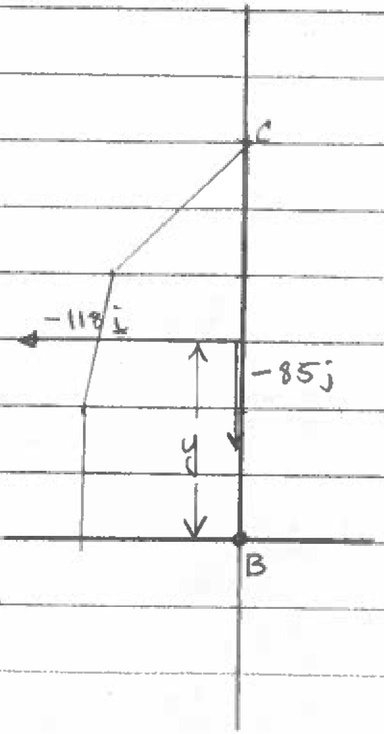
let

$\underline{\hat{r}} = y j$

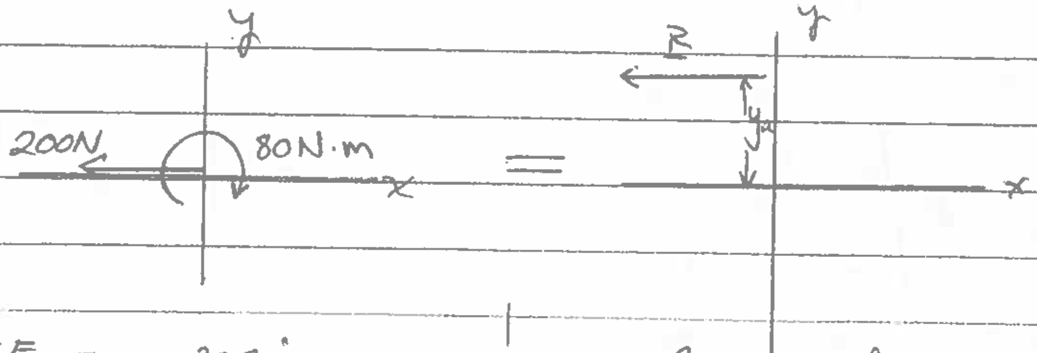
$1090 \underline{k} = (y j) \times (-118 \underline{i} - 85 j)$

$1090 = 118 y \underline{k}$

$\frac{1090}{118} = y = 9.24m$ above B.



2.55 Replace force-couple system at "O" with a single force.
Specify the coordinate ya through which the line of action of this force passes



$\underline{R} = \Sigma \underline{F} = -200\,\underline{i}$

$\Sigma M_o = -80\ \text{N·m}\ \underline{k}$

$R = -200\,\underline{i}$

$\Sigma M_o = y_a \underline{j} \times -200\,\underline{i}$

$= 200 y_a\ \underline{k}$

$\therefore\ -80\ \text{N·m}\ \underline{k} = 200\, y_a\ \underline{k}$

$\dfrac{-80\ \text{N·m}}{200\ \text{N}} = y_a\ \text{m} = \underline{\underline{-0.4\text{m}}}$ ←

2.62 Replace the 10 kN force as shown with an equivalent force-couple system at point "O"

$\underline{R} = \Sigma \underline{F}$

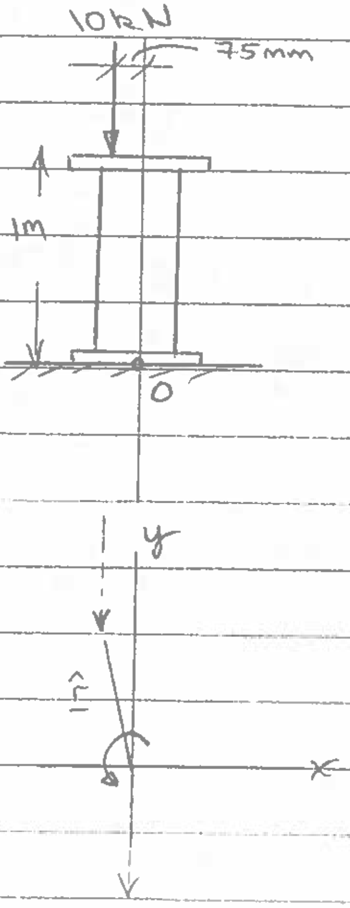
$= -10\,kN\,\underline{j}$

$\Sigma \underline{M}_O = \hat{r} \times \underline{F}$

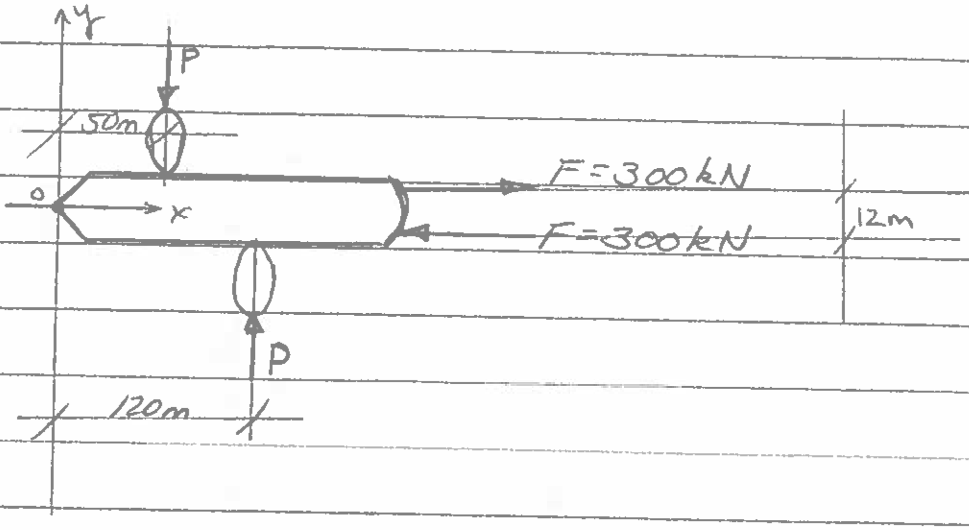
$= (-0.075\underline{i} + 1\underline{j}) \times (-10\,kN\,\underline{j})$

$= (0.075\,m)(10\,kN)\,\underline{k}$

$= 0.75\,kN \cdot m\,\underline{k}$



2.59 Ship with tugs as shown.
Find thrust P for each tug
to prevent rotation.



Bow is "O"

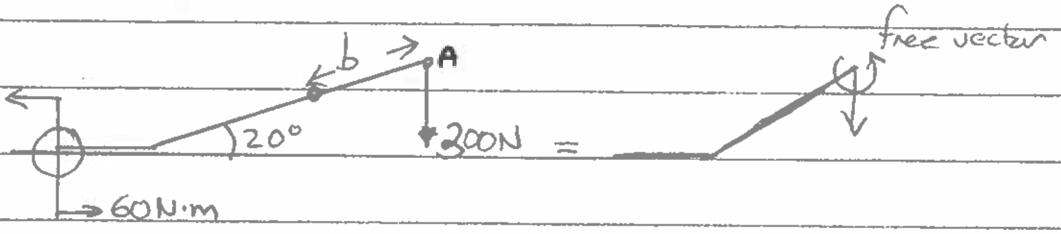
To prevent rotation $\Sigma \underline{M}_o = 0$

$$\Sigma \underline{M}_o = \Sigma \underline{r} \times \underline{F} = 6\underline{j} \times 300\underline{i} + -6\underline{j} \times -300\underline{i}$$
$$+ 50\underline{i} \times -P\underline{j} + 120\underline{i} \times P\underline{j} = 0$$

$$-1800\underline{k} - 1800\underline{k} - 50P\underline{k} + 120P\underline{k} = 0$$
$$-3600\underline{k} + 70P\underline{k} = 0$$

$$P = \frac{3600}{70} = \underline{\underline{51.4 kN}} \longleftarrow$$

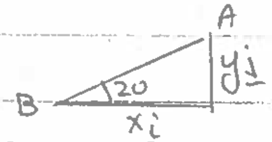
2.67 Replace the couple and force shown by a single force F applied at a point D. Locate D by finding distance b.



$\underline{R} = \Sigma \underline{F} = -300N\,j$

$\Sigma \underline{M}_A = 60 N\cdot m\ \underline{k}$ (Couple is a free vector)

Now move force to point "b"



$\Sigma M_A = \cancel{200N j \times x}$

$(-x\cos 20\,\underline{i} - y\sin 20\,\underline{j}) \times -300N\,j$

$= (x\cos 20)(300)\,\underline{k} = 60\,N\cdot m\,\underline{k}$

$$x = \frac{60\,N\cdot m}{300\,N\cos 20} = 0.2128\,m$$

$= \underline{213mm}$ ←

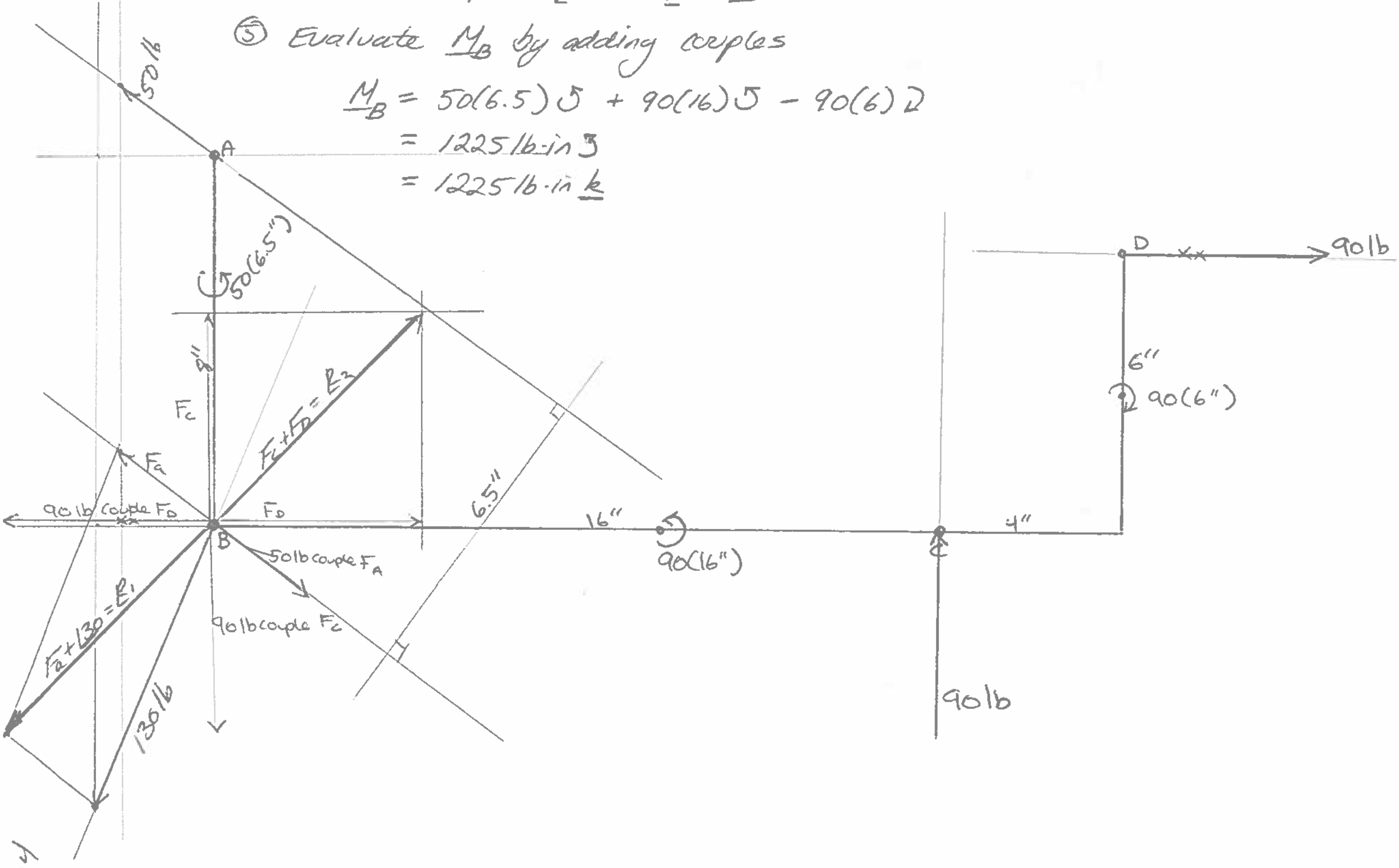
determine resultant graphically & then check by vector analysis

① Replace force at D with couple
② Replace force at C with couple
③ Replace force at A with couple
④ Evaluate force polygon at B

$$\underline{R_1} = -\underline{R_2} \quad \therefore \quad \underline{R} = \underline{0}$$

⑤ Evaluate $\underline{M_B}$ by adding couples

$$\underline{M_B} = 50(6.5)\circlearrowleft + 90(16)\circlearrowleft - 90(6)\circlearrowright$$
$$= 1225\ \text{lb·in} \circlearrowleft$$
$$= 1225\ \text{lb·in}\ \underline{k}$$



2.84

6. P.4

2.84(p2)

Vector analysis

$$\underline{R} = \Sigma \underline{F}$$

$$R_x = 90\,\text{lb}\,\underline{i} - 50\,\text{lb}\left(\tfrac{4}{5}\right)\underline{i} - 130\,\text{lb}\left(\tfrac{5}{130}\right)\underline{i}$$

$$= \underline{0}\,\underline{i}$$

$$R_y = 90\,\text{lb}\,j + 50\,\text{lb}\left(\tfrac{3}{5}\right)j - 130\,\text{lb}\left(\tfrac{12}{130}\right)j$$

$$= \underline{0}\,j$$

$$\Sigma \underline{M}_B = 8''\underline{j} \times (-40\underline{i} + 30j) + 16''\underline{i} \times (90j)$$

$$+ (20''\underline{i} + 6''j) \times (90\,\text{lb}\,\underline{i})$$

$$= \overset{320}{\cancel{240}}\,\underline{k} + 16(90)\underline{k} - 90(6)\underline{k}$$

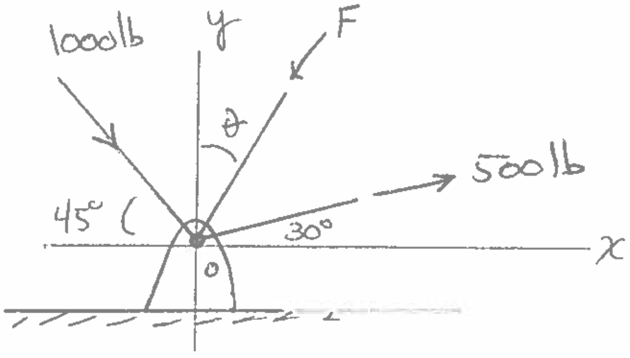
$$= 1220\,\underline{k}\ \text{lb}\cdot\text{in}$$

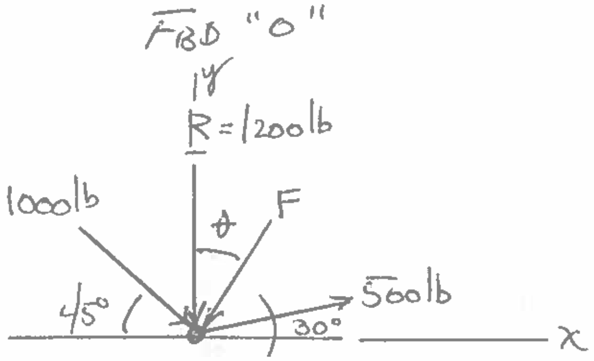
6.P.5

Resultants

2.73 Given: force system shown

Find: F, θ so that $\underline{R}$ is vertical down @1200lb.





$$\underline{R} = \Sigma \underline{F}$$

(1) $R_x = 1000\cos 45 + 500\cos 30 - F\sin\theta = 0$

(2) $R_y = -1000\sin 45 + 500\sin 30 - F\cos\theta = -1200\text{ lb}$

(1) $1140.12 = F\sin\theta$

(2) $742.89 = F\cos\theta$

(1)/(2) $\Rightarrow \tan\theta = \dfrac{1140.12}{742.89}$

$\tan\theta = 1.535$

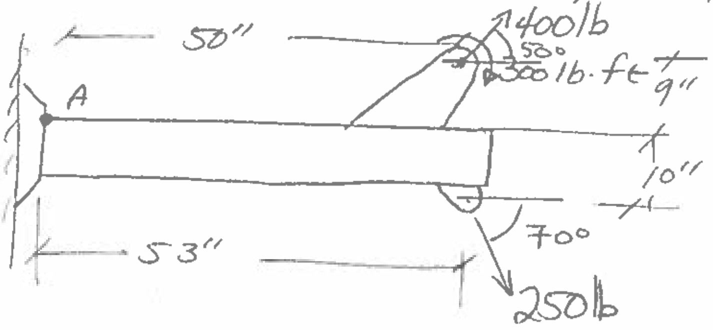
$\tan^{-1}(1.535) \approx 56.95°$

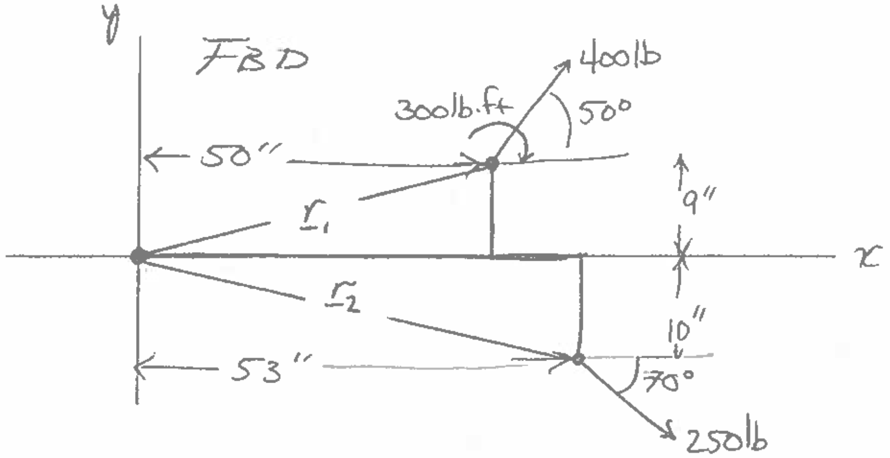
$\therefore$ (1) $F = \dfrac{1140.12}{\sin(56.95°)} = 1360.$

(2) $F = \dfrac{742.89}{\cos(56.95°)} = 1362.$

2.76 Given: Bracket, forces, couple shown.

Find: Equivalent force-couple system at A





$$\underline{R} = \Sigma \underline{F} = 400lb\cos 50\,\underline{i} + 400lb\,\overset{sin}{\cancel{cos}}50\,\underline{j} + 250lb\cos 70\,\underline{i} - 250lb\sin 70\,\underline{j}$$

$$= 257.12\,\underline{i} + 85.51\,\underline{i} + 306.42\,\underline{j} - 234.92\,\underline{j}$$

$$= 342.63\,\underline{i} + 71.5\,\underline{j}$$

$$\Sigma M_A = -300 lb\cdot ft\,\underline{k} + \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2$$

$$= -300\,\underline{k} + [(4.16\underline{i} + 0.75\underline{j}) \times (257.12\underline{i} + 306.42\underline{j})]$$

$$+ [(4.42\underline{i} - 0.83\underline{j}) \times (85.51\underline{i} - 234.9\underline{j})]$$

$$= -300\underline{k} + (4.16)(306.42)\underline{k} - (0.75)(257.12)\underline{k}$$

$$- (4.42)(234.9)\underline{k} + (0.83)(85.51)\underline{k}$$

$$= -300\,\underline{k} + 1274.7\underline{k} - 192.84\underline{k}$$

$$- 1038.26\underline{k} + 70.97\underline{k}$$

$$= -185.43\,\underline{k}$$

$$\underline{r}_1 = \frac{50}{12}\underline{i} + \frac{9}{12}\underline{j}$$

$$= 4.16\underline{i} + 0.75\underline{j}$$

$$\underline{r}_2 = \frac{53}{12}\underline{i} - \frac{10}{12}\underline{j}$$

$$= 4.42\underline{i} - 0.83\underline{j}$$

$$\underline{F}_1 = 257.12\underline{i} + \overset{306.42}{\cancel{85.51}}\,\underline{j}$$

$$\underline{F}_2 = 85.51\underline{i} - 234.92\underline{j}$$

FORCE-COUPLE SYSTEM

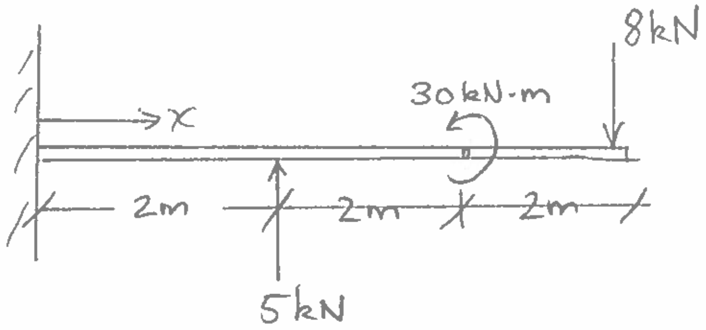
$\therefore$

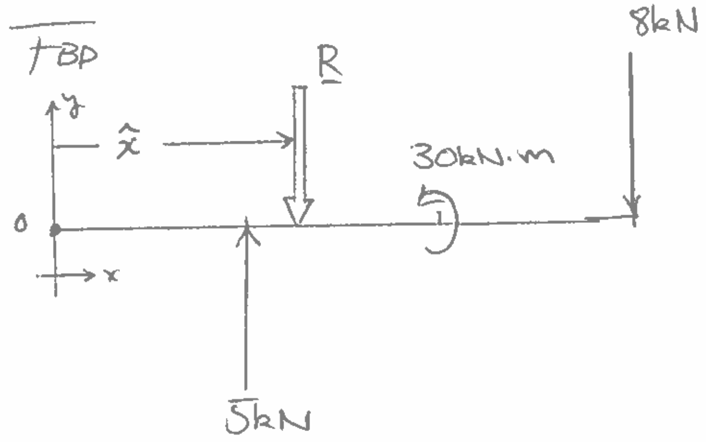
$$\underline{R} = 343\,\underline{i} + 71.5\,\underline{j}$$

$$\underline{M}_A = -185.4\ lb\cdot ft\ \underline{k}$$

6.P.2 ~~2.76~~

2.77 Determine and locate the single force $\underline{R}$ that replaces the force-couple system on the beam





$$\underline{R} = \Sigma \underline{F}$$
$$= 5\,kN\,\underline{j} - 8\,kN\,\underline{j}$$
$$= \underline{-3\,kN\,\underline{j}}$$

$$\Sigma \underline{M}_o = 30\,kN\cdot m\,\underline{k} + 2m\,\underline{i} \times 5kN\,\underline{j}$$
$$+ 6m\,\underline{i} \times -8kN\,\underline{j}$$
$$= 30kN\cdot m\,\underline{k} + 10kN\cdot m\,\underline{k}$$
$$-48\,kN\cdot m\,\underline{k}$$
$$= \overset{-8}{\cancel{22}}\,kN\cdot m\,\underline{k}$$

$$\Sigma \underline{M}_o = \hat{x} \times \underline{R}$$
$$= \hat{x}\,\underline{i} \times -3kN\,\underline{j}$$
$$= -3\hat{x}\,kN\cdot m\,\underline{k} = -8\,kN\cdot m\,\underline{k}$$
$$\hat{x} = \frac{-8\,kN\cdot m}{-3\,kN} = \underline{+2.67\,m}$$

$\therefore$ $\underline{R} = -3kN\,\underline{j}$

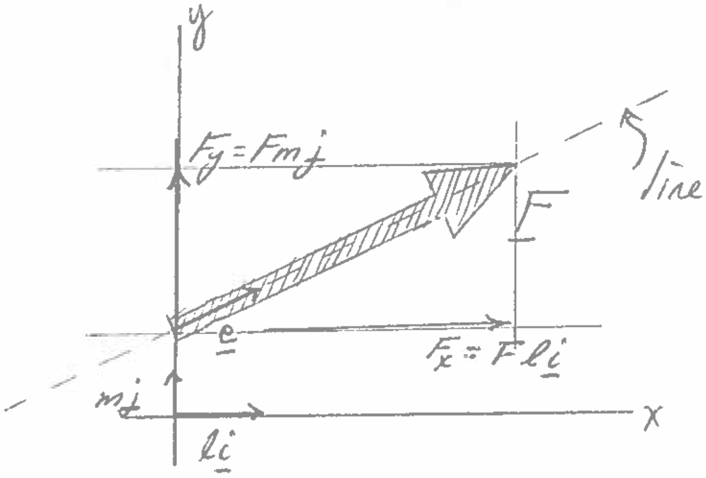
located 2.67m from support

let $l = \cos\theta_x$, $m = \cos\theta_y$, $n = \cos\theta_z$

then $$\underline{F} = F\underbrace{(l\underline{i} + m\underline{j} + n\underline{k})}_{\text{vector of magnitude 1}}$$

∴ it is a unit vector along the line of action of force $\underline{F}$

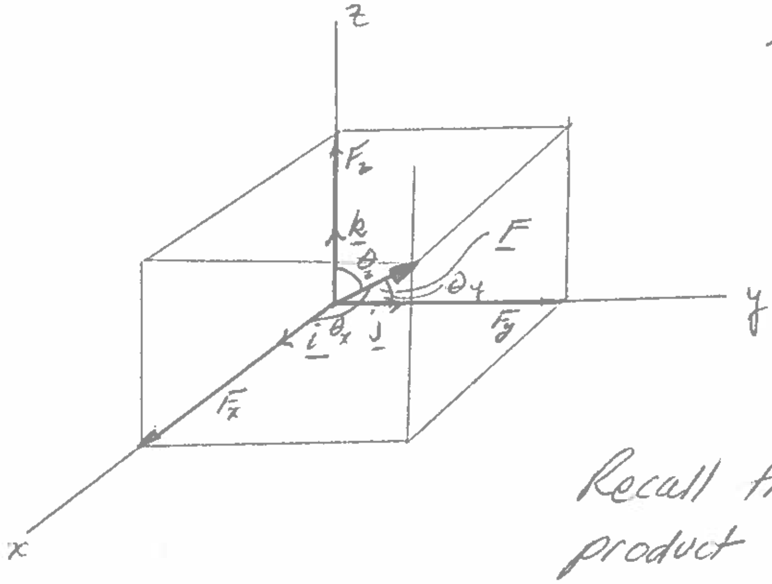
So $\underline{F} = F\underline{e}$ where $\underline{e}$ is a unit vector in the direction of the force



The *projection* of a force $\underline{F}$ in a given direction is defined as the scalar product of $\underline{F}$ and a unit vector in the given direction $\underline{e}_\lambda$.

$$F_\lambda = \underline{F}\cdot\underline{e}_\lambda = F\underline{e}\cdot\underline{e}_\lambda$$

Three dimensional force systems – Rectangular Components



In 3D

$$\underline{F} = F_x \underline{i} + F_y \underline{j} + F_z \underline{k}$$

$$F_x = F \cos\theta_x$$

$$F_y = F \cos\theta_y$$

$$F_z = F \cos\theta_z$$

Recall from defn. of dot product

$$\left.\begin{aligned} \cos\theta_x &= \frac{\underline{i}\cdot\underline{F}}{(\underline{F}\cdot\underline{F})^{1/2}} \\ \cos\theta_y &= \frac{\underline{j}\cdot\underline{F}}{(\underline{F}\cdot\underline{F})^{1/2}} \\ \cos\theta_z &= \frac{\underline{k}\cdot\underline{F}}{(\underline{F}\cdot\underline{F})^{1/2}} \end{aligned}\right\}$$

direction cosines of force $\underline{F}$

Thus $\underline{F}$ can also be written as

$$\underline{F} = F(\cos\theta_x \underline{i} + \cos\theta_y \underline{j} + \cos\theta_z \underline{k})$$

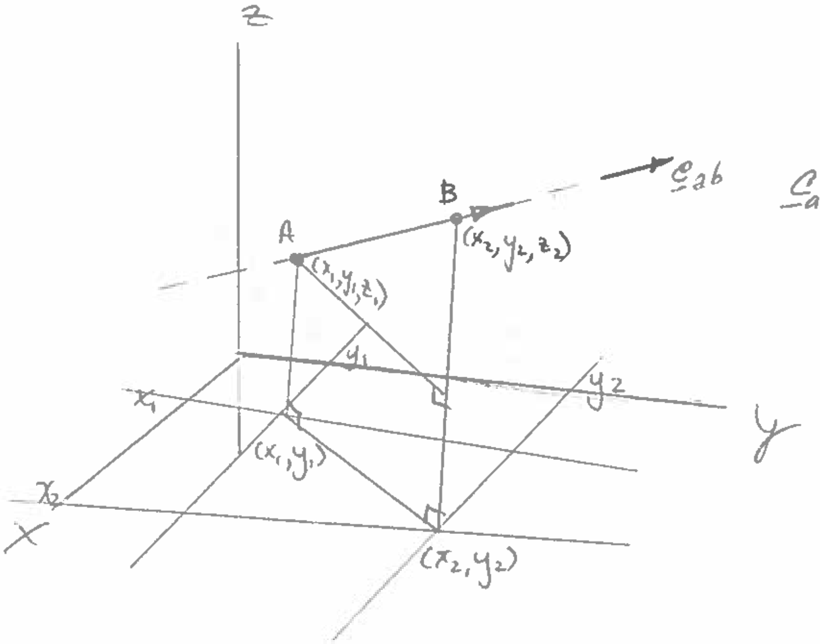
Observe

$$(\underline{F}\cdot\underline{F})^{1/2} = (F^2(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z))^{1/2} = F$$

$$\Rightarrow \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

Applications

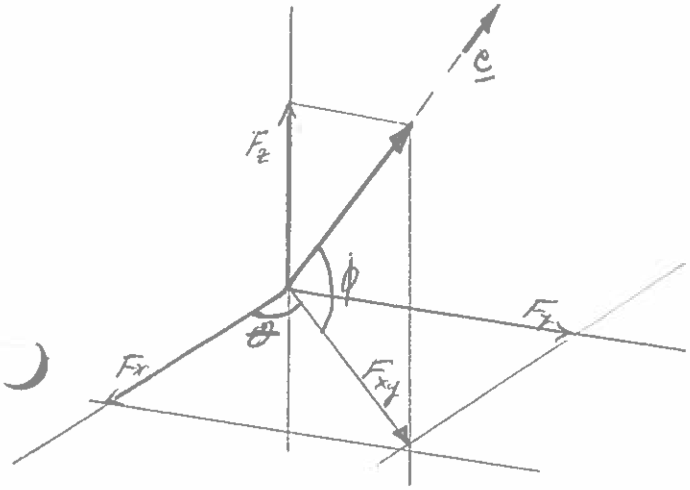
(a) Know two points along the line of action of the force



$$\underline{F} = F\underline{e}_{ab}$$

$$\underline{e}_{ab} = \frac{(x_2 - x_1)\underline{i} + (y_2 - y_1)\underline{j} + (z_2 - z_1)\underline{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

(b) You know the angle the force makes with a plane and the orientation of the projection of the force onto the plane



$$F_{xy} = F\cos\phi$$

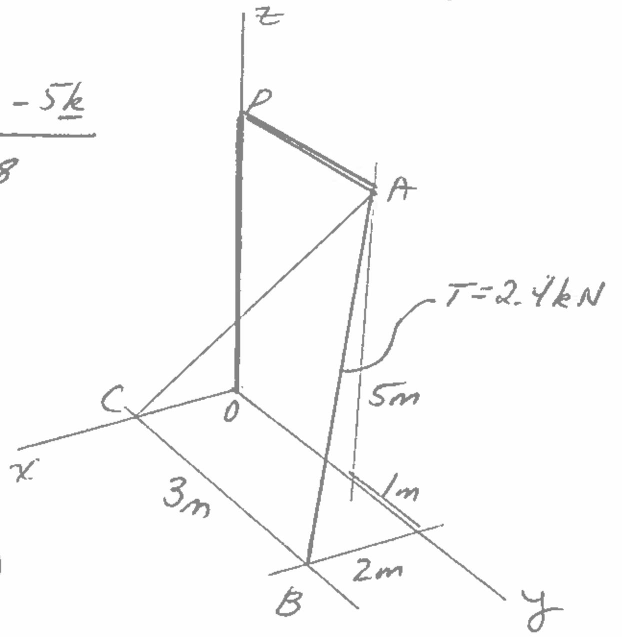
$$F_z = F\sin\phi$$

$$F_x = F_{xy}\cos\theta = F\cos\phi\cos\theta$$

$$F_y = F_{xy}\sin\theta = F\cos\phi\sin\theta$$

$$\underline{e} = \cos\phi\cos\theta\,\underline{i} + \cos\phi\sin\theta\,\underline{j} + \sin\phi\,\underline{k}$$

2.97 The turnbuckle is set so $T = 2.4\text{kN}$ in cable AB.
Find $\underline{T}$ as a force acting on member AD.
Find magnitude of projection of T along AC.



$$\underline{e}_{ab} = \frac{-2\underline{i} + \underline{j} - 5\underline{k}}{\sqrt{2^2+1^2+5^2}} = \frac{-2\underline{i}+\underline{j}-5\underline{k}}{5.48}$$

$$= -0.365\underline{i} + 0.183\underline{j} - 0.912\underline{k}$$

$$\underline{T} = T\underline{e}_{ab}$$

$$= 2.4(-0.365\underline{i} + 0.183\underline{j} - 0.912\underline{k})$$

$$\underline{T} = \underline{\underline{-0.876\underline{i} + 0.439\underline{j} - 2.19\underline{k}}} \longleftarrow$$

Projection of $\underline{T}$ on AC

$$\underline{e}_{ac} = \frac{-2\underline{i} - 2\underline{j} - 5\underline{k}}{\sqrt{2^2+2^2+5^2}} = \frac{-2\underline{i} - 2\underline{j} - 5\underline{k}}{5.745}$$

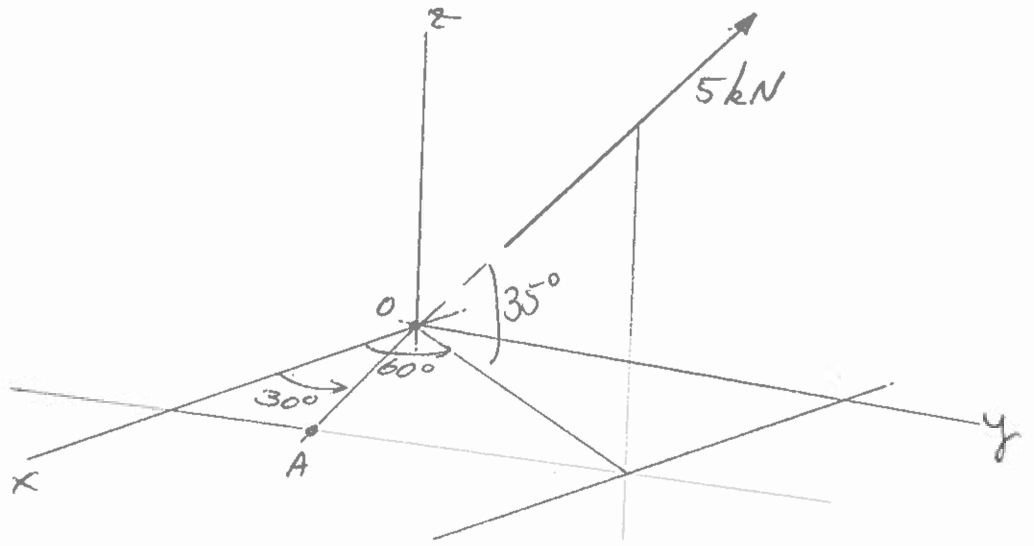
$$= -0.348\underline{i} - 0.348\underline{j} - 0.870\underline{k}$$

$$T_{ac} = \underline{T} \cdot \underline{e}_{ac} = T\underline{e}_{ab} \cdot \underline{e}_{ac}$$

$$= 2.4(-0.365\underline{i} + 0.183\underline{j} - 0.912\underline{k}) \cdot (-0.348\underline{i} - 0.348\underline{j} - 0.870\underline{k})$$

$$= 2.4(0.856) = \underline{\underline{2.056\text{ kN}}} \longleftarrow$$

2.95 Express $\underline{F}$ in rectangular (x,y,z) components. Find projection of $\underline{F}$ onto line $\underline{OA}$.



Find direction cosines

$F_z = 5kN \sin 35°$
$= 2.867\ kN$

$F_{xy} = 5kN \cos 35°$
$F_x = F_{xy} \cos 60°$
$F_y = F_{xy} \sin 60°$

$\therefore F_x = 5 \cos 35 \cos 60 = 2.048\ kN$
$F_y = 5 \cos 35 \sin 60 = 3.547\ kN$

$\underline{e}_{OA} = \cos 30 \underline{i} + \sin 30 \underline{j}$

$F_{OA} = \underline{F} \cdot \underline{e}_{OA}$

$= (2.048\underline{i} + 3.547\underline{j} + 2.867\underline{k}) \cdot (\cos 30 \underline{i} + \sin 30 \underline{j} + 0\underline{k})$

$= 3.547$

$\therefore$
$F_x = 2.05kN$
$F_y = 3.55kN$
$F_z = 2.87kN$

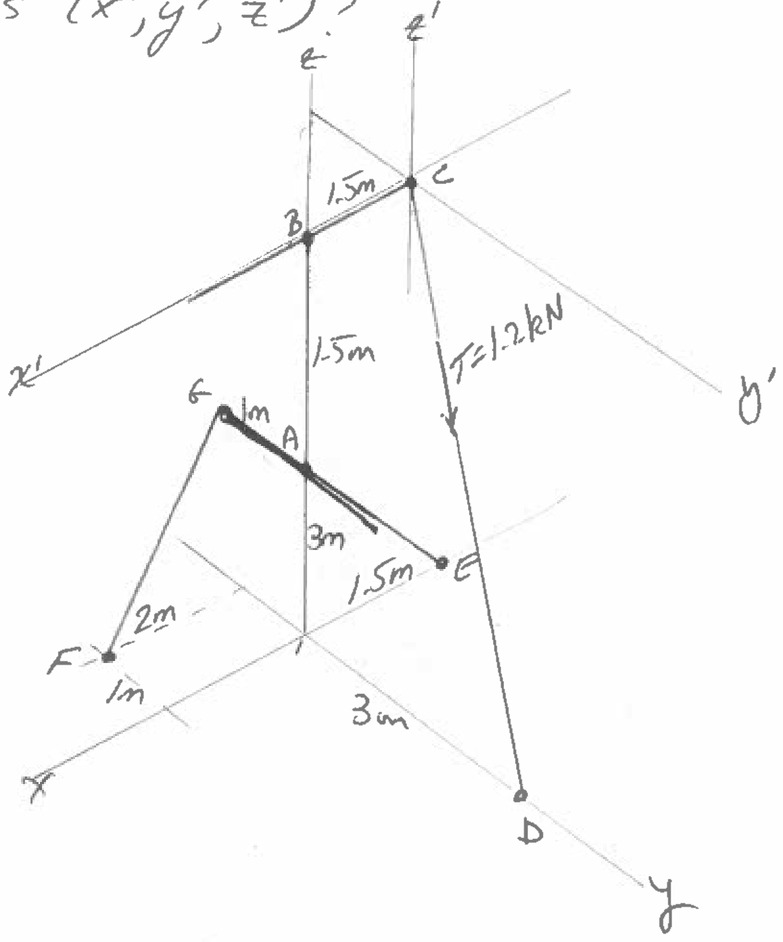
$F_{OA} = 3.55kN$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\} \longleftarrow$

2.99 Given: Assembly shown

Find: $\underline{T}$ as a vector.

Does it matter which coordinate system is used? (x, y, z) vs (x', y', z')?



In x, y, z system

$$\underline{e}_{CD} = \frac{1.5\underline{i} + 3\underline{j} - 4.5\underline{k}}{\sqrt{1.5^2+3^2+4.5^2}}$$

$$= \frac{1.5}{5.61}\underline{i} + \frac{3}{5.61}\underline{j} - \frac{4.5}{5.61}\underline{k}$$

In x', y', z' system

$$\underline{e}_{CD} = \frac{1.5\underline{i}' + 3\underline{j}' - 4.5\underline{k}'}{\sqrt{1.5^2+3^2+4.5^2}}$$

$$= \frac{1.5}{5.61}\underline{i}' + \frac{3}{5.61}\underline{j}' - \frac{4.5}{5.61}\underline{k}'$$

∴ both systems are equivalent for <u>this</u> problem

$$\underline{T} = T\underline{e}_{CD} = 1.2\,kN\,(0.267\underline{i} + 0.535\underline{j} - 0.802\underline{k})$$

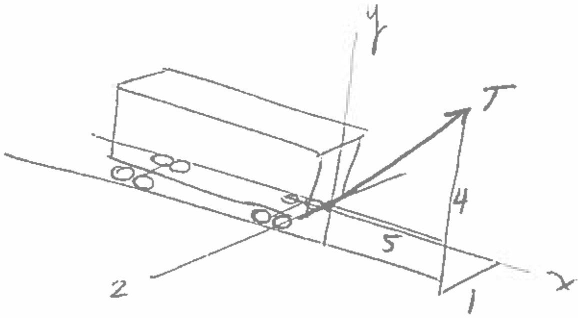
$$= 0.320\underline{i} + 0.642\underline{j} - 0.962\underline{k}$$

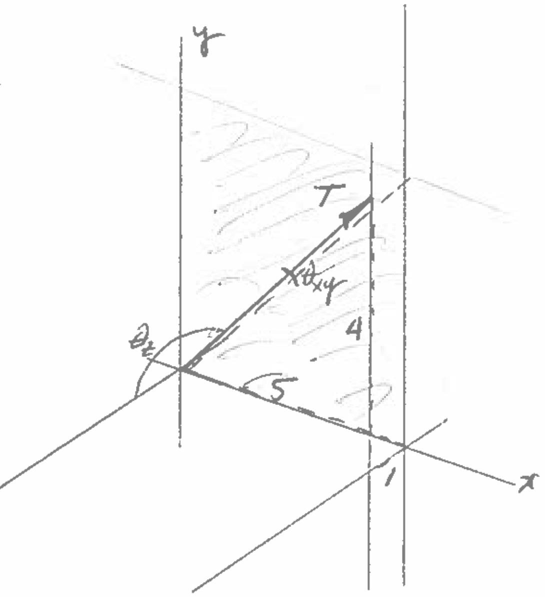
2.104

Given: Boxcar as shown ~~—~~ crane used to reposition.

Find: T required in cable so $T_x = 600$ lb.

θ_{xy} between cable & x-y plane





$$\underline{T} = T\underline{e}_2$$

$$\underline{e}_2 = \frac{5\underline{i} + 4\underline{j} + 1\underline{k}}{\sqrt{5^2+4^2+1^2}}$$

$$= \frac{5}{6.48}\underline{i} + \frac{4}{6.48}\underline{j} + \frac{1}{6.48}\underline{k}$$

$$= 0.772\underline{i} + 0.617\underline{j} + 0.154\underline{k}$$

$$\underline{T} = T(0.772\underline{i} + 0.617\underline{j} + 0.154\underline{k})$$

$T(0.772)\underline{i} = 600$ lb (given)

$\therefore T = \frac{600}{.772} = 777.2$ lb

$\theta_{xy} = 90 - \theta_z$

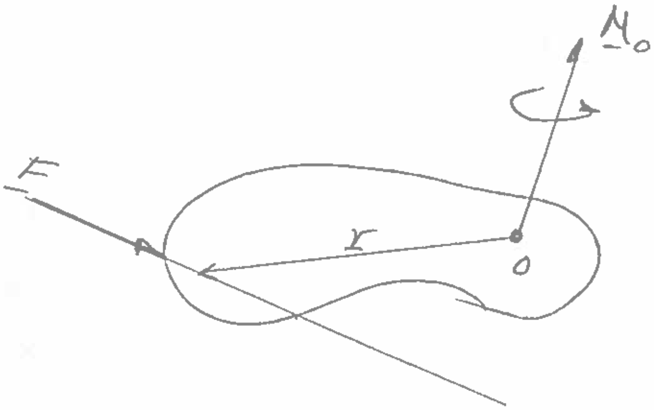
$\cos\theta_z = 0.154$

$\theta_z = 81.50°$

$\therefore \theta_{xy} = 8.5°$

3D Force Systems – Moment and Couple

3D moments and couples are determined using the vector relationships already developed



The moment of force $\underline{F}$ about moment center O is $\underline{M}_o = \underline{r} \times \underline{F}$ where $\underline{r}$ is a position vector <u>from</u> "O" to <u>any point</u> on the line of action of $\underline{F}$.

The moment $\underline{M}_\lambda$ about any axis λ passing through 0 is given by

$$\underline{M}_\lambda = (\underline{M}_o \cdot \underline{e}_\lambda)\underline{e}_\lambda$$

$$= (\underline{r} \times \underline{F} \cdot \underline{e}_\lambda)\underline{e}_\lambda$$

Where $\underline{e}_\lambda$ is a unit vector in the λ-direction

$\underline{r} \times \underline{F} \circ \underline{e}_\lambda$ can be evaluated as a mixed triple product

$$\underline{r} \times \underline{F} \cdot \underline{e}_\lambda = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ e_x & e_y & e_z \end{vmatrix} = \begin{aligned} & r_x F_y e_z - r_x F_z e_y \\ & + r_y F_z e_x - r_y F_x e_z \\ & + r_z F_x e_y - r_z F_y e_x \end{aligned}$$

The mixed triple product is a scalar!

$$\therefore \; |\underline{M}_\lambda| = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ e_x & e_y & e_z \end{vmatrix}$$

Varingon's theorem is unchanged.

The sum of moments about a given moment center of a system of concurrent forces is equal to the moment of their resultant.

$$\underline{M}_o = \underline{r} \times \underline{R} = \Sigma(\underline{r} \times \underline{F})$$

where $\underline{R} = \Sigma \underline{F}$

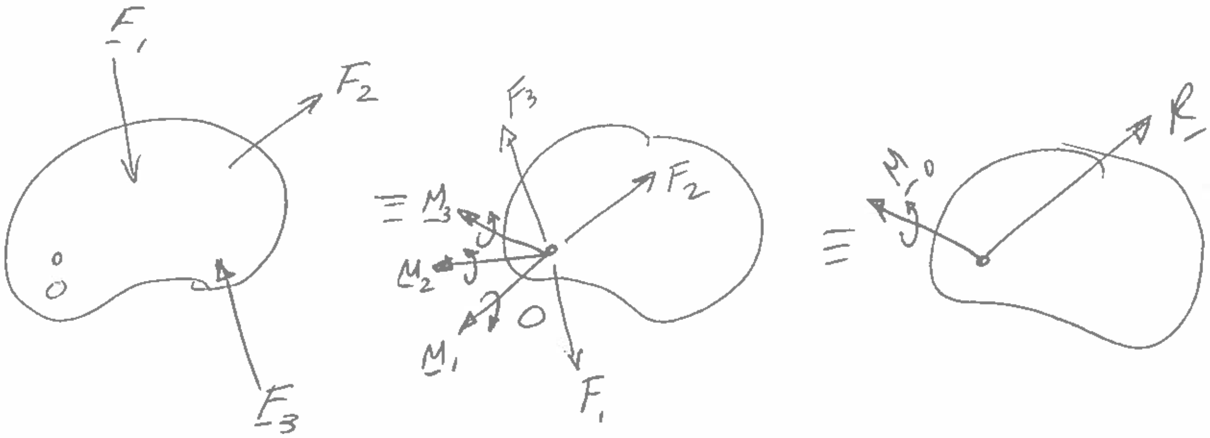
Couples are unchanged in 3D. The moment of the couple is

$$\underline{M} = \underline{r}' \times \underline{F}$$

where $\underline{r}'$ is a vector joining <u>any two points</u> on the lines of action of $^{+}\underline{F}$ and $-\underline{F}$.

Resultants

The concept of resultant is unchanged. Arbitrary 3D force systems may be reduced to a resultant force $\underline{R}$ and a resultant couple $\underline{M}_0$ about a specified point



Each force in the system is replaced by a force and couple acting at a concurrent point, then their individual effects summed:

$$\underline{R} = \Sigma \underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3 + \cdots$$

$$\underline{M}_o = \underline{M}_1 + \underline{M}_2 + \underline{M}_3 + \cdots = \Sigma(\underline{r} \times \underline{F})$$

The selection of "O" is arbitrary, the value of $\underline{M}_o$ (but not $\underline{R}$) will depend upon this point.

Special Cases

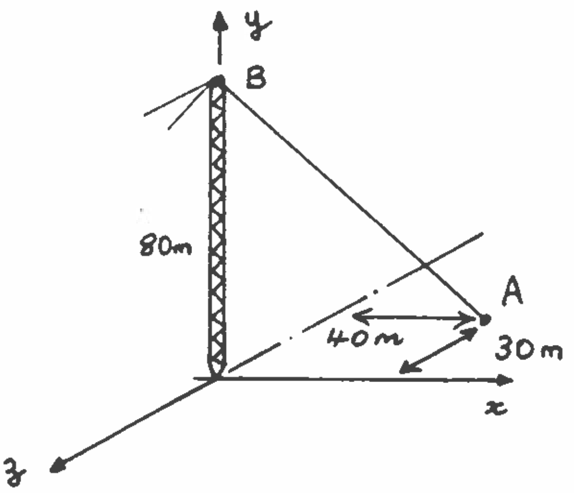
Concurrent forces – when all forces act at a single point, only $\underline{R}$. $\underline{M}_o = 0$

parallel forces – resultant $\underline{R}$ and moment $\underline{M}_o$ will be perpindicular.

wrench resultant – when $\underline{R}$ and $\underline{M}_o$ are parallel, the resultant is called a wrench, either positive or negative.

EXAMPLE 1 : Rectangular Components (3D) - Forces

A tower guy wire is anchored by a bolt at A. The tension in the wire is 2500 N. Find (a) the components F_x, F_y, F_z of the force acting on the bolt; (b) the angles $\theta_x, \theta_y, \theta_z$ defining the force direction.



Force on bolt is directed from A to B. The components of the vector $\overrightarrow{AB} = \underline{r}_B - \underline{r}_A$ are $\underline{r}_{AB} = -40\underline{i} + 30\underline{k} + 80\underline{j}$.

A unit vector in the direction AB is $\underline{e}_{AB} = \underline{r}_{AB} / r_{AB} \Rightarrow$

$$\underline{e}_{AB} = \{-40\underline{i} + 80\underline{j} + 30\underline{k}\} / \{40^2 + 30^2 + 80^2\}^{1/2}$$

$$= -.424\,\underline{i} + .848\,\underline{j} + .318\,\underline{k}$$

$$\underline{F} = F\,\underline{e}_{AB} = 2500\,\underline{e}_{AB} = -1060\,\underline{i} + 2120\,\underline{j} + 795\,\underline{k}$$

So $\quad \underline{\underline{F_x = -1060 \text{ N}, \quad F_y = 2120 \text{ N}, \quad F_z = 795 \text{ N.}}}$ (a)

Also,

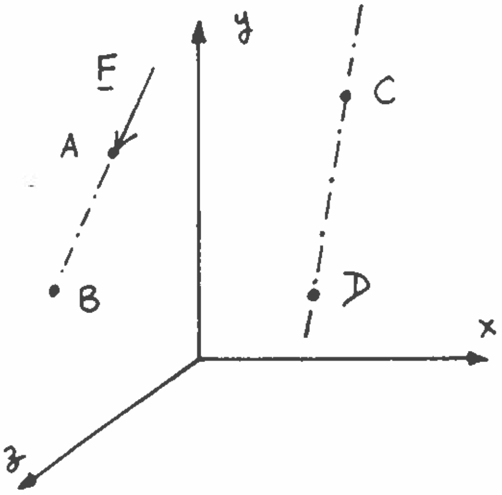
$$\cos\theta_x = \underline{e}_{AB} \cdot \underline{i} = -.424 \rightarrow \underline{\underline{\theta_x = 115.1^\circ}}$$

$$\cos\theta_y = \underline{e}_{AB} \cdot \underline{j} = .848 \rightarrow \underline{\underline{\theta_y = 32.0^\circ}}$$

$$\cos\theta_z = \underline{e}_{AB} \cdot \underline{k} = .318 \rightarrow \underline{\underline{\theta_z = 71.5^\circ}}$$ (b)

EXAMPLE 2 : Rectangular Components (3D) - Forces

A force $\underline{F}$ has a magnitude of 8 N and a line of action through points A and B with coordinates (0, 6, 5) and (-4, 0, 2), respectively. $\underline{F}$ is directed from A to B. Find the component of $\underline{F}$ along the line joining points C and D with coordinates (0, 3, -4) and (4, 1, 2). All coordinates are in meters.



Let $\underline{r}_{AB}$ and $\underline{r}_{CD}$ be vectors directed from A → B and C → D

$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = (-4\underline{i} + 2\underline{k}) - (6\underline{j} + 5\underline{k}) = -4\underline{i} - 6\underline{j} - 3\underline{k} \quad m$$

$$\underline{r}_{CD} = \underline{r}_D - \underline{r}_C = (4\underline{i} + \underline{j} + 2\underline{k}) - (3\underline{j} - 4\underline{k}) = 4\underline{i} - 2\underline{j} + 6\underline{k} \quad m$$

Corresponding unit vectors are

$$\underline{e}_{AB} = \underline{r}_{AB} / r_{AB} = -.512\,\underline{i} - .768\,\underline{j} - .384\,\underline{k}$$

$$\underline{e}_{CD} = \underline{r}_{CD} / r_{CD} = .535\,\underline{i} - .267\,\underline{j} + .802\,\underline{k}$$

Vector representation of $\underline{F} = F\underline{e}_{AB} = 8\,\underline{e}_{AB}$

$$= -4.10\,\underline{i} - 6.14\,\underline{j} - 3.07\,\underline{k} \quad N$$

Component of $\underline{F}$ in direction C → D $\quad F_{CD} = \underline{F} \cdot \underline{e}_{CD}$, so

$$F_{CD} = (-4.10\,\underline{i} - 6.14\,\underline{j} - 3.07\underline{k}) \cdot (.535\,\underline{i} - .267\,\underline{j} + .802\,\underline{k})$$

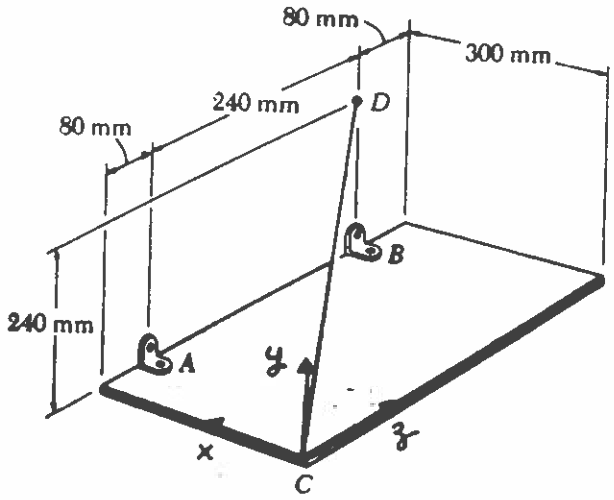
$$= -3.02 \quad N$$

Vector $\underline{F}_{CD} = F_{CD}\,\underline{e}_{CD} = -3.02\,\underline{e}_{CD}$

$$= -.162\,\underline{i} + .806\,\underline{j} - 2.42\,\underline{k} \quad N$$

EXAMPLE 1 : Moments in Three Dimensions

A rectangular plate is supported by brackets at A and B and by a wire CD. If the tension in the wire is 800 N, find the moment about A of the force exerted by the wire on point C.



Use rectangular coordinates, origin at C.

First, determine $\underline{F}$ in vector form. Let $\underline{r}_{CD} = \underline{r}_D - \underline{r}_C$, then
$\underline{r}_{CD} = .3\underline{i} + .24\underline{j} + .32\underline{k}$ m

Unit vector $\underline{e}_{CD} = \underline{r}_{CD} / r_{CD}$
$= .6\underline{i} + .48\underline{j} + .64\underline{k}$

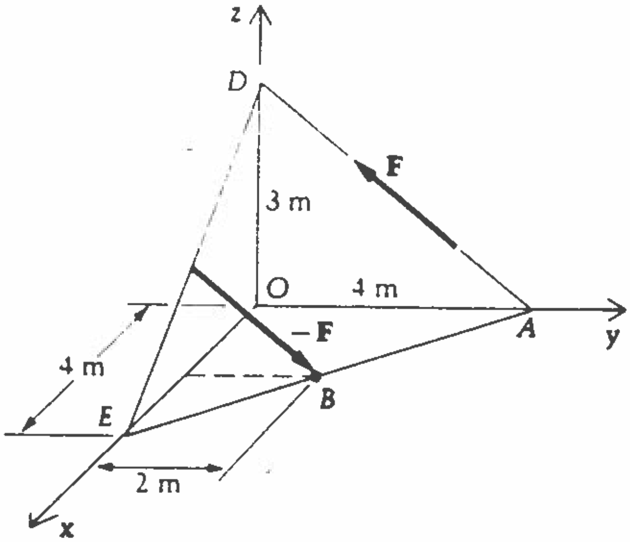
Tension force $\underline{F} = 200\,\underline{e}_{CD} = 120\underline{i} + 96\underline{j} + 128\underline{k}$ N

Position vector of C from A, $\underline{r} = \underline{r}_C - \underline{r}_A = -(.3\underline{i} + .08\underline{k})$ m

Moment $\underline{M}_A = \underline{r} \times \underline{F}$
$= \{-.3\underline{i} - .08\underline{k}\} \times \{120\underline{i} + 96\underline{j} + 128\underline{k}\}$ Nm
$= 7.68\underline{i} + 28.8\underline{j} - 28.8\underline{k}$

EXAMPLE 2 : Moments in Three Dimensions

The two forces shown have magnitudes of 50N and are oppositely directed. Find the moment of the couple that they constitute. By taking moments about two different points, show that the moment is independent of the point of calculation.



First, express $\underline{F}$ in vector form. Let $\underline{r}_{AD} = \underline{r}_D - \underline{r}_A$, then

$\underline{r}_{AD} = 3\underline{k} - 4\underline{j}$ m.

Unit vector $\underline{e}_{AD} = \underline{r}_{AD} / r_{AD} = \cdot 6\underline{k} - \cdot 8\underline{j}$.

Force $\underline{F} = 50\,\underline{e}_{AD} = 30\underline{k} - 40\underline{j}$ N

Choose origin as one point :

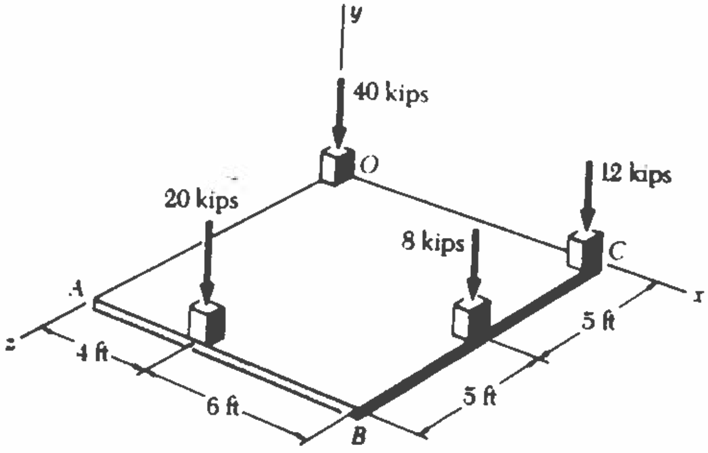
$$
\begin{aligned}
\underline{M}_o &= \underline{r}_{OA} \times \underline{F} + \underline{r}_{OB} \times -\underline{F} \\
&= (\underline{r}_{OA} - \underline{r}_{OB}) \times \underline{F} \\
&= \{4\underline{j} - (2\underline{i} + 2\underline{j})\} \times \{30\underline{k} - 40\underline{j}\} \\
&= (2\underline{j} - 2\underline{i}) \times (30\underline{k} - 40\underline{j}) \\
&= 60\underline{i} + 60\underline{j} + 80\underline{k} \quad \text{Nm}
\end{aligned}
$$

Choose A as second point :

$$
\begin{aligned}
\underline{M}_A &= 0 + \underline{r}_{AB} \times -\underline{F} \\
&= (\underline{r}_B - \underline{r}_A) \times -\underline{F} \\
&= \{2\underline{i} + 2\underline{j} - 4\underline{j}\} \times \{-30\underline{k} + 40\underline{j}\} \\
&= 60\underline{i} + 60\underline{j} + 80\underline{k} \quad \text{N m} \\
&= \underline{M}_o
\end{aligned}
$$

EXAMPLE 1 : Resultants in Three Dimensions

A square foundation mat supports the four columns shown. Find the magnitude and point of application of the resultant of the 4 loads.



First, reduce system of forces to a force couple system at O, then $\underline{R} = \Sigma \underline{F}$ and $\underline{M}_o = \Sigma (\underline{r} \times F)$

$$\underline{R} = -40\underline{j} - 12\underline{j} - 20\underline{j} - 8\underline{j} = -80\underline{j} \quad \text{kips}$$

$$\begin{aligned} \underline{M}_o &= 10\underline{i} \times (-12\underline{j}) + (10\underline{i} + 5\underline{k}) \times (-8\underline{j}) + (4\underline{i} + 10\underline{k}) \times (-20\underline{j}) \\ &= -120\underline{k} + (40\underline{i} - 80\underline{k}) + (200\underline{i} - 80\underline{k}) \\ &= 240\underline{i} - 280\underline{k} \quad \text{kip.ft} \end{aligned}$$

Since the force $\underline{R}$ and couple $\underline{M}_o$ are mutually perpendicular, system may be further reduced to a single force $\underline{R}$ acting at a point A (x, z). This is the point of application of the resultant. Then, to give the same moment about O,

$$\underline{r} \times \underline{R} = \underline{M}_o$$

$$(x\underline{i} + z\underline{k}) \times (-80\underline{j}) = 240\underline{i} - 280\underline{k}$$

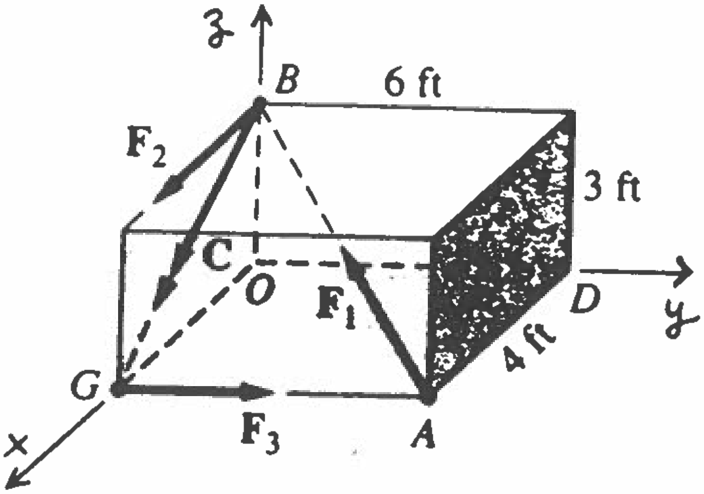
$$-80x\underline{k} + 80z\underline{i} = 240\underline{i} - 280\underline{k}$$

Equating components : $x = 3.5$ ft, $z = 3.0$ ft

Resultant of system of forces is force of 80 kips acting vertically downwards at a point $(x, z) = (3.5 \text{ ft}, 3.0 \text{ ft})$ relative to point O.

EXAMPLE 2 : Resultants in Three Dimensions

Determine the wrench equal to the force system shown. Find the coordinates of the point where the axis of the wrench crosses the xy-plane. Use as data, $C = 200$ lb.ft ; $F_1 = 100$ lb ; $F_2 = 90$ lb ; $F_3 = 120$ lb.



First reduce the system to a force-couple system acting at O.

Express $\underline{F}_1$, $\underline{F}_2$ and $\underline{F}_3$ in vector form :

$$\underline{F}_1 = 100\, \underline{e}_{AB} = 100\,(\underline{r}_B - \underline{r}_A) / |\underline{r}_B - \underline{r}_A|$$

$$= 100\,(3\underline{k} - 4\underline{i} - 6\underline{j}) / 7.81$$

$$= -51.2\,\underline{i} - 76.8\,\underline{j} + 38.4\,\underline{k} \quad \text{lb}$$

$$\underline{F}_2 = 90\,\underline{i} \quad \text{lb}$$

$$\underline{F}_3 = 120\,\underline{j} \quad \text{lb}$$

Also express the couple $\underline{C}$ in vector form

$$\underline{C} = 200\, \underline{e}_{BG} = 200\,(\underline{r}_G - \underline{r}_B) / |\underline{r}_G - \underline{r}_B|$$

$$= 200\,(4\underline{i} - 3\underline{k})/5$$

$$= 160\,\underline{i} - 120\,\underline{k} \quad \text{lb.ft}$$

Resultant : $\underline{R} = \Sigma \underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$

$$= 38.8\,\underline{i} + 43.2\,\underline{j} + 38.4\,\underline{k} \quad \text{lb}$$

Moments about O : $\Sigma \underline{M}_o = \underline{r}_{OB} \times \underline{F}_1 + \underline{r}_{OB} \times \underline{F}_2$

$$+ \underline{r}_{OG} \times \underline{F}_3 + \underline{C}$$

$$
\begin{aligned}
\Sigma \underline{M}_o &= 3\underline{k} \times (-51.2\underline{i} - 76.8\underline{j} + 38.4\underline{k}) + 3\underline{k} \times 90\underline{i} \\
&+ 4\underline{i} \times 120\underline{j} + (160\underline{i} - 120\underline{k}) \\
&= -153.6\underline{j} + 230.4\underline{i} + 270\underline{j} + 480\underline{k} \\
&\quad + 160\underline{i} - 120\underline{k} \\
&= 390\underline{i} + 116\underline{j} + 360\underline{k}
\end{aligned}
$$

The axis of the wrench is in the direction of $\underline{R}$, a unit vector in this direction, $\underline{e}_\lambda$, is defined by

$$
\begin{aligned}
\underline{e}_\lambda &= \underline{R}/R = (38{\cdot}8\underline{i} + 43{\cdot}2\underline{j} + 38{\cdot}4\underline{k})/69{\cdot}6 \\
&= {\cdot}557\underline{i} + {\cdot}621\underline{j} + {\cdot}552\underline{k}
\end{aligned}
$$

The component of $\underline{M}_o$ in the λ-direction has magnitude

$$
\begin{aligned}
M_\lambda = \underline{M}_o . \underline{e}_\lambda &= (390\underline{i} + 116\underline{j} + 360\underline{k}).({\cdot}557\underline{i} + {\cdot}621\underline{j} + {\cdot}552\underline{k}) \\
&= 488 \text{ lb.ft}
\end{aligned}
$$

giving

$$\underline{M}_\lambda = M_\lambda \underline{e}_\lambda = 272\underline{i} + 303\underline{j} + 269\underline{k} \quad \text{lb.ft}$$

<u>Wrench system</u> consists of

$$
\begin{aligned}
\underline{R} &= 38{\cdot}8\underline{i} + 43.2\underline{j} + 38{\cdot}4\underline{k} \quad \text{lb} \\
\underline{M}_\lambda &= 272\underline{i} + 303\underline{j} + 269\underline{k} \quad \text{lb.ft}
\end{aligned}
$$

Above moment $\underline{M}_\lambda$ is parallel to $\underline{R}$ (wrench), component perpendicular to $\underline{R}$ is given by

$$
\begin{aligned}
\underline{M}_\mu &= \underline{M}_o - \underline{M}_\lambda \\
&= (390\underline{i} + 116\underline{j} + 360\underline{k}) - (272\underline{i} + 303\underline{j} + 269\underline{k}) \\
&= 118\underline{i} - 187\underline{j} + 91\underline{k} \quad \text{lb. ft}
\end{aligned}
$$

Since moment caused by $\underline{R}$ is perpendicular to $\underline{R}$, we require that this moment only be balanced by $\underline{M}_\mu$, i.e.

$$\underline{r} \times \underline{R} = \underline{M}_\mu$$

Given $\underline{r} = x\underline{i} + y\underline{j}$ ($z = 0$ for xy-plane) then

$$(x\underline{i} + y\underline{j}) \times (38.8\underline{i} + 43.2\underline{j} + 38.4\underline{k}) = 118\underline{i} - 187\underline{j} + 91\underline{k}$$

$$(38.4y)\underline{i} - (38.4x)\underline{j} + (43.2x - 38.8y)\underline{k} = 118\underline{i} - 187\underline{j} + 91\underline{k}$$

Equating components :

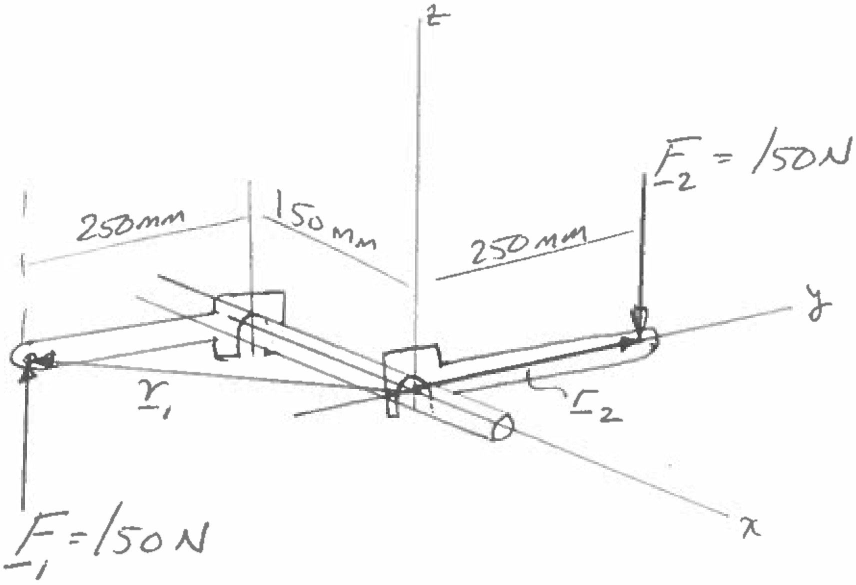
$\underline{i}$: $38.4y = 118 \Rightarrow y = 3.07$ ft

$\underline{j}$: $-38.4x = -187 \Rightarrow x = 4.87$ ft

$\underline{k}$: Equation not independent of x,y equations (can verify by substitution).

Wrench axis intersects xy-plane at $(x, y) = (4.87, 3.07)$ ft.

2.114 The two forces acting on the handles of the pipe wrenches are a couple $\underline{M}$. Express this couple as a vector



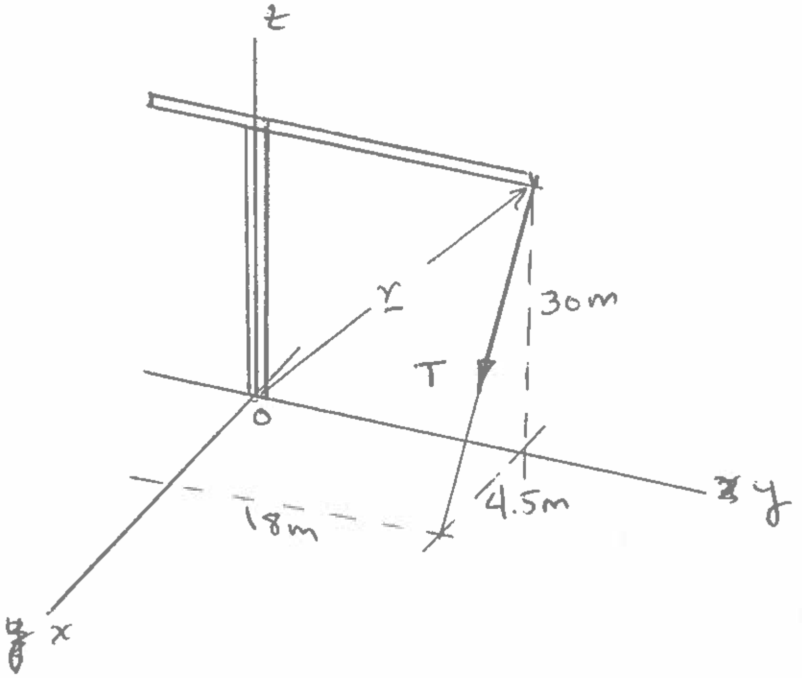
$\underline{F}_1 = 150N\,\underline{k} \quad \underline{r}_1 = -.150m\,\underline{i} - .250m\,\underline{j}$

$\underline{F}_2 = -150\,\underline{k} \quad \underline{r}_2 = \quad .250m\,\underline{j}$

$\underline{R} = \Sigma\underline{F} = 150N\,\underline{k} - 150N\,\underline{k} = \underline{0}$

$$\begin{aligned}\underline{M}_o &= \Sigma\,\underline{r}\times\underline{F}\\ &= (-.15\,\underline{i} - .25\,\underline{j}) \times (150\,\underline{k}) + (.25\,\underline{j}) \times (-150\,\underline{k})\\ &= -37.5\,\underline{i} + 22.5\,\underline{j} - 37.5\,\underline{i}\\ &= -75\,\underline{i} + 22.5\,\underline{j} \text{ N·m}\end{aligned}$$

2.126 Crane as shown. $T = 21\,kN$ is developed. Find moment about base caused by T.



$$\underline{M}_o = \underline{r} \times \underline{F}$$

$$\underline{r} = 0\underline{i} + 18\underline{j} + 30\underline{k}$$

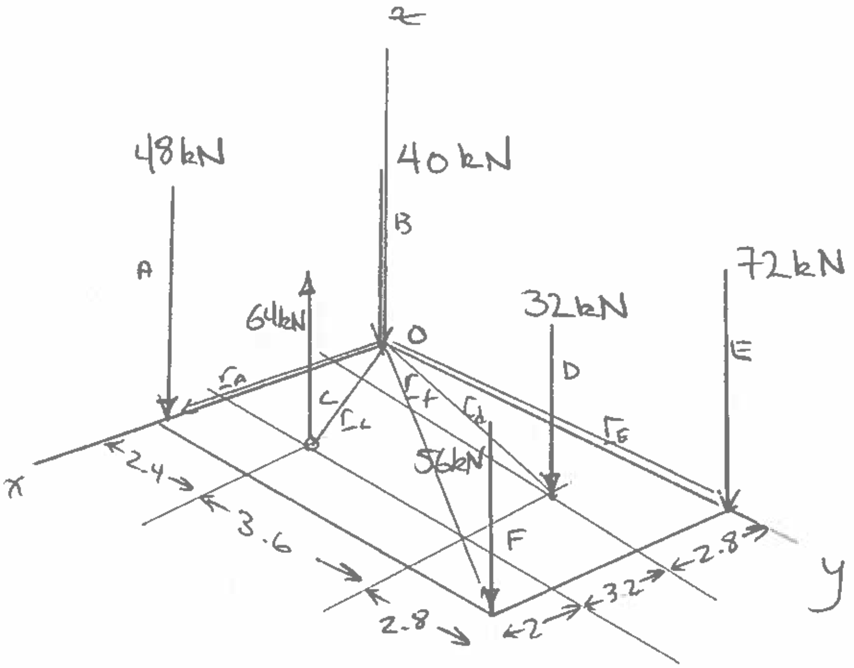
$$\underline{F} = 21\left(\frac{4.5}{30.33}\,\underline{i} - \frac{30}{30.33}\,\underline{j}\right)$$

$$= 3.115\underline{i} - 20.767\underline{j}$$

$$\underline{M}_o = \begin{array}{ccc|cc} i & j & k & i & j \\ 0 & 18 & 30 & 0 & 18 \\ 3.115 & 0 & -20.767 & 3.115 & 0 \\ -56.07\underline{k} & 0 & 0 & -373.8\underline{i} & 93.45\underline{j} \quad 0 \end{array}$$

$$\underline{M}_o = -373.8\underline{i} + 93.45\underline{j} - 56.07\underline{k}$$

2.139 Slab supports six vertical loads shown. Find (x, y) coordinate of point on slab through which resultant passes.



$$\underline{R} = \Sigma \underline{F} = -48\underline{k} - 40\underline{k} + 64\underline{k} - 32\underline{k} - 72\underline{k} - 56\underline{k}$$

$$= -184\underline{k}$$

$$\underline{M}_o = \Sigma \underline{r} \times \underline{F}$$

$$= 8\underline{i} \times -48\underline{k} \quad + \quad (6\underline{i} + 2.4\underline{j}) \times (64\underline{k})$$

$$+ (2.8\underline{i} + 6\underline{j}) \times (-32\underline{k}) \quad + \quad (8.8\underline{j}) \times (-72\underline{k})$$

$$+ (8\underline{i} + 8.8\underline{j}) \times (-56\underline{k})$$

$$= 384\underline{j} - 384\underline{j} + 153.6\underline{i} + 89.6\underline{j} - 192\underline{i}$$

$$- 633.6\underline{i} + 448\underline{j} - 492.8\underline{i}$$

$$= -1164.8\underline{i} + 537.6\underline{j}$$

$$\underline{M}_o = \underline{\hat{r}} \times \underline{R}$$

$$= (x\underline{i} + y\underline{j}) \times (-184\underline{k})$$

$$= 184x\,\underline{j} - 184y\,\underline{i}$$

$184x = 537.6; x = 2.921$ m

$-184y = -1164.8; y = 6.331$ m

Show $\underline{R} \perp \underline{M}_o$

$$\underline{R} \cdot \underline{M}_0 = (0\underline{i} + 600\underline{j} + 0\underline{k}) \cdot (1200\underline{i} + 0\underline{j} + 4800\underline{k})$$

$$= (1200)(0) + (600)(0) + (4800)(0) = 0$$

$\therefore$ Perpindicular.

$$\underline{M}_0 = \hat{r} \times \underline{R}$$

$$= (x\underline{i} + y\underline{j} + z\underline{k}) \times (600\underline{j})$$

$$= 600x\underline{k} + 0 - 600z\underline{i}$$

$$\therefore \quad 1200\underline{i} + 4800\underline{k} = -600z\underline{i} + 600x\underline{k}$$

$$z = -\frac{1200}{600} = -2\text{ m}$$

$$x = \frac{4800}{600} = 8\text{ m}$$

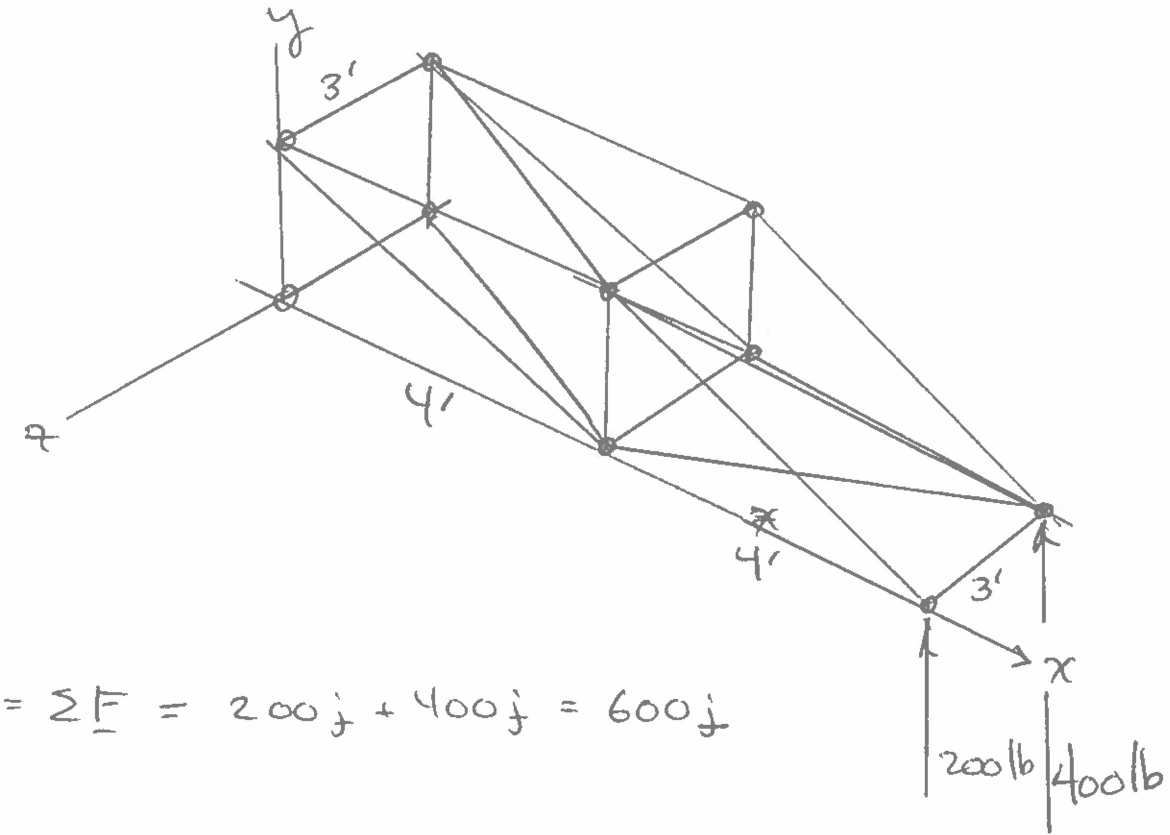
Solutn

$\underline{R} = 600\underline{j}$ lbs

$\underline{M}_o = 1200\underline{i} + 4800\underline{k}$ lb-ft

$(x, y, z) = (8, 0, 2)$ meters

2.143 Two loads on truss. Reduce to single force-couple system at O. Show that $\underline{R}$ is perpindicular to $\underline{M}_o$. Find point in (x-z) plane through which resultant passes



$\underline{R} = \Sigma \underline{F} = 200j + 400j = 600j$

$\underline{M}_o = \Sigma r \times F$

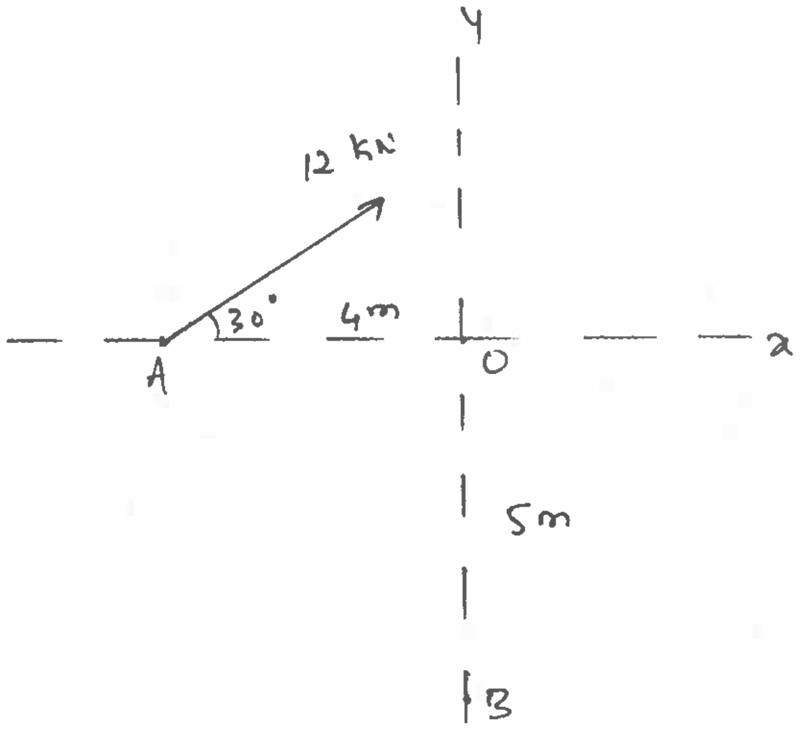
$= 8\underline{i} \times 200j + (8\underline{i} - 3\underline{k}) \times 400j$

$= 1600\underline{k} + 3200\underline{k} + 1200\underline{i}$

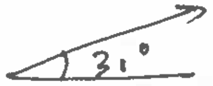
$= 1200\underline{i} + 0j + 4800\underline{k}$

CIVE 2330 HW # 5

Prob # 2.54



At pt O

F = 12 kN 

Moment @ O

$\overset{+}{\circlearrowright} M_o = 12(\sin 30°)(4)$

$= 24$ kN-m clock wise.

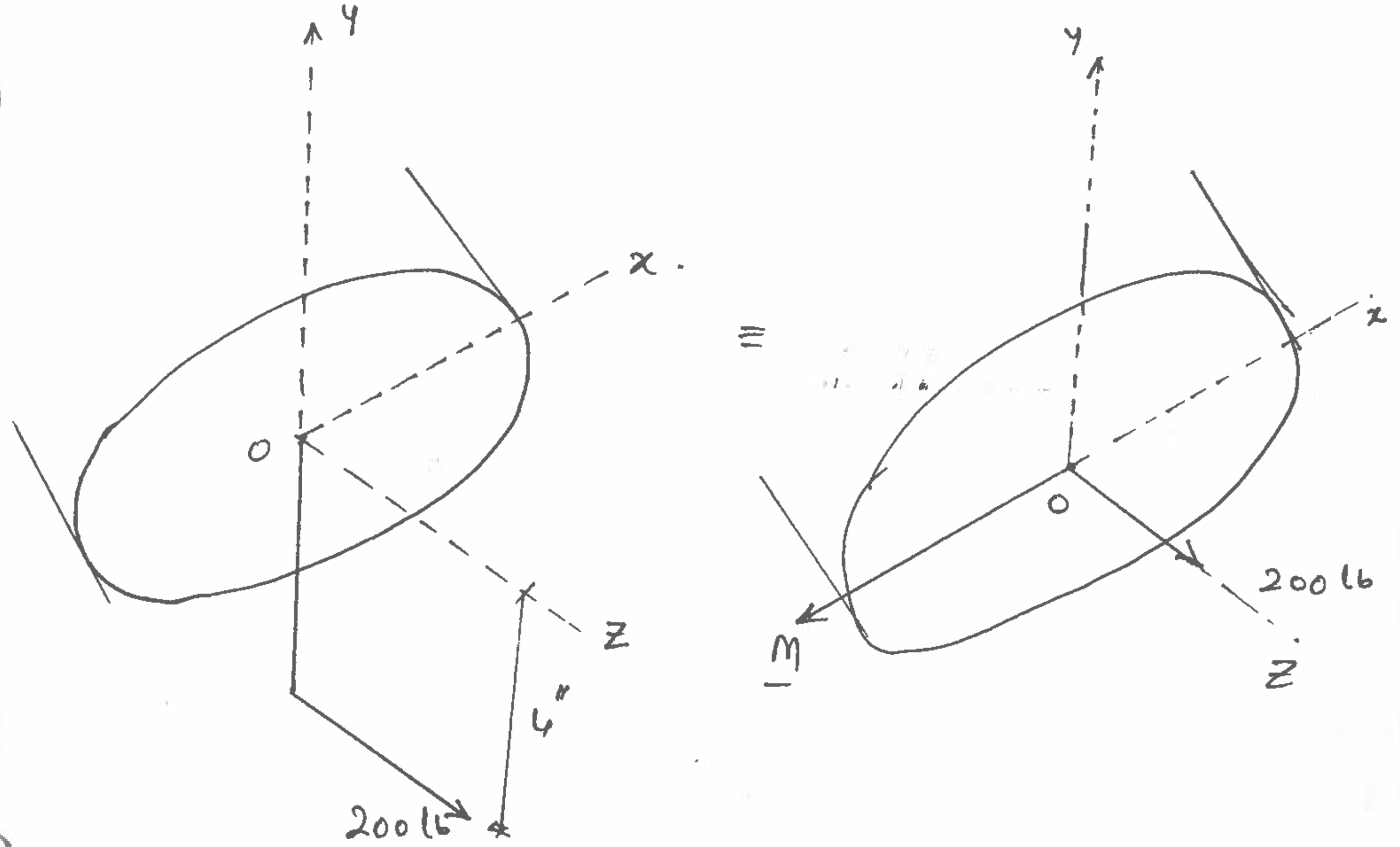
At pt B

F = 12 kN 

$\overset{+}{\circlearrowright} M_B = (12 \sin 30°)(4) + (12 \cos 30°)(5)$

$= 76.0$ kN-m clock wise.

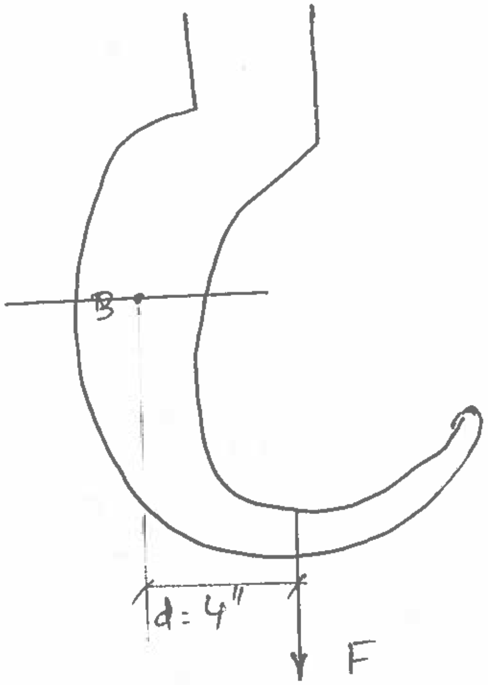
Prob # 2.58



$$M = 200(4)(-\underline{j})$$

$$= -800\,\underline{j} \text{ lb-in}$$

Prob 2.61



$M = 4{,}000 \text{lb-ft}$

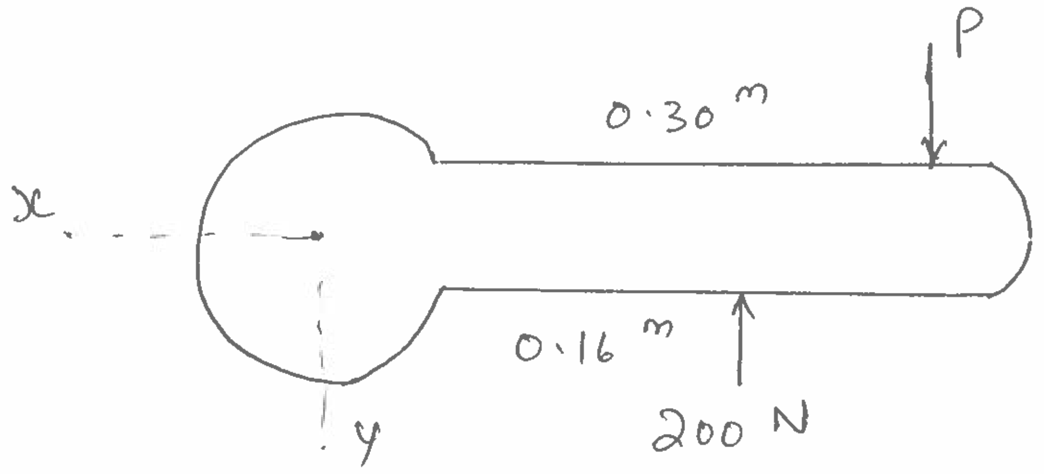
$M = F \cdot d$

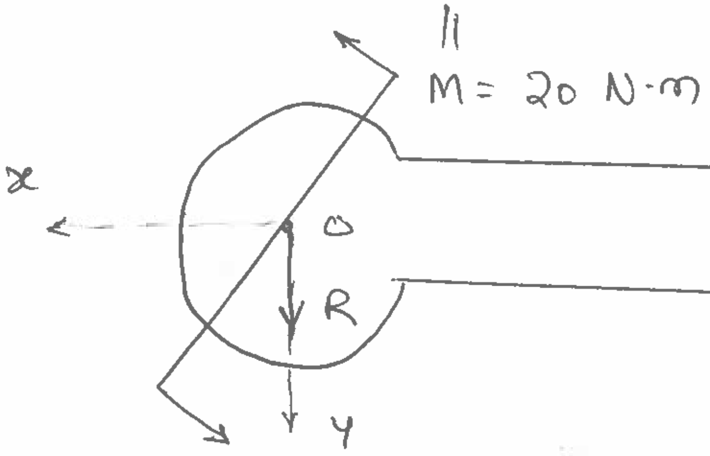
$F = \dfrac{M}{d}$

$F = \dfrac{4000 \times 12}{4}$

$F = 12.000 \text{ lb}$

Prob 2.66





M = 20 N·m

$M = \sum F \cdot d$

$20 = 200(0.16) - 0.30P$

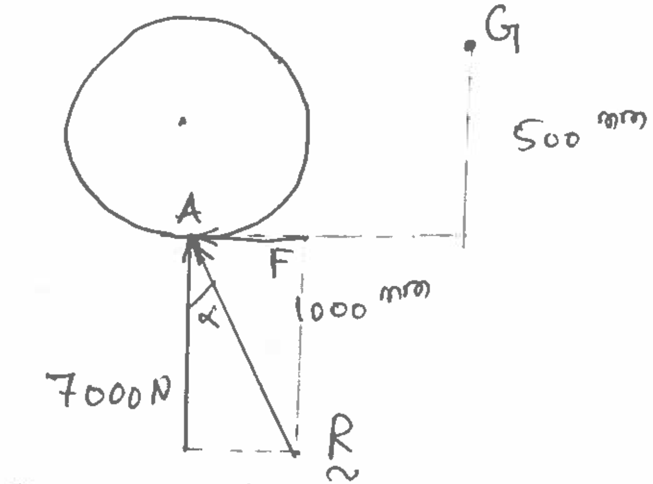
$P = 40 \text{ N}$

$\underline{P} = 40 \underline{j} \text{ N}.$

$\underline{R} = -200\underline{j} + 40\underline{j} = -160\underline{j} \text{ N}.$

CIVE 2330 HW # 6

Prob # 2.72



(α = 15°)

$$\tan 15° = \frac{F}{7000}$$

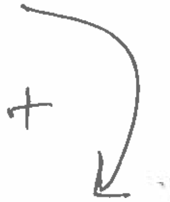
$$F = 1876\ N.$$

At Point G

$$\underset{\sim}{R} = \Sigma \underset{\sim}{F} = 7247\ N$$

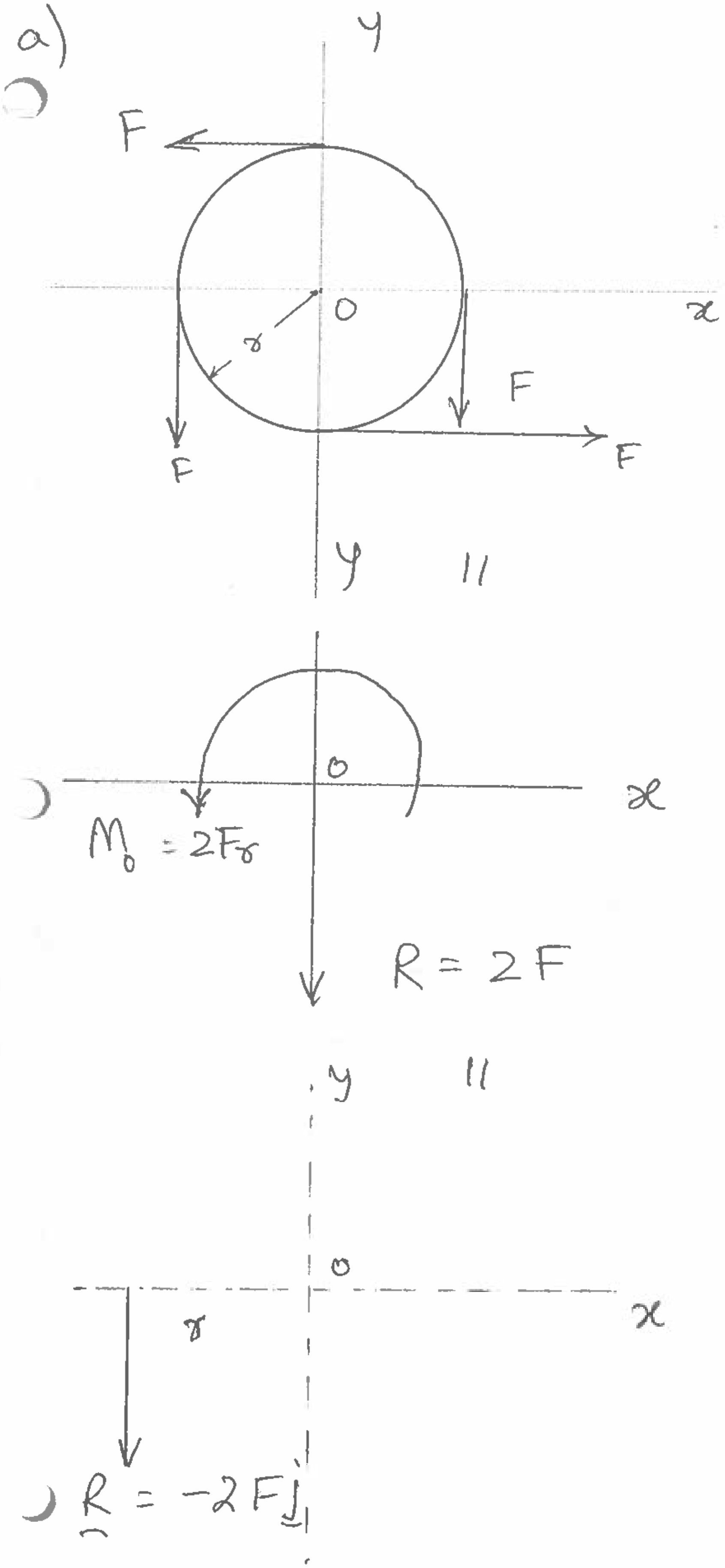
$$M_G = 7000(1) + 1876(0.5)$$

$$= 7938\ N \cdot m$$

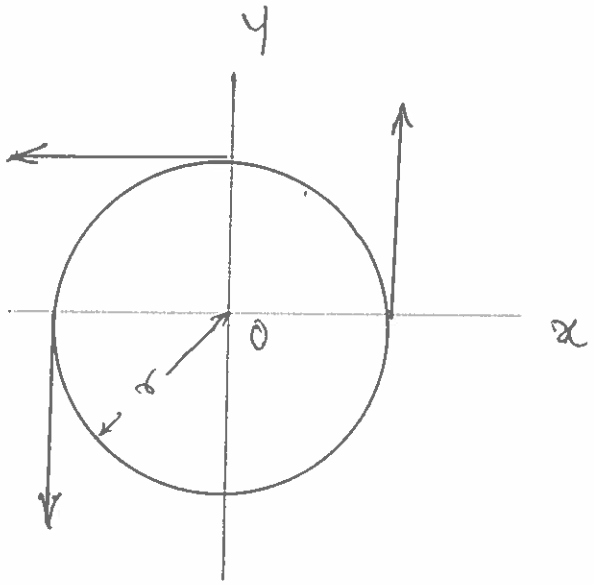


Prob # 2.75

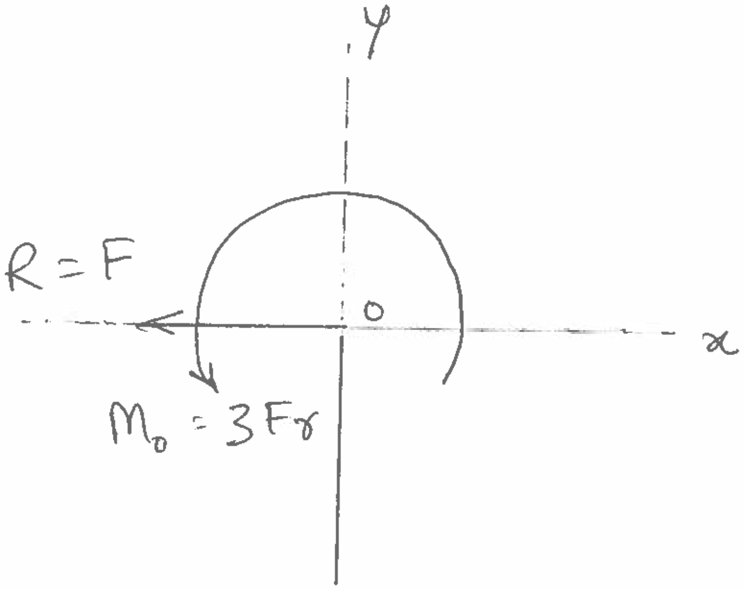
a)



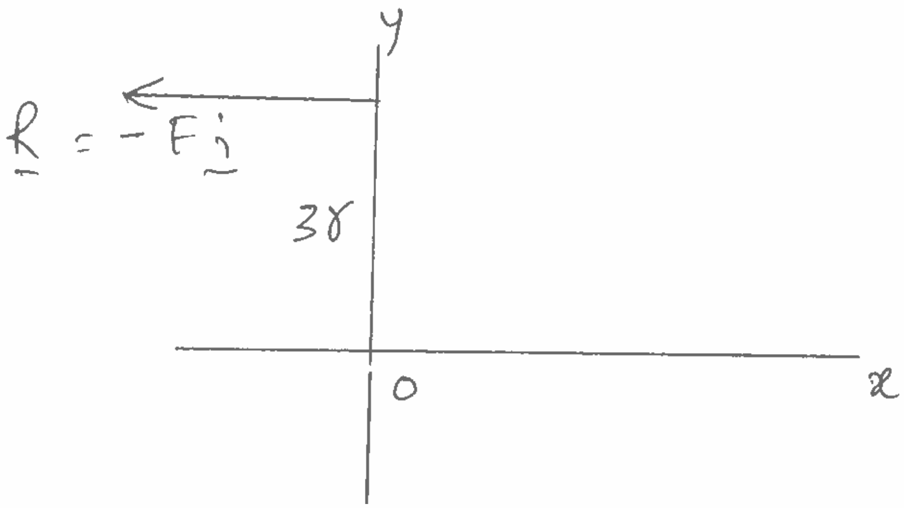
(b)



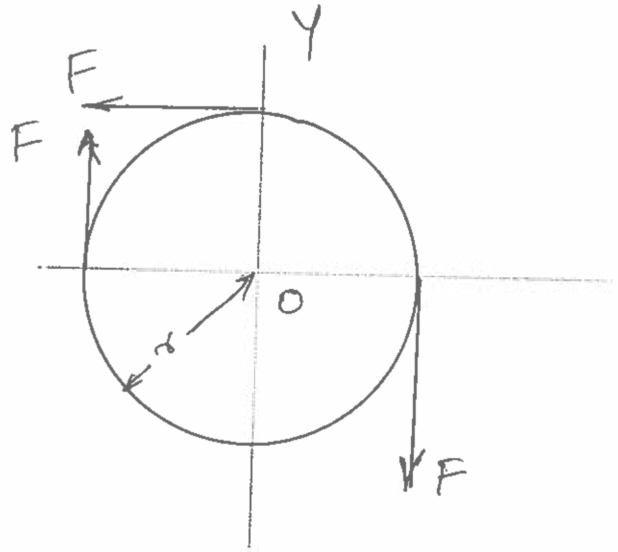
||

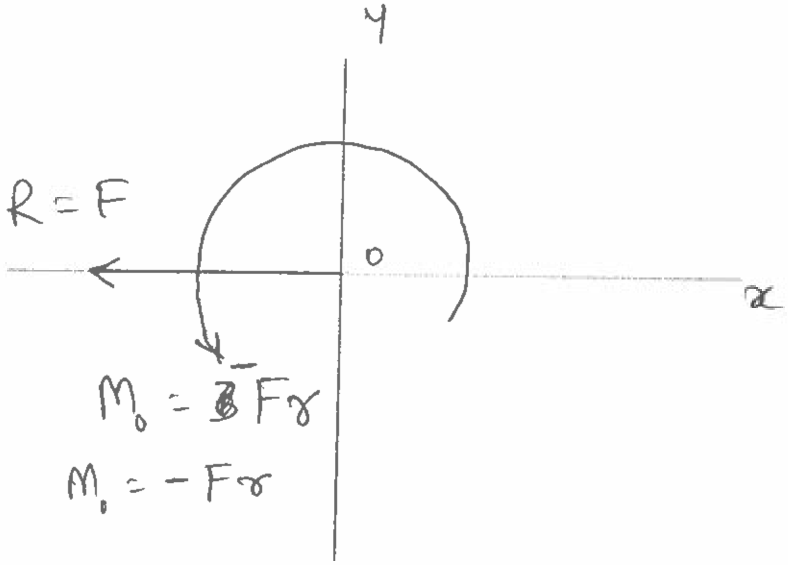


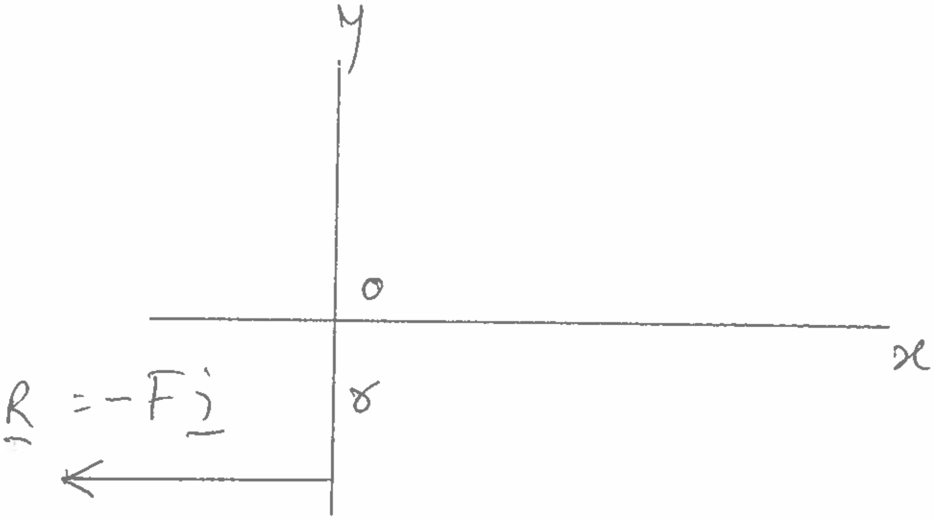
||



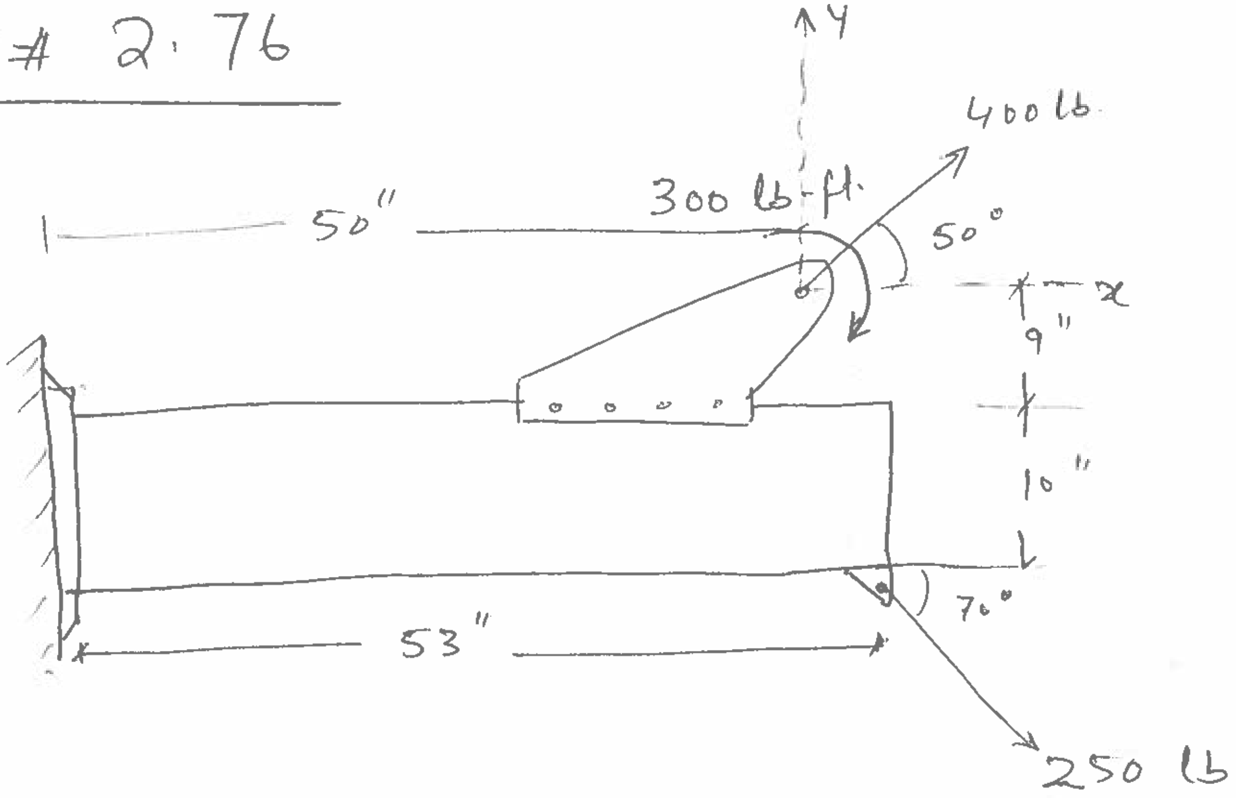
c)







Prob # 2.76



$$R_x = \sum F_x = 400 \cos 50^\circ + 250 \cos 70^\circ$$
$$= 343 \text{ lb.}$$

$$R_y = \sum F_y = 400 \sin 50^\circ \cancel{+} - 250 \sin 70^\circ$$
$$= 71.5 \text{ lb}$$

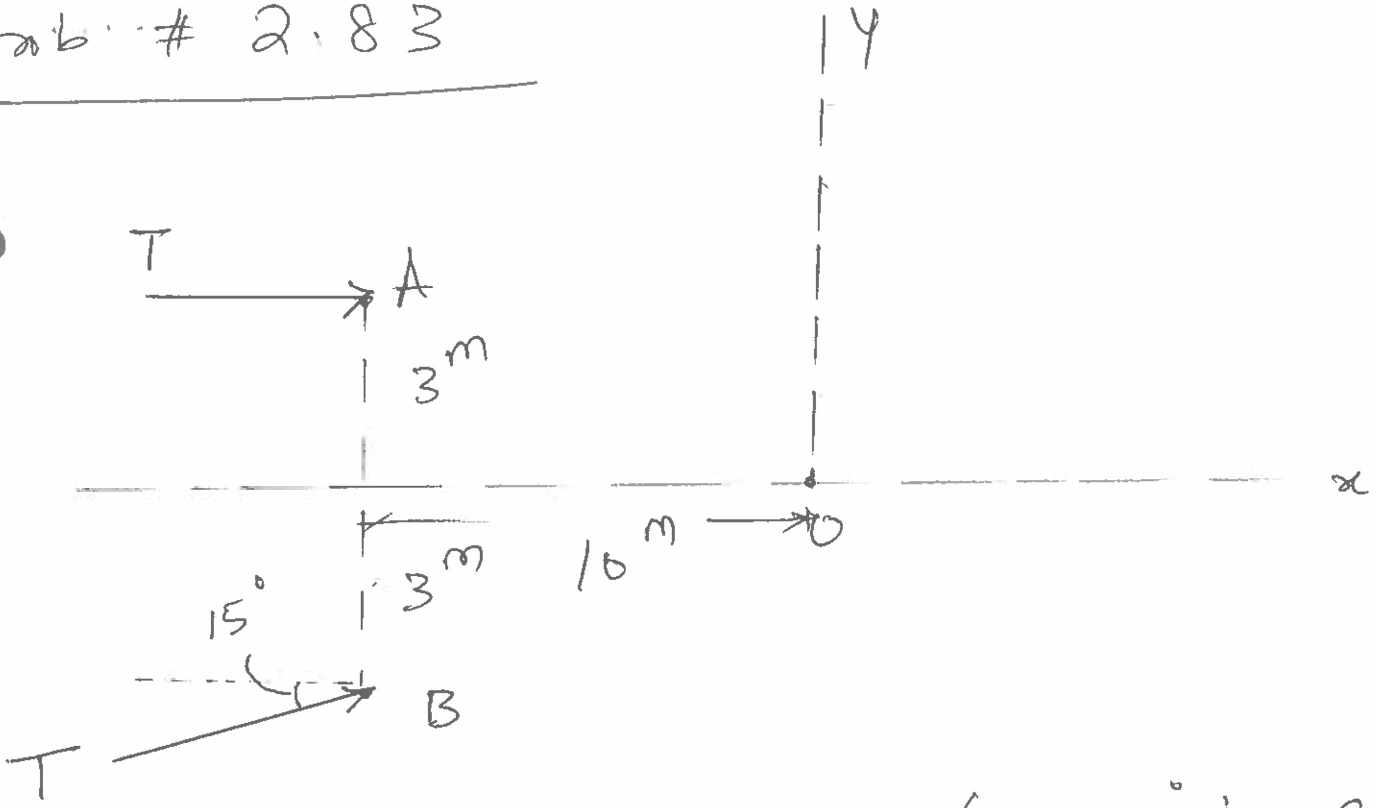
$$+\circlearrowright M_A = 300 + (400 \cos 50^\circ)\left(\frac{9}{12}\right)$$
$$- (400 \sin 50^\circ)\left(\frac{50}{12}\right)$$
$$+ (250 \sin 70^\circ)\left(\frac{53}{12}\right)$$
$$- (250 \cos 70^\circ)\left(\frac{10}{12}\right)$$
$$= 182.1 \text{ lb-ft}$$

Contd 2

Force Couple System at A is

$$\begin{cases} \underline{R} = 343\,\underline{i} + 71.5\,\underline{j} \ \text{lb} \\ M = 182.4 \ \text{lb-ft} \ \ \text{CW} \end{cases}$$

Prob # 2.83



$$\underline{R} = \sum \underline{F} = T\underline{i} + T(\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j})$$

$$= 1.966\, T\underline{i} + 0.259\, T\underline{j}$$

$$+\curvearrowright\ M_o = 3T - T\cos 15^\circ (3) + T\sin 15^\circ (10)$$

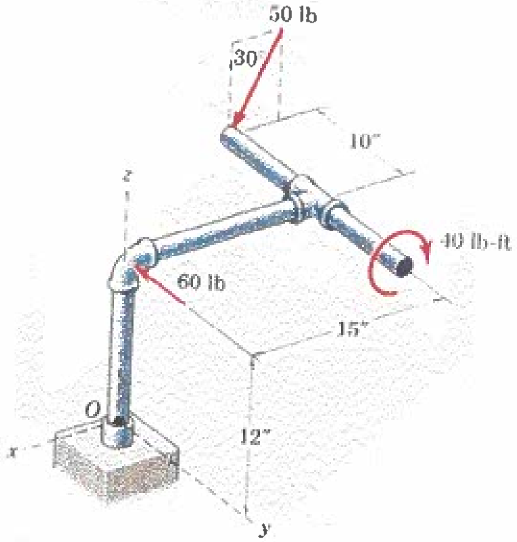
$$= 2.69\, T$$

$$-R_y X = M_o : -0.259T(x) = 2.69\, T$$

$$X = -10.39\ m$$

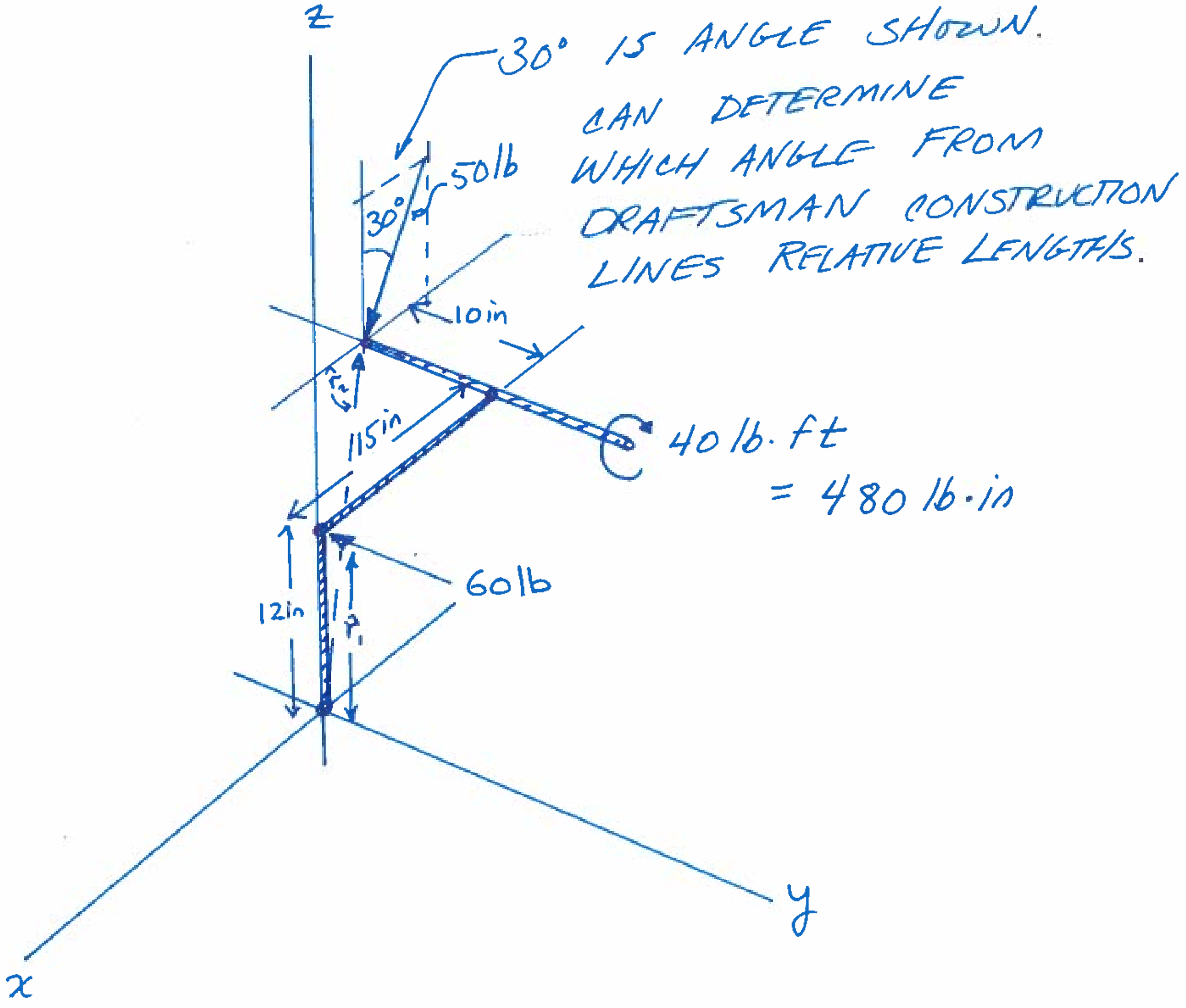
Problem #2

2/144 Replace the two forces and one couple acting on the rigid pipe frame by their equivalent resultant force **R** acting at point *O* and a couple $\mathbf{M}_O$.



Problem 2/144

DIAGRAM



30° IS ANGLE SHOWN. CAN DETERMINE WHICH ANGLE FROM DRAFTSMAN CONSTRUCTION LINES RELATIVE LENGTHS.

EXPRESS ALL FORCES AS VECTORS:

$$50 \sin 30\, \underline{i} - 50 \cos 30\, \underline{k}$$

and

$$-60\, \underline{j}$$

EXPRESS COUPLE AS VECTOR: $-480 \text{lb}\cdot\text{in}\, \underline{j}$

$$\underline{R} = \Sigma \underline{F}$$

$$= 50\sin 30\underline{i} - 50\cos 30\underline{k} - 60\underline{j}$$

$$= \underline{\underline{25\underline{i} - 60\underline{j} - 43.3\underline{k} \quad \text{lb}}}$$

$$\underline{M}_o = \sum \hat{r} \times \underline{F} + \underline{M}_c$$

$$= 12\underline{k} \times -60\underline{j}$$

$$+ (-15\underline{i} - 10\underline{j} + 12\underline{k}) \times (25\underline{i} - 43.3\underline{k})$$

$$+ (-480\,\text{lb}\cdot\text{in})\underline{j}$$

$$= 720\underline{i}$$

$$- 649.5\underline{j} - 250\underline{k} + 433\underline{i} + 300\underline{j}$$

$$- 480\underline{j}$$

$$= \underline{\underline{1153\underline{i} - 830\underline{j} + 250\underline{k} \quad \text{lb}\cdot\text{in}}} \longleftarrow$$

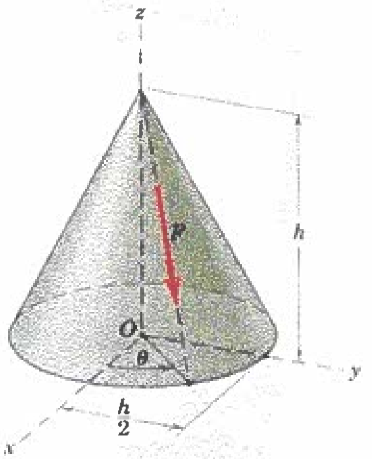
or

$$96\underline{i} - 69\underline{j} + 20.8\underline{k} \quad \text{lb}\cdot\text{ft}$$

CIVE 2330 Fall 1998 Examination #1 Name:______________________________
Page 1 of 5 Student Number:______________________________

Problem # 1

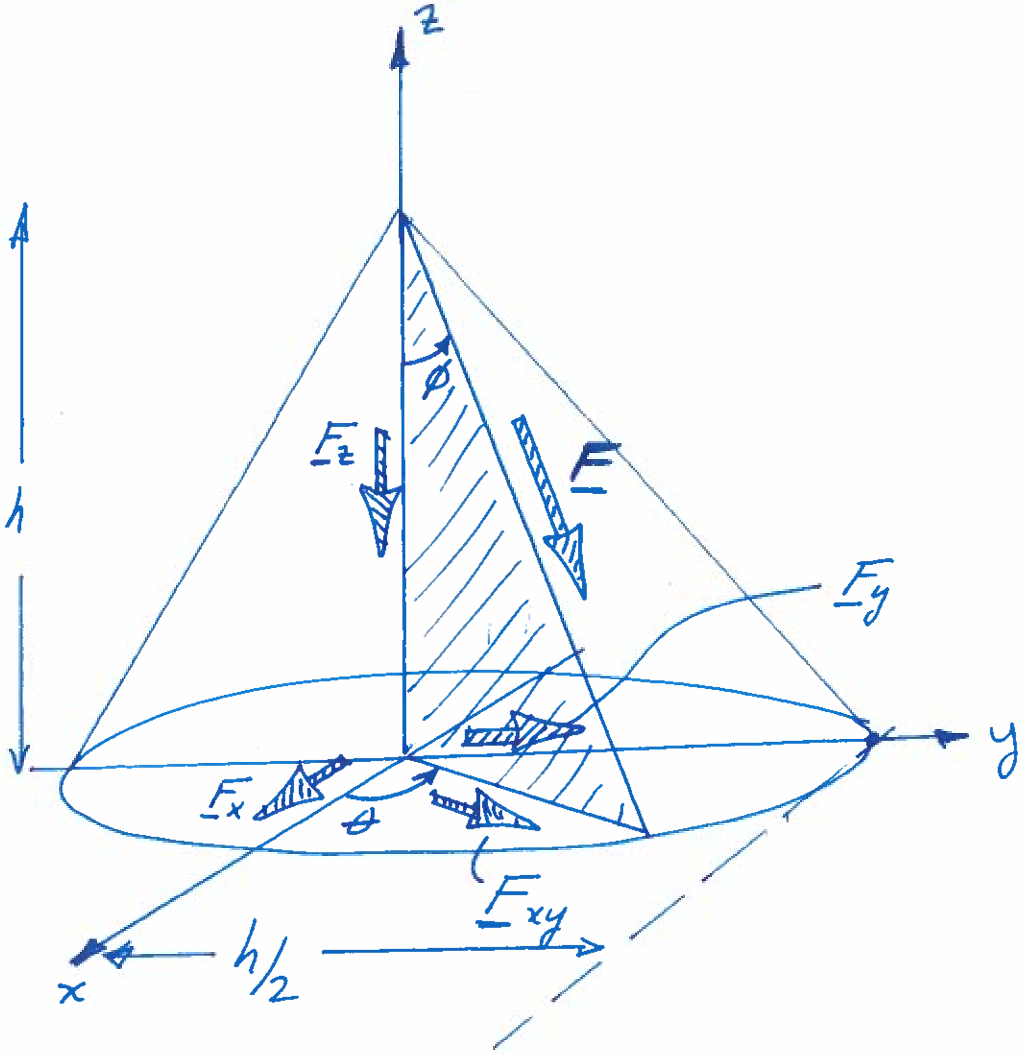
2/132 The force F acts along an element of the right circular cone as shown. Determine the equivalent force–couple system at point O.



Problem 2/132

PROBLEM 1

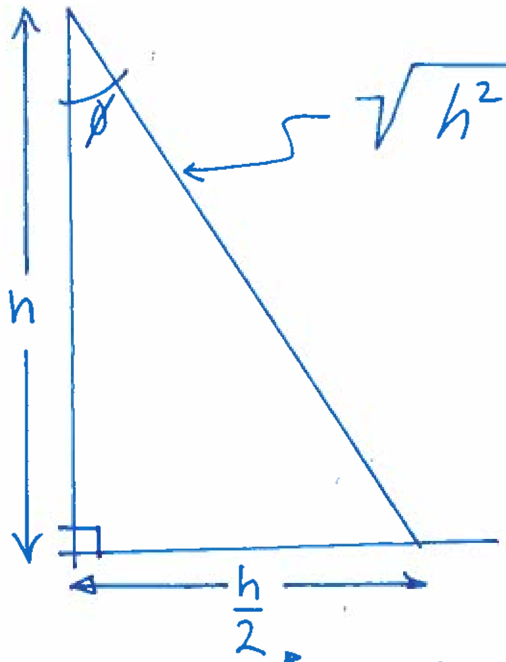
DIAGRAM



PROJECTION (BASE) OF CONE IS A CIRCLE OF RADIUS $h/2$.

HEIGHT TO VERTEX IS h.

TRIGONOMETRY ON SHADED TRIANGLE

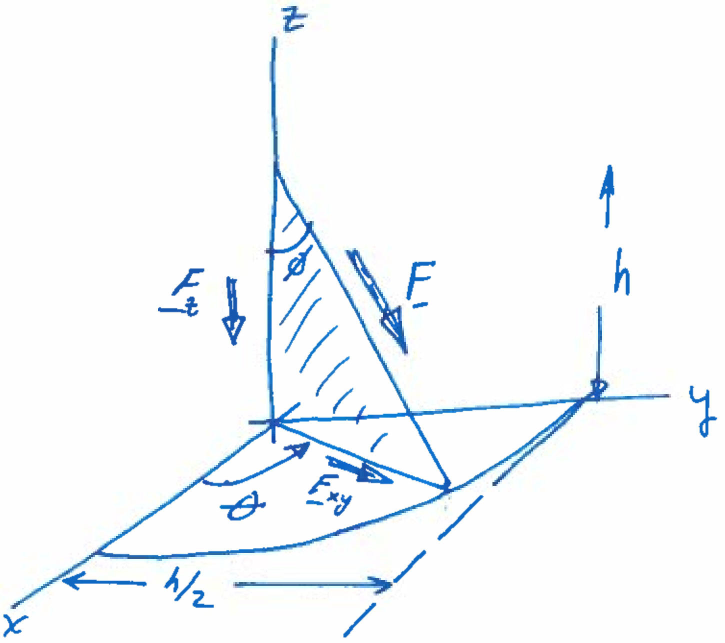


$$\sqrt{h^2 + \frac{h^2}{4}} = \frac{\sqrt{5}\,h}{2}$$

(RADIUS OF CIRCULAR BASE)

$$\sin\phi = \frac{\frac{h}{2}}{\frac{\sqrt{5}\,h}{2}} = \frac{1}{\sqrt{5}}$$

$$\cos\phi = \frac{h}{\frac{\sqrt{5}\,h}{2}} = \frac{2}{\sqrt{5}}$$



$$\underline{F}_z = -F\cos\phi\,\underline{k} = \frac{-2F}{\sqrt{5}}\,\underline{k}$$

$$F_{xy} = F\sin\phi = \frac{F}{\sqrt{5}}$$

$$\underline{F}_x = F_{xy}\cos\theta\,\underline{i} = \frac{F}{\sqrt{5}}\cos\theta\,\underline{i}$$

$$\underline{F}_y = F_{xy}\sin\theta\,\underline{j} = \frac{F}{\sqrt{5}}\sin\theta\,\underline{j}$$

$$\underline{M}_o = \underline{r} \times \underline{F}$$

$\underline{r}$ IS POSITION VECTOR FROM O TO LINE OF APPLICATION OF $\underline{F}$. ONE OBVIOUS CHOICE IS ALONG THE Z-AXIS

$$\therefore \quad \underline{r} = h\underline{k}$$

$$\underline{r} \times \underline{F} = h\underline{k} \times \left(\frac{F\cos\theta}{\sqrt{5}}\underline{i} + \frac{F\sin\theta}{\sqrt{5}}\underline{j} - \frac{2F}{\sqrt{5}}\underline{k} \right)$$

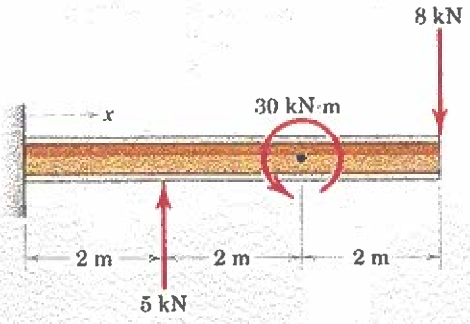
$$= -\frac{Fh}{\sqrt{5}}\cos\theta\,\underline{i} + \frac{Fh}{\sqrt{5}}\sin\theta\,\underline{j} + 0\underline{k}$$

$$\underline{F} = \frac{F\cos\theta}{\sqrt{5}}\underline{i} + \frac{F\sin\theta}{\sqrt{5}}\underline{j} - \frac{2F}{\sqrt{5}}\underline{k}$$

$$\underline{M_o} = -\frac{Fh}{\sqrt{5}}\cos\theta\,\underline{i} + \frac{Fh}{\sqrt{5}}\sin\theta\,\underline{j} + 0\underline{k}$$

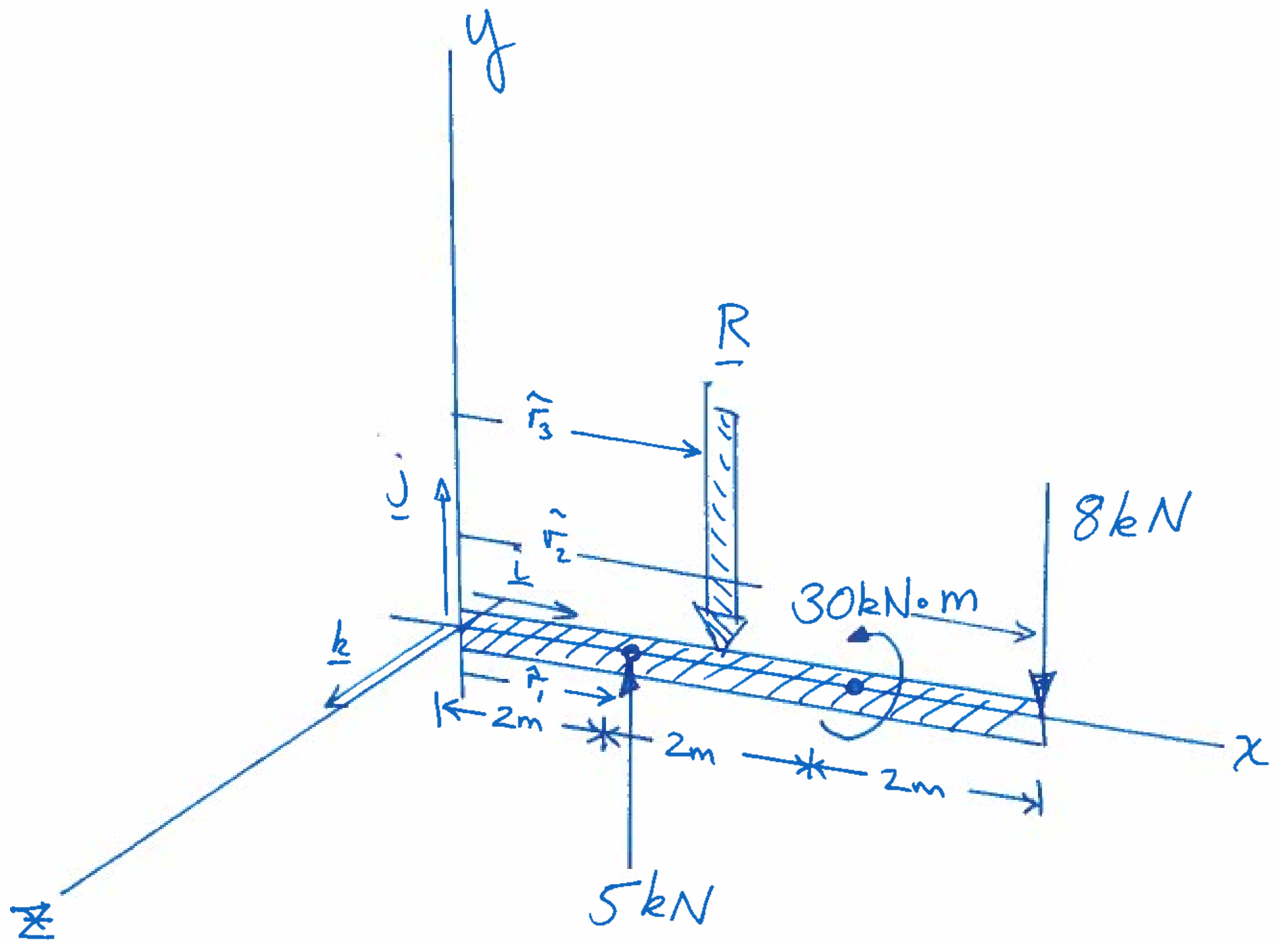
Problem #3

2/77 Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.



Problem 2/77

DIAGRAM



EXPRESS FORCES AS VECTORS

$-8kN\,j$ $\quad 5kN\,j$

EXPRESS COUPLE AS VECTOR

$30kN\cdot m\ \underline{k}$

$$\underline{R} = \Sigma \underline{F}$$

$$= -8j + 5j = \underline{\underline{-3\,kN\,j}}$$

$$\underline{M}_o = \Sigma \hat{r} \times \underline{F} + \underline{M}_c$$

$$= 2\underline{i} \times 5j$$

$$+6\underline{i} \times -8j$$

$$+30\underline{k}$$

$$= 10\underline{k} - 48\underline{k} + 30\underline{k}$$

$$\underline{\underline{= -8\underline{k}\ kN\cdot m}}$$

$$\underline{M}_o = \hat{r}_3 \times \underline{R}$$

$$= x\underline{i} \times -3kNj = -8\,kN\cdot m\,\underline{k}$$

$$-3x\underline{k} = -8\underline{k} \qquad \therefore \hat{r}_3 = 2.67\underline{i}\ m$$

$$x = \frac{-8}{-3} = \underline{\underline{2.67m}}$$

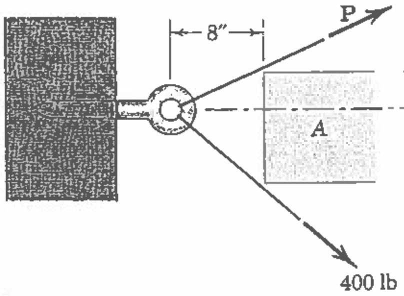
SOLUTIONS 8/27/99

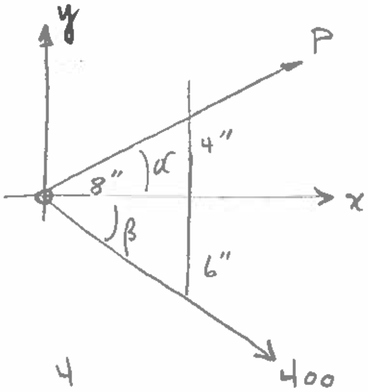
CIVE 2330 Fall 1999 Examination 1

Page 1 of 4

Problem 1:

It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction *A* prevents direct access, so that two forces, one 400 lb and the other **P**, are applied by cables as shown. Compute the magnitude of **P** necessary to ensure a resultant **T** directed along the spike. Also find *T*.





$\tan\alpha = \frac{4}{8}$

$\tan\beta = \frac{6}{8}$

$\alpha = \tan^{-1}(4/8) = 26.56°$

$\beta = \tan^{-1}(6/8) = 36.86°$

$\underline{R} = \Sigma \underline{F} = T\underline{i} + 0\underline{j}$

$\Sigma \underline{F} = P\cos\alpha\,\underline{i} + P\sin\alpha\,\underline{j} + 400\cos\beta\,\underline{i} - 400\sin\beta\,\underline{j} = T\underline{i} + 0\underline{j}$

Separate into component equations

$P\sin\alpha = 400\sin\beta$

$P = \frac{400\sin\beta}{\sin\alpha} = 536.6$

$P\cos\alpha + 400\cos\beta = T$

$537\cos(26.56) + 400\cos(36.86) = T$

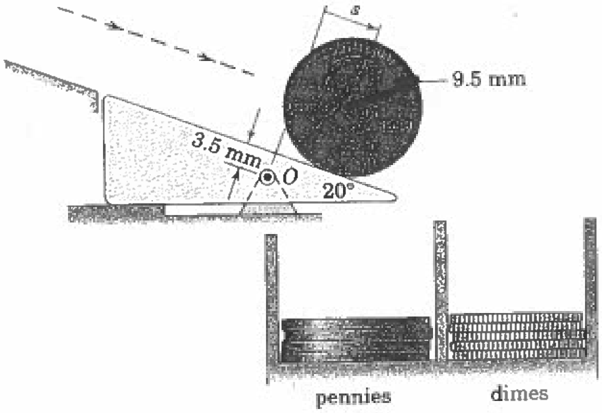
$T = 8000$

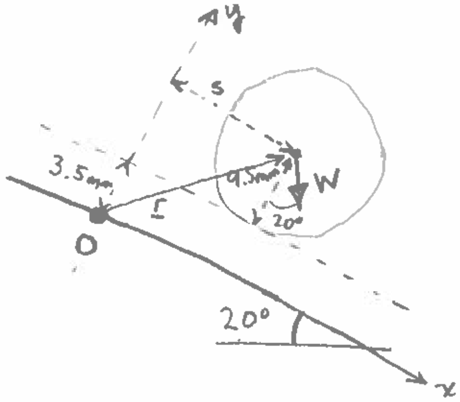
$T = 800$ lbs ←

$P = 537$ lbs ←

Problem 2:

A portion of a mechanical coin sorter works as follows: Pennies and dimes roll down the 20° incline, the last triangular portion of which pivots freely about a horizontal axis through *O*. Dimes are light enough (2.28 grams each) so that the triangular portion remains stationary, and the dimes roll into the right collection column. Pennies, on the other hand, are heavy enough (3.06 grams each) so that the triangular portion pivots clockwise, and the pennies roll into the left collection column. Determine the moment about *O* of the weight of the penny in terms of the slant distance *s* in millimeters.





$$\underline{M}_o = \underline{r} \times \underline{F}$$

$$\underline{r} = s\,\underline{i} + (3.5+9.5)\,\underline{j}$$

$$\underline{F} = W\sin 20\,\underline{i} - W\cos 20\,\underline{j}$$

$$\underline{r}\times\underline{F} = \begin{vmatrix} i & j & k \\ s & 13 & 0 \\ W\sin 20 & -W\cos 20 & 0 \end{vmatrix}$$

$$-Ws\cos 20\,\underline{k} - 13W\sin 20\,\underline{k}$$

$$W = 3.06g\left(\frac{kg}{1000g}\right)(9.8\,m/s^2)$$

$$= 0.0299\,N$$

$$\underline{M}_0 = \begin{matrix} -13(0.0299)\sin 20 \\ -s(0.0299)\cos 20 \end{matrix} \quad \underline{k}$$

$$\left.\begin{matrix} \underline{M}_0 = -0.133\ N\cdot mm\ \underline{k} \\ -s(0.028)\ N\cdot mm\ \underline{k} \\ s \text{ in millimeters} \end{matrix}\right\} \longleftarrow$$

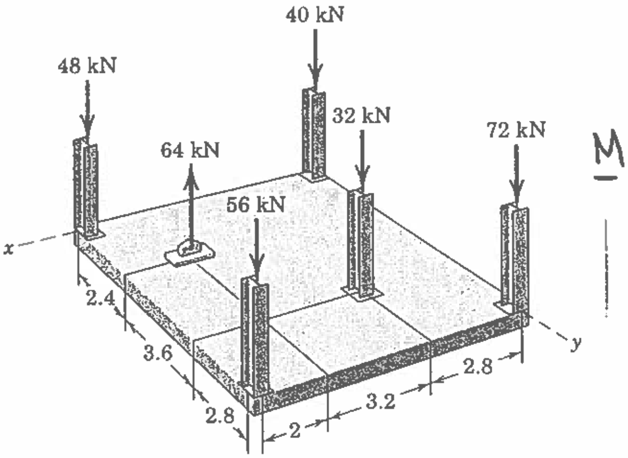
$$M_o = 0.133 + 0.028s \circlearrowright N\cdot mm \longleftarrow$$

or

$$\underline{M}_0 = -0.133 - 0.028s\ \underline{k}\ N\cdot mm \longleftarrow$$

Problem 3:

The concrete slab supports the six vertical loads shown. Determine the *x*- and *y*-coordinates of the point on the slab through which the resultant of the loading system passes.



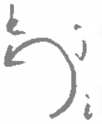
Dimensions in meters

$\underline{R} = \Sigma \underline{F}$

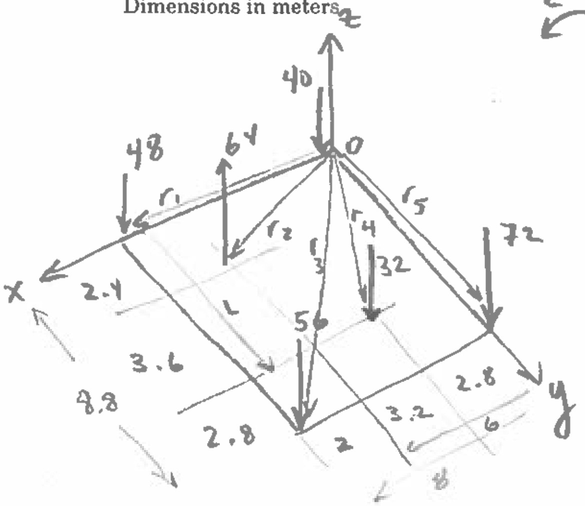
$\underline{M}_o = \Sigma \underline{r} \times \underline{F} = \hat{\underline{r}} \times \underline{R}$

$\underline{R} = -40\underline{k} - 48\underline{k} + 64\underline{k} - 32\underline{k}$
$-56\underline{k} - 72\underline{k}$ kN
$= -184$ kN $\underline{k}$

$\underline{M}_o = 8\underline{i} \times -48\underline{k} \quad = -(-384)\underline{j}$
$+(6\underline{i} + 2.4\underline{j}) \times 64\underline{k} = -(384)\underline{j} + (153.6)\underline{i}$
$+(8\underline{i} + 8.8\underline{j}) \times -56\underline{k} = -(-448)\underline{j} + (-492.8)\underline{i}$
$+(2.8\underline{i} + 6\underline{j}) \times -32\underline{k} = -(-89.6)\underline{j} + (-192)\underline{i}$
$+(8.8\underline{j}) \times -72\underline{k} = (-633.6)\underline{i}$



$\underline{M}_o = 384\underline{j} + 153.6\underline{i}$
$-384\underline{j} - 492.8\underline{i}$
$+448\underline{j} - 192\underline{i}$
$+89.6\underline{j} - 633.6\underline{i}$
$= 537.6\underline{j} - 1164.8\underline{i}$ kN·m Ⓐ



$\underline{M}_o = \hat{\underline{r}} \times \underline{R}$
$= (x\underline{i} + y\underline{j}) \times (-184\underline{k})$
$= -(-184x)\underline{j} + (-184y)\underline{i}$ Ⓑ

Set Ⓐ = Ⓑ, Solve for x & y

$184x = 537.6$ & $-184y = -1164.8$

x = 2.92 meters   y = 6.33 m ←

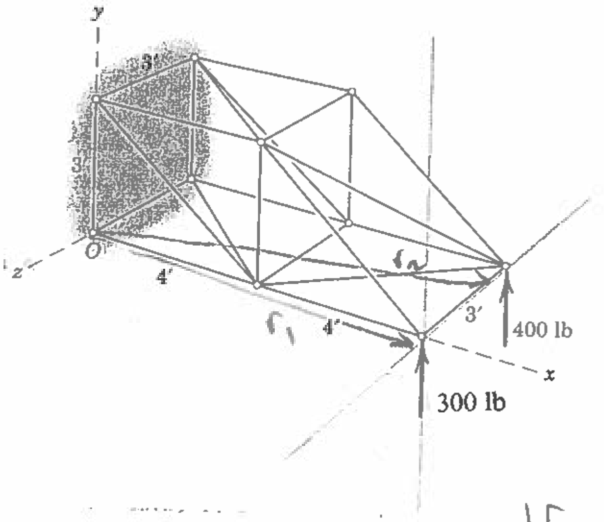
Problem 4:

Two upward loads are exerted on the small three-dimensional truss. Reduce these two loads to a single force–couple system at point *O*. Show that **R** is perpendicular to $\mathbf{M}_O$. Then determine the point in the *x-z* plane through which the resultant passes.

$\underline{R} = \Sigma \underline{F}$

$\underline{M}_o = \Sigma \underline{r} \times \underline{F}$

$\underline{R} = 300\underline{j} + 400\underline{j} = \underline{700\underline{j}\ \text{lb}}$ ←



$\underline{M}_o = 8'\underline{i} \times 300\underline{j} = 300(8)\ \underline{k}$

$+ (8\underline{i} - 3\underline{k}) \times 400\underline{j} = (8(400))\ \underline{k} - (-3(400))$

$= 2400\underline{k} + 3200\underline{k} + 1200\underline{i}$

$= \underline{1200\underline{i} + 5600\underline{k}}$ Ⓐ ←

If perpindicular then $\underline{R} \circ \underline{M}_o = 0$

$\underline{\underline{R} \cdot \underline{M}_o = (700\underline{j}) \cdot (1200\underline{i} + 5600\underline{k}) = 0}$ ←

$\underline{M}_o = \hat{\underline{r}} \times \underline{R}$

$= (x\underline{i} + y\hat{j} + z\underline{k}) \times (700\underline{j})$

$= 700x\ \underline{k} + 0 - 700z\ \underline{i}$ Ⓑ

Set Ⓐ = Ⓑ, Solve for x & z

$-700z = 1200$; $700x = 5600$

$\underline{z = -1.71\text{ft}}$ $\underline{x = 8\text{ft}}$ ←

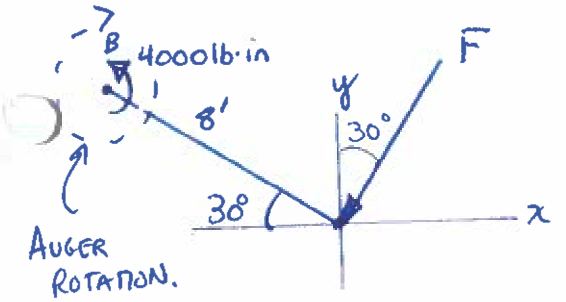
5.164

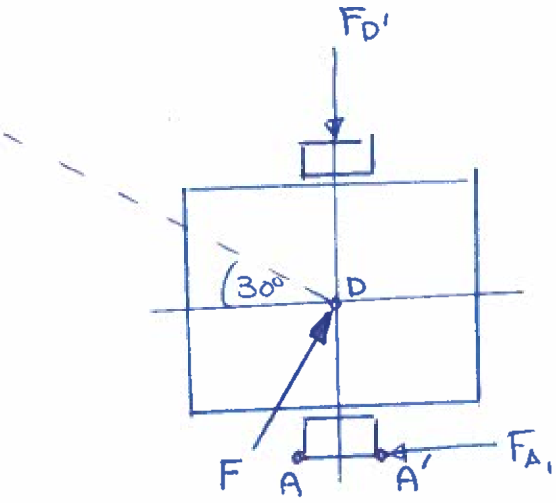
TORQUE = 4000 lb·in

B slides, but not rotate.

Unit rotates about D.

Find force against A or A'





TORQUE (MOMENT) ON AUGER IS OPPOSITE TO TORQUE APPLIED TO SOIL

AUGER

$\Sigma M_B = 0 = 4000\,\text{lb}\cdot\text{in}\,\underline{k} +$

$(12)(8\cos 30\underline{i} - 8\sin 30\underline{j}) \times (-F\sin 30\underline{i} - F\underset{\cos}{\cancel{\sin}}30\underline{j})$

inches / feet   feet

$0 = 4000\underline{k} + (12)(8)(\cos 30)(-\underset{\cos}{\cancel{F\sin}}30)\underline{k} - (12)(-8)(\sin 30)(-F\sin 30)\,\underline{k}$

$0 = 4000 + (12)(8)(-F\cos^2 30 - F\sin^2 30)$ Recall $\cos^2\theta + \sin^2\theta = 1$

$0 = 4000 - (12)(8)(F)$ Solve for F

$F = \frac{4000}{(12)(8)} = 41.7$ lbf.

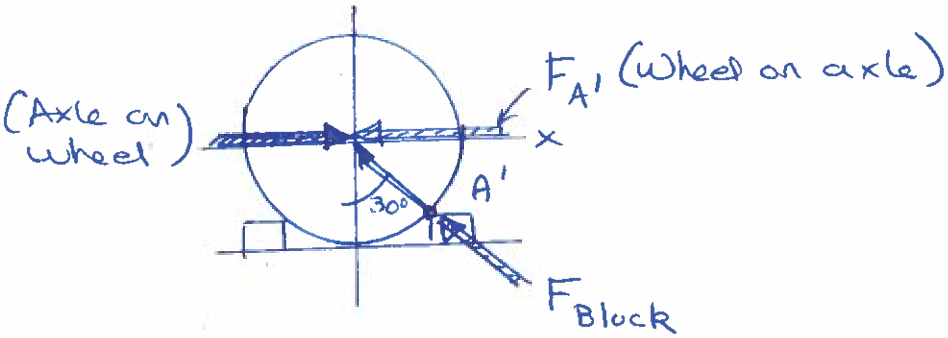
1 of 2

TRAILER

$$\Sigma F_x = 0 = F\sin 30 - F_{A'}$$

$$F_{A'} = F\sin 30$$

WHEEL



$$\Sigma F_x = 0 = F_{A'} - F_{Block}\sin 30^\circ$$

But $F_{A'} = F\sin 30$

$$\therefore \quad 0 = F\sin 30 - F_{Block}\sin 30^\circ$$

$F_{Block} = F = 41.7$ lbs.

Structures - Plane Trusses

A framework of members joined at their ends to form a rigid structure is called a truss.

When all the members of the truss lie in a single plane, the truss is called a plane-truss

The basic element of a plane truss is the triangle, composed of 3 rigid bars joined by pins at their ends. This configuration creates a rigid frame (stable) Four or more bars, pin-joined to form a polygon create a non-rigid (unstable) frame.

Structures built from triangles are called simple trusses.

When there are more members than necessary to prevent collapse, the truss is statically indeterminate. These structures cannot be analyzed by the equilibrium equations alone.

Additional members not necessary for equilibrium are redundant.

Assumptions used to analyze simple trusses

- Weight of member is negligible compared to force supported
- All connections are pin connections and cannot transmit a moment
- All external forces are applied at the pin connections
- All members are two-force members and are either in tension or compression

FORCES IN MEMBERS

TENSION

COMPRESSION

FORCES ON PINS

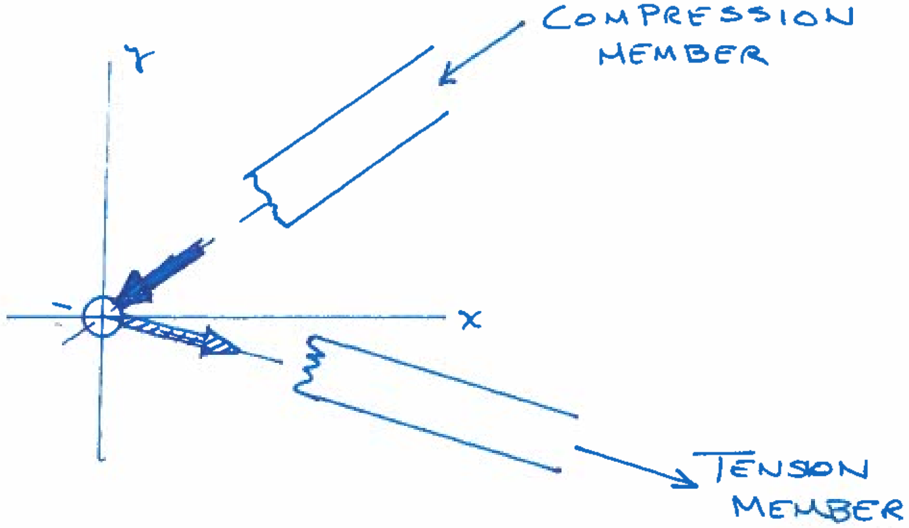
TENSION @ PIN

COMPRESSION @ PIN

ANALYSIS OF SIMPLE TRUSS – METHOD OF JOINTS

Apply equilibrium conditions for the forces acting on the connecting pin of each joint. Method only uses the force balance at each joint ($\Sigma \underline{F} = \underline{0}$)

Use consistent notations for tension and compression



In many cases it is not possible to assign the correct direction of an unknown force. Simply make an arbitrary assignment. A negative result means the actual direction is <u>opposite</u> of the arbitrary assingment.

Usually arbritary tensions are most convienent because angles and projections are easier to draw & infer.

The <u>tension/compression</u> refers to a given <u>member</u> not to the joint.

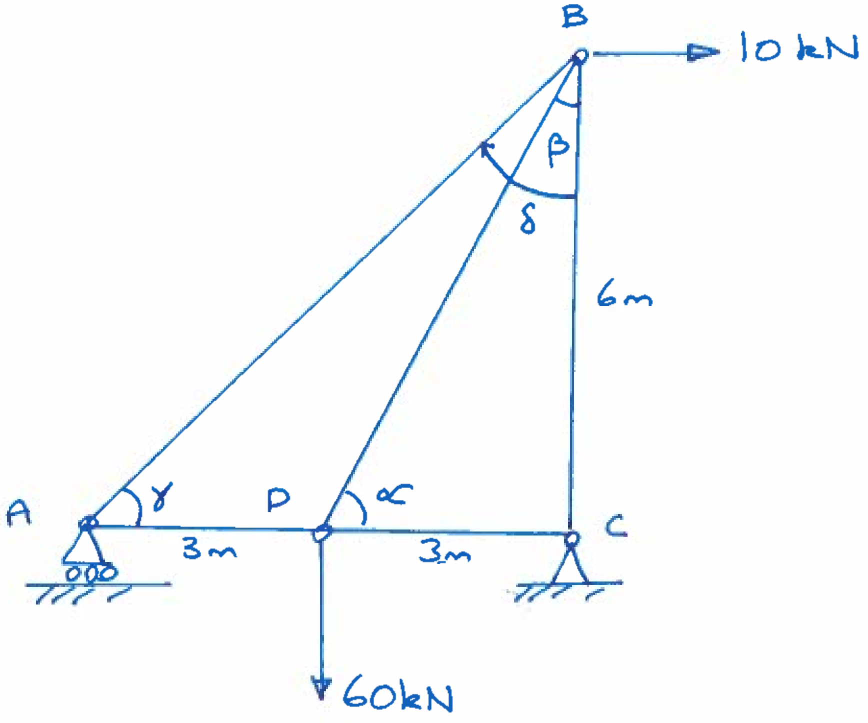
Because the entire truss is a rigid body inequilibrium, additional equations to find reaction forces at supports can be written using equilibrium principles.

Suggested procedure

- Draw space diagram
- Use trigonometery to find all angles in the truss
- Draw FBD of entire truss, apply equilibrium conditions to find reactions forces.
- Analyze truss joint-by-joint to find member forces.

Example

Find force in each member of truss shown



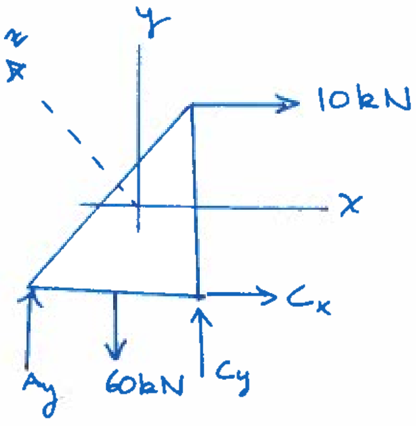
$\tan \alpha = \frac{6}{3}$

$\alpha = 63.4°$

$\tan \beta = \frac{3}{6}$

$\beta = 26.6°$

$\tan \gamma = \frac{6}{6}$ $\quad \gamma = 45°$



$\Sigma M_c = 0$

$= -A_y(6)\underline{k} + 60(3)\underline{k}$

$-10(6)k \quad = 0$

$A_y = 20\text{kN} \uparrow$

$\Sigma F_y = 0 = A_y + C_y - 60$

$C_y = 40\text{kN} \uparrow$

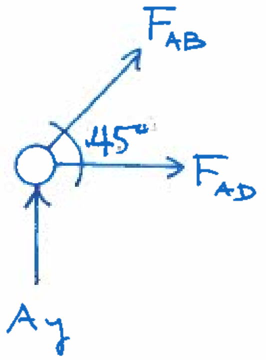
$\Sigma F_x = 0 = C_x + 10$

$C_x = -10\text{ kN}$

OR

$C_x = 10\text{kN} \leftarrow$

JoinT A



$$\sum F_x = 0 = F_{AB}\cos 45 \quad + F_{AD} = 0$$

$$\sum F_y = 0 = F_{AB}\sin 45 + A_y$$

$$F_{AB} = \frac{-A_y}{\sin 45} = \frac{-20}{\sin 45} = -28.3\,kN$$

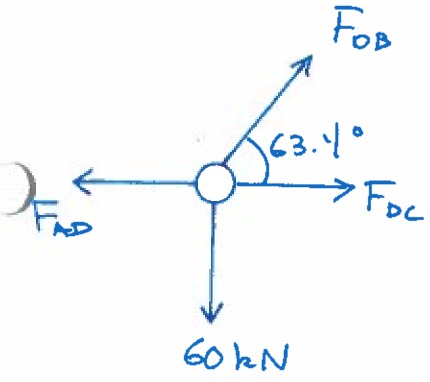
Use THIS RESULT AS DRAWN

$$\boxed{F_{AB} = 28.3\,kN \text{ Compression}}$$

$$F_{AD} = -F_{AB}\cos 45 = 20\,kN$$

$$\boxed{F_{AD} = 20\,kN \text{ Tension}}$$

JoINT D



$$\sum F_x = 0 = F_{DC} + F_{DB}\cos 63.4 - F_{AD}$$

$$\sum F_y = 0 = F_{DB}\sin 63.4 - 60$$

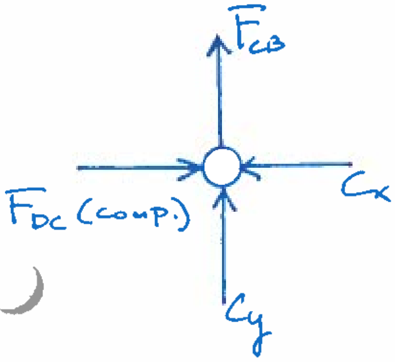
$$\boxed{F_{DB} = 67.1\,kN\,(T)}$$

$$F_{DC} = F_{AD} - F_{DB}\cos 63.4$$

$$= -10\,kN$$

$$\boxed{F_{DC} = 10\,kN\,(C)}$$

JoINT C



$$\sum F_y = 0 = F_{CB} + 40$$

$$F_{CB} = -40\,kN$$

$$\boxed{F_{CB} = 40\,kN\,(C)}$$

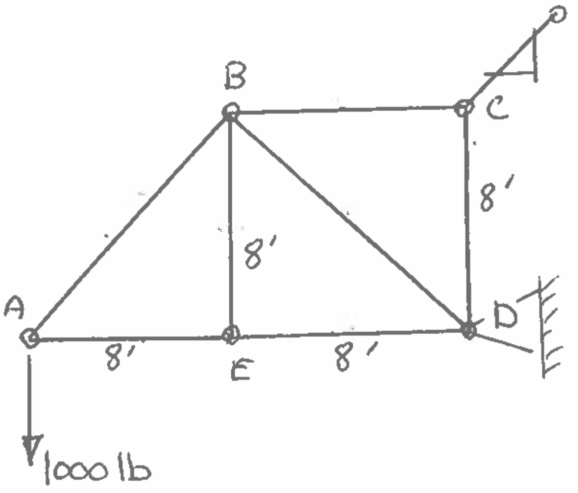
$$\sum F_x = F_{DC} - C_x = 0$$

$$\underline{F_{DC} = C_x \quad = 10\,kN}$$ ✓

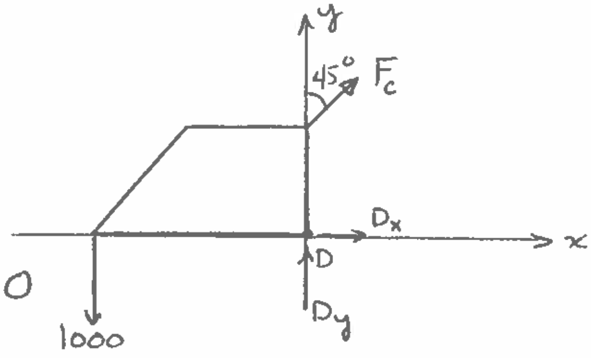
F_{DC} is SHOWN AS COMPRESSION.

MAGNITUDE OK!
DIRECTION OK.

4.3 FIND FORCES IN BE & BD.



FIND REACTION FORCES



$\Sigma \underline{F} = 0$

$\Sigma \underline{M} = 0$

$\Sigma F_x = D_x + F_c \sin 45 = 0$

$\Sigma F_y = D_y + F_c \cos 45 - 1000 = 0$

$+\circlearrowleft \Sigma M_b = 0 = 1000(16)\underline{k} - F_c \sin 45 (8)\, \underline{k}$

$F_c \sin 45(8) = 1000(16)$

$F_c = \dfrac{1000(16)}{\sin 45 (8)} = 2828$ lbs

$D_x = -2828(\sin 45) = -2000$ lb

$D_y = 1000 - 2828(\cos 45) = -1000$ lbs

$D_x = 2000$ lb ←

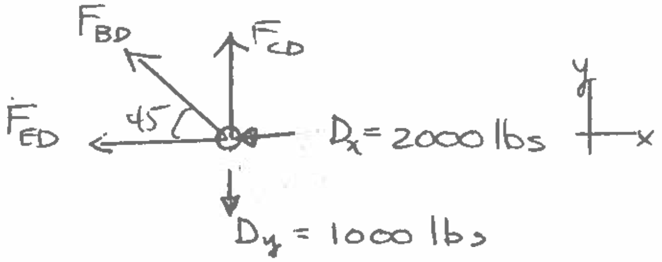
$D_y = 1000$ lb ↓

SHOW ALL FORCES AS TENSION.
(−) TENSION = COMPRESSION

JOINT D

$$\Sigma F_x = -2000 - F_{ED} - F_{BD}\cos 45 = 0$$

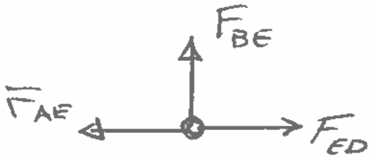
$$\Sigma F_y = -1000 + F_{BD}\sin 45 + F_{CD} = 0$$



JOINT E

$$\Sigma F_x = -F_{AE} + F_{ED} = 0$$

$$\Sigma F_y = F_{BE} = 0$$



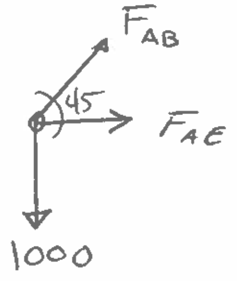
$\therefore$ $\underline{\underline{F_{BE} = 0}}$ (ZERO FORCE MEMBER) ←

$$F_{AE} = F_{ED}$$

JOINT A

$$\Sigma F_x = F_{AE} + F_{AB}\cos 45 = 0$$

$$\Sigma F_y = F_{AB}\sin 45 - 1000 = 0$$



$$F_{AB} = \frac{1000}{\sin 45} = 1414 \text{ lbs}$$

$$F_{AE} = -F_{AB}\cos 45 = -1000 \text{ lb}$$

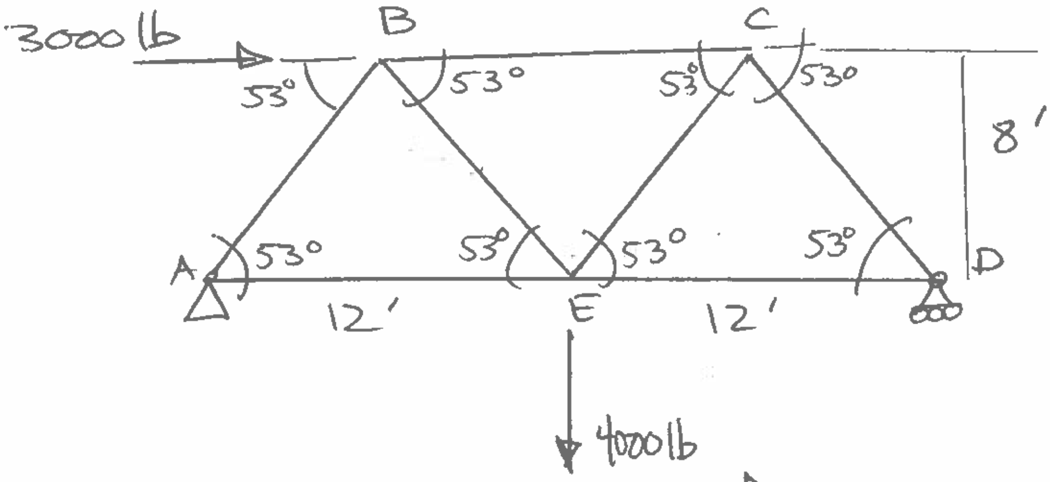
NOW SUBSTITUTE

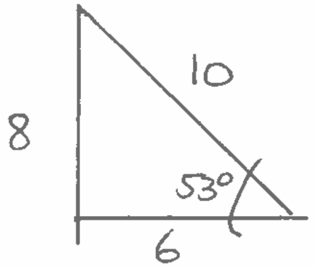
$$F_{AE} = F_{ED} = -1000 \text{ lb}$$

$$\therefore \frac{-2000 - F_{ED}}{\cos 45} = F_{BD} \qquad \frac{-2000 - (-1000)}{\cos 45} = \frac{-1000}{\cos 45} = -1414 \text{lbs}$$

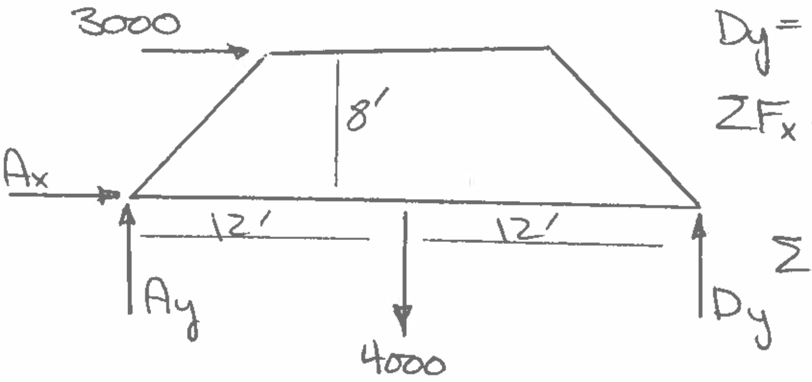
$\therefore$ $\underline{\underline{F_{BD} = 1414 \text{ lbs COMPRESSION}}}$ ←

4.6 CALCULATE FORCE IN EACH MEMBER OF TRUSS





FIND REACTIONS



$$\Sigma M_A = -3000(8) - 4000(12) + D_y(24)$$

$$D_y = \frac{3000(8) + 4000(12)}{24} = 3000 \text{ lb}$$

$$\Sigma F_x = 3000 + A_x = 0$$

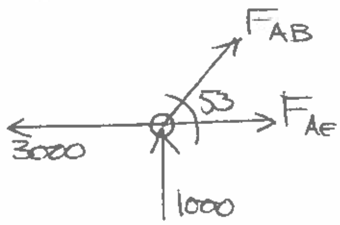
$$A_x = -3000 \text{ lb}$$

$$\Sigma F_y = A_y + D_y - 4000$$

$$A_y = 4000 - D_y = 1000 \text{ lb}$$

JOINT A

$\Sigma F_x = 0$
$\Sigma F_y = 0$



$$\Sigma F_x = F_{AB}\cos 53 + F_{AE} - 3000 = 0$$

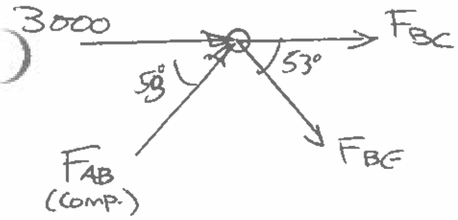
$$\Sigma F_y = F_{AB}\sin 53 + 1000 = 0$$

$$F_{AB} = \frac{-1000}{\sin 53} = -1252$$

$$F_{AE} = 3000 - F_{AB}\cos 53$$
$$= 3000 - (-1252)\cos 53 = 3753$$

$\therefore F_{AB} = 1252\ C$
$F_{AE} = 3753\ T$

JOINT B



$$\Sigma F_x = 3000 + F_{AB}\cos 53 + F_{BC} + F_{BE}\cos 53 = 0$$

$$\Sigma F_y = F_{AB}\sin 53 - F_{BE}\sin 53 = 0$$

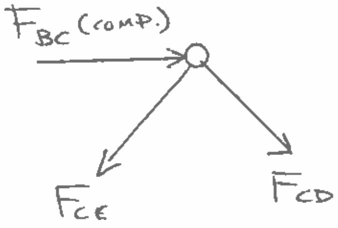
$$F_{Be} = F_{AB} = 1252$$

$$F_{BE} = 1252\ T$$

$$F_{BC} = -F_{AB}\cos 53 - F_{BE}\cos 53 - 3000$$
$$= -4507 \text{ lbs}$$

$F_{BC} = 4507$ lbs C

JOINT C



$$\Sigma F_x = F_{BC} - F_{CE}\cos 53 + F_{CD}\cos 53 = 0$$

$$\Sigma F_y = -F_{CE}\sin 53 - F_{CD}\sin 53 = 0$$

$$F_{CE} = -F_{CD}$$

$$F_{CD} = \frac{-F_{BC}}{2\cos 53} = \frac{-4507}{2\cos 53} = -3744 \text{ lbs}$$

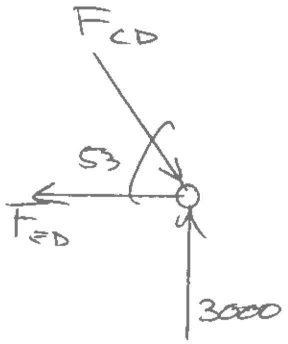
$F_{CD} = 3744\ C$
$F_{CE} = 3744\ T$

JOINT D

$$\Sigma F_x = -F_{ED} + F_{CD}\cos 53 = 0$$

$$F_{ED} = F_{CD}\cos 53$$

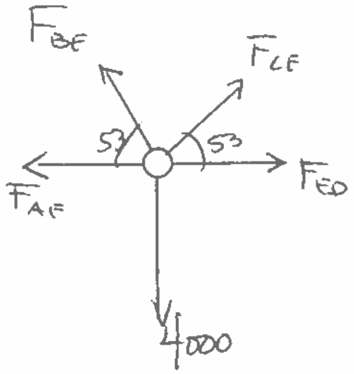
$$= 2253 \text{ lb } T$$



$$\Sigma F_y = -F_{CD}\sin 53 + 3000$$

$$F_{CD} = \frac{3000}{\sin 53} = 3756 \quad \underline{\underline{\text{OK}}}$$

JOINT E (CHECK)



$$\Sigma F_x = F_{ED} + F_{CE}\cos 53 - F_{BE}\cos 53 - F_{AE}$$

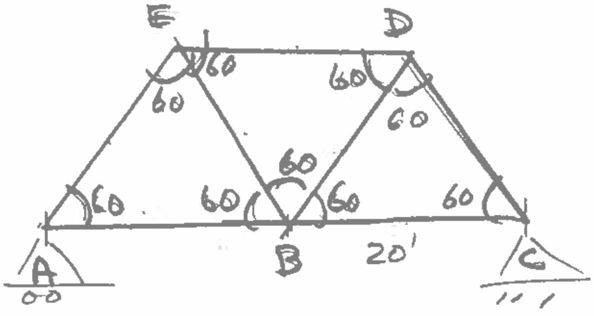
$$= 2253 + 3744\cos 53 - 1252\cos 53 - 3753$$

$$= 2253 + 2253 - 753 - 3753 = 0 \checkmark$$

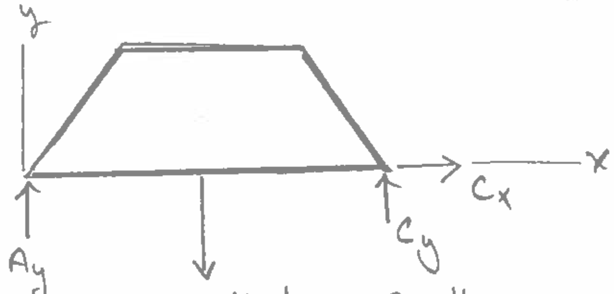
$$\Sigma F_y = F_{BE}\sin 53 + F_{CE}\sin 53 - 4000$$

$$1252\sin 53 + 3744\sin 53 - 4000 = 0 \checkmark$$

4-9 Each member is a 20ft, 400lb bar. Find average tension & compression in each member



1) FIND REACTION FORCES



W = 7*400 = 2800lb

$\Sigma F_x = 0$

$C_x = 0$

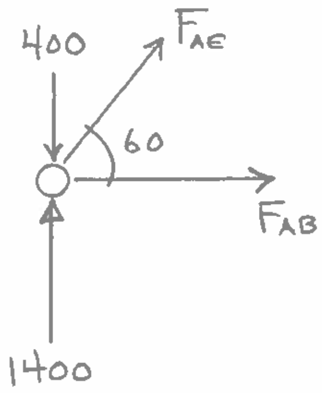
$\Sigma F_y = 0 = A_y + C_y - W$

$+\circlearrowleft \Sigma M_A = 0 = -W(20) + C_y(40) = 0$

$C_y = \frac{W(20)}{40} = \frac{1}{2}W = 1400\text{lb}$

$A_y = 1400$ lb

2) JOINT A



$$\Sigma F_x = 0 = F_{AB} + F_{AE}\cos 60$$

$$F_{AB} = -F_{AE}\cos 60$$

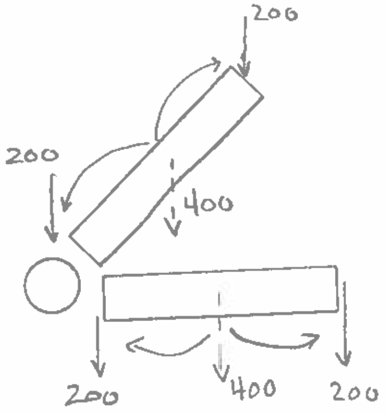
$$\Sigma F_y = 0 = 1400 + F_{AE}\sin 60 - 400$$

$$F_{AE} = \frac{400}{\sin 60} - \frac{1400}{\sin 60} = 461.8 - 1616.5$$

$$= \underline{\underline{-1154.78 \text{ lb}}}$$

$$F_{AB} = -(-1154.78)\cos 60$$

$$= \underline{\underline{577.4 \text{ lb}}}$$



$F_{AE} = 1154.8$ lb C

$F_{AB} = 577.4$ lb T

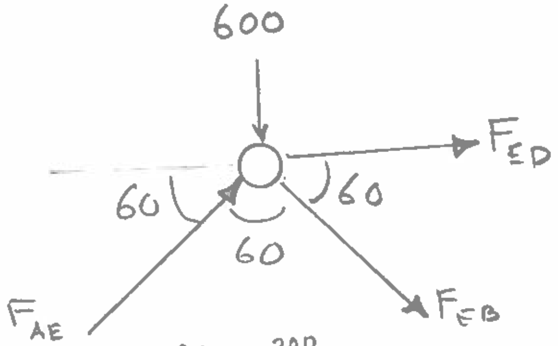
JOINT E

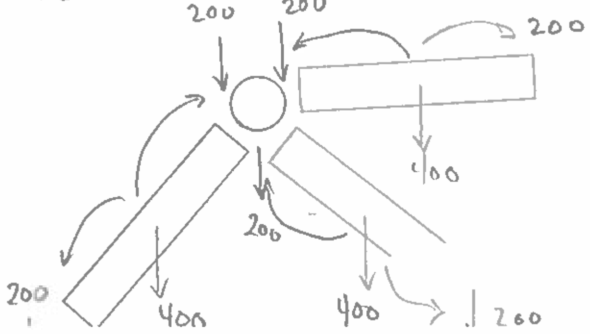
$$\Sigma F_y = 0 = F_{AE}\sin 60 - F_{EB}\sin 60 - 600$$

$$F_{BE} = \frac{F_{AE}\sin 60 - 600}{\sin 60}$$

$$= \frac{1154.8\cancel{\sin 60}}{\cancel{\sin 60}} - \frac{600}{\sin 60}$$

$$= 461.97 \text{ lb}$$





$$\Sigma F_x = 0 = F_{ED} + F_{AE}\cos 60 + F_{EB}\cos 60$$

$$F_{ED} = -F_{AE}\cos 60 - F_{EB}\cos 60$$

$$= -1154.8\cos 60 - 461.97\cos 60$$

$$= -808.3 \text{ lb}$$

BY SYMMETRY

$F_{DC} = F_{AE}$

$F_{DB} = F_{BE}$

$F_{BC} = F_{AB}$

$\therefore$ $F_{AB} = 577.4$ lb T

$F_{AE} = 1154.8$ lb C

$F_{ED} = 808.3$ lb C

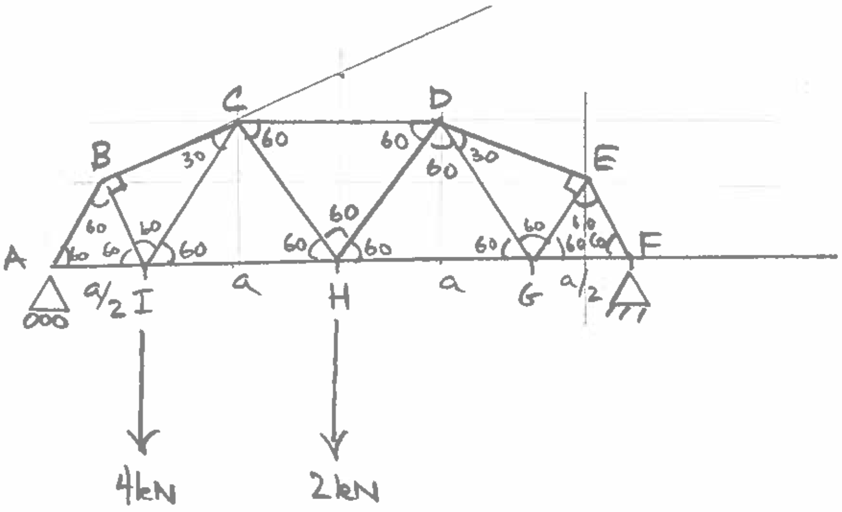
$F_{BE} = 461.97$ lb T

$F_{DC} = 1154.8$ lb C

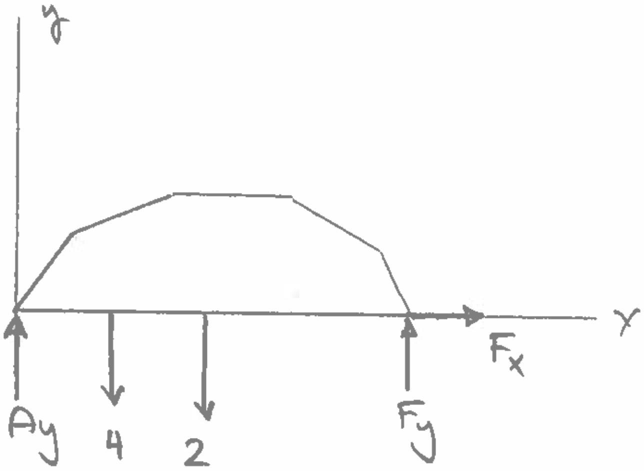
$F_{BD} = 461.97$ lb T

$F_{BC} = .577.4$ lb T

4-15 FIND FORCES IN BI, CI & HI



1) FIND REACTIONS



$$\Sigma F_x = 0 = F_x$$

$$\circlearrowleft \Sigma M_A = 0 = -4\,a/2 - 2\,3a/2 + F_y\,3a$$

$$F_y\,3\cancel{a} = 2\cancel{a} + 3\cancel{a}$$

$$F_y = 5/3 \text{ kN}$$

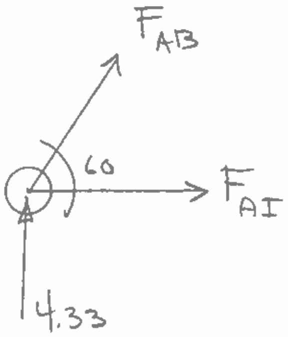
$$\Sigma F_y = 0 = F_y + A_y - 6$$

$$A_y = 6 - 5/3 = \frac{18}{3} - \frac{5}{3} = 13/3$$

$F_y = 1.67\text{ kN}$

$A_y = 4.33\text{ kN}$

JOINT A



$\Sigma F_y = 0 = 4.33 + F_{AB}\sin 60$

$$F_{AB} = \frac{-4.33}{\sin 60} = -4.99\text{ kN}$$

$\Sigma F_x = 0 = F_{AI} + F_{AB}\cos 60$

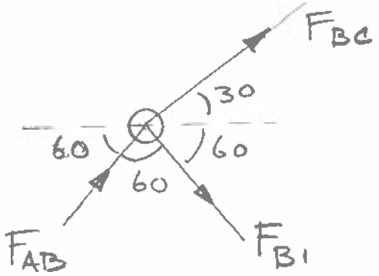
$F_{AI} = -F_{AB}\cos 60$

$= -(-4.99)\cos 60 = 2.499\text{ kN}$

$F_{AB} = 5.0\text{ kN}$ C

$F_{AI} = 2.5\text{ kN}$ T

JOINT B



$\Sigma F_x = 0 = F_{AB}\cos 60 + F_{B1}\cos 60 + F_{BC}\cos 30$

$\Sigma F_y = 0 = F_{AB}\sin 60 - F_{B1}\sin 60 + F_{BC}\sin 30$

2 eqn. 2 unk.

$F_{AB}\cos 60 = 2.5$

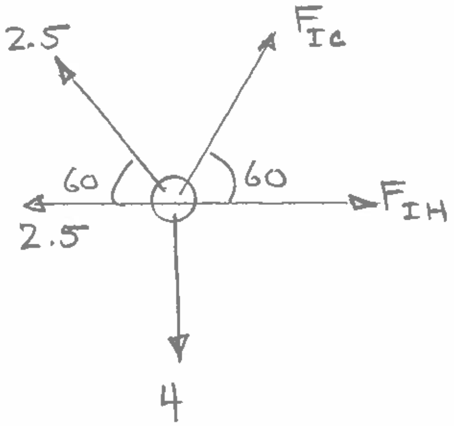
$F_{AB}\sin 60 = 4.33$

$$\begin{pmatrix} \cos 60 & \cos 30 \\ -\sin 60 & \sin 30 \end{pmatrix}\begin{pmatrix} F_{B1} \\ F_{BC} \end{pmatrix} = \begin{pmatrix} -2.5 \\ -4.33 \end{pmatrix}$$

$F_{B1} = 2.5\text{ kN}$

$F_{BC} = -4.33\text{ kN}$

JOINT I



$$\Sigma F_y = 0 = 2.5 \sin 60 + F_{Ic} \sin 60 - 4 = 0$$

$$F_{Ic} = \frac{4 - 2.5 \sin 60}{\sin 60}$$

$$= \frac{4}{\sin 60} - 2.5 = 2.118 \text{ kN}$$

$$\Sigma F_x = 0 = -2.5 - 2.5 \cos 60 + 2.118 \cos 60 + F_{IH}$$

$$F_{IH} = 2.5 + 2.5 \cos 60 - 2.118 \cos 60$$

$$= 2.69 \text{ kN}$$

$\therefore$

$F_{BI} = 2.5$ kN T

$F_{cN} = 2.12$ kN T

$F_{HI} = 2.69$ kN T

ANALYZE CASTER

$$+\circlearrowright \Sigma M_A = 0 = -(4\text{kN} + \Delta B)(70) + (4\text{kN} - \Delta B)(70) - 3(25)$$

SOLVE FOR ΔB

$$-\cancel{4(70)} - \Delta B(70) + \cancel{4(70)} - \Delta B(70) = 75$$

$$-2\Delta B(70) = 75$$

$$\Delta B = \frac{-75}{2(70)} = -0.536 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0 = (4\text{kN} + \Delta A)(70) - (4\text{kN} - \Delta A)(70) - 3(95) = 0$$

$$\cancel{4(70)} + \Delta A\,70 - \cancel{4(70)} + \Delta A\,70 = 285$$

$$\Delta A = \frac{285}{2(70)} = 2.04 \text{ kN}$$

ANALYZE BOLTS

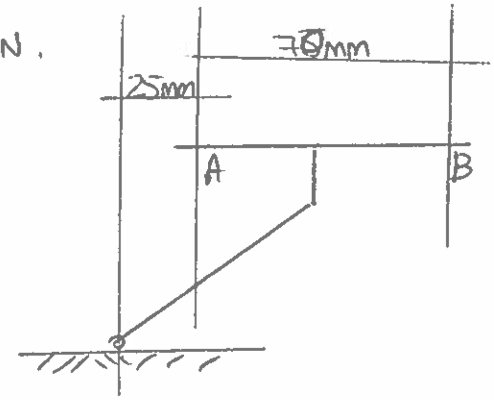
$$T_A = 4\text{kN} - \Delta A = 4 - 2.04 = 1.96 \text{ kN (TENSION)}$$

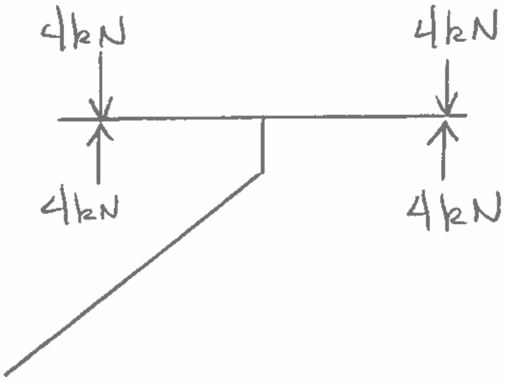
$$T_B = 4\text{kN} - \Delta B = 4 - (-0.54) = 4.54 \text{ kN (TENSION)}$$

3.110

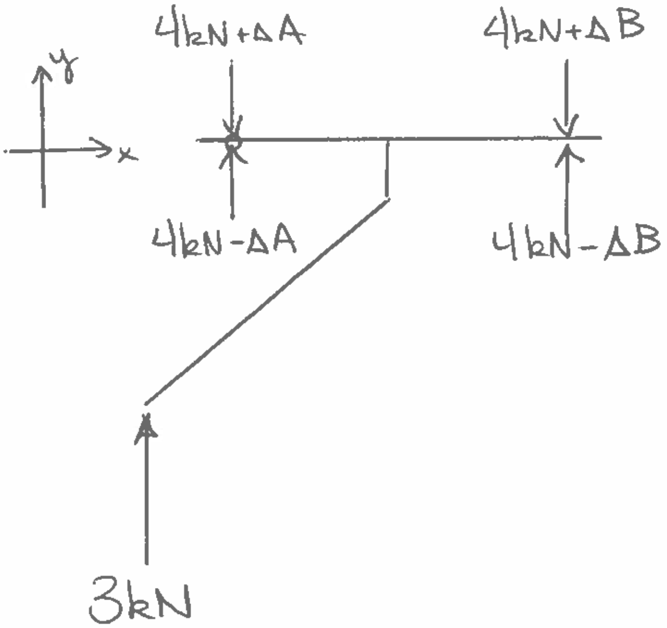
A & B ARE INITIALLY STRESSED TO 4 kN TENSION.

FIND FORCE IN EACH BOLT WHEN CASTER SUPPORTS 3 kN LOAD



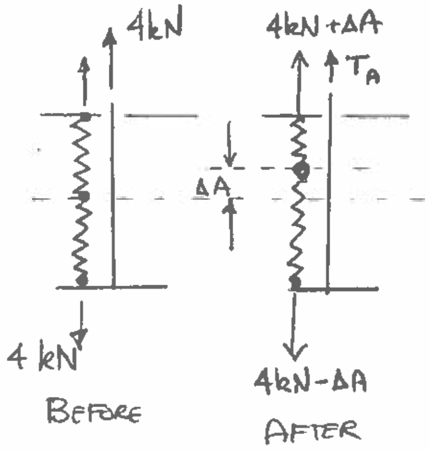


BEFORE

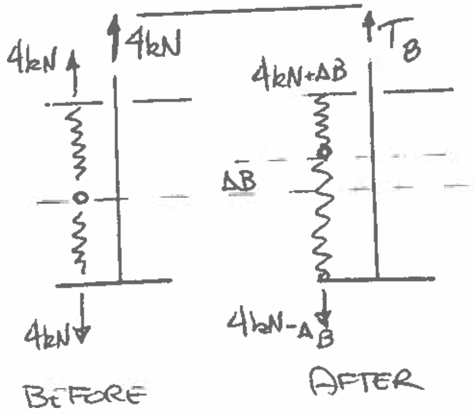


AFTER

BOLT A

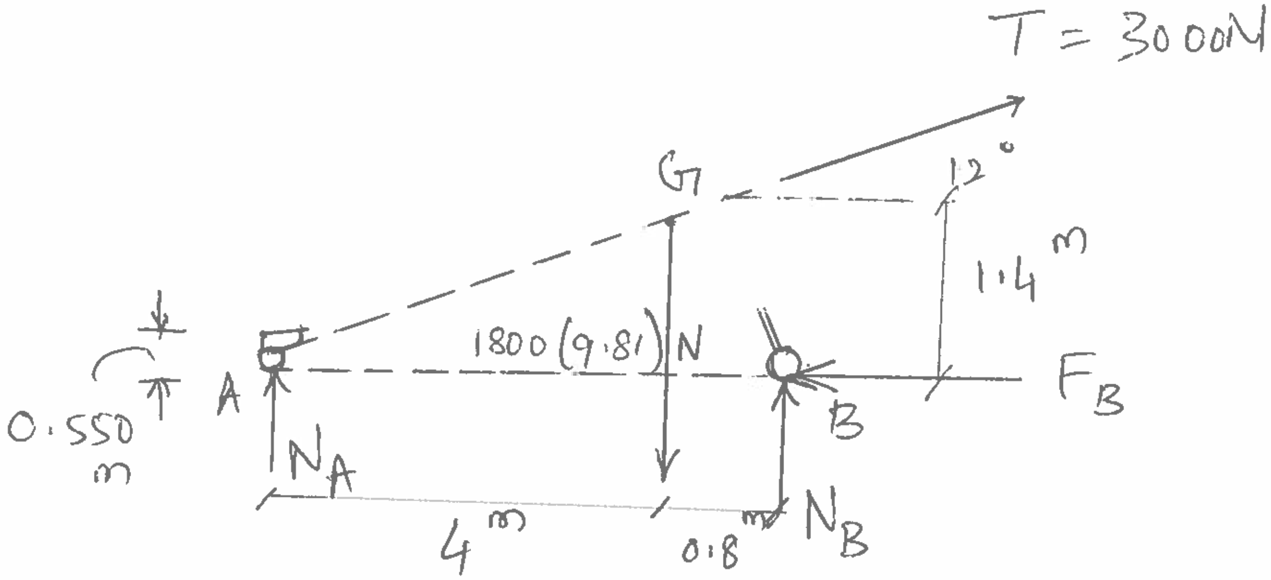


BOLT B



HW #10

Prob # 3.24



Engine off : $T = 0$; $F_B = 0$

$\circlearrowright \Sigma M_A = 0 : 1800(9.81)4 - N_B(4.8)0$;

$N_B = 14720\ N$

$\Sigma F_y = 0 : N_A + 14720 - 1800(9.81) = 0$;

$N_A = 2940\ N$

$\Sigma M_A = 0 : 1800(9.81)4 - N_B'(4.8)$

$+ 3000 \cos 12° (0.550) = 0$

$N_B' = 15{,}050\ N$

$\sum F_y = 0:$

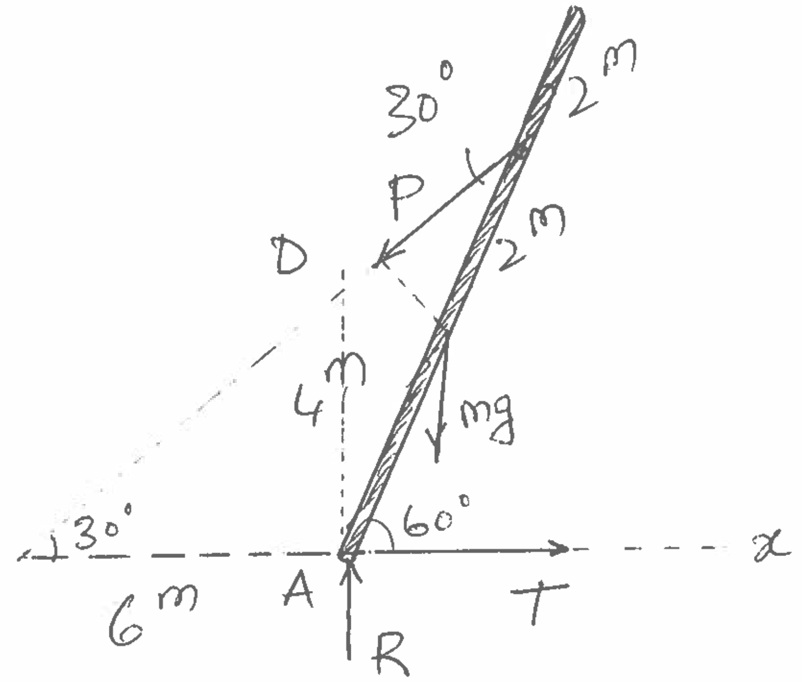
$$N_A' + 15050 - 1800(9.81) + 3000 \sin 12^\circ = 0$$

$$\underline{N_A' = 1983\,N}$$

$$A = \frac{N_A' - N_A}{N_A}(100) = \underline{-32.6\%}$$

$$B = \frac{N_B' - N_B}{N_B} = \underline{2.28\%}$$

Prob # 3.25



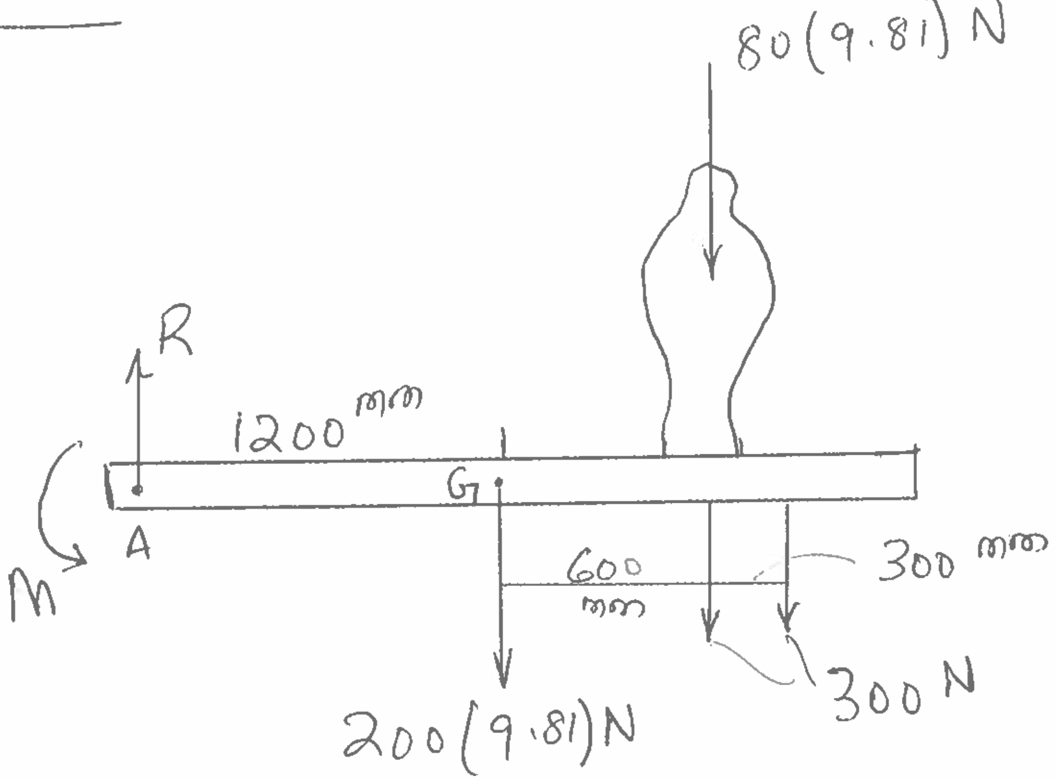
$$mg = 25(9.81) = 245 \text{ kN}$$

$$\Sigma M_D = 0;$$

$$T\left[\left(\frac{4+2}{2}\right) / \cos 30^\circ\right] - 245(4 \cos 60^\circ) = 0$$

$$T = \frac{490}{3/0.866} = 141.6 \text{ kN.}$$

Prob 3.40

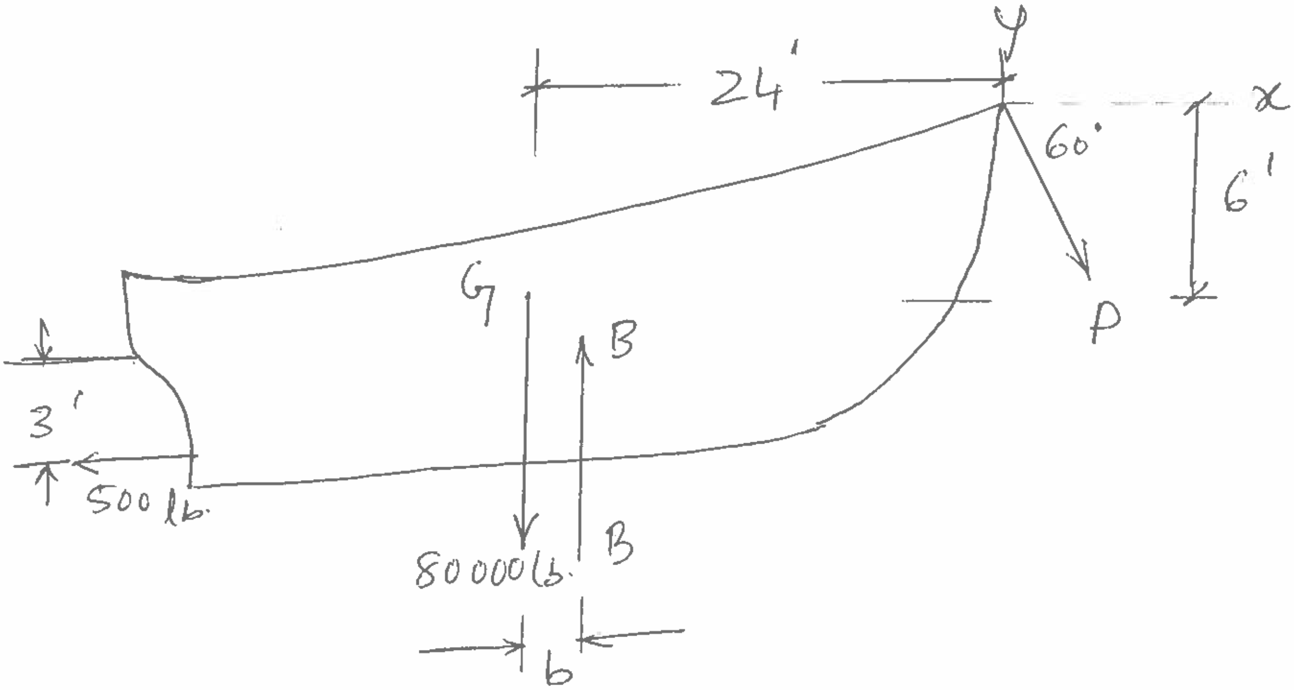


$$\circlearrowright \Sigma M_A = 0; \quad 80(9.81)(1800) + 200(9.81)(1200) + 300(1800 + 2100) - M = 0$$

$$\underline{M = 4.94 (10^6)\ N \cdot mm}$$

or $\underline{M = 4.94\ kN \cdot m}$

Prob # 3.54



$\sum F_x = 0$; $P \cos 60^\circ - 500 = 0$

$P = 1000$ lb

$\sum F_y = 0$; $B - 80{,}000 - 1000 \sin 60^\circ = 0$

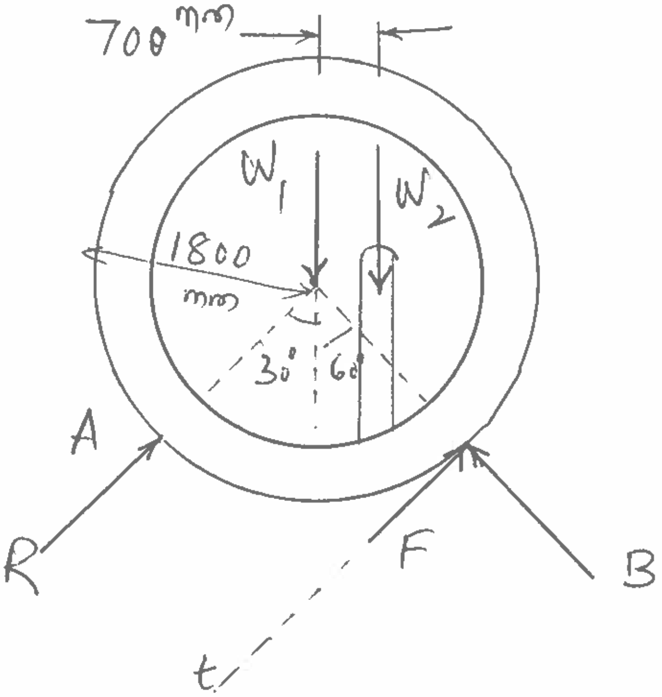
$\underline{B = 80{,}866 \text{ lb}}$

$\sum M_A = 0$; $80000(24) - 80{,}866(24 - b) - 500(6+3) = 0$

$1920{,}000 - 1{,}940{,}784 + 80{,}866b - 4500 = 0$

$b = \dfrac{25284}{80866} = \underline{0.3127 \text{ ft}}$ or $\underline{b = 3.75 \text{ in}}$

3.55



$W_1 = m_1 g = 400(9.81) = 3924\ N$

$W_2 = m_2 g = 80(9.81) = 785\ N$

$\Sigma M_o = 0;\ 785(0.7) - 1.8F = 0$

$F = 305 N$

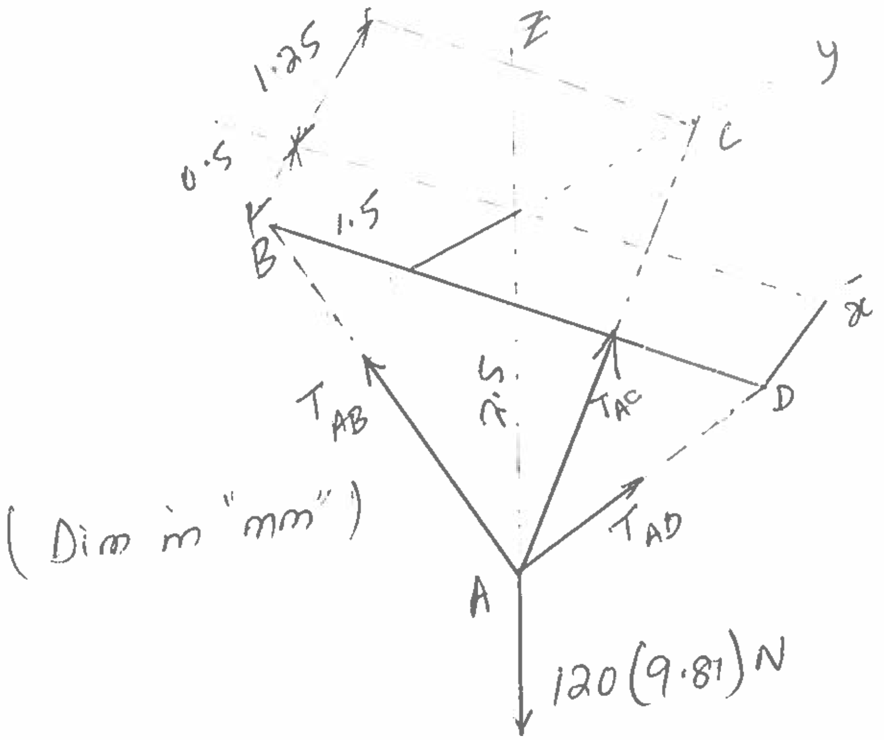
$\Sigma F_t = 0;\ R + 305 - (3924 + 785)\cos 30^\circ = 0$

$\underline{R = 3770 N}$

CIVE : 2330

HW # 11

Prob # 3.61



$$\underline{T}_{AB} = T_{AB} \frac{-1.5\underline{i} - 0.5\underline{j} + 2.5k}{\sqrt{1.5^2 + 0.5^2 + 2.5^2}}$$

$$= T_{AB}\left(-0.507\underline{i} - 0.169\underline{j} + 0.845\underline{k}\right)$$

$$\underline{T}_{AC} = T_{AC} \frac{1.25\underline{j} + 2.5\underline{k}}{\sqrt{1.25^2 + 2.5^2}}$$

$$= T_{AC}\left(0.447\underline{j} + 0.894\underline{k}\right)$$

$$\underline{T}_{AD} = T_{AD} \frac{2\underline{i} - 0.5\underline{j} + 2.5\underline{k}}{\sqrt{2^2 + 0.5^2 + 2.5^2}}$$

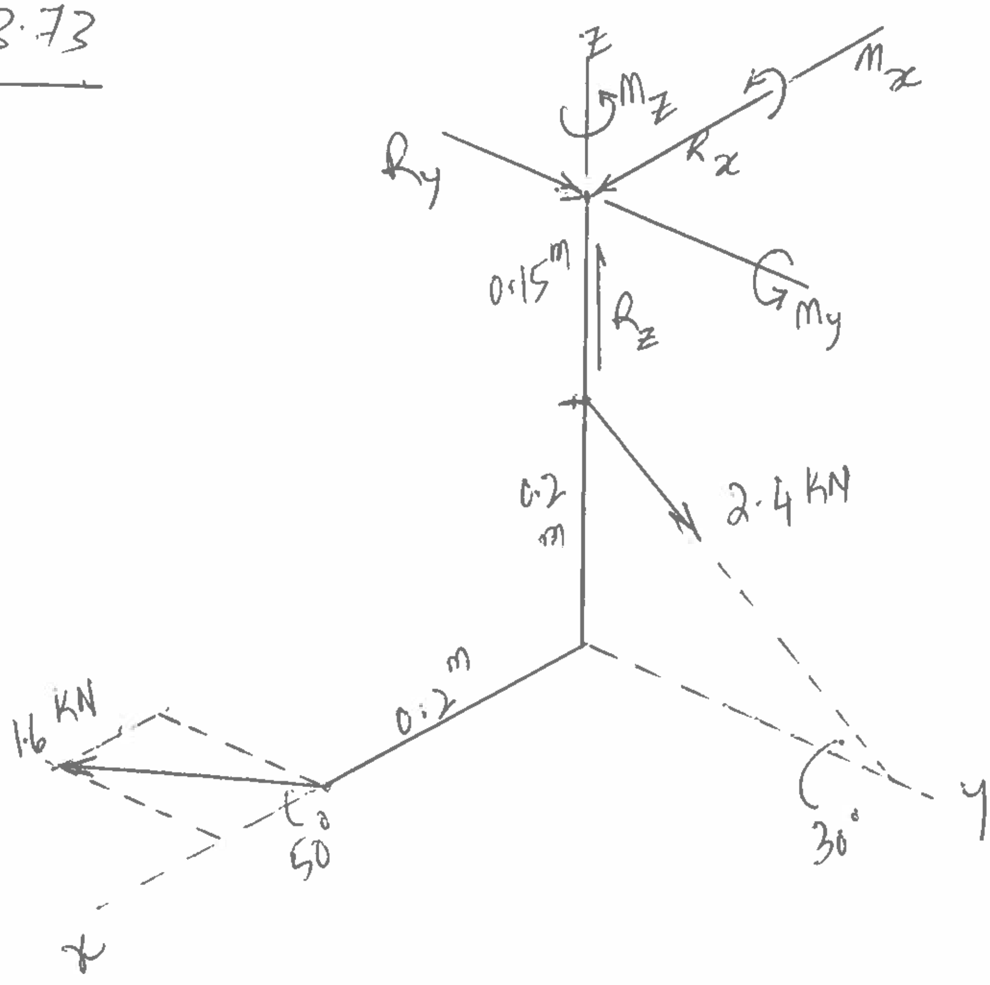
$$= T_{AD}\left(0.617\,\underline{i} - 0.1543\,\underline{j} + 0.772\,\underline{k}\right)$$

$$\left\{\begin{aligned}
&\Sigma F_x = 0 : -0.507\,T_{AB} + 0.617\,T_{AD} = 0. \\
&\Sigma F_y = 0 : -0.1690\,T_{AB} + 0.447\,T_{AC} - 0.1543\,T_{AD} = 0 \\
&\Sigma F_z = 0 : 0.845\,T_{AB} + 0.894\,T_{AC} + 0.772\,T_{AD} \\
&\qquad\qquad - 120(9.81) = 0.
\end{aligned}\right.$$

Solution :

$$\left[\begin{aligned}
T_{AB} &= 569\text{ N} \\
T_{AC} &= 376\text{ N} \\
T_{AD} &= 467\text{ N}
\end{aligned}\right]$$

Prob # 3.73



$\Sigma F_x = 0: \; R_x + 1.6 \cos 50^\circ; \quad \underline{R_x = -1.028 \text{ kN}}$

$\Sigma F_y = 0: \; R_y + 2.4 \cos 30^\circ - 1.6 \sin 50^\circ = 0;$

$\underline{R_y = -0.853 \text{ kN}}$

$\Sigma F_z = 0 \; : \; R_z - 2.4 \sin 30^\circ = 0 \quad \underline{R_z = 1.2 \text{ kN}}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = 1.796 \text{ kN}$

$$\Sigma M_{o_x} = 0 : \quad M_x + 2.4 \cos 30^\circ (0.15) - 1.6 \sin 50^\circ (0.35) = 0$$

$$\underline{M_x = 0.1172 \text{ kN-m}}$$

$$\Sigma M_{o_y} = 0 : \quad M_y - 1.6 \cos 50^\circ (0.35) = 0 \,;$$

$$\underline{M_y = 0.360 \text{ kN-m}}$$

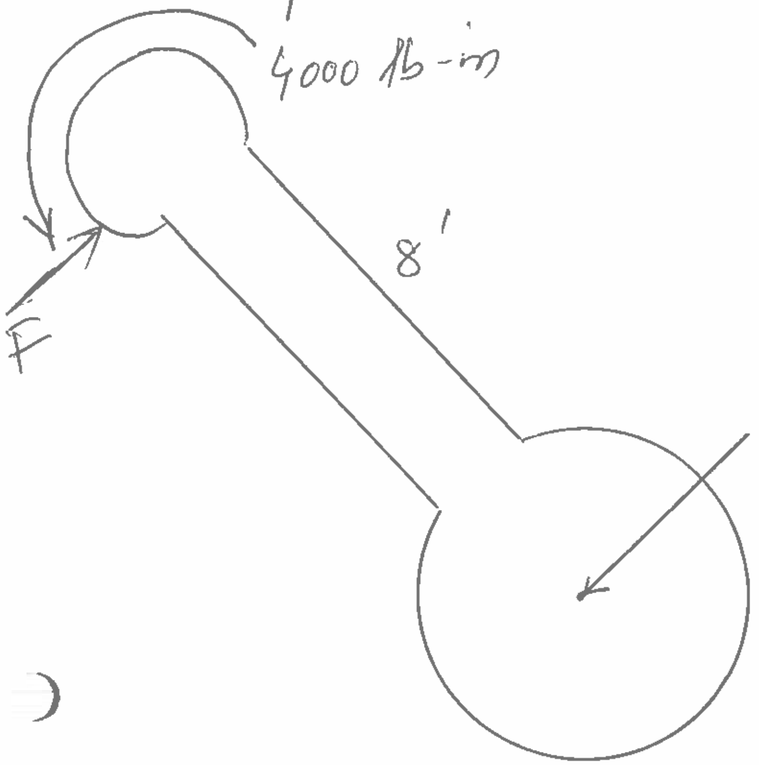
$$\Sigma M_{o_z} = 0 : \quad M_z - 1.6 \sin 50^\circ (0.2) = 0 \,;$$

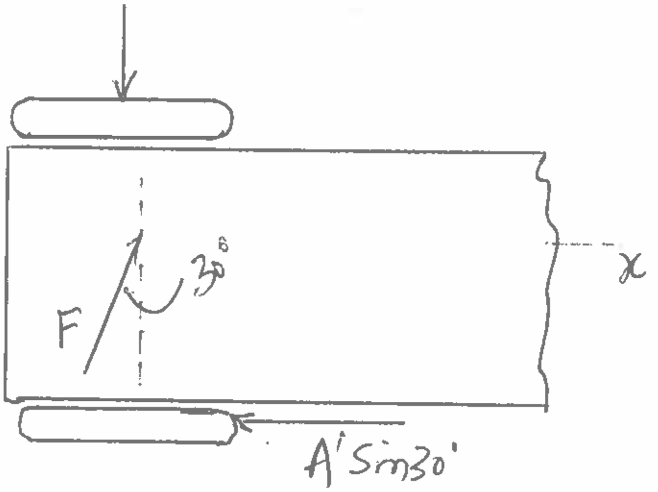
$$\underline{M_z = 0.245 \text{ kN.m}}$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = 0.451 \text{ kN.m}$$

Prob # 3.104

Torque on auger is opposite to applied torque.





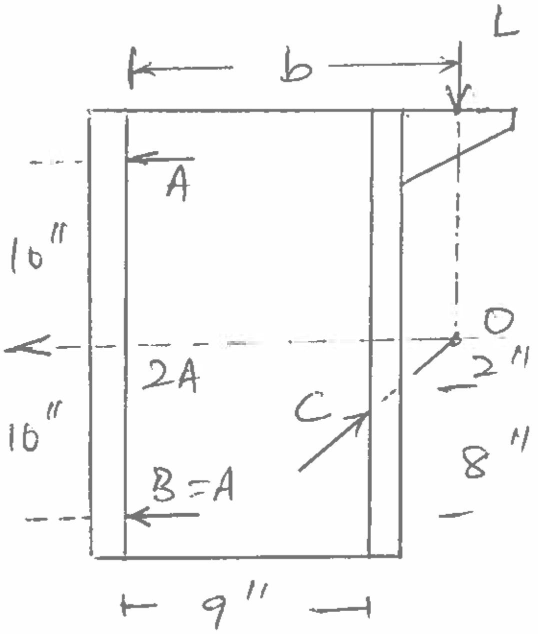
Arm : $\Sigma M = 0$; $8(12)F - 4{,}000 = 0$;

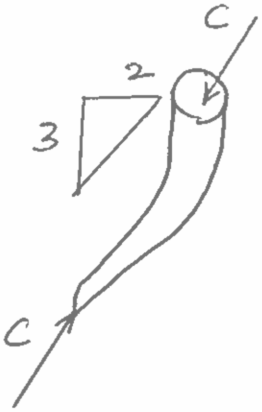
$F = 41.7$ lb.

Truck: $\Sigma F_x = 0$; $A' \sin 30^\circ - 41.7 \sin 30^\circ$

$A' = 41.7$ lb

Prob 3.109

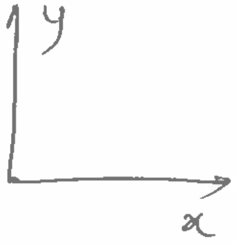




Reduce to three forces Concurrent at O

~~b + 9~~ $b = 9 + \frac{2}{3}(2) = 10.33$ in

Alternative Solutions



$\Sigma M_C = 0$; $L(b-9) - A(12) + A(8) = 0$

$L(b-9) = 4A$ ---(i)

$\Sigma F_y = 0$; $\frac{3}{\sqrt{13}} C = L$

$\Sigma F_y = 0$; $\frac{2}{\sqrt{13}} C = 2A$

$\Big\}$ $L = 3A$ (ii)

(1) & (2) give $b - 9 = \frac{4}{3}$; $b = 10.33$ in.

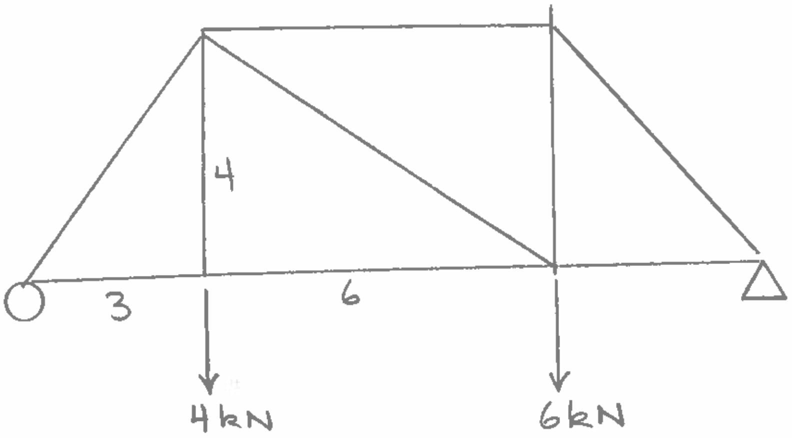
Method of Sections

Method of joints only uses two of the three equilibrium equations. (no moment equation - concurrent forces). Method of sections involves isolating a selected portion (or section) of a truss and applying equilibrium conditions to that section. Since the section will involve at least 2 joints forces will not be concurrent.

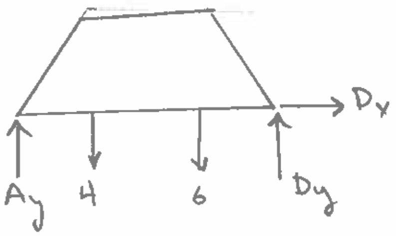
Method of sections has the advantage that the force in any member may be found directly from an analysis of a section which has cut that member.

In choosing a section of a truss, not more than 3 members whose forces are unknown may be cut - only have 3 equilibrium equations (in plane truss)

In method of sections, can either assign unknown forces as tensile/compressive by inspection, or can assume all unknown forces are positive in the tension direction (i.e. away from the section), negative answer indicates a compressive force.



REACTIONS



$\overset{+}{\curvearrowright} \Sigma M_A = 0$

$0 = -4(3) - 6(9) + D_y(12)$

$D_y = \frac{4(3) + 6(9)}{12} = \frac{12+54}{12}$

$= 5.5$ kN

$\Sigma F_y = 0 = A_y + D_y - 4 - 6$

$A_y = 4 + 6 - D_y$

$= 10 - 5.5 = 4.5$ kN

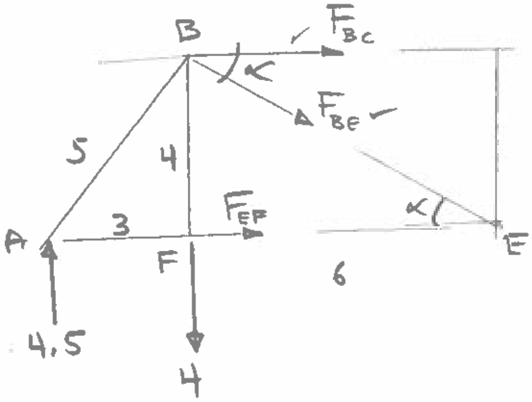
$\therefore A_y = 4.5$ kN

$D_y = 5.5$ kN

$D_x = 0$ kN

$\Sigma F_x = 0 = D_x$

$D_x = 0$



$\tan\alpha = \frac{4}{6}$

$\alpha = \tan^{-1}\left(\frac{4}{6}\right) = 33.69°$

$$+\circlearrowleft \Sigma M_E = 0 = 4(6) - 4.5(9) - F_{BC}(4) = 0$$

$$F_{BC} = \frac{24 - 40.5}{4} = -4.125 \text{ kN}$$

$$\therefore F_{BC} = 4.125 \text{ kN C}$$

$$\Sigma F_y = 0 = 4.5 - 4 - F_{BE} \sin 33.69$$

$$F_{BE} = \frac{4.5-4}{\sin 33.69} = 0.901 \text{ kN}$$

$$\therefore F_{BE} = 0.901 \text{ kN T}$$

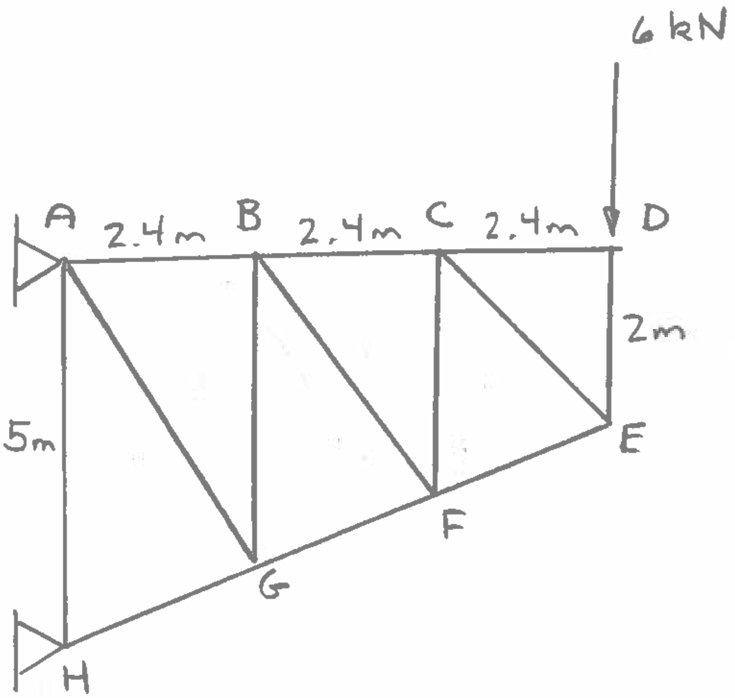
$$\Sigma F_x = 0 = F_{EF} + F_{BC} + F_{BE} \cos 33.69$$

$$F_{EF} = -F_{BC} - F_{BE} \cos 33.69$$

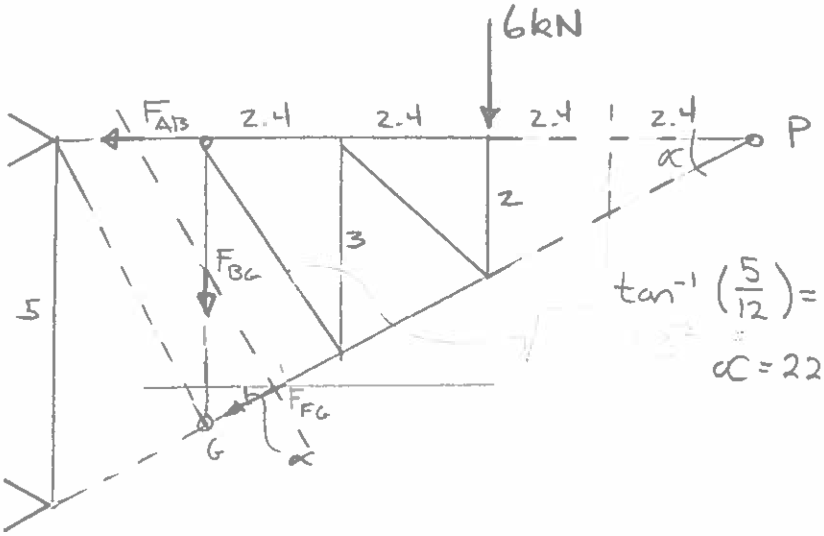
$$= -(-4.125) - (0.901)\cos 33.69 = 3.375 \text{ kN}$$

$$\therefore F_{EF} = 3.375 \text{ kN T}$$

4-32 FIND F_{AB}, F_{BG}, F_{GF}



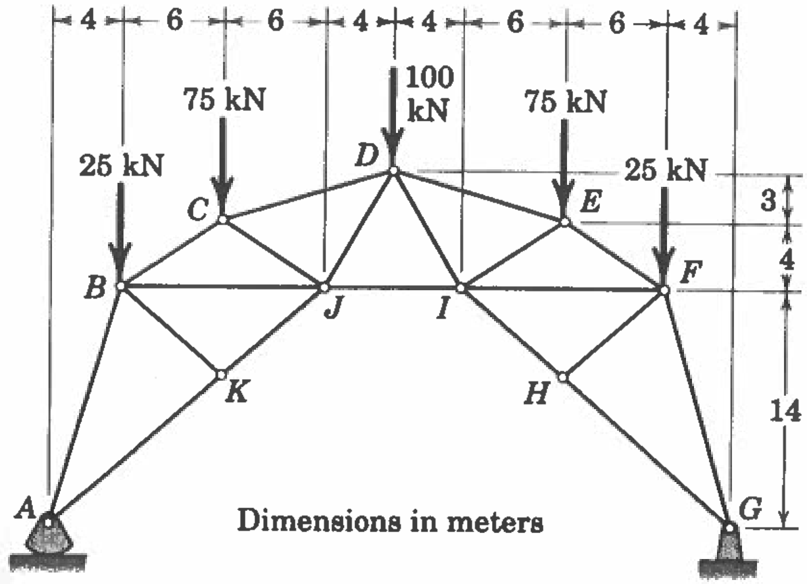
DIRECT APPLICATION OF SECTIONS



$\tan^{-1}\left(\frac{5}{12}\right)=\alpha$

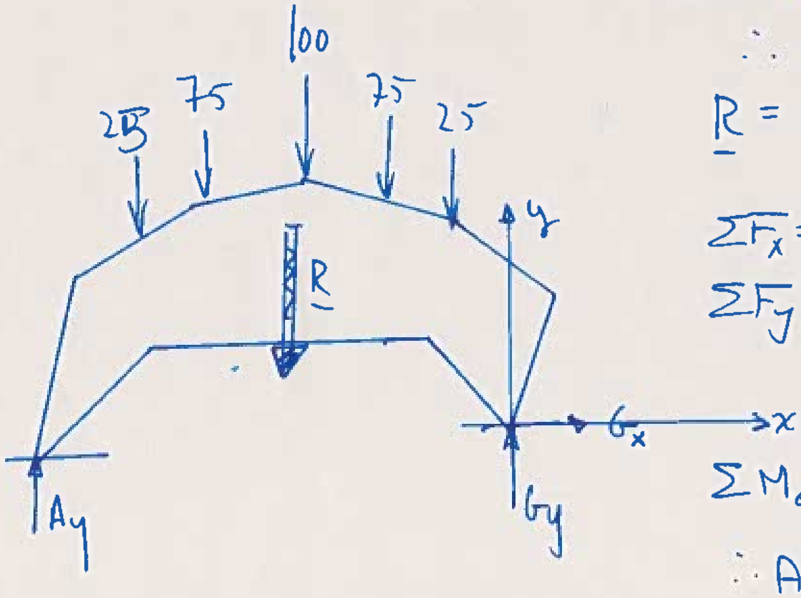
$\alpha = 22.619°$

4/46 Determine the forces in members *DE*, *EI*, *FI*, and *HI* of the arched roof truss.



Problem 4/46

1) FIND REACTIONS



- SYMMETRIC LOAD & TRUSS

∴ $\underline{R} = 300$ kN ↓ @ 20m from G

$\Sigma F_x = 0 \quad G_x = 0$

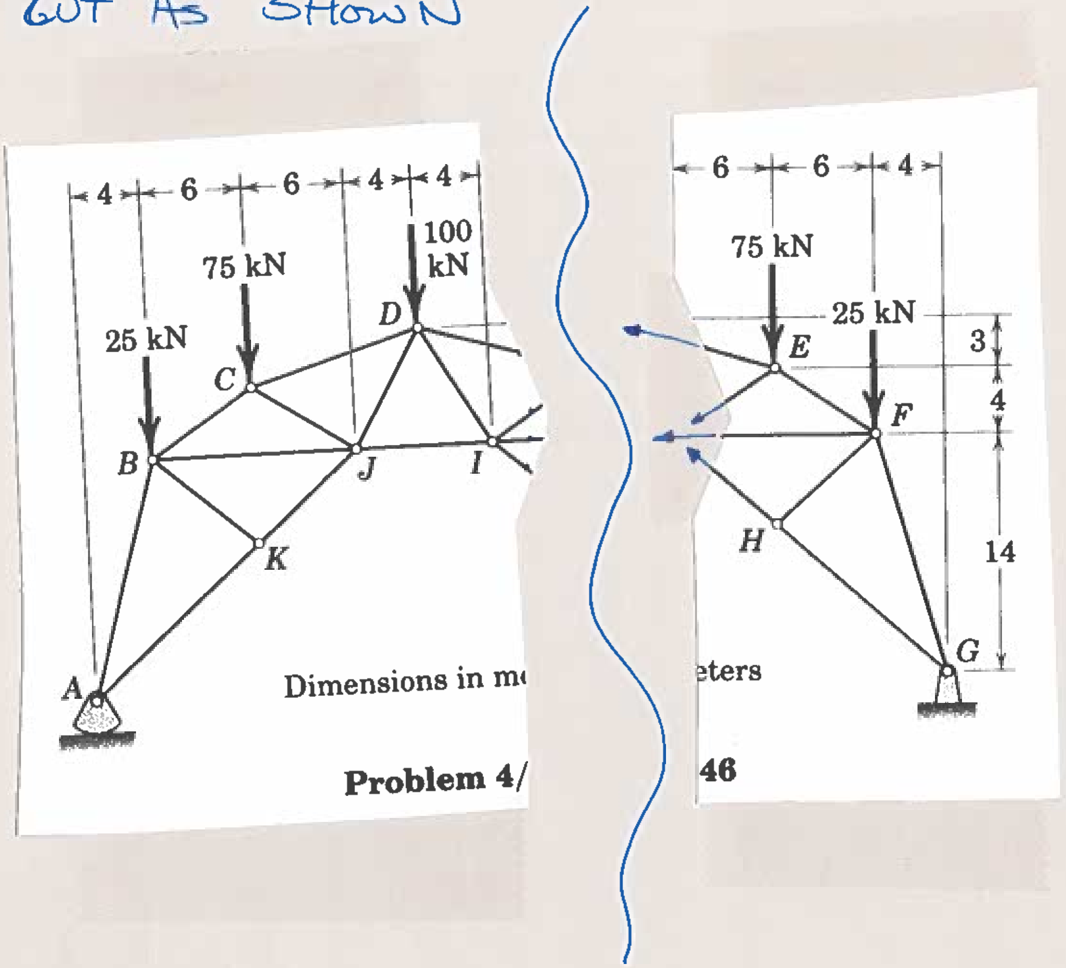
$\Sigma F_y = A_y + G_y - 300$

$\Sigma M_G = 0 = 20(300) - 40(A_y)$

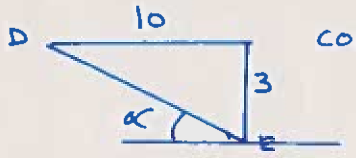
∴ $A_y = 150$ kN

$G_y = 150$ kN

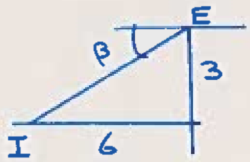
CUT AS SHOWN



TRIGONOMETRY TO FIND DIRECTION COSINES

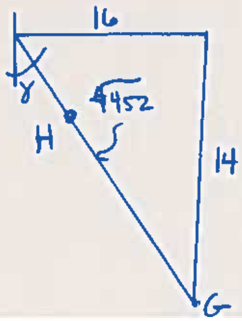


$\cos\alpha = \frac{10}{\sqrt{109}}$ $\sin\alpha = \frac{3}{\sqrt{109}}$

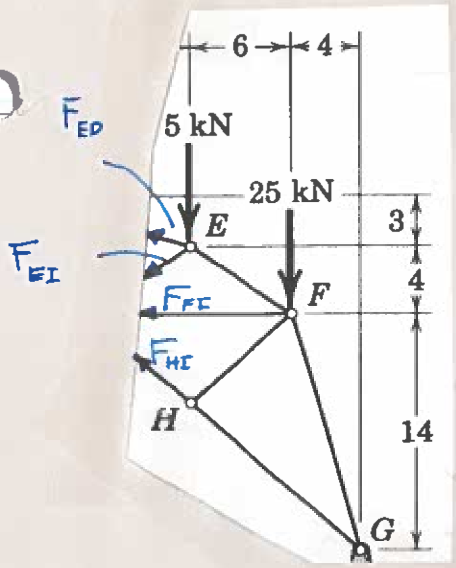


$\cos\beta = \frac{6}{\sqrt{45}}$ $\sin\beta = \frac{3}{\sqrt{45}}$

$6^2+3^2 = 36+9=45$



$\cos\gamma = \frac{14}{\sqrt{452}}$ $\sin\gamma = \frac{16}{\sqrt{452}}$



$$\Sigma M_I = 0 = 16\underline{i} \times 150\underline{j} \quad + (6\underline{i} + 4\underline{j}) \times (-ED\cos\alpha\,\underline{i} + ED\sin\alpha\,\underline{j})$$

$$+ (6\underline{i} + 4\underline{j}) \times -75\underline{j}$$

$$+ (12\underline{i}) \times -25\underline{j}$$

$$= 16(150) - 6(75) - 25(12)$$

$$+ 6ED\left(\frac{3}{\sqrt{109}}\right) + 4ED\left(\frac{10}{\sqrt{109}}\right) = 0$$

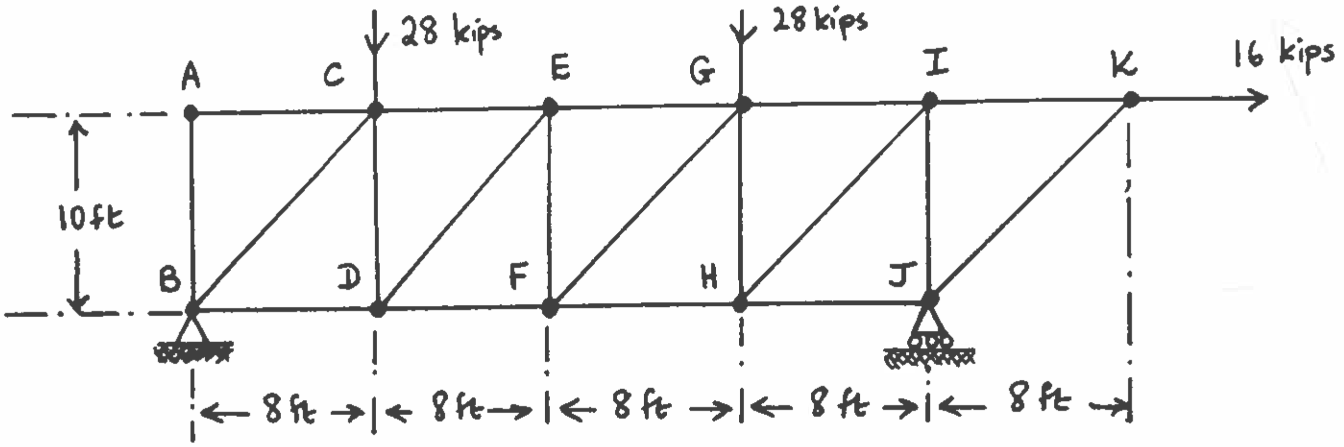
$$2400 - 450 - 300 + ED\left(\frac{58}{\sqrt{109}}\right) = 0$$

$$ED = \frac{2400 - 450 - 300}{5.55} = 297\text{ kN}$$

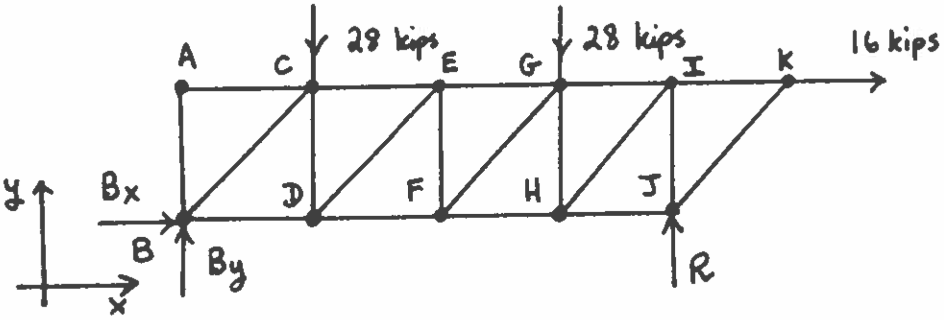
$$\Sigma M_E = 0 =$$

EXAMPLE 1 : Method of Sections

Find the forces in members EF and GI of the truss shown



FBD of Entire Truss:



Equilibrium of entire truss gives

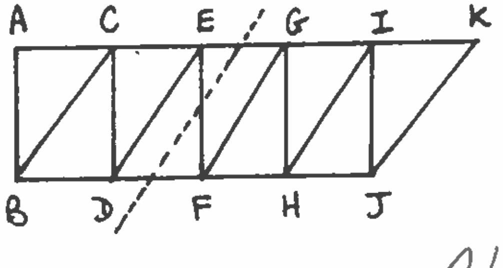
$$\circlearrowleft + \sum M_B = 0 \quad \Rightarrow \quad -(28)(8) - (28)(24) - (16)(10) + R(32) = 0$$

$$\Rightarrow \quad R = 33 \text{ kips}$$

$$\sum F_x = 0 \quad \Rightarrow \quad B_x + 16 = 0 \quad \Rightarrow \quad B_x = -16 \text{ kips}$$

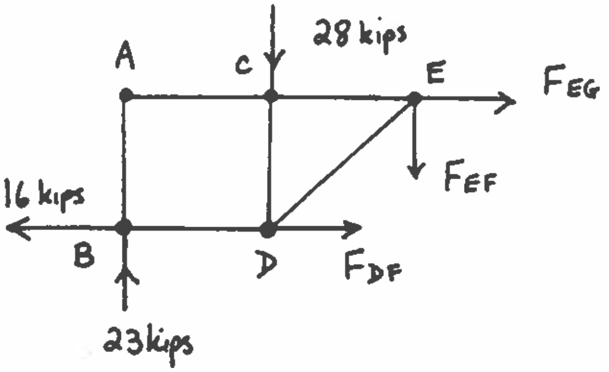
$$\sum F_y = 0 \quad \Rightarrow \quad B_y + R - 56 = 0 \quad \Rightarrow \quad B_y = 23 \text{ kips}$$

To calculate force in member EF, cut along section shown



Consider LH section.

FBD of LH section :



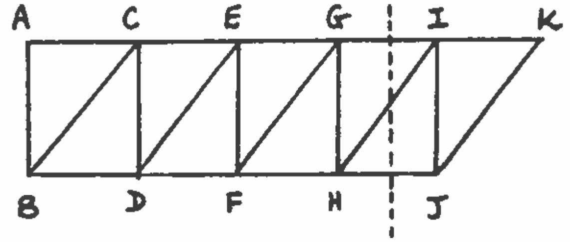
Equilibrium for section gives

$$\Sigma F_y = 0 \Rightarrow 23 - 28 - F_{EF} = 0$$

$$\Rightarrow \underline{\underline{F_{EF} = -5 \text{ kips}}}$$

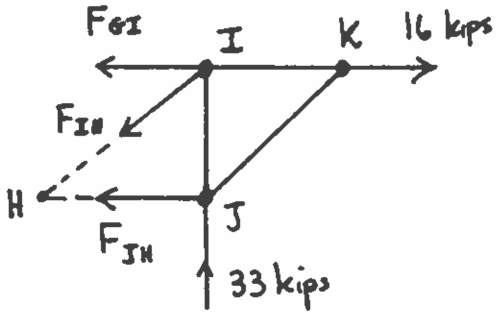
(ie. Member EF actually in compression)

To calculate force in member GI, cut along section shown



Consider RH section

FBD of RH section :



Equilibrium for section gives

$$\curvearrowleft+ \ \Sigma M_H = 0 \Rightarrow 33(8) - 16(10) + F_{GI}(10) = 0$$

$$\Rightarrow \underline{\underline{F_{GI} = -10.4 \text{ kips}}}$$

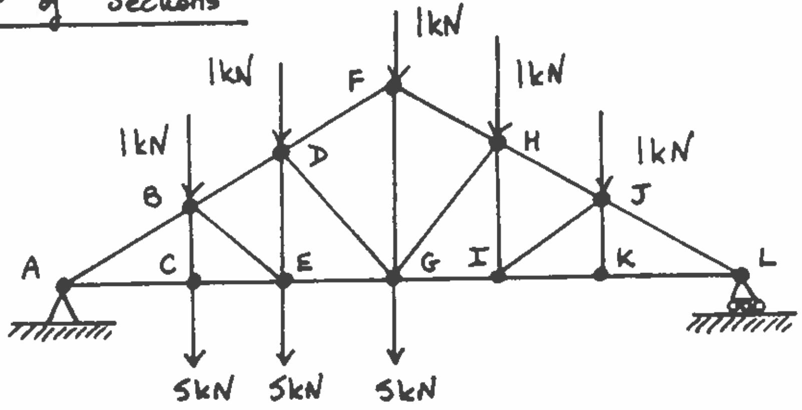
(ie. Member GI actually in compression)

<u>Solutions</u> :

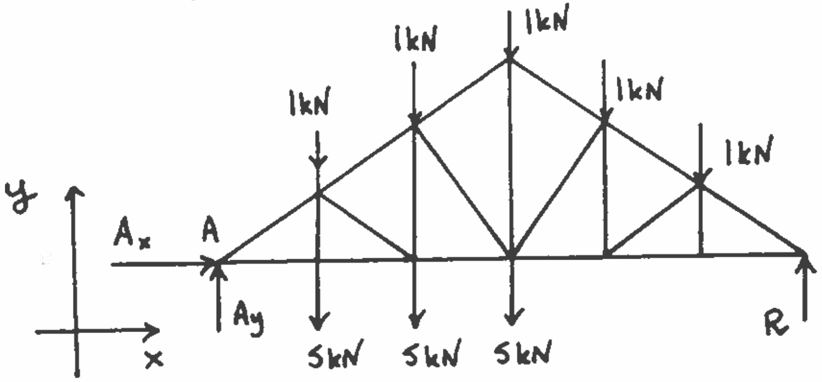
F_{EF} = 5 kips , compression .
F_{GI} = 10.4 kips , compression .

EXAMPLE 2 : Method of Sections

Find the forces in members FH, GH and GI of the roof truss shown. Use FG = 8m and AL = 6 panels @ 5m = 30m



FBD of Entire Truss :



$\Sigma F_x = 0 \Rightarrow \underline{A_x = 0}$

$\circlearrowleft^{+} \Sigma M_A = 0 \Rightarrow$

$-6(5) - 6(10) - 6(15)$

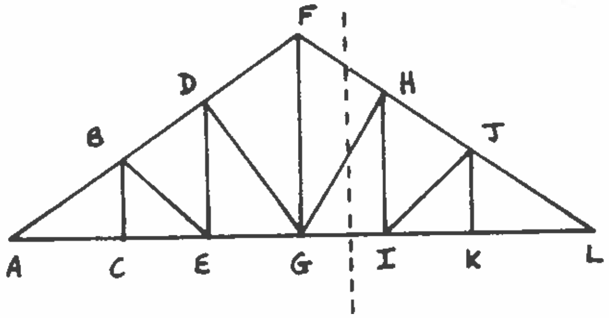
$-1(20) - 1(25) + R(30) = 0$

$\underline{R = 7.5 \text{ kN}}$

$\Sigma F_y = 0 \Rightarrow A_y + R - 20 = 0$

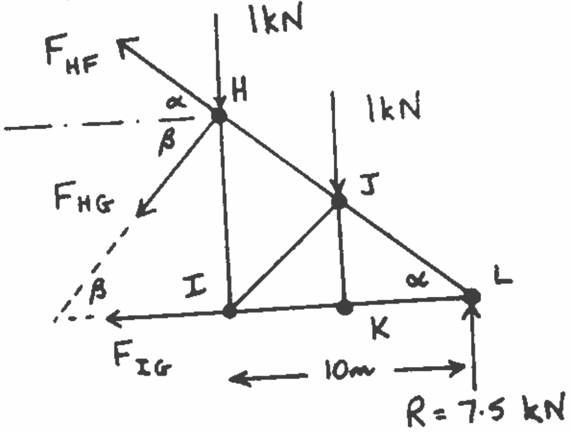
$\underline{A_y = 12.5 \text{ kN}}$

To calculate forces in members, cut along section shown



Consider RH section.

FBD of RH section :



Angle $\alpha = \tan^{-1}(8/15) = 28.07°$

Equilibrium for section gives

$$\circlearrowleft + \; \sum M_H = 0 \Rightarrow$$

$$7.5(10) - 1(5) - F_{IG}(10 \tan\alpha) = 0$$

$$\Rightarrow \underline{\underline{F_{IG} = 13.13 \text{ kN} \quad (T)}}$$

$$\sum F_x = 0 \quad \Rightarrow \quad -F_{HF}\cos\alpha - F_{HG}\cos\beta - F_{IG} = 0$$

$$\sum F_y = 0 \quad \Rightarrow \quad F_{HF}\sin\alpha + 7.5 - F_{HG}\sin\beta - 2 = 0$$

Angle β : $\beta = \tan^{-1}(HI/5) = \tan^{-1}(10\tan\alpha/5) = 46.84°$

Equilibrium equations become

$$-.882\, F_{HF} - .684\, F_{HG} = 13.13$$

$$.471\, F_{HF} - .729\, F_{HG} = -5.5$$

Solving gives

$$\underline{\underline{F_{HF} = -13.82 \text{ kN} \quad (C)}}$$

$$\underline{\underline{F_{HG} = -1.38 \text{ kN} \quad (C)}}$$

$$+\circlearrowleft \Sigma M_p = 0 = -4.8(6) - 9.6(F_{BG}) = 0$$

$$9.6 F_{BG} = -28.8$$

$$F_{BG} = \frac{-28.8}{9.6} = -3 \text{ kN}$$

$$+\circlearrowleft \Sigma M_G = 0 = 4(F_{AB}) - 4.8(6)$$

$$F_{AB} = \frac{28.8}{4} = 7.2 \text{ kN}$$

$$+\circlearrowleft \Sigma M_B = 0 = -4(F_{GF} \cos 22.619°) - 4.8(6)$$

$$F_{GF} \cos 22.619 = -\frac{28.8}{4}$$

$$F_{GF} = \frac{-28.8}{4 \cos 22.619} = -7.79 \text{ kN}$$

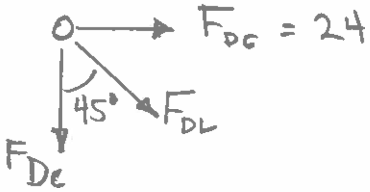
$\therefore$

$F_{AB} = 7.2$ kN T

$F_{BG} = 3.0$ kN C

$F_{GF} = 7.8$ kN C

JOINT D



$$\Sigma F_x = 0 = 24 + F_{DL} \sin 45$$

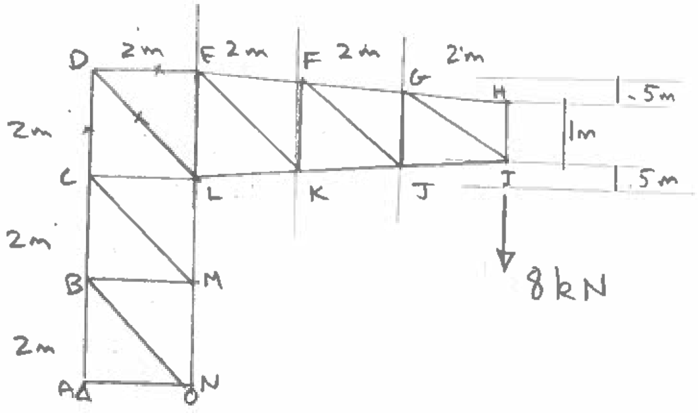
$$F_{DL} = \frac{-24}{\sin 45} = -33.9\ kN$$

$$\therefore \quad F_{DE} = 24\ kN\ T$$

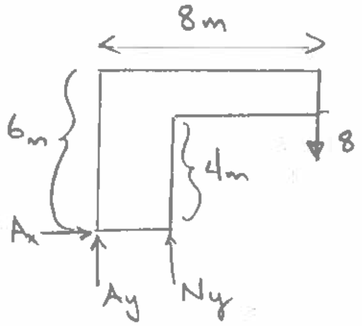
$$F_{DL} = 33.9\ kN\ C$$

NOTE: COMBINE SECTIONS & JOINTS

4-35 FIND F_{DE}, F_{DL}



FBD ENTIRE TRUSS



$$\Sigma F_x = 0 = A_x$$

$$+\circlearrowleft \Sigma M_A = 0 = -8(8) + 2(N_y)$$

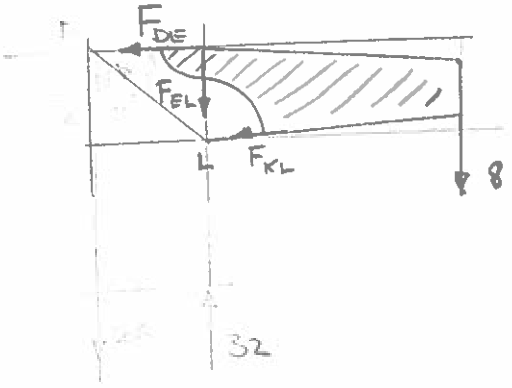
$$N_y = \frac{64}{2} = 32$$

$$\Sigma F_y = 0 = A_y + N_y - 8$$

$$A_y = 8 - N_y$$

$$A_y = -26$$

SECTION

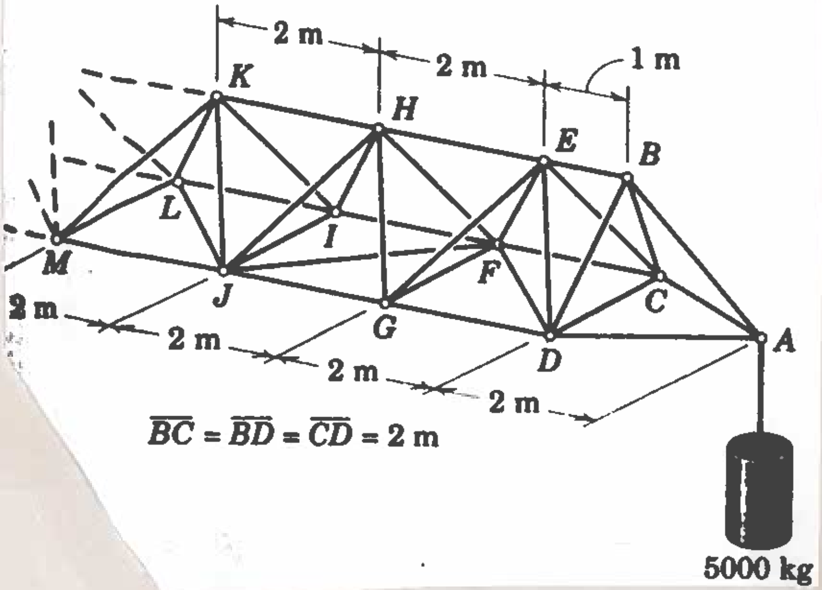


$$\circlearrowleft \Sigma M_L = 0 = 2(F_{DE}) - 6(8)$$

$$F_{DE} = \frac{48}{2} = 24 \text{ kN}$$

4/63 The lengthy boom of an overhead construction crane, a portion of which is shown, is an example of a periodic structure—one which is composed of repeated and identical structural units. Use the method of sections to find the forces in members *FJ* and *GJ*.

Ans. *FJ* = 0, *GJ* = −70.8 kN



$\overline{BC} = \overline{BD} = \overline{CD} = 2$ m

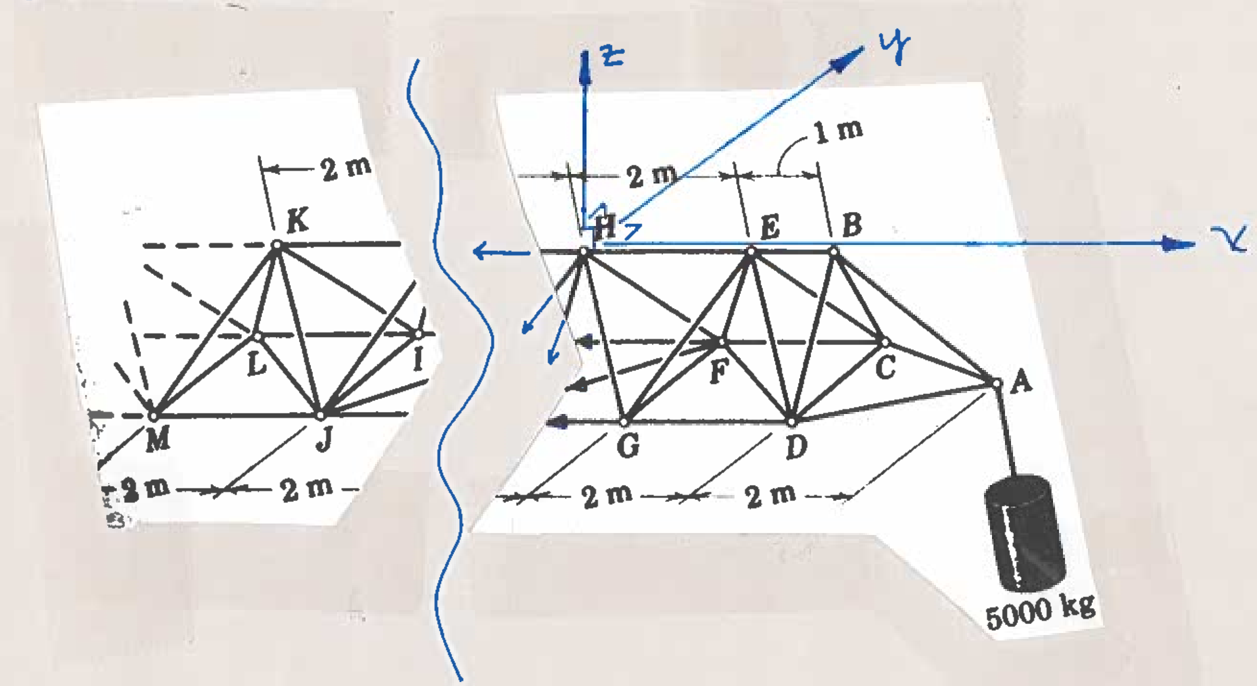
1) EXPRESS 5000kg LOAD AS A FORCE

$W = 5000\ (9.8) = 49000\text{N}$

$= 49\text{kN}$

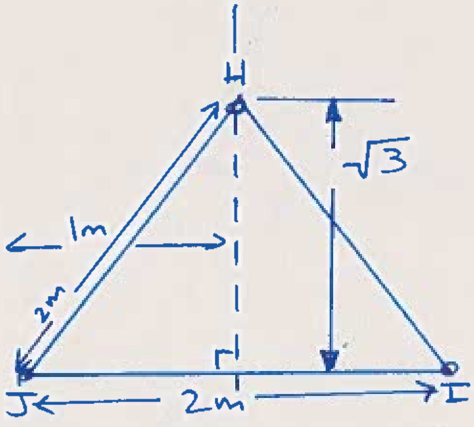
ALL FORCES WILL BE EXPRESSED IN kN

2) MAKE CUT BETWEEN H & JI AXIS

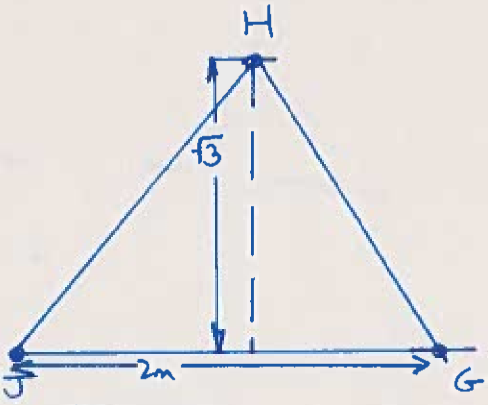


3) ESTABLISH X-Y-Z COORDINATE SYSTEM AT H.

4) TRIGONOMETRY TO HELP IDENTIFY DIRECTION COSINES



PROJECTION ONTO Y-Z PLANE



PROJECTION ONTO X-Z PLANE

LIST UNKNOWN FORCES

F_{HK}, F_{FI}, F_{GJ}, F_{HJ}, F_{HI}, F_{FJ}

6 UNKNOWNS, NEED 6 EQUATIONS

$\Sigma \underline{F} = 0$ WILL PROVIDE 3 EQN.

$\Sigma \underline{M} = 0$ WILL PROVIDE 3 EQN.

$\therefore$ SOLVABLE.

UNIT VECTORS ALONG LINES OF ACTION FOR UNKNOWN FORCES

$$e_{HJ} = \frac{-\underline{i} - \underline{j} - \sqrt{3}\,\underline{k}}{\sqrt{5}}$$

$$e_{HI} = \frac{-\underline{i} + \underline{j} - \sqrt{3}\,\underline{k}}{\sqrt{5}}$$

$$e_{HK} = -\underline{i}$$

$$e_{FI} = -\underline{i}$$

$$e_{GJ} = -i$$

$$e_{FJ} = \frac{-2\underline{i} - 2\underline{j} + 0\,\underline{k}}{\sqrt{8}}$$

$\Sigma F_x = 0$

$$-F_{HK} - F_{FI} - F_{GJ} - \frac{1}{\sqrt{5}}F_{HJ} - \frac{1}{\sqrt{5}}F_{HI} - \frac{2}{\sqrt{8}}F_{FJ} = 0$$

$\Sigma F_y = 0$

$$-\frac{1}{\sqrt{5}}F_{HJ} + \frac{1}{\sqrt{5}}F_{HI} - \frac{2}{\sqrt{8}}F_{FJ} = 0$$

$\Sigma F_z = 0$

$$-\frac{\sqrt{3}}{\sqrt{5}}F_{HJ} - \frac{\sqrt{3}}{\sqrt{5}}F_{HI} = W$$

$\Sigma M_H = 0$

$$(-1\underline{i} - 1\underline{j} - \sqrt{3}\underline{k}) \times F_{GJ}(-\underline{i}) + (1\underline{i} + 1\underline{j} - \sqrt{3}\underline{k}) \times F_{FI}(-\underline{i})$$

$$+ (1\underline{i} + 1\underline{j} - \sqrt{3}\underline{k}) \times F_{FJ}\left(\frac{-2}{\sqrt{8}}\underline{i} - \frac{2}{\sqrt{8}}\underline{j}\right) + 5\underline{i} \times -W\underline{k} = 0$$

EVALUATE CROSS PRODUCTS, EXPRESS RESULT AS 3 SCALAR (x,y,z) MOMENTS

$$-\frac{2\sqrt{3}}{\sqrt{8}}F_{FJ} = 0$$

$$\sqrt{3}F_{FI} \quad \sqrt{3}F_{GJ} \quad \frac{2\sqrt{3}}{\sqrt{8}}F_{FJ} = -5W$$

$$F_{FI} - F_{GJ} = 0$$

EXPRESS AS SYSTEM OF LINEAR EQUATIONS

$$\begin{array}{lllllll}
-F_{HK} & -F_{FI} & -F_{GJ} & -\frac{1}{\sqrt{5}}F_{HJ} & -\frac{1}{\sqrt{5}}F_{HI} & -\frac{2}{\sqrt{8}}F_{FJ} & =0 \\
 & & & -\frac{1}{\sqrt{5}}F_{HJ} & +\frac{1}{\sqrt{5}}F_{HI} & -\frac{2}{\sqrt{8}}F_{FJ} & =0 \\
 & & & -\frac{\sqrt{3}}{\sqrt{5}}F_{HJ} & -\frac{\sqrt{3}}{\sqrt{5}}F_{HI} & & =W \\
 & & & & & -\frac{2\sqrt{3}}{\sqrt{8}}F_{FJ} & =0 \\
 & \sqrt{3}\,F_{FI} & \sqrt{3}\,F_{GJ} & & & \frac{2\sqrt{3}}{\sqrt{8}}F_{FJ} & =-5W \\
 & F_{FI} & -F_{GJ} & & & & =0
\end{array}$$

SOLVE BY INSPECTION OR COMPUTER

FROM ROW 4

$$F_{FJ} = 0$$

FROM ROW 6

$$F_{FI} = F_{GJ}$$

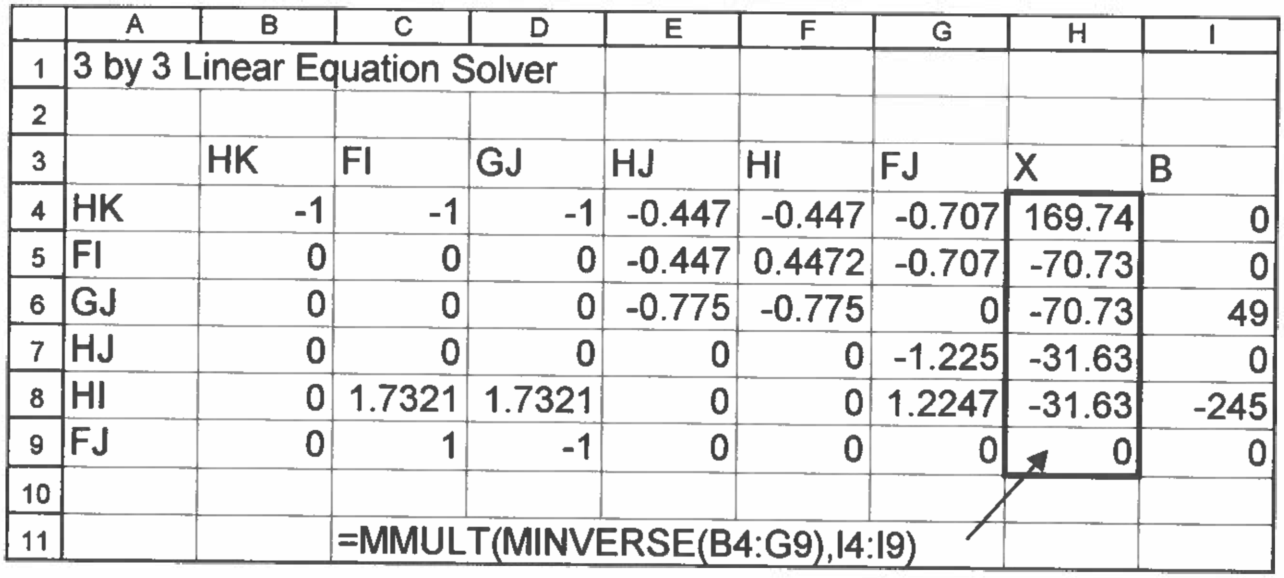
FROM ROW 5

$$2\sqrt{3}\,F_{GJ} = -5W$$

$$\therefore \quad F_{GJ} = \frac{-5W}{2\sqrt{3}} = \frac{-5\,(49\,kN)}{2\sqrt{3}} = -70.725\ kN$$

$$\therefore \quad \underline{\underline{F_{FJ} = 0}} \longleftarrow$$

$$\underline{\underline{F_{GJ} = -70.73\,kN}} \longleftarrow$$



	A	B	C	D	E	F	G	H	I
1	3 by 3 Linear Equation Solver								
2									
3		HK	FI	GJ	HJ	HI	FJ	X	B
4	HK	-1	-1	-1	-0.447	-0.447	-0.707	169.74	0
5	FI	0	0	0	-0.447	0.4472	-0.707	-70.73	0
6	GJ	0	0	0	-0.775	-0.775	0	-70.73	49
7	HJ	0	0	0	0	0	-1.225	-31.63	0
8	HI	0	1.7321	1.7321	0	0	1.2247	-31.63	-245
9	FJ	0	1	-1	0	0	0	0	0
10									
11			=MMULT(MINVERSE(B4:G9),I4:I9)						

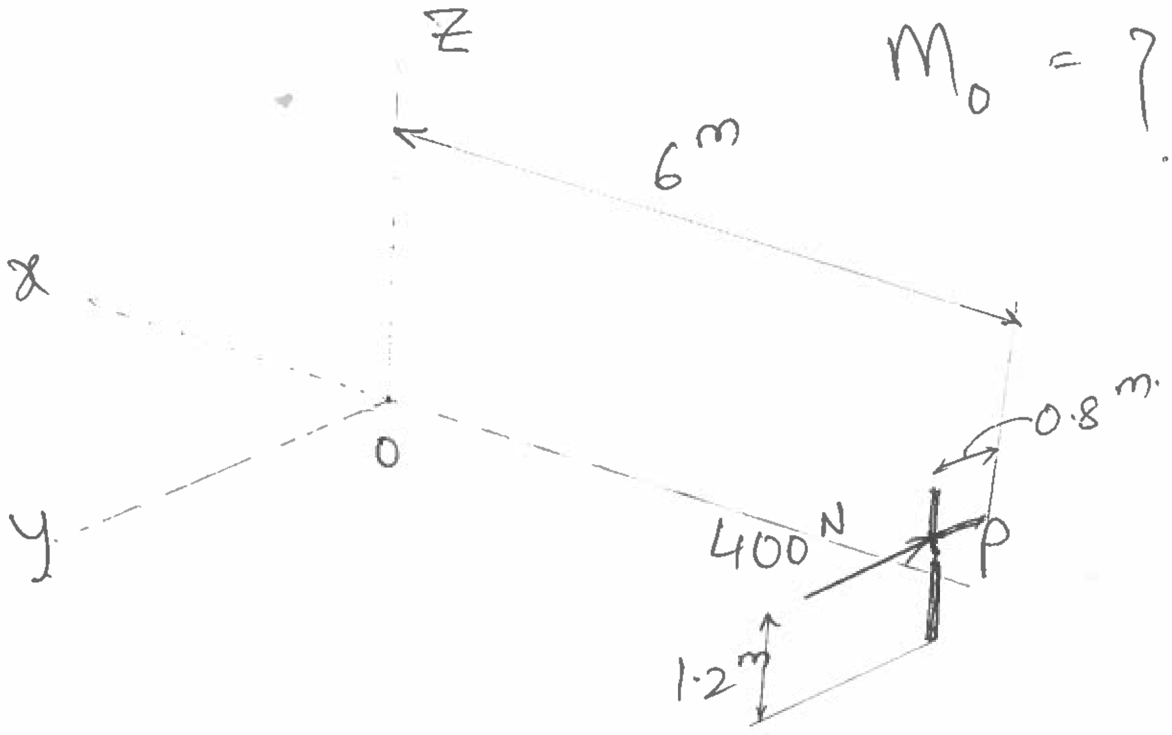
CIVE 2330

HW #8

Prob # 2.113

$P = 400\,N$

$M_o = ?$



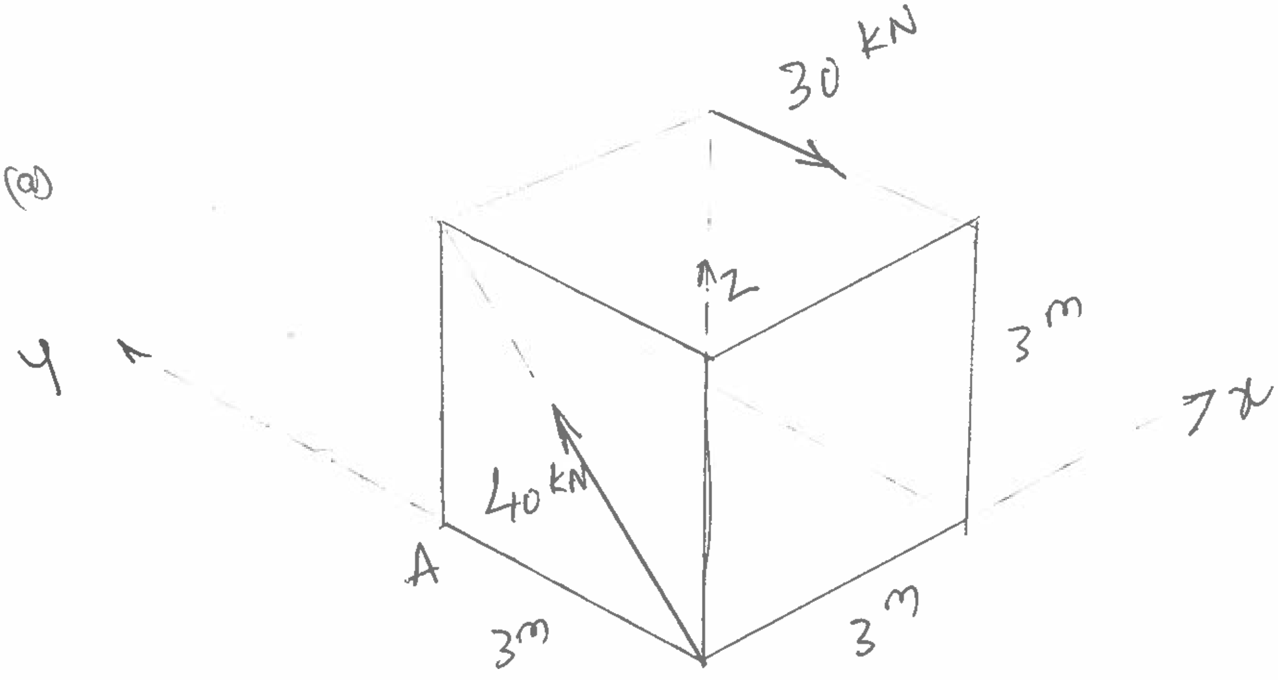
$$M_o = \underline{r} \times \underline{F}$$

$$= (-6\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \times (-400\underline{j})$$

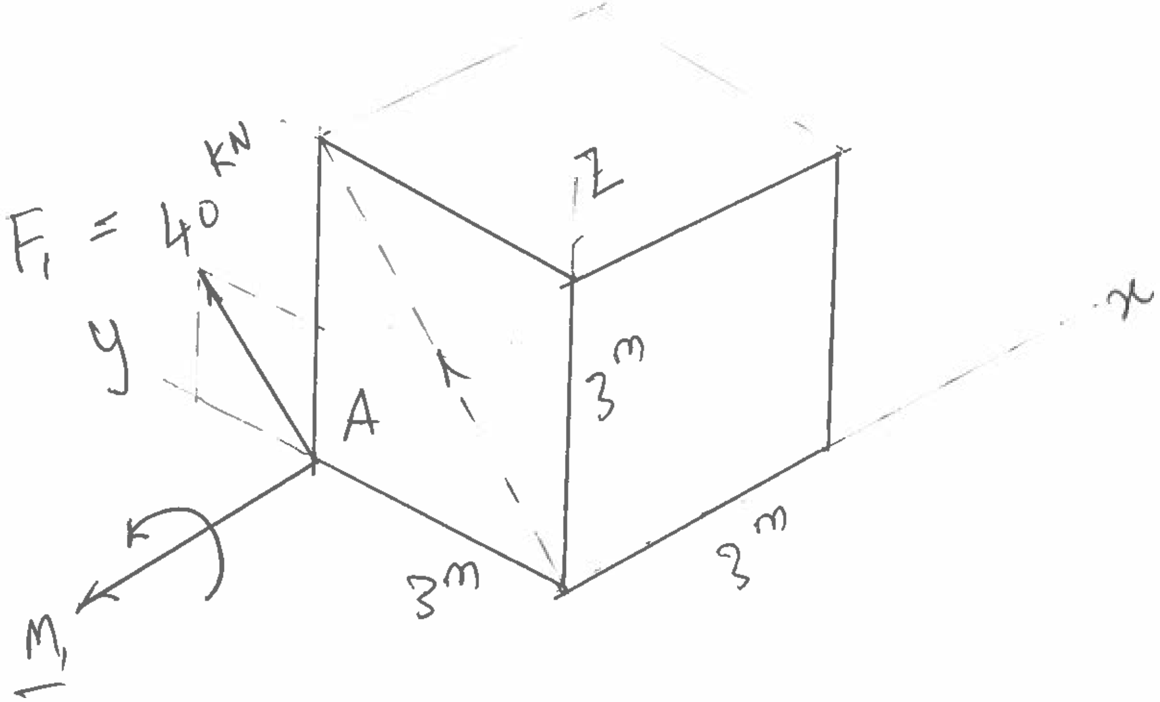
$$= 480\,\underline{i} + 2400\,\underline{k} \quad N.m$$

2.125

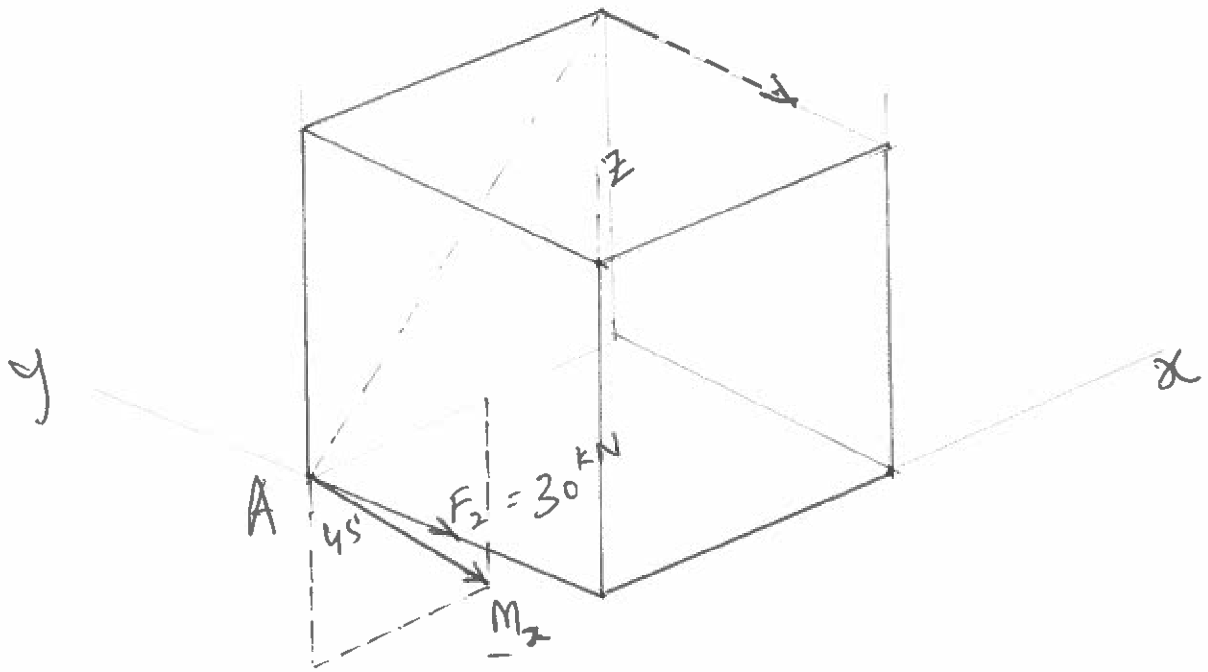
(a)



(b)



2(c)



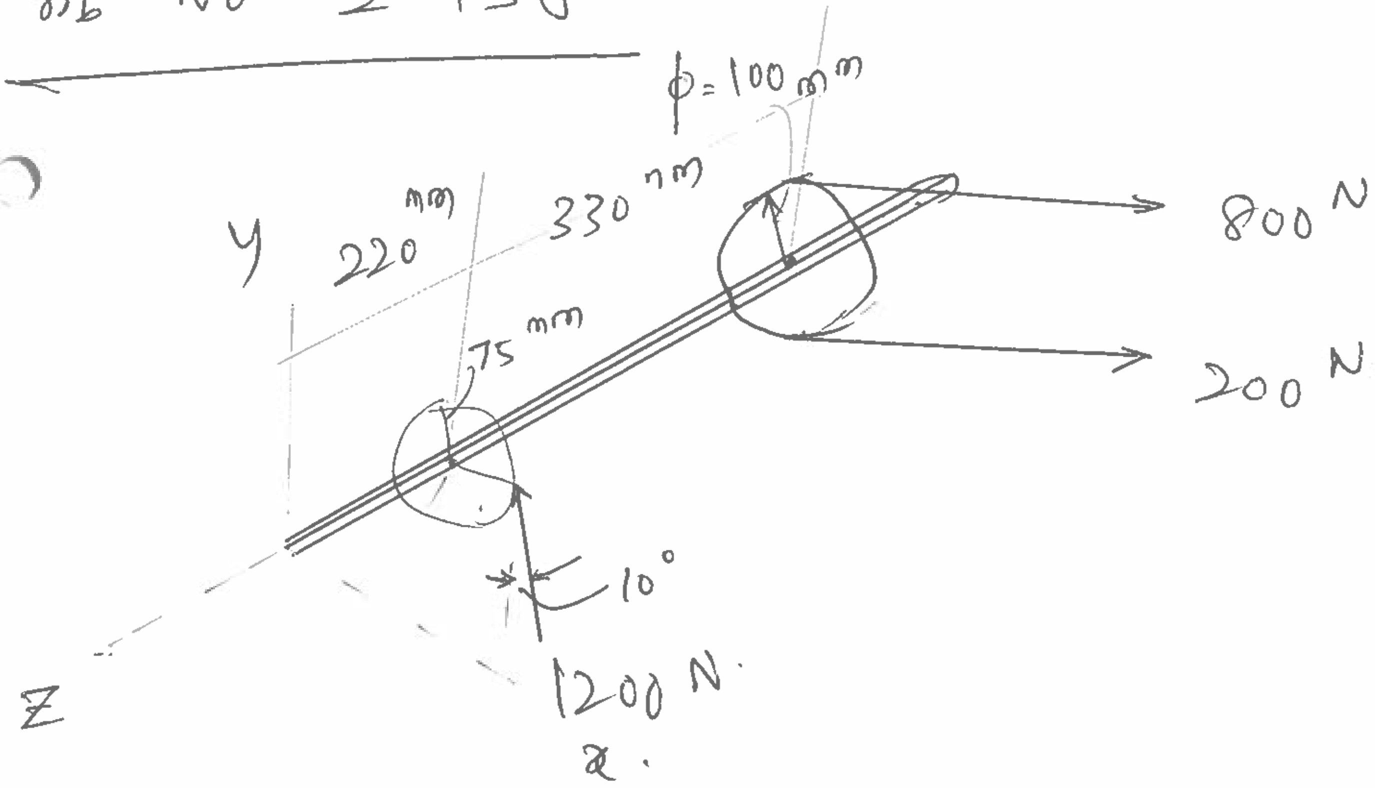
$$\underline{F} = \underline{F}_1 + \underline{F}_2 = \left(\frac{40}{\sqrt{2}}\underline{i} + \frac{40}{\sqrt{2}}\underline{k}\right) - 30\underline{j}$$

$$= -1.716\underline{j} + 28.3\underline{k} \quad \text{kN.}$$

$$\underline{M} = M_1 + M_2 = \left(\frac{-40}{\sqrt{2}}\right)(3)\underline{i} + \left[30(3)\underline{i} - 30(3)\underline{k}\right]$$

$$= 5.15\underline{i} - 90\underline{k} \quad \text{kN-m}$$

Prob NO 2.138



$$\underline{R} = (200+800)\,\underline{i} + 1200\,(\cos 10^\circ\,\underline{j} - \sin 10^\circ\,\underline{i})$$

$$= 792\,\underline{i} + 1182\,\underline{j}\ \text{N}$$

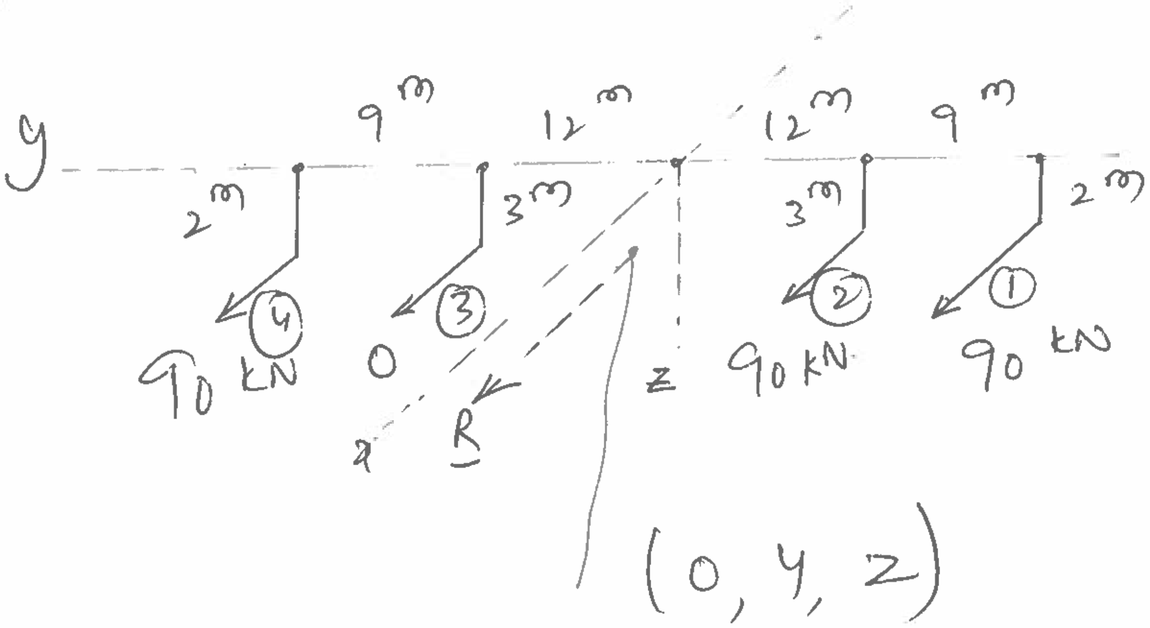
$$M_o = [(200-800)(0.1) + (1200\cos 10^\circ)(0.075)]\,\underline{k}$$

$$+ [-(200+800)(0.220+0.330) + 1200\sin 10^\circ(0.220$$

$$+ [1200\cos 10^\circ\,(0.220)]\,\underline{i}$$

$$= 260\,\underline{i} - 504\,\underline{j} + 286\,\underline{k}\ \text{N.m}$$

Prob # 2.142



$$R = \Sigma F = 3(90) = 270 \text{ kN}$$

$$\Sigma M_z = -Ry = 90(21) + 90(12) - \cancel{+}\, 90(21),$$

$$y = -4.00 \text{ m.}$$

$$\Sigma M_y = Rz = 2(90)(2) + 1(90)(3),$$

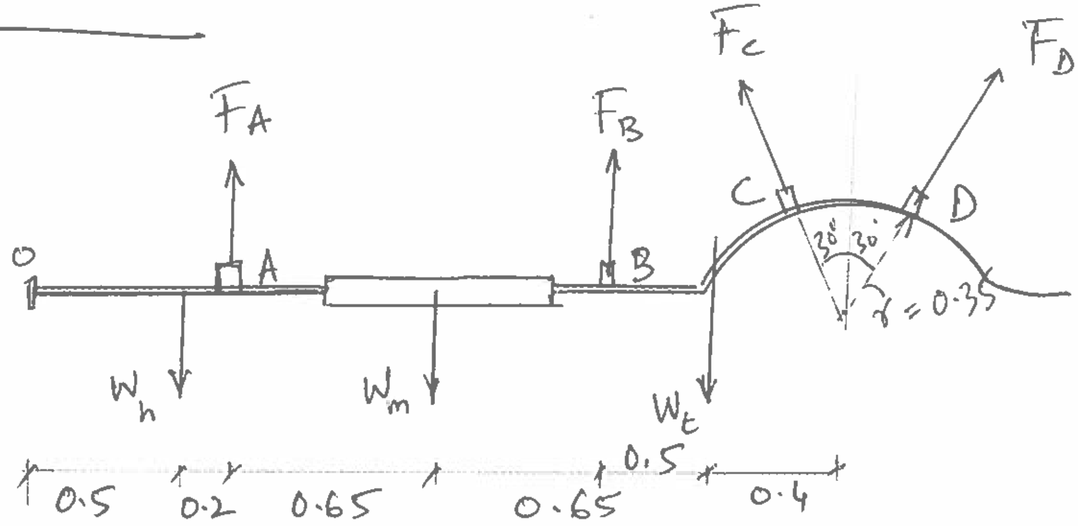
$$z = 2.33 \text{ m}$$

CIVE 2330

HW # 7.

3D Moments & Couples

prob # 2.92



$W_h = 10$ N

$W_m = 100$ N

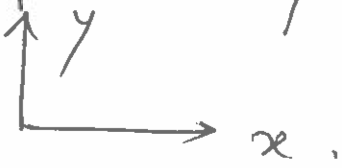
$W_t = 50$ N

$F_A = 50$ N

$M_0 = 0$

$F_B = F_C = F_D = ?$

For a zero force-couple system at point O :



$R_O = \sum \underline{F} = (-F_C \sin 30° + F_D \sin 30°)\ \underline{i}$

$+ (50 - 10 - 100 - 50 + F_B$

$$+ F_C \cos 30^\circ + F_D \cos 30^\circ) \underline{j} = 0$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft + M_O = -10(0.5) + 50(0.7)$$

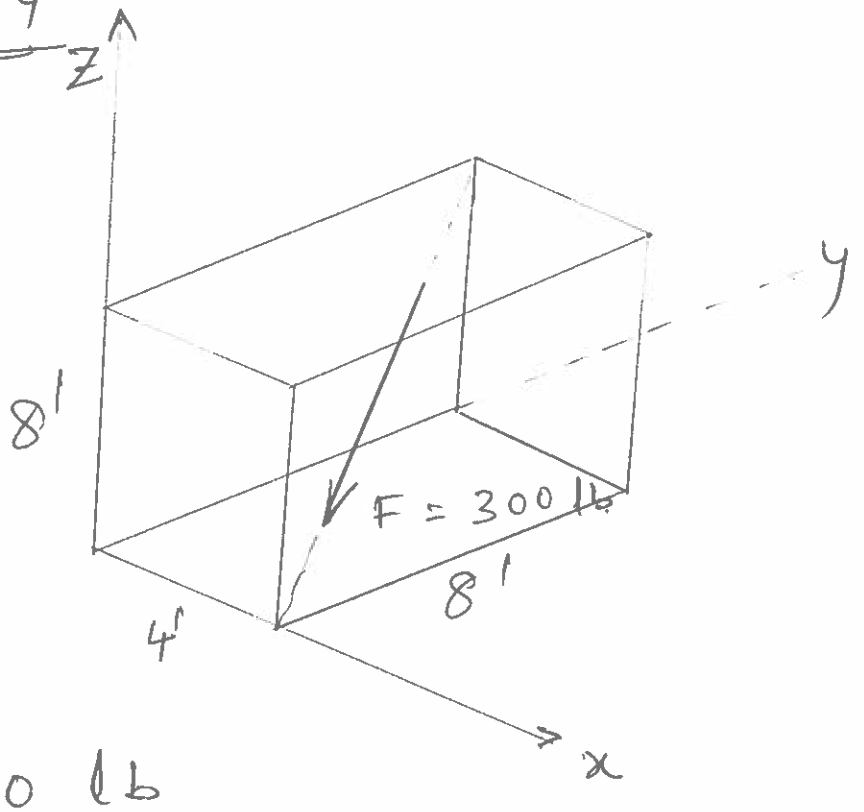
$$- 100(1.35) + F_B(2) - 50(2.5$$

$$+ 2F\cos 30^\circ (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42\,N}$$

$$\underline{F_B = 98.9\,N.}$$

Prob 2.94



F = 300 lb

$$F = F_n = 300\left[\frac{4\underline{i} - 8\underline{j} - 8\underline{k}}{\sqrt{4^2+8^2+8^2}}\right]$$

$$= 300\left[\frac{1}{3}\underline{i} - \frac{2}{3}\underline{j} - \frac{2}{3}\underline{k}\right] \text{ lb}$$

$\underline{F_x = 100 \text{ lb}}$ $\underline{F_z = -200 \text{ lb}}$

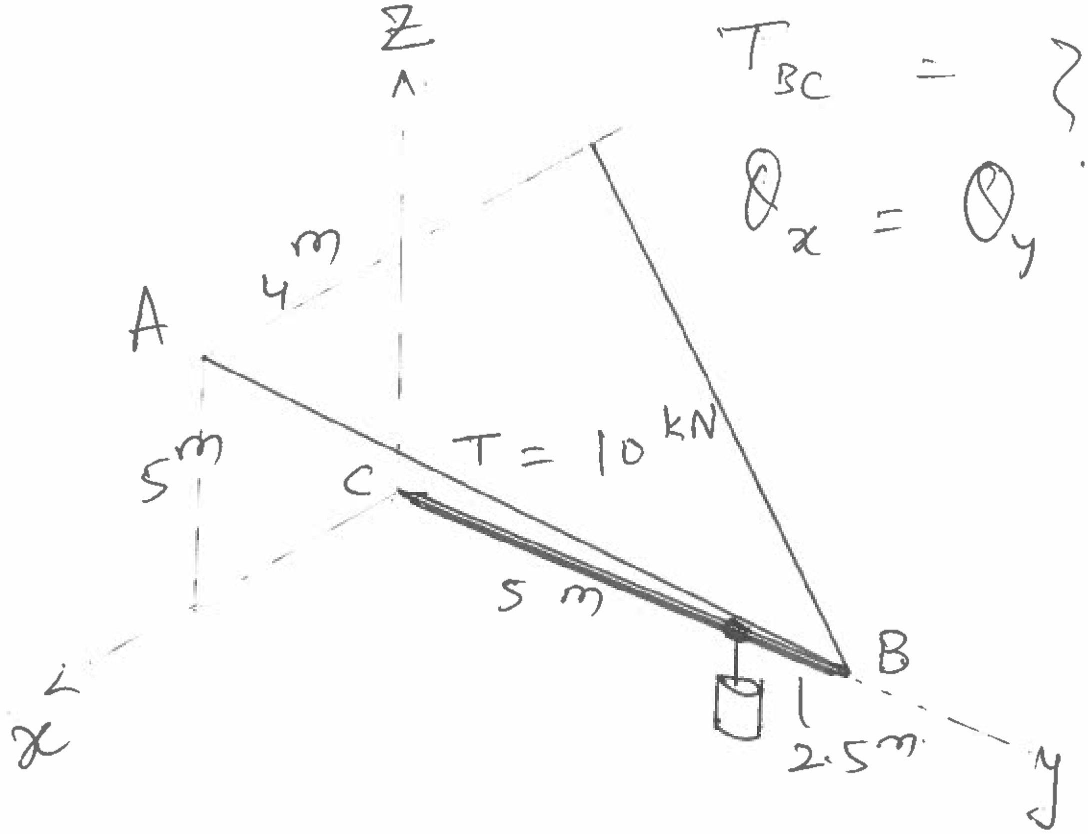
$\underline{F_y = -200 \text{ lb}}$

2.98

$T_{AB} = 10 \text{ kN}$

$T_{BC} = ?$

$\theta_x = \theta_y = \theta_z = ?$



$$T = T\underline{n}_{AB} = 10\left[\frac{4\underline{i} - 7.5\underline{j} + 5\underline{k}}{\sqrt{4^2 + (-7.5)^2 + 5^2}}\right]$$

$$= 10\left(0.406\,\underline{i} - 0.761\,\underline{j} + 0.507\,\underline{k}\right) \text{kN}$$

$\cos\theta_x = 0.406$; $\theta_x = 66.1^\circ$

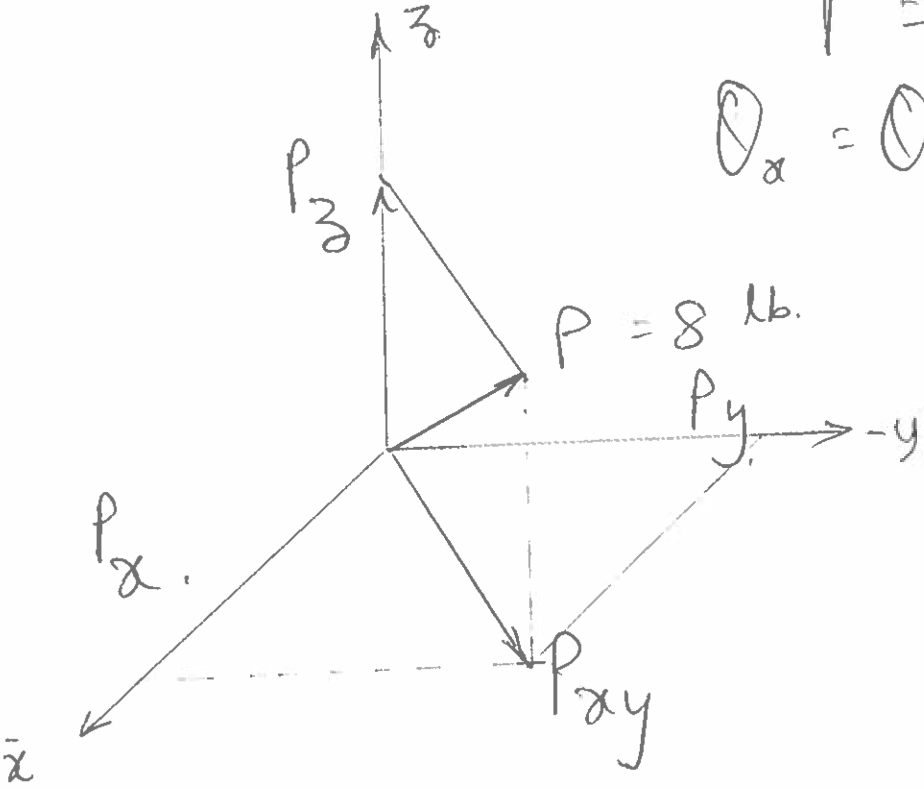
$\cos\theta_y = -0.761$; $\theta_y = 139.5^\circ$

$\cos\theta_z = 0.507$; $\theta_z = 59.5^\circ$

Prob 2.103

$P = 8\ lb$

$\theta_x = \theta_y = \theta_z = ?$



$P_z = P \sin 30° = 8 \sin 30°$

$= \underline{4\ lb.}$

$P_{xy} = P \cos 30°$

$= \underline{6.93\ lb}$

$P_y = P_{xy} \sin 20°$

$= \underline{2.37\ lb}$

$$P_x = P_{xy} \cos 20^\circ = 6.51 \text{ lb}$$

$$\underline{P} = 6.51\,\underline{i} + 2.37\,\underline{j} + 4.00\,\underline{k} \text{ lb}$$

$$\cos \theta_x = P_x / P = 6.51/8 \; ; \; \theta_x = 35.5^\circ$$

$$\cos \theta_y = P_y / P = 2.37/8 \; ; \; \theta_y = 72.8^\circ$$

$$\cos \theta_z = P_z / P = 4/8 \; ; \; \theta_z = 60.0^\circ$$

Equilibrium

Statics is concerned with the force conditions required to maintain the equilibrium of engineering structures.

A body is at equilibrium when the resultant of all forces acting on it is zero and the couple acting on it is zero.

The equilibrium equations are

$$\underline{R} = \Sigma \underline{F} = \underline{0}$$

$$\underline{M} = \Sigma \underline{M} = 0$$

These are the fundamental equations of statics.

Mechanical System Analysis

Isolate each component of the system and show all forces acting on the component

A component may be a particle, single rigid body, single deformable body, or a combination of connected bodies

Forces acting on each system component are shown using a free-body diagram.

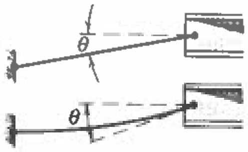
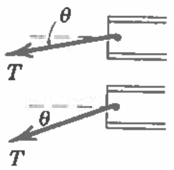
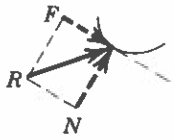
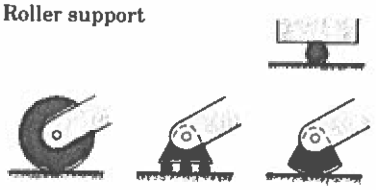
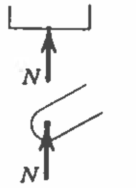
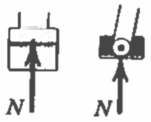
All forces are shown: both contact & body forces.

The FBD is the most important step in mechanics.

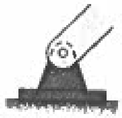
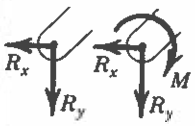
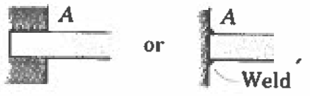
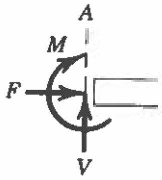
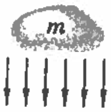
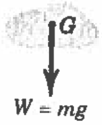
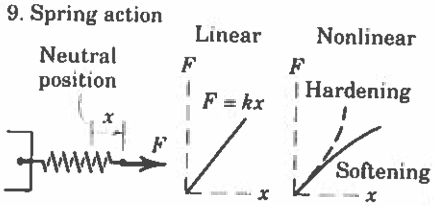
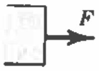
Free-Body Diagrams

1) Determine which component(s) to isolate
2) Draw a diagram that represents the complete external boundary
3) All forces acting <u>on</u> the body are represented as vectors
4) Coordinate axes are indicated directly on the diagram

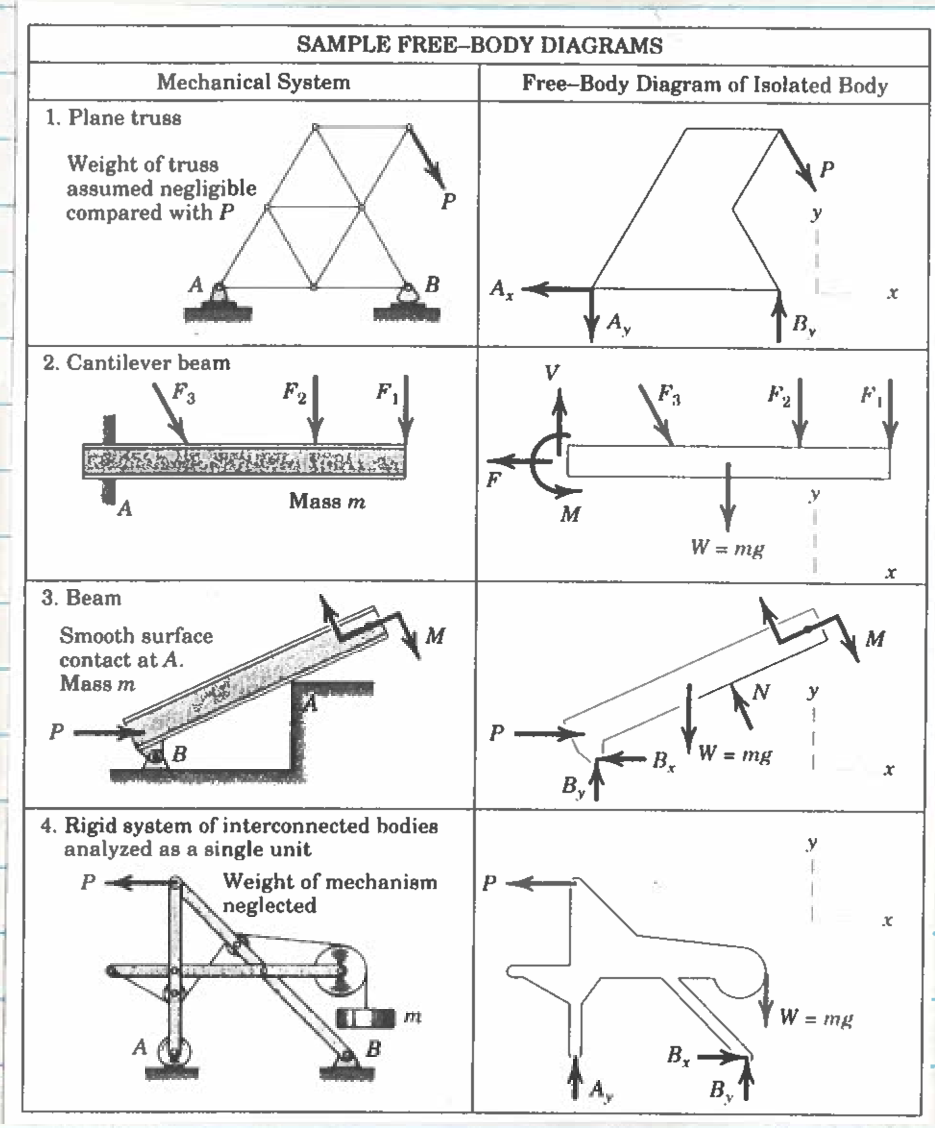
Support reactions are the most difficult part of a FBD.

<table>
<tr><th colspan="2">MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS</th></tr>
<tr><th>Type of Contact and Force Origin</th><th>Action on Body to be Isolated</th></tr>
<tr><td>1. Flexible cable, belt, chain, or rope
Weight of cable negligible
Weight of cable not negligible
 </td><td>
Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.</td></tr>
<tr><td>2. Smooth surfaces
</td><td>
Contact force is compressive and is normal to the surface.</td></tr>
<tr><td>3. Rough surfaces
</td><td>
Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.</td></tr>
<tr><td>4. Roller support
</td><td>
Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.</td></tr>
<tr><td>5. Freely sliding guide
</td><td>
Collar or slider free to move along smooth guides; can support force normal to guide only.</td></tr>
</table>

All forces on the body act in a direction opposite to that which the body would move if the support were removed

Type of Contact and Force Origin	Action on Body to be Isolated
6. Pin connection 	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

All forces on the body act in a direction opposite to that which the body would move if the support were removed.



SAMPLE FREE–BODY DIAGRAMS

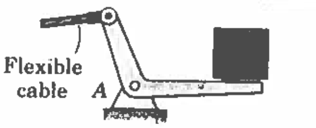
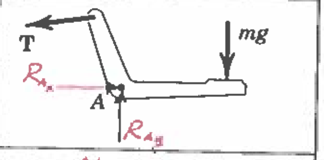
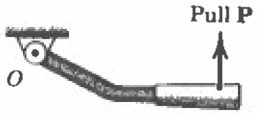
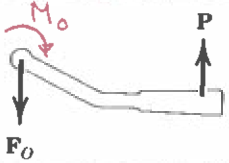
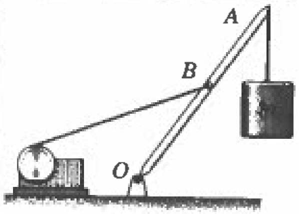
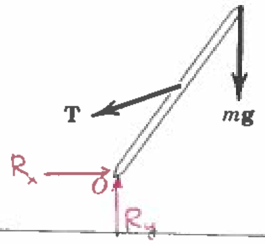
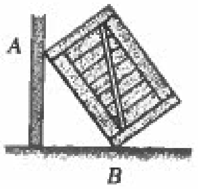
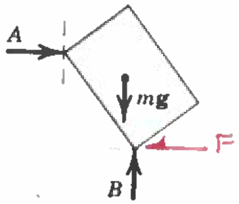
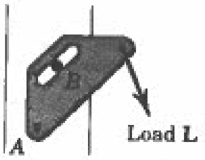
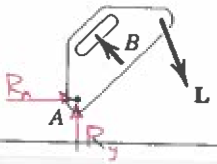
Mechanical System	Free–Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P	
2. Cantilever beam Mass m	
3. Beam Smooth surface contact at A. Mass m	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected	

When an entire system is analyzed, components can be treated as a single unit.

Force locations are extremely important.

FREE-BODY DIAGRAM EXERCISES

3/A In each of the five following examples, the body to be isolated is shown in the left-hand diagram, and an *incomplete* free-body diagram (FBD) of the isolated body is shown on the right. Add whatever forces are necessary in each case to form a complete free-body diagram. The weights of the bodies are negligible unless otherwise indicated. Dimensions and numerical values are omitted for simplicity.

	Body	Incomplete *FBD*
1. Bell crank supporting mass *m* with pin support at *A*.		
2. Control lever applying torque to shaft at *O*.		
3. Boom *OA*, of negligible mass compared with mass *m*. Boom hinged at *O* and supported by hoisting cable at *B*.		
4. Uniform crate of mass *m* leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at *A* and fixed pin in smooth slot at *B*.		

reaction at A

Moment at O

reactions at O

friction force at B

reactions at A

Complete the FBD

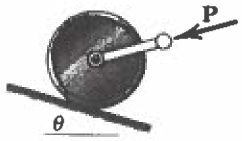
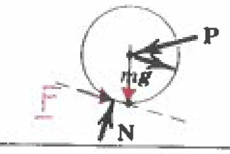
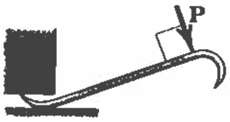
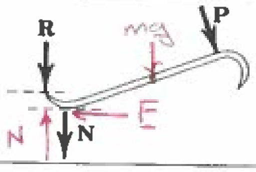
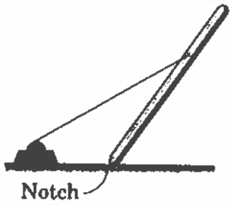
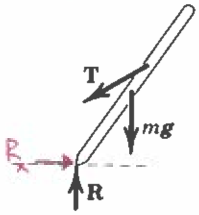
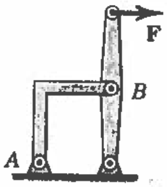
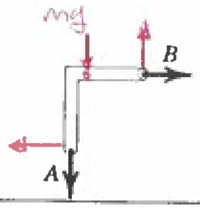
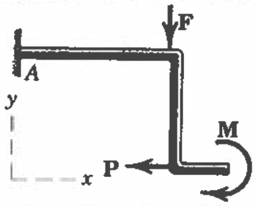
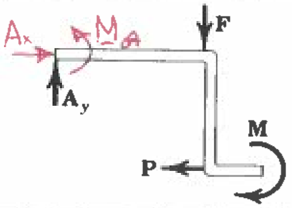
	Body	Wrong or Incomplete *FBD*
1. Lawn roller of mass *m* being pushed up incline θ.		
2. Pry bar lifting body *A* having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass *m* being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame. Pin joints		
5. Bent rod welded to support at *A* and subjected to two forces and couple.		

Figure 3/B

g wrong direction
Friction on incline

N wrong direction
Friction at surface
body force

Horizontal notch force

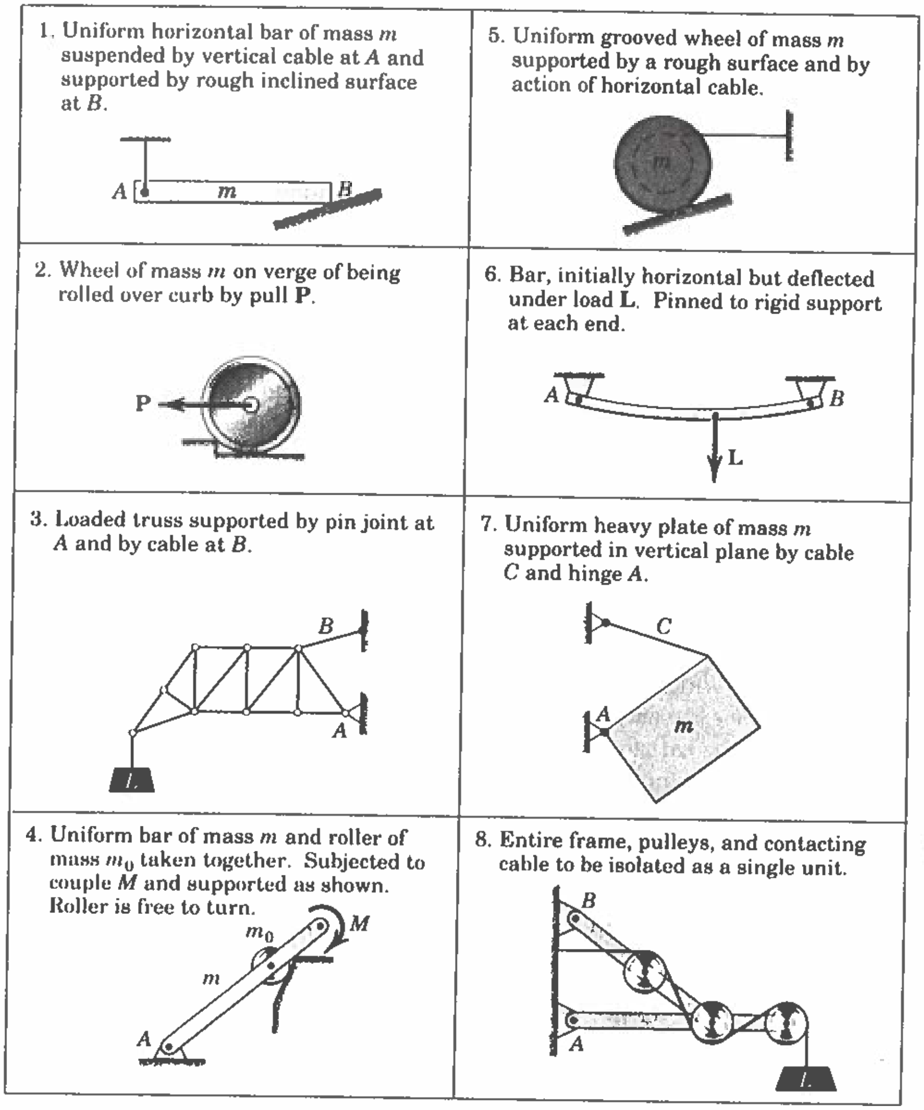
Horizontal force at A
Body force
Vertical force at B

Horizontal force at
~~Moment at A~~

Find incorrect forces; Complete the diagrams

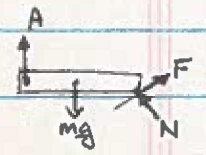
8.7

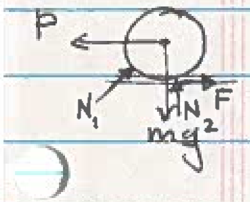
3/C Draw a complete and correct free-body diagram of each of the bodies designated in the statements. The weights of the bodies are significant only if the mass is stated. All forces, known and unknown, should be labeled. (*Note:* The sense of some reaction components cannot always be determined without numerical calculation.)

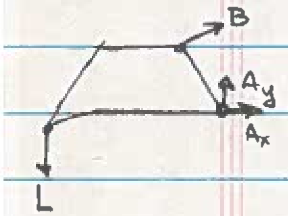


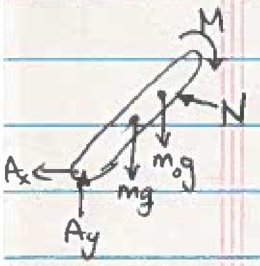
1. Uniform horizontal bar of mass *m* suspended by vertical cable at *A* and supported by rough inclined surface at *B*.
2. Wheel of mass *m* on verge of being rolled over curb by pull **P**.
3. Loaded truss supported by pin joint at *A* and by cable at *B*.
4. Uniform bar of mass *m* and roller of mass m_0 taken together. Subjected to couple *M* and supported as shown. Roller is free to turn.
5. Uniform grooved wheel of mass *m* supported by a rough surface and by action of horizontal cable.
6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.
7. Uniform heavy plate of mass *m* supported in vertical plane by cable *C* and hinge *A*.
8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.

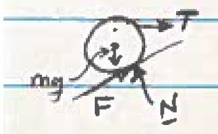
Figure 3/C

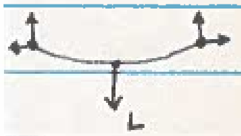


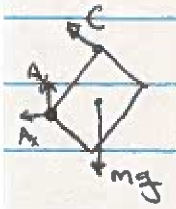


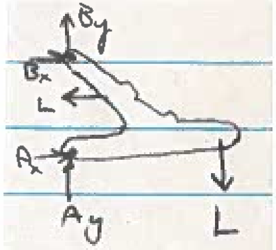






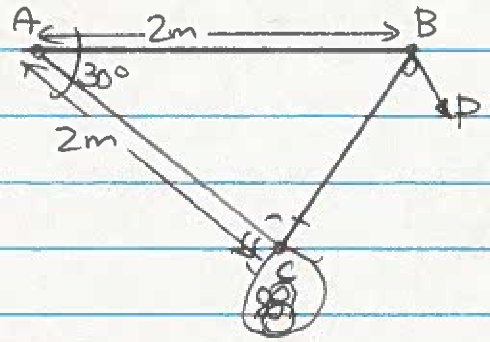


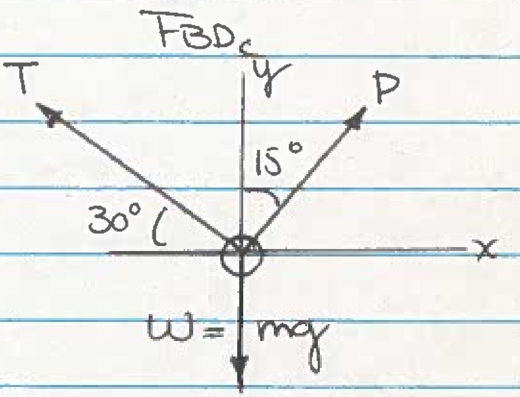




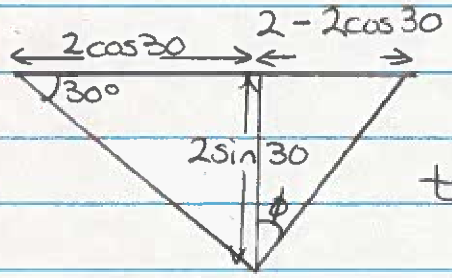
8.8

C3.2 *Find P required to hold 200kg engine. $\theta = 30°$*





$W = 200kg(9.8m/s^2) = 1960N$



$$\tan\phi = \frac{2-2\cos 30}{2\sin 30} = \frac{0.268}{1}$$

$$\phi = \tan^{-1}(0.268) = 15°$$

$\Sigma \underline{F} = \underline{0}$

$\Sigma F_x = P\sin 15 - T\cos 30 = 0$

$T\cos 30 = P\sin 15$

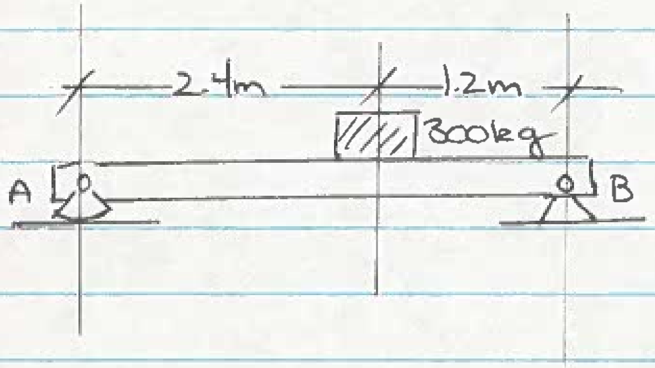
$T = P\,\frac{\sin 15}{\cos 30}$

$\Sigma F_y = P\cos 15 + T\sin 30 - W = 0$

$\Rightarrow P\cos 15 + P\,\frac{\sin 15\sin 30}{\cos 30} = W$

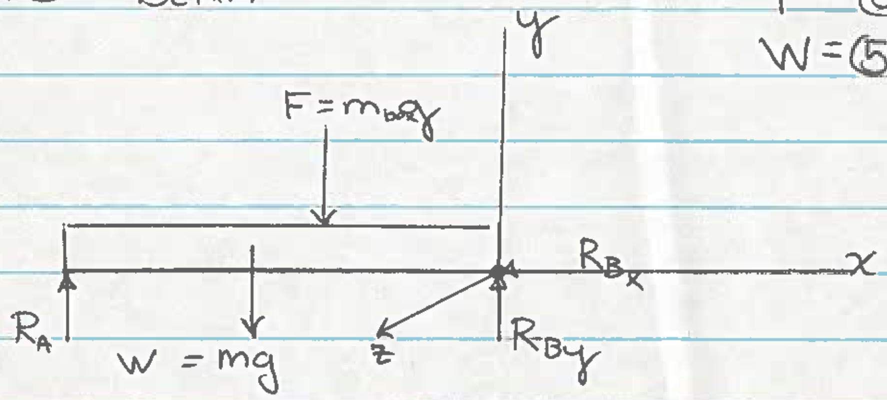
$\Rightarrow P(1.115) = W \qquad P = \frac{1960N}{1.115} = \underline{\underline{1757N}} \leftarrow$

The uniform beam has m = 50kg/meter
Find reactions at supports.



FBD BEAM

$F = (300)(9.8) = 2940\ N$
$W = (50)(3.6)(9.8) = 1764\ N$



$\Sigma \underline{F} = 0$

$\Sigma F_x = -R_{B_x} = 0$

$\Sigma F_y = R_A + R_{B_y} - W - F = 0$

$\Rightarrow R_A + R_{B_y} = 4707N$

$R_{B_y} = 4707 - R_A$

$\underline{R}_A = 1862N\,j$

$\underline{R}_B = 2845N\,j$

$\Sigma \underline{M}_B = 0$

$= \Sigma \underline{r} \times \underline{F}$

$0 = -3.6\underline{i} \times R_A j \; +$

$-1.8\underline{i} \times -Wj \; +$

$-1.2\underline{i} \times -Fj$

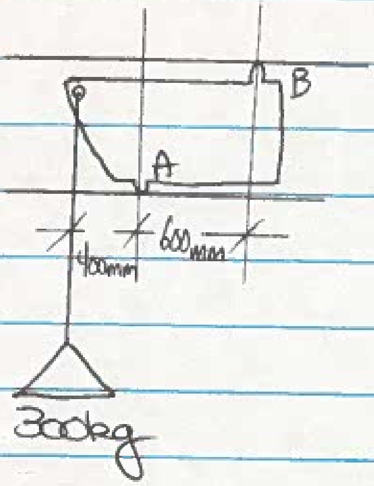
$= (1.8W + 1.2F - 3.6R_A)$

$\therefore R_A = \dfrac{1.8W + 1.2F}{3.6} \dfrac{(N \cdot m)}{(m)}$

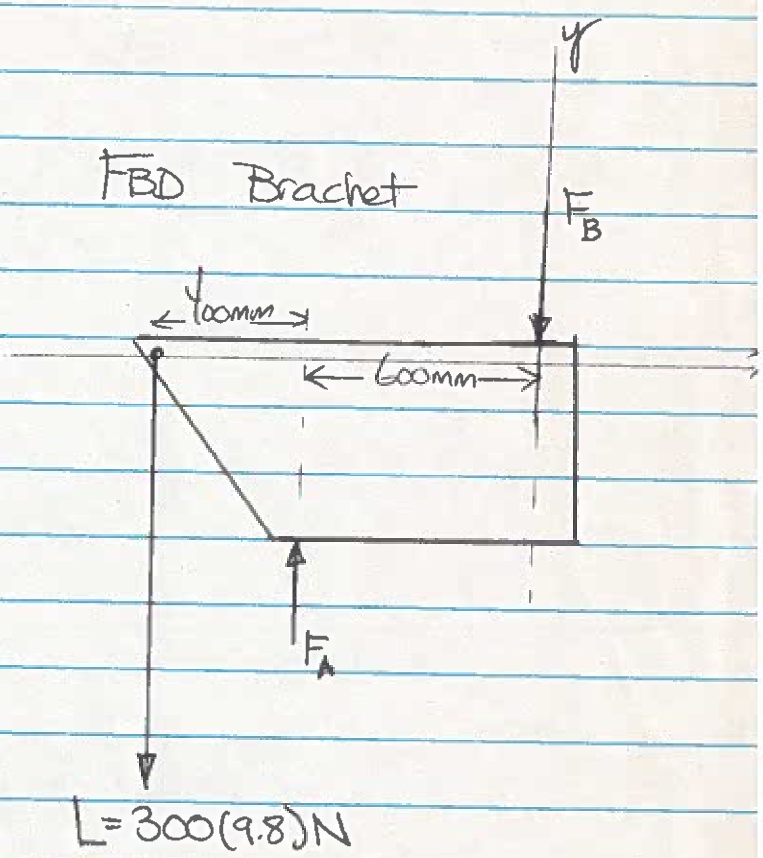
$= \dfrac{1.8(1764) + 1.2(2940)}{3.6}$ (N

$= 1862N$

3.9 Forces At A&B for system shown



FBD Bracket



$\Sigma \underline{F} = \underline{0}$

$\Sigma F_x = \underline{0}$

$\Sigma F_y = -300(9.8) + F_A - F_B$

$F_B = F_A - 300(9.8)$ N

$\Sigma \underline{M}_B = \underline{0} = \Sigma \underline{r} \times \underline{F}$

$= -1000\text{mm}\,\underline{i} \times -300(9.8)\,\underline{j} +$

$- 600\text{mm}\,\underline{i} \times F_A\,\underline{j}$

$= (1000)(300)(9.8)\,\underline{k} - 600 F_A\,\underline{k} = 0$ (N·mm)

$\therefore$

$$F_A = \frac{(1000)(300)(9.8)}{600} = 4900\text{N}$$

$F_B = 4900 - 300(9.8) = 1960$ N

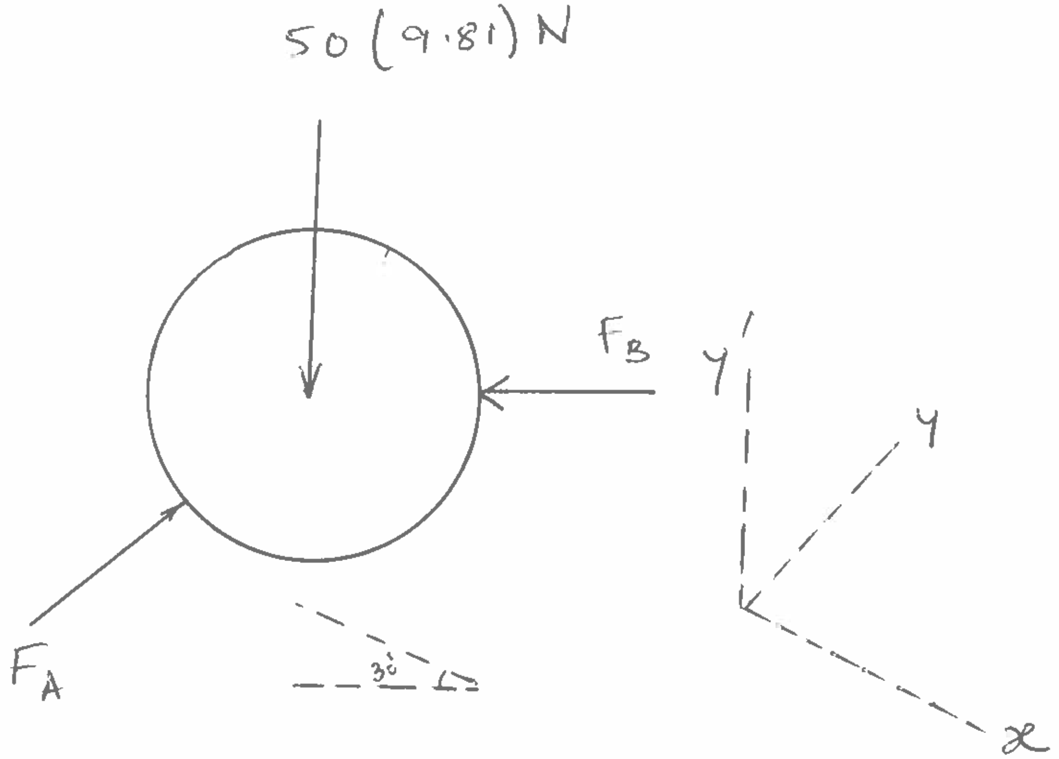
$\underline{F_A} = 4900\text{N}\,\underline{j}$

$\underline{F_B} = -1960\text{N}\,\underline{j}$ ⟵

CIVE 2330

HW # 9

Prob # 3.1



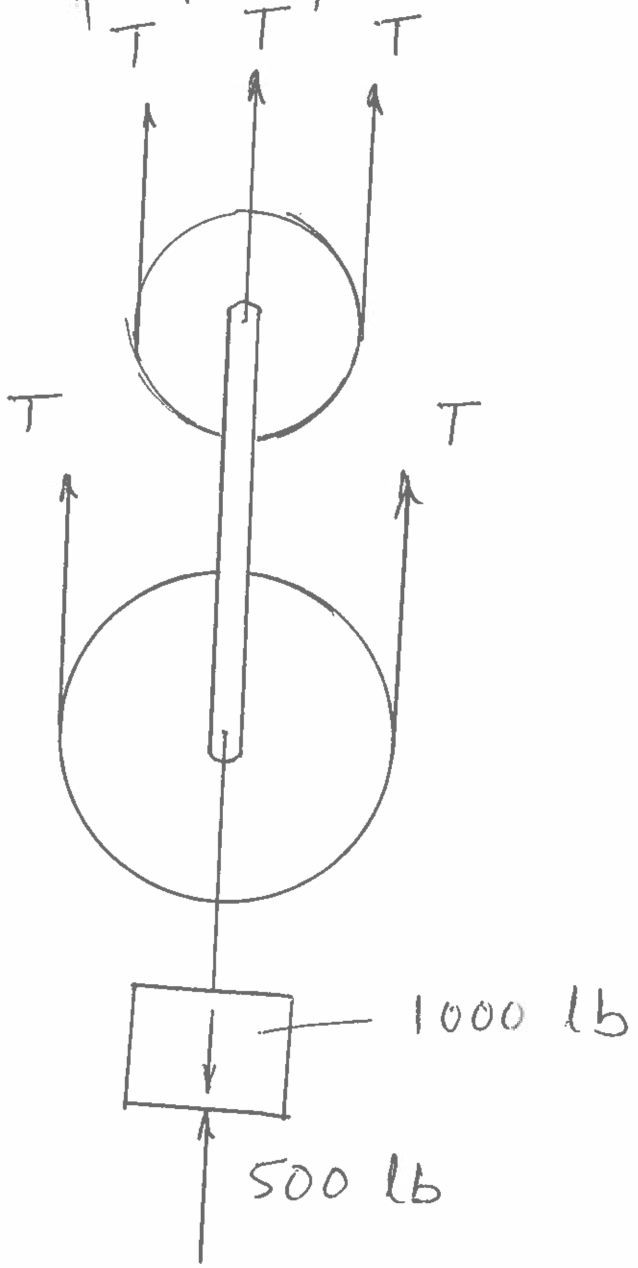
$$\Sigma F_x = 0 \, ; \quad 50(9.81) \sin 30^\circ - F_B \cos 30^\circ =$$

$$F_B = 283 \text{ N}$$

$$\Sigma F_{y'} = 0 \, ; \quad F_A \cos 30^\circ = 50(9.81) = 0$$

$$F_A = 566 \text{ N}$$

Prob # 37 FBD of 1000-lb weight and lower pair of pulleys:

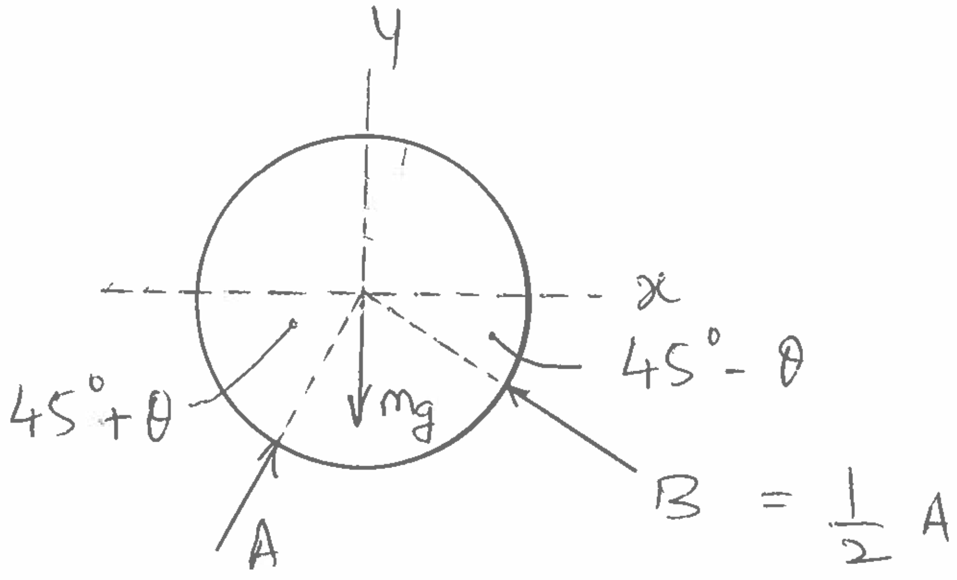


$$\uparrow \sum F = 0 : \quad 5T + 500 - 1000 = 0,$$

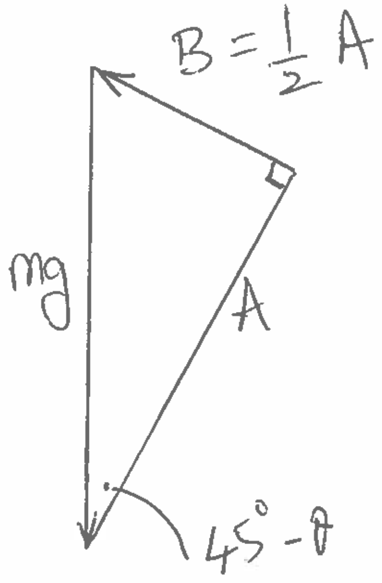
$$T = 100 \text{ lb}$$

(We assume that the nonverticality of some of the cables is negligible.)

Prob#3.8



$\Sigma F = 0$:

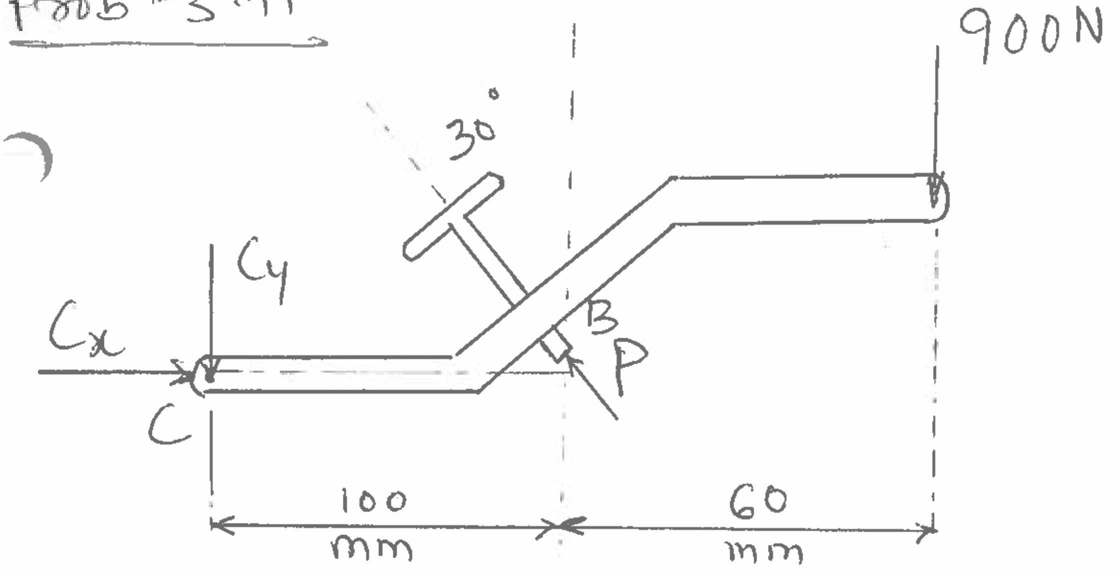


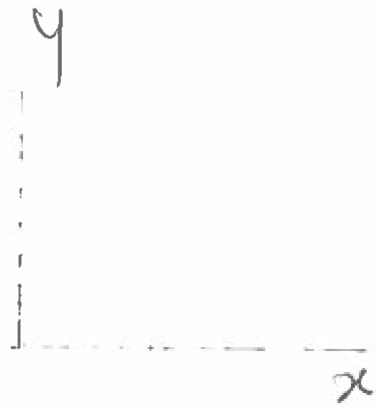
$$\tan(45^\circ - \theta) = \frac{A/2}{A} = \frac{1}{2}$$

$$45^\circ - \theta = 26.6^\circ$$

$$\theta = 18.43^\circ$$

Prob # 3.11





$$\circlearrowleft^{+}\ \Sigma M_c = 0 : P(100\cos 30^\circ) - 900(160) = 0;$$

$$\underline{P = 1663\ N}$$

$$\Sigma F_x = 0 : C_x - 1663 \sin 30^\circ = 0;$$

$$\underline{C_x = 831 N}$$

$$\Sigma F_y = 0 : 1663 \cos 30^\circ - 900 - C_y = 0;$$

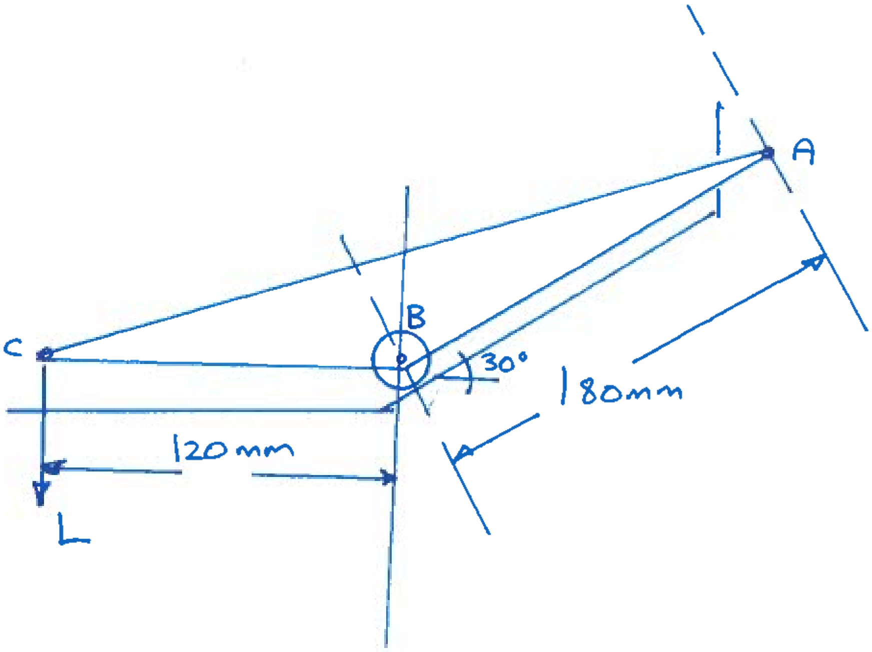
$$C_y = 540 N.$$

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{831^2 + 540^2}$$

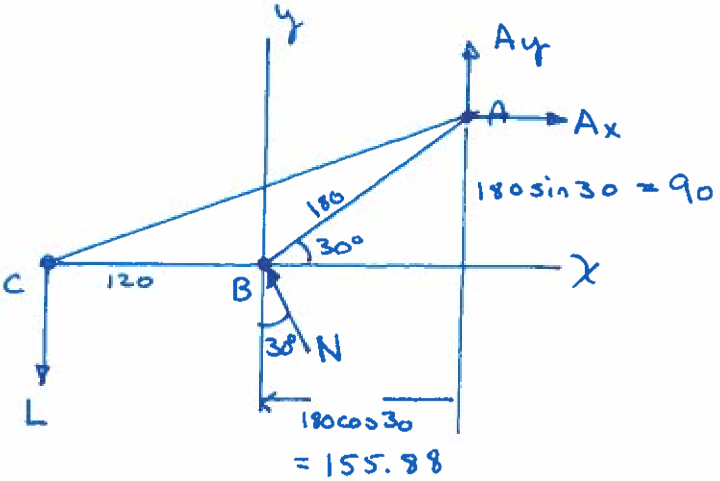
$$= 991 N.$$

3.12 CALCULATE MAGNITUDE OF FORCE IN EACH PIN (A & B)

L = 1.6 kN



FBD CRANK



$\Sigma \underline{F} = 0$

$\Sigma F_x = A_x - N \quad 30 = 0$

$A_x = N \sin 30$

$\Sigma F_y = A_y - L + N \cos 30 = 0$

$A_y = L - N \cos 30$

$N \sin 30 = 2.13 + 1.73(L - N \cos 30)$

$= 2.13 + 1.73(1.6) - 1.73 N \cos 30$

$N(\sin 30 + 1.73 \cos 30) = 2.13 + 1.73(1.6)$

$$N = \frac{2.13 + 1.73(1.6)}{(\sin 30 + 1.73 \cos 30)}$$

$\underline{N = 2.45 \text{ kN}}$

$\underline{A_x = 2.45(\sin 30) = 1.23 \text{ kN}}$

$\underline{A_y = 1.6 - 2.45(\cos 30) = -0.524 \text{ kN}}$

$\Sigma \underline{M} = 0$

$\Sigma M_B = 0$

$0 = -120 \underline{i} \times -L \underline{j}$

$+ 90 \underline{j} \times A_x \underline{i}$

$+ 155.8 \underline{i} \times A_y \underline{j}$

$0 = 120 L \underline{k} - 90 A_x \underline{k}$

$+ 155.8 A_y \underline{k}$

$90 A_x = 120 L + 155.8 A_y$

$$A_x = \frac{120(1.6)}{90} + \frac{155.8}{90} A_y$$

$A_x = 2.13 + 1.73 A_y$

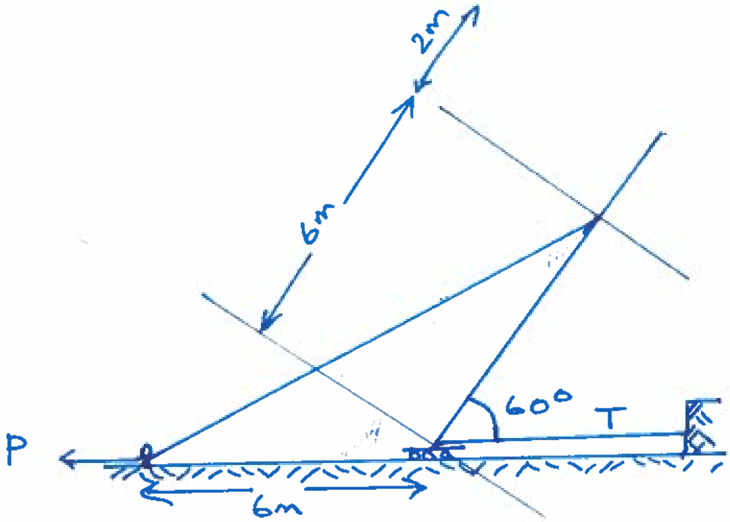
←

3.25

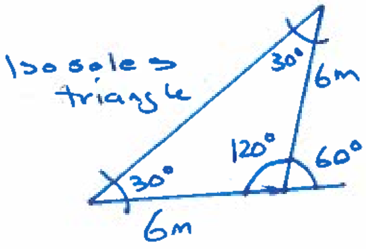
CONCRETE SLAB M = 25 Mg

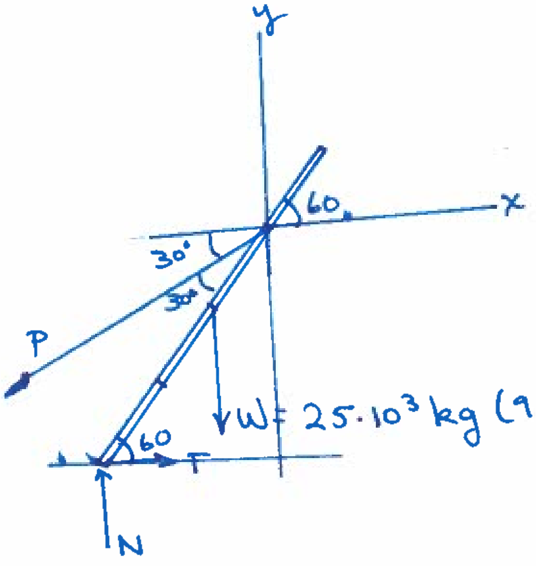
HOISTED AS SHOWN

$\theta = 60°$ FIND T

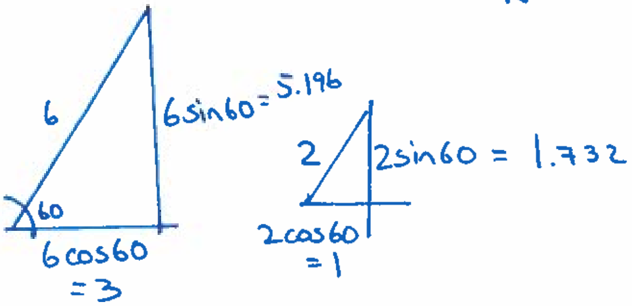


FBD SLAB





$W = 25 \cdot 10^3 \text{ kg} (9.8 \text{ m/s}^2) = 245 \text{kN}$



$$\Sigma F_x = 0 = T - P\cos 30 = 0$$

$$T = P\cos 30 \quad (1)$$

$$\Sigma F_y = N - W - P\sin 30 = 0 \quad (2)$$

$$\Sigma M_o = 0 = (-\underline{i} - 1.732\underline{j}) \times (-245\underline{j})$$

$$+ (-3\underline{i} - 5.196\underline{j}) \times (T\underline{i})$$

$$+ (-3\underline{i} - 5.196\underline{j}) \times (N\underline{j})$$

$$0 = 245\underline{k} + 5.196T\underline{k} - 3N\underline{k} \quad (3)$$

$$N = \frac{245}{3} + \frac{5.196T}{3} = 81.67 + 1.732T$$

SUBSTITUTE INTO (2)

$$81.67 + 1.732T - P\sin 30 = 245$$

$$T = \frac{163.3}{1.732} + \frac{P\sin 30}{1.732} = 94.301 + 0.288P$$

SUBSTITUTE INTO (1)

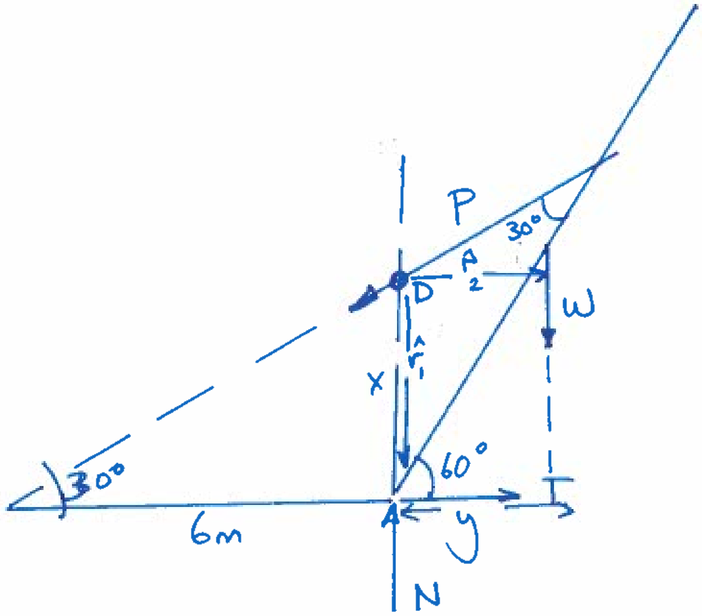
$$94.301 + 0.288P = P\cos 30$$

$$94.301 = P(0.866 - 0.288)$$

$$P = \frac{94.301}{(0.578)} = 163.14\,kN$$

$$T = 163.14\,kN\,(0.866) = \underline{141.28\,kN} \longleftarrow$$

WITH A SINGLE MOMENT CENTER



$$\frac{x}{6} = \tan 30$$

$$x = 6 \tan 30$$
$$= 3.46 \text{ m}$$

$$y = 4 \cos 60$$
$$= 2. \text{ m}$$

$$\Sigma \underline{M}_D = 0$$

$$= \hat{r}_1 \times \underline{T} + \hat{r}_2 \times \underline{W} = 0$$

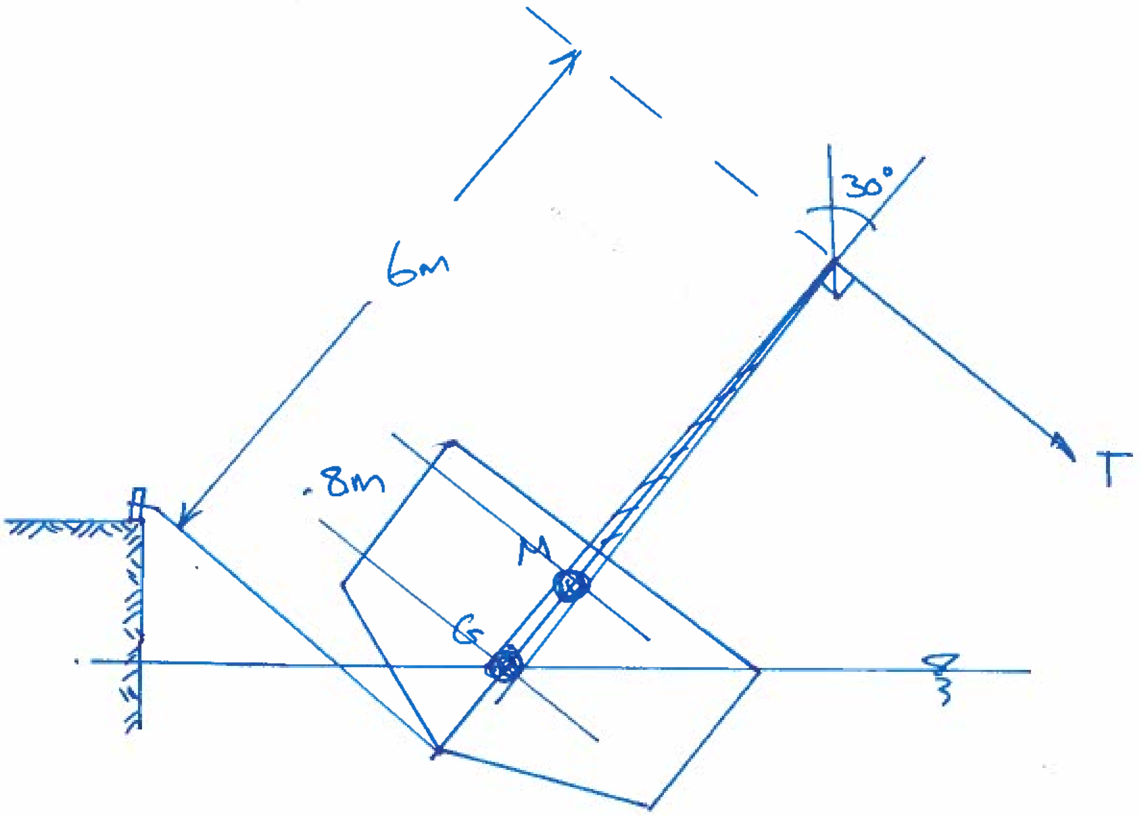
$$= -3.46 j \times T\underline{i} + 2\underline{i} \times -Wj$$

$$= 3.46 T\underline{k} - 2W\underline{k} = 0$$

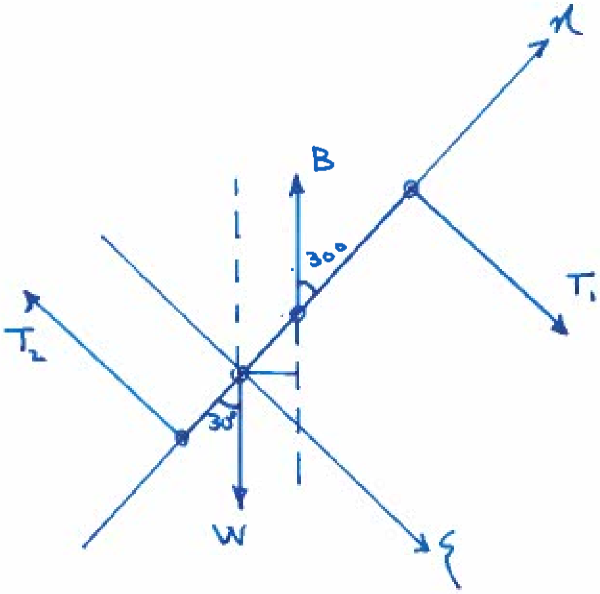
$$T = \frac{2W}{3.46} = \frac{2(245)}{3.46} = \underline{141.6 \text{ kN}} \leftarrow$$

3.26 SAILBOAT AS SHOWN.

FIND T REQUIRED TO MAINTAIN POSITION m = 5000kg



FBD



1

$$\Sigma F_\xi = 0 = W \sin 30 - B \sin 30 + T_1 - T_2 \quad (1)$$

$$\Sigma F_\eta = 0 = B\cos 30 - W \cos 30 = 0 \quad (2)$$

$$B \cos 30 = W \cos 30$$

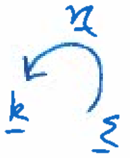
$$\therefore B = W$$

SUBSTITUTE INTO (1)

$$T_1 - T_2 = 0 \Rightarrow T_1 = T_2 = T$$

T_1 & T_2 ARE A COUPLE

W & B ARE A COUPLE



$$M_{T_1 - T_2} = -T\,6\text{m}\ \underline{k}$$

$$M_{WB} = -0.8\text{m}\,\underline{\eta} \times W(-\cos 30\,\eta + \sin 30\,\xi)$$

$$= +0.8\text{m}\ W \sin 30\ \underline{k}$$

$$\therefore \Sigma M = 0 = (0.8)(0.5)W - 6T$$

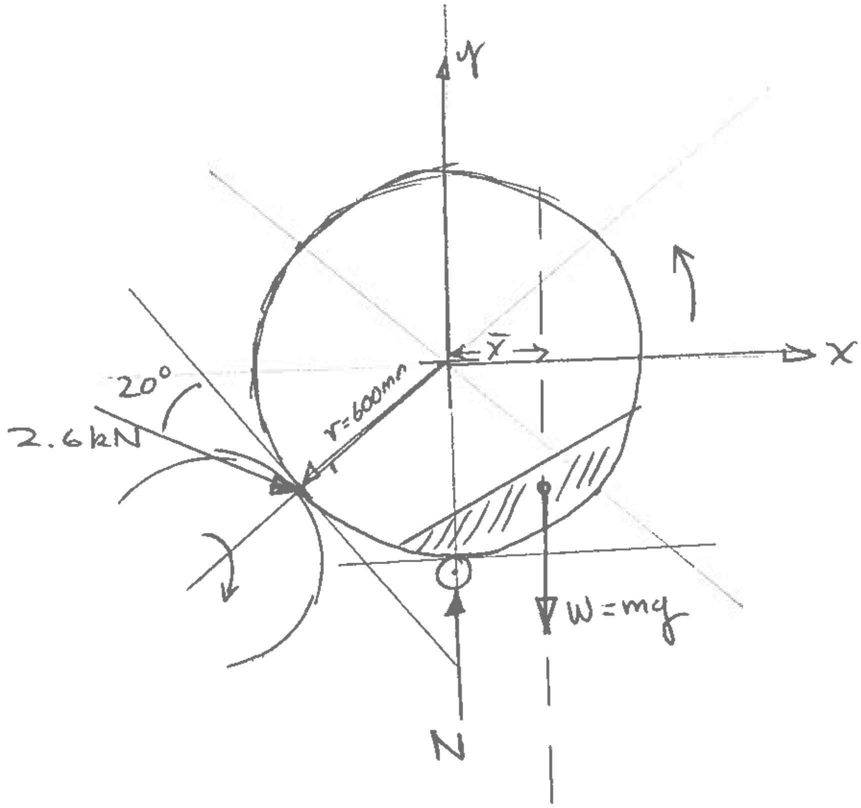
$$T = \frac{(0.8)(0.5)W}{6} = \frac{(0.4)(5000)(9.8)}{6}$$

3266 N

$\underline{3.26\text{kN}}$ ←

3.105 Cement mixer. $M_{sand} = 750\text{kg}$.

→ Find $\bar{x}$



$\Sigma \underline{F} = 0$, but don't know points of application of gear drive

$$\Sigma M_o = 0 = -\bar{x}\, W\, \underline{k} + r\ 2.6\cos 20\ \underline{k}$$

$$\bar{x} = \frac{600\text{mm}\,(2.6\cos 20)}{(750)(9.8)} = 119.2\text{mm}$$

3-41 LAWNMOWER UPHILL

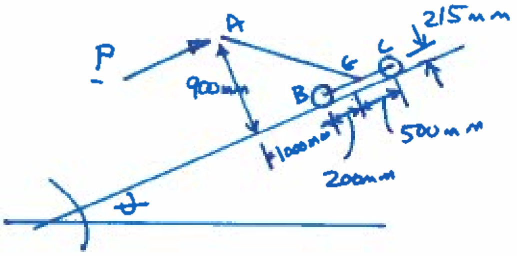
FIND NORMAL FORCES WHEN AT WHEELS

$\theta = 15°$

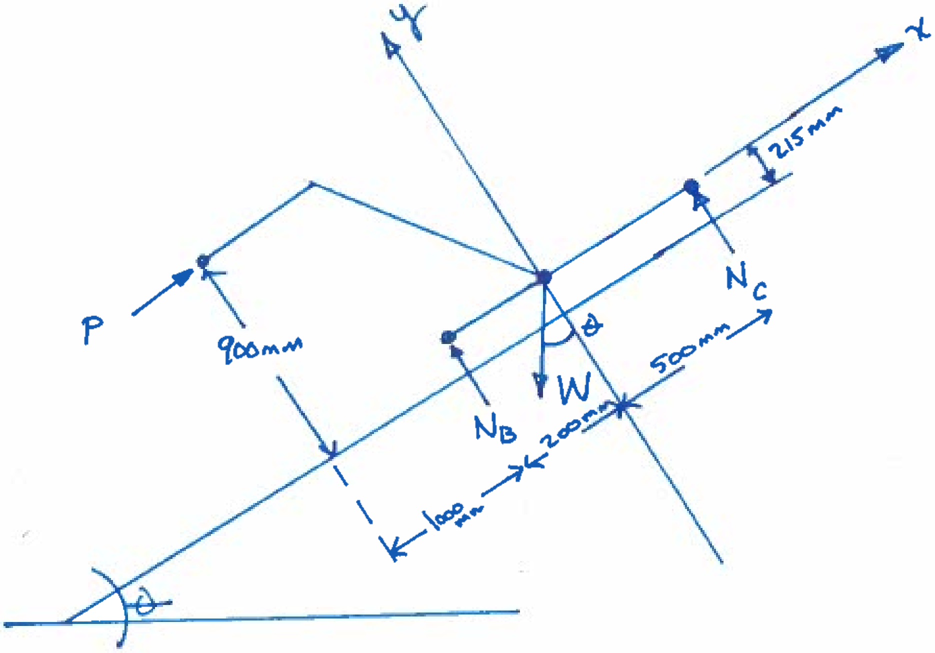
and

$\theta, P = 0$

$M = 50kg$

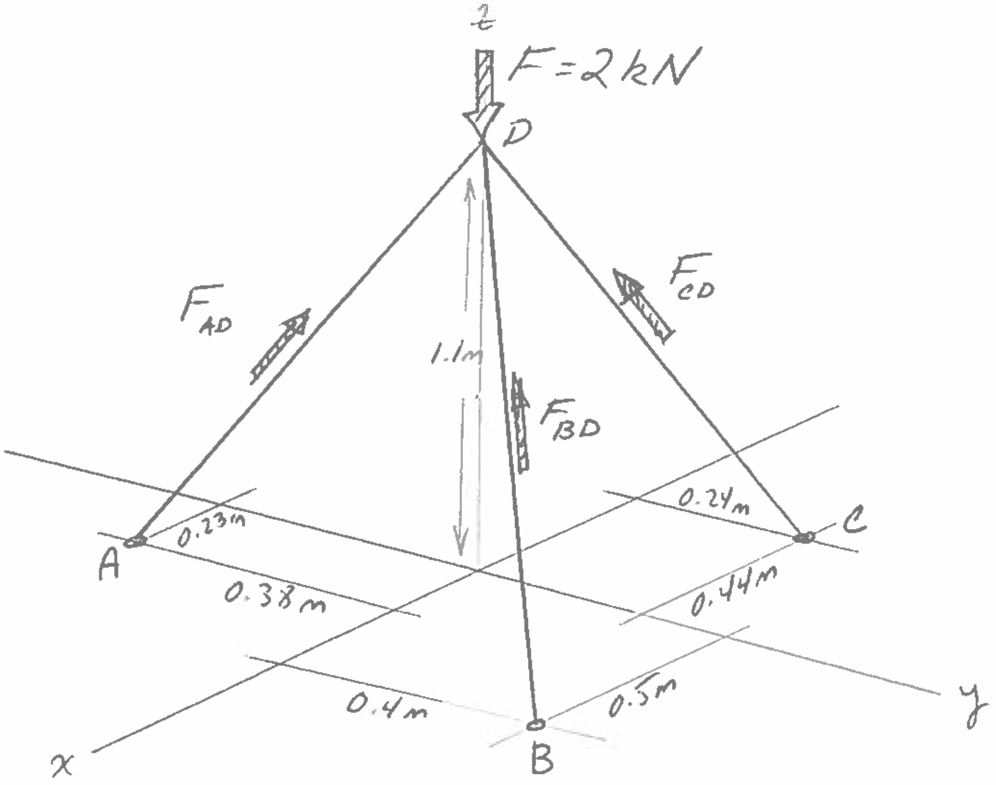


FBD MOWER



F

3.62 Determine Compression in each leg of tripod.



Diagram

To find $\underline{F}_{AD}$, $\underline{F}_{BD}$, $\underline{F}_{CD}$ need to find uni vectors along line of action of these forces

POINT	X	Y	Z
A	0.23	-0.38	0
B	0.5	0.4	0
C	-0.44	0.24	0
D	0	0	1.1

$$e_{AD} = \frac{(0-0.23)\underline{i} + (0+0.38)\underline{j} + (1.1-0)\underline{k}}{\sqrt{0.23^2 + 0.38^2 + 1.1^2}}$$

	A	B	C	D	E
1	Unit Vector Calculator				
2		Start	End	End-Start	Direction Cosine
3	i	0.23	0	-0.23	-0.194
4	j	-0.38	0	0.38	0.320
5	k	0	1.1	1.1	0.927
6	Length			1.186297	
7		Start	End	End-Start	Direction Cosine
8	i	0.5	0	-0.5	-0.393
9	j	0.4	0	-0.4	-0.314
10	k	0	1.1	1.1	0.864
11	Length			1.272792	
12		Start	End	End-Start	Direction Cosine
13	i	-0.44	0	0.44	0.364
14	j	0.24	0	-0.24	-0.199
15	k	0	1.1	1.1	0.910
16	Length			1.208801	

← SPREADSHEET TO DO CALCULATIONS

$e_{AD} = -0.194\underline{i} + .320\underline{j} + .927\underline{k}$

$e_{BD} = -.393\underline{i} - .314\underline{j} + .864\underline{k}$

$e_{CD} = .364\underline{i} - .199\underline{j} + .910\underline{k}$

NOW VECTORS ARE:

$$\underline{F}_{AD} = F_{AD}(-0.194\underline{i} + .320\underline{j} + 0.927\underline{k})$$

$$\underline{F}_{BD} = F_{BD}(-.393\underline{i} - .314\underline{j} + .864\underline{k})$$

$$\underline{F}_{CD} = F_{CD}(.364\underline{i} - .199\underline{j} + .910\underline{k})$$

$$\underline{F}_{D} = -2\underline{k}$$

STATIC FORCE BALANCE

$$\Sigma\underline{F} = \underline{0}$$

$$\Sigma F_x = -0.194F_{AD} - 0.393F_{BD} + 0.364F_{CD} = 0$$

$$\Sigma F_y = 0.320F_{AD} - 0.314F_{BD} - 0.199F_{CD} = 0$$

$$\Sigma F_z = 0.927F_{AD} + 0.864F_{BD} + 0.910F_{CD} - 2 = 0$$

SIMULTANEOUS SYSTEM OF LINEAR EQNS.

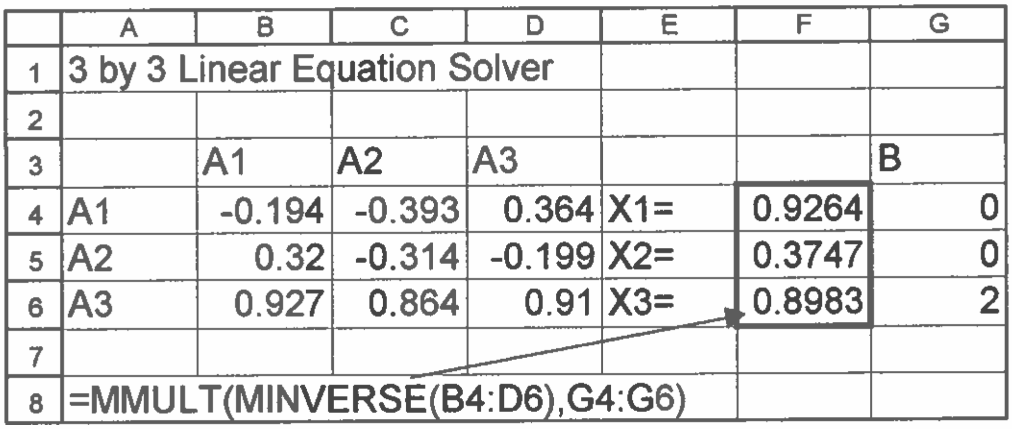
$$\underbrace{\begin{pmatrix} -0.194 & -0.393 & 0.364 \\ 0.320 & -0.314 & -0.199 \\ 0.927 & 0.864 & 0.910 \end{pmatrix}}_{\underline{A}} \underbrace{\begin{pmatrix} F_{AD} \\ F_{BD} \\ F_{CD} \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}}_{\underline{b}}$$

RECALL FROM LINEAR ALGEBRA

$$\underline{A}\,\underline{x} = \underline{b}$$

$$\therefore \underline{x} = \underline{A}^{-1}\underline{b}$$

SIMPLY PROGRAM A SPREADSHEET TO DO THE CALCULATIONS



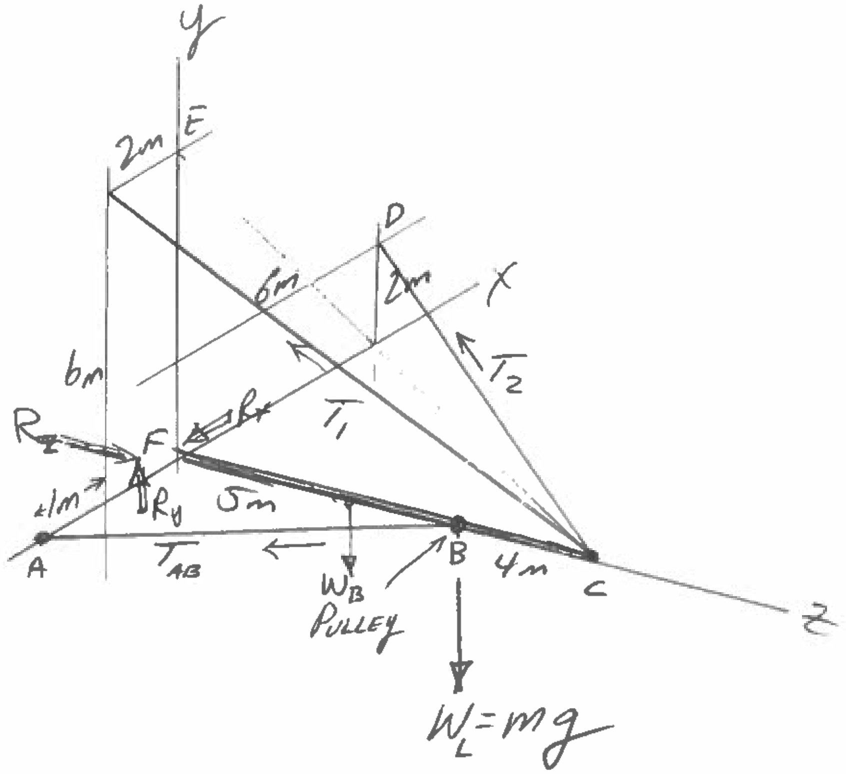
	A	B	C	D	E	F	G
1	3 by 3 Linear Equation Solver						
2							
3		A1	A2	A3			B
4	A1	-0.194	-0.393	0.364	X1=	0.9264	0
5	A2	0.32	-0.314	-0.199	X2=	0.3747	0
6	A3	0.927	0.864	0.91	X3=	0.8983	2
7							
8	=MMULT(MINVERSE(B4:D6),G4:G6)						

$$\therefore \left.\begin{aligned} F_{AD} &= 0.926\,kN \\ F_{BD} &= 0.376\,kN \\ F_{CD} &= 0.898\,kN \end{aligned}\right\} \text{ ALL COMPRESSION}$$

3.74 Boom Mass = 600kg

Load = 2000kg

FIND T_1



DIAGRAM

FIND LINES OF ACTION FOR T_1, T_2, T_{AB}

	$x_2 - x_1$	$y_2 - y_1$	$z_2 - z_1$
T_1	−2	6	−9
T_2	6	2	−9
T_{AB}	−3	0	−5

	A	B	C	D	E
1	Unit Vector Calculator				
2		Start	End	End-Start	Direction Cosine
3	i	0	-2	-2	-0.182
4	j	0	6	6	0.545
5	k	0	-9	-9	-0.818
6	Length			11	
7		Start	End	End-Start	Direction Cosine
8	i	0	6	6	0.545
9	j	0	2	2	0.182
10	k	0	-9	-9	-0.818
11	Length			11	
12		Start	End	End-Start	Direction Cosine
13	i	0	-3	-3	-0.514
14	j	0	0	0	0.000
15	k	0	-5	-5	-0.857
16	Length			5.830952	

$\underline{T}_1 = T_1(-.182\underline{i} + 0.545\underline{j} - 0.818\underline{k})$

$\underline{T}_2 = T_2(0.545\underline{i} + 0.182\underline{j} - 0.818\underline{k})$

$\underline{T}_{AB} = T_{AB}(-.514\underline{i} + 0\underline{j} - 0.857\underline{k})$

FROM KINEMATICS, $T_{AB} = W_L = 2000(9.8) = 19{,}600N$

$W_B = 600(9.8) = 5{,}880N$

$\Sigma \underline{M}_F = \underline{0} = \Sigma \underline{\hat{r}} \times \underline{F}$

$$\Sigma \underline{M}_F = 9\underline{k} \times T_1(-.182\underline{i} + .545\underline{j} - .818\underline{k}) \; +$$

$$9\underline{k} \times T_2(.545\underline{i} + .182\underline{j} - .818\underline{k}) \; +$$

$$5\underline{k} \times -W_L\,\underline{j} \; +$$

$$4.5\underline{k} \times -W_B\,\underline{j} \; +$$

$$5\underline{k} \times T_{AB}(-.514\underline{i} - .857\underline{k}) \quad = 0$$

$$T_{AB} = W_L$$

EVALUATE CROSS PRODUCTS

$$9T_1(-.182) + 9T_2(.545) - 5W_L(.514) \quad \underline{j}$$

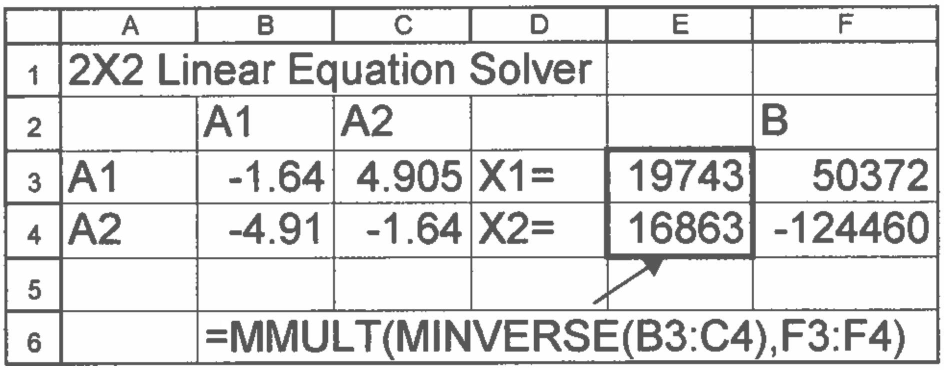
$$-9T_1(.545) - 9T_2(.182) + 5W_L + 4.5W_B \quad \underline{i} \qquad = 0$$

$$-1.638T_1 + \overset{4.91}{\cancel{2.673}}T_2 - 50372 = 0$$

$$-\underset{4.91}{\cancel{2.673}}T_1 - 1.638T_2 + 98000 + 26460 = 0$$

SET UP LINEAR SYSTEM

$$\begin{pmatrix} -1.638 & 4.91 \\ 4.91 & -1.638 \end{pmatrix} \begin{pmatrix} T_1 \\ T_1 \end{pmatrix} = \begin{pmatrix} 50372 \\ -124460 \end{pmatrix}$$



	A	B	C	D	E	F
1	2X2 Linear Equation Solver					
2		A1	A2			B
3	A1	-1.64	4.905	X1=	19743	50372
4	A2	-4.91	-1.64	X2=	16863	-124460
5						
6		=MMULT(MINVERSE(B3:C4),F3:F4)				

$\therefore T_1 = 19.74$ kN

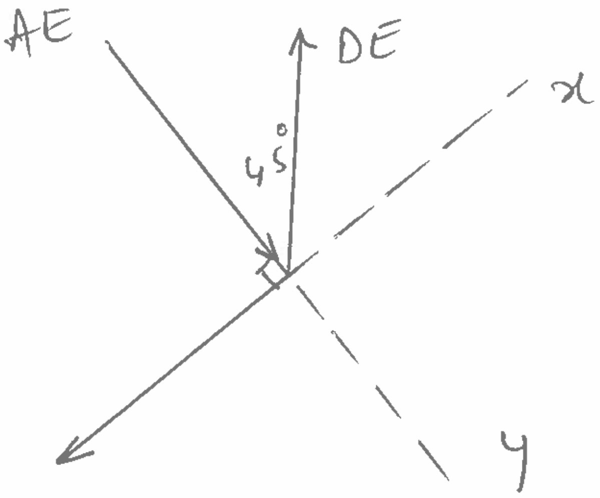
$T_2 = 16.86$ kN

CIVE 2330

HW #12

Prob # 4.4

Joint E



$\Sigma F_x = 0$:

$DE \cos 45^\circ - 2 = 0$

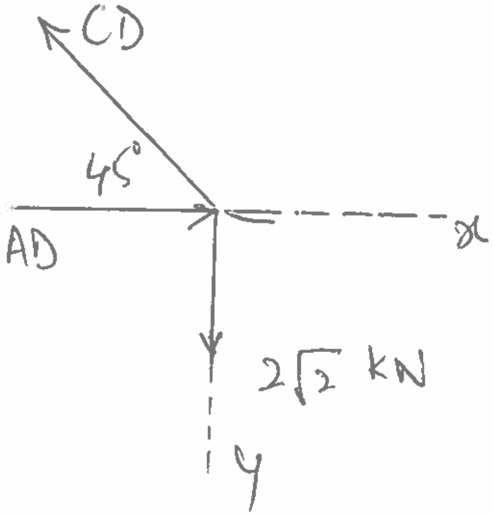
$DE = 2\sqrt{2}$ kN T

$\Sigma F_y = 0$:

$AE - 2\sqrt{2} \cos 45^\circ = 0$

$AE = 2$ kN C

Joint D



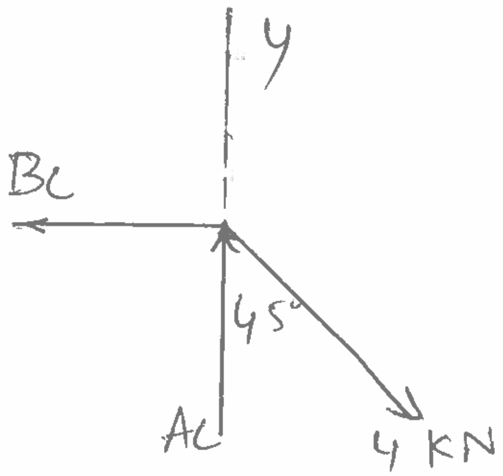
$\Sigma F_y = 0 : 2\sqrt{2} - CD \sin 45^\circ = 0$

$CD = 4$ kN T

$\Sigma F_x = 0 : AD - 4 \cos 45^\circ = 0$

$AD = 2\sqrt{2}$ kN C

Joint C

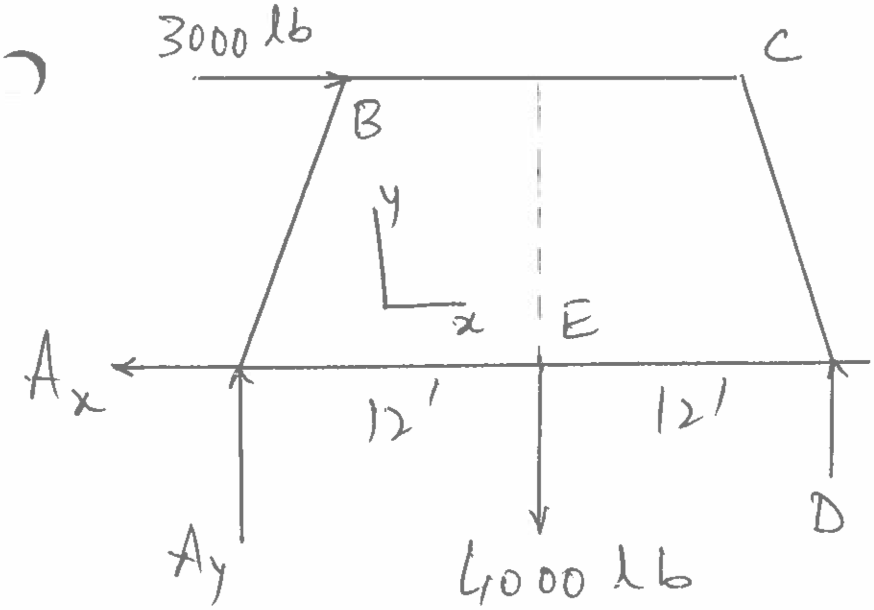


$\Sigma F_y = 0$:

$AC - 4 \cos 45^\circ = 0$

$AC = 2\sqrt{2}$ kN C

Prob # 4.6



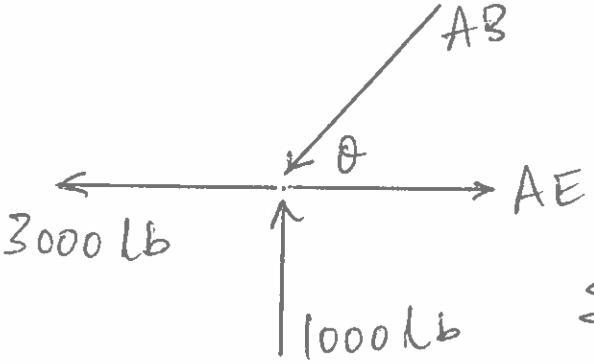
$\Sigma M_A = 0;$ $4000(12) + 3000(8) - 24D = 0$

$D = 3000$ lb.

$\Sigma F_x = 0;$ $A_x = 3000$ lb.

$\Sigma F_y = 0;$ $A_y = 4000 - 3000 = 1000$ lb.

Joint A



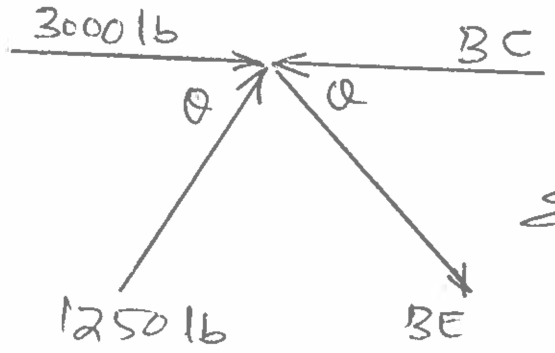
$\Sigma F_y = 0;$ $AB\left(\frac{4}{5}\right) - 1000 = 0.$

$AB = 1250$ lb C

$\Sigma F_x = 0;$ $AE - 1250\left(\frac{3}{5}\right) - 3000 =$

$AE = 3750$ lb T

Joint B



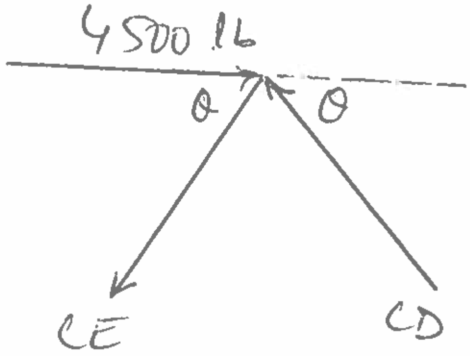
$\Sigma F_y = 0 : 1250\left(\frac{4}{5}\right) - BE\left(\frac{4}{5}\right) = 0$

$\underline{BE = 1250 \text{ lb } T}$

$\Sigma F_x = 0 ; \; BC - 3000 - 1250\left(\frac{3}{5}\right) - 1250\left(\frac{3}{5}\right) = 0$

$\underline{BC = 4500 \text{ lb } C}$

Joint C



$\Sigma F_y = 0 : CE \sin\theta - CD \sin\theta = 0$

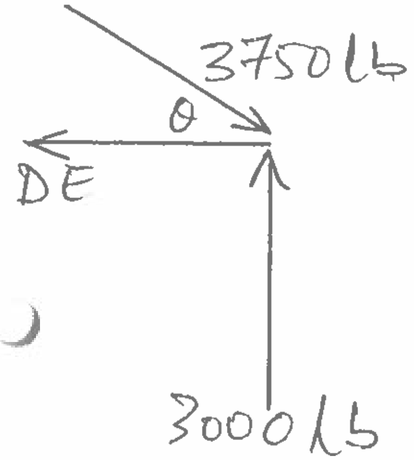
$CE = CD$

$\Sigma F_x = 0 ; \; 2\,CE\left(3/5\right) - 4500 = 0$

$\underline{CE = 3750 \text{ lb } T}$

$\underline{CD = 3750 \text{ lb } C}$

Joint D



$\Sigma F_x = 0 ;$

$DE - 3750\left(\frac{3}{5}\right) = 0$

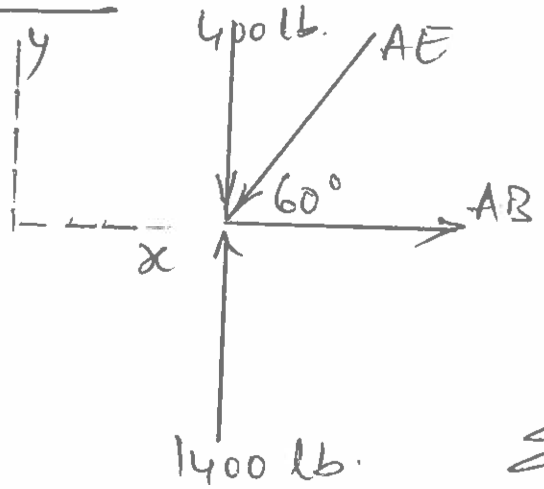
$\underline{DE = 2250 \text{ lb } T}$

Prob # 4.9

Total weight of truss = 7(400) = 2800 lb

By Symmetry reactions at A & C are 1400 lb.

Joint A



$\Sigma F_y = 0$: $AE \cos 30° + 400 - 1400 = 0$

$AE = 2000/\sqrt{3}$ lb C

$\Sigma F_x = 0$: $AB - \frac{2000}{\sqrt{3}} \cos 60° = 0$

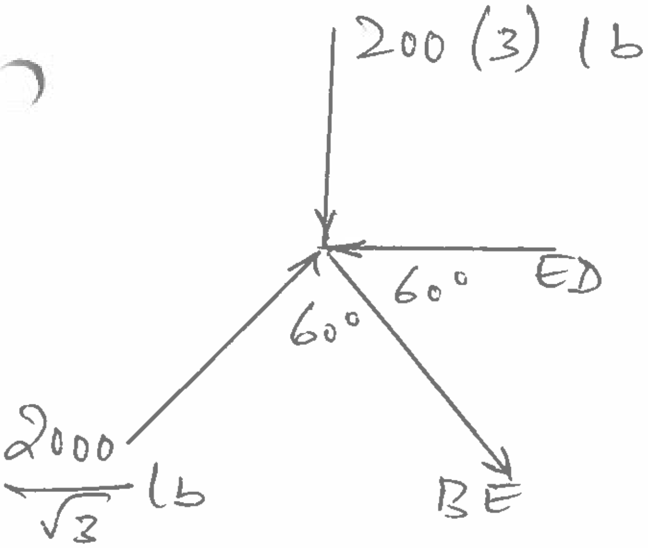
$AB = 1000/\sqrt{3}$ lb T

By Symmetry $BC = 1000/\sqrt{3}$ lb T

$CD = 2000/\sqrt{3}$ lb C

Joint E

Joint E



$\Sigma F_y = 0$;

$$BE \sin 60° - \frac{2000}{\sqrt{3}} \sin 60° + 600 = 0.$$

$$\underline{BE = 800/\sqrt{3} \text{ lb T}}$$

& by Symmetry $\underline{BD = 800/\sqrt{3} \text{ lb T}}$

$$\Sigma F_x = 0 : \; ED - \frac{2000}{\sqrt{3}} \sin 30° - \frac{800}{\sqrt{3}} \sin 30°$$

$$\underline{ED = 1400/\sqrt{3} \text{ lb C}}$$

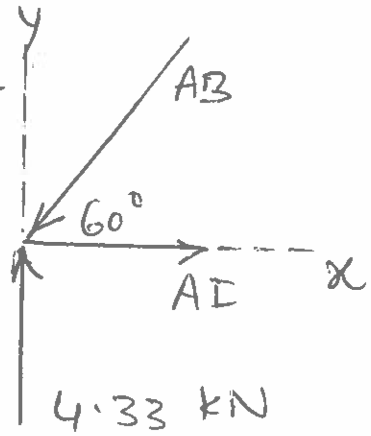
Prob # 4.15

Truss as a whole

$$\Sigma M_F = 0 \; ; \; 2\left(a + \frac{a}{2}\right) + 4\left(2a + \frac{a}{2}\right) - A(3a) =$$

$$A = 13/3 = 4.33 \text{ kN.}$$

Joint A



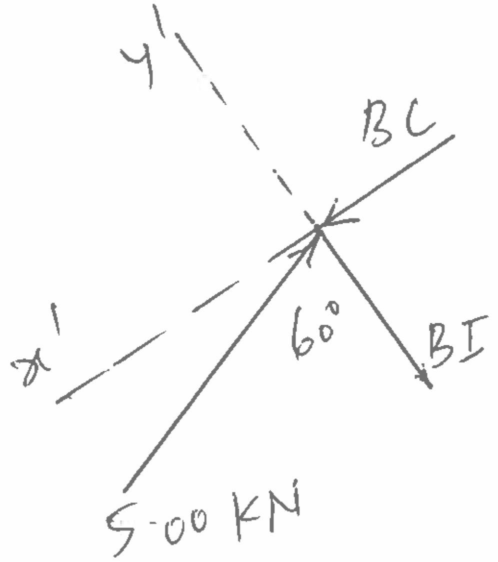
$$\Sigma F_y = 0 ; \; AB \sin 60° - 4.33 =$$

$$AB = 5.00 \text{ kN C}$$

$$\Sigma F_x = 0 ; \; AI - 5 \cos 60° = 0$$

$$AI = 2.50 \text{ kN T}$$

Joint B

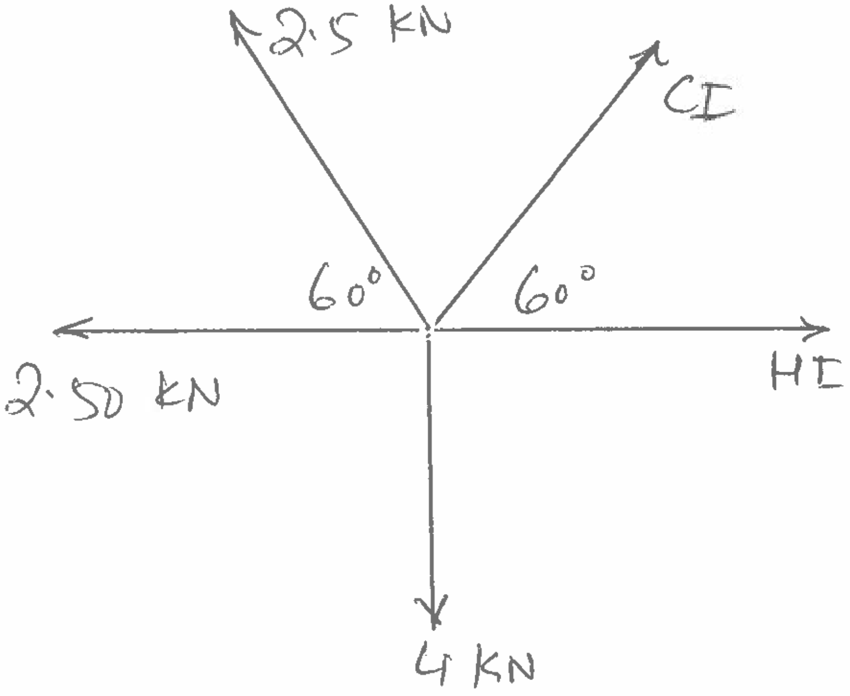


$$\Sigma F_{y'} = 0 ;$$

$$5.00 \cos 60° - BI = 0$$

$$BI = 2.50 \text{ kN T}$$

Joint I



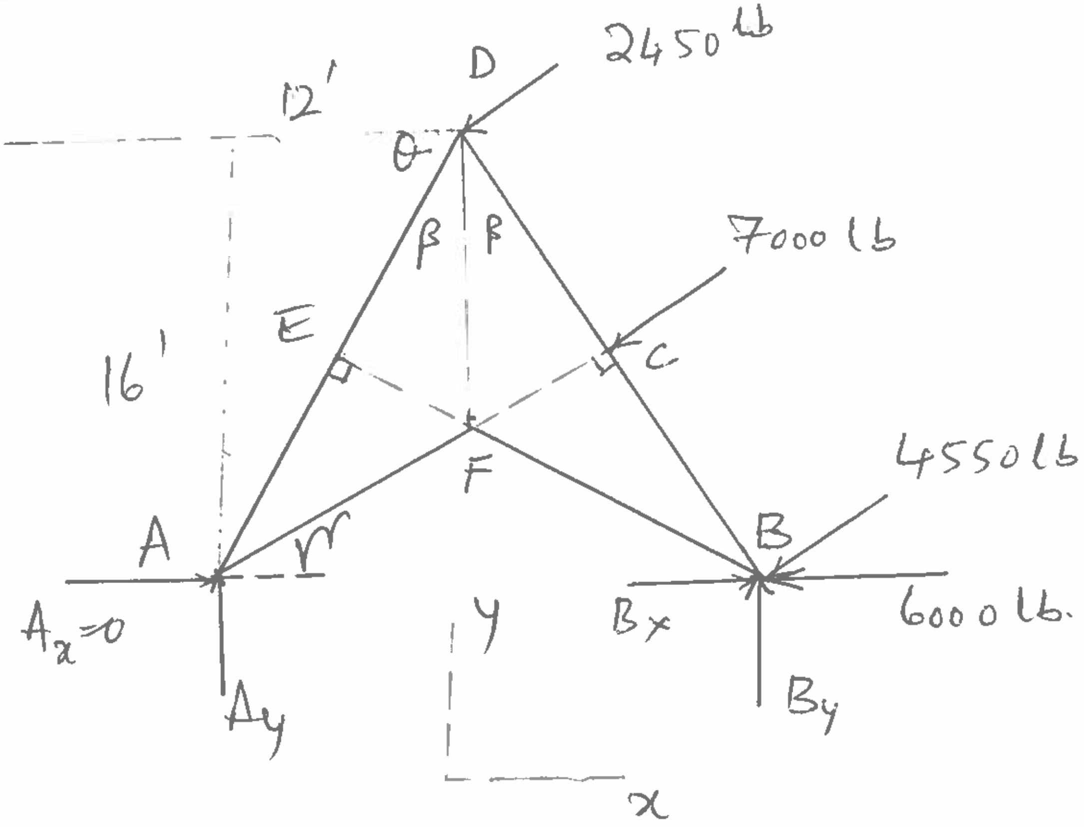
$\Sigma F_y = 0 \quad (CI + 2.5)\sin 60° - 4 = 0$

$\underline{CI = 2.12 \text{ KN T}}$

$\Sigma F_x = 0 \;; \quad HI + 2.12 \cos 60° - 2.50$
$\qquad - 2.5 \cos 60° = 0$

$\underline{HI = 2.69 \text{ KN T}}$

Prob 4.26



Geometry

$$\theta = \tan^{-1}\frac{16}{12} = 53.1^\circ$$

$$\beta = 90 - \theta = 36.9^\circ$$

$$\alpha = 90 - 2\beta = 16.26^\circ$$

$$\overline{CD} = 20 \sin\alpha = 5.60 \text{ ft.}$$

$$\overline{CB} = 20 - \overline{CD} = 14.40 \text{ ft.}$$

$$\gamma = 90 - (90 - \theta) - \alpha = 36.9^\circ$$

Truss as a whole :

$\circlearrowright + \sum M_B = 0 : \quad 7000(14.4) + 2450(20) - 24 A_y = 0$

$A_y = 6240 \text{ lb}.$

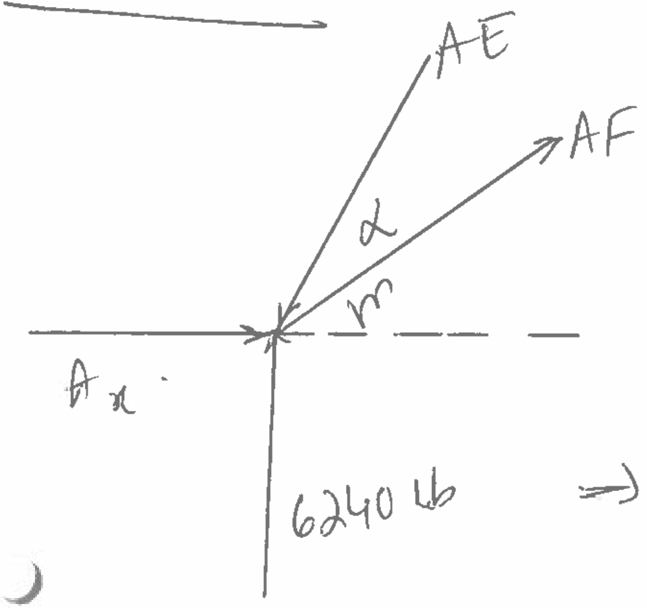
$\sum F_y = 0 : \quad B_y + 6240 - (2450 + 7000 + 4550) \sin 36.9° = 0$

$B_y = 2160 \text{ lb}.$

$\sum F_x = 0 : \quad B_x - (2450 + 7000 + 4550) \cos 36.9° - 6000 = 0$

$B_x = 17,200 \text{ lb}$

Joint A



$\sum F_x = 0 : \quad -AE \cos 53.1° + AF \cos 36.9° = 0$

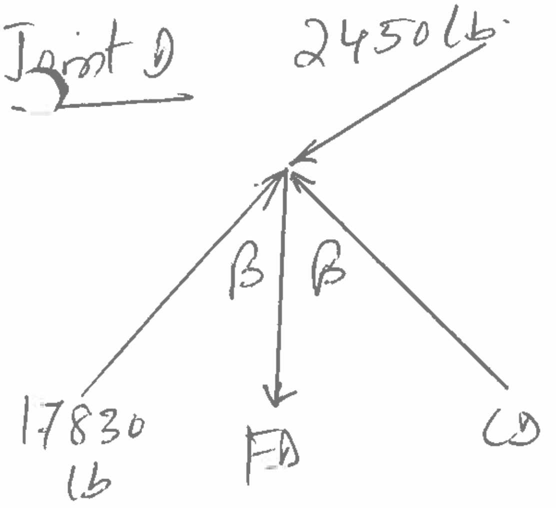
$\sum F_y = 0 : \quad 6240 - AE \sin 53.1° + AF \sin 36.9° = 0$

$\Rightarrow \quad AF = 13,380 \text{ lb } T$

$AE = 17830 \text{ lb } C$

From Joint E, ED = AE = 17,830 lb C

Joint D



$\Sigma F_x = 0$: 17830 sin 36.9° − CD sin 36

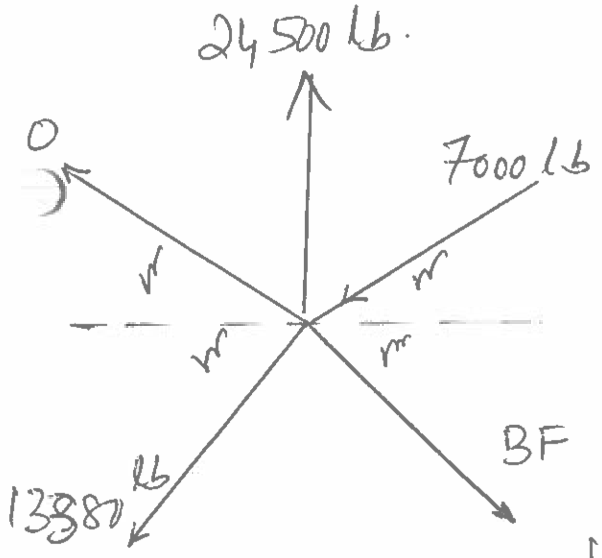
− 2450 cos 36.9° = 0

CD = 14,570 lb C

$\Sigma F_y = 0$: (14570 + 17830) cos 36

− FD − 2450 sin 36.9° = 0; FD = 24,500 lb

Joint F :



$\Sigma F_x = 0$: BF cos 36.9° − (13,380 + 7000) cos 36.9° = 0

BF = 20,400 lb T

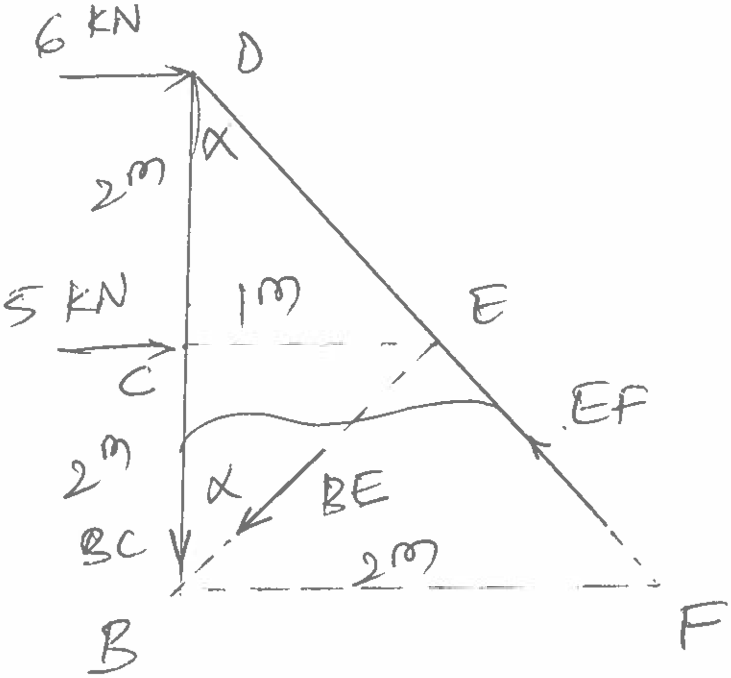
(Joint B Checks)

The maximum force occurs in member FD

FD = 24,500 lb T

HW # 13

Prob 4.30

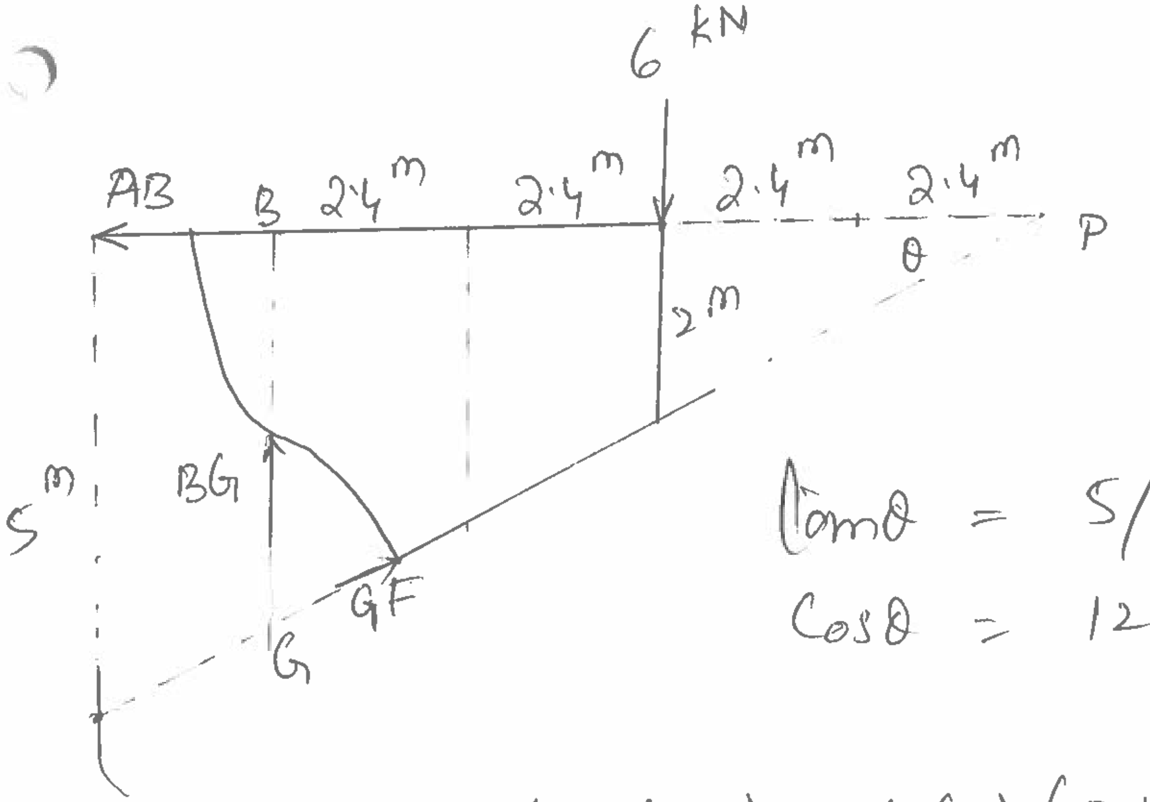


$$\alpha = \tan^{-1}\frac{1}{2} = 26.6^\circ$$

$$\circlearrowright + \Sigma M_D = 0 \; : \; 5(2) - BE \sin 26.6^\circ (4) = 0$$

$$BE = 5.59 \text{ kN } T$$

Prob # 4.32



$\tan\theta = 5/12$

$\cos\theta = 12/13.$

$\sum M_P = 0 ; \quad BG(4)(2.4) - 6(2)(2.4) = 0$

$\underline{BG = 3 \text{ kN C}}$

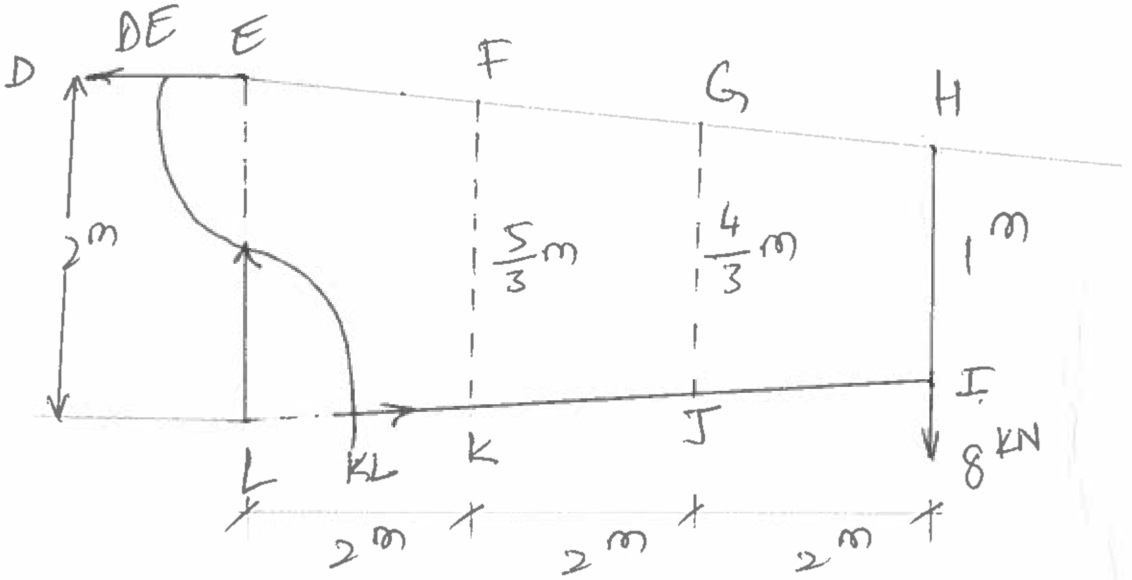
$\sum M_B = 0; \quad GF\left(\frac{12}{13}\right)(4) - 6(2)(2.4) = 0$

$\underline{GF = 7.8 \text{ kN C}}$

$\sum M_G = 0 ; \quad AB(4) - 6(2)(2.4) = 0$

$\underline{AB = 7.2 \text{ kN T}}$

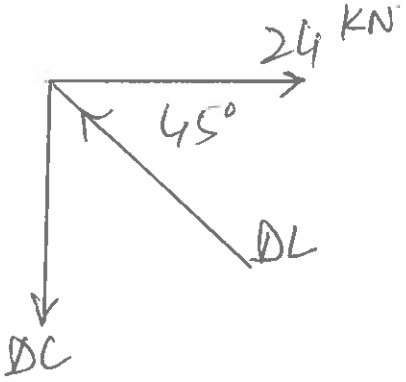
Prob #4.35



$\Sigma M_L = 0$: $DE(2) - 8(6) = 0$;

$DE = 24$ KN T

Joint D :

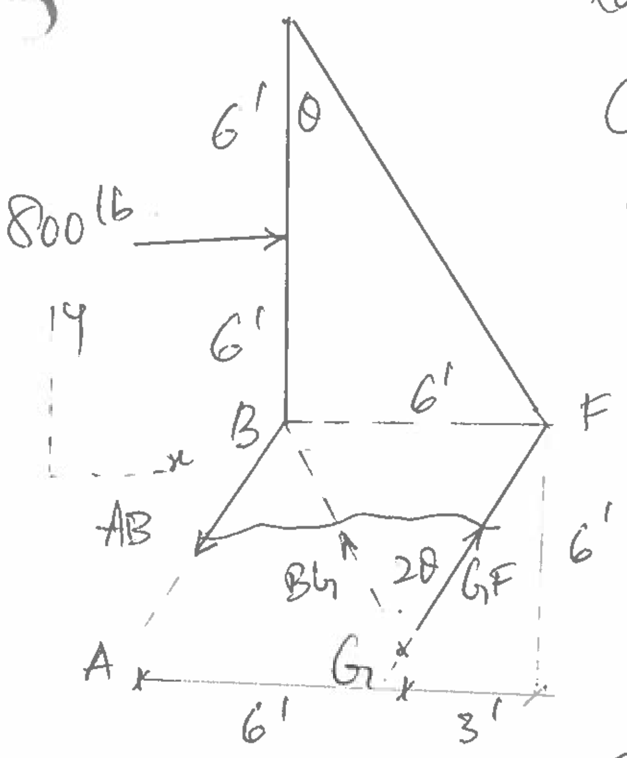


$\xrightarrow{+} \Sigma F_x = 0$:

$24 - DL \cos 45° = 0$

$DL = 33.9$ KN C

Prob # 4.37



$\tan\theta = \frac{1}{2}$

$\cos\theta = 2/\sqrt{5}$

$\sin\theta = 1/\sqrt{5}$

$\Sigma M_G = 0\ ;\ 800(12) - AB\left(\frac{2}{\sqrt{5}}\right)6$

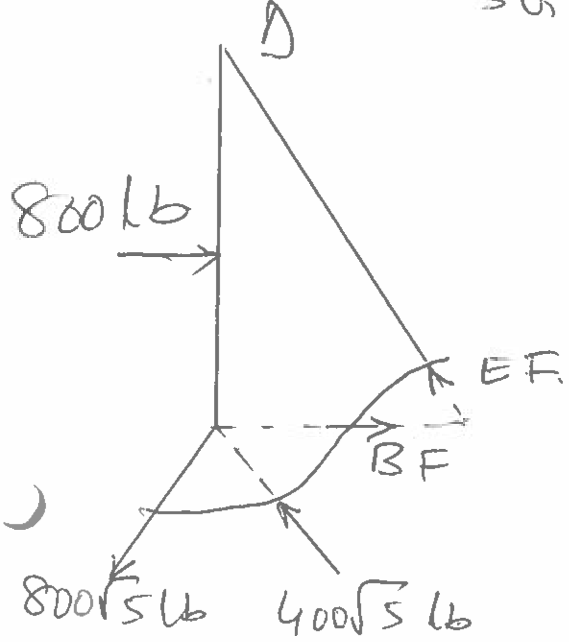
$AB = 800\sqrt{5}$ lb T

$\Sigma M_B = 0\ ;\ 800(6) - GF\left(\frac{2}{\sqrt{5}}\right)6 = 0$

$GF = 400\sqrt{5}$ lb C

$\Sigma F_y = 0\ ;\ BG\cos\theta + GF\cos\theta - AB\cos\theta = 0$

$BG = AB - GF = \underline{400\sqrt{5} \text{ lb C}}$



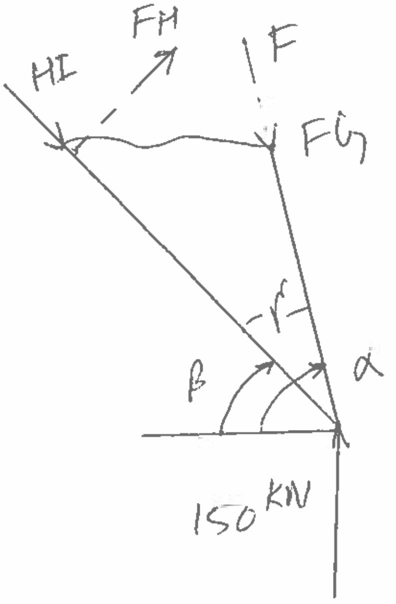
$\Sigma M_D = 0\ ;\ 800(6) + BF(12)$

$- 800\sqrt{5}\left(\frac{12}{\sqrt{5}}\right) - 400\sqrt{5}\left(\frac{12}{\sqrt{5}}\right) = 0$

$\underline{BF = 800 \text{ lb T}}$

Prob #6.46

By Symmetry $A = G = 150\text{ kN}$



$\curvearrowleft \Sigma M_F = 0$

$150(4) + HI(7.902) = 0$

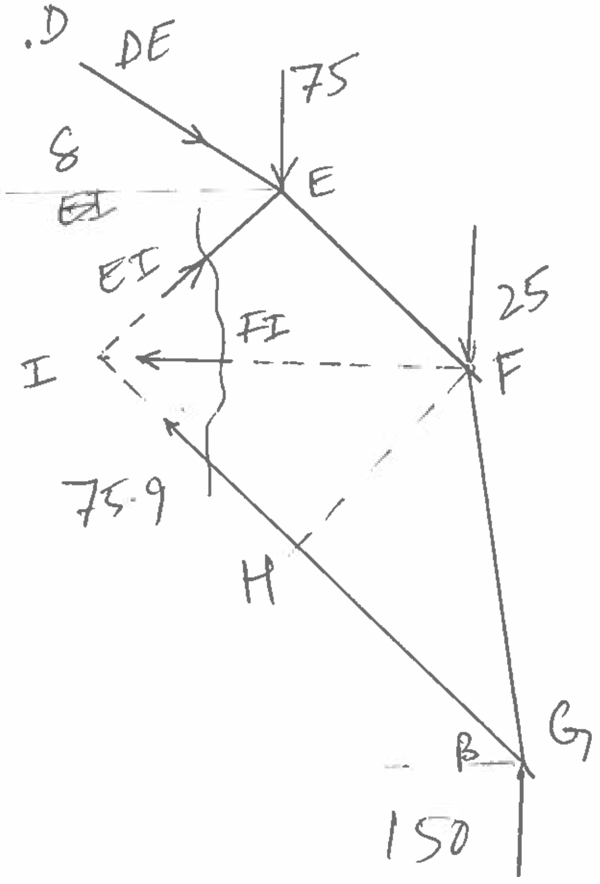
$\underline{HI = -75.9 \text{ kN T}}$

(Note: $\alpha = \tan^{-1}\left(\frac{14}{4}\right) = 74.1°$

$\beta = \tan^{-1}\left(\frac{14}{16}\right) = 41.2°$

$\gamma = \alpha - \beta = 32.9°$.

Then $d_\perp = FG \sin\gamma = \sqrt{14^2+4^2} \sin\gamma = 7.902\text{m})$.



$\delta = \tan^{-1}\left(\frac{3}{10}\right) = 16.7°$.

$$\circlearrowleft + \quad \Sigma M_E = 0 : -25(6) + 150(10)$$

$$- FI(4) - (75.9 \sin\beta)(6)$$

$$-(75.9 \cos\beta)(4) = 0$$

$$\underline{FI = 205 \text{ kN T}}$$

$$\Sigma M_I = 0 : -75(6) - 25(12) + 150(16)$$

$$-(DE \cos\delta)(4) - (DE \sin\delta)(6) = 0$$

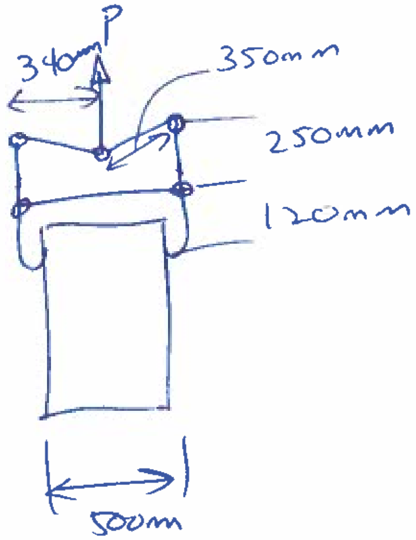
$$\underline{DE = 297 \text{ kN C}}$$

$$\Sigma F_y = 0 : -75 - 25 + 150 - 297 \sin\delta$$

$$+ 75.9 \sin\beta + EI \frac{4}{\sqrt{52}} = 0$$

$$\underline{EI = -26.4 \text{ kN T}}$$

4/95



JOINT G

$\Sigma F_y = 0 = P + GB\sin\theta + GA\sin\theta$

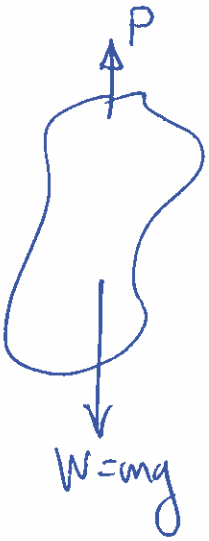
$\Sigma F_x = 0 = GA\cos\theta - GB\cos\theta$

$GA = GB$

$P = -2GB\sin\theta$

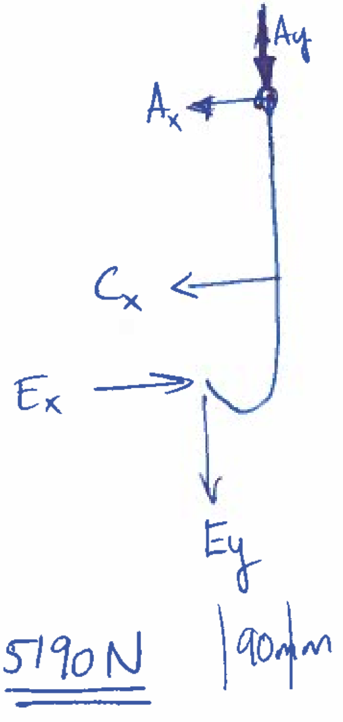
$\cos\theta = \frac{340}{350} = 0.97$

$\theta = 13.73^\circ$



$P = 135(9.8) = 1323N$

$GA = GB = -\frac{1323}{2\sin 13.73} = -2787N$



$A_y = -2787\sin 13.73 = -662\,N$

$A_x = -2787\cos 13.73 = -2710N$

$\Sigma F_y = 0 = -A_y - E_y$

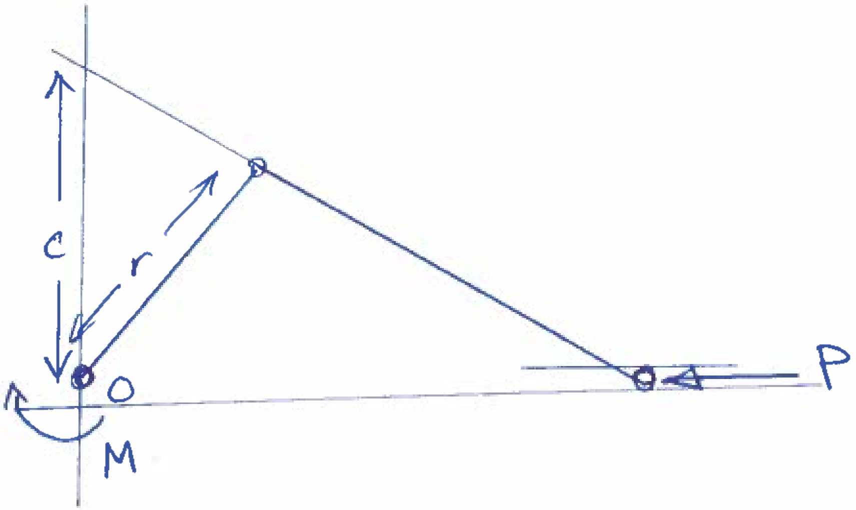
$E_y = -A_y = 662N$

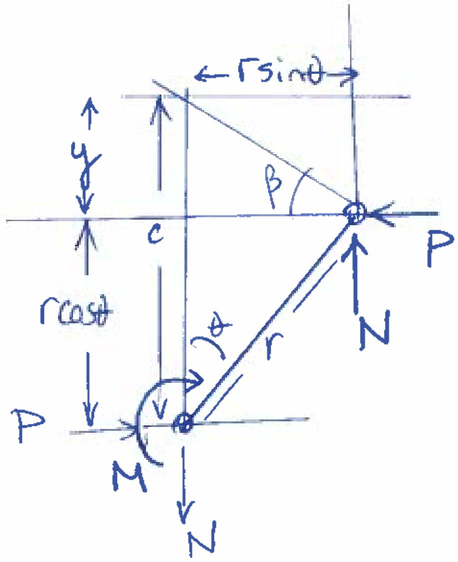
$\Sigma M_c = 0 = -2710(250)\underline{k} - 662(90)\underline{k} - E_x(120)$

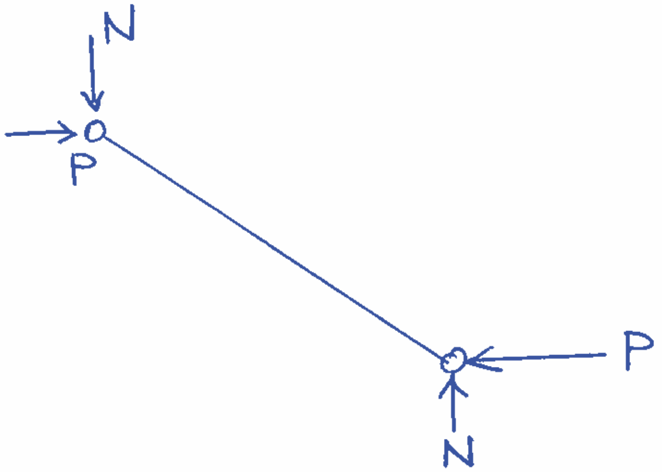
$E_x = 5150N$

$E = \sqrt{E_x^2 + E_y^2}$

$= \sqrt{5150^2 + 662^2} = \underline{\underline{5190N}}$







$$\Sigma M_O = r\cos\theta\, P\, \underline{k} + r\sin\theta\, N\, \underline{k} \quad - M\, \underline{k} = 0$$

$$M = Pr\cos\theta + Nr\sin\theta$$

But $\frac{N}{P} = \tan\beta$ $\therefore$ $N = P\tan\beta$

So $M = Pr\cos\theta + Pr\tan\beta\sin\theta$

Also $\tan\beta = \dfrac{y}{r\sin\theta}$

So $y = r\sin\theta\tan\beta$

$$c = r\cos\theta + y = r\cos\theta + r\sin\theta\tan\beta$$

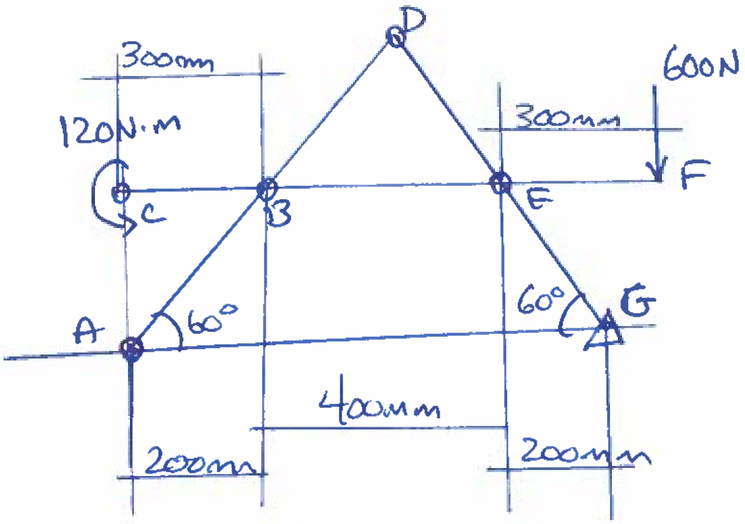
$\therefore$

$$M = P\underbrace{(r\cos\theta + r\sin\theta\tan\beta)}_{c}$$

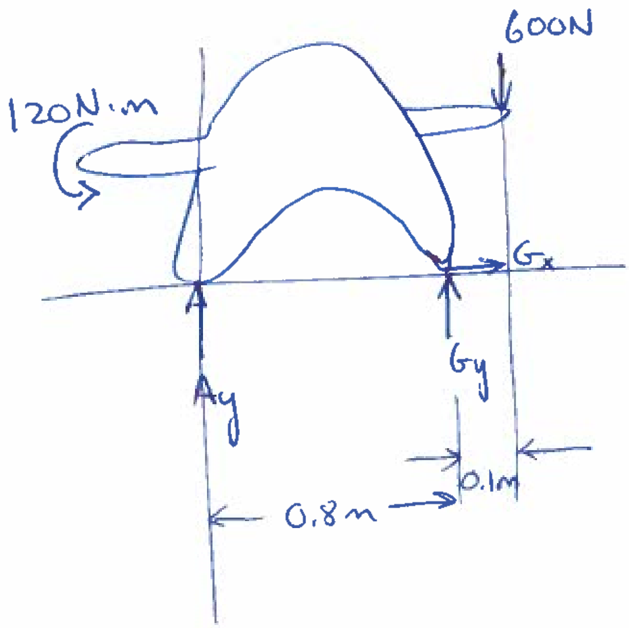
So

$$\underline{\underline{M = Pc}} \longleftarrow$$

4/108 FIND FORCES IN EACH MEMBER



ENTIRE STRUCTURE



$$\Sigma M_G = 0 = -0.8 A_y \underline{k} - 0.1(600)\underline{k} + 120$$

$$A_y = \frac{-60 + 120}{0.8} = 75N$$

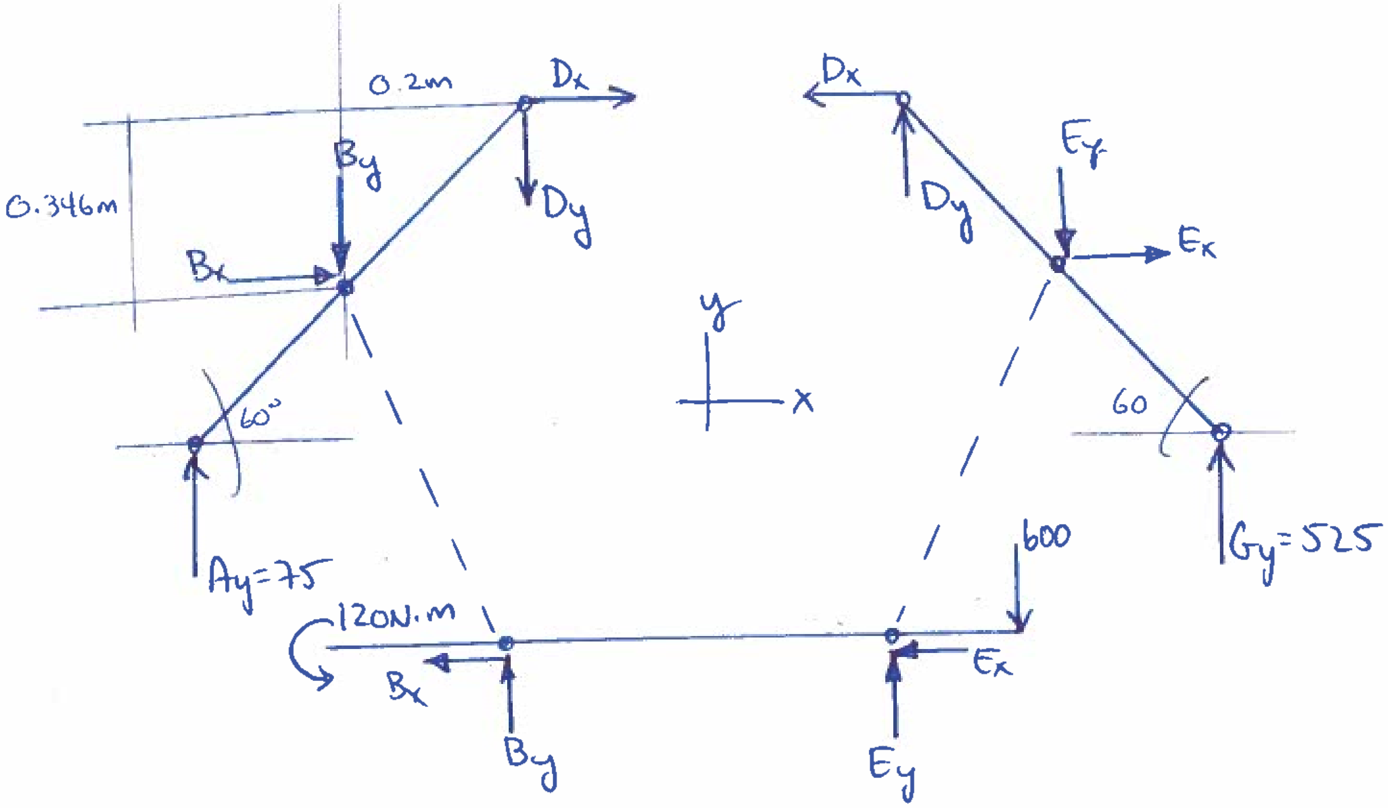
$$\Sigma F_y = 0 = G_y + A_y - 600$$

$$G_y = 600 - A_y$$
$$= 600 - 75 = 525N$$

$$\Sigma F_x = 0 = G_x$$

$$G_x = 0$$

②



CF

$$\Sigma \underline{M}_B = \underline{0} = 120\underline{k} + 0.4E_y\underline{k} - 0.7(600)\underline{k}$$

$$E_y = 750\text{N}$$

$$\Sigma F_y = 0 = E_y + B_y - 600$$

$$B_y = 600 - E_y = -\underline{\underline{150\text{N}}} \longleftarrow B_y$$

AD

$$\Sigma F_y = 0 = A_y - B_y - D_y$$

$$D_y = A_y - B_y = 75 - (-150) = \underline{\underline{225\text{N}}} \longleftarrow D_y$$

$\Sigma M_B = 0 = -0.346 D_x \underline{k} - 0.2 D_y \underline{k} - 0.2 A_y \underline{k}$

$\underline{\underline{D_x = -173.2\,N}}$ ← D_x

$\Sigma F_x = 0 = B_x + D_x$

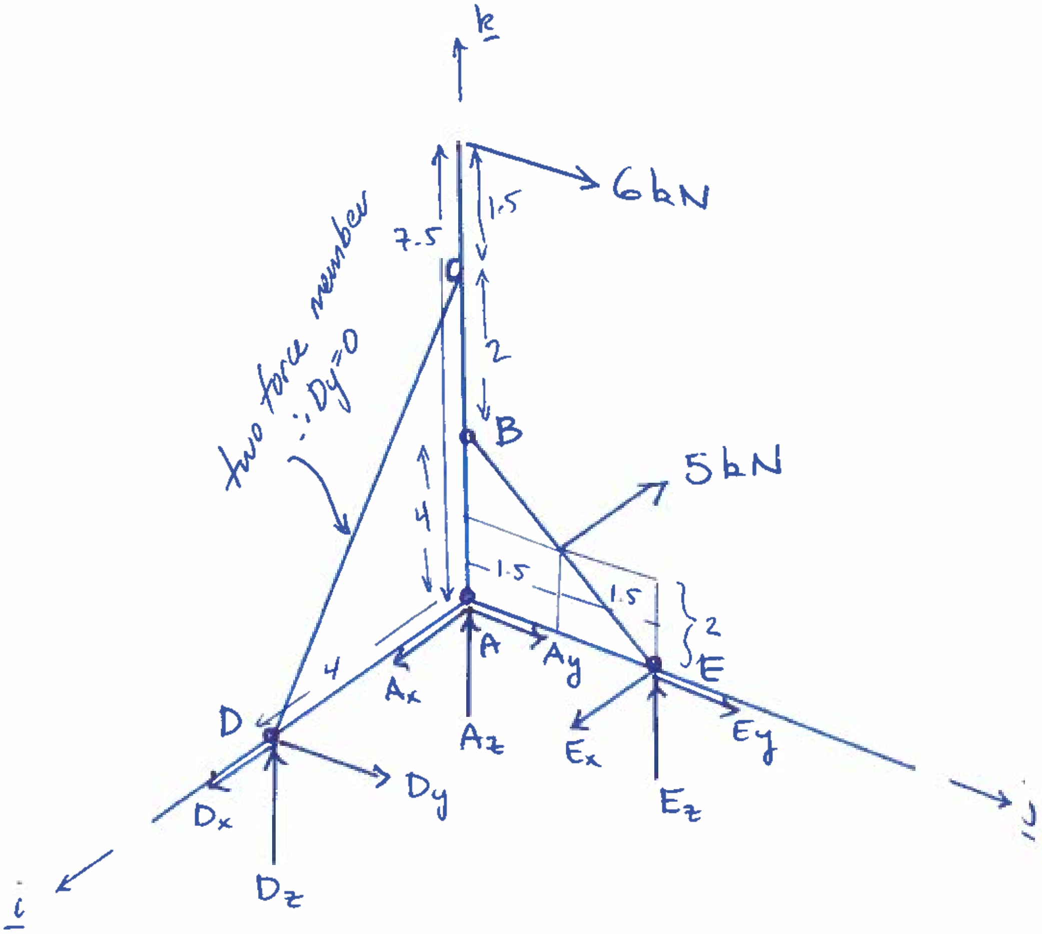
$B_x = -D_x = -(-173.2\,N) = \underline{\underline{173.2\,N}}$ ← B_x

<u>DG</u>

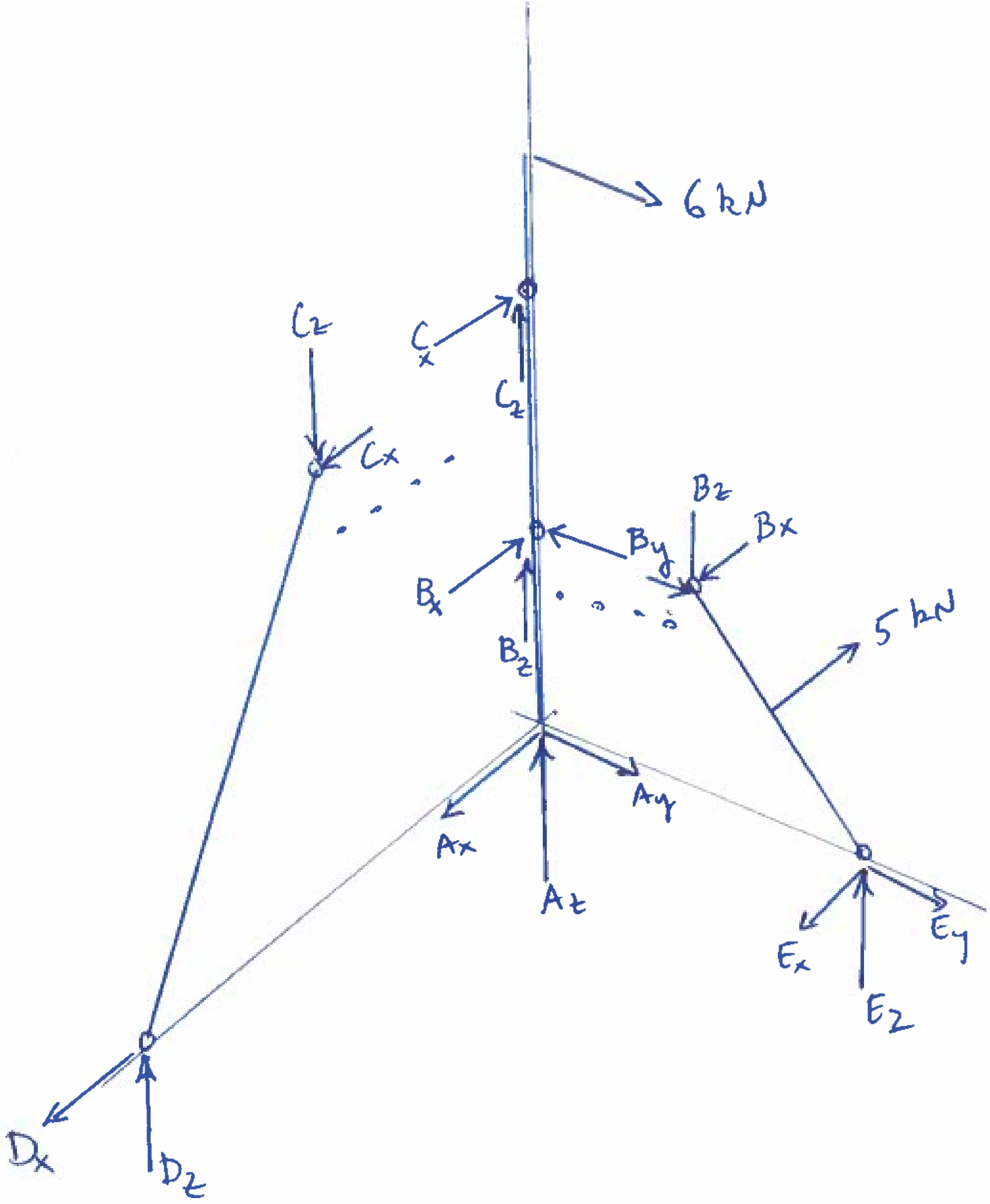
$\Sigma F_x = 0 = E_x - D_x$

$E_x = D_x = \underline{\underline{-173.2\,N}}$ ← $\underline{E_x}$

4.117 FIND REACTIONS AT $\underline{A}$



DIVIDE INTO 3 MEMBERS



③

MEMBER BE

$\Sigma M_E = 0 = (-1.5\underline{j} + 2\underline{k}) \times -5\underline{i}$

$+ (-3\underline{j} + 4\underline{k}) \times (B_x\underline{i} + B_y\underline{j} - B_z\underline{k})$

$= \cancel{-7.5\underline{k}} - \cancel{10\underline{j}} + \cancel{3B_x\underline{k}} + \cancel{4B_x\underline{j}}$

$- \cancel{4B_y\underline{i}} + \cancel{3B_z\underline{i}} = 0$

$\Sigma M_{E_x} = 3B_z - 4B_y = 0 \Rightarrow B_y = \frac{3}{4}B_z$

$\Sigma M_{E_y} = 4B_x - 10 = 0$

$\Sigma M_{E_z} = 3B_x - 7.5 = 0 \Rightarrow B_x = 2.5\,kN$

$\Sigma F_z = 0 = E_z - B_z \Rightarrow E_z = B_z$

$\Sigma F_x = 0 = E_x + B_x - 5 = 0 \Rightarrow E_x = 2.5\,kN$

$\Sigma F_y = 0 = B_y + E_y \Rightarrow B_y = E_y = \frac{3}{4}B_z$

MEMBER ABC

$\Sigma M_A = 0 = (4\underline{k} \times -B_x\underline{i}) + (4\underline{k} \times -B_y\underline{j}) + (6\underline{k} \times -C_x\underline{i})$

$+ (7.5\underline{k} \times 6\underline{j}) = 0$

$= -4B_x\underline{j} + \cancel{4B_y\underline{i}} - 6C_x\underline{j} - \cancel{45\underline{i}} = 0$

$4B_y = 45$

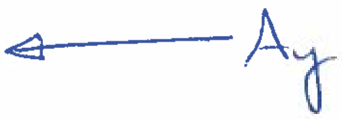
$B_y = 11.25\,kN$

$-4B_x = 6C_x$

$C_x = -\frac{4}{6}B_x$

$\Sigma F_y = A_y - B_y + 6$

$A_y = B_y - 6 = 11.25 - 6 = \underline{\underline{5.25\,kN}}$



$$C_x = -\frac{4}{6}(2.5) = -1.667\,kN$$

$$\Sigma F_x = A_x + B_x - C_x = 0$$

$$A_x = B_x + C_x = 2.5 - 1.667 = \underline{\underline{0.833\,kN}} \leftarrow A_x$$

$$\Sigma F_z = A_z + B_z + C_z = 0$$

MEMBER CD

$$\Sigma \underline{M}_D = 0 = -4\underline{i} \times -C_z\underline{k} = -4C_z\,\dot{j}$$

$$+ \; 6\underline{k} \times C_x\underline{i} = 6C_x\,\dot{j}$$

$$4C_z = 6C_x$$

$$C_z = \frac{6}{4}C_x = \frac{6}{4}(-1.667) = -2.5\,kN$$

RETURN TO ABC

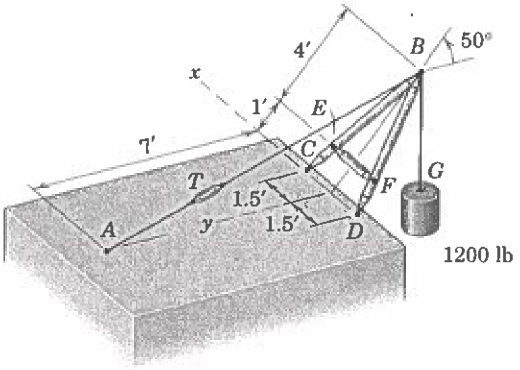
$$\Sigma F_z = A_z + B_z + C_z$$

$$A_z = -B_z - C_z = -\left(\frac{4}{3}B_y\right) - C_z$$

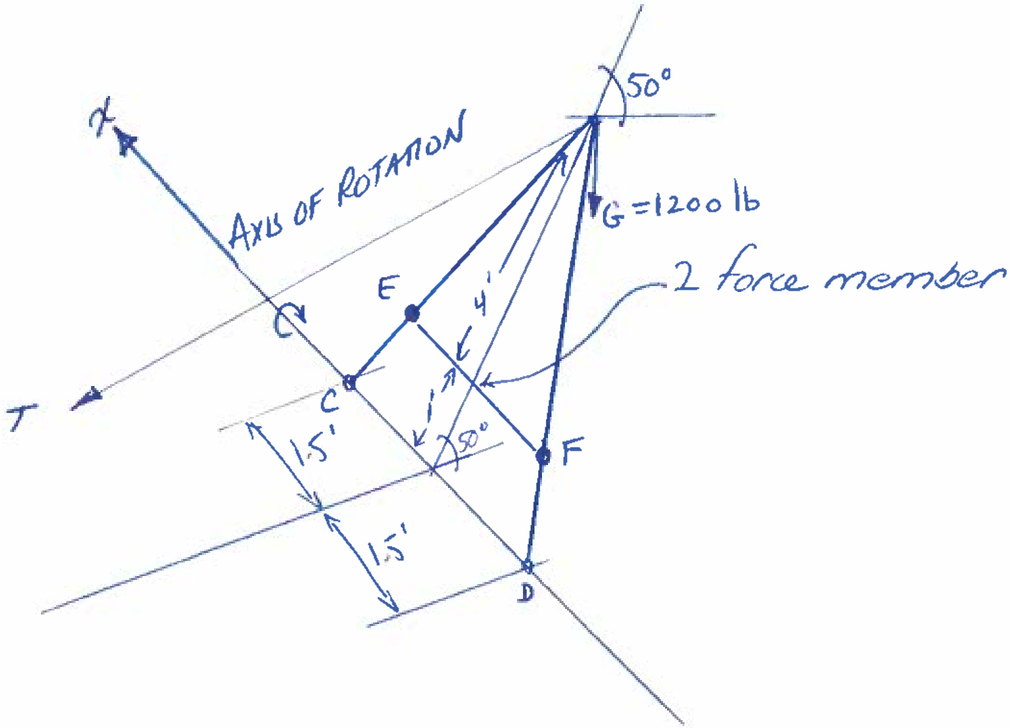
$$= -\left(\frac{4}{3}\right)(11.25) - (-2.5) = \underline{\underline{-12.5\,kN}} \; \overset{\leftarrow A_z}{}$$

①

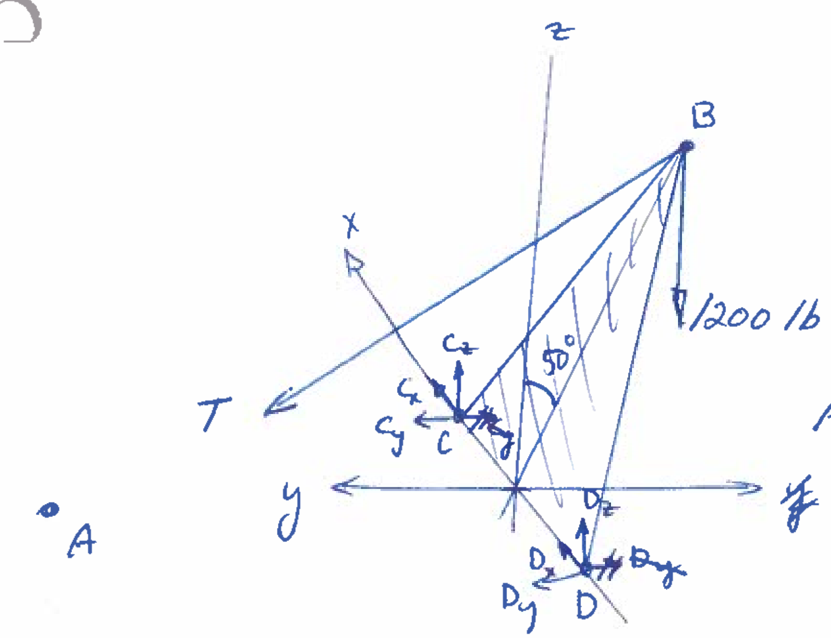
4/118 In the schematic representation of an actual structure, T represents a turnbuckle, C and D are non-thrust-bearing hinges whose axes are along the line CD, and B, E, and F are ball-and-socket joints. Determine the tension T in the turnbuckle and the force in member EF.



Problem 4/118



REACTIONS IN ENTIRE FRAME



LOCATE $\underline{B}$ COORDINATES

$$B = (0, -5\cos 50, 5\sin 50)$$

$$= 0, -3.214, 3.831$$

LOCATE $\underline{A}$ COORDINATES

$$A = (0, +7, 0)$$

$$\underline{T} = T\left(\frac{0\underline{i} + (7+3.214)\underline{j} + (0-3.831)\underline{k}}{\sqrt{10.214^2 + 3.831^2}} \right)$$

$$= T\left(0\underline{i} + \frac{10.214}{10.90}\underline{j} - \frac{3.831}{10.90}\underline{k}\right)$$

$$\Sigma \underline{M}_o = 0 = 1.5\underline{i} \times C_y\underline{j} + 1.5\underline{i} \times C_z\underline{k} - 1.5\underline{i} \times D_y\underline{j} - 1.5\underline{i} \times D_z\underline{k}$$

$$+ (-5\cos 50\underline{j}) \times (-1200\underline{k})$$

$$+ (-5\cos 50\underline{j} + 5\sin 50\underline{k}) \times (T)(0.936\underline{j} - 0.351\underline{k})$$

$$0 = 1.5C_y\underline{k} + 1.5C_z(-\underline{j}) - 1.5D_y\underline{k} - 1.5D_z(-\underline{j})$$

$$\underbrace{+ (3.214)(1200)\underline{i} + T(-3.214)(-0.351)\underline{i} + T(3.831)(0.936)(-}_{\text{consider only } i\text{-component}}$$

$0 = 3856.8 + T(1.128) - T(3.585)$

Solve for T

$$T(3.585 - 1.128) = 3856.8$$

$$T = \frac{3856.8}{2.457} = 1569.2 \text{ lbs}$$

Now find other reactions

$$\Sigma F_z = 0 = -0.351(1569.2) - 1200 + C_z + D_z = 0$$

$$\Sigma M_y = 0 = -1.5 C_z + 1.5 D_z = 0$$

$$\therefore C_z = D_z$$

so $1750.7 \text{ lbs} = 2C_z$

$$C_z = 875.4$$

$$D_z = 875.4$$

$$\Sigma F_y = 0 = C_y + D_y + (1569.2)(0.936)$$

$$\Sigma M_z = 0 = 1.5 C_y - 1.5 D_y = 0$$

$$\therefore C_y = D_y$$

$$C_y = D_y = -734.38 \text{ lbs}$$

9/19

ANALYZE SECTION OF FRAME

$C_y = -734.4 \text{ lbs } \underline{j}$

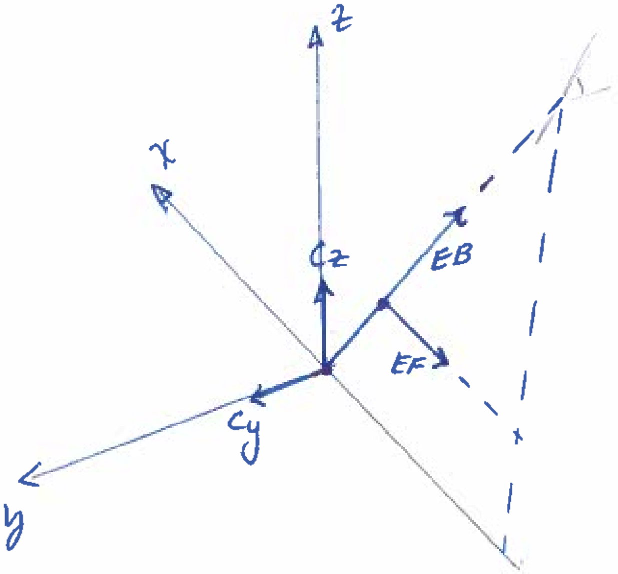
$C_z = 875.4 \text{ lbs } \underline{k}$

$EF = EF(-\underline{i})$

$$EB = EB\left(\frac{-1.5\underline{i} + (-3.214\underline{j}) + (3.831\underline{k})}{\sqrt{1.5^2 + 3.214^2 + 3.831^2}}\right) = \cancel{EB(0.287\underline{i}}$$

$\boxed{5.22}$

$$= EB(-0.287\underline{i} - 0.616\underline{j} + 0.734\underline{k})$$



$\Sigma F_x = 0 = -EF - 0.287EB$

$\Sigma F_y = 0 = -734.4 \text{lbs} - 0.616EB \Rightarrow EB = -1192.2$

$\Sigma F_z = 0 = 875.4 \text{lbs} + 0.734EB \Rightarrow EB = -1192.1$

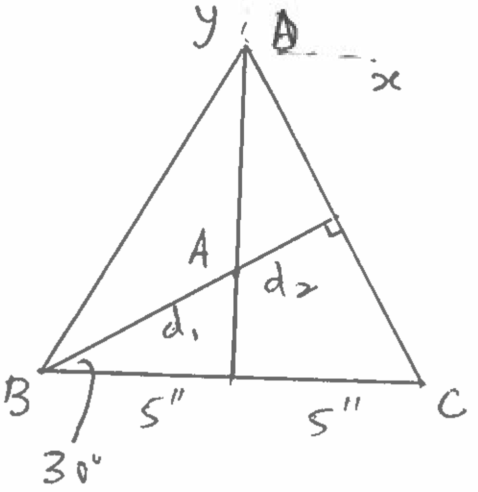
$\therefore \quad EF = -0.287(-1192.2) = 342.1 \text{ lbs}$

CIVE 2330

HW # 14

Prob # 4.53

Top View of base :



$\cos 30^\circ = \dfrac{d_1 + d_2}{10}$, $d_1 + d_2 = 8.66'$

$\cos 30^\circ = \dfrac{5}{d_1}$, $d_1 = 5.77''$

$d_2 = 8.66 - 5.77 = 2.89''$

For Joint A, assuming Symmetry:

$$\underline{F}_{BA} = P\left[\frac{5\underline{i} + 2.89\underline{j} + 16\underline{k}}{(5^2 + 2.89^2 + 16^2)^{1/2}}\right] = P(0.294\,\underline{i} + 0.17\underline{j} + 0.941\,\underline{k})$$

$$\underline{F}_{CA} = P(-0.294\,\underline{i} + 0.170\,\underline{j} + 0.941\,\underline{k})\ \underline{F}_{DA} = P(-0.339\,j + 0.941\,\underline{k})$$

$\Sigma F_z = 0$ at A: $3(0.941P) - 1000 = 0$

$P = 354$ lb.

For joint C, assuming symmetry:

$\underline{F}_{BC} = -Q\underline{i}$, $\underline{F}_{CD} = Q(-\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j})$

Normal $\underline{N} = 333\,\underline{k}$

$\Sigma \underline{F} = 0$ at C: $\underline{N} + \underline{F}_{BC} + \underline{F}_{CD} + \underline{F}_{AC} = \underline{0}$

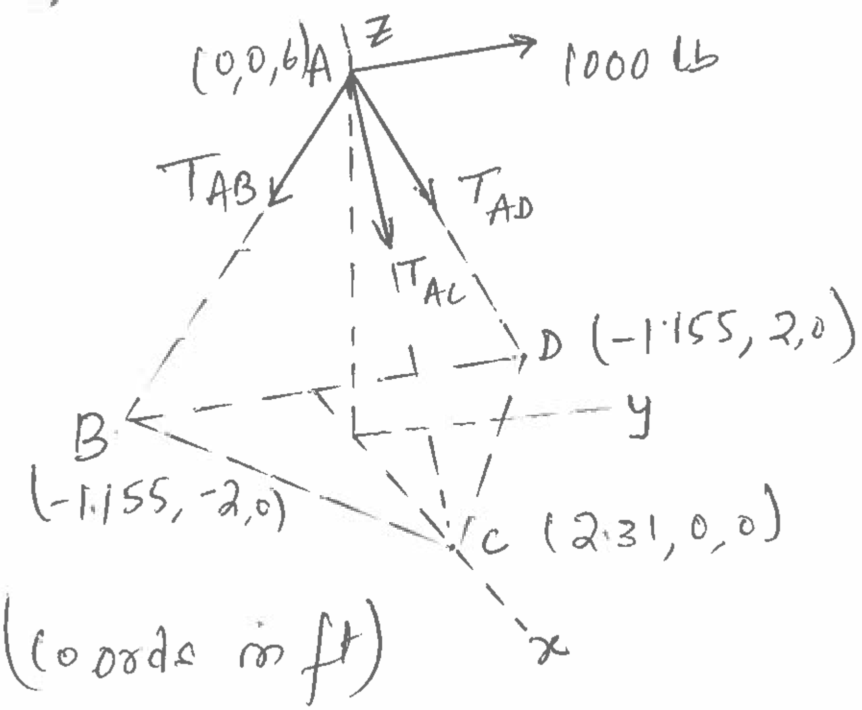
$333\,\underline{k} - Q\underline{i} + 354(0.294\,\underline{i} - 0.170\,\underline{j} + 0.941\,\underline{k})$

$+ Q(-0.5\,\underline{i} + 0.866\,\underline{j}) = \underline{0}$

Solving, $Q = 69.4$ lb

$\Rightarrow$ BC = BD = CD = 69.4 lb T

Prob 4.54



$$\underline{T}_{AB} = T_{AB}\,\frac{-1.155\,\underline{i} - 2\underline{j} - 6\underline{k}}{\sqrt{1.155^2 + 2^2 + 6^2}}$$

$$= T_{AB}\left[-0.1796\,\underline{i} - 0.311\,\underline{j} - 0.933\,\underline{k}\right]$$

$$\underline{T}_{AC} = T_{AC}\,\frac{2.31\,\underline{i} - 6\,\underline{k}}{\sqrt{2.31^2 + 6^2}}$$

$$= T_{AC}\left[0.359\,\underline{i} - 0.933\,\underline{k}\right]$$

$$\underline{T}_{AD} = T_{AD} \frac{-1.155\underline{i} + 2\underline{j} - 6\underline{k}}{\sqrt{1.155^2 + 2^2 + 6^2}}$$

$$= T_{AD}\left[-0.1796\underline{i} + 0.311\underline{j} - 0.933\underline{k}\right]$$

$$\Sigma F_x = 0: \quad -0.1796\, T_{AB} + 0.359\, T_{AC} - 0.1796\, T_{AD} = 0 \quad (1$$

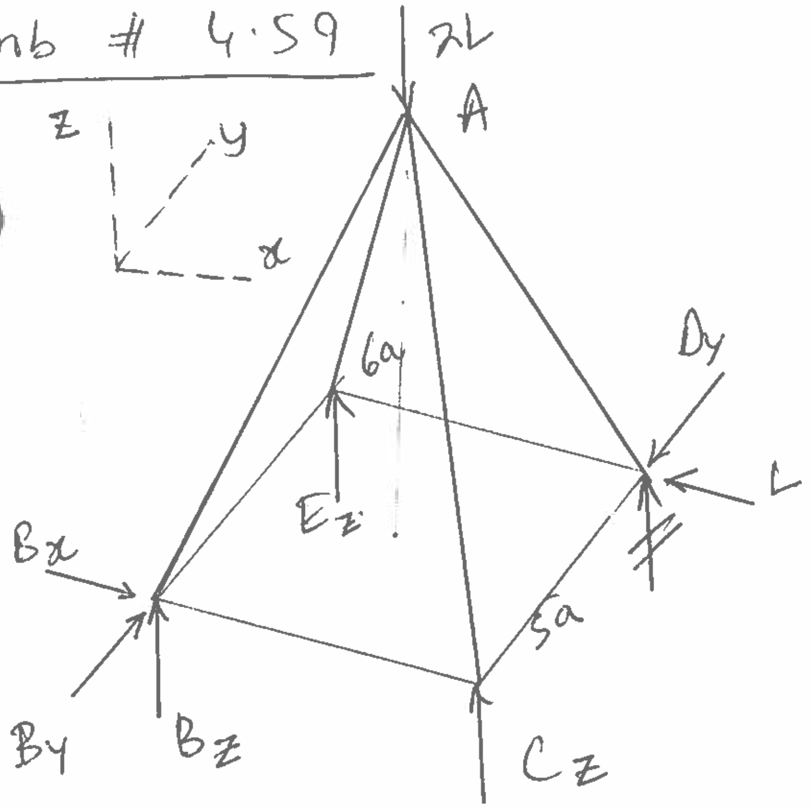
$$\Sigma F_y = 0: \quad 1000 - 0.311\, T_{AB} + 0.311\, T_{AD} = 0 \quad (2)$$

$$\Sigma F_z = 0: \quad -0.933\, T_{AB} - 0.933\, T_{AC} - 0.933\, T_{AC} = 0($$

Solve Eqs (1)- (3)

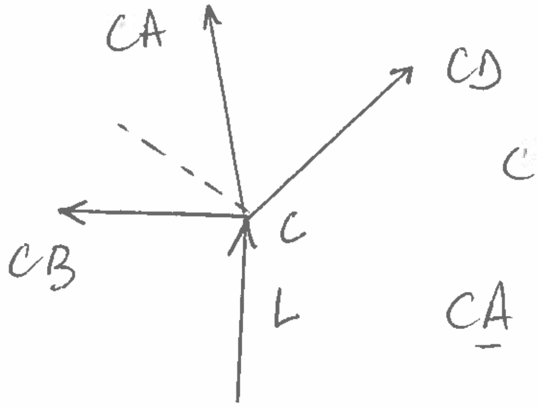
$$\begin{cases} T_{AB} = 1607 \text{ lb (T)} \\ T_{AC} = 0 \\ T_{AD} = -1607 \text{ lb (C)} \end{cases}$$

Prob # 4.59



$$\left\{ \begin{array}{l} \Sigma M_{BE} = 0 \Rightarrow C_z = L \\ \Sigma M_{Bz} = 0 \Rightarrow D_y = L \end{array} \right.$$

Joint c: (tensions assumed).



$\underline{CB} = -CB\,\underline{i}$, $\underline{CD} = CD\,\underline{j}$, $\underline{L} = L\,\underline{k}$

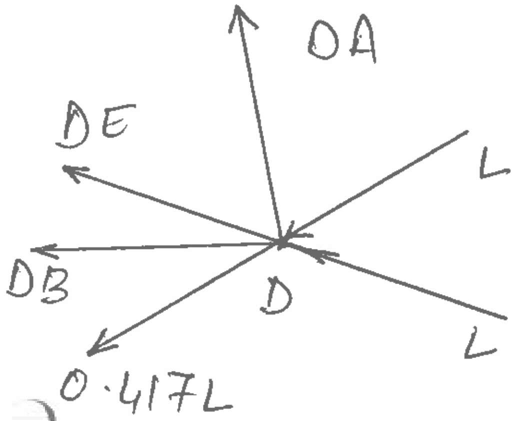
$$\underline{CA} = CA\left(\frac{-2.5a\,\underline{i} + 2.5a\,\underline{j} + 6a\,\underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}}\right)$$

$$= CA(-0.359\,\underline{i} + 0.359\,\underline{j} + 0.862\,\underline{k})$$

$\sum \underline{F} = \underline{0}$ yields :

$$\left.\begin{aligned} \underline{i}&: -CB - 0.359\,CA = 0 \\ \underline{j}&: CD + 0.359\,CA = 0 \\ \underline{k}&: L + 0.862\,CA = 0 \end{aligned}\right\} \begin{aligned} CA &= 0.1667\,L \\ CD &= +0.417\,L \end{aligned}$$

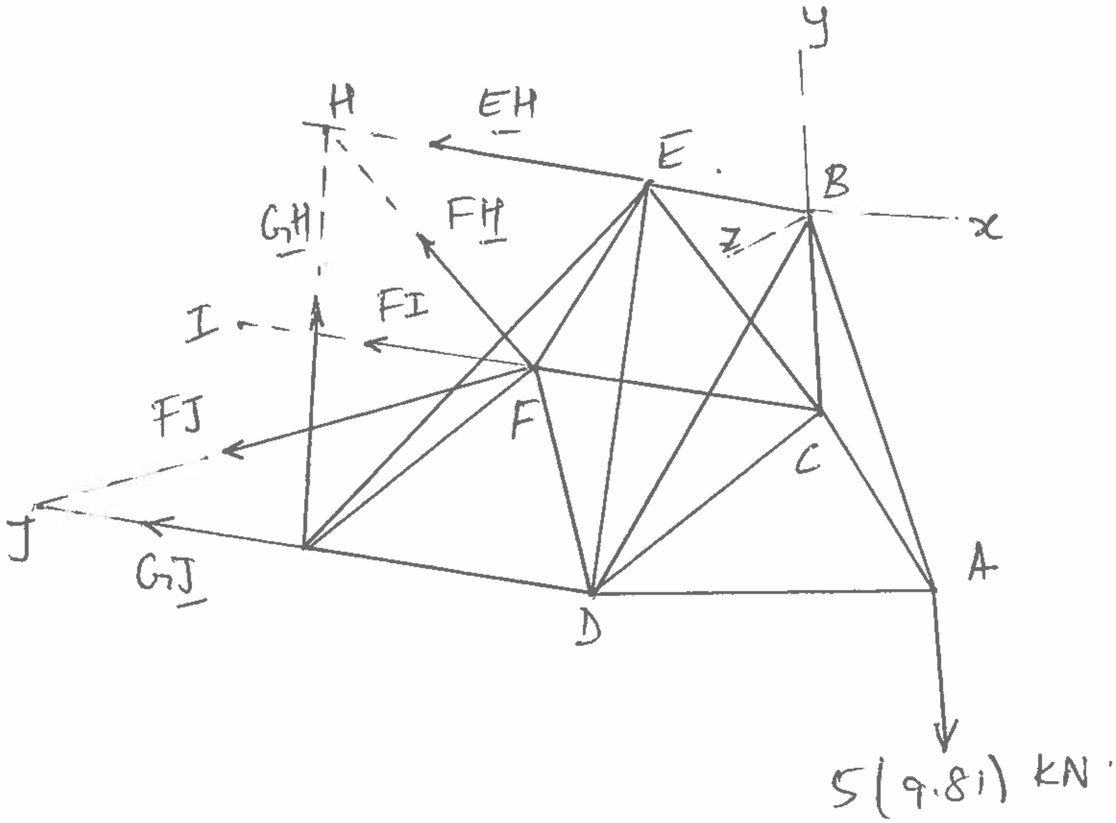
<u>Joint D :</u> $\sum \underline{F} = \underline{0}$ yields.



$$\left\{\begin{aligned} \underline{i}&: -DE - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}&: -0.417L - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}&: 0.862\,DA = 0 \end{aligned}\right.$$

$DA = 0$, $\underline{DB = -2.00L}$,

Prob 4.63



All members assumed in tension
Note that six members are out!

$\underline{GJ} = -GJ\,\underline{i}$; $\underline{FI} = -FI\,\underline{i}$; $\underline{FJ} = \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k})$

$$\sum \underline{M}_H = 0: \; -49.05(5)\underline{k} + (-2\cos 30^\circ \underline{j} + 2\sin 30^\circ \underline{k}) \times (-GJ)\underline{i} + (-2\cos 30^\circ \underline{j} - 2\sin 30^\circ \underline{k}) \times (-FI)\underline{i} + (\underline{j} - 2\cos 30^\circ \underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k}) = \underline{0}$$

∴ Equating Unit Vector Coefficients to zero

$$-1.225\ FJ = 0 \Rightarrow FJ = 0$$

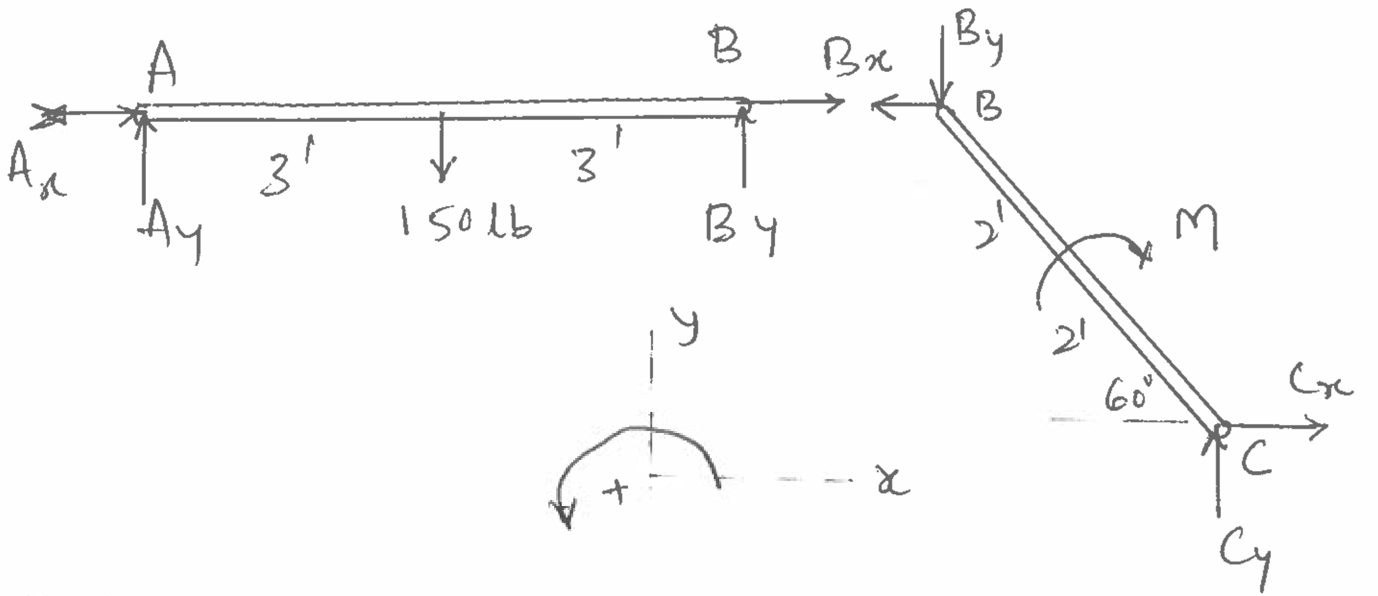
$$\left.\begin{array}{l} -GJ + FI = 0 \\ -1.732\ GJ - 1.732\ FI = 245 \end{array}\right\}$$

$$FI = GJ = -70.8\,kN$$

∴ Force in GJ = 70.8 kN C

HW # 15

Prob # 4.67



(AB)

For $A_x = 0$, $B_x = 0$; $A_y = B_y = 75$ lb by inspection

(BC) $\Sigma M_c = 0 : B_y(4\cos 60°) - M = 0$, $M = 150$ lb-ft.

Now apply $M = 150$ lb-ft CCW, $A_x, B_x \neq 0$.

but $A_y = B_y = 75$ lb.

(BC) $\Sigma M_c = 0 : B_y(4\cos 60°) + B_x(4\sin 60°) + 150 = 0$

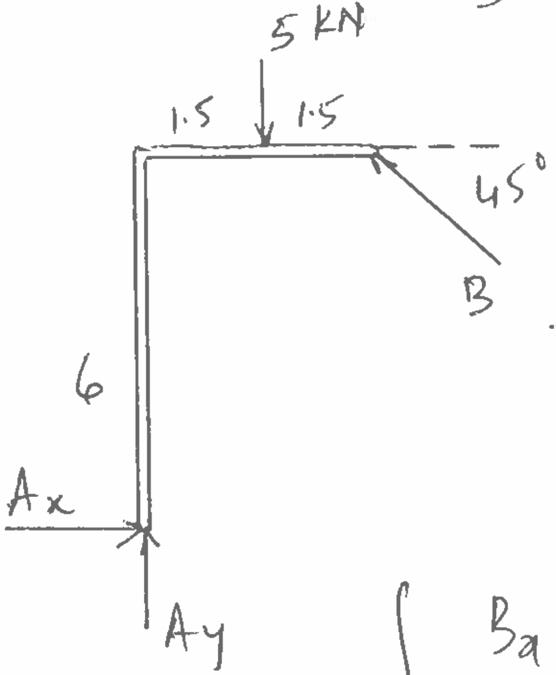
$\Rightarrow B_y = -86.6$ lb

(AB) $\Sigma F_x = 0 : A_x + B_x = 0$, $A_x = -B_x = -(-86.6$

or $A_x = 86.6$ lb.

Prob # 4.74

BC is a two-force member.



$\circlearrowleft + \Sigma M_A = 0 : -1.5(5) + B\frac{\sqrt{2}}{2}(3) + B\frac{\sqrt{2}}{2}(6) = 0$

$B = 1.179$ kN.

$$\left\{ \begin{array}{l} B_x = -0.833 \text{ kN} \\ B_y = 0.833 \text{ kN} \end{array} \right\}.$$

$\Sigma F_x = 0 : A_x - 1.179\left(\frac{\sqrt{2}}{2}\right) = 0 ; A_x = 0.833$ kN

$\Sigma F_y = 0 : A_y - 5 + 1.179\frac{\sqrt{2}}{2} = 0$

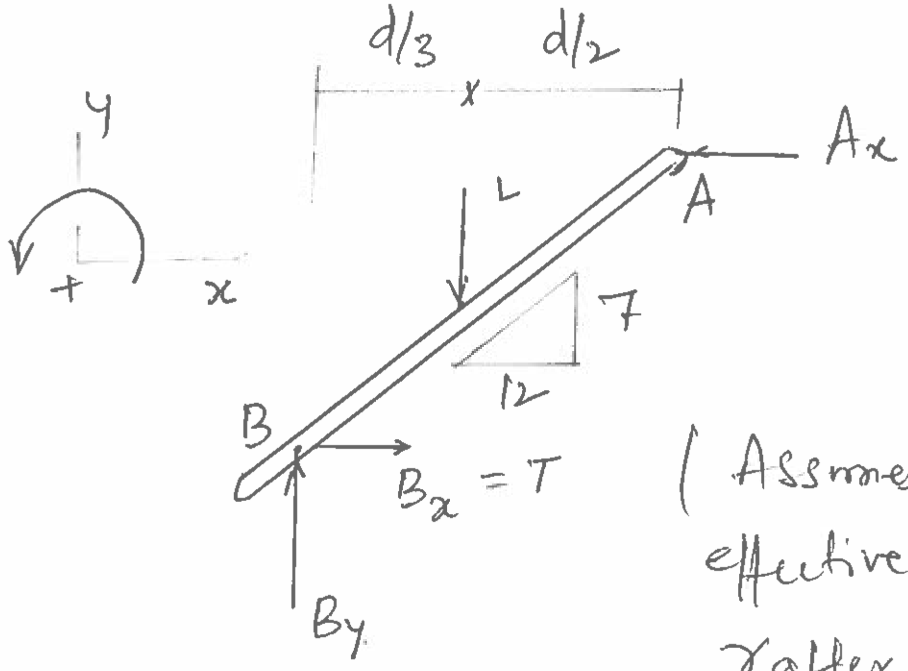
$A_y = 4.16$ kN.

On member BC

at B : $\left\{ \begin{array}{l} x : 0.833 \text{ kN} \\ y : -0.833 \text{ kN} \end{array} \right.$

at C : $\left\{ \begin{array}{l} x : -0.833 \text{ kN} \\ y : 0.833 \text{ kN} \end{array} \right.$

Prob # 4.77



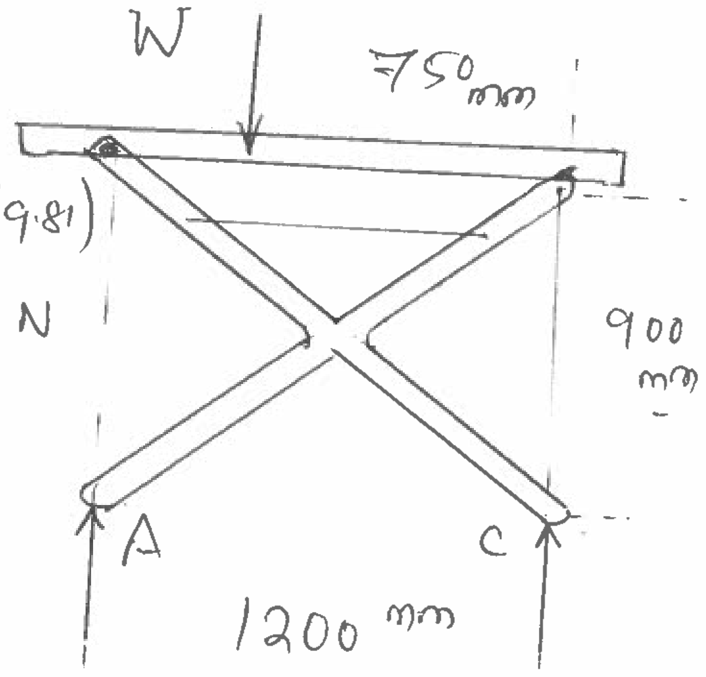
(Assumes Cable tension effectively transferred to rafter in proper fastener

$$\Sigma F_x = 0 \Rightarrow T = A_x$$

$$\Sigma M_B = 0: \quad -L\left(\frac{d}{3}\right) + T\left(\frac{5d}{6} \cdot \frac{7}{12}\right) = 0$$

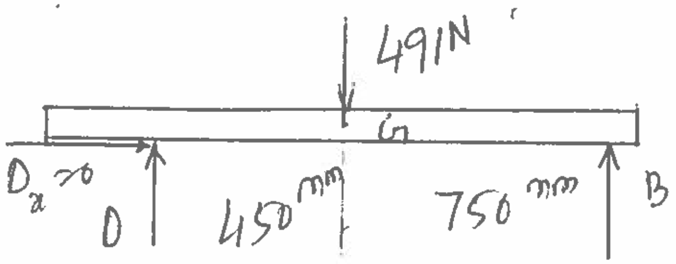
$$T = \frac{24}{35} L$$

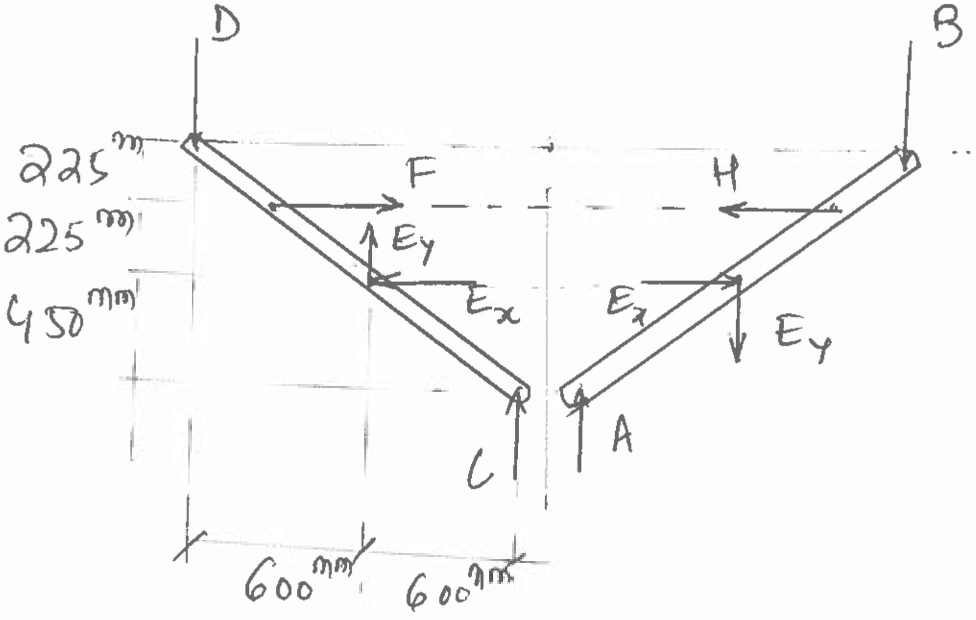
Prob # 4.94



$W = 50(9.81)$

$= 491\,N$





(Top) $\circlearrowleft + \sum M_B = 0$:

$491(750) - 1200D = 0$; $D = 307$ N

$+\uparrow \sum F_y = 0$: $B + 307 - 491 = 0$; $B = 183.9$ N

Same for entire frame: $\begin{cases} A = D = 307 \text{ N} \\ C = B = 183.9 \text{ N.} \end{cases}$

(DEC) $\sum F_y = 0$: $183.9 + E_y - 307 = 0$

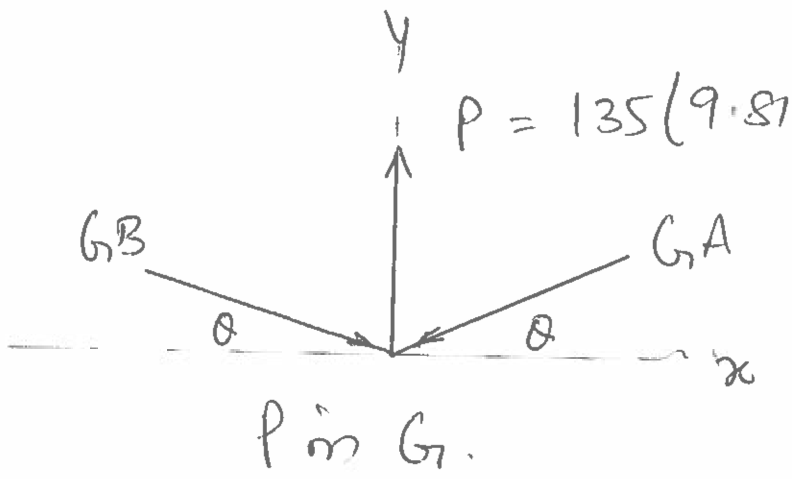
$\underline{E_y = 122.6 \text{ N}}$

$\sum M_E = 0$: $225F - 307(600) - 184(600) = 0$

$F = 1308$ N

$\sum F_x = 0$: $\underline{E_x = F = 1308 \text{N}}$

Prob # 4.95

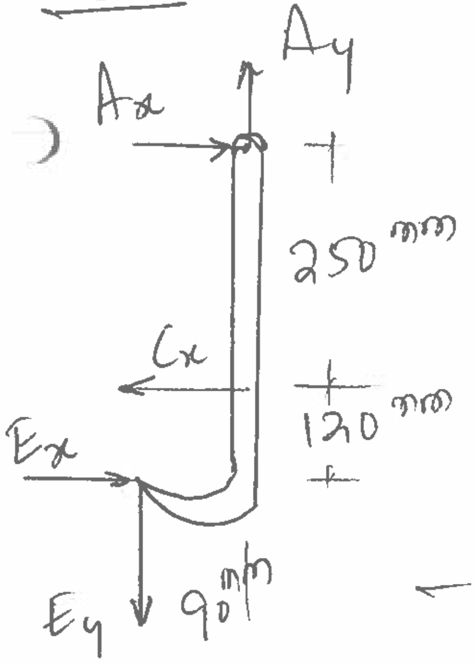


$\theta = \cos^{-1}\frac{340}{350} = 13.73^\circ$

$\Sigma F_y = 0 : 135(9.81) - 2GA \sin 13.73^\circ = 0$

$GA = GB = 2790$ N.

ACE:



$A_x = 2790 \cos 13.73^\circ = 2710$ N

$A_y = 2790 \sin 13.73^\circ = 662$ N

$\Sigma F_y = 0 \Rightarrow E_y = 662$ N

$\Sigma M_C = 0 : 2710(250) - 662(90)$

$- E_x(120) = 0,$ $E_x = 5150$ N.

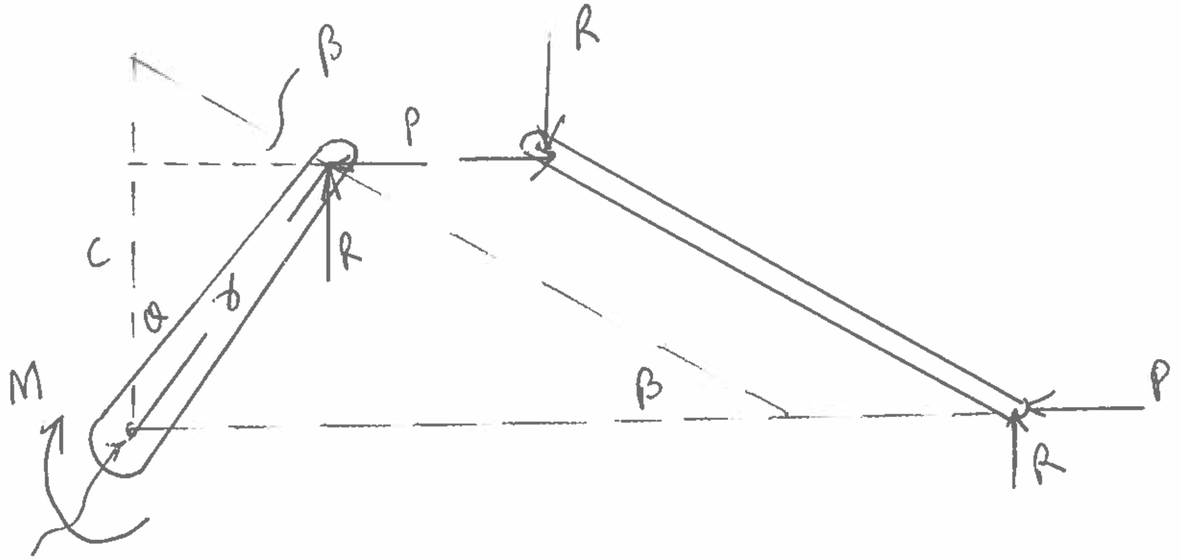
$E_x = \sqrt{E_x^2 + E_y^2} = 5190$ N

or 5.19 kN

CIVE 2330

HW #16

Prob # 4.104



For the two force member, $R = P \tan\beta$

Lever: $\overset{\curvearrowright}{+} \sum M_0 = 0 : M - Rr\sin\theta - Pr\cos\theta = 0$

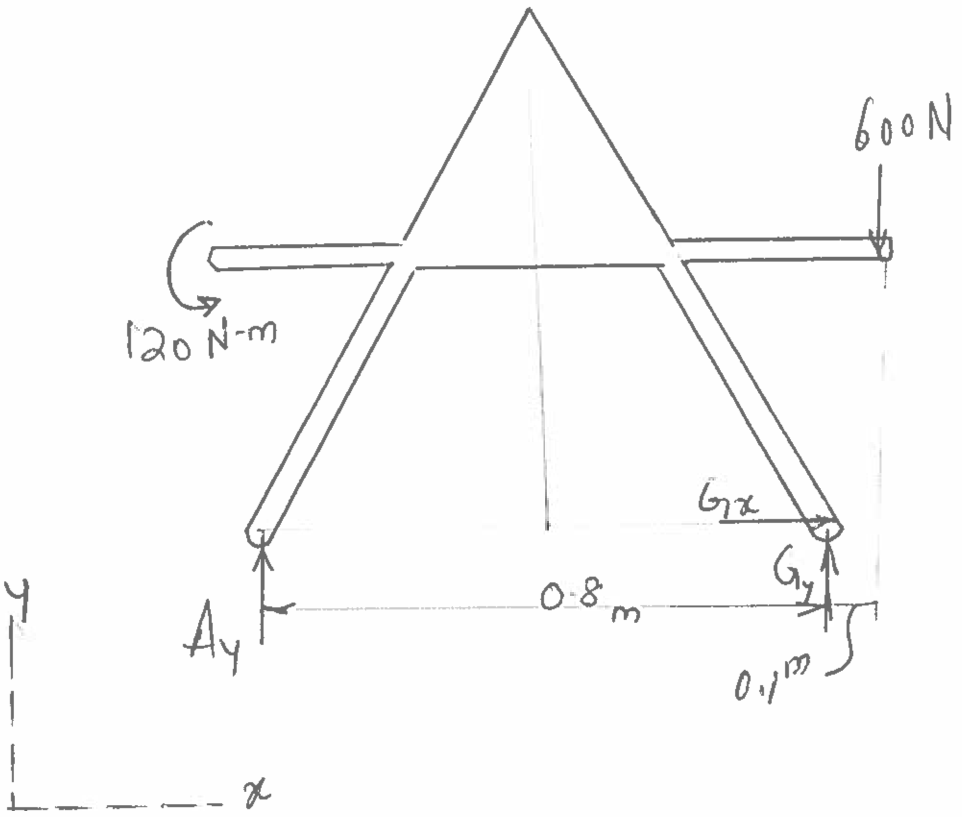
$$M = P\tan\beta(r\sin\theta) + Pr\cos\theta$$

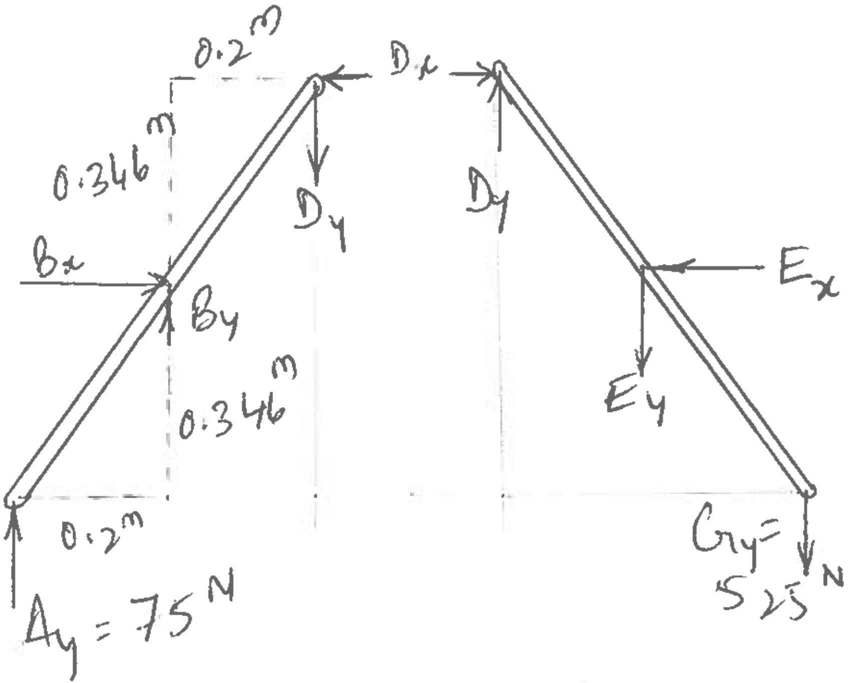
$$= Pr(\sin\theta\tan\beta + \cos\theta)$$

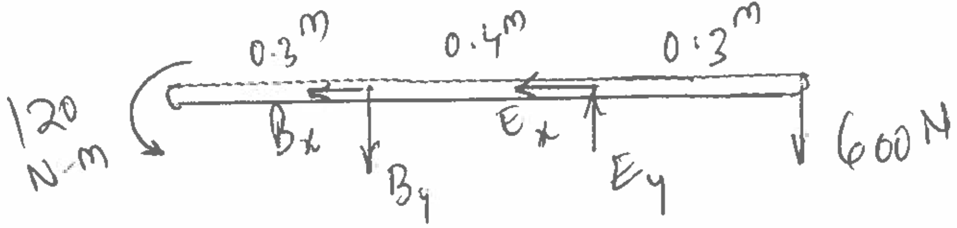
But $C = r\cos\theta + r\sin\theta\tan\beta$

So $\underline{M = PC}$

Prob # 4.108


600 N
120 N-m
Gx
Gy
Ay
0.8 m
0.1 m
y
x


0.2 m
Dx
0.346 m
Dy
Dy
Bx
By
Ex
Ey
0.346 m
0.2 m
Ay = 75 N
Gy = 525 N


0.3 m
0.4 m
0.3 m
120 N-m
Bx
By
Ex
Ey
600 N

As a whole

$\circlearrowleft + \Sigma M_G = 0 : \quad 120 - 600(0.1) - A_y(0.8) = 0,$

$\Sigma F_y = 0 : \quad 75 + G_y - 600 = 0,$

$\Sigma F_x = 0 :$

$$\left\{ \begin{array}{l} A_y = 75\,N \\ G_y = 525\,N \\ G_x = 0 \end{array} \right\}$$

CF: $\circlearrowleft + \Sigma M_B = 0 : \quad 120 + 0.4E_y - 0.7(600) = 0,$

$\Sigma F_y = 0 : \quad 750 - 600 - B_y = 0,$

$$\left\{ \underline{E_y = 750\,N \,;\, B_y = 150\,N} \right\}$$

AD: $\Sigma F_y = 0 : \quad 75 + 150 - D_y = 0$

$\circlearrowleft + \Sigma M_B = 0 : \quad 0.346 D_x - 0.2(225) - 0.2(75) = 0.$

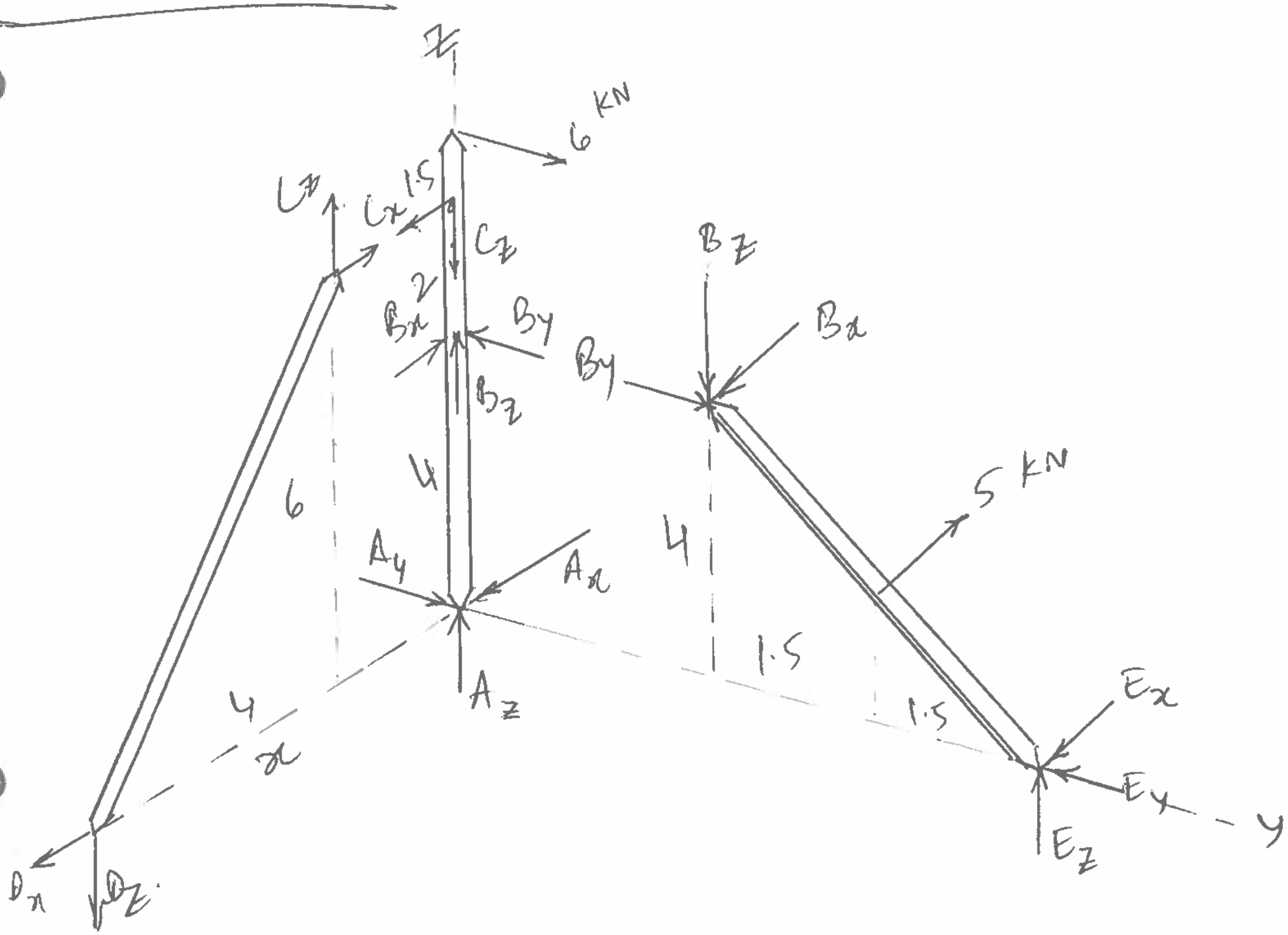
$$\left\{ \begin{array}{ll} D_x = 173.2\,N & \underline{B_x = 173.2\,N} \\ D_y = 225\,N & \end{array} \right\}$$

$\Sigma F_x = 0 : \quad B_x - 173.2 = 0,$

DG: $\Sigma F_x = 0 : \quad 173.2 - E_x = 0,$

$$\underline{E_x = 173.2\,N}$$

Prob # 4.117



BE:

$\Sigma M_{E_x} = 0: \; 3B_z - 4B_y = 0, \qquad B_y = \frac{3}{4} B_z$

$\Sigma F_y = \Sigma F_z = 0 : \; E_y = B_y = \frac{3}{4} B_z, \quad E_z = B_z$

$\Sigma M_{E_z} = 0 : \; 3B_x - 1.5(5) = 0, \quad B_x = 2.5$ kN.

$\Sigma F_x = 0 : \; E_x + 2.5 - 5 = 0, \quad E_x = 2.5$ kN.

ABC: $\Sigma M_{Ax} = 0$: $4 B_y - 7.5(6) = 0$;

$\underline{B_y = 11.25 \text{ kN}}$

$\Sigma F_y = 0$: $6 - 11.25 + A_y = 0$.

$\underline{A_y = 5.25 \text{ kN}}$

$\Sigma M_{Ay} = 0$: $6 C_x - 4(2.5) = 0$;

$\underline{C_x = 1.667 \text{ kN}}$

$\Sigma F_x = 0$: $1.667 + A_x - 2.5 = 0$;

$\underline{A_x = 0.833 \text{ kN}}$

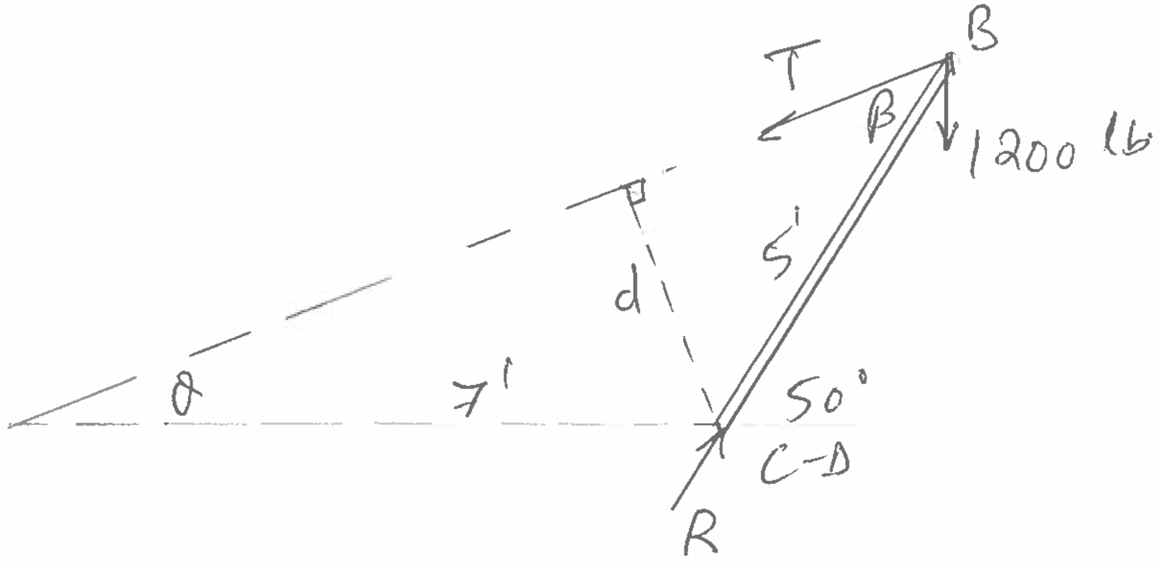
CD: $\Sigma M_{Dy} = 0$: $4 C_z - 6(1.667) = 0$

$\underline{C_z = 2.50 \text{ kN}}$

ABC: $\Sigma F_z = 0$: $A_z + \frac{4}{3}(11.25) - 2.50 = 0$

$\underline{A_z = -12.50 \text{ kN}}$

Prob # 4.118



Frame as a whole:

$$\theta = \tan^{-1}\frac{5\sin 50^\circ}{7+5\cos 50^\circ} = 20.6^\circ.$$

$$d = 7\sin 20.6^\circ = 2.46\ \text{ft}.$$

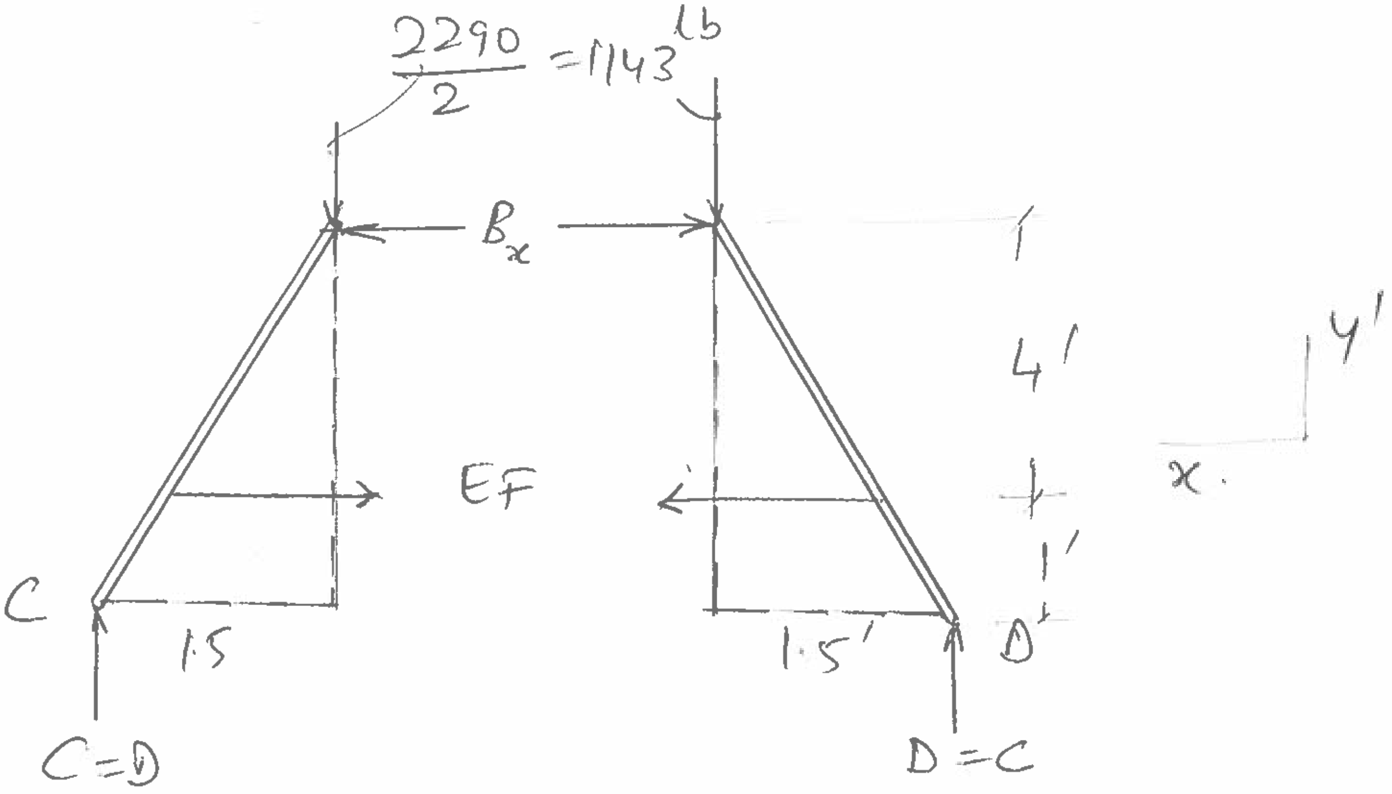
$$\beta + 20.6^\circ = 50^\circ, \qquad \beta = 29.4^\circ.$$

$$\Sigma M_x = 0: (5\cos 50^\circ)(1200) - 2.46T = 0$$

$$\underline{T = 1569\ \text{lb}}$$

$$\Sigma F_{(C-D)-B} = 0: R - 1200\cos 40^\circ - 1569\cos 29.4^\circ = 0$$

$$R = 2290\ \text{lb}.$$



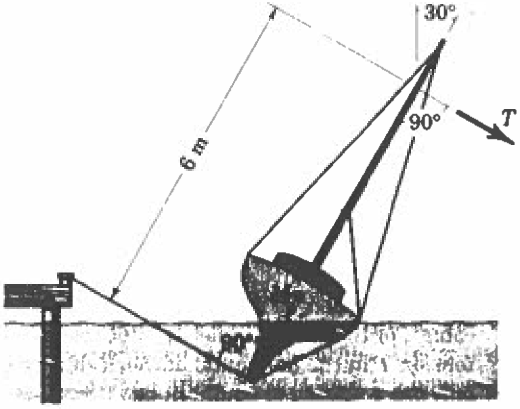
Plane of frames.

$\Sigma F_y = 0 ! \; C = D = 1143 \text{ lb.}$

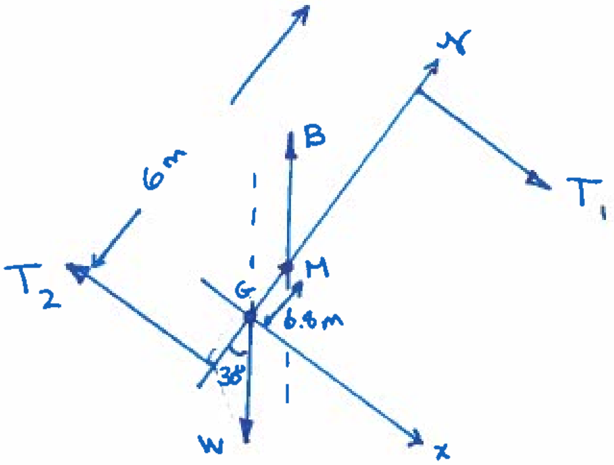
$\Sigma M_B = 0 ; \; 1143(1.5) - 4EF = 0$

$EF = 429 \text{ lb}$

3/26 The small sailboat may be tipped at its moorings as shown to effect repairs below the waterline. One attached rope is passed under the keel and secured to the dock. The other rope is attached to the mast and is used to tip the boat. The boat shown has a displacement (which equals the total mass) of 5000 kg with mass center at G. The metacenter M is the point on the centerline of the boat through which the vertical resultant of the buoyant forces passes, and $\overline{GM} = 0.8$ m. Calculate the tension T required to hold the boat in the position shown.



Problem 3/26



$$\Sigma F_x = 0 = W\sin 30 - B\sin 30 + T_1 - T_2$$

$$\Sigma F_y = 0 = B\cos 30 - W\cos 30 = 0$$

$$\Rightarrow B = W \;; \quad \Rightarrow T_1 = T_2 = T$$

B & W; T_1 & T_2 are couples

$$+\circlearrowleft \Sigma M = 0 = -T(6m) + W\sin 30(0.8m)$$

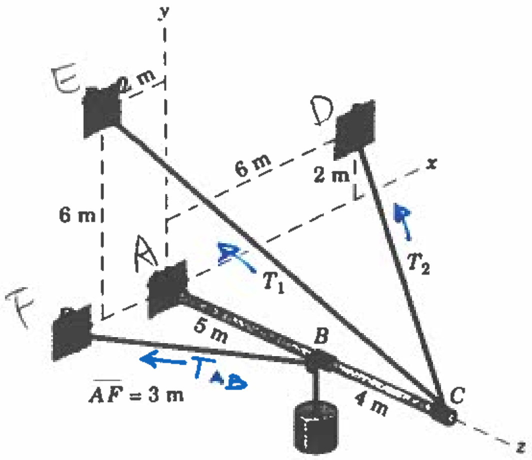
$$T = \frac{W\sin 30(0.8m)}{6m} = \frac{(5000)(9.8)(0.5)(0.8)}{6}$$

$$= 3266\,N$$

$$= \underline{\underline{3.26\,kN}} \longleftarrow$$

CIVE 2330 Engineering Mechanics I.
Examination 2, Pg 1 Name: Solution

3/74 The 9-m steel boom has a mass of 600 kg with center of mass at midlength. It is supported by a ball-and-socket joint at A and the two cables under tensions T_1 and T_2. The cable which supports the 2000-kg load leads through a sheave (pulley) at B and is secured to the vertical x-y plane at F. Calculate the magnitude of the tension T_1. (*Hint:* Write a moment equation which eliminates all unknowns except T_1.)



Problem 3/74

LINES OF ACTION

	$x_2 - x_1$	$y_2 - y_1$	$z_2 - z_1$
T_1	-2	6	-9
T_2	6	2	-9
T_{AB}	-3	0	-5

DIRECTION COSINES

$$\underline{T_1} = T_1\left(-\frac{2}{11}i + \frac{6}{11}j - \frac{9}{11}k\right)$$

$$\underline{T_2} = T_2\left(\frac{6}{11}i + \frac{2}{11}j - \frac{9}{11}k\right)$$

$$T_{AB} = T_{AB}\left(\frac{-3}{5.8}i + \frac{0}{5.8}j - \frac{5}{5.8}k\right)$$

KINEMATICS

$$T_{AB} = W = 2000(9.8) = 19{,}600$$

$$W_{Boom} = 600(9.8) = 5{,}880\ N$$

$$\Sigma M_F = 0 = \Sigma r \times \underline{F} = 9\underline{k} \times T_1(-.182\underline{i} + .545\underline{j} - .818\underline{k}) +$$
$$9\underline{k} \times T_2(.545\underline{i} + .182\underline{j} - .818\underline{k}) +$$
$$5\underline{k} \times -W\underline{j} + 4.5\underline{k} \times -W_{Boom}\underline{j} +$$
$$5\underline{k} \times T_{AB}(-.514\underline{i} - .857\underline{k}) = 0$$

EVALUATE

$$(-1.638T_1 + 4.91T_2 - 50372)\underline{j} + (-4.91T_1 - 1.638T_2 + 124460)\underline{i} =$$

TWO EQN – TWO UNK

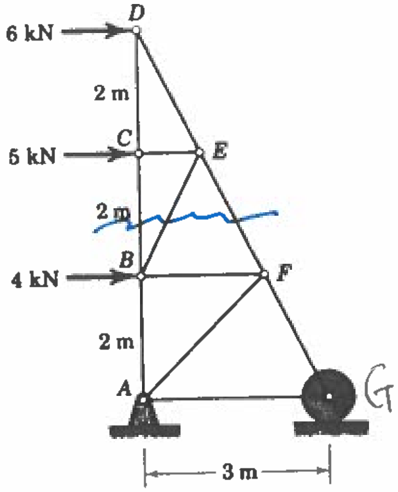
$$-1.638T_1 + 4.91T_2 = 50372$$

$$4.91T_1 - 1.638T_2 = -124460$$

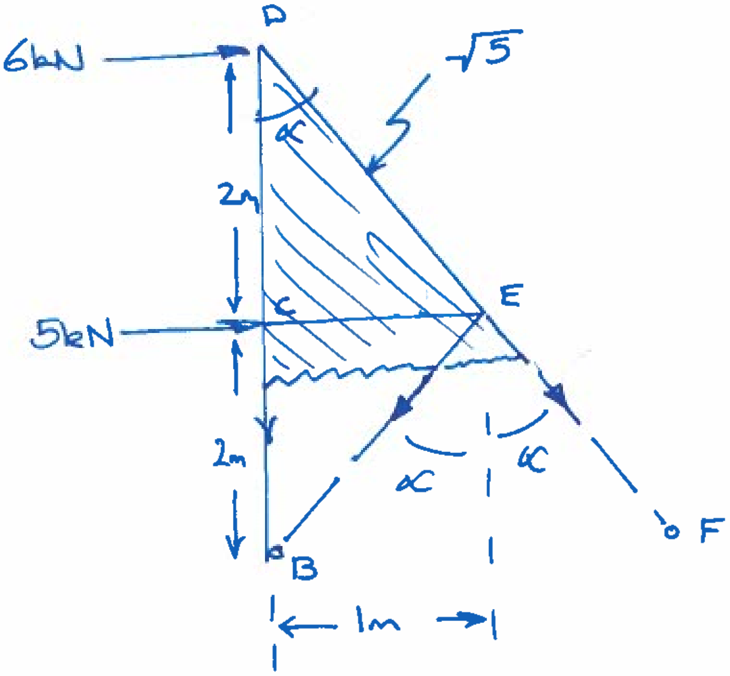
$$\underline{\underline{T_1 = 19.74kN}} \leftarrow$$

$$T_2 = 16.86kN$$

4/30 Determine the force in member *BE* of the loaded truss



Problem 4/30



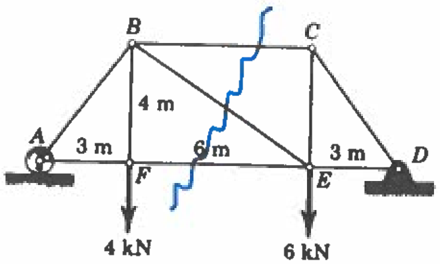
$$+\circlearrowleft \Sigma M_D = 0 = 5\text{kN}(2\text{m}) - EB\sin\alpha\,(2\text{m}) - EB\cos\alpha\,(1\text{m})$$

$$10 = EB\left(\underbrace{2\cdot\frac{1}{\sqrt{5}}}_{2\sin\alpha} + \underbrace{1\cdot\frac{2}{\sqrt{5}}}_{1\cos\alpha}\right)$$

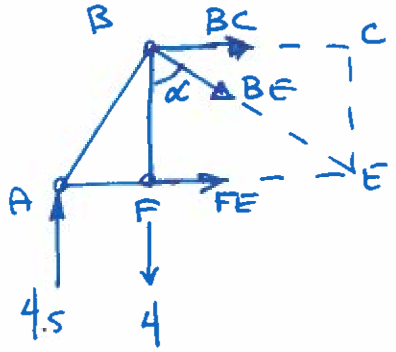
$$10 = EB\,\frac{4}{\sqrt{5}}$$

$$EB = \frac{10\sqrt{5}}{4} = \underline{\underline{5.59\text{ kN}}}\ (T) \leftarrow$$

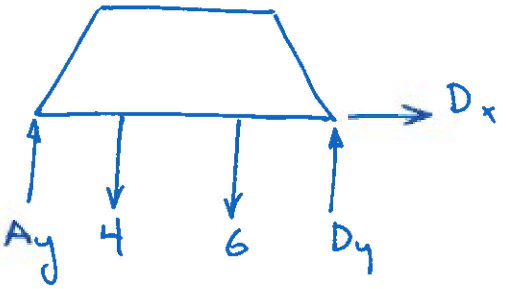
4/29 Compute the forces in members *BC*, *BE*, and *EF* of the truss



Problem 4/29



REACTIONS



$$\Sigma F_x = 0 = D_x$$

$$\Sigma F_y = A_y + D_y - 10$$

$$+\circlearrowleft \Sigma M_D = 0 = 6(3) + 4(9) - A_y(12)$$

$$\frac{54}{12} = A_y = \underline{\underline{4.5\,kN}}$$

$$D_y = 10 - 4.5 = \underline{\underline{5.5\,kN}}$$

$$+\circlearrowleft \Sigma M_F = 0 = 4(6) - 4.5(9) - BC(4$$

$$BC = \frac{24 - 40.5}{4} = \underline{\underline{-4.125k}}$$

$$\Sigma F_y = 0 = 4.5 - 4 - BE \underbrace{\frac{4}{\sqrt{52}}}_{\cos\alpha}$$

$$BE = \underline{\underline{0.901\,kN}}$$

$$\Sigma F_x = 0 = EF + BC + BE \underbrace{\frac{6}{\sqrt{52}}}_{\sin\alpha}$$

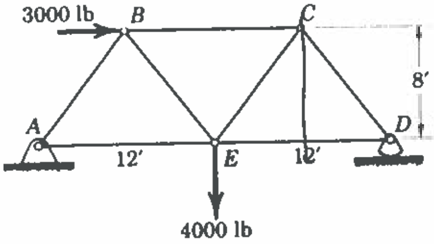
$$EF = -(-4.125) - (0.901)\frac{6}{\sqrt{52}}$$

$$= \underline{\underline{3.375\,kN}}$$

$$\Rightarrow \boxed{\begin{aligned} BC &= 4.125\,kN\,(C) \\ BE &= 0.901\,kN\,(T) \\ EF &= 3.375\,kN\,(T) \end{aligned}}$$

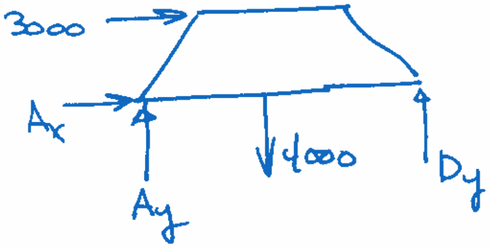
Name:______________________________

4/6 Calculate the force in each member of the loaded truss. All triangles are isosceles.



Problem 4/6

Reactions



$\Sigma F_x = 0 = A_x + 3000$

$\underline{A_x = -3000\,lb}$ ⇐

$+\circlearrowleft \Sigma M_A = 0 = -3000(8) - 4000(12) + D_y(24)$

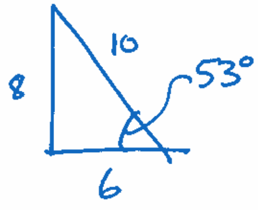
$D_y = \dfrac{3000(8)+4000(12)}{24} = \underline{3000\,lb}$ ⇐

$\Sigma F_y = 0 = A_y + D_y - 4000$

$= A_y + 3000 - 4000$

$\underline{A_y = 1000\,lb}$ ⇐

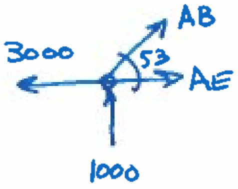
Trigonometry



$\sin 53 = \dfrac{8}{10}$

$\cos 53 = \dfrac{6}{10}$

Joint A



$\Sigma F_y = AB\sin 53 + 1000 = 0$

$\underline{AB} = -\dfrac{1000}{0.8} = \underline{-1252}$

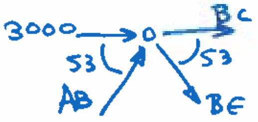
$\Sigma F_x = AB\cos 53 + AE - 3000$

$AE = 3000 - (-1252)(0.6)$

$\underline{AE} = \underline{3753}$

⇒ $\begin{bmatrix} AB = 1252 \\ AE = 3753 \end{bmatrix}$

Joint B



$\Sigma F_y = 0 = AB\sin 53 - BE\sin 53$

$\underline{BE} = AB = \underline{1252}$

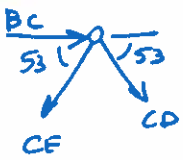
$\Sigma F_x = 3000 + AB\cos 53 + BC + BE\cos 5$

$BC = -AB(.6) - BE(.6) - 3000$

$= -4587$

⇒ $\begin{bmatrix} BE = 1252\ T \\ BC = 4507\ C \end{bmatrix}$

Joint C



$\Sigma F_y = 0 = -CE\sin 53 - CD\sin 53$

$CE = -CD$

$\Sigma F_x = 0 =$

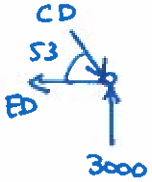
$BC - CE\cos 53 + CD\cos 5$

$CD = \dfrac{-BC}{2(0.6)} = -3744$

$CE = 3744$

⇒ $\begin{bmatrix} CD = 3744\ C \\ CE = 3744\ T \end{bmatrix}$

Joint D



$\Sigma F_y = -CD\sin 53 + 3000$

$\Sigma F_x = -ED + CD\cos 53 = 0$

$ED = CD(0.6) = 2253$

⇒ $\begin{bmatrix} ED = 2253\ T \end{bmatrix}$

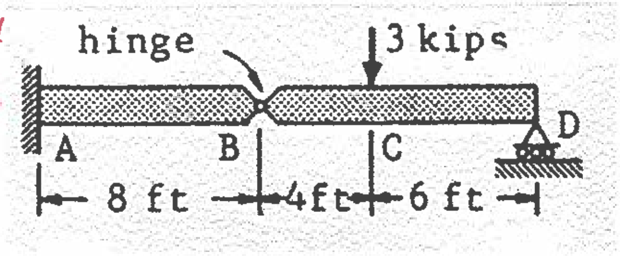
Name:__________________________

SOLUTION

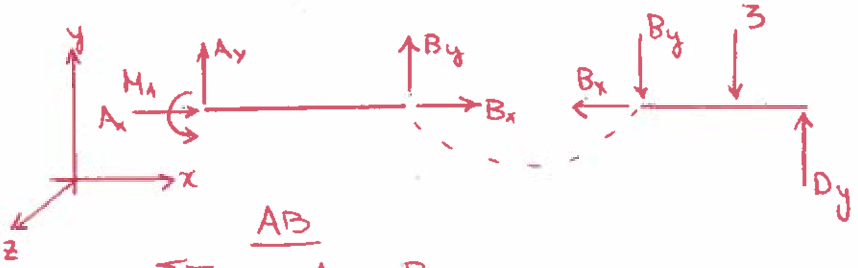
Problem 1: Determine the reactions at the supports.

Cantilever at A

∴ A_x, A_y, M_A



Hint: Draw free body diagram(s) of beam to the left and right of the hinge. Remember to include the hinge forces on each diagram.



ALL FORCES IN KIPS.

AB

$\Sigma F_x = 0 = A_x + B_x$

$\therefore A_x = -B_x$

$\Sigma F_y = 0 = A_y + B_y$

$\therefore A_y = -B_y$

$\Sigma M_A = 0 = M_A\underline{k} + (8ft\,\underline{i}) \times (B_y\,\underline{j})$

$M_A = -8B_y$

BD

$\Sigma F_x = 0 = B_x$

$\Sigma F_y = 0 = D_y - B_y - 3$

$\Sigma M_B = 0 = (10\underline{i} \times D_y\underline{j}) + (4\underline{i} \times -3\underline{j}$

$10D_y\underline{k} - 12\underline{k} = 0$

$D_y = \frac{12}{10} = 1.2$

$B_y = D_y - 3 = -1.8$

∴ $A_x = 0$ kip

$A_y = -B_y = -(-1.8) = 1.8$ kip

$M_A = -8(-1.8) = 14.4$ kip-ft

SOLUTION

CIVE 2330 Fall 1999 Examination 2
Page 2 of 4

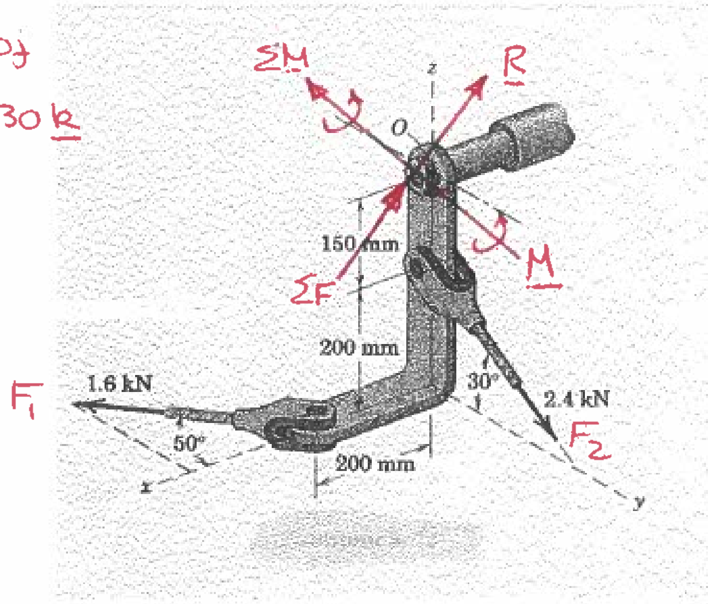
Problem 2: Determine the force **R** and the couple **M** exerted by the nut and bolt at *O* to maintain the loaded bracket in equilibrium.

$F_1 = 1.6\cos 50\,\underline{i} - 1.6\sin 50\,\underline{j}$

$F_2 = 2.4\cos 30\,\underline{j} - 2.4\sin 30\,\underline{k}$

$r_1 = 0.2\underline{i} - 0.35\underline{k}$

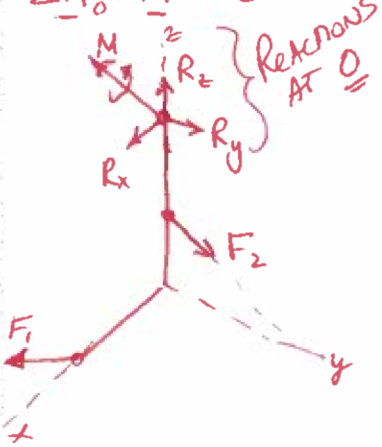
$r_2 = \qquad -0.15\underline{k}$



EQUILIBRIUM

$\Sigma \underline{F} + \underline{R} = 0$

$\Sigma \underline{M}_o + \underline{M} = 0$



$\Sigma \underline{F} + \underline{R} = 1.6\cos 50\,\underline{i} + R_x\underline{i} - 1.6\sin 50\,\underline{j} + 2.4\cos 30\,\underline{j} + R_y\underline{j} - 2.4\sin 30\,\underline{k} + R_z\underline{k} = 0$

$R_x = -1.6\cos 50$ $\qquad R_y = 1.6\sin 50 - 2.4\cos 30$ $\qquad R_z = 2.4\sin 30$

$\underline{\underline{R_x = -1.028\text{ kN}}}$ $\qquad \underline{\underline{R_y = -0.853\text{ kN}}}$ $\qquad \underline{\underline{R_z = 1.2\text{ kN}}}$

$$\Sigma \underline{M}_o + \underline{M} = (0.2\underline{i} - 0.35\underline{k}) \times (1.6\cos 50\,\underline{i} - 1.6\sin 50\,\underline{j}) + (\qquad -0.15\underline{k}) \times (2.4\cos 30\,\underline{j} - 2.4\sin 30\,\underline{k}) + M_x\underline{i} + M_y\underline{j} + M_z\underline{k} = 0$$

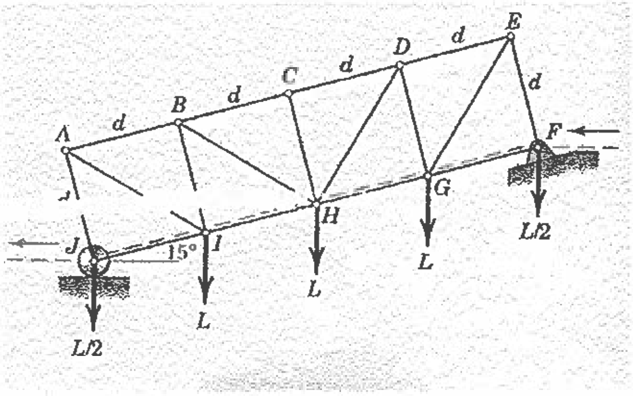
$$0 = (0.2)(-1.6)(\sin 50)(\underline{k}) + (-0.35)(1.6)(\cos 50)(\underline{j}) + (-0.35)(-1.6)(\sin 50)(-\underline{i}) + (-0.15)(2.4)(\cos 30)(-\underline{i}) + M_x\underline{i} + M_y\underline{j} + M_z\underline{k} = 0$$

$M_z = (0.2)(1.6)(\sin 50) = \underline{\underline{0.245\text{ kN}\cdot\text{m}}}$

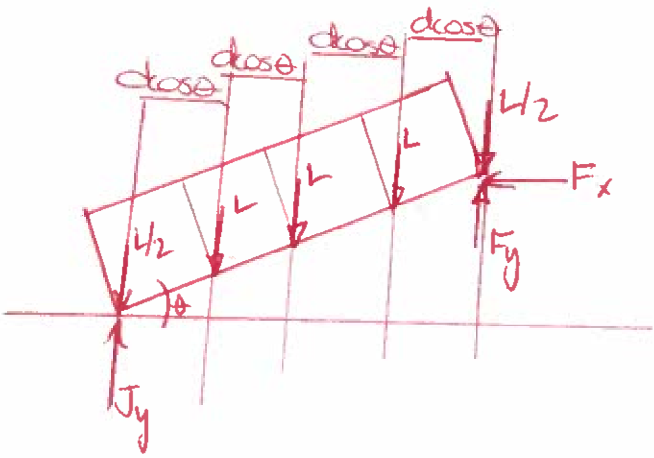
$M_y = (0.35)(1.6)(\cos 50) = \underline{\underline{0.359\text{ kN}\cdot\text{m}}}$

$M_x = (0.35)(1.6)(\sin 50) - (0.15)(2.4)(\cos 30) = \underline{\underline{0.117\text{ kN}\cdot\text{m}}}$

Problem 3: The truss supports a ramp (shown with a dashed line) that extends from a fixed approach level near F to an exit level near J. The drawing shows the ramp angle as 15 degrees. The loads shown represent the weight of the ramp. Determine the forces in members BH and CD when the ramp angle is 15 degrees and when the ramp angle is 45 degrees.



ENTIRE TRUSS



$\Sigma F_x = 0 = F_x$

$\Sigma F_y = 0 = J_y + F_y - 4L$

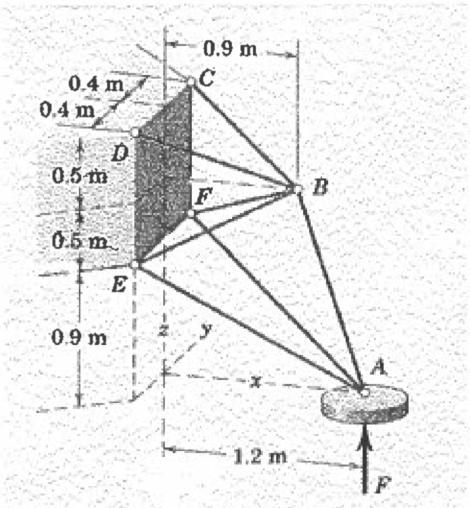
$\Sigma M_F = 0 = L d \cos\theta + 2 L d \cos\theta + 3 L d \cos\theta$
$+ 4 \frac{L}{2} d \cos\theta - J_y 4 d \cos\theta$

$J_y = \frac{8L}{4} = 2L$ (independent of θ!)

OVER

Problem 4: Each landing strut for a spacecraft is designed as a space truss symmetrical about the x-z plane. For a landing force of 2.2 kN, determine the force in member BE. Assume the mass of the truss itself is negligible.

JOINT A



$$\underline{AB} = AB\left(\frac{-0.3\underline{i} + 1.4\underline{k}}{\sqrt{.3^2 + 1.4^2}}\right)$$

$$= AB\left(-0.209\underline{i} + 0.977\underline{k}\right)$$

$$\underline{AE} \quad AE\left(\frac{-1.2\underline{i} - 0.4\underline{j} + 0.9\underline{k}}{\sqrt{1.2^2 + .4^2 + .9^2}}\right)$$

$$= AE\left(-0.773\underline{i} - 0.257\underline{j} + 0.579k\right)$$

$$\underline{AF} = AF\left(-0.773\underline{i} + 0.257\underline{j} + 0.579k\right)$$

$$\Sigma F_x = 0 = -0.209AB - 0.773AE - 0.773AF \Rightarrow AE = \frac{-0.209\,AB}{1.546} = -0.135AB$$

$$\Sigma F_y = 0 = -0.257AE + 0.257AF \Rightarrow AE = AF$$

$$\Sigma F_z = 0 = 2.2 + 0.977AB + 0.579AE + 0.579AF$$

$$AB = -2.2 - 1.159AE$$

$$AB = -2.2 - 1.159(-0.135)AB$$

$$AB - 0.156AB = -2.2$$

$$AB = -2.608$$

Distributed Forces

Up to now have treated forces as concentrated along their lines of action and at their points of application.

When forces are applied over a region whose dimensions are <u>not</u> negligible compared to other problem dimensions then the way in which forces are distributed over the region must be accounted for.

The effect of a distributed force can be replaced in analysis by a concentrated force acting at an appropriate location on the structure. The magnitude of this equivalent force is determined by integration

Categories of distributed forces

Line distribution, eg continuous load supported by a suspended cable. Intensity of load is N/m or lb/ft.

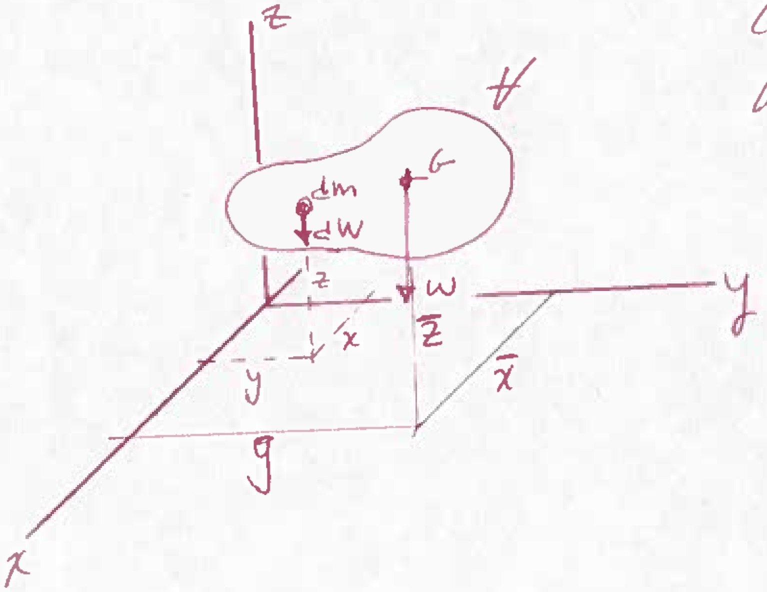
area distribution, eg hydraulic pressure of water against the inner face of a dam. Intensity of load is N/m^2 or lbs/ft^2 (lb/in^2). N/m^2 is a Pascal (Pa). It is called "pressure" in a fluid and "normal stres in a solid.

Volume distribution, eg force of gravitational attraction which acts on all elements of a body (weight). Intensity is specific weight (ρg). N/m^3, lbs/ft^3. ρ is the density (mass/volume) and g is the accelerati from gravity. A force distributed over the entire volume of a body is known as a body force.

Center of Mass

Weight of a body is a distributed load acting on all mass elements of the body. This distributed load may be replaced by a resultant (concentrated) force acting at a single point. This point is known as the <u>center of gravity</u> of the body.

Consider an arbitrary 3D body shown. To find the center of mass apply the principle of moments



Coordinates of G are $(\bar{x}, \bar{y}, \bar{z})$

Weight of element dm is

$$dW = dm\,g$$

Weight of entire body is

$$W = mg = \iiint_{\forall} dW$$

Principle of moments means that the moment of W is equal to the sum of the moments of the infinitesimal weight elements dW about the same moment center.

$$\therefore \quad \bar{x} \iiint_{\forall} dW = \iiint x\,dW$$

$$\bar{y} \int dW = \int y\,dW \quad \longrightarrow \quad \bar{x} = \frac{\int x\,dW}{\int dW}$$

$$\bar{z} \int dW = \int z\,dW \qquad \bar{y} = \frac{\int y\,dW}{\int dW} \qquad \bar{z} = \frac{\int z\,dW}{\int dW}$$

Recall $dw = dm\, g$

Then

$$\bar{x} = \frac{\int x\, dm \cancel{g}}{\int dm \cancel{g}}, \quad \bar{y} = \frac{\int y\, dm \cancel{g}}{\int \cancel{g}\, dm}, \quad \bar{z} = \frac{\int z \cancel{g}\, dm}{\int \cancel{g}\, dm}$$

So $\bar{x} = \dfrac{\int x\, dm}{m}$, $\bar{y} = \dfrac{\int y\, dm}{m}$, $\bar{z} = \dfrac{\int z\, dm}{m}$

Since g no longer appears the location $(\bar{x}, \bar{y}, \bar{z})$ is called the center of mass.

The expression may also be written in vector form as $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$, $\bar{\underline{r}} = \bar{x}\underline{i} + \bar{y}\underline{j} + \bar{z}\underline{k}$

$$\bar{\underline{r}} = \frac{\int \underline{r}\, dm}{m}$$

From definition of density

$dm = \varphi\, d\forall$ where $d\forall$ is a volume element.

$$\bar{\underline{r}} = \frac{\int \varphi \underline{r}\, d\forall}{\int \varphi\, d\forall}$$

If the density is constant throughout the body then

$$\bar{\underline{r}} = \frac{\int \underline{r}\, dV}{\int dV}$$

$\bar{\underline{r}}$ is now a geometrical property of the body. In this case $\bar{\underline{r}}$ is called the centroid of the body

Centroids of Lines, Areas, Volumes

a) Lines. Slender rod of length L, cross sectional area A and density ρ.

If ρ, A are constant then

$$\bar{\underline{r}} = \frac{\int \underline{r}\, dm}{\int dm} = \frac{\int \underline{r}\, \rho A\, dL}{\int \rho A\, dL} = \frac{\int \underline{r}\, dL}{\int dL}$$

($\bar{\underline{r}}$ is not necessarily on the rod)

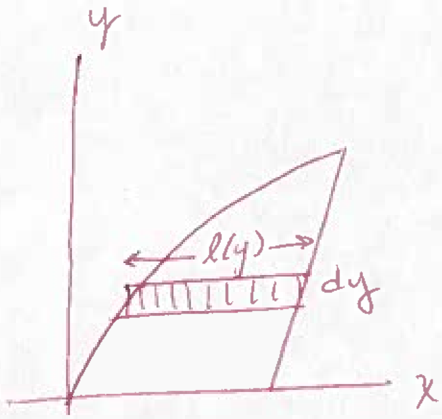
b) Areas. Finite body, small constant thickness, t, area A.
If ρ, t are constant

$$\bar{\underline{r}} = \frac{\int \underline{r}\, dm}{\int dm} = \frac{\int \underline{r}\, \rho t\, dA}{\int \rho t\, dA} = \frac{\int \underline{r}\, dA}{\int dA}$$

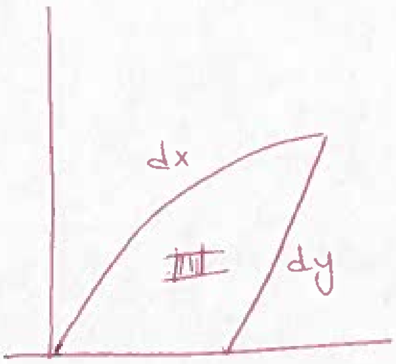
The term $\int \underline{r}\, dA$ is called the first moment of area

Finding Centroids

1) Choose a low order integration element



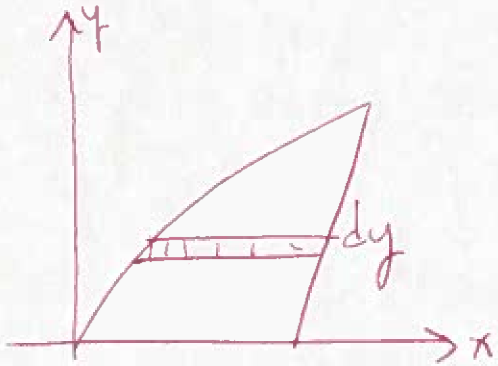
$dA = l(y)\,dy$

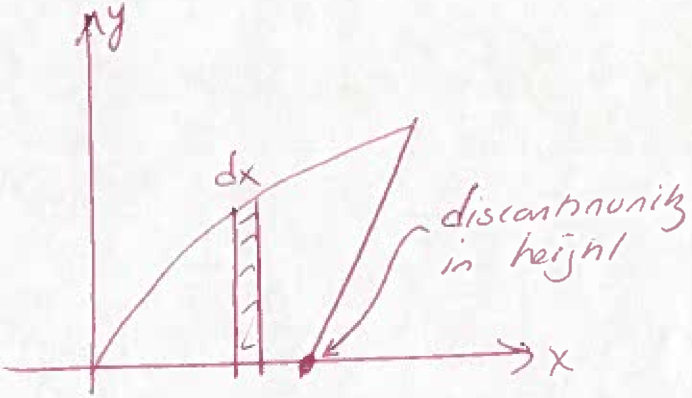


$dA = dx\,dy$

Try to choose element so only one variable of integration is required.

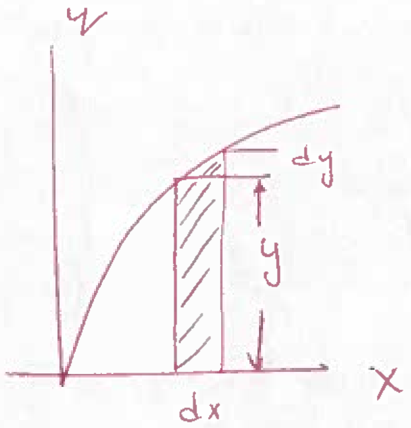
2) Integrate over single continuous function





3) Choose coordinate system that best matches the area under consideration. Use symmetry to reduce integration domains or to locate components of $\underline{\bar{r}}$

4) Neglect higher order differential terms in differential elements

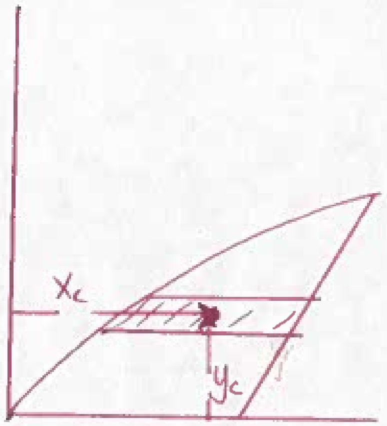


$$dA \approx y\,dx + \underbrace{\tfrac{1}{2}\,dx\,dy}_{\text{neglect}}$$

$$\therefore \; dA = y\,dx$$

5) Centroidal Coordinate of an Element

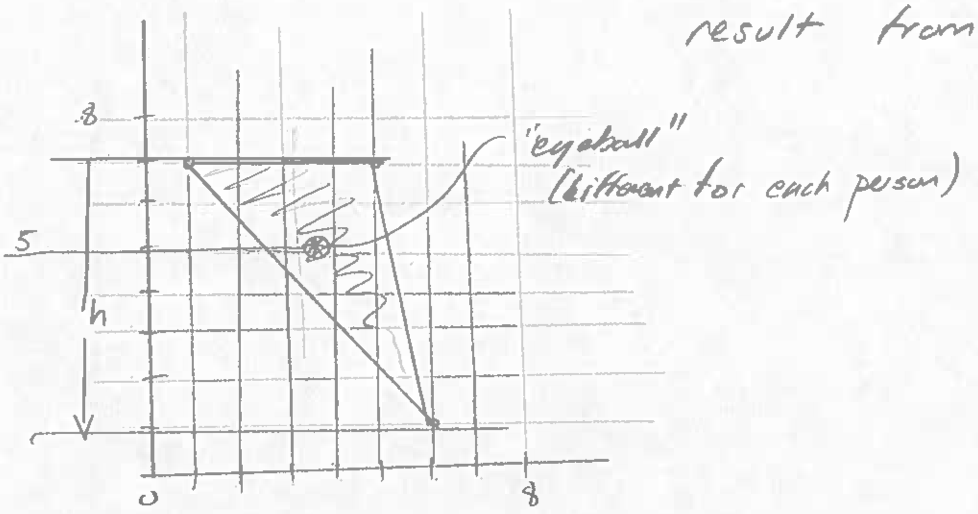
When using a first or second order element the centroidal coordinate is used for the moment arm in setting up the moment of the differential element.



Centroid of shaded area is x_c, y_c. Moment of dA about x axis is $x_c\,dA$, which is not the x that describes the boundary of the shaded region

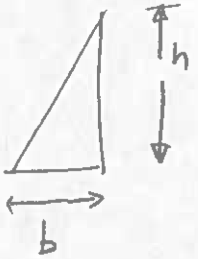
$$\bar{x} = \frac{\int x_c\,dA}{\int dA} \qquad \bar{y} = \frac{\int y_c\,dA}{\int dA}$$

5-1 "Eyeball" the location of centroid. Compare vertical co-ordinate with result from sample 5-2



Vertical co-ordinate by "eye" ≈ 5

5-2 $\bar{y} = \frac{h}{3}$

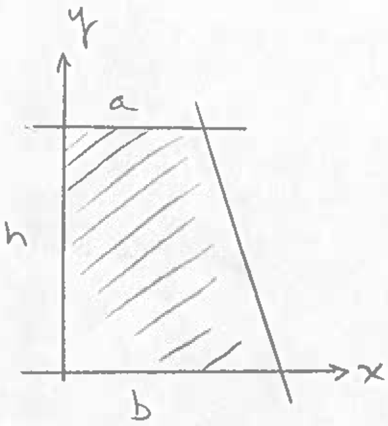


$h = 6$

$h/3 = 2$

$y = 7 - (2) = 5$ good agreement!

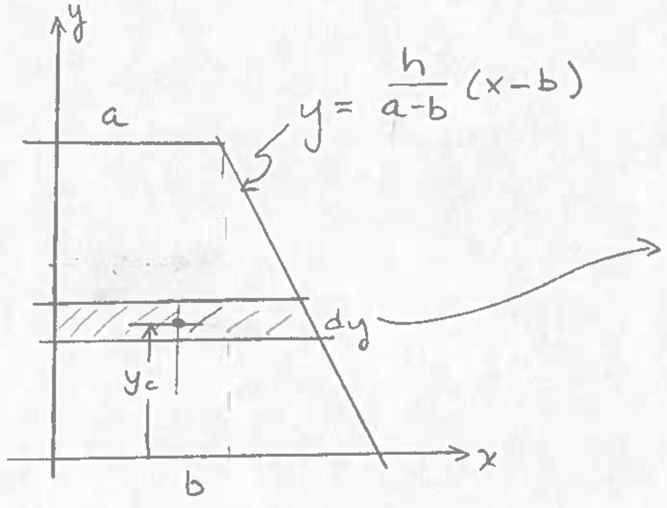
5-5 Find $\bar{x}$ & $\bar{y}$ of area below

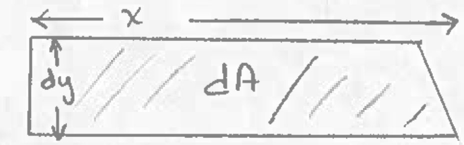


General Formula

$$\bar{X} = \frac{\int x_c \, dA}{\int dA} \qquad \bar{y} = \frac{\int y_c \, dA}{\int dA}$$

Solution by Integration





$$x = y\left(\frac{a-b}{h}\right) + b$$

$$\therefore dA = y\left(\frac{a-b}{h}\right) + b \; dy$$

$$\int dA = \int_0^h y\left(\frac{a-b}{h}\right) + b \, dy = \frac{y^2}{2}\left(\frac{a-b}{h}\right) + by \Big|_0^h$$

$$= \frac{h^2}{2}\left(\frac{a-b}{h}\right) + bh = \frac{h}{2}(a-b) + bh$$

$$= \frac{ha}{2} - \frac{hb}{2} + \frac{2hb}{2} = \frac{1}{2}h(a+b)$$

$$\int y\,dA = \int_0^h y^2\left(\frac{a-b}{h}\right) + by\,dy = \frac{y^3}{3}\left(\frac{a-b}{h}\right) + \frac{by^2}{2}\Big|_0^h$$

$$= \frac{h^3}{3}\left(\frac{a-b}{h}\right) + \frac{bh^2}{2} = \frac{h^2}{3}a - \frac{h^2b}{3} + \frac{h^2b}{2}$$

$$= \frac{2h^2a}{6} - \frac{2h^2b}{6} + \frac{3h^2b}{6} = \frac{2h^2a}{6} + \frac{h^2b}{6} = \frac{h^2}{6}(2a+b)$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\frac{h^2}{6}(2a+b)}{\frac{h}{2}(a+b)} = \frac{h}{3}\,\frac{(2a+b)}{(a+b)} \longleftarrow \bar{y}$$

$$\int x_c\,dA = \int_0^h \underbrace{\frac{\left(y\left(\frac{a-b}{h}\right)+b\right)}{2}}_{x_c}\underbrace{\left(y\cdot\left(\frac{a-b}{h}\right)+b\right)dy}_{dA}$$

$$= \frac{1}{2}\int_0^h \left(y\left(\frac{a-b}{h}\right)+b\right)^2 dy = \frac{1}{2}\int_0^h y^2\left(\frac{a-b}{h}\right)^2 + 2yb\left(\frac{a-b}{h}\right) + b^2\,dy$$

$$= \left(\frac{y^3}{\cancel{3}}\left(\frac{a-b}{h}\right)^2\Big|_0^h + \frac{2y^2b}{\cancel{2}}\left(\frac{a-b}{h}\right)\Big|_0^h + b^2y\Big|_0^h\right)\tfrac{1}{2}$$

$$= \left(\frac{h^3}{6}\left(\frac{a-b}{h}\right)^2 + \frac{h^2b}{2}\left(\frac{a-b}{h}\right) + \frac{b^2h}{2}\right)$$

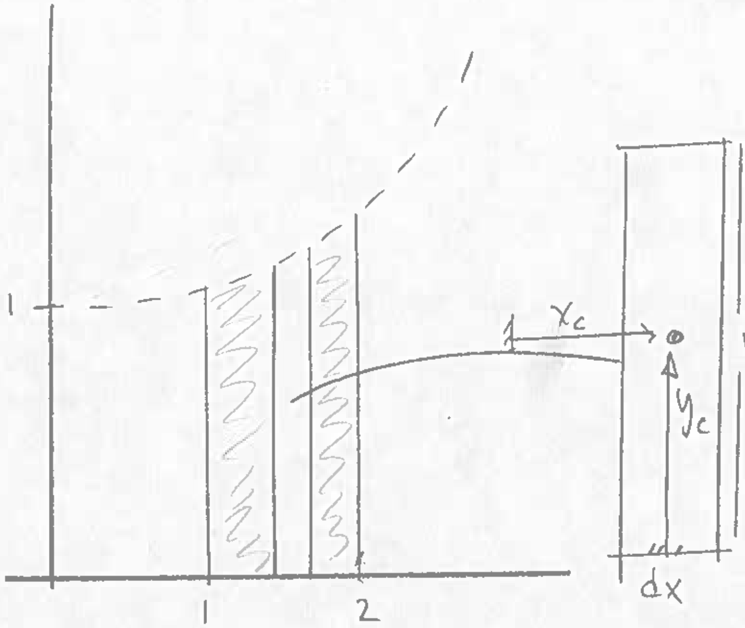
$$= \frac{h^3}{6}\left(\frac{a-b}{h}\right)^2 + \frac{h^2b}{2}\left(\frac{a-b}{h}\right) + \frac{hb^2}{2} = \frac{h}{6}(a-b)^2 + \frac{h}{2}b(a-b) + \frac{h}{2}b^2$$

$$= \frac{h}{6}(a-b)^2 + \frac{3h}{6}b(a-b) + \frac{3hb^2}{6} = \frac{h}{6}\left((a-b)^2 + 3b(a-b) + 3b^2\right)$$

$$\frac{h}{6}\left(a^2 - \cancel{2}ab + b^2 + \cancel{3}ab - 3\cancel{b^2} + 3\cancel{b^2}\right) = \frac{h}{6}\left(a^2+b^2+ab\right)$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{\cancel{h}}{6}(a^2+b^2+ab)}{\frac{\cancel{h}}{2}(a+b)} = \frac{a^2+b^2+ab}{3(a+b)}$$

5-7 FIND $\bar{x}$ & $\bar{y}$ OF AREA SHOWN



$x_c = x$

$y_c = y/2$

$y = 1 + \frac{1}{6}x^3$

$dA = y\,dx$

$= (1+\frac{1}{6})x^3 dx$

$$\int dA = \int_1^2 y\,dx = \int_1^2 1+\frac{1}{6}x^3 dx = x + \frac{1}{24}x^4\Big|_1^2 = 2+\frac{16}{24}-1-\frac{1}{24} = \frac{39}{24} = 1.625$$

$$\int x_c\,dA = \int_1^2 xy\,dx = \int_1^2 x+\frac{1}{6}x^4 dx = \frac{x^2}{2}+\frac{1}{30}x^5\Big|_1^2 = \frac{4}{2}+\frac{32}{30}-\frac{1}{2}-\frac{1}{30} = 2.533$$

$$\int y_c\,dA = \frac{1}{2}\int_1^2 yy\,dx = \frac{1}{2}\int_1^2 (1+\frac{1}{6}x^3)^2 dx = \frac{1}{2}\int_1^2 1+\frac{x^3}{3}+\frac{x^6}{36}\,dx$$

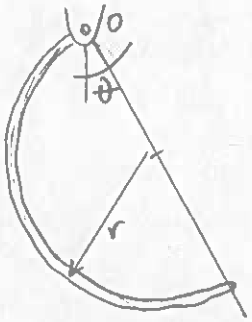
$$= \frac{1}{2}\left(x+\frac{x^4}{12}+\frac{x^7}{252}\right)\Big|_1^2 = \frac{1}{2}\left(2-1+\frac{16}{12}-\frac{1}{12}+\frac{128}{252}-\frac{1}{252}\right) = 1.377$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{2.533}{1.625} = \underline{\underline{1.559}}$$

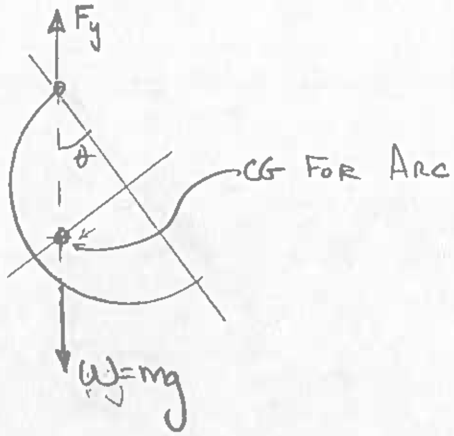
$$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{1.377}{1.625} = \underline{\underline{0.847}}$$

5-8 UNIFORM ROD AS SHOWN.

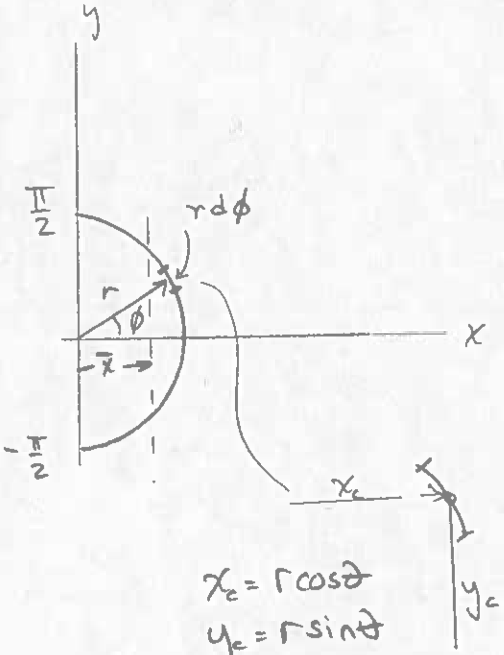
FIND θ FOR EQUILIBRIUM



FBD



(d) FIND CENTROID (C.G.) OF CIRCULAR ARC



$x_c = r\cos\theta$

$y_c = r\sin\theta$

$d\ell = r\,d\phi$

$$\bar{x} = \frac{\int x\,d\ell}{\int d\ell} \qquad \ell = \text{arc length}$$

$$\int x\,d\ell = \int_{-\frac{\pi}{2}}^{\pi/2} r\cos\phi\, r\,d\phi = 2\int_0^{\frac{\pi}{2}} r\cos\phi\, r\,d\phi$$

$$= 2r^2\sin\phi \Big|_0^{\pi/2} = 2r^2$$

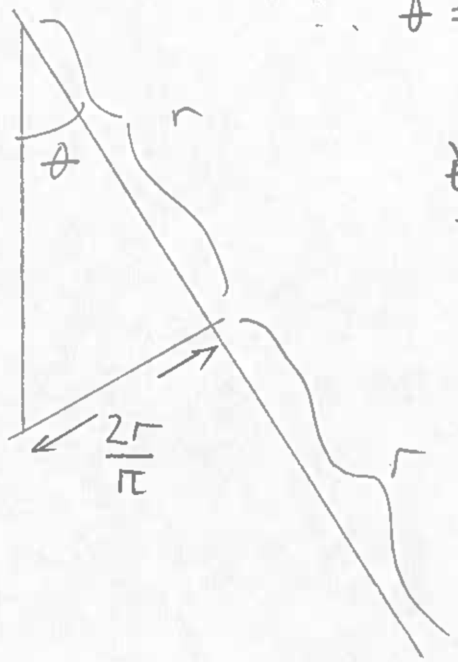
$$\int d\ell = 2\int_0^{\pi/2} r\,d\phi = 2r\phi\Big|_0^{\pi/2}$$

$$= 2r\,\pi/2$$

$$\therefore \bar{x} = \frac{2r^2}{2r\,\pi/2} = \frac{2r}{\pi}$$

b) APPLY RESULT

FROM FBD

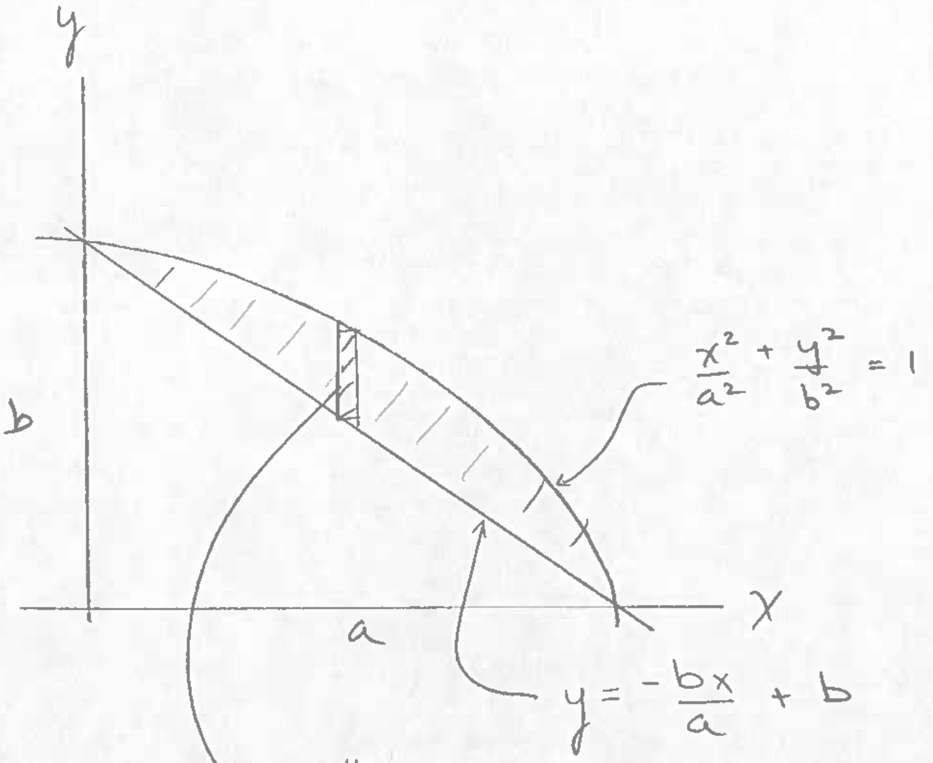


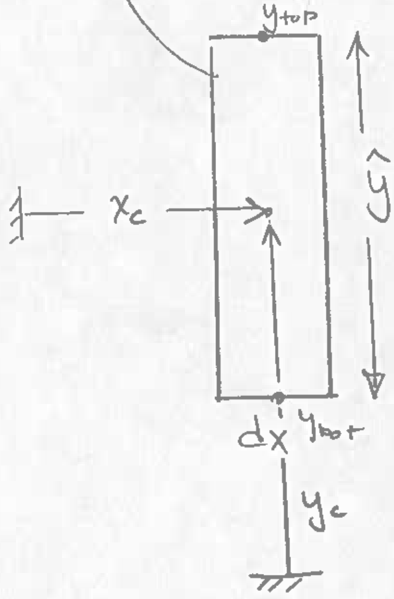
$$\therefore \theta = \tan^{-1}\left(\frac{\frac{2r}{\pi}}{r}\right) = \tan^{-1}\left(\frac{2}{\pi}\right)$$

$$\underline{\theta = 32.5^\circ}$$

5-13

FIND $\bar{x}$ & $\bar{y}$ OF AREA SHOWN





$$\hat{y} = y_{top} - y_{bottom}$$

$$y_{top}^2 = b^2 - \frac{x^2}{a^2} b^2$$

$$= \frac{a^2b^2}{a^2} - \frac{b^2x^2}{a^2} = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y_{top} = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$y_{bottom} = -\frac{bx}{a} + b = -\frac{bx}{a} + \frac{ba}{a}$$

$$= \frac{b}{a}(a - x)$$

$$\therefore \hat{y} = \frac{b}{a}\left(\sqrt{a^2 - x^2} - (a - x)\right)$$

$$dA = dx\,\hat{y} = \frac{b}{a}\left(\sqrt{a^2 - x^2} - (a - x)\right) dx$$

$$x_c\,dA = x\,\hat{y}\,dx = x\,\frac{b}{a}\left(\sqrt{a^2-x^2}-(a-x)\right)dx$$

$$y_c\,dA = \left(y_{bot}+\frac{\hat{y}}{2}\right)\hat{y}\,dx = \left[\frac{b}{a}\left(\frac{\sqrt{a^2-x^2}-(a-x)}{2}\right)+\frac{b}{a}(a-x)\right]\left[\frac{b}{a}\left(\sqrt{a^2-x^2}-(a-x\right.\right.$$

$$\int dA = \int_0^a \frac{b}{a}\left(\sqrt{a^2-x^2}-(a-x)\right)dx = \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}-\left(ax-\frac{x^2}{2}\right)\right]_0^a$$

$$= \frac{ab}{2}\left(\frac{\pi}{2}-1\right)$$

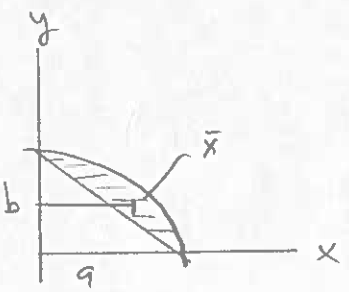
$$\int x_c\,dA = \int_0^a x\;\frac{b}{a}\left(\sqrt{a^2-x^2}-(a-x)\right)dx$$

$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3}-\left(\frac{ax^2}{2}-\frac{x^3}{3}\right)\right]_0^a = \frac{ba^2}{6}$$

$$\bar{x} = \frac{\int x\,dA}{\int dA} = \frac{\frac{ba^2}{6}}{\frac{ab}{2}\left(\frac{\pi}{2}-1\right)} = \frac{a}{3\left(\frac{\pi}{2}-1\right)}$$

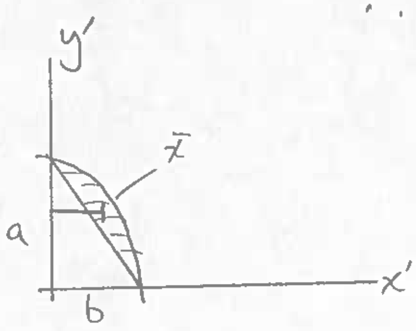
FIND $\bar{Y}$ FROM OBSERVATION:

$$\therefore\ \bar{y} = \frac{b}{3\left(\frac{\pi}{2}-1\right)}$$



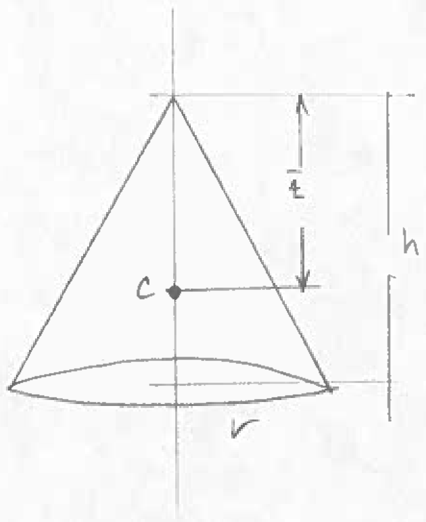
THIS PROBLEM

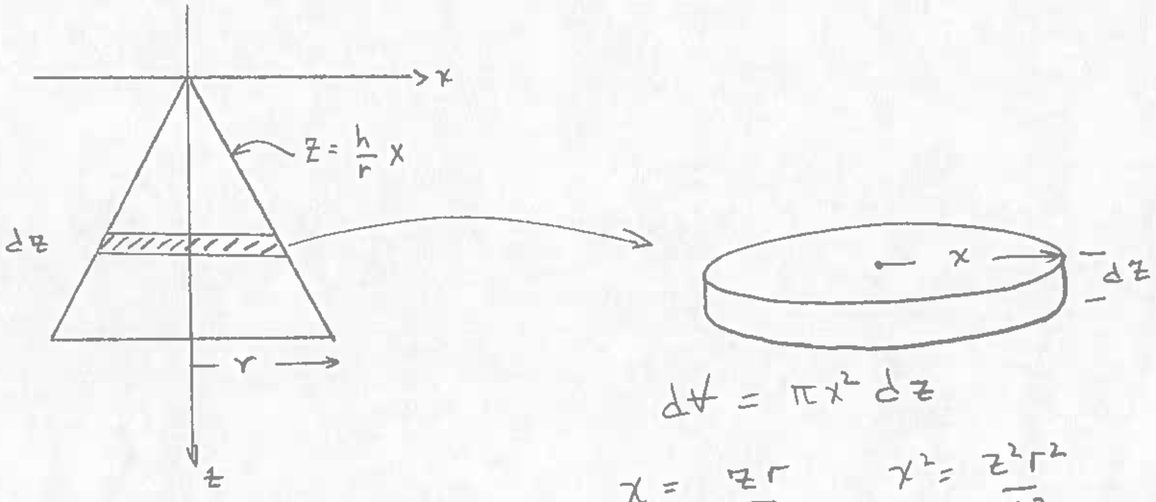
$\sim$



ROTATED AXIS

-16 FIND $\bar{Z}$ OF RIGHT CIRCULAR CONE





$$d\forall = \pi x^2 \, dz$$

$$x = \frac{zr}{h} \qquad x^2 = \frac{z^2r^2}{h^2}$$

$$\therefore \; d\forall = \pi \frac{z^2r^2}{h^2} \, dz$$

$$\int d\forall = \int_0^h \pi \frac{z^2r^2}{h^2} \, dz = \pi \frac{z^3r^2}{3h^2} \Big|_0^h$$

$$= \pi \frac{h^3r^2}{3h^2} = \frac{\pi r^2}{3} h$$

$$z\,d\forall = z\,\frac{\pi z^2 r^2}{h^2}\,dz$$

$$\int z\,d\forall = \int_0^h \frac{\pi z^3 r^2}{h^2}\,dz = \frac{\pi z^4 r^2}{4h^2}\Big|_0^h = \frac{\pi h^4 r^2}{4h^2}$$

$$= \frac{\pi h^2 r^2}{4}$$

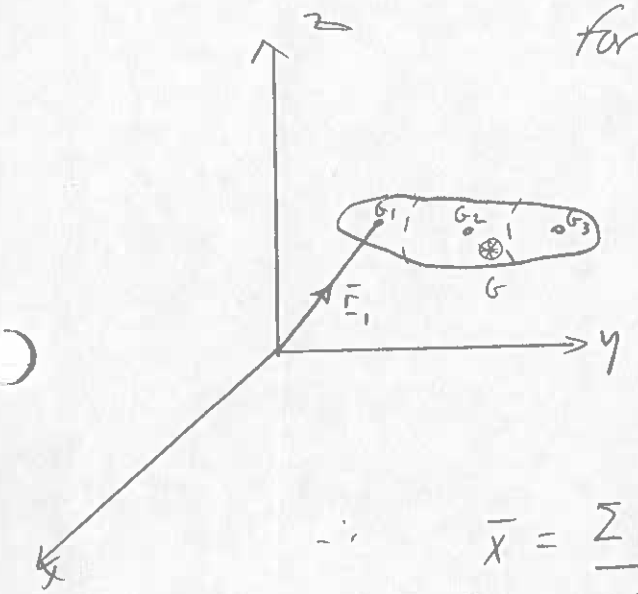
$$\therefore \quad \bar{z} = \frac{\int z\,d\forall}{\int d\forall} = \frac{\frac{\cancel{\pi} h^{\cancel{2}} \cancel{r^2}}{4}}{\frac{\cancel{\pi} \cancel{r^2} \cancel{h}}{3}} = \underline{\underline{\frac{3h}{4}}} \quad \longleftarrow$$

5-16-1

Centroids of Composite Bodies

When a body can be divided into several elements that are easy to analyze, we can use the method of composite bodies.

Consider body shown, each part has centroid $\bar{\underline{r}}_i$ and mass m_i. Let coordinates of G, the mass center for the entire body be $\bar{\underline{r}} = (\bar{x}, \bar{y}, \bar{z})$.



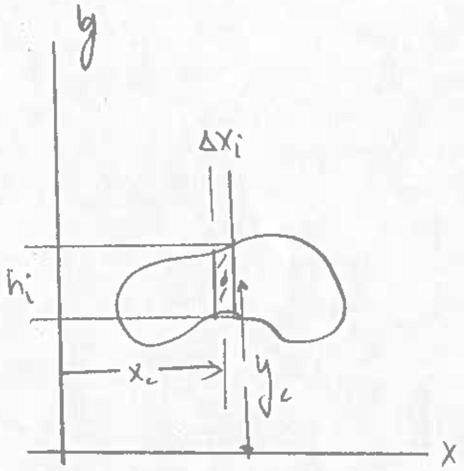
Can find G by method of moments

$$\sum m_i \bar{\underline{r}} = \sum m_i \bar{\underline{r}}_i$$

$$\therefore \quad \bar{x} = \frac{\sum m_i \bar{x}_i}{\sum m_i}, \quad \bar{y} = \frac{\sum m_i \bar{y}_i}{\sum m_i}, \quad \bar{z} = \frac{\sum m_i \bar{z}_i}{\sum m_i}$$

if a hole or cavity is present, it is treated as a negative mass. Similar formulas can be written for composite line areas and volumes.

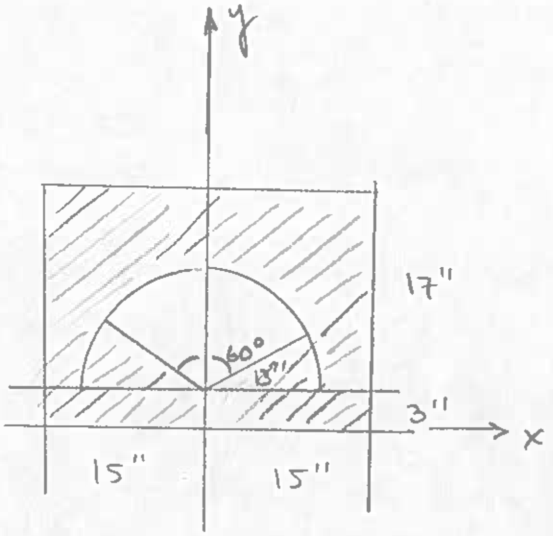
Method can be used to estimate centroid of body whose shape cannot be described in terms of simple geometric shapes.

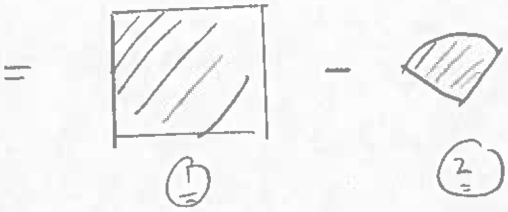


$$\bar{X} \approx \frac{\sum x_{c_i} A_i}{\sum A_i} = \frac{\sum x_{c_i} h_i \Delta x_i}{\sum h_i \Delta x_i}$$

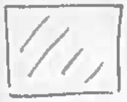
Approximation improves as $\Delta x_i \rightarrow 0$

5-46 FIND $\bar{y}$ OF SHADED AREA





SECTION 1

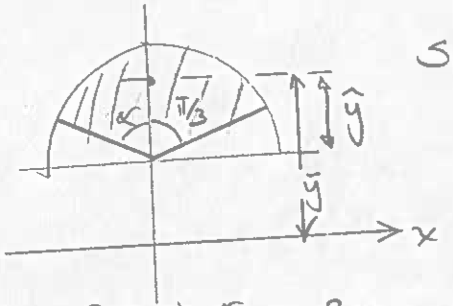


$A = 20(30) = 600$

$\bar{x} = 0$

$\bar{y} = 10$

SECTION 2



See Sample Problem 5-3 pg 251

$$\hat{y} = \frac{2}{3} r \frac{\sin\alpha}{\alpha} = \frac{2}{3}(13)\frac{\sin \pi/3}{\pi/3}$$

$$\bar{y} = 3 + \frac{2}{3}(13)\frac{\sin \pi/3}{\pi/3} = 10.17 \text{ in}$$

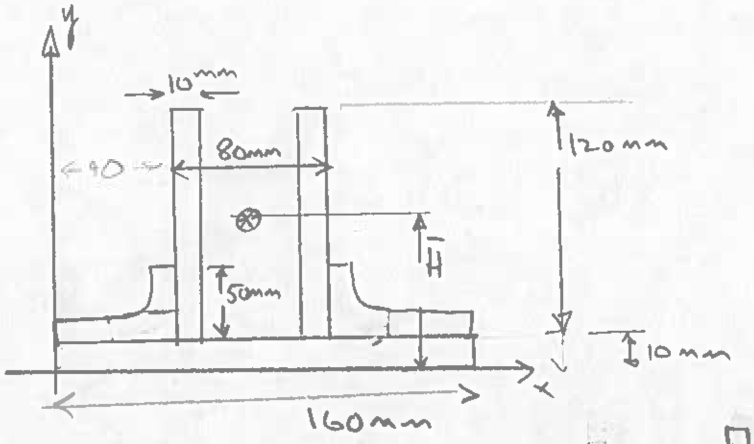
$$A = r^2\alpha = (13)^2\left(\frac{\pi}{3}\right) = 177 \text{ in}^2$$

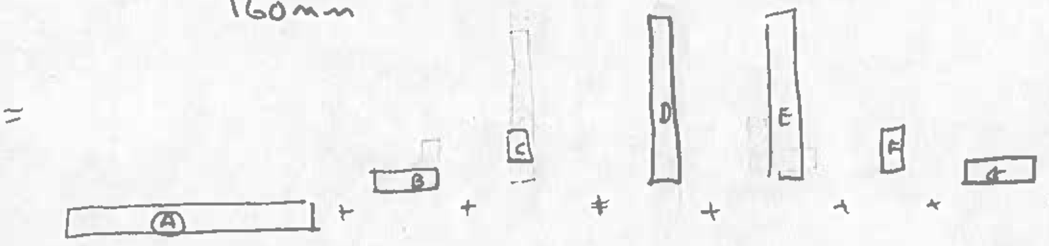
SECTION	A	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
①	600	0	10	0	6000
②	-177	0	10.17	0	-1800.09
Σ	423			0	4199

$\bar{x} = 0$

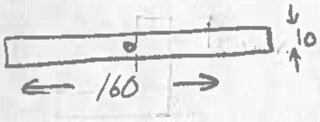
$$\bar{y} = \frac{4199}{423} = \underline{\underline{9.93 \text{ in}}} \leftarrow$$

5-51 Find $\bar{H}$ of composite section





Ⓐ

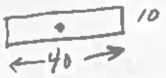


$A = 1600$

$\bar{x}_c = 80$

$\bar{y}_c = 5$

Ⓑ, Ⓖ

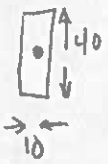


$A = 400$

$\bar{x}_c = 20$

$\bar{y}_c = 5$

Ⓒ, Ⓕ

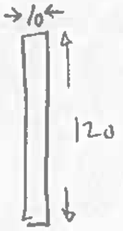


$A = 400$

$\bar{x}_c = 5$

$\bar{y}_c = 20$

Ⓓ, Ⓔ



$A = 1200$

$\bar{x}_c = 5$

$\bar{y}_c = 60$

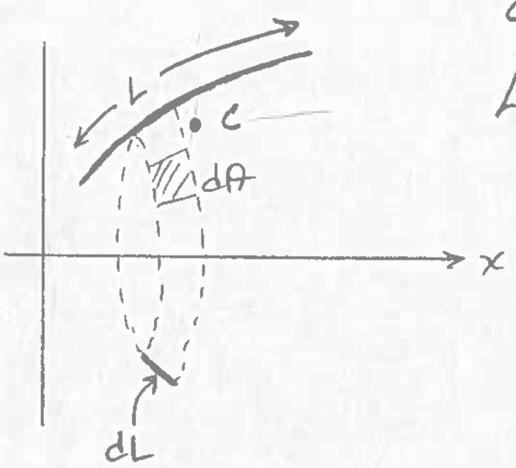
SECTION	A	$\bar{x}$	$\bar{y}$	$A\bar{y}$
A	1600	80	5	8000
B	400	20	15	6000
C	400	35	40	16 000
D	1200	45	70	84 000
E	1200	115	70	84 000
F	400	125	40	16000
G	400	140	15	6000
Σ	5600			220,000

$$\bar{y} = \frac{A\bar{y}}{A} = \frac{220,000}{5600}$$

$$= \underline{39.3\text{mm}}$$

Areas & Volumes of Rotation

Method for calculating surface <u>area</u> generated by revolving a curve about an axis (theorem of Pappus)



C is centroid of $L = (\bar{x}, \bar{y})$

Line segment has length L, generates a surface when revolved about the x axis. Surface area

$$dA = dL\,y\,d\theta$$

Surface area for whole line is

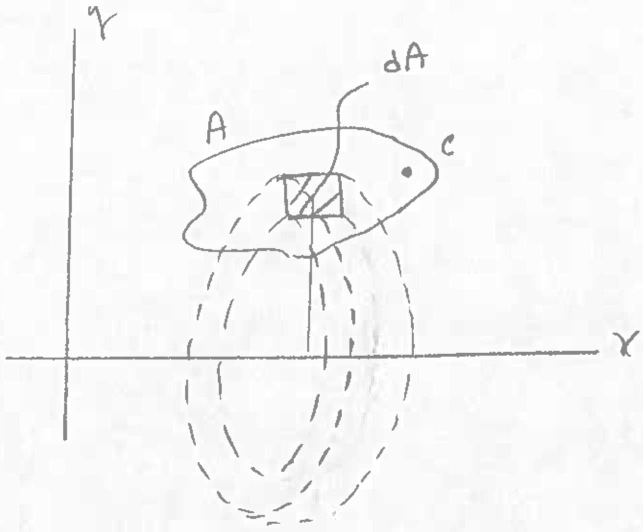
$$A = \int_0^{2\pi} \int_L y\,dL\,d\theta = 2\pi \int_L y\,dL$$

But by definition $\bar{y}L = \int_L y\,dL$ so area of surface of revolution is

$$A = 2\pi \bar{y} L$$

(For partial rotation $A = \theta \bar{y} L$ (in $\overset{\theta}{\text{radians}}$))

For a volume a similar formula exists



C= centroid of A $(\bar{x},\bar{y})$

Area A generates a Volume when rotated about x-axis

Volume $d\forall = dA\,y\,d\theta$

Volume of whole object is

$$V = \int_0^{2\pi}\int_A y\,dA\,d\theta$$

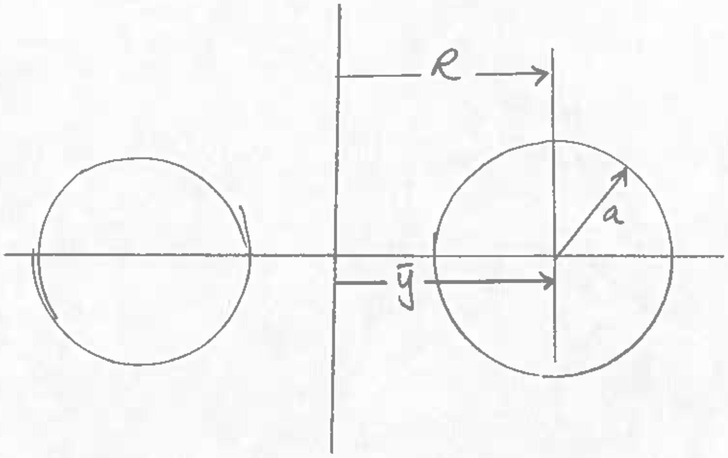
$$= 2\pi \int_A y\,dA$$

$$= 2\pi\,\bar{y}\,A$$

For partial rotation

$$V = \int_0^{\theta}\int_A y\,dA = \theta\,\bar{y}\,A$$

5-74 Find $\forall$ and A (surface area) of torus shown



Surface area

① Find circumference of circle $L = 2\pi a$

② Apply Pappus theorem to find area

$$A = 2\pi \bar{y} L = 2\pi R L = 2\pi R \, 2\pi a$$
$$= 4\pi^2 R a$$

Volume

① Find area of circle $\underline{\underline{A = \pi a^2}} \longleftarrow$

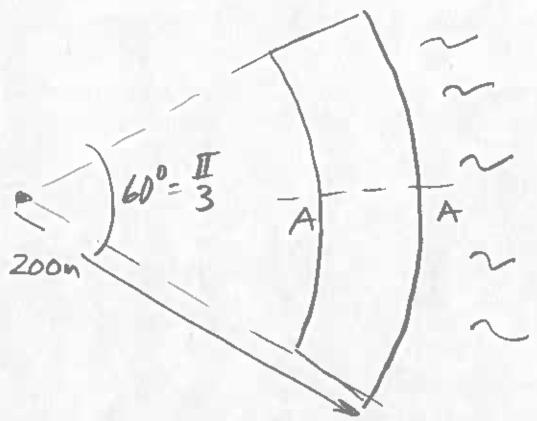
② Apply Pappus' theorem to find volume

$$V = 2\pi \bar{y} A = 2\pi R \pi a^2$$
$$= \underline{\underline{2\pi^2 R a^2}} \longleftarrow$$

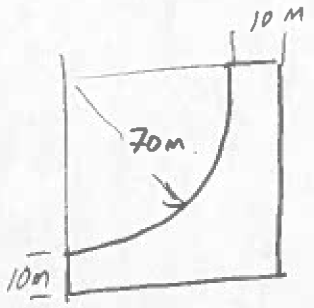
5-92 Find mass m of concrete arch dam.

Concrete has density of 2.40 Mg/m³ = 2400 kg/m³

Plan View



Section A-A

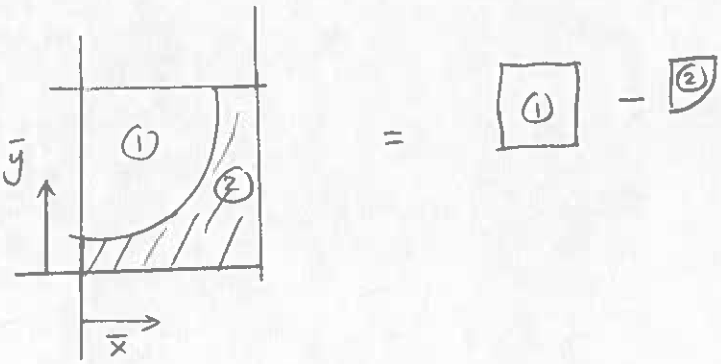


Steps

① Need volume of dam

② Use section A-A. Find Area & centroid.

③ Use Pappus' theorem to get volume.



Section ①

$A = (80)(80) = 6400 m^2$

$\bar{X} = 40$

Section ②

$$A = \frac{\pi (70)^2}{4} = 3848.4 m^2$$

$$\bar{X} = \frac{4(70)}{3\pi} = 29.70$$

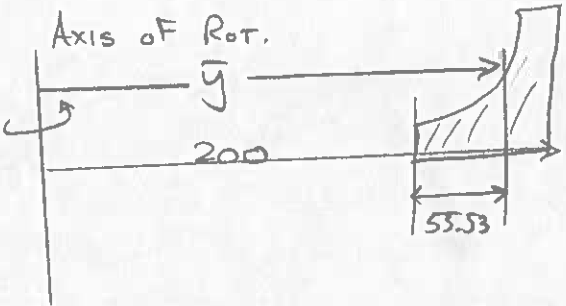
(Table D3)

Section	A	$\bar{x}$	$A\bar{x}$
1	6400	40	256000
2	-3848.4	29.70	-114297
	Σ 2551.6		141703

$$\bar{x} = \frac{141703}{2551.6} = 55.53$$

$$A = 2551.6$$

Now use Pappus' theorem



$$\bar{y} = 200 - 80 + 55.53 = 175.53$$

$$\mathcal{V} = \theta \bar{y} A = \frac{\pi}{3}(175.53)(2551.6) = 469034.13 \, m^3$$

$$\text{mass} = \rho V = 2400 \, kg/m^3 \cdot 469034.13 \, m^3$$

$$= 1.126 \cdot 10^9 \, kg$$

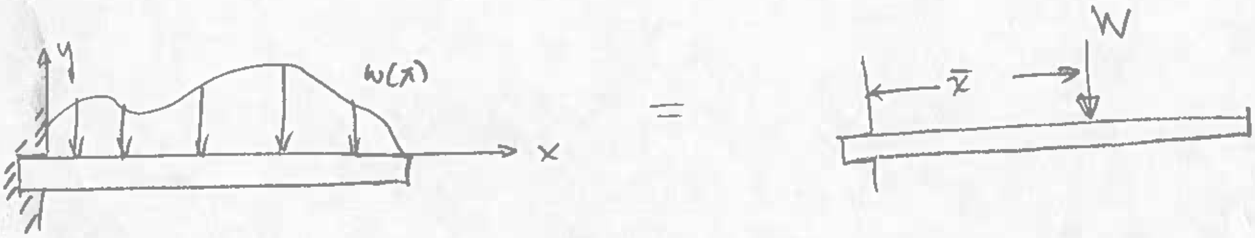
Statically determinate beams can be analyzed using equilibrium equations alone.

Statically indeterminate beams also need load-deformation properties to be analyzed.

In this <u>course</u> we only consider statically determinate beams

Distributed Loads

Expressed as load intensity, $w(x)$, which is the force per unit length of beam and is a function of location



For static analysis of external effects, one can determine the magnitude of the load as $W = \int w(x)\,dx$.

The load acts at its centroid $\bar{x}$.

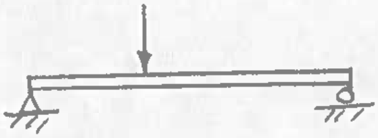
$$\bar{x} = \frac{\int x\, w(x)\,dx}{\int w(x)\,dx}$$

Beams - External Effects

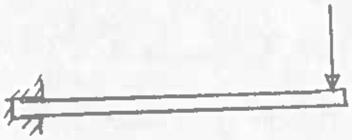
A member that offers resistance to bending from an applied load is called a <u>beam</u>

Beam analysis uses entire beam to find external reactions, then uses a portion of the beam to calculate internal (shear, bending, torsion) effects

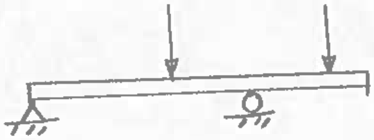
<u>Types of Beams</u>



simple

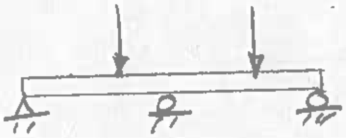


cantilever

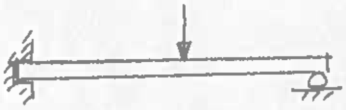


combination

Statically determinate



continuous

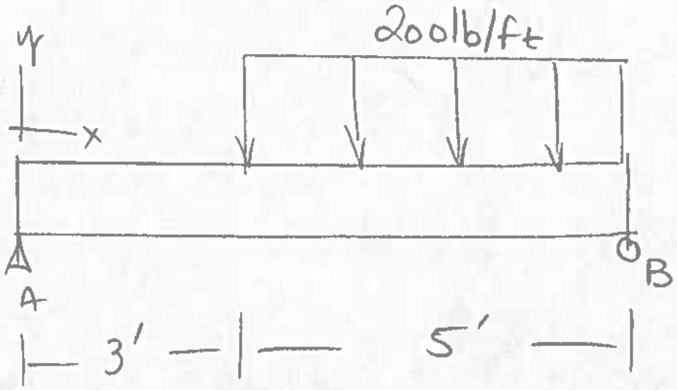


end-supported cantilever

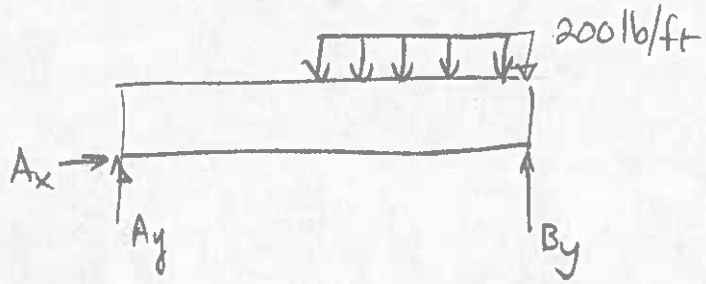
fixed

statically indeterminate

5.93 Find reactions at A & B



FBD Beam



$$\Sigma F_x = 0 = A_x$$

$$\Sigma F_y = 0 = A_y + B_y - \int_3^8 200\ \text{lb/ft}\, dx$$

$$= A_y + B_y - 1000\ \text{lb}$$

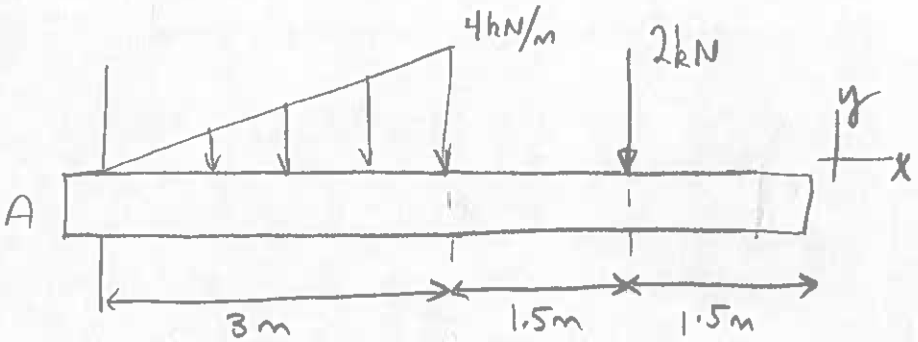
$$\circlearrowleft \Sigma M_A = 0 = 8(B_y) - \int_3^8 200\ \text{lb/ft}\ x\ dx$$

$$= 8B_y - 200 \frac{x^2}{2}\Big|_3^8 = 8B_y - 100(64) + 100(9)$$

$$B_y = \frac{100(64) - 100(9)}{8} = 687.5\ \text{lb}$$

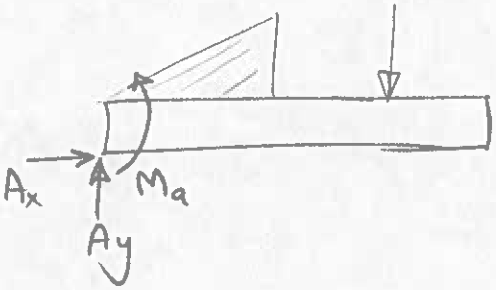
$$A_y = 1000 - 687.5 = 312.5\ \text{lb}$$

5-96 Find reactions at A



Distributed force is $\frac{4}{3}x$ kN/m

FBD



$$\underline{\underline{\Sigma F_x = 0 = A_x}} \longleftarrow A_x$$

$$\Sigma F_y = 0 = A_y - 2kN - \int_0^3 \frac{4}{3} x\, dx$$

$$A_y = 2kN + \frac{4}{3}\frac{x^2}{2}\Big|_0^3 = \underline{\underline{8kN}} \longleftarrow A$$

$$\circlearrowleft \Sigma M_A = 0 = -\int_0^3 \frac{4}{3}x^2 dx - 2kN(4.5) + M_A$$

$$= -\frac{4}{3}\overset{3\cdot 3\cdot 3}{\frac{x^3}{3}}\Big|_0^3 - 9\text{ kN}\cdot\text{m} + M_A$$

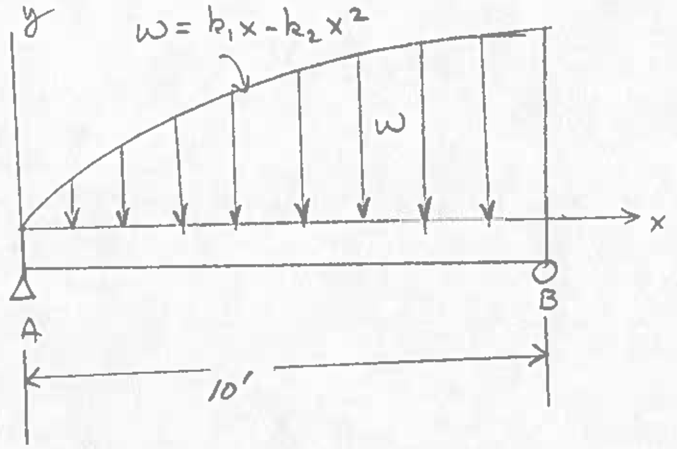
$$= -21 + M_A$$

$$\therefore \underline{\underline{M_A = 21 N\cdot m}} \longleftarrow M_A$$

5-105) $x = 10, \; w = 300 \text{ lb/ft}$

$x = 0, \; \frac{dw}{dx} = 50 \text{ lb/ft/ft}$

Find reactions at A&B



$$x = 10, \quad w = 300 \text{ lb/ft} = k_1 10 - k_2 100$$

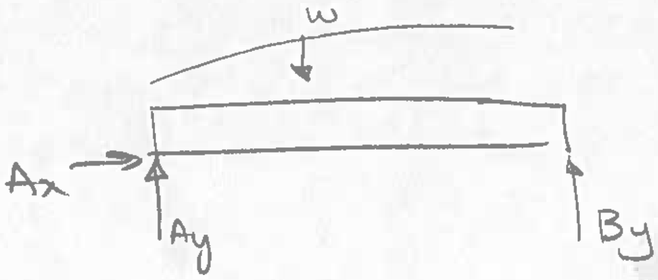
$$\frac{dw}{dx} = k_1 - 2k_2 x \qquad @\, x = 0, \quad \frac{dw}{dx} = 50$$

$$\therefore k_1 = 50$$

$$300 \text{ lb/ft} = 50(10) - k_2 100$$
$$= 500 - k_2 100 \quad \therefore \quad k_2 = 2$$

Thus $w(x) = 50x - 2x^2$

FBD BEAM



$$\Sigma F_x = 0 = A_x \longleftarrow A_x$$

$$\Sigma F_y = 0 = A_y + B_y - \int w \, dx$$

$$↺ \Sigma M_A = 0 = 10(B_y) - \int x w \, dx$$

Evaluate integrals

$$\int w\,dx = \int_0^{10} 50x - 2x^2\,dx$$

$$= \frac{50x^2}{2} - \frac{2x^3}{3}\Big|_0^{10} = \frac{50(10)^2}{2} - \frac{2(10)^3}{3}$$

$$= \frac{5000}{2} - \frac{2000}{3}$$

$$= 2500 - 666.6 = 1833.3 \text{ lbs}$$

$$\int xw\,dx = \int_0^{10} 50x^2 - 2x^3\,dx$$

$$= \frac{50x^3}{3} - \frac{2x^4}{4} = \frac{50(10)^3}{3} - \frac{2(10)^4}{4}$$

$$= \frac{50{,}000}{3} - \frac{20{,}000}{4} = 16{,}666.6 - 5000$$

$$= 11{,}666.6 \text{ ft.lb}$$

Substitute into equilibrium equations

$$\circlearrowright \Sigma M_A = 10 B_y - 11{,}666.6$$

$$\therefore B_y = \frac{11{,}666.6}{10} = \underline{\underline{1166.6 \text{ lbs}}} \longleftarrow$$

$$A_y = 1833.3 - 1166.6 = \underline{\underline{666.7 \text{ lb}}} \longleftarrow$$

$$+\circlearrowleft \Sigma M_A = 0 = 30\text{ kip}\cdot\text{ft} - 8(12) + \int_5^{10} x\left(2+\frac{3}{5}x-3\right)dx + M_A$$

$$M_A = -30 + 96 - \int_5^{10} 2x + \frac{3x^2}{5} - 3x \; dx$$

$$= -30+96 - \left(\frac{2x^2}{2} + \frac{3x^3}{15} - \frac{3x^2}{2}\right)\Bigg|_5^{10}$$

$$-\left(\frac{\cancel{2}(100)}{2} + \frac{3(1000)}{15} - \frac{3(100)}{2} - \frac{2(25)}{2} - \frac{3(125)}{15} + \frac{3(25)}{2}\right)$$

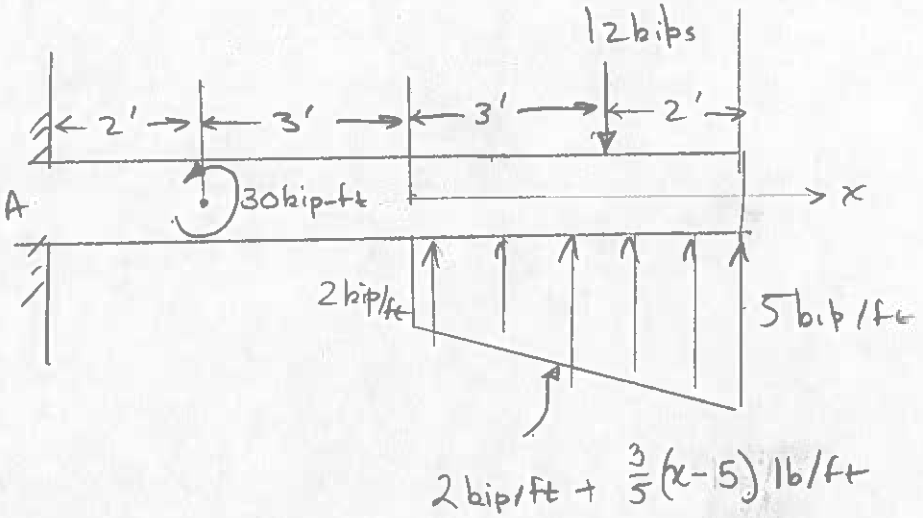
$$-\left(100 + \cancel{200} - \cancel{150} - \cancel{25} - \cancel{25} + 37.5\right)$$

$$= -30+96 - 137.5$$

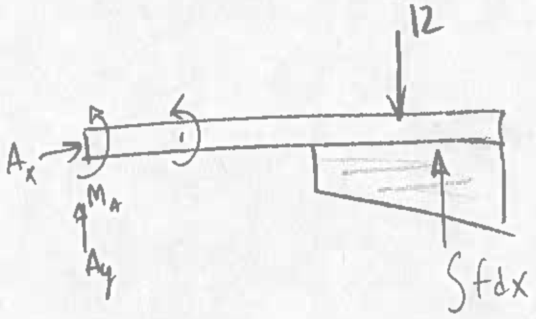
$$= \underline{\underline{-71.5\text{ kip-ft}}} \longleftarrow M_A$$

5-102

FIND REACTIONS AT A



FBD



$\Sigma F_x = 0 = \underline{A_x}$ ← A_x

$\Sigma F_y = 0 = -12 + A_y + \int f dx$

$0 = -12 + A_y + \int_5^{10} 2 + \frac{3}{5}(x-5)\,dx$

$A_y = 12 - \int_5^{10} 2 + \frac{3}{5}(x-5)\,dx$

$12 - \left(2x + \frac{3}{5}\frac{x^2}{2} - 3x \Big|_5^{10}\right)$

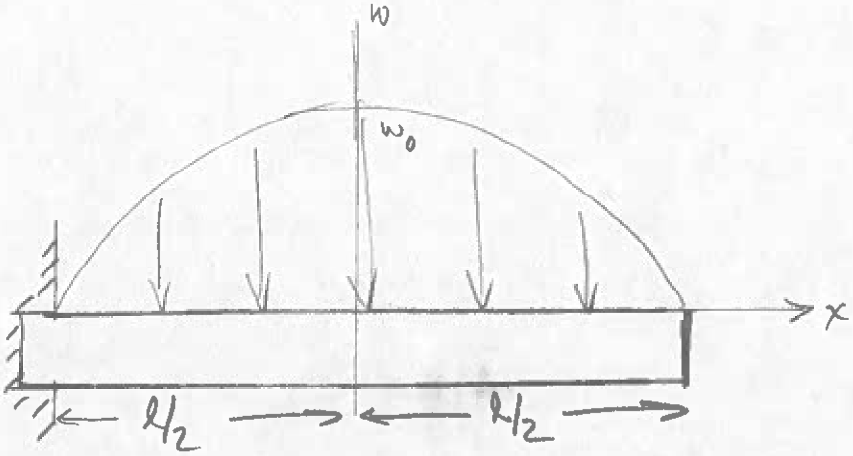
$= 12 - \left(2(10) + \frac{3(100)}{10} - 3(10) - 2(5) - \frac{3(25)}{10} + 3(5)\right)$

$= 12 - (\overset{10}{\cancel{20}} + \cancel{30} - \cancel{30} - \cancel{10} - \cancel{7.5} + \overset{7.5}{\cancel{15}})$

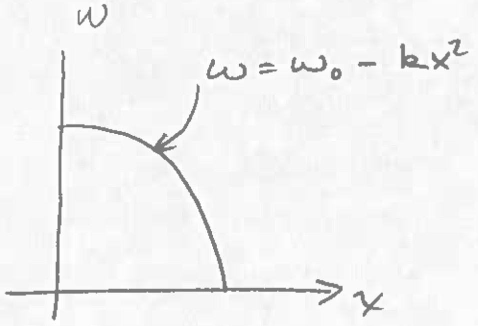
$= 12 - 17.5 \; \underline{\underline{= -5.5 \text{ kips}}}$ ← A_y

5-107

FIND REACTIONS AT A



By symmetry distributed load can be replaced by point load at $l/2$. Find magnitude of point load by integration



$$0 = w_0 - k\frac{l^2}{4}$$

$$\therefore k = \frac{4w_0}{l^2}$$

$$\therefore w(x) = w_0\left(1 - \frac{4}{l^2}x^2\right)$$

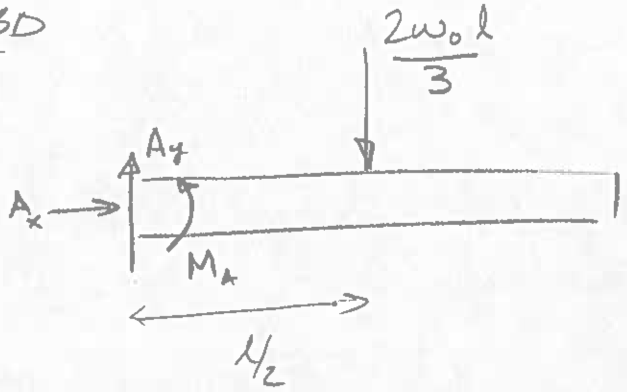
$$2\int w(x)\,dx = \text{Magnitude of load}$$

$$= 2\int_0^{l/2} w_0\left(1 - \frac{4}{l^2}x^2\right)dx$$

$$= 2w_0\left(x - \frac{4x^3}{l^2 3}\right)\Bigg|_0^{l/2} = \cancel{2}w_0\frac{l}{\cancel{2}} - \frac{\cancel{8}w_0 l^3/\cancel{8}}{l^2 3}$$

$$= w_0 l - \frac{w_0 l^{\cancel{3}}}{\cancel{l^2}3} = \frac{2w_0 l}{3}$$

FBD



$$\Sigma F_x = 0 = \underline{\underline{A_x}} \longleftarrow A_x$$

$$\Sigma F_y = 0 = A_y - \frac{2w_0 l}{3}$$

$$\underline{\underline{A_y = \frac{2w_0 l}{3}}} \longleftarrow A_y$$

$$\circlearrowleft + \; \Sigma M_A = 0 = M_A - \frac{l}{\cancel{2}}\left(\frac{\cancel{2}w_0 l}{3}\right)$$

$$\underline{\underline{M_A = \frac{w_0 l^2}{3}}} \longleftarrow M_A$$

Friction

Previously, forces of action / reaction have been taken to act normal to contacting surfaces. This assumption is valid for smooth surfaces. Real (rough) surfaces can also generate tangential forces where they are in contact - these forces are known as friction forces. Whenever a tendency exists for two contacting surfaces to slide relative to each other, the friction forces developed oppose this tendency, ie. they oppose the (impending) motion.

TYPES OF FRICTION

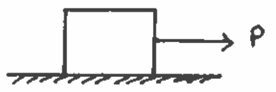
* Dry Friction - friction between two unlubricated surfaces. Friction force is developed to oppose any tendency of surfaces to slide. Also called Coulomb friction.

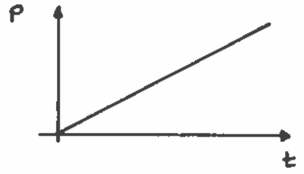
Fluid Friction - friction developed when adjacent layers of fluid are moving at different velocities. Friction forces developed depend on the relative velocity between the fluid layers, and the viscosity of the fluid.

Internal Friction - friction found in all solid materials subjected to cyclic loading. For highly elastic materials, the recovery from deformation occurs with very little loss of energy from internal friction. For materials with low elasticity this energy loss may be significant.

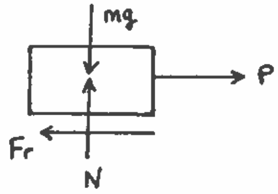
Dry Friction

Block of mass m on rough surface. Apply horizontal force P varying continuously from zero upto a value to give the block a significant velocity





FBD :



Fr = friction force, opposes motion or tendency to motion of block.

- When P = 0, equilibrium → Fr = 0.
- As P is increased with no motion of block, equilibrium => Fr = P.
- As P is increased further, reach a value of P such that block starts to move. At this time friction force also drops abruptly to an essentially constant value.

The region upto the point of slippage (impending motion) is the range of static friction. Value of Fr is found from equilibrium equations and is less than some maximum value, $(Fr)_{max}$.
When slipping is about to occur the friction force equals its maximum value

$$(Fr)_{max} = \mu_s N$$

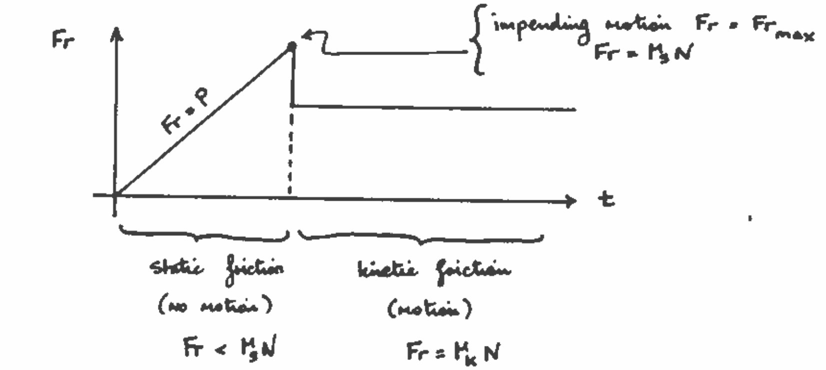
μ_s = coefficient of static friction.

When slipping occurs, friction is known as kinetic friction, and has a constant value given by

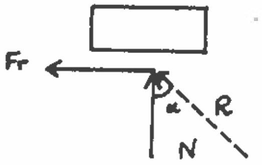
$$F_r = \mu_k N$$

μ_k = coefficient of kinetic friction. Usually $\mu_k < \mu_s$.

Behavior of friction force is shown below :



Resultant force R on block by supporting surface is combination of Fr and N. Angle α is given by $\tan\alpha = F_r/N$.

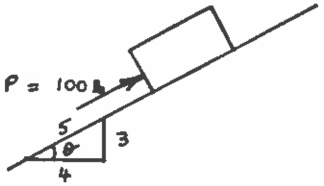


When $F_r = F_{r_{max}} = \mu_s N$ then α reaches a maximum value ϕ_s and $\mu_s = \tan\phi_s$. The angle ϕ_s is known as the <u>angle of static friction</u>.

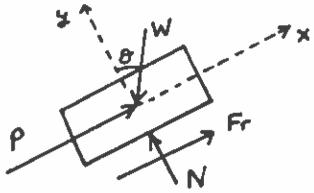
If motion is impending, R must lie on a right circular cone of vertex angle $2\phi_s$ (<u>cone of static friction</u>), if motion is not impending R will be within the cone.

<u>EXAMPLE 1</u> : <u>Friction</u>

A 100 lb force acts on a 300 lb block on an inclined plane. If $\mu_s = \cdot 25$ and $\mu_k = \cdot 2$, determine whether the block is in equilibrium, and find the value of the friction force.



First determine value of Fr to maintain equilibrium.



$\Sigma F_x = 0 \Rightarrow$

$P + F_r - W\sin\theta = 0$

$100 + F_r - 300\,(3/5) = 0$

$\underline{\underline{F_r = 80 \text{ lbs}}}$

$\Sigma F_y = 0 \Rightarrow$

$N - W\cos\theta = 0$

$N - 300\,(4/5) = 0$

$\underline{\underline{N = 240 \text{ lbs}}}$

Max. static friction force $(F_r)_{max} = \mu_s N = \cdot 25\,(240) = 60$ lbs. Since Fr required for equilibrium is greater than $(F_r)_{max}$, block will move (down plane).

Friction force on block is kinetic friction $F_r = \mu_k N = \cdot 2(240)$ $\underline{\underline{= 48 \text{ lbs}}}$

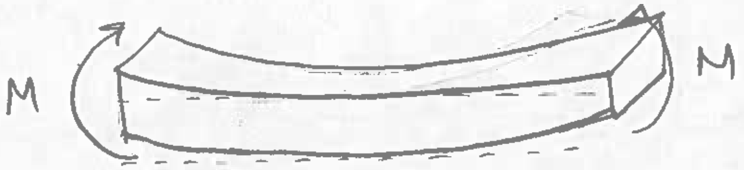
Resultant force on block $R = \Sigma F_x = P + F_r - W\sin\theta$

$= 100 + 48 - 300\,(3/5)$

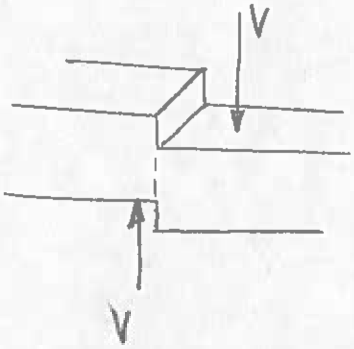
$= \underline{-32 \text{ lbs}}$ <u>(ie. down plane)</u>

Beams - Internal Effects

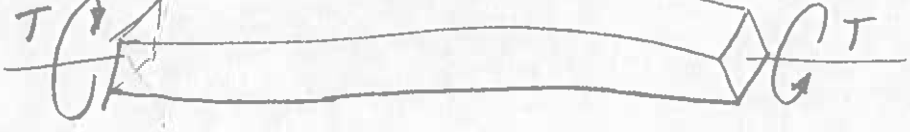
In addition to supporting tension and compression a beam can also resist bending, shear, and torsion



Bending

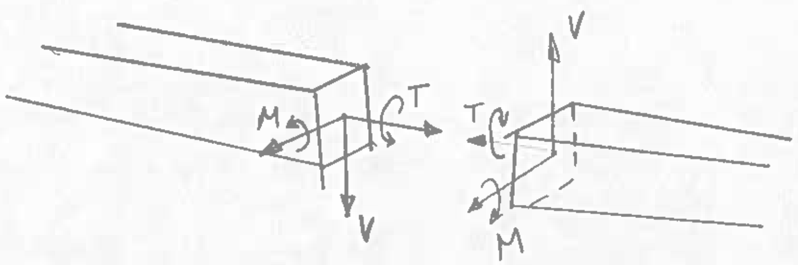


Shear



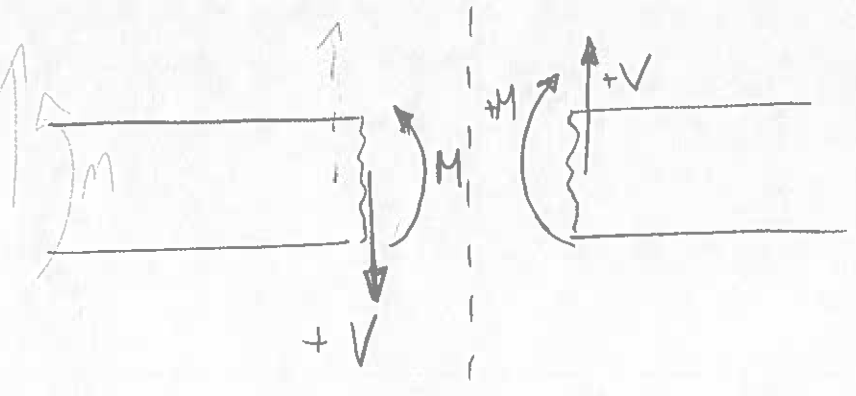
Torsion

The combined loading consists of all 3 components



Principle of action-reaction means that forces/moments acting on each side of an internal surface are equal and opposite

For forces/moments caused by in-plane loading only shear force V and bending moment M are of interest. Convention for positive V, M are



Generally represent V & M as positive in FBD and let algebraic signs of computed values indicate proper directions

Variation of V and M along a beam gives information necessary for design of the beam.

Typically results are shown graphically as shear and bending moment diagrams, where V and M are plotted against x (beam coordinate)

Construction of shear & bending moment diagrams:

1) Find all external reactions on beam

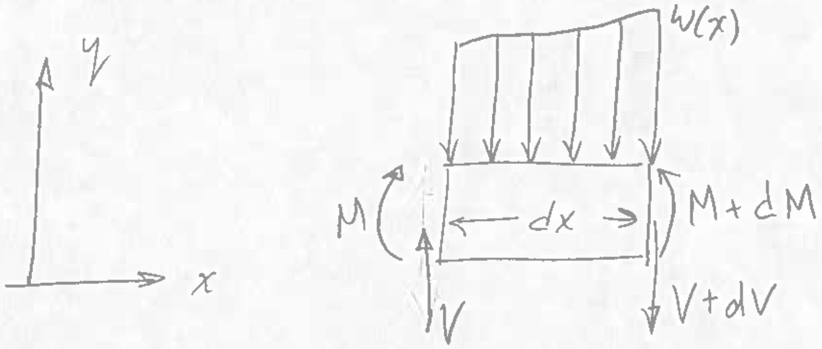
2) Isolate a section of beam and apply equilibrium to that portion to find V and M.

3) Move section boundary, repeat step (2) to obtain V and M at new location. By continually moving section one can "draw" the diagrams

Avoid using a section that coincides with a concentrated load or couple because V or M will be discontinuous at this location

General relations for V, M

Beam loaded by distributed load w(x).
Isolate small portion of beam & draw FBD



$\Sigma F_y = 0 \Rightarrow V - w(x)\,dx - (V + dV) = 0$

$\Rightarrow -dV - w(x)\,dx = 0$

$\therefore \dfrac{dV}{dx} = -w(x)$ (slope of V is equal to negative of load)

$$\int dV = \int -w(x)\,dx$$

$$V = V_0 - \int_{x_0}^{x} w(x)\,dx \qquad (V_0 = V(x_0))$$

$\therefore$ V is shear force at x_0 less area under load curve from x_0 to x.

Often the integration can be performed by inspection.

Now consider the moment balance

$\Sigma M_L = 0$ (Left edge of element)

$$M + (w\,dx)\left(\frac{dx}{2}\right) + (V + dV)\,dx - (M + dM) = 0$$

$$M + \frac{w}{2}\cancel{dx^2} + V\,dx + \cancel{dV\,dx} - M - dM = 0$$

neglect higher order differentials

$$V\,dx - dM = 0$$

$$\frac{dM}{dx} = V$$

∴ Slope of bending moment is equal to shear force

$$\int dM = \int V\,dx$$

$$M = M_0 + \int_{x_0}^{x} V\,dx \qquad (M_0 = M(x_0))$$

∴ bending moment at x is the bending moment at x_0 plus area under shear force curve from x_0 to x

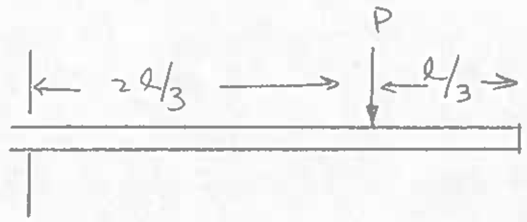
The shear & bending equations can be combined to give

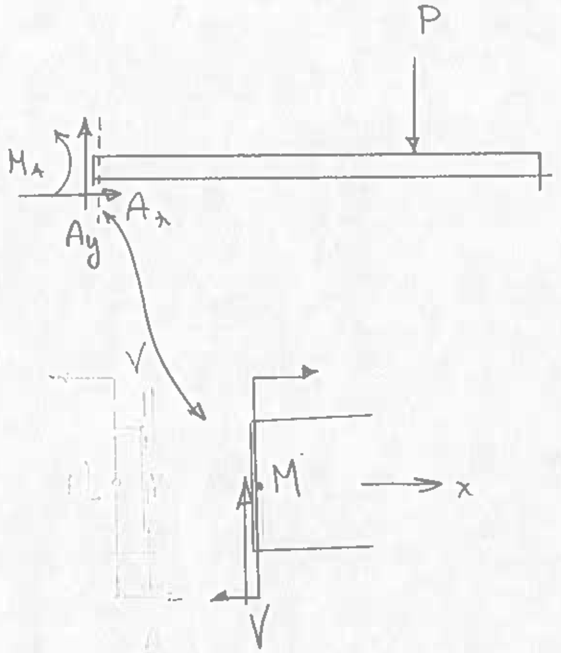
$$\frac{d^2M}{dx^2} = -\omega(x)$$

For point loads, change in V at location of load equals magnitude of point load, then V is constant until next point load.

Max/min values of M occur when $V=0$ and $\frac{dV}{dx} \neq 0$. Critical values of M also occur when V crosses the zero axis discontinuously (point loads)

5-113 Find V&M in beam shown





(SIGN CONVENTION FOR SHEAR & BENDING

$$\Sigma F_x = A_x = 0$$

$$\Sigma F_y = A_y - P = 0$$

$$A_y = P$$

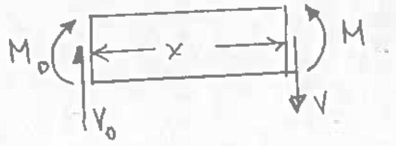
$$\circlearrowright \Sigma M_A = 0 = M_A - P\frac{2l}{3}$$

$$M_A = \frac{P2l}{3}$$

$$M_0 = -M_A = -\frac{2Pl}{3}$$

$$V_0 = A_y = P$$

$$0 < x < \frac{2l}{3}$$



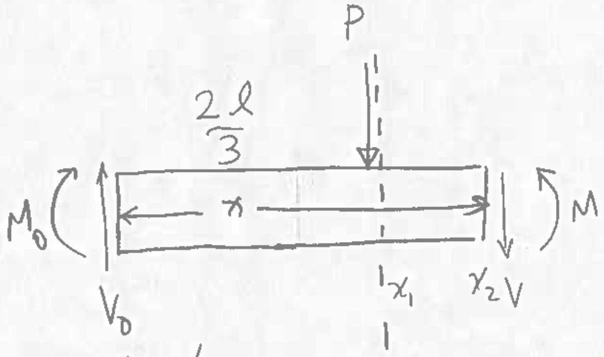
$$V = V_0 + \cancelto{= 0 \text{ for } 0 < x < \frac{2\ell}{3}}{\int_{x_0}^{x} - w(x)\, dx}$$

$$M = M_0 + \int_{x_0}^{x} V\, dx \;=\; M_0 + V_0 (x - x_0)$$

$$\therefore V(x) = V_0 = P$$

$$M(x) = -\frac{2P\ell}{3} + Px$$

$$\frac{2\ell}{3} < x < \ell$$

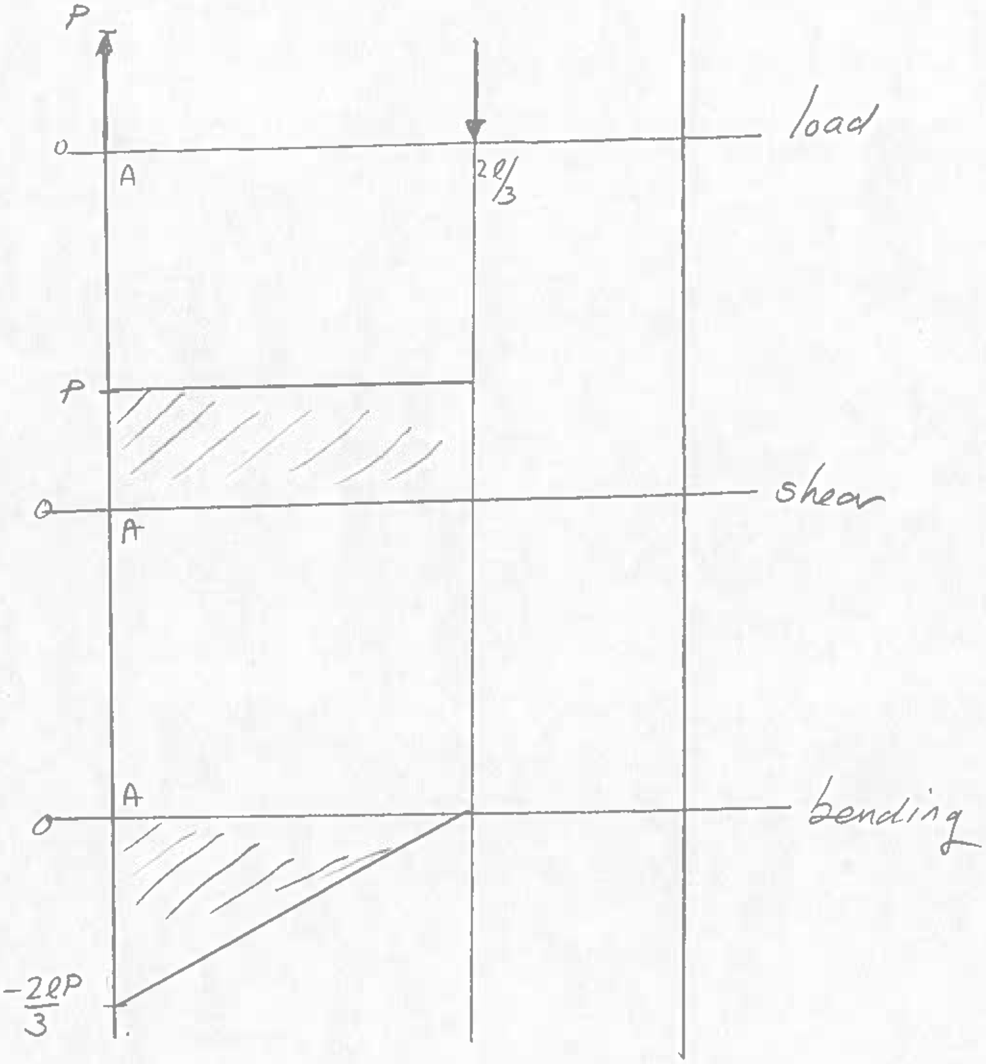


$$V = V_0 - P - \cancelto{0}{\int_{x_1}^{x_2} w\, dx} \;=\; V = P - P = 0$$

$$M = M_0 + M(x_1) + \cancelto{0}{\int_{x_2}^{x_2} V\, dx}$$

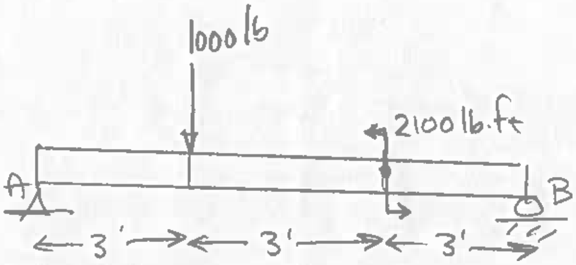
$$= -\frac{2P\ell}{3} + \frac{2P\ell}{3} = 0$$

Use these results to draw diagrams

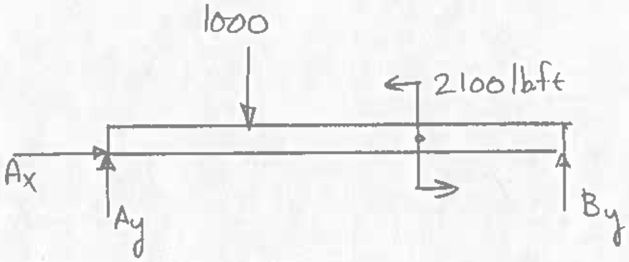


Maximum bending moment at A.

5-116 Find V & M for beam



FBD



$\Sigma F_x = 0 = A_x$

$+\circlearrowleft \Sigma M_A = 0 = 2100 - 1000(3) + B_y(9)$

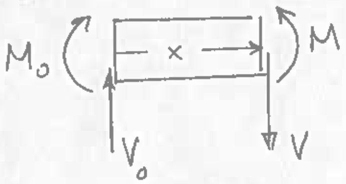
$B_y(9) = 3000 - 2100$

$B_y(9) = 900$

$B_y = 100 \text{ lb}$

$\Sigma F_y = 0 = A_y + B_y - 1000$

$A_y = 900$



$V_1 = V_0 - \int_{x_0}^{x_1} w(dx) = A_y \quad (0 < x < 3)$

$V_2 = V_1 - \int_{x_1}^{x_2} w(dx) = A_y - 1000 \quad (3 < x < 9)$

$V_3 = V_2 - \int_{x_2}^{x_3} w(d) = A_y - 1000 + B_y \quad (x \to 9)$

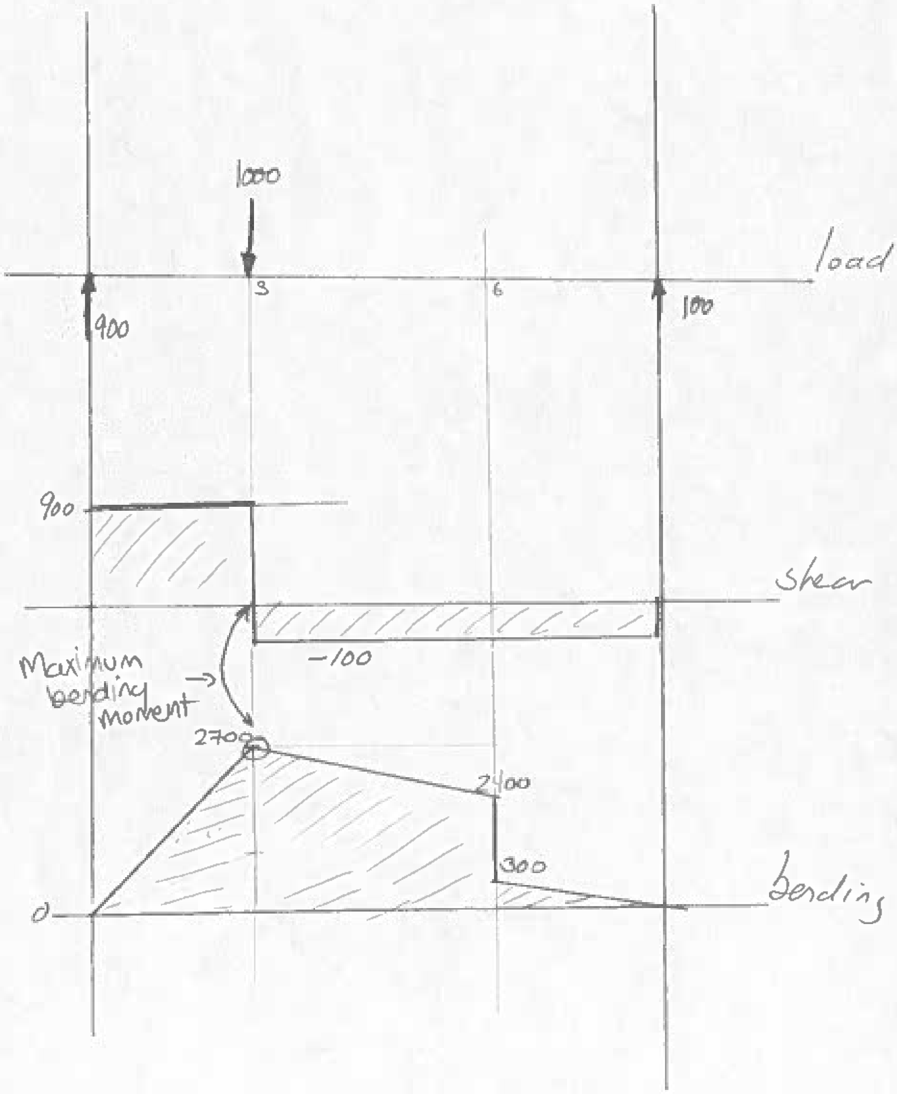
$$M_1 = M_0 + \int_{x_0}^{x_1} V\,dx = 0 + A_y x \quad (0<x<3)$$

$$M_2 = M_1 + \int_{x_1}^{x_2} V\,dx = 3A_y + (A_y-1000)(x-x_1)(3<x<6)$$

$$M_3 = M_2 + M = 3A_y + (A_y-1000)3 - 2100 \quad x \to 6$$

$$M_4 = M_3 + \int_{x_3}^{x_4} V\,dX = 3A_y + (A_y-1000)3 - 2100 + (A_y-1000)(X-X_3$$

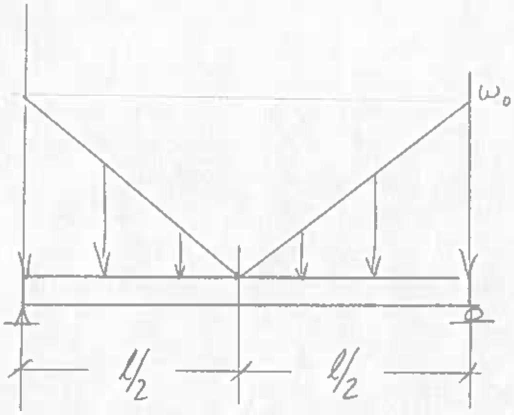
Now draw diagram



Maximum bending moment @ $x=3$

At 1000 lb load

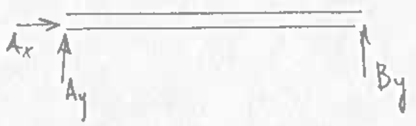
5-121-1 Draw V & M diadrams



$$w(x) = w_0 - \frac{2w_0 x}{l} \quad (0 < x < l/2)$$

$$w(x) = \frac{2w_0 x}{l} - w_0 \quad (l/2 < x < l)$$

Reachins



$\Sigma F_x = 0 = A_x$

$\Sigma F_y = 0 = A_y + B_y - \frac{w_0 l}{2}$

by symmetry

$A_y = B_y = \frac{w_0 l}{4}$

$$V(x) = V_0 - \int_0^x w(x)\,dx$$

$$= A_y - w_0 x + \frac{w_0 x^2}{l} \quad (0 < x < l/2)$$

$$V(x) = V_1 - \int_{x_1}^x w(x)\,dx$$

$$= V_1 - \frac{w_0 x^2}{l} + w_0 x + \frac{w_0 x_1^2}{l} - w_0 x_1$$

$$V_1 = A_y - \frac{w_0 l}{2} + \frac{w_0 l^2}{4l} = \frac{w_0 l}{4} - \frac{2w_0 l}{4} + \frac{w_0 l}{4}$$

$$= 0$$

$\therefore V(x) =$

$$\frac{w_0 l}{4} - w_0 x + \frac{w_0 x^2}{l} \quad 0 < x < l/2$$

$$-\frac{w_0 x^2}{l} + w_0 x + \frac{w_0 l^2}{4l} - \frac{w_0 l}{2} \quad l/2 < x < l$$

$$V(0) = \frac{w_0 l}{4}$$

$$V\left(\frac{l}{2}\right) = 0$$

$$V(l) = -\frac{\cancel{w_0} l^2}{\cancel{l}} + \cancel{w_0} l + \frac{w_0 l^2}{4l} - \frac{w_0 l}{\cancel{}2}$$

$$= \frac{w_0 l}{4} - \frac{2 w_0 l}{4} = -\frac{w_0 l}{4}$$

but at $x = l$ $\quad V = V_0 + \int w\,dx + B_y$

$$\therefore V(l) = 0$$

$$M(x) = M_0 + \int_0^x V\,dx = \frac{w_0 l x}{4} - \frac{w_0 x^2}{2} + \frac{w_0 x^3}{3l} \qquad 0 < x < l/2$$

$$M_1 = M(l/2) = \frac{\cancel{w_0 l^2}}{\cancel{8}} - \frac{\cancel{w_0 l^2}}{\cancel{8}} + \frac{w_0 l^3}{3 \cdot 8 \cdot l} = \frac{w_0 l^2}{24}$$

$$M(x) = M_1 + \int_{x_1}^x V\,dx = \frac{w_0 l^2}{24} + \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3l} + \frac{w_0 l^2 x}{4l} - \frac{w_0 l x}{2} \qquad \frac{l}{2} < x < l$$

$$-\frac{w_0 l^2}{8} + \frac{w_0 l^3}{3 \cdot 8 \cdot l} - \frac{w_0 l^3}{4 \cdot 2 \cdot l} + \frac{w_0 l^2}{4}$$

$$M(x=l) = \frac{w_0 l^2}{24} + \frac{w_0 l^2}{2} - \frac{w_0 l^3}{3l} + \frac{w_0 l^3}{4l} - \frac{w_0 l^2}{2}$$

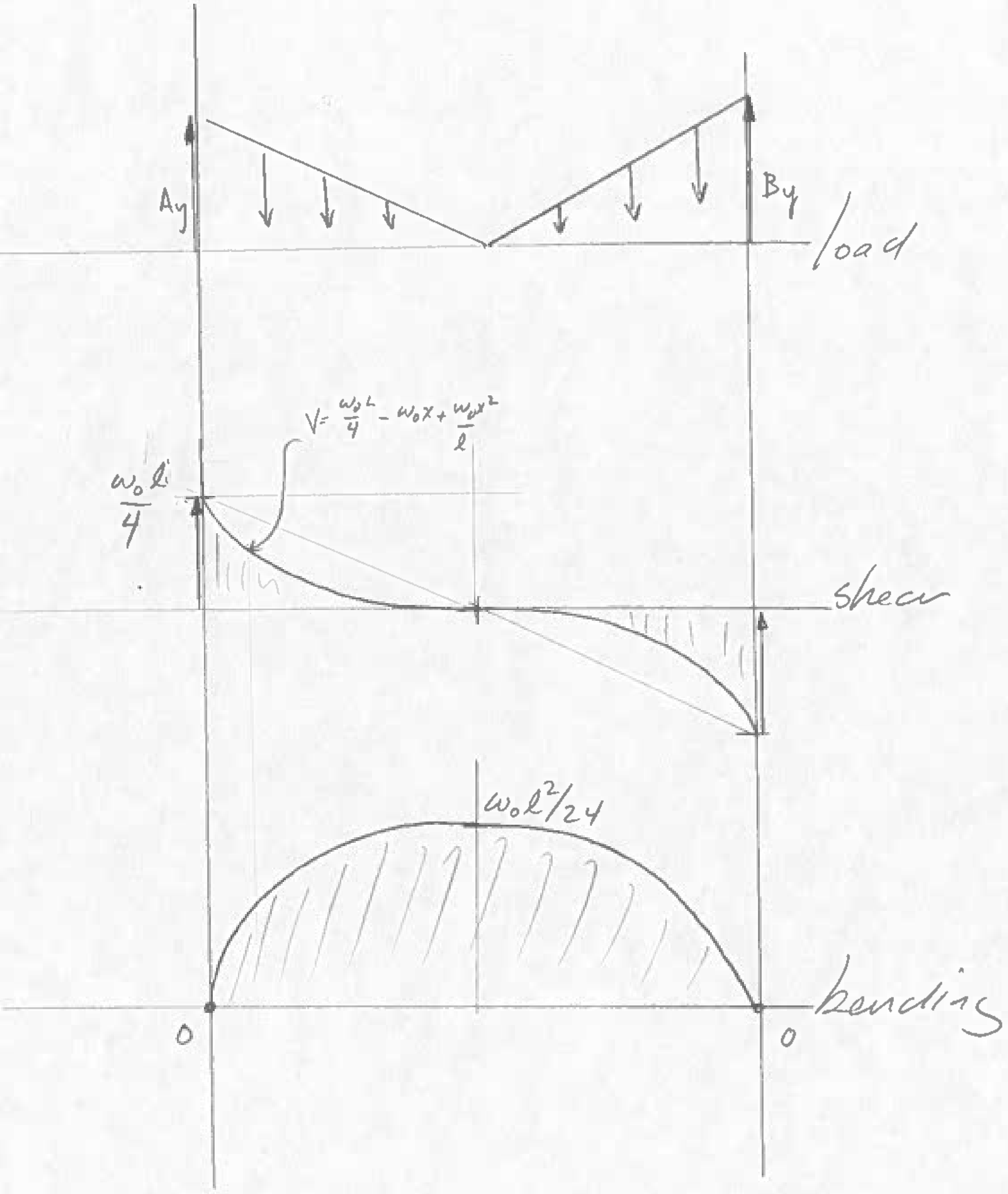
$$-\frac{w_0 l^2}{8} + \frac{w_0 l^3}{24} - \frac{w_0 l^3}{8l} + \frac{w_0 l^2}{4}$$

$$+14 - 14$$

$$= \frac{w_0 l^2}{24} + \frac{12 \cancel{w_0} l^2}{\cancel{24}} - \frac{8 w_0 l^2}{24} + \frac{6 w_0 l^2}{24} - \frac{12 \cancel{w_0} l^2}{\cancel{24}} - \frac{3 w_0 l^2}{24} + \frac{w_0 l^2}{24}$$

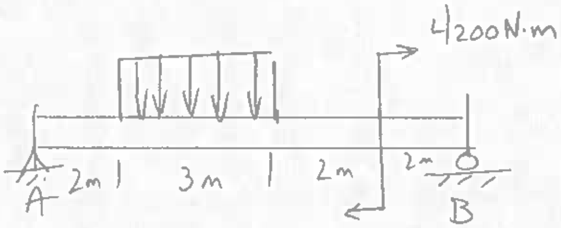
$$-3 w_0 l^2 + 6 w_0 l^2 \quad = 0$$

5-171-7

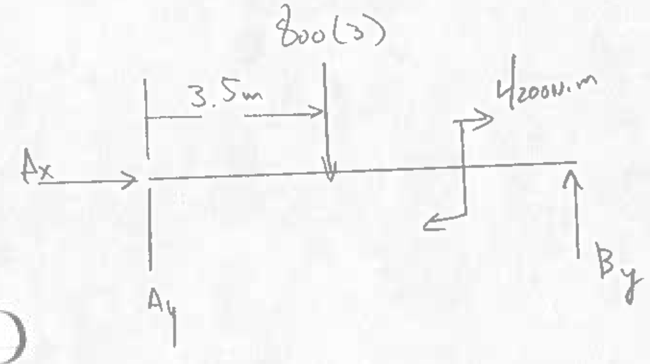


5-121-3

5-131 Draw V & M diagrams



Reactions



$\Sigma F_x = 0 = A_x$

$\Sigma F_y = A_y + B_y - 2400$

$+\circlearrowleft \Sigma M_A = 0 = -4200 + 9B_y - 3.5(2400)$

$9B_y = 3.5(2400) + 4200$

$B_y = \frac{8400 + 4200}{9} = 1400$

$A_y = 1000$

FIND M^*

$V^* = 1000 - \int_2^x 800\,dx$

$= 1000 - 800x + 1600 = 0$

Solve for x $\quad x = 3.25$

$M^* = M_1 + \int_2^{3.25} V\,dx$

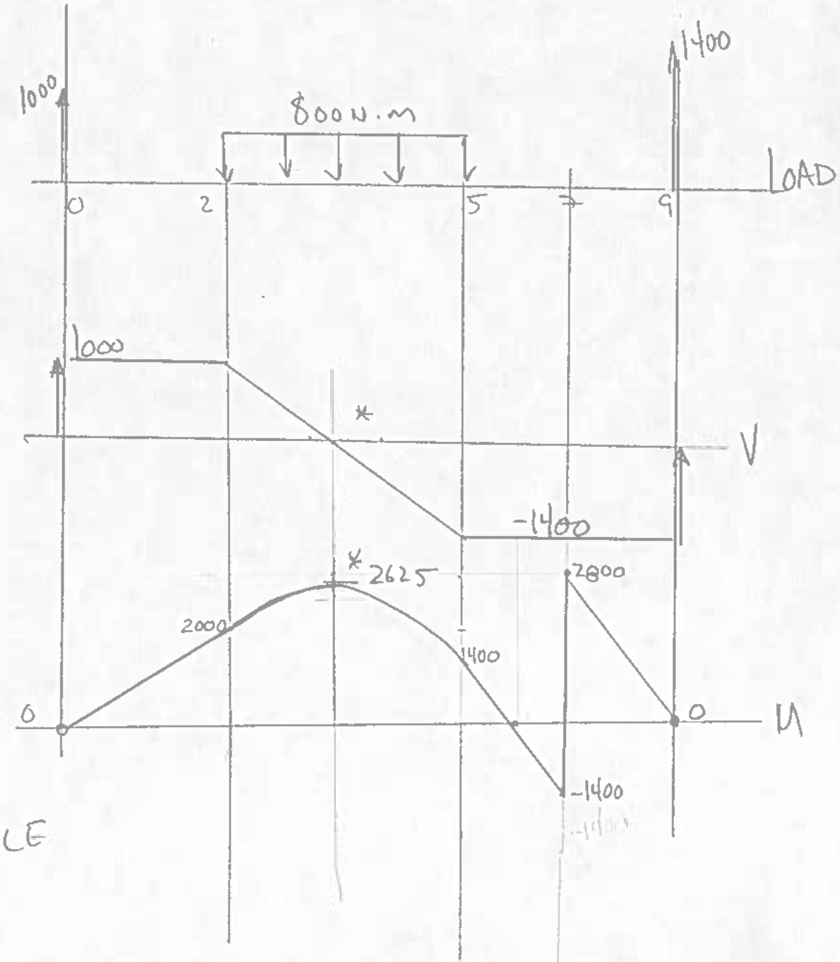
$= 2000 + 1000(3.25) - \frac{800(3.25)^2}{2} + 1600(3.25))$

$- 1000(2) + 800(2) - 1600(2)$

$= 2625$

$M_{MAX} = 2800$ kN·m

@ LOCATION OF COUPLE



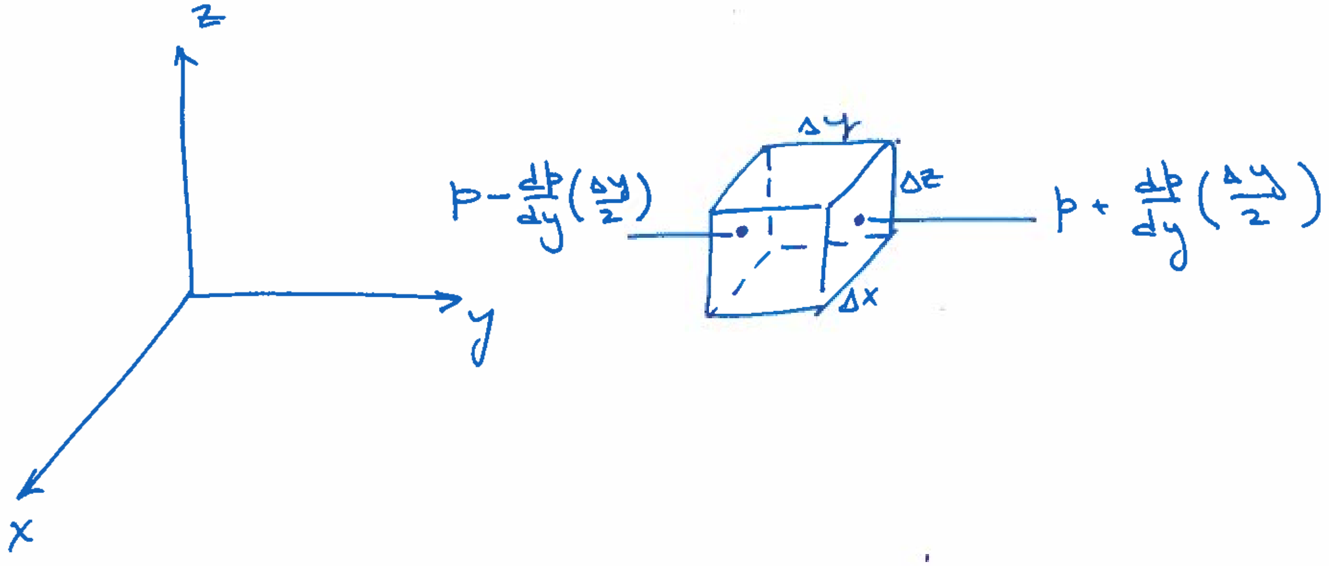
A fluid is a continuous substance which at rest cannot support shear force. A fluid at rest exerts only normal forces on a bounding surface.

Fluid statics in known as hydrostatics.

Pressure

The pressure at any point in a fluid is the same in all directions (Pascal's Law).

Consider a small element of fluid, from a force balance we can find the equation of hydrostatic pressure



Weight of this fluid element is

$$\rho g \, \Delta x \Delta y \Delta z$$

Pressure force on a face is product of pressure & face area

e.g. Force on left face is

$$\left[p - \frac{dp}{dy}\left(\frac{\Delta y}{2}\right)\right]\left[\Delta x \, \Delta z\right]$$

Now write a force balance for all three directions.

$$\Sigma F_x = \left\{p - \frac{dp}{dx}\left(\frac{\Delta x}{2}\right)\right\}\Delta y \Delta z - \left\{p + \frac{dp}{dx}\left(\frac{\Delta x}{2}\right)\right\}\Delta y \Delta z = 0$$

$$\Rightarrow -\frac{dp}{dx} = 0$$

$$\Sigma F_y = \left\{p - \frac{dp}{dy}\left(\frac{\Delta y}{2}\right)\right\}\Delta x \Delta z - \left\{p + \frac{dp}{dy}\left(\frac{\Delta y}{2}\right)\right\}\Delta x \Delta z = 0$$

$$\Rightarrow -\frac{dp}{dy} = 0$$

$$\Sigma F_z = \left\{p - \frac{dp}{dz}\left(\frac{\Delta z}{2}\right)\right\}\Delta x \Delta y - \left\{p + \frac{dp}{dz}\left(\frac{\Delta z}{2}\right)\right\}\Delta x \Delta y - \rho g \Delta x \Delta y \Delta z = 0$$

$$\Rightarrow -\frac{dp}{dz} - \rho g = 0$$

$$\underline{\frac{dp}{dz} = -\rho g}$$

This expression means that hydrostatic pressure is a function of z only.

For an <u>incompressible</u> fluid (water)

ρ = constant.

$$\int_{p_0}^{p} \frac{dp}{dz}\,dz = \int_{p_0}^{p} dp = \int_{z_0}^{z} -\rho g\,dz$$

$$p - p_0 = -\rho g\,(z - z_0)$$

OR

$$p = p_0 - \rho g\,(z - z_0)$$

If $z = z_0$ is the surface of the fluid, the depth (h) positive downward into the fluid $(h = -z)$ allows us to write

$$p = p_0 + \rho g h$$

Where h is depth below fluid surface of point of interest

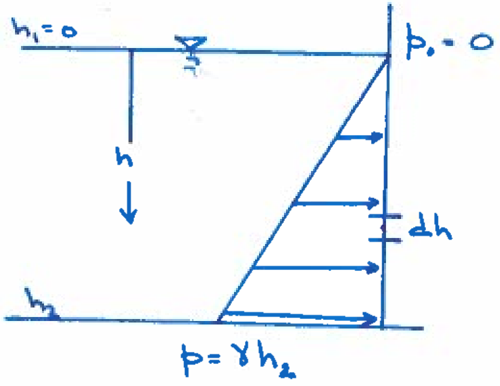
Units of pressure in SI is kilopascal kPa which is 1 kN/m².

In U.S. customary units pressure is expressed as lb/in²

Sometimes the quantity $\gamma = \rho g$ the specific weight of the fluid is used in the pressure formula

$$p = p_o + \gamma h$$

Hydrostatic Pressure on A WALL



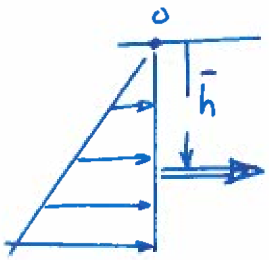
$$p = \gamma h$$

$$dF_p = p\, dA = p \cdot dh \quad \text{(per unit length of wall)}$$

$$F_p = \int dF_p = \int_{h_1}^{h_2} p\, dh = \int_{h_1}^{h_2} \gamma h\, dh$$

$$= \frac{\gamma h^2}{2}\Big|_0^{h_2} = \frac{\gamma h_2^2}{2}$$

LINE OF ACTION



$$\Sigma M_0 = 0$$

$$\Rightarrow \bar{h} F_p = \int_{h_1}^{h_2} \gamma h^2\, dh$$

$$= \frac{\gamma h^3}{3}\Big|_{h_1}^{h_2}$$

$$\bar{h} = \frac{\frac{\gamma}{3} h_2^3 - h_1^3}{\frac{\gamma}{2} h_2^2 - h_1^2} = \frac{2}{3} h_2$$

Hydrostatic Pressure on Submerged Rectangular Plate

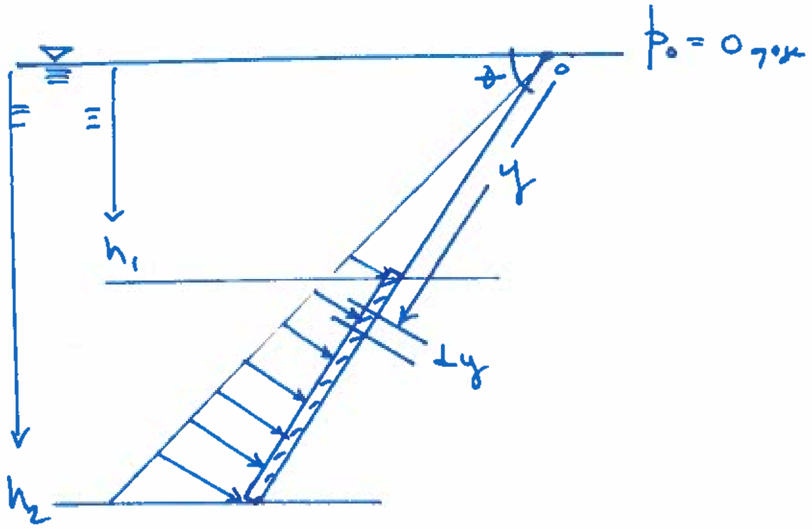


Plate of length L.

Plate has width B.

Pressure at any point y

$$p = \gamma h = \gamma y \sin\theta$$

Differential force at any point y

$$dF = p\,dA = p\,B\,dy$$

Total force on one side of plate.

$$F = \int_{y_0}^{y} dF = \int_{y_0}^{y} \gamma y \sin\theta\, B\,dy$$

$$= \frac{\gamma \sin\theta\, B}{2}(y_2^2 - y_1^2)$$

Center of pressure. (Moment @ O = 0)

$$F\bar{y} = \int_{y_0}^{y} \gamma y^2 \sin\theta\, B\,dy$$

FSA

$$F\bar{y} = \frac{\gamma \sin\theta B}{3} y_2^3 - y_1^3$$

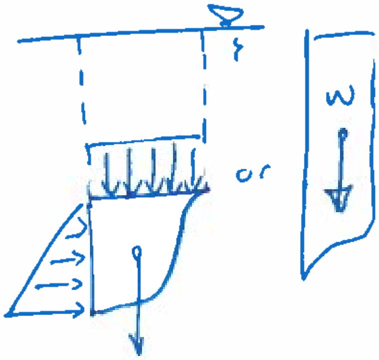
$$\bar{y} = \frac{\frac{\gamma \sin\theta B}{3} y_2^3 - y_1^3}{\frac{\gamma \sin\theta B}{2} y_2^2 - y_1^2} \quad \left(= \frac{\int_{y_1}^{y_2} \gamma y^2 \sin\theta B \, dy}{\int_{y_1}^{y_2} \gamma y \sin\theta B \, dy} \right)$$

$$= \frac{2}{3} \frac{y_2^3 - y_1^3}{y_2^2 - y_1^2} = \frac{2}{3} \frac{y_2^3 - y_1^3}{(y_2 - y_1)(y_2 + y_1)}$$

In terms of h

$$y_2 = \frac{h_2}{\sin\theta}, \quad y_1 = \frac{h_1}{\sin\theta}$$

$$\bar{y} = \frac{2}{3} \frac{y_2^3 - y_1^3}{\frac{h_2^2}{\sin^2\theta} - \frac{h_1^2}{\sin^2\theta}} = \frac{2}{3} \frac{\frac{h_2^3}{\sin^3\theta} - \frac{h_1^3}{\sin^3\theta}}{\frac{h_2^2 - h_1^2}{\sin^2\theta}}$$



$$\bar{y} = \frac{2}{3} \frac{h_2^3 - h_1^3}{(h_2 - h_1)(h_2 + h_1)} \cdot \frac{1}{\sin\theta}$$

but $\bar{y} = \frac{\bar{h}}{\sin\theta}$

∴

$$\frac{\bar{h}}{\sin\theta} = \frac{2}{3} \cdot \frac{1}{\sin\theta} \cdot \frac{h_2^3 - h_1^3}{(h_2 - h_1)(h_2 + h_1)}$$

but $(h_2 - h_1)\cancel{\sin\theta} = L \sin\theta$

$$\bar{h} = \frac{2}{3 \sin\theta L} \frac{h_2^3 - h_1^3}{h_1 + h_2}$$

FCB

Buoyancy

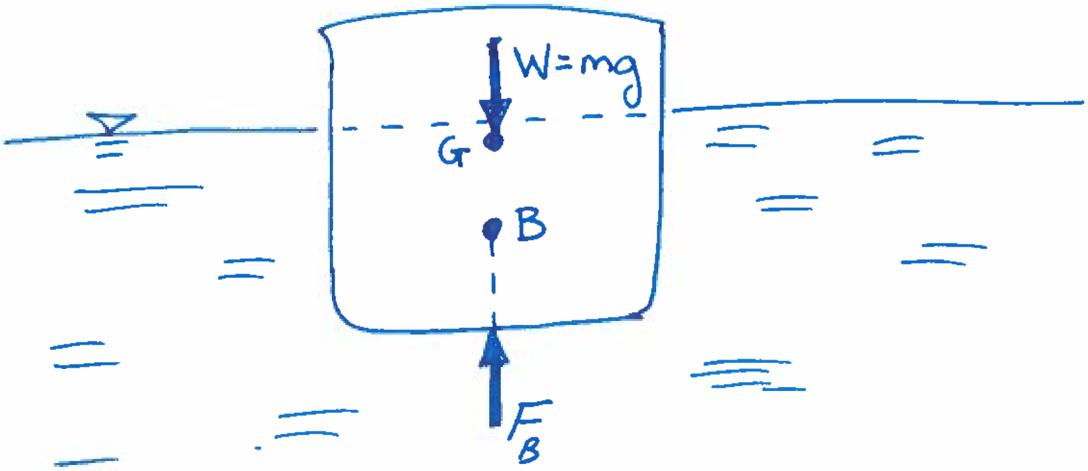
When a body is submerged in a fluid it experiences a vertical force called the buoyancy force, F_B, that is equal to the weight of the fluid displaced (Archimedes's principle) and acts through the centroid of the displaced fluid (if $\rho = \text{const.}$).

The magnitude of the force is

$F_B = \rho g \forall$ where $\forall$ is the volume of fluid displaced.

If the buoyancy force on an immersed body is greater than its weight, then the body will rise until it reaches equilibrium i.e. the buoyant force will equal the weight. This is how a body finds its own "depth of submergence".

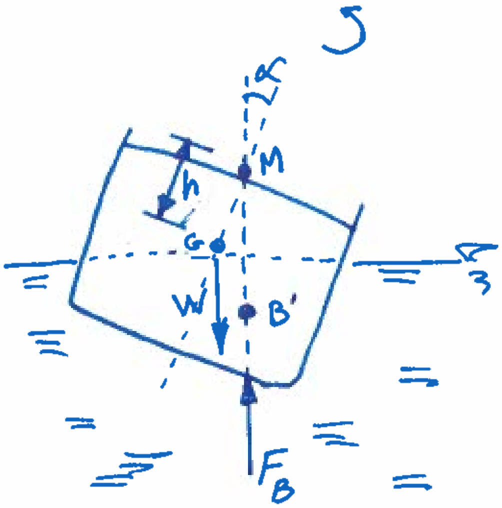
For a floating body such as a ship the buoyant force acts through a point B called the <u>center of buoyancy</u> (the centroid of the displaced fluid volume)



The bouyant force F_B is equal and opposite the weight W of the ship

Stability

Consider a ship whose section is shown



STABLE

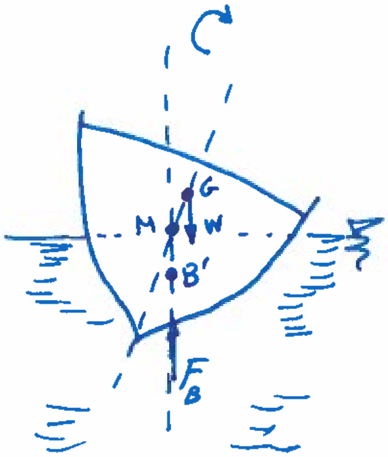
(COUPLE ROTATION DIRECTION OPPOSITE ROLL DIRECTION)

If the ship rolls through angle α, the shape of the displaced fluid changes and the center of buoyancy moves to B'.

The intersection of the vertical line through B' with the centerline of the ship is called the <u>metacenter</u>, M. The distance, h between the metacenter, M and the mass center G, is called the <u>metacentric height</u>.

When the distance h is positive (M lies above G) the F causes a righting moment about G that tends to bring the ship back into its upright position. The ship is stable

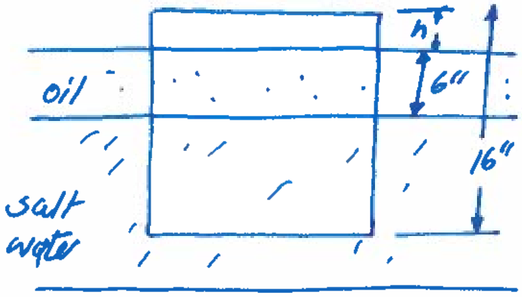
If the distance h is negative (M lies below G) the force F causes an overturning moment, and the ship is unstable



UNSTABLE

(COUPLE ROTATION DIRECTION SAME AS ROLL DIRECTION)

5.171 Block of wood $\gamma = 56 \text{ lb/ft}^3$ floats in oil-water system. Find h. $\gamma_w = 64 \text{ lb/ft}^3$, $\gamma_{oil} = 50 \text{ lb/ft}^3$



Weight of cube

$$W_c = \left(\frac{16}{12}\right)^3 (56 \text{ lb/ft}^3) = 132.74 \text{ lbs}$$

Weight of oil displaced

$$W_{oil} = \left(\frac{16}{12}\right)^2 \left(\frac{6}{12}\right)(50 \text{ lb/ft}^3) = 44.44 \text{ lbs}$$

Weight of water displaced

$$W_{water} = \left(\frac{16}{12}\right)^2 \left(\frac{16-h-6}{12}\right)(64)$$

$$= (1.78)\left(\frac{16}{12} - \frac{h}{12} - \frac{6}{12}\right)(64)$$

$$= (113.78)\left(1.33 - 0.5 - \frac{h}{12}\right)$$

$$= (113.78)\left(0.83 - \frac{h}{12}\right)$$

Equilibrium

$$132.74 = 44.44 + (113.78)\left(0.83 - \frac{h}{12}\right)$$

Solve for h

$$\frac{132.74-44.44}{113.78} = 0.83 - \frac{h}{12}$$

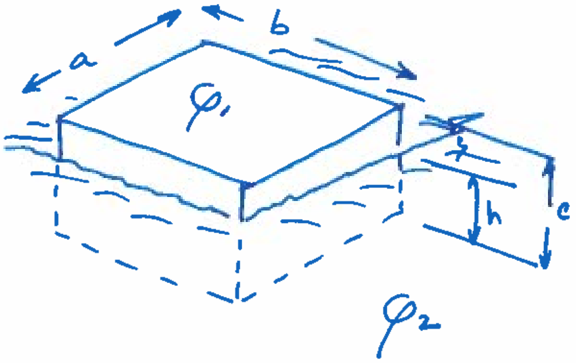
$$9.3125 = 10 - h$$

$$\therefore h = 10 - 9.3125 = 0.6875 \text{ ft}$$

$$h = (12)(0.6875) = \boxed{8.25 \text{ inches}}$$

Block has density ρ_1 Fluid has density ρ_2

a) Find $r = h/c$ where h is depth submerged.



b) Find r for oak in water

c) Find r for steel in mercury

a) Block weight

$W = \rho_1 g\, abc$

Displaced Fluid Weight

$F_B = \rho_2 g\, ab\, h$

Equilibrium

$$W = F_B$$

$$\rho_1 g abc = \rho_2 g ab h$$

$$\rho_1 c = \rho_2 h$$

$$\boxed{\frac{\rho_1}{\rho_2} = \frac{h}{c} = r}$$

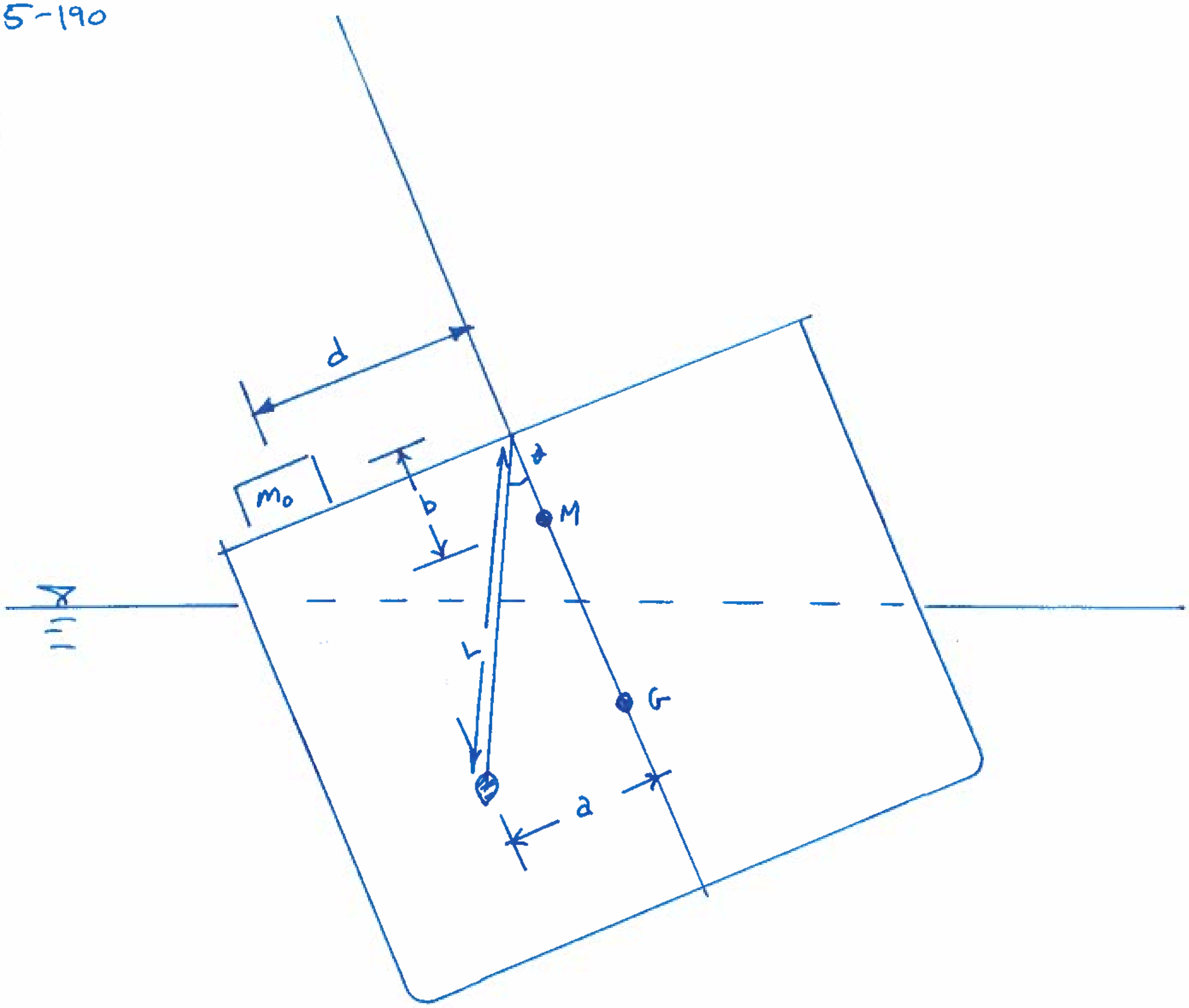
b) $\rho_{oak} = 800 kg/m^3$ $\rho_{water} = 1000 kg/m^3$

$$\gamma = \frac{800}{1000} = 0.8$$

c) $\rho_{steel} = 7830\ kg/m^3$ $\rho_{mercury} = 13{,}570 kg/m^3$

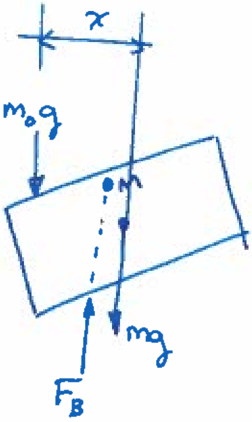
$$\gamma = \frac{7830}{13570} = 0.577$$

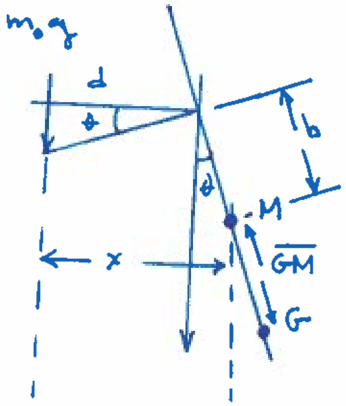
5-190



FINDING G IS PERFORMED EXPERIMENTALLY BY SHIFTING m_0 OUTBOARD A DISTANCE d. THE LIST ANGLE θ IS MEASURED WITH A PLUMB BOB OF LENGTH L. THE DISPLACEMENT & METACENTER ARE KNOWN.

FIND THE METACENTRIC HEIGHT $\overline{GM}$ FOR 12,000-t ship, with 27-t MASS SHIFTED 7.8 m. THE MASS IS 1.8 M ABOVE M. THE 6 m PLUMB BOB IS DISPLACED 0.2 m.





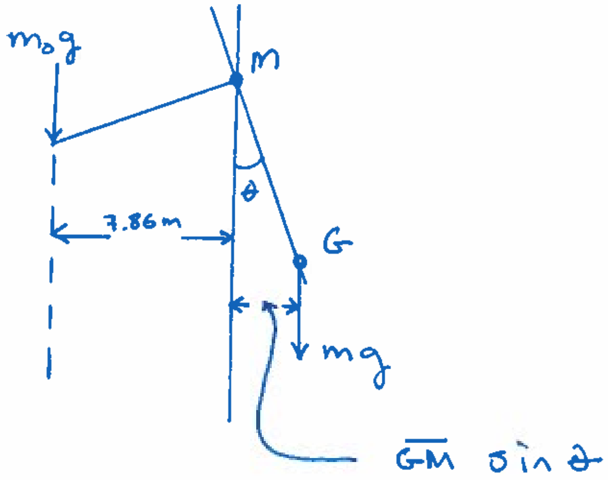
$$F_B = mg + m_0 g = (m + m_0)g$$

$$\theta = \sin^{-1}\frac{a}{L}$$

$$= \sin^{-1}\frac{0.2m}{6m}$$

$$= 1.91°$$

Now Take Moments @ M



$$x = d\cos\theta + b\sin\theta$$

$$= 7.8\cos 1.91 + 1.8\sin 1.91$$

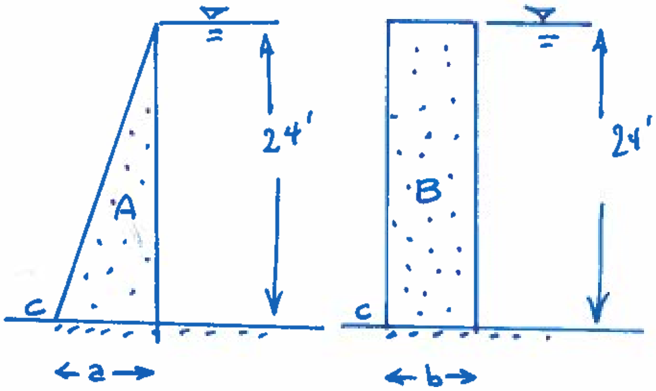
$$= 7.86\ m$$

$$+\circlearrowleft \Sigma M_M = 0 = m_0 g\,(7.86m) - mg\,(\overline{GM}\sin 1.91)$$

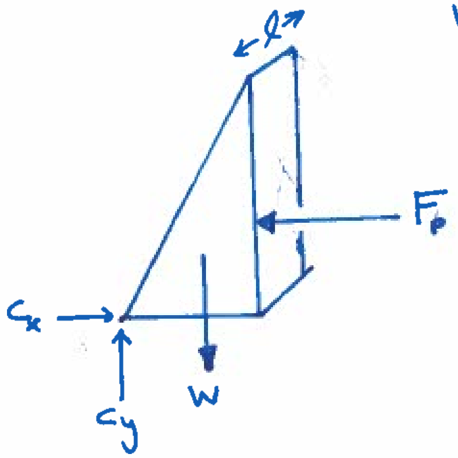
$$\overline{GM} = \frac{m_0\ 7.86}{m\sin 1.91} = \frac{27t}{12{,}000t}\cdot\frac{7.86}{0.0333} = 0.530m.$$

5-177 $\rho_c g = 150\ lb/ft^3.$ $\gamma_w = 62.4\ lb/ft^3$

Find a & b so that overturning moment at C=0.



FBD A

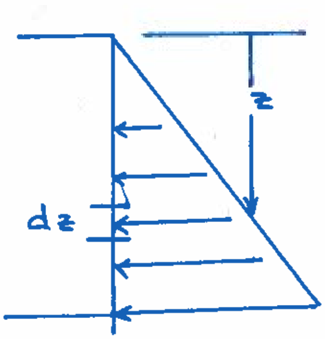


$W = \gamma \forall = \gamma \cdot A \cdot l$ express per unit length

$= 150(24)(a)(\frac{1}{2}) = 1800a\ lb/ft$

$F_p = 17,970\ lb/ft$

Pressure Force



$p = p_o + \gamma z$

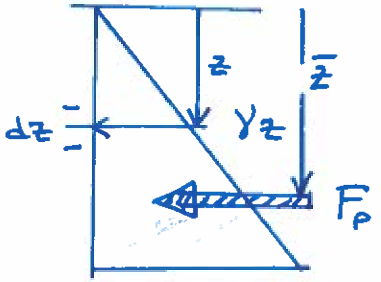
$p = \gamma z$

$dF_p = l\, dz\, p$

$F_p = \int_0^{24} \gamma z\, dz$ (per foot of wall)

$= \frac{\gamma z^2}{2} = \frac{62.4}{2}(24)^2 = 17,970\ \frac{lb}{ft}$

Line of Action of Pressure Force



$$F_p \bar{z} = \int_0^{24} \gamma z^2 \, dz$$

$$F_p \bar{z} = \frac{\gamma (24)^3}{3}$$

$$\bar{z} = \frac{\cancel{\gamma}(24)^{\cancel{3}}}{3} \cdot \frac{2}{\cancel{\gamma (24)^2}} = \frac{2}{3}\,24$$

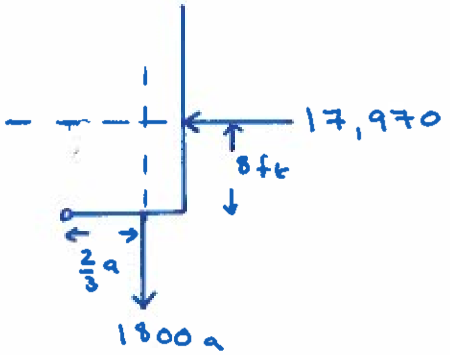
$$\bar{z} = 16 \text{ ft}$$

MOMENT @ C

$$+\circlearrowright M_c = 17{,}970(8) - \frac{2}{3}a^2\,1800 = 0$$

solve for a

$$a = 10.95 \text{ ft}$$



$$W_A = 1800(10.95) = \boxed{19{,}700 \text{ lb/ft of wall}}$$

FBD B

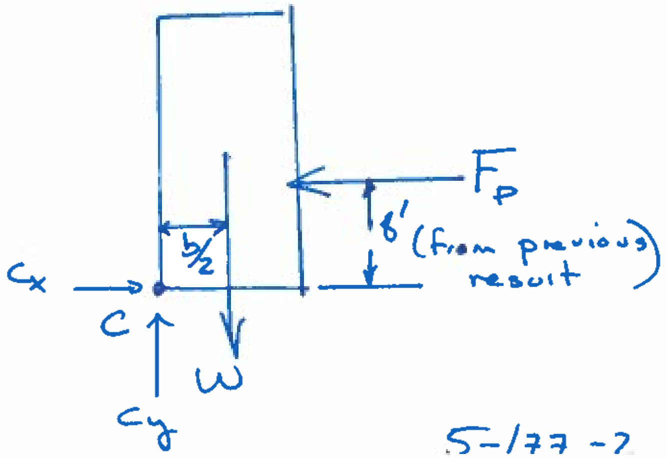
$$W_B = \gamma \forall$$

$$= 150(24)(b) = 3600b \text{ lb/ft}$$

$$+\circlearrowright M_c = 17{,}970(8) - \frac{b^2}{2}3600 = 0$$

Solve for b

$$b = 8.94 \text{ ft}$$



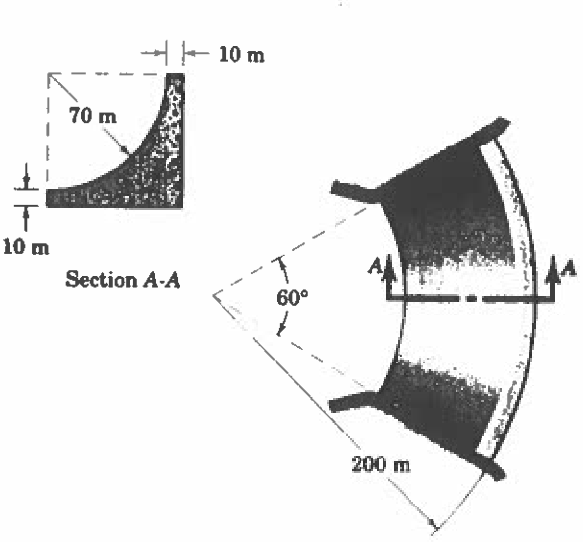
$W_B = 3600(8.94) = 32{,}200$ lb/ft

$\therefore$ A Requires 32,200 lb/ft − 19,700 lb/ft

= 12,470 lb/ft less material than B.

D03 CIVE 2330 FALL 98 FINAL EXAM

5/92 Calculate the mass m of concrete required to construct the arched dam shown. Concrete has a density of 2.40 Mg/m^3.



Problem 5/92

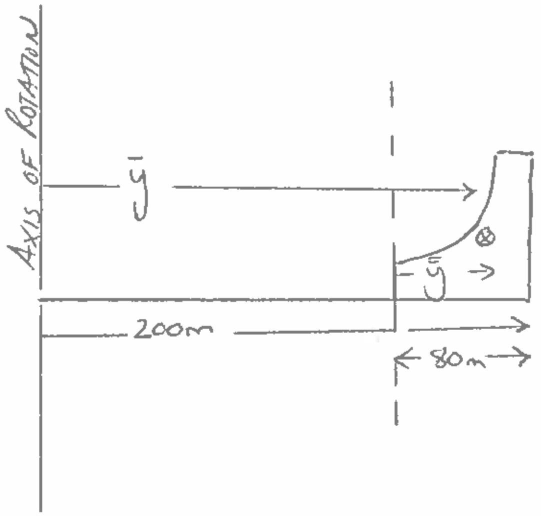
Solutions

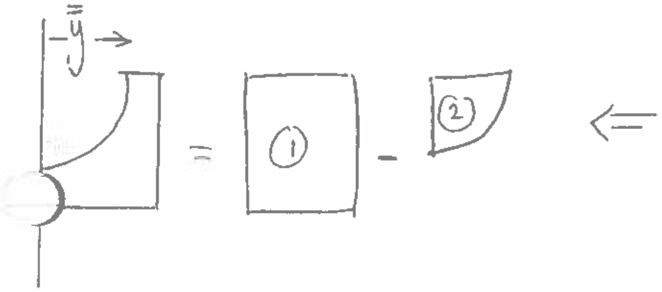
Theorem of Pappus

$$\forall = \theta \bar{y} A$$

$\bar{y}$ is centroid of rotated section.

① Find $\bar{y}$





Section 1

$A = (80)(80)\,m^2 = 6400\,m^2$

$\bar{\bar{y}} = 40\,m$ (by inspection)

Section 2

$A = \dfrac{\pi(70m)^2}{4} = 3848.4\,m^2$

$\bar{\bar{y}} = \dfrac{4(70m)}{3\pi} = 29.70\,m$

Composite

	A	$\bar{\bar{y}}$	$A\bar{\bar{y}}$
1	6400	40	256000
	−3848.4	29.7	−114297
	2551.6		141703

$\bar{\bar{y}} = 141703/2551.6 = 55.53\,m$

$\Rightarrow$

$\bar{y} = 200m - 80m + \bar{\bar{y}}$

$= 200 - 80 + 55.53$

$= 175.53$

$\forall = \dfrac{\pi}{3}(175.53)(2551.6)$

$= 469034.1\,m^3$

$\text{Mass} = \rho\forall = 2400\,kg/m^3 \cdot 469034$

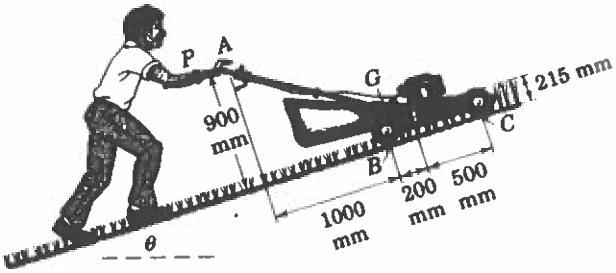
$= 1.126 \cdot 10^9\,kg$

or

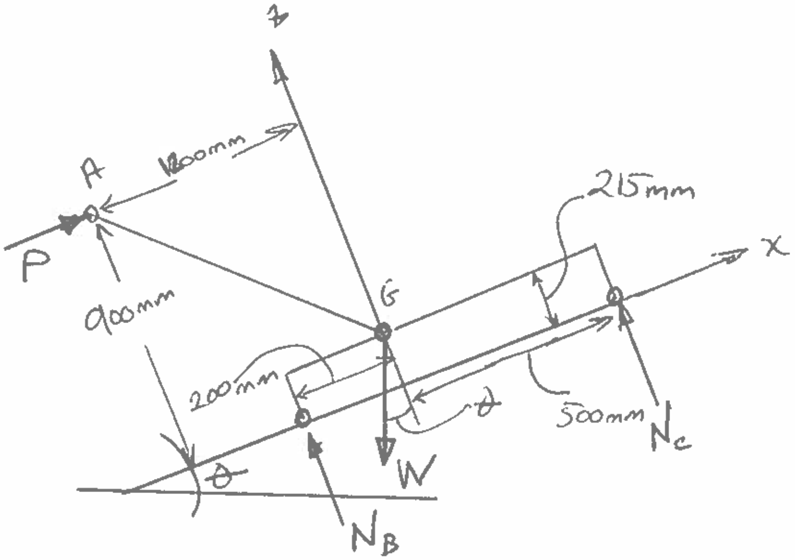
$1.126 \cdot 10^6\,Mg$

CIVE 2330 FALL 98 FINAL EXAM

3/41 The man pushes the lawn mower at a steady speed with a force P that is parallel to the incline. The mass of the mower with attached grass bag is 50 kg with mass center at G. If $\theta = 15°$, determine the normal forces N_B and N_C under each pair of wheels B and C. Neglect friction. Compare with the normal forces for the conditions of $\theta = 0$ and $P = 0$.



Problem 3/41



FBD Lawnmower

$$\Sigma F_x = 0 = P - W\sin\theta$$

$$P = W\sin\theta$$

$$P = (50)(9.8)\sin\theta$$

$$\Sigma F_z = 0 = N_B + N_c - W\cos\theta$$

$$N_B + N_c = W\cos\theta = (50)(9.8)\cos\theta$$

$$\circlearrowright \Sigma M_G = 0 = N_c\,500\text{mm} - N_B\,200\text{mm} - P\,685\text{mm}$$

$$\therefore P = 490\sin\theta$$

$$N_B = 490\cos\theta - N_c = \underbrace{490\cos\theta - N_B\frac{2}{5} - P\frac{6.85}{5}}$$

$$N_c = N_B\frac{200}{500} + P\frac{685}{500}$$

$$N_B = \frac{490\cos\theta - P\frac{6.85}{5}}{1 + \frac{2}{5}}$$

$\theta = 15°$

$P = 126.82N$

$N_B = 214N$

$N_c = 260N$

$\theta = 0°$

$P = 0$

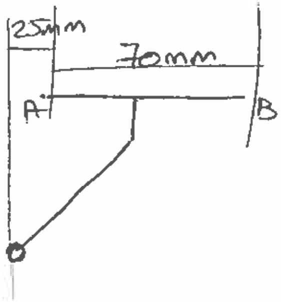
$N_B = 350N$

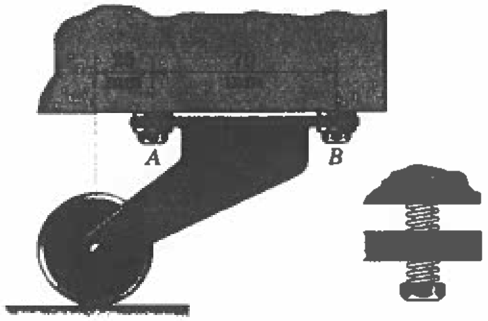
$N_c = 140N$

PROBLEM 2

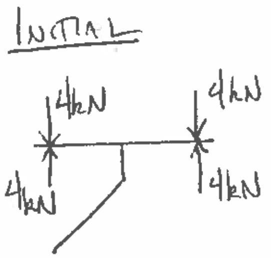
CIVE 233: FALL '98 FINAL EXAM

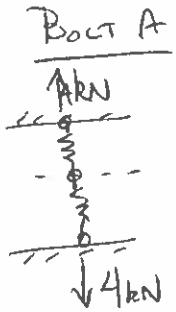
▶3/110 The base plate of a special industrial caster is mounted to the carriage, with soft elastic washers on either side of the plate in order to absorb shock during operation. Bolts *A* and *B* are initially tightened to a tension of 4 kN with no load on the caster. Determine the force in each bolt when the caster supports a load of 3 kN. The forces acting at each bolt location may be modeled by replacing the washers by identical stiff springs as shown in the separate view. Consider the bolts and base plate to be perfectly rigid. (*Caution:* Draw your free-body diagrams of each part carefully.)

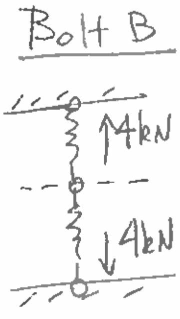


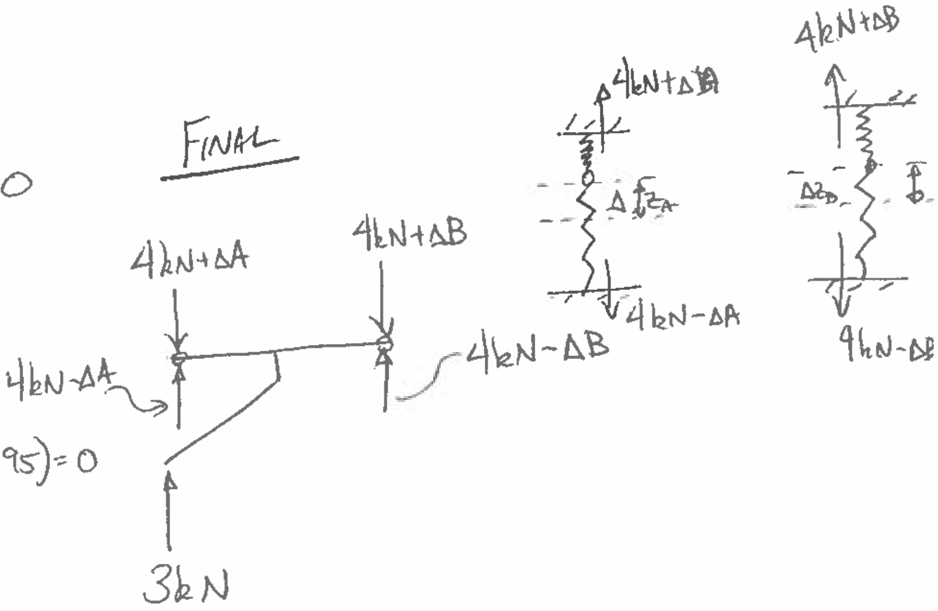


Problem 3/110









$\Sigma M_A = -(4kN+\Delta B)(70) + (4kN-\Delta B)70 = 0$
$-3(25)$

Solve for ΔB

$\Delta B = \frac{-75}{140} = -0.536kN$

$\Sigma M_B = (4kN+\Delta A)(70) - (4kN-\Delta A)(70) - 3(95) = 0$

Solve for ΔA

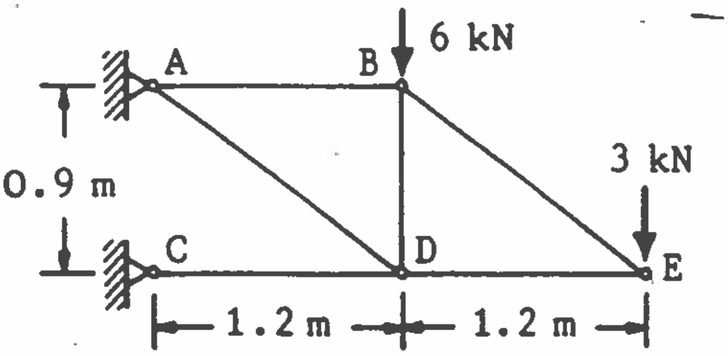
$\Delta A = \frac{285}{140} = 2.04kN$

ANALYZE BOLTS

$T_A = 4kN - \Delta A = 1.96kN$ (Tension)

$T_B = 4kN - \Delta B = 4 - (-0.54) = 4.54$ (TENSION)

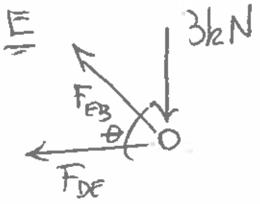
CIVE 2330 FALL 98 FINAL EXAM



Determine the force in each member of the truss shown.

Method of Joints

E



$\tan\theta = \frac{0.9}{1.2}$

$\theta = 36.86°$

$\Sigma F_y = F_{EB}\sin\theta - 3kN = 0$

$\Sigma F_x = -F_{EB}\cos\theta - F_{DE} = 0$

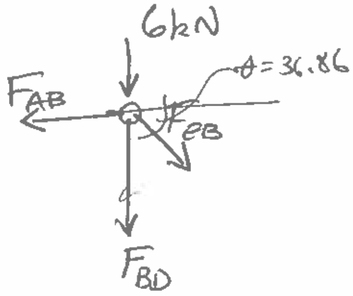
$F_{EB} = \frac{3}{\sin 36.86} = 5kN$

$F_{DE} = -\frac{3}{\sin 36.86} \cdot \cos 36.86 = -4kN$

$\therefore F_{EB} = 5kN$ T

$F_{DE} = 4kN$ C

B



$\Sigma F_y = -6 - F_{BD} - 5(\sin 36.86)$

$F_{BD} = -6 - 5\sin 36.86$

$= -9kN$

$\Sigma F_x = -F_{AB} + F_{EB}\cos 36.86$

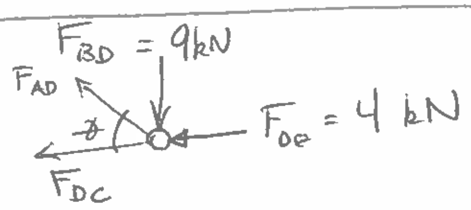
$F_{AB} = F_{EB}\cos 36.86 = 5\cos 36.86$

$= 4kN$

$F_{BD} = 9kN$ C

$F_{AB} = 4kN$ T

D



$\Sigma F_y = 0 = -9 + F_{AD}\sin 36.86$

$F_{AD} = \frac{9}{\sin 36.86} = 15kN$

$\Sigma F_x = 0 = -F_{DC} - F_{DE} - F_{AD}\cos 36.86$

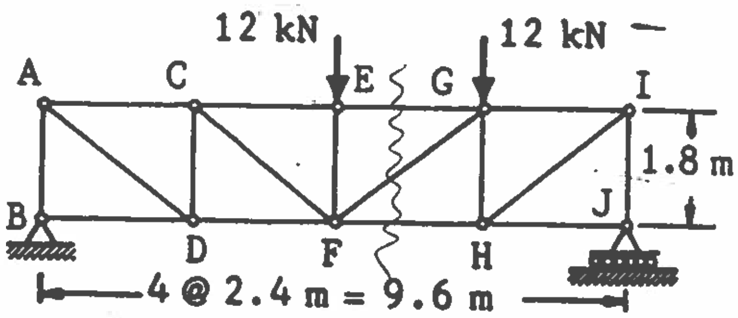
$F_{DC} = -4kN - 15\cos 36.86$

$= -16kN$

$\therefore F_{AD} = 15kN$ T

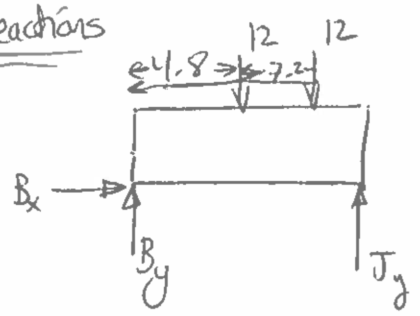
$F_{DC} = 16kN$ C

CIVE 2330 FALL 98 FINAL EXAM



Determine the force in members FG and FH.

Reactions



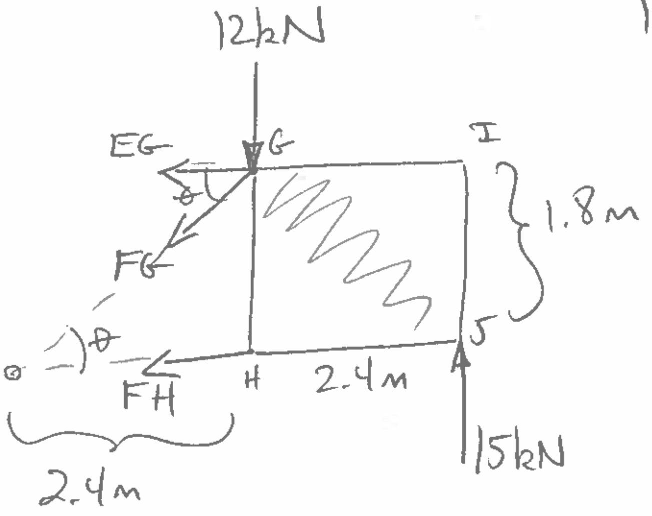
$\Sigma F_x = 0 = B_x$

$\Sigma F_y = 0 = -24\text{kN} + B_y + J_y$

$\circlearrowleft \Sigma M_B = 0 = -4.8(12) - 7.2(12) + 9.6 J_y$

$J_y = +15\text{kN}$

$B_y = 24 - J_y = 9\text{ kN}$



$\circlearrowleft \Sigma M_G = 0 = 2.4(15) - 1.8(FH)$

$FH = \dfrac{2.4(15)}{1.8} = 20\text{kN}$

$\Sigma F_y = 0 = 15 - 12 - FG \sin 36.86$

$FG = \dfrac{3}{\sin 36.86} = 5\text{ kN}$

$\Sigma F_x = 0 = -EG - FH - FG \cos 36.86$

$EG = -(20) - (5)\cos 36.86$

$= -24\text{kN}$

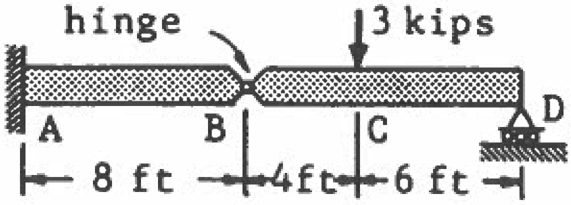
$\tan\theta = \dfrac{1.8}{2.4}$

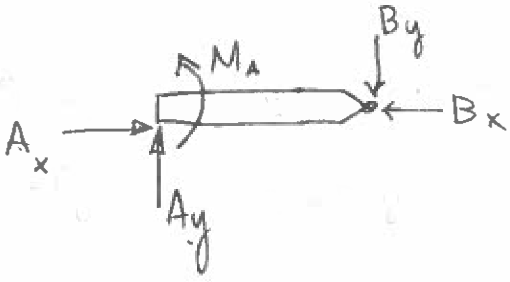
$\theta = 36.86°$

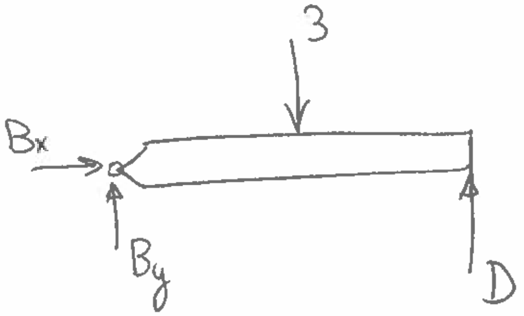
$\therefore$ FG = 5kN T

FH = 20kN T

Determine the reactions at the supports.







$\Sigma M_A = 0 = M_A - 8 B_y$

$M_A = 8(1.8)$

$= 14.4$ kip.ft

$\Sigma M_B = 0 = 10(D) - 4(3)$

$D = \frac{12}{10} = 1.2$ kips.

$\Sigma F_x = 0 = B_x$

$\Sigma F_y = 0 = B_y + D - 3$

$B_y = 3 - D = 3 - 1.2 = 1.8$ kip

$\overset{+}{\Sigma} F_x = 0 = A_x - B_x$

$A_x = B_x$

$A_x = 0$

$\Sigma F_y = A_y - B_y = 0$

$A_y = B_y$

$= 1.8$ kips

$D = 1.2$ kips ↑

∴ $A_x = 0$

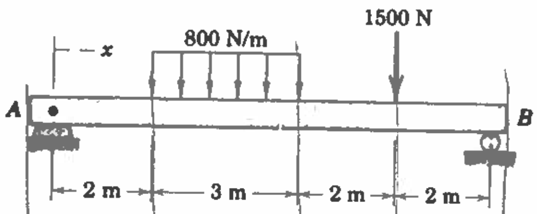
$A_y = 1.8$ kips ↑

$M_A = 14.4$ kip.ft ↻

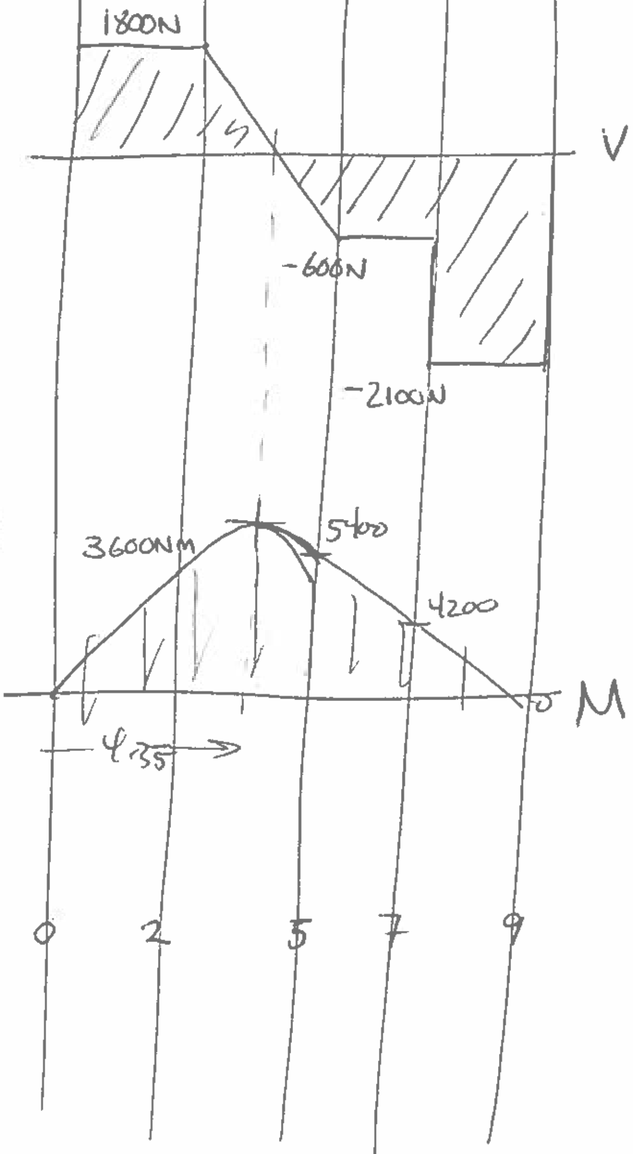
PROBLEM 6

CIVE 2330 FALL 98 FINAL EXAM

5/130 Plot the shear and moment diagrams for the beam loaded with both the distributed and point loads. What are the values of the shear and moment at $x = 6$ m? Determine the maximum bending moment M_{max}.



Problem 5/130



$$\frac{dM}{dx} = 0 = -800x + 3400 = 0$$

$$x = 4.25$$

$$M_{MAX} = 5625 \text{ N·m}$$

Reactions

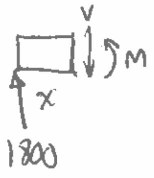
$$\Sigma F_y = -800(3) - 1500 + A_y + B_y = 0$$

$$\Sigma M_A = -3.5(2400) - 7(1500) + 9B_y = 0$$

$$B_y = 2100\text{N}$$

$$A_y = 1800\text{N}$$

$0 < x < 2$



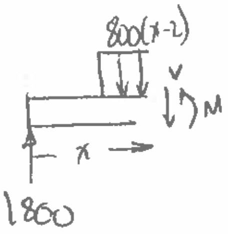
$\Sigma F_y = 0 = 1800 - V$

$V = 1800$

$\Sigma M = 0 = M - 1800x$

$M = 1800x$

$2 < x < 5$



$\Sigma F_y = 0 = 1800 - 800(x-2) - V$

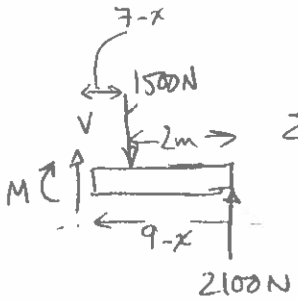
$V = 3400 - 800x$

$\Sigma M = 0 = M - 1800x$ - ~~800(x-2~~

$-800(x-2)\frac{x-2}{2}$

$M = 3400x - 400x^2 - 1600$

$5 < x < 7$



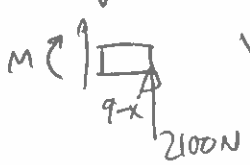
$\Sigma F_y = 0 = V - 1500 + 2100$

$V = -600\text{N}$

$\Sigma M = 0 = -M - 1500(7-x) + 2100(9-$

$M = 8400 - 600x$

$7 < x < 9$



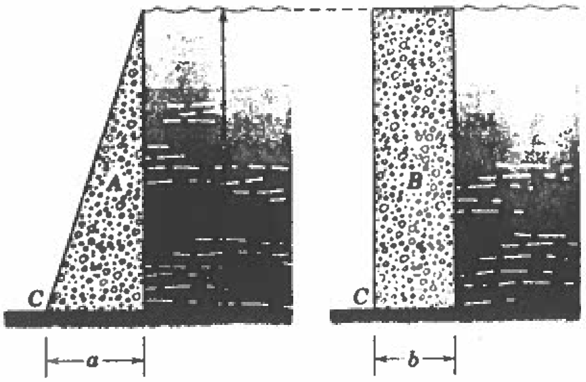
$V = -2100\text{N}$

$M = 18900 - 2100x$

PROBLEM 7

CIVE 2330 FALL 98 FINAL EXAM

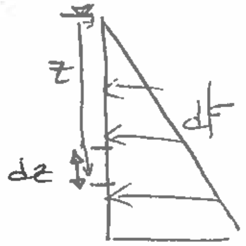
5/177 The triangular and rectangular sections are being considered for the design of a small fresh-water concrete dam. From the standpoint of resistance to overturning about *C*, which section will require less concrete, and how much less per foot of dam length? Concrete weighs 150 lb/ft³.



Problem 5/177

$\gamma_c = 150\ lb/ft^3 \quad \gamma_w = 62.4\ lb/ft^3$

Find pressure force on wall



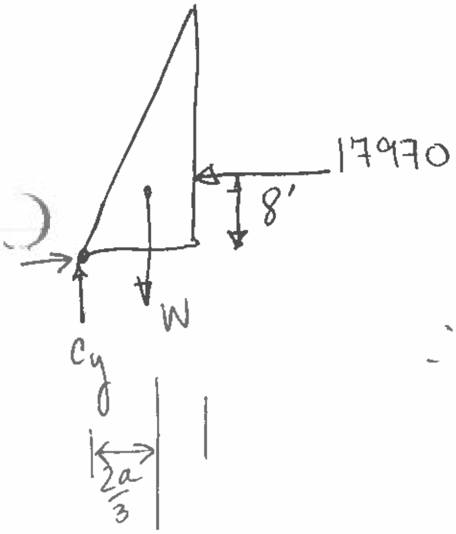
$dF = l\,dz\,p$

$p = \gamma_w z$

l = length of wall

$F = \int_0^{24} \gamma_w z\, l\, dz$

$= \frac{62.4}{2}(24)^2 l = 17970 \frac{lb}{ft} \cdot l\ ft$



$\Sigma M_c = 8(17970) - W\left(\frac{2a}{3}\right) = 0$

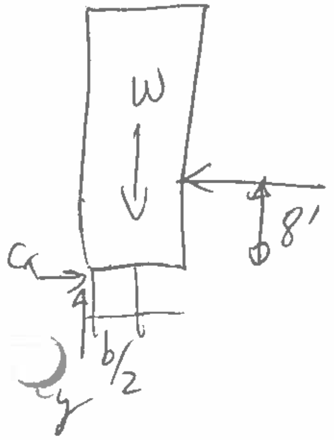
$W = \gamma_c (24)(a)(\tfrac{1}{2})$

$= 12\gamma_c a$

$\therefore 17970 = \frac{12(150)a^2\left(\frac{2}{3}\right)}{8}$

$a = 10.95\ ft$

$\therefore W = 19700\ lb/ft$ of wall



$\Sigma M_c = 8(17970) - W\frac{b}{2} = 0$

$W = \gamma_c(24)(b)$

$17970 = \frac{(150)(24)(b)^2}{2(8)}$

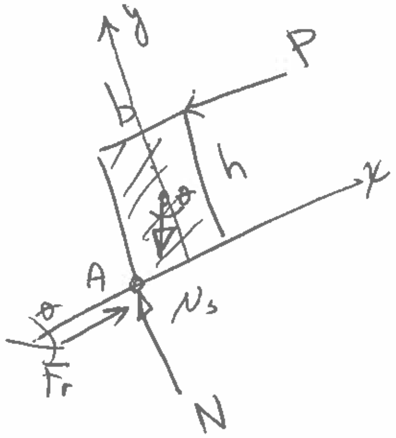
$b = 8.94\ ft \quad W = 32{,}200\ lb/ft$ of wall

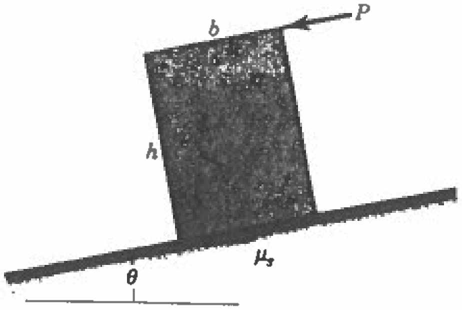
PROBLEM 8

$\therefore$ A uses $32200 - 19700$
$= 12470\ lb/ft$ less

CIVE 2330 FALL 98 FINAL EXAM

6/5 Determine the maximum ratio h/b for which the homogeneous block will slide without tipping under the action of force P. The coefficients of static friction between the block and the incline is μ_s.





Problem 6/5

$$\Sigma F_x = \mu_s W \cos\theta - W\sin\theta - P = 0$$

$$\Sigma F_y = \underbrace{W\cos\theta}_{N} - W\cos\theta = 0$$

$$\Sigma M_A = hP + \frac{Wh}{2}\sin\theta - \frac{Wb}{2}\cos\theta = 0$$

$$\mu_s W\cos\theta = P + W\sin\theta$$

$$\therefore Ph = h\mu_s W\cos\theta - hW\sin\theta$$

Substitute into ΣM_A

$$h\mu_s W\cos\theta - hW\sin\theta + \frac{Wh}{2}\sin\theta - \frac{Wb}{2}\cos\theta = 0$$

divide by b

$$\frac{h}{b}\mu_s W\cos\theta - \frac{h}{b}W\sin\theta + \frac{h}{b}\frac{W\sin\theta}{2} - \frac{W}{2}\cos\theta = 0$$

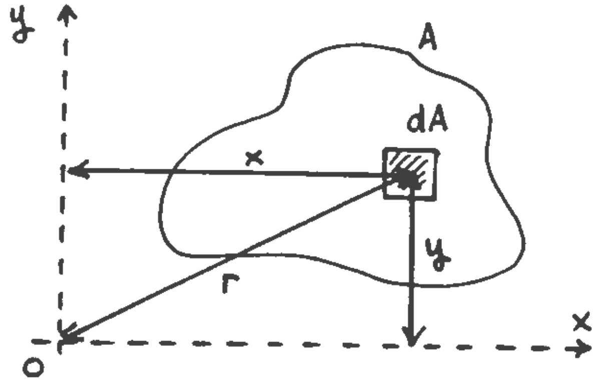
divide by $W\cos\theta$

$$\frac{h}{b}\mu_s - \frac{h}{b}\tan\theta + \frac{h}{b}\frac{\tan\theta}{2} - \frac{1}{2} = 0$$

$$\frac{h}{b} = \frac{1}{2}\left(\frac{1}{\mu_s - \frac{\tan\theta}{2}}\right) = \frac{1}{2\mu_s - \tan\theta}$$

Area Moments of Inertia

Consider area A in xy-plane. Define moments of inertia of element dA about x and y axes, respectively, as



$$dI_x = y^2\,dA$$
$$dI_y = x^2\,dA$$

The moments of inertia of A about the same axes are

$$I_x = \int y^2\,dA \qquad \text{and} \qquad I_y = \int x^2\,dA$$

(rectangular moments of inertia)

The moment of inertia about the z-axis (passing through O) is

$$I_z = \int r^2\,dA \qquad \text{(polar moment of inertia)}$$

Since $x^2+y^2 = r^2 \Rightarrow I_x + I_y = I_z$. Units of inertia are $(\text{length})^4$, ie. ft^4 or m^4, usually.

Can define the radius of gyration of an area about a given axis by

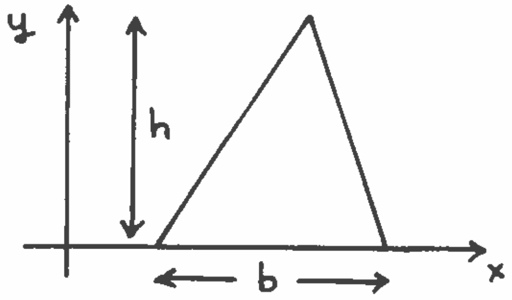
$$I_x = A k_x^2 \Rightarrow k_x = \sqrt{\frac{I_x}{A}}$$

$$I_Y = A k_y^2 \Rightarrow k_y = \sqrt{\frac{I_Y}{A}}$$

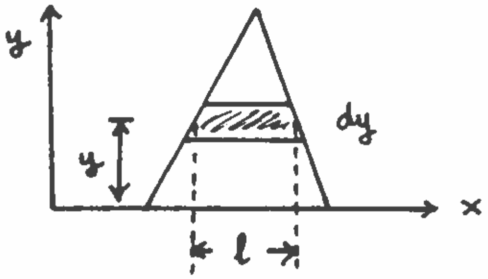
$$I_z = A k_z^2 \Rightarrow k_z = \sqrt{\frac{I_z}{A}}$$

EXAMPLE 1 : Moments of Inertia

Determine the moment of inertia of the given triangle with respect to its base. Find also the radius of gyration.



Choose a horizontal differential area element as shown.



$$dI_x = y^2\,dA \qquad dA = l\,dy$$

Value of l can be found from similar triangles

$$\frac{l}{b} = \frac{h-y}{h} \Rightarrow l = \frac{b(h-y)}{h}$$

Then $dI_x = y^2\,l\,dy = \dfrac{y^2 b(h-y)}{h}\,dy$

Integrating gives $I_x = \dfrac{b}{h}\displaystyle\int_0^h y^2(h-y)\,dy = \dfrac{b}{h}\left(y^3\dfrac{h}{3} - \dfrac{y^4}{4}\right)\Bigg|_0^h$

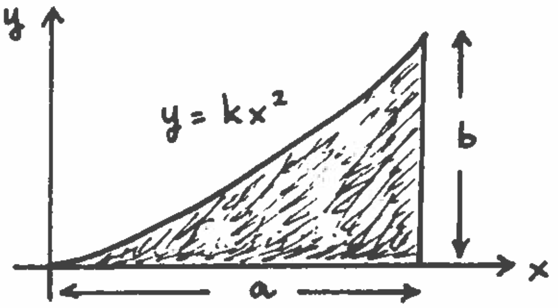
$$= \underline{\underline{\frac{bh^3}{12}}}$$

Radius of gyration $k_x = \sqrt{I_x/A}$ $\qquad A = \frac{1}{2}bh$

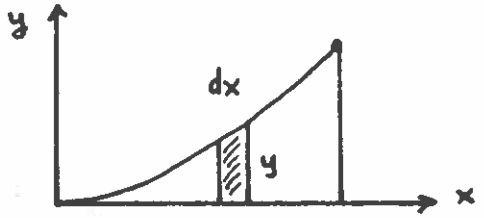
$$\Rightarrow \underline{\underline{k_x = h/\sqrt{6}}}$$

EXAMPLE 2 : Moments of Inertia

Find the moment of inertia of the shaded area w.r.t. each of the coordinate axes. Also, find the radius of gyration w.r.t. each of the coordinate axes.



First, constant k is found from $b = ka^2 \Rightarrow k = b/a^2$.



$$A = \int_0^a y\,dx = \int_0^a \frac{bx^2}{a^2}\,dx$$

$$= \frac{b}{a^2}\,\frac{x^3}{3}\Big|_0^a = \underline{\underline{\frac{1}{3}ab}}.$$

$$I_y = \int x^2 dA = \int_0^a x^2 y\,dx = \int_0^a \frac{b}{a^2}x^4\,dx = \frac{b}{a^2}\,\frac{x^5}{5}\Big|_0^a = \underline{\underline{\frac{a^3b}{5}}}$$

I_x can be calculated using same vertical area element. First calculate inertia of this element about x axis,

$$dI_x = \int_0^y y^2\,dx\,dy = \frac{1}{3}y^3\,dx$$

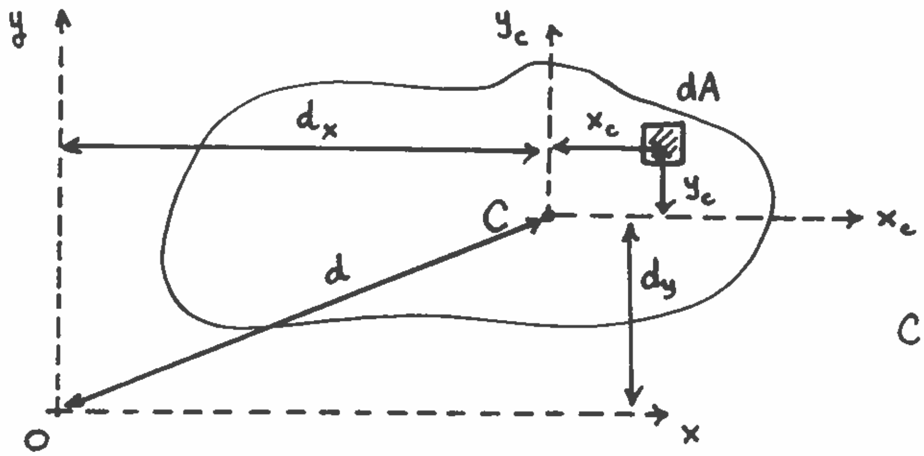
Then integrate this over all elements, $I_x = \int dI_x$, to give

$$I_x = \int_0^a \frac{1}{3}y^3\,dx = \frac{b^3}{3a^6}\int_0^a x^6\,dx = \frac{b^3}{3a^6}\cdot\frac{x^7}{7}\Big|_0^a = \underline{\underline{\frac{ab^3}{21}}}$$

Radii of gyration : $k_x = \sqrt{I_x/A} = \underline{b/\sqrt{7}}$; $k_y = \sqrt{I_y/A} = \underline{\sqrt{3/5}\,a}$

Transfer of Axes:

Moments of inertia about 2 parallel axes can be related, provided one of the axes passes through the centroid of the area.



C = centroid of area A

Moment of inertia of area element dA about x-axis is

$$dI_x = (y_c + d_y)^2 \, dA$$

For the entire area A, the moment of inertia about the x-axis is

$$I_x = \int (y_c + d_y)^2 \, dA = \int y_c^2 \, dA + 2 d_y \int y_c \, dA + d_y^2 \int dA$$

$$= \bar{I}_x + 0 + A d_y^2$$

$$\Rightarrow \quad I_x = \bar{I}_x + A d_y^2$$

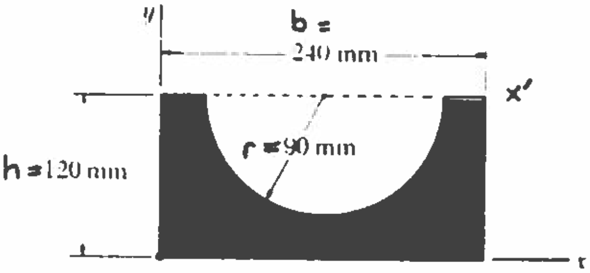
Similarly $\quad I_y = \bar{I}_y + A d_x^2$

Summing $\Rightarrow \quad I_z = \bar{I}_z + A d^2$

These are known as the <u>parallel axis theorems</u>, where $\bar{I}_x$ and $\bar{I}_y$ are rectangular moments of inertia about the x_c and y_c axes, respectively. $\bar{I}_z$ is polar moment of inertia about the z-axis passing through C

EXAMPLE 1 : Composite Area Moments of Inertia

Find the x-moment of inertia of the shaded region, given I_x for the rectangle is $\frac{1}{3} b h^3$ and $I_{x'}$ for the semi-circle is $\frac{1}{8} \pi r^4$.



Treat as composite area, rectangle minus semi-circle.

I_{xx} for rectangle :
$$I_x = \tfrac{1}{3} b h^3 = \tfrac{1}{3}(240)(120)^3$$
$$= \underline{138.2 \times 10^6 \text{ mm}^4}$$

I_{xx} for semicircle :
$$I_{x'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi(90)^4 = 25.76 \times 10^6 \text{ mm}^4$$
$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi(90)^2 = 12.72 \times 10^3 \text{ mm}^2$$

Use parallel axis theorem to find $\bar{I}_x$: $I_{x'} = \bar{I}_x + A\bar{r}^2$
where $\bar{r} = 4r/3\pi = 38.2$ mm, so
$$25.76 \times 10^6 = \bar{I}_x + (12.72 \times 10^3)(38.2)^2$$
$$\bar{I}_x = 7.20 \times 10^6 \text{ mm}^4$$

Use parallel axis theorem again to find I_x : $I_x = \bar{I}_x + A(h-\bar{r})^2$, so
$$I_x = 7.20 \times 10^6 + (12.72 \times 10^3)(120-38.2)^2$$
$$\underline{= 92.3 \times 10^6 \text{ mm}^4}$$

I_x for composite area : subtract I_{circle} from $I_{rectangle}$
$$I_x^c = 138.2 \times 10^6 - 92.3 \times 10^6$$
$$\underline{= 45.9 \times 10^6 \text{ mm}^4}$$

Composite Areas

Sometimes a complicated geometric shape can be decomposed into a number of simple shapes, each of which has a calculable moment of inertia.

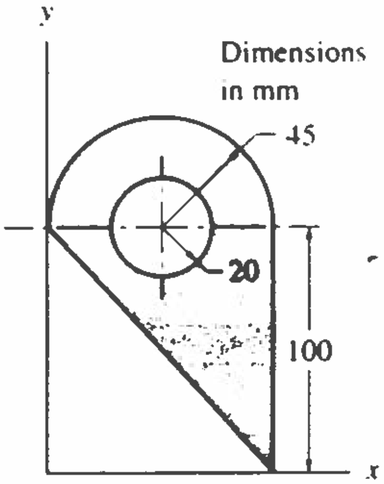
The moment of inertia of a composite area about a given axis is the sum of the moments of inertia of its composite areas about the same axis.

In the case of a void (hole), that piece can be considered as a negative area when computing its moment of inertia.

It is convenient to tabulate the areas, radii of gyration and moments of inertia of the composite parts in the same manner as was done when computing centroids of composite bodies.

EXAMPLE 2 : Composite Area Moments of Inertia

Calculate the radii of gyration of the given area about the x and y axes.



Treat body as composite area consisting of triangle, semi-circle and circle. Circle inertia will be treated as negative.

Formulas:

Triangle :

$$\text{Area} = bh/2$$
$$\bar{I}_x = bh^3/36$$
$$I_x = \bar{I}_x + A\bar{y}^2$$
$$\bar{I}_y = hb^3/36$$
$$I_y = \bar{I}_y + A\bar{x}^2$$

Semicircle :

$$\text{Area} = \pi R^2/2$$
$$\bar{I}_x = (9\pi^2 - 64)R^4/72\pi$$
$$I_x = \bar{I}_x + A\bar{y}^2$$
$$\bar{I}_y = \pi R^4/8$$
$$I_y = I_y + A\bar{x}^2$$

Circle :

$$\text{Area} = \pi r^2$$
$$\bar{I}_x = \pi r^4/4$$
$$I_x = \bar{I}_x + A\bar{y}^2$$
$$\bar{I}_y = \pi r^4/4$$
$$I_y = \bar{I}_y + A\bar{x}^2$$

Calculations :

	TRIANGLE	SEMICIRCLE	CIRCLE
A (mm^2)	4500	3181	1257
$\bar{I}_x$ (mm^4)	2.5×10^6	$.45 \times 10^6$	$.1257 \times 10^6$
$\bar{I}_y$ (mm^4)	2.025×10^6	1.61×10^6	$.1257 \times 10^6$
$\bar{x}$ (mm)	60	45	45
$\bar{y}$ (mm)	66.7	119.1	100
I_x (mm^4)	22.52×10^6	45.57×10^6	12.7×10^6
I_y (mm^4)	18.23×10^6	8.05×10^6	2.67×10^6

Composite Area.

$$A = \Sigma A = 4500 + 3181 - 1257 = \underline{6424 \ mm^2}$$

$$I_x = \Sigma I_x = (22.52 + 45.57 - 12.7) \times 10^6 = \underline{55.39 \times 10^6 \ mm^4}$$

$$I_y = \Sigma I_y = (18.23 + 8.05 - 2.67) \times 10^6 = \underline{23.61 \times 10^6 \ mm^4}$$

Radii of Gyration :

$$k_x = \sqrt{I_x / A} = \underline{\underline{92.9 \ mm}}$$

$$k_y = \sqrt{I_y / A} = \underline{\underline{60.6 \ mm}}$$

Products of Inertia

Product of inertia is defined as

$$I_{xy} = \int xy \, dA$$

and may be positive, negative or zero. I_{xy} is zero if x, y axis is an axis of symmetry.

Can transfer axes from centroidal $x_c y_c$-axes to general (parallel) xy-axes using

$$I_{xy} = \int (x_c + d_x)(y_c + d_y) \, dA$$

$$= \int x_c y_c \, dA + d_x \int y_c \, dA + d_y \int x_c \, dA + d_x d_y \int dA$$

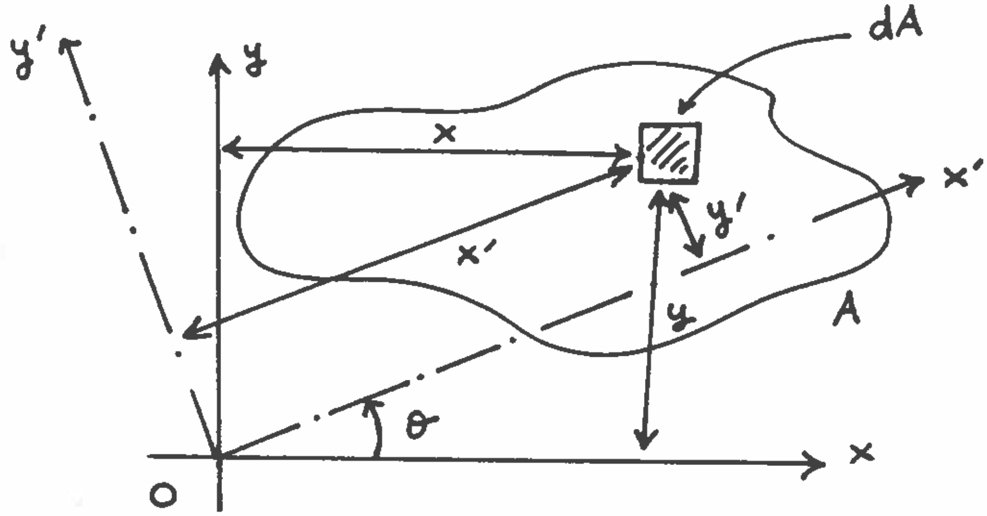
$$= \bar{I}_{xy} + 0 + 0 + d_x d_y A$$

$$\Rightarrow \quad I_{xy} = \bar{I}_{xy} + d_x d_y A$$

Products of inertia are only important for unsymmetrical cross-sections and are essentially a measure of the asymmetry of the inertia of that section about the given axis.

Products of Inertia - Rotation of Axes

Product of inertia is useful in calculating inertia about inclined axes.



Need to calculate moments of inertia of area A about axes x', y' inclined at an angle θ to x, y axes.

$$I_{x'} = \int (y')^2 \, dA = \int (y \cos\theta - x \sin\theta)^2 dA$$

$$I_{y'} = \int (x')^2 dA = \int (y \sin\theta + x \cos\theta)^2 dA$$

expanding gives

$$I_x' = \int (y^2 \cos^2\theta + x^2 \sin^2\theta - 2xy \sin\theta \cos\theta) \, dA$$

$$= \cos^2\theta \; I_Y + \sin^2\theta \; I_x - \sin 2\theta \; I_{xy}$$

and

$$I_y' = \sin^2\theta \; I_Y + \cos^2\theta \; I_x + \sin 2\theta \; I_{xY}$$

Using the double angle formulas for $\cos^2\theta$, $\sin^2\theta$ =>

$$I_{x'} = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y'} = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta$$

Similarly, $I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta$

The angle which makes $I_{x'}$, $I_{y'}$ a maximum or a minimum can be found from

$$\frac{dI_{x'}}{d\theta} = -(I_x - I_y)\sin 2\theta_m - 2I_{xy}\cos 2\theta_m = 0$$

$$\Rightarrow \quad \tan 2\theta_m = \frac{2I_{xy}}{I_y - I_x}$$

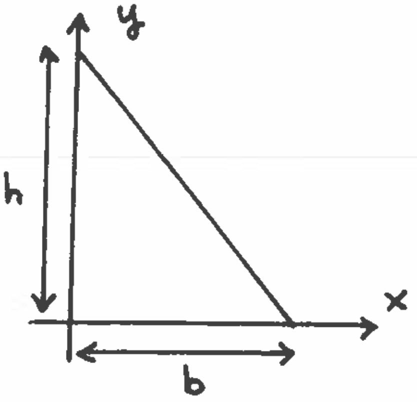
This equation gives 2 values for $2\theta_m$ which differ by π, so the 2 critical values of θ_m will differ by $\pi/2$. One value defines the value of θ_m for $\max\{I_{x'}, I_{y'}\}$ the other defines the value of θ_m for $\min\{I_{x'}, I_{y'}\}$.

The sets of axes that give maximum or minimum moments of inertia are known as the <u>principal axes of inertia</u>. Substituting for $\theta = \theta_m$ into $I_{x'}$, $I_{y'}$ gives

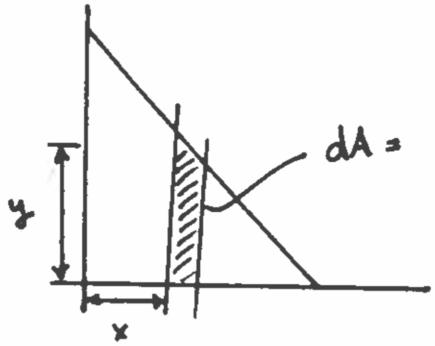
$$I = \frac{1}{2}(I_x + I_y) \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

EXAMPLE 1 : Products of Inertia

Find the product of inertia of the triangle shown about the x and y axes. What is its product of inertia about centroidal axes parallel to the x and y axes?



Choose a vertical strip as a differential element of area, dA



$$dI_{xy} = x_e\, y_e\, dA$$

where (x_e, y_e) is centroid of rectangula element dA,

$$x_e = x$$

$$y_e = \tfrac{1}{2} y = \tfrac{h}{2}(1 - x/b)$$

$$dA = y\,dx = h\,(1 - x/b)\,dx$$

Then $I_{xy} = \int dI_{xy} = \int_{x=0}^{b} x \,.\, \frac{h}{2}(1 - x/b) \,.\, h\,(1 - x/b)\,dx$

$$= \frac{h^2}{2}\int_{x=0}^{b} x\,(1 - x/b)^2\,dx = \underline{\underline{b^2h^2/24}}$$

The coords of the centroid of the triangle are $\bar{x} = b/3$, $\bar{y} = h/3$.

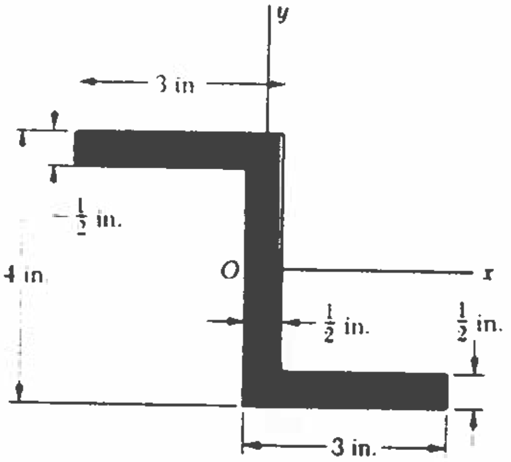
By the parallel axis theorem $I_{xy} = \bar{I}_{xy} + \bar{x}\,\bar{y}\,A$

$$\Rightarrow \quad b^2h^2/24 = \bar{I}_{xy} + (bh/9)(bh/2)$$

$$\underline{\underline{\bar{I}_{xy} = -\,b^2h^2/72}}$$

EXAMPLE 2: Products of Inertia

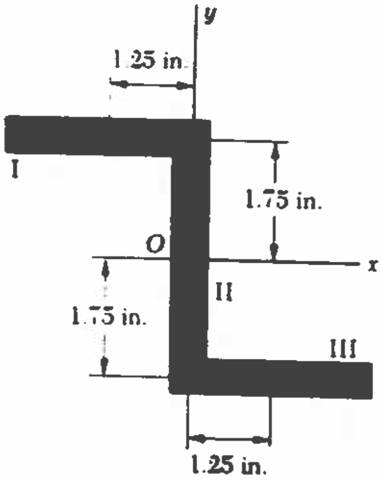
The moments of inertia of the given section are $I_x = 10.38$ in^4 and $I_y = 6.97$ in^4. Find the principal axes of the section about O, and the values of the principal moments of inertia of the section about O



First compute product of inertia about x,y axes. Divide area into 3 rectangles as shown. Note that $\bar{I}_{xy} = 0$ for all 3 rectangles, so by parallel axis theorem

$$I_{xy} = \bar{x}\,\bar{y}\,A$$

for each rectangle



In tabular form:

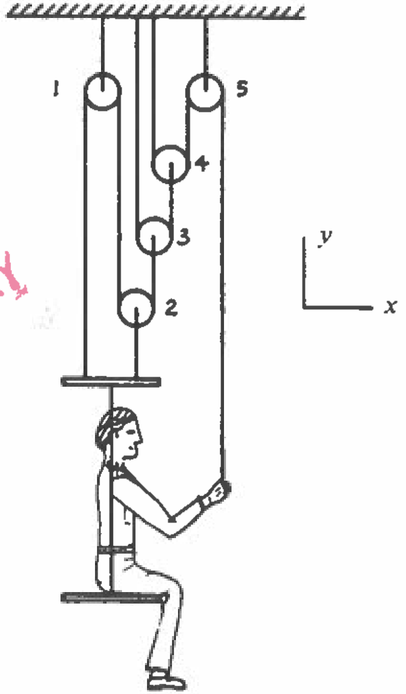
Rect.	A (in^2)	$\bar{x}$ (in)	$\bar{y}$ (in)	I_{xy}
I	1.5	−1.25	+1.75	−3.28
II	1.5	0	0	0
III	1.5	+1.25	−1.75	−3.28

(in^4)

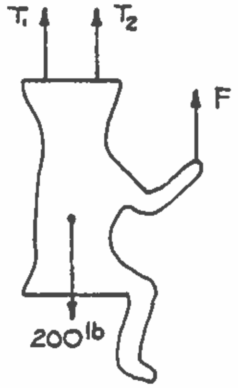
giving $I_{xy} = \sum I_{xy}$

$= -6.56$ in^4

① 11.1 Determine the force F that the 200-lb man must exert on the end of the rope to support himself.

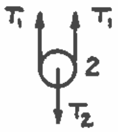


②



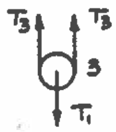
$$\Sigma F_y = 0: \quad T_1 + T_2 + F - 200 = 0$$

③



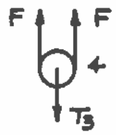
$$\Sigma F_y = 0: \quad T_1 + T_1 - T_2 = 0, \qquad T_2 = 2T_1$$

④



$$\Sigma F_y = 0: \quad T_3 + T_3 - T_1 = 0, \qquad T_1 = 2T_3$$

⑤



$$\Sigma F_y = 0: \quad F + F - T_3 = 0, \qquad T_3 = 2F$$

⑥

$$F = 200 - T_1 - T_2 = 200 - T_1 - 2T_1 = 200 - 3T_1$$

$$F = 200 - 3(2T_3) = 200 - 6(2F)$$

$$F = \frac{200}{13} = \underline{15.4 \text{ lb}}$$

① We suspect that as you read this simple looking problem you immediately tried to solve it intuitively. When you were first introduced to mechanics in beginning physics classes, you probably learned rules about counting the cords in pulley systems, etc. Be careful! It is always safer to fall back on basic principles. ①

Caution: Don't loose track of the simplifying assumptions which you must make in working mechanics problems. We will assume here that the weight of the ropes and the weight of the pulleys are to be neglected and the ropes can transmit only tension.

② As you begin this problem, it may not be obvious what free-body diagrams to use and in which order to use them. Since we are looking for the force F between the rope and the man's hand, we must have at least one free-body diagram which contains that force. We also know that the weight of the man will be related to F. Therefore, one logical free-body diagram contains the man and the sling in which he is sitting. The force equation in the horizontal direction for the FBD of the man and the sling is already satisfied. Since we are assuming the horizontal dimensions to be negligible, in effect, we have a collinear force system which yields only one equation. ②

③ Since the isolation of pulley 2 involves two unknowns, T_1 and T_2 which also appear in the first free-body diagram, we can now obtain an additional relationship between these two forces.

Caution: For a free-body diagram this simple, it may seem natural just to write down the result without sketching the free-body diagram and formerly writing the relationship. Include all steps in your problem solution. It will eliminate errors and allow you to recheck your work. ③

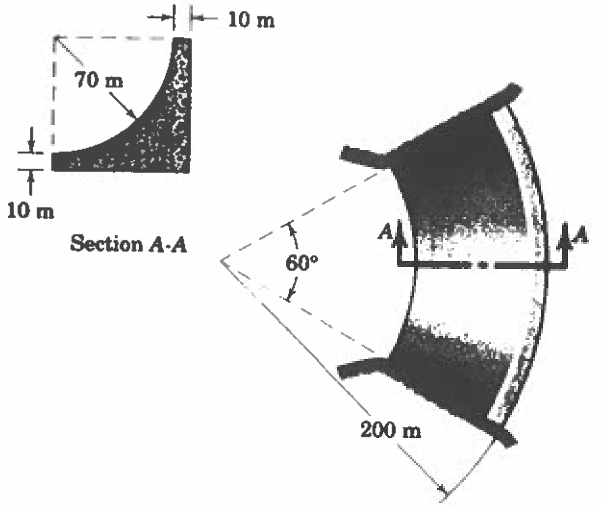
④ We can see by looking at the pulley system that to relate the forces T_1 and T_2 to F, we must introduce another force, T_3. Again, it may appear obvious to you that T_3 is one-half of T_1, but include this step in the solution. ④

⑤ This free-body diagram now relates T_3 and F. Note that since we are dealing with the one-dimensional system, or a system in which we can assume the forces to be collinear, each free-body diagram gives us only one independent equation. Therefore, we were able to write four independent equations involving four unknowns, T_1, T_2, T_3 and F. ⑤

⑥ This step solves the four equations which have been written for F. In spite of what we said about organizing and solving problems as simple as this one, you as an engineer must be able to check your work by intuition or inspection. Now that we see that the force exerted on the rope by the man is $\frac{1}{13}$ of his weight, can you, by inspection of the pulley system, check to see if this is, in fact, correct? ⑥

CIVE 2330 FALL 98 FINAL EXAM

5/92 Calculate the mass m of concrete required to construct the arched dam shown. Concrete has a density of 2.40 Mg/m^3.



Problem 5/92

Friction

Forces of action/reaction have been treated as <u>normal</u> forces at contact surfaces.

Assumption is valid for <u>smooth</u> surfaces. <u>Rough</u> surfaces can also generate tangential forces at contact points.

These forces are called <u>friction forces</u>.

Whenever two contacting surfaces have a tendency to slide relative to each other, the friction forces developed oppose this tendency, i.e. they oppose the impending motion.

Types of friction

Dry friction – friction between two unlubricated surfaces. Forces oppose any tendency of surfaces to slide. <u>Coulomb friction</u>

Fluid friction – friction developed between adjacent layers of fluid moving at different velocities. Friction forces developed depend on relative velocity ~~between~~ fluid layers and fluid viscosity.

When motion occurs friction is called kinetic friction and has a constant value

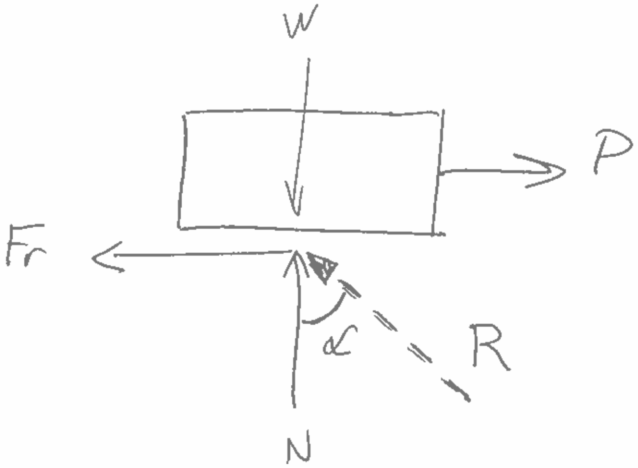
$$F_r = \mu_k N$$

Usually $\mu_k < \mu_s$

The resultant force R on the block by supporting surface is a combination of F_r and N. The ~~to~~ angle α is given by

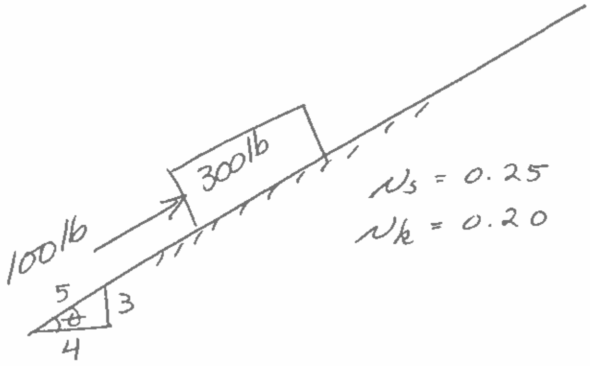
$\tan\alpha = F_r/N$.

When $F_r = F_{r\,max} = \mu_s N$ then α reaches a maximum value ϕ_s and $\mu_s = \tan\phi_s$. The angle ϕ_s is known as the angle of static friction.



If motion is about to occur, R must lie on a right circular cone of vertex angle $2\phi_s$ (cone of static friction).
If motion is not about to occur, R will lie within the cone.

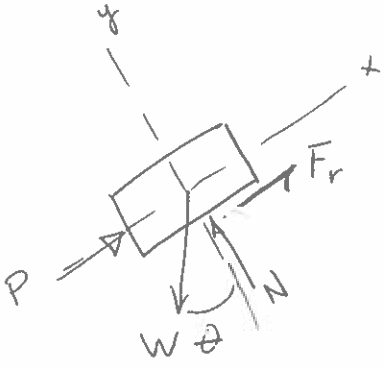
Example Friction



$\mu_s = 0.25$

$\mu_k = 0.20$

Is block in equilibrium? What is value of F_r.



$\Sigma F_x = 0 = P - W\sin\theta + F_r$

$100 + F_r - 300(3/5) = 0$

$F_r = 80\text{lbs}$

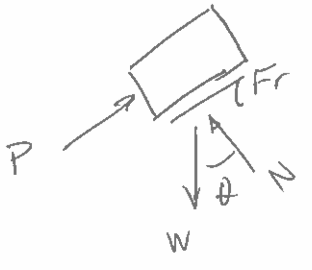
$\Sigma F_y = 0$

$N - W\cos\theta = 0$

$N = 300(4/5) = 240\text{lbs}$

$F_{r_{max}} = \mu_s N = 0.25(240) = 60\text{ lbs}$

∴ Block will move down plane.



$F_r = \mu_R N = 0.2(240)$

$= 48\text{ lbs}$

Net force on block

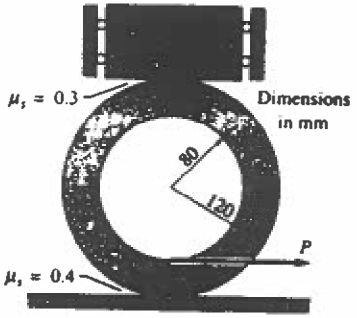
$\Sigma F_x = P + F_r - W\sin\theta$

$= 100 + 48 - 300(3/5)$

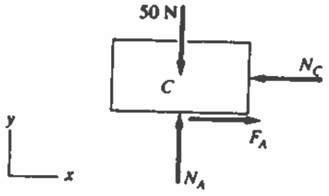
$= -32\text{ lbs}$ (down ~~ble~~ plane)

EXAMPLE 2 : Friction

The spool weighs 25 N and its center of gravity is located at the geometric center. The weight of block C is 50N. Find the largest horizontal force P that can be applied without causing motion of the system.



Motion can be caused either by sliding at A or sliding at B



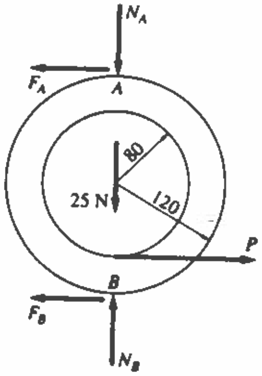
Assume Sliding Impending at A:

Then $F_A = \mu_{s_A} N_A = .3(50) = 15\ N.$

$\circlearrowleft + \sum M_B = 0$ for spool →

$F_A(240) - P(40) = 0$

$\underline{P = 90\ N}$



Assume Sliding Impending at B:

Then $F_B = \mu_{s_B} N_B = .4(75) = 30\ N.$

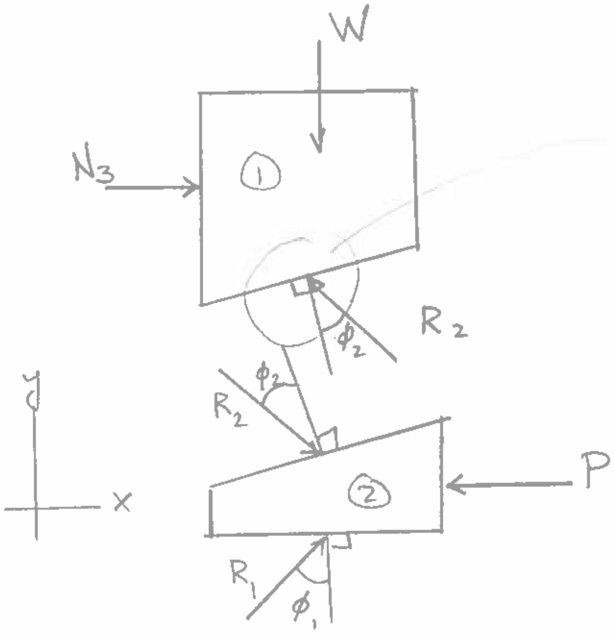
$\circlearrowleft + \sum M_A = 0$ for spool →

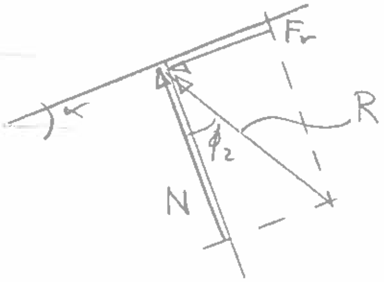
$-F_B(240) + P(200) = 0$

$\underline{P = 36\ N}$

So sliding at A will occur for $P > 90$ and sliding at B will occur for $P > 36$.

Max. force that can be applied is 36 N (sliding impending at B)





$$\tan\phi_2 = \frac{Fr}{N}$$

$$Fr = \mu N$$

$$\therefore \frac{Fr}{N} = \mu$$

Thus $\tan\phi_2 = \mu_2$

and $\tan\phi_1 = \mu_1$

①

$$\Sigma F_x = 0 = N_3 - R_2 \sin(\phi_2 + \alpha) = 0$$

$$\Sigma F_y = 0 = R_2 \cos(\phi_2 + \alpha) - W = 0$$

②

$$\Sigma F_x = 0 : R_1 \sin\phi_1 + R_2 \sin(\alpha + \phi_2) - P = 0$$

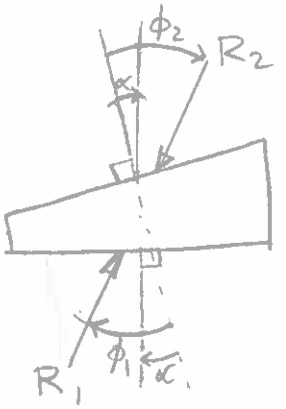
$$\Sigma F_y = 0 = R_1 \cos\phi_1 - R_2 \cos(\alpha + \phi_2) = 0$$

If P is removed, either the wedge remains (self locking) or the wedge slips.

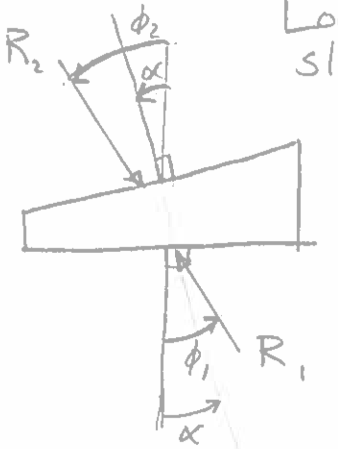
If P is removed and wedge remains in place equilibrium requires

$R_1 = R_2$ & R_1, R_2 collinear

Upper slipping



Lower slipping



$\alpha < \phi$

$x: R_1 \sin(\phi_1 - \alpha) = R_2 \sin(\phi_2 - \alpha)$

$y: R_1 \cos(\phi_1 - \alpha) = R_2 \cos(\phi_2 - \alpha)$

$\Rightarrow \quad R_1 = R_2$

$\phi_1 - \alpha = \phi_2 - \alpha$

$\Rightarrow \phi_1 = \phi_2 = \phi$

$x: R_1 \sin\phi_1 = R_2 \sin\phi_2$

$y: R_1 \cos\phi_1 = R_2 \cos\phi_2$

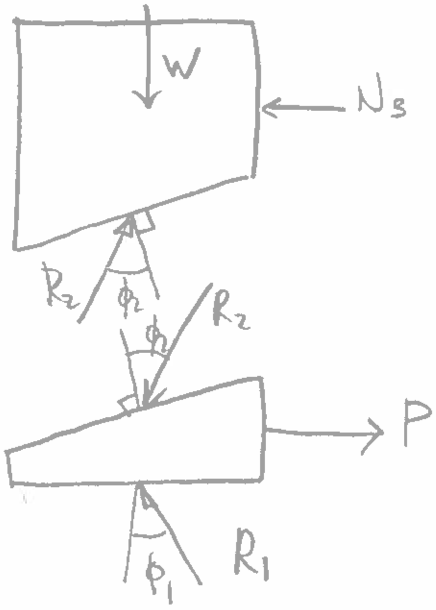
$\Rightarrow \quad R_1 = R_2$

$\phi_1 = \phi_2 = \phi$

For Wedge to slip, both faces must slip simultaneously.

It can be shown that wedge will not slip provided $\alpha < 2\phi$

For self-locking wedge a force P is required to lower W



x: $R_2 \sin(\phi_2 - \alpha) - N_3 = 0$

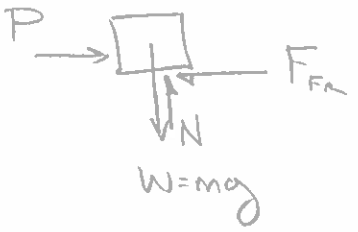
y: $R_2 \cos(\phi_2 - \alpha) - W = 0$

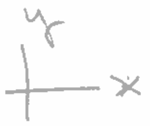
x: $P - R_1 \sin\phi_1 - R_2 \sin(\phi_2 - \alpha) = 0$

y: $R_1 \cos\phi_1 - R_2 \cos(\phi_2 - \alpha) = 0$

6.1 FIND F_{Fr} for $P = 300N, 400N, 500N$

<u>FBD</u>





$\Sigma F_x = 0 = P - F_{FR}$ (No Motion)

$P = F_{FR}$

$\Sigma F_y = 0 = N - W$

$N = W = 90(9.8) = 882N$

$F_{FR} = \mu N = 0.5(882)$ MAX - No MOTION $= 441$

$0.4(882)$ KINETIC - MOTION $= 352.8$

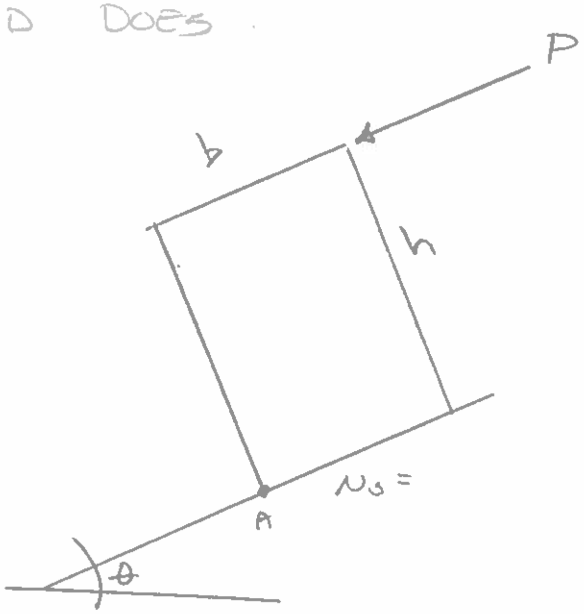
$P = 300,\ F_{FR} = 300$

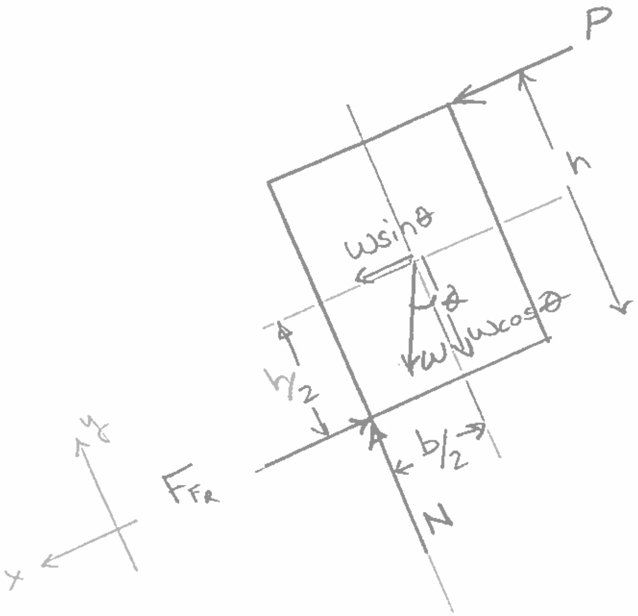
$P = 400,\ F_{FR} = 400$

$P = 500,\ F_{FR} = 352.8$ — BOX WILL ACCELERATE

6/5

FIND h/b SO BLOCK SLIDES AND DOES NOT TIP





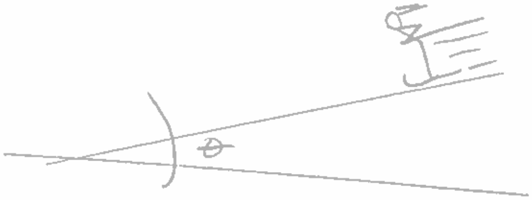
$$\Sigma F_x = P + W\sin\theta - \mu_s W\cos\theta = 0$$

$$\Sigma F_y = W\cos\theta - W\cos\theta = 0$$

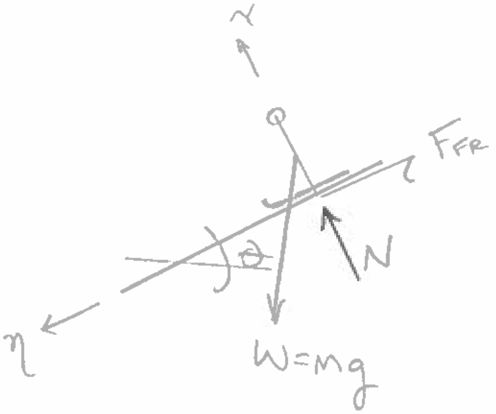
$$\Sigma M_A = P(h) + W\sin\theta\left(\frac{h}{2}\right) - W\cos\theta\left(\frac{b}{2}\right) = 0$$

6.4 JILL PHYSICS ON SKI SLOPE.

$\mu_s = 0.1$, $\mu_k = 0.08$



WANT CONSTANT SPEED SECTION, FIND θ



$\Sigma F_\nu = 0 = N - mg\cos\theta$

$N = mg\cos\theta$

$\Sigma F_\eta = 0 = mg\sin\theta - F_{FR}$

CONSTANT (NON-ZERO) SPEED USE μ_k

$\cancel{mg}\sin\theta = \mu_k N = \mu_k \cancel{mg}\cos\theta$

$\tan\theta = \mu_k$

$\tan\theta = 0.08 \qquad \theta = 4.57$

ASIDE

$\tan\theta = 0.10 \qquad \theta = 5.71$

∴ SKIER MUST BE MOVING TO MAINTAIN CONSTANT SPEED,

WEIGHT NOT ENOUGH TO OVERCOME STATIC FRICTION!

$\mu_s W\cos\theta = P + W\sin\theta$

$\therefore\ Ph = h\mu_s W\cos\theta - hW\sin\theta$

$0 = h\mu_s W\cos\theta - hW\sin\theta + W\sin\theta\,\frac{h}{2} - W\cos\theta\,\frac{b}{2} = 0$

divide by b

$$\frac{h}{b}\mu_s w\cos\theta - \frac{h}{b}w\sin\theta + \frac{h}{b}\,\frac{w\sin\theta}{2} - \frac{1}{2}w\cos\theta = 0$$

divide by $W\cos\theta$

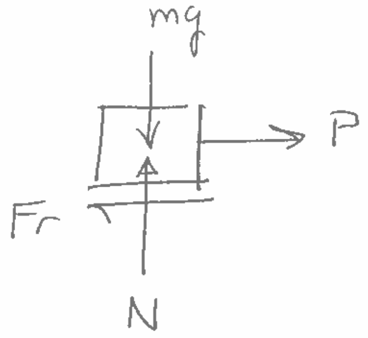
$$\frac{h}{b}\mu_s - \frac{h}{b}\tan\theta + \frac{h}{b}\,\frac{\tan\theta}{2} - \frac{1}{2} = 0$$

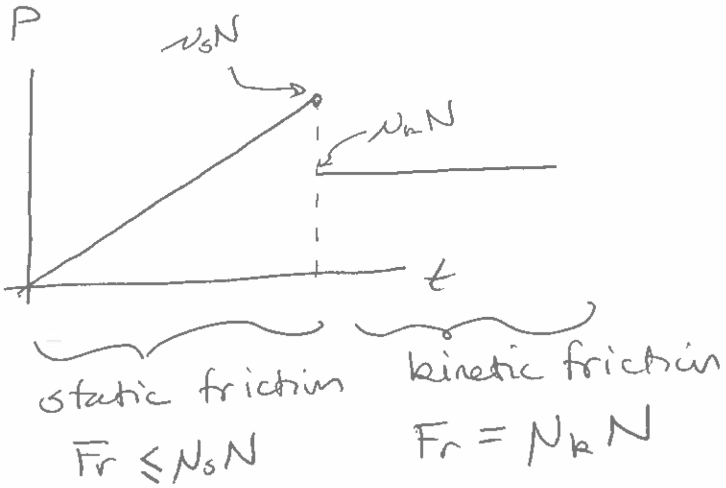
$$\frac{h}{b}\left(\mu_s - \frac{\tan\theta}{2}\right) = \frac{1}{2}$$

$$\therefore\ \frac{h}{b} = \frac{1}{2}\left(\frac{1}{\mu_s - \frac{\tan\theta}{2}}\right) = \frac{1}{2\mu_s - \tan\theta}$$

Internal friction - Friction found in all solid materials subjected to cyclic loading.

<u>Dry friction</u> Block of mass m on rough surface. Apply force P from 0 until block moves





- $P = 0$, $Fr = 0$
- P increases, with no motion $Fr = P$
- P increases further block starts to move.

At this point the frictional force drops abrubtly and remains constant.

Region up to point of slippage is called static fricti Value Fr is found from equilibrium equations and is less than some maximum value, Fr_{max}.

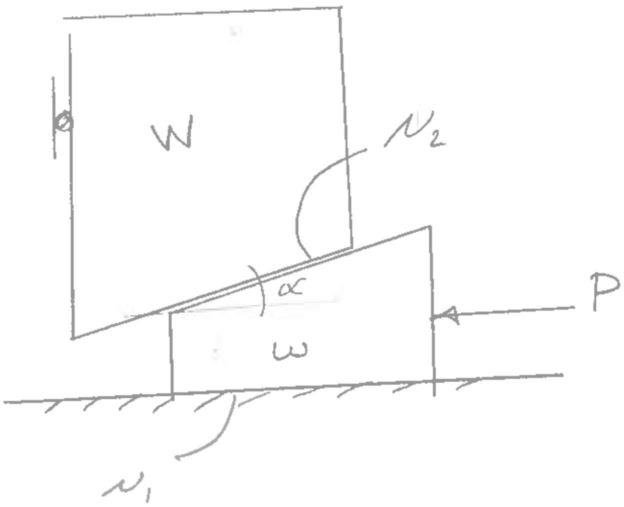
When motion is about to occur the friction force equals its maximum value

$$Fr_{max} = \mu_s N$$

Wedges

A wedge is a simple machine to raise/lower a large body or as a method to apply large forces. Wedges depend on friction

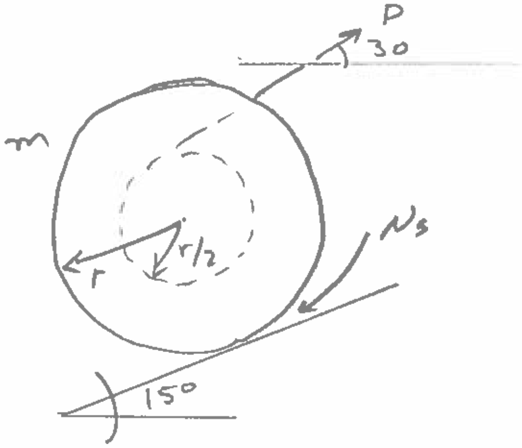
When sliding is impending, the resultant on each sliding surface is inclined from the normal by an angle ϕ_s, the friction angle



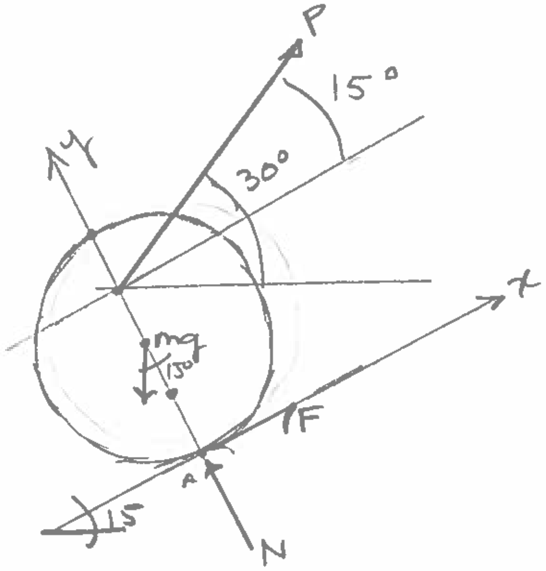
Usually can ignore weight of the wedge, ω.

$\therefore \ \omega \ll W$

FIND SMALLEST μ_s SO DRUM WILL **ROLL** UPHILL



FBD



$$\Sigma F_x = P\cos 15 - mg\sin 15 + F = 0$$

$$\Sigma F_y = P\sin 15 - mg\cos 15 + N = 0$$

$$F = \mu_s N$$

$$\circlearrowleft \Sigma M_A = -mg\sin 15\, r + P\cos 15\, \frac{3r}{2} = 0$$

a) $P\cos 15 - mg\sin 15 + N_s N = 0$

b) $P\sin 15 - mg\cos 15 + N = 0$

c) $-mg\sin 15 r + P\cos 15 \frac{3r}{2} = 0$

Solve (c) for P

$$P = \frac{2mg\sin 15 \cancel{r}}{3\cancel{r}\cos 15} = \frac{2}{3}mg\tan 15 = 0.1786\,mg$$

Solve (b) for N

$$-\frac{2}{3}mg\tan 15\sin 15 + mg\cos 15 = N$$

$$mg\left(\cos 15 - \frac{2}{3}\frac{\sin^2 15}{\cos 15}\right) = N = 0.919\,mg$$

Solve (a) for N_s

$$\frac{-\frac{2}{3}mg\tan 15\cos 15 + mg\sin 15}{mg\left(\cos 15 - \frac{2}{3}\frac{\sin^2 15}{\cos 15}\right)} = N_s$$

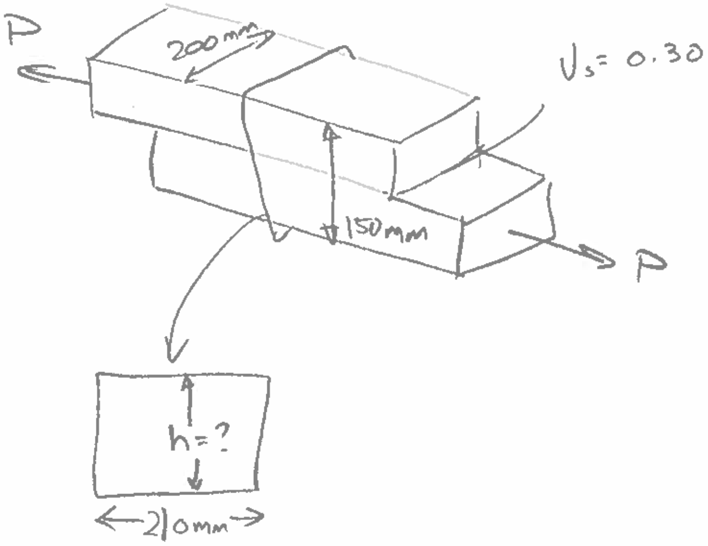
$$\frac{\cancel{mg}\left(\sin 15 - \frac{2}{3}\sin 15\right)}{\cancel{mg}\left(\cos 15 - \frac{2}{3}\frac{\sin^2 15}{\cos 15}\right)} = N_s = 0.0938$$

$P = 0.178\,mg$

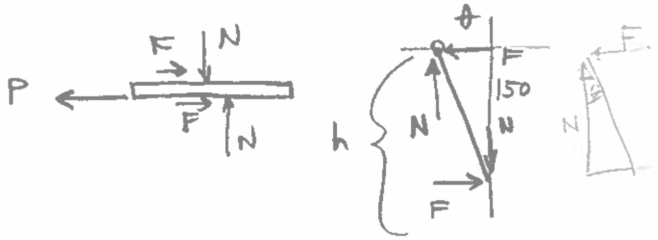
$F = 0.086\,mg$

$N = 0.938\,mg$

6.23



P = 800N find h for no slip



$\frac{F}{N} = \tan\theta$ at impending motion

$F = \mu_s N = 0.30 N$

Solve for θ

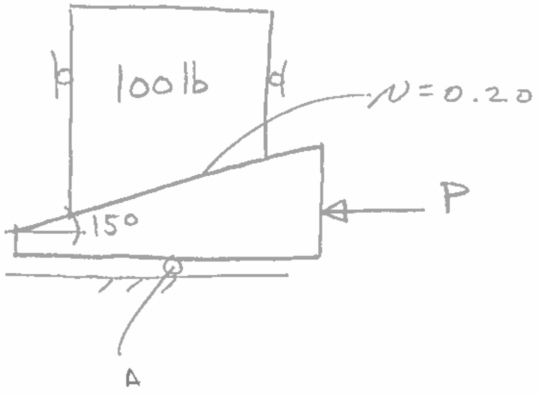
$\tan\theta = 0.3 = \frac{\sqrt{h^2 - 150^2}}{150}$ $h = 156.6 \text{ mm}$

$P = 800 N$

$P - 2F = 0 \Rightarrow F = 400 N$

$N = \frac{400}{0.3} = 1333 N$

6 53 FIND FORCE TO RAISE BLOCK IF $\nu_A = 0$ (frictionless)
$\nu_A = 0.2$.



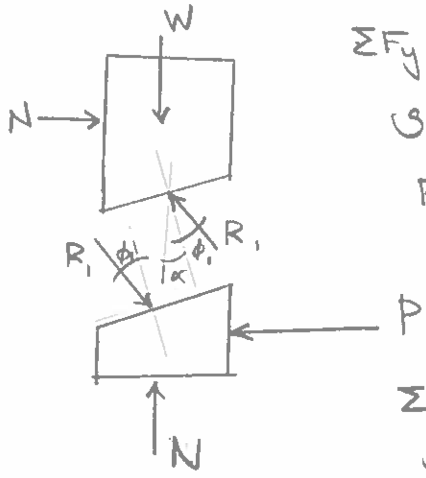
$\tan\phi_1 = 0.2$

$\phi_1 = 11.3°$

$\alpha = 15$

$\phi_1 + \alpha = 26.3$

a)



$\Sigma F_x = N - R_1 \sin(\phi_1 + \alpha)$

$\Sigma F_y = R_1 \cos(\phi_1 + \alpha) - W$

SOLVE FOR R_1

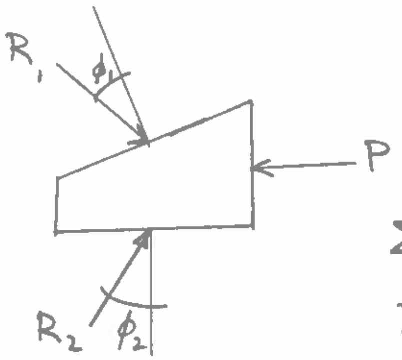
$$R_1 = \frac{W}{\cos(26.3)} = \frac{100}{0.89} = 111.55 \text{ lbs}$$

$\Sigma F_x = R_1 \sin(\phi_1 + \alpha) - P$

solve for P

$P = 111.55 \sin(26.3) = \underline{\underline{49.43 \text{ lbs}}}$

b)



$\Sigma F_x = R_1 \sin(\phi_1 + \alpha) + R_2 \sin(\phi_2) - P$

$\Sigma F_y = R_2 \cos\phi_2 - R_1 \cos(\phi_1 + \alpha)$

Solve for R_2 $\quad R_2 = \dfrac{R_1 \cos(26.3)}{\cos(11.3)} = 101.98$

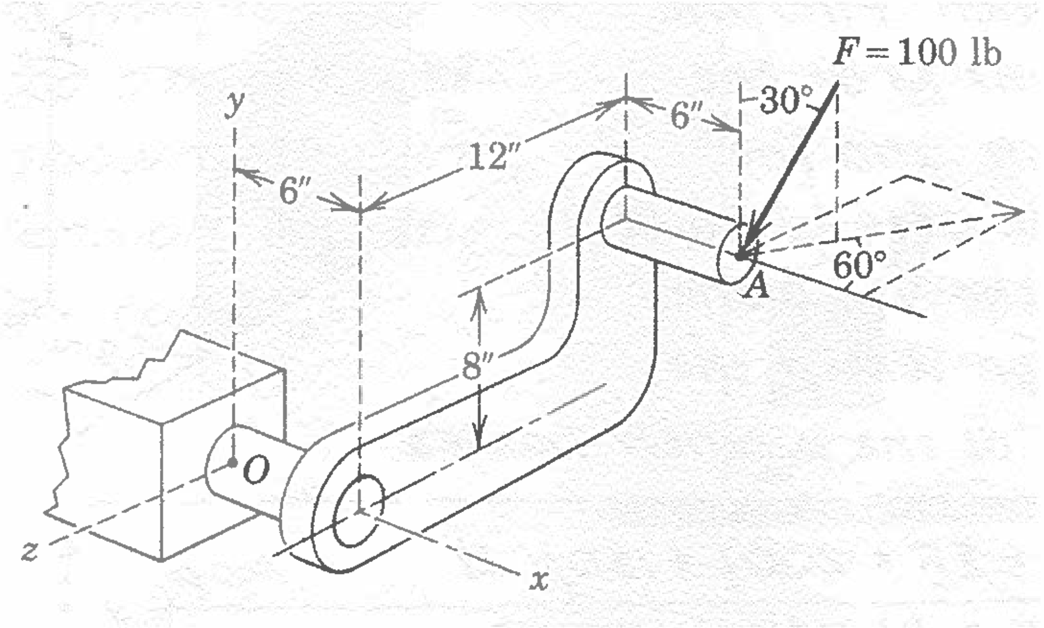
Solve for P

$P = 49.4 + 101.98 \sin(11.3) = \underline{\underline{69.4 \text{ lbs}}}$

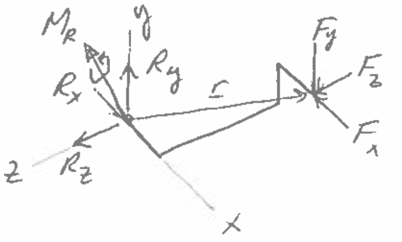
6.53

SOLUTION (10 min)

Problem 2: Determine the reaction force and couple at *O* required to keep the lever in equilibrium. All dimensions are inches.



Hints: Draw a free body diagram of the lever and show three positive reaction forces and a positive restraining moment vector. Use equilibrium conditions to determine the magnitudes of the reaction forces and the restraining moment vector components.



$F_x = -F\sin 30\cos 60 = -25$

$F_z = F\sin 30\sin 60 = 43.3$

$F_y = -F\cos 30 = -86.6$

ALL FORCES IN kN

$\therefore \underline{F} = -25\underline{i} - 86.6\underline{j} + 43.3\underline{k}$

$\underline{r} = 12\underline{i} + 8\underline{j} - 12\underline{k}$

$\Sigma \underline{F} = 0 = \underline{F} + \underline{R} = -25\underline{i} - 86.6\underline{j} + 43.3\underline{k} + R_x\underline{i} + R_y\underline{j} + R_z\underline{k}$

$\therefore R_x = 25, \; R_y = 86.6, \; R_z = -43.3$

$\Sigma M_o = 0 = \underline{M}_R + \underline{r} \times \underline{F}$

$\underline{M}_R = -\underline{r} \times \underline{F}$

$$\underline{r} \times \underline{F} = \begin{vmatrix} i & j & k \\ 12 & 8 & -12 \\ -25 & -86.6 & 43.3 \end{vmatrix}$$

$\rightarrow (8)(43.3) - (-12)(-86.6)\underline{i} = -692.8\underline{i}$

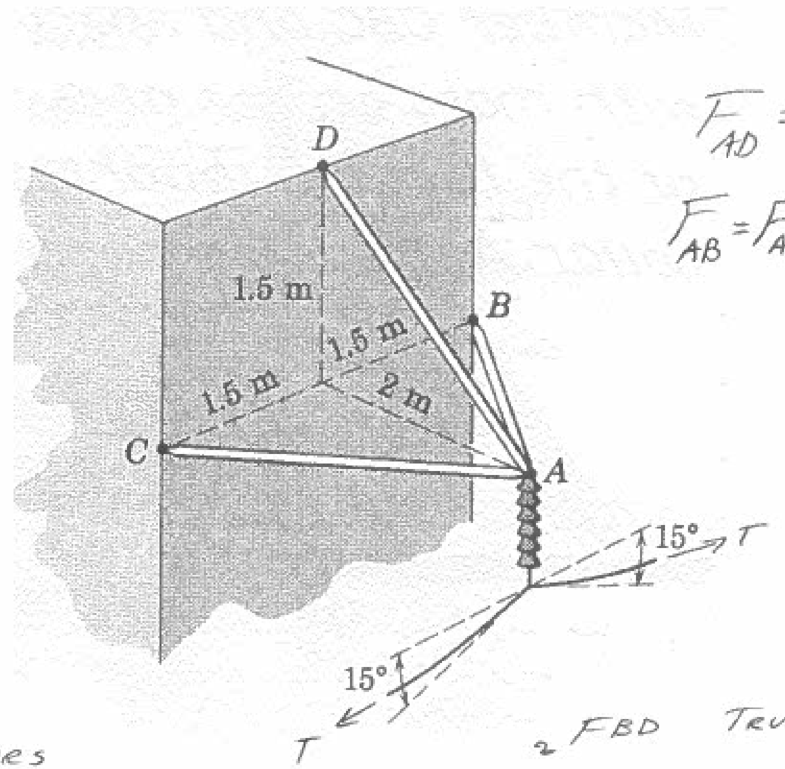
$+ (-12)(-25) - (12)(43.3)\underline{j} = -219.6\underline{j}$

$= 839.2\,k$

$\therefore \underline{M}_R = 692.8\underline{i} + 219.6\underline{j} + 839.2\underline{k}$

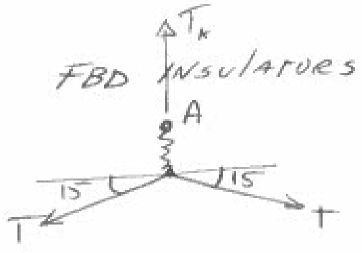
SOLUTION (10 min)

Problem 3: A high-voltage power line is suspended as shown. Tension in the line at the insulators is *3kN*. Determine the forces in links *AD, AC, AB*.



$F_{AD} = 2.58$ kN Tension

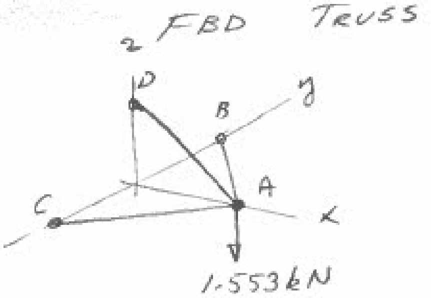
$F_{AB} = F_{AC} = 1.29$ kN Comp.



$\Sigma F_z = 0 = T_A - 2T\sin 15$

$T_A = 2(3)\sin 15$

$= 1.553$ kN



ALL TENSIONS

$$\underline{F_{AC}} = F_{AC}\left(\frac{-2\underline{i} - 1.5\underline{j} + 0\underline{k}}{\sqrt{2^2+1.5^2}}\right) = F_{AC}\left(\frac{-2}{2.5}\underline{i} - \frac{1.5}{2.5}\underline{j}\right)$$

$$\underline{F_{AB}} = F_{AB}\left(-\frac{2}{2.5}\underline{i} + \frac{1.5}{2.5}\underline{j}\right)$$

$$\underline{F_{AD}} = F_{AD}\left(\frac{-2\underline{i}}{2.5} + \frac{0\underline{j}}{2.5} + \frac{1.5\underline{k}}{2.5}\right)$$

EQUILIBRIUM AT A

$\Sigma F_x = 0 = F_{AC}\frac{-2}{2.5} + F_{AB}\frac{-2}{2.5} + F_{AD}\frac{-2}{2.5}$

$\Sigma F_y = 0 = F_{AC}\frac{-1.5}{2.5} + F_{AB}\frac{1.5}{2.5} \Rightarrow F_{AC} = F_{AB}$

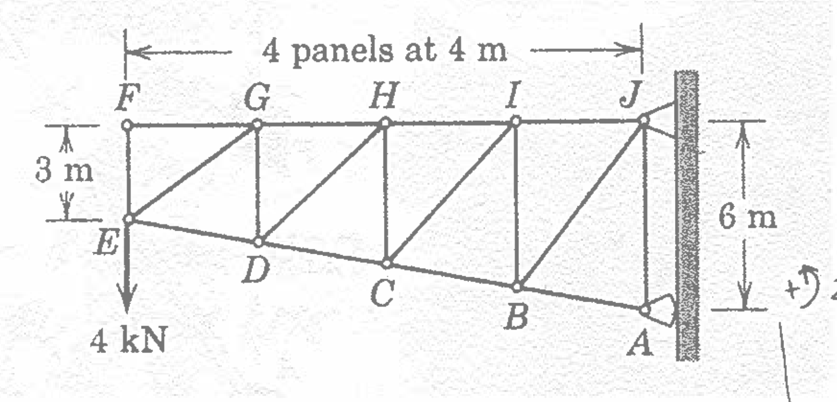
$\Sigma F_z = 0 = F_{AD}\,1.5 - 1.553$

$F_{AD} = \frac{1.553(2.5)}{1.5} = \underline{2.58\text{ kN}}$

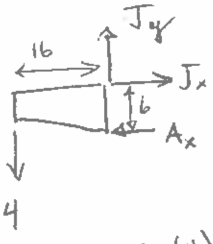
$F_{AB} = -F_{AD} = -\underline{1.29\text{ kN}}$ *

SOLUTION (10 min)

Problem 4: Determine the reaction forces at *A* and *J*, and the forces in members *FG*,*EG*, and *DG*, for the simple truss shown.



REACTIONS



$+\circlearrowleft \Sigma M_j = 0 = 16(4)\underline{k} - 6A_x$

$\therefore \underline{A_x} = \frac{16(4)}{6} = \underline{10.66k}$

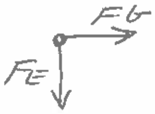
$\Sigma F_y = 0 = J_y - 4$

$J_y = 4$

$\Sigma F_x = 0 = J_x - A_x$

$J_x = A_x = 10.66k$

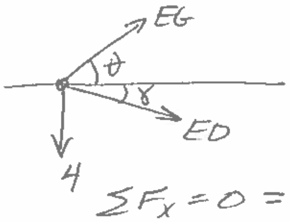
JOINT F



$\Sigma F_x = 0 = FG$

$\Sigma F_y = 0 = FE$ $\therefore$ FE & FG ARE ZERO FORCE MEMBERS

JOINT E



$\theta = \tan^{-1}\frac{3}{4} = 36.87°$

$\gamma = \tan^{-1}\frac{3}{16} = 10.62°$

$\Sigma F_x = 0 = EG\cos\theta + ED\cos\gamma \rightarrow EG = \frac{-ED\cos\gamma}{\cos\theta}$

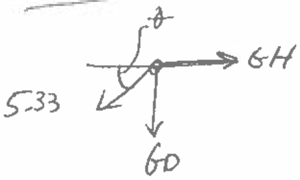
$\Sigma F_y = 0 = EG\sin\theta - ED\sin\gamma - 4 \rightarrow -ED\cos\gamma\tan\theta - ED\sin\gamma = 4$

$ED = \frac{-4}{\cos 10.62(0.75) + \sin 10.62}$

$= -4.34\ kN$

$\rightarrow \underline{EG} = 4.34\ kN\frac{\cos 10.62}{\cos 36.87} = \underline{5.33 kN}$

JOINT G



$\Sigma F_y = 0 = -GD - 5.33\sin\theta$

$GD = -5.33\sin 36.87 = \underline{-3.19\ kN}$

SUMMARY

REACTIONS $J_x = 10.66\ kN \rightarrow$

$J_y = 4\ kN \uparrow$

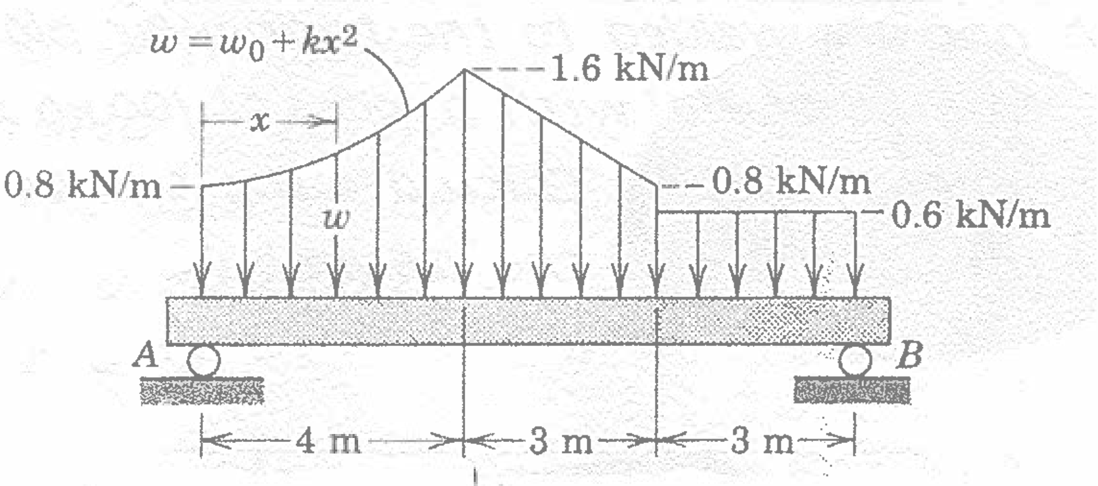
$A_x = 10.66\ kN \leftarrow$

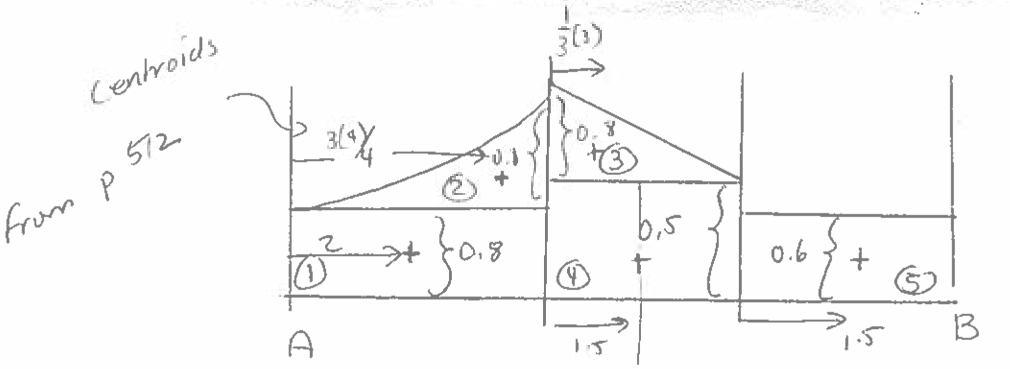
MEMBERS

FG = 0 ; EG = 5.33kN (Tension) ; DG = 3.19kN(Comp

10 min

Problem 5. Determine the support reactions at A and B for the beam shown. Sketch the shear and bending moment diagrams for this beam.





USE COMPOSITE AREA & CENTROITDS

AREAS

$F_① = 4(0.8) = 3.2$

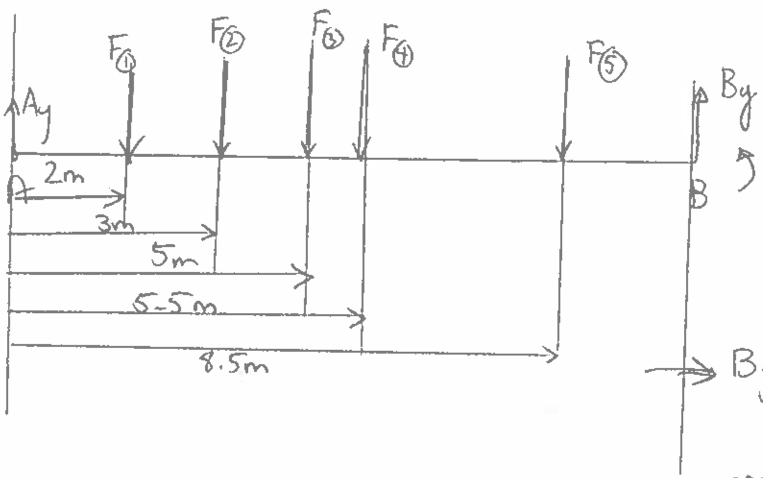
$F_② = \frac{(4)(0.8)}{3} = 1.06$

$F_③ = \frac{3(0.8)}{2} = 1.2$

$F_④ = 3(0.8) = 2.8$

$F_⑤ = 3(0.6) = 1.8$

Σ 10.06 kN



$\circlearrowright \Sigma M_A = 0$

$= -2(3.2) - 3(1.06) - 5(1.2) - 5.5(2.8) - 8.5(1.8) + 10(B_y)$

$\rightarrow B_y = \frac{46.28}{10} = \underline{\underline{4.63k}}$

$\Sigma F_y = 0 = A_y + B_y - \Sigma F_i$

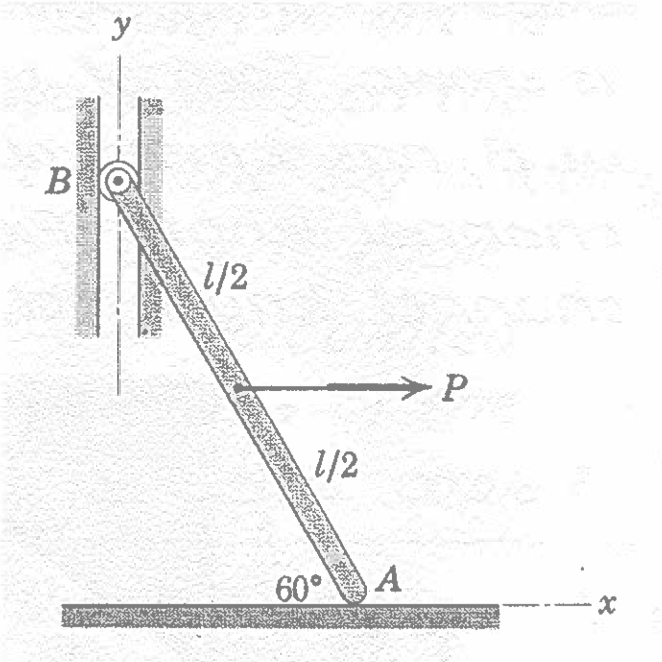
$\rightarrow A_y = 10.06 - 4.63 = \underline{\underline{5.43}}$

30min

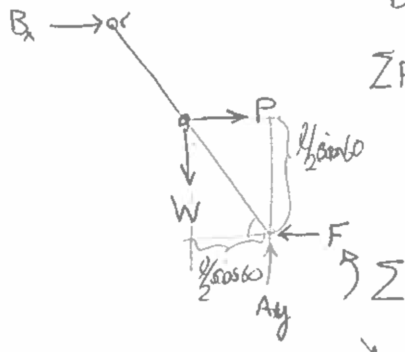
SOLUTION

CIVE 2330 Fall 1999 Final Examination

Problem 6. A uniform *60-kg* bar *AB* is subjected to force *P*. Guides a *B* are smooth. At *A* the coefficient of static friction is *0.8*. Determine the friction force at *A* if *P=400N*. Find the magnitude of *P* required to cause slippage at *A*.



FBD BAR



$\Sigma F_x = 0 = B_x + P - F$

$B_x = F - P$

$F_{max} = \mu_s A_y$

$\Sigma F_y = 0 = -W + A_y$

$A_y = W$

$= 60kg(9.8) = 588N$

$F_{max} = 0.8(588) = 470.7N$

$\circlearrowleft \Sigma M_g = 0 = A_y \cdot l/2 \cos 60 - F \, l/2 \sin 60 - B_x \, l/2 \sin 60$

$A_y \cancel{l/2} \cancel{\cos} 60 = F \frac{\cancel{l/2} \sin 60}{\cos 60} + (F - P) \frac{\cancel{l/2} \sin 60}{\cos 60} = \{F + F - P\} \tan 60$

$A_y / \tan 60 = F + F - P = 2F - P$

$\left(\frac{A_y}{\tan 60} + P \right) \frac{1}{2} = F$

Case 1 $P = 400$; $F = 369N$

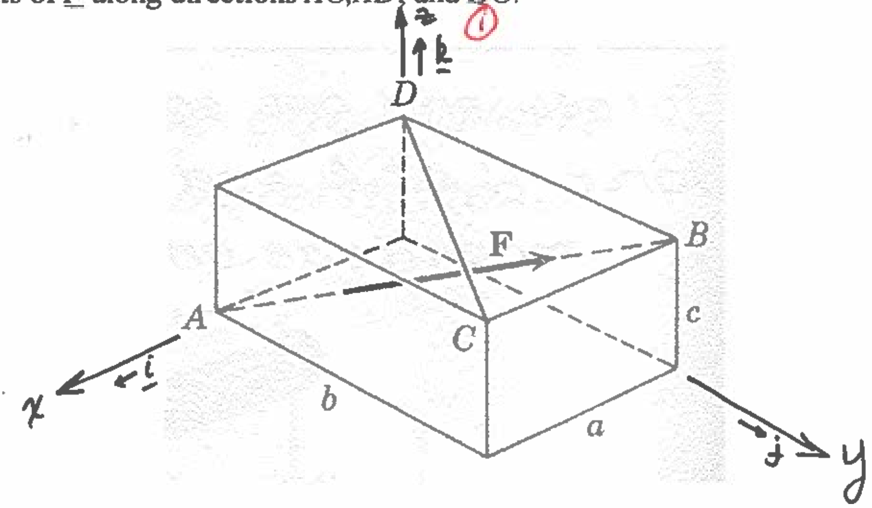
Case 2 $P = 602$; $F = 470.7$

SOLUTION (10min)

CIVE 2330 Fall 1999 Final Examination
Page 1 of 8

coordinates -2
$\underline{F}$ - 2
$_{AC}$ - 2
$_{AD}$ - 2
$_{DC}$ - 2
$_{AC}\cdot\underline{F}$ - 2
$_{AD}\cdot\underline{F}$ - 2
$_{DC}\underline{F}$ - 2
- 2
- 2
- 2

Problem 1: For *a=3m*, *b=6m*, *c=2m*, *F=10kN* determine the magnitudes of the components of $\underline{F}$ along directions *AC*,*AD*, and *DC*.



$$\underline{F} = 10kN\left(\frac{-3\underline{i}+6\underline{j}+2\underline{k}}{\sqrt{3^2+6^2+2^2}}\right) = -\frac{30}{7}\underline{i} + \frac{60}{7}\underline{j} + \frac{20}{7}\underline{k}$$

$$e_{AC} = \frac{0\underline{i}+6\underline{j}+2\underline{k}}{\sqrt{6^2+2^2}} = 0\underline{i} + \frac{6}{\sqrt{40}}\underline{j} + \frac{2}{\sqrt{40}}\underline{k}$$

$$e_{AD} = \frac{-3\underline{i}+0\underline{j}+2\underline{k}}{\sqrt{3^2+2^2}} = -\frac{3}{\sqrt{13}}\underline{i} + 0\underline{j} + \frac{2}{\sqrt{13}}\underline{k}$$

$$e_{DC} = \frac{3\underline{i}+6\underline{j}+0\underline{k}}{\sqrt{3^2+6^2}} = \frac{3}{\sqrt{45}}\underline{i} + \frac{6}{\sqrt{45}}\underline{j} + 0\underline{k}$$

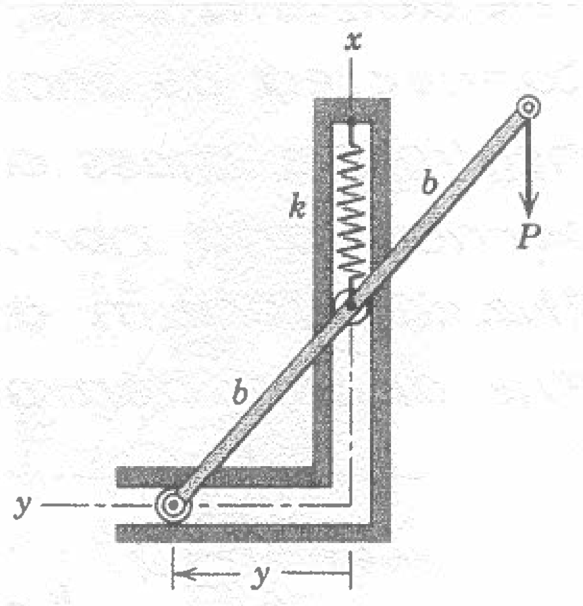
$$F_{AC} = \underline{F}\cdot\underline{e}_{AC} = -\frac{30}{7}\cdot 0 + \frac{60}{7}\cdot\frac{6}{\sqrt{40}} + \frac{20}{7}\cdot\frac{2}{\sqrt{40}} = \frac{400}{7\sqrt{40}} = \underline{\underline{9.04\,kN}}$$

$$F_{AD} = \underline{F}\cdot\underline{e}_{AD} = -\frac{30}{7}\cdot\frac{-3}{\sqrt{13}} + \frac{60}{7}\cdot 0 + \frac{20}{7}\cdot\frac{2}{\sqrt{13}} = \frac{130}{7\sqrt{13}} = \underline{\underline{5.15\,kN}}$$

$$F_{DC} = \underline{F}\cdot\underline{e}_{DC} = -\frac{30}{7}\cdot\frac{3}{\sqrt{45}} + \frac{60}{7}\cdot\frac{6}{\sqrt{45}} + \frac{20}{7}\cdot 0 = \frac{270}{7\sqrt{45}} = \underline{\underline{5.74\,kN}}$$

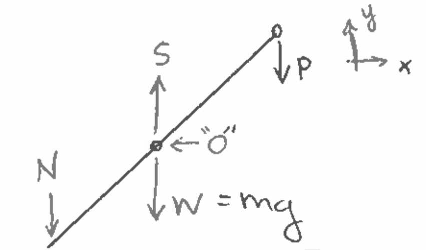
SOLUTION (30 min)

Problem 8: Determine *y* for equilibrium of the mechanism in the vertical plane under the load P. The spring of stiffness *k* is unstretched when *y=0*. The mass of the uniform link is *m*.



Hints: Determine the deformation of the spring as a function of y from trigonometry. Then draw free body diagram of the bar, don't forget the weight of the bar and normal forces at rollers. Use equilibrium conditions to determine the requested quantity.

FBD Bar



$$+\circlearrowleft\ \Sigma M_o = 0 = Ny - Py$$

$$\Rightarrow N = P$$

$$\Sigma F_y = 0 = S - W - N - P$$

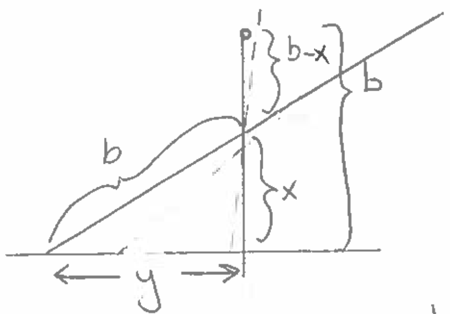
$$S = 2P + W$$

$$k(b-x) = 2P + W$$

$$b - x = \frac{2P+W}{k}$$

$$x = b - \frac{2P+W}{k}$$

GEOMETRY



Spring deforms $b - x$

substitute ← $S = k(b - x)$ linear spring

$$x = \sqrt{b^2 - y^2}$$

$$\therefore\ y^2 = b^2 - x^2$$

$$x^2 = b^2 - 2b\left(\frac{2P+W}{k}\right) + \left(\frac{2P+W}{k}\right)^2$$

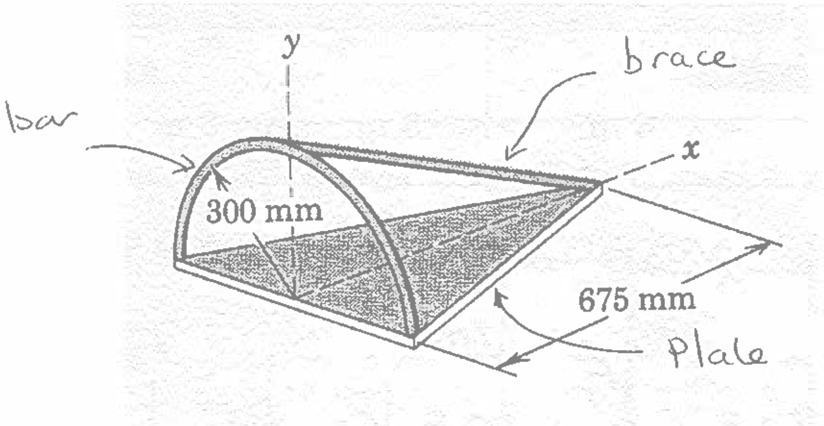
over

10 min

CIVE 2330 Fall 1999 Final Examination

Page 7 of 8

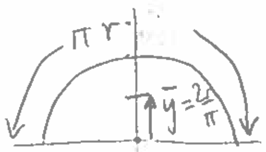
Problem 7. The semicircular and straight bars are made from stock with a mass of *7.5kg* per meter of length and are welded to the triangular plate made from material with a mass of *100kg* per square meter. Determine the coordinates of the center of gravity of the assembly.

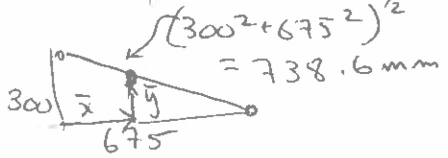


	m	$\bar{x}$	$\bar{y}$	$m\bar{x}$	$m\bar{y}$
Circular Bar	7.06kg	0	190.98	0	1348.36
Brace	5.54	337.5	150	1869.75	831.
Plate	20.25	225	0	4556.25	0
Σ	32.85			6426	2179.36

$$\bar{x} = \frac{m\bar{x}}{m} = \frac{6426}{32.85} = \underline{\underline{195.6\ mm}} \leftarrow$$

$$\bar{y} = \frac{m\bar{y}}{m} = \frac{2179.36}{32.85} = \underline{\underline{66.34\ mm}} \leftarrow$$





$(300^2+675^2)^{1/2} = 738.6mm$

$A = 300 \cdot 675 = 0.2025 m^2$

$\bar{x} = \frac{675}{3} =$

CIVE 2330

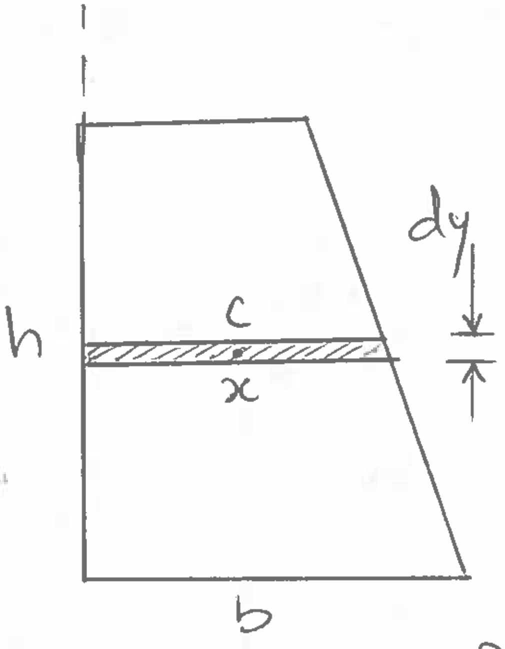
HW # 17

Prob # 5.1

The vertical coordinate is

$$1 + \frac{2}{3}(7-1) = \underline{5}$$

Prob # 5.5



$$x = \left(\frac{a-b}{h}\right) y + b.$$

$$dA = x\,dy = \left[\left(\frac{a-h}{h}\right) y + b\right] dy$$

$$A = \left(\frac{a+b}{2}\right) h = \frac{h}{2}(a+b)$$

$$\int x_c \, dA = \int_0^h \frac{x}{2} \left[\left(\frac{a-b}{h} \right) y + b \right] dy$$

$$= \frac{1}{2} \int_0^h \left[\left(\frac{a-b}{h} \right) y + b \right]^2 dy$$

$$= \frac{1}{2} \int_0^h \left(\frac{a-b}{h} \right)^2 y^2 + 2b \left(\frac{a-b}{h} \right) y + b^2 \Big] dy$$

$$= \frac{1}{2} \left[\left(\frac{a-b}{h} \right)^2 \frac{y^3}{3} + 2b \left(\frac{a-b}{h} \right) \frac{y^2}{2} + b^2 y \right]_0^h$$

$$= \frac{h}{6} \left[a^2 + b^2 + ab \right]$$

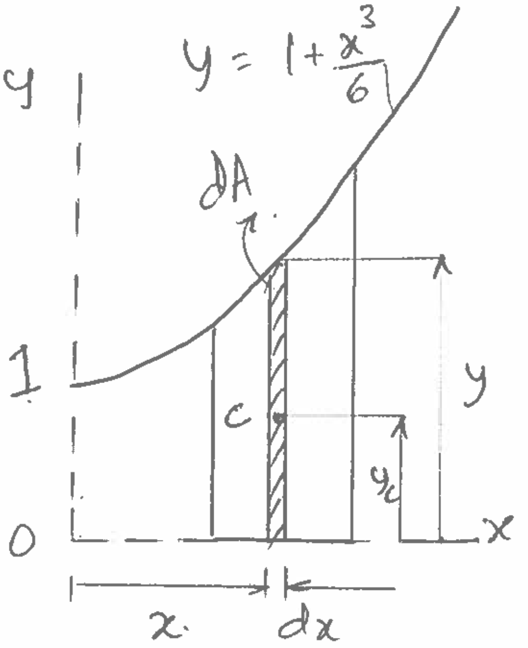
$$\int y_c \, dA = \int_0^h y \left[\left(\frac{a-b}{h} \right) y + b \right] dy = \int_0^h \left[\left(\frac{a-b}{h} \right) y^2 + by \right] dy$$

$$= \left[\left(\frac{a-b}{h} \right) \frac{y^3}{3} + b \frac{y^2}{2} \right]_0^h = \frac{h^2}{3} \left(a + \frac{b}{2} \right)$$

$$\bar{x} = \frac{\int x_c \, dA}{A} = \frac{h/6 \left(a^2 + b^2 + ab \right)}{h/2 \, (a+b)} = \underline{\frac{a^2 + b^2 + ab}{3 \, (a+b)}}$$

$$\bar{y} = \frac{\int y_c \, dA}{A} = \frac{h^2/3 \left(a + b/2 \right)}{h/2 \, (a+b)} = \underline{\frac{h(2a+b)}{3 \, (a+b)}}$$

Prob # 5.7



$$dA = y\,dx = \left(1 + \frac{x^3}{6}\right) dx$$

$$A = \int dA = \int_{1\,\cancel{0}}^{2} \left(1 + \frac{x^3}{6}\right) dx$$

$$= x + \frac{x^4}{24} \Big|_1^2 = \frac{39}{24}$$

$$\int x_c\, dA = \int_1^2 x\left(1 + \frac{x^3}{6}\right) dx.$$

$$= \frac{x^2}{2} + \frac{x^5}{30} \Big|_1^2 = \frac{38}{15}$$

$$\int y_c\, dA = \int \frac{y}{2}\, y\, dx = \int \frac{y^2}{2}\, dx = \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{6}\right)^2 d$$

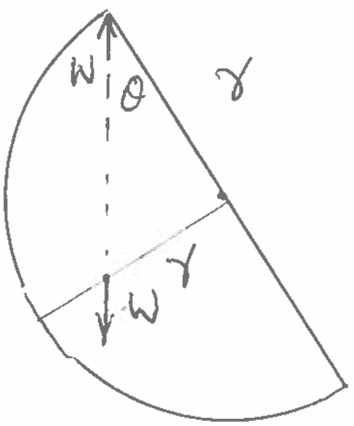
$$= \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{3} + \frac{x^6}{36}\right) dx$$

$$= \frac{1}{2}\left(x + \frac{x^4}{12} + \frac{x^7}{252}\right)\Big|_1^2 = \frac{347}{252}$$

So $\bar{x} = \dfrac{\int x_c\, dA}{\int dA} = {}^{38/15}\big/_{39/24} = 1.559$

$$\bar{y} = \frac{\int y_c\, dA}{\int dA} = \frac{347/252}{39/24} = 0.847$$

Prob # 5.8



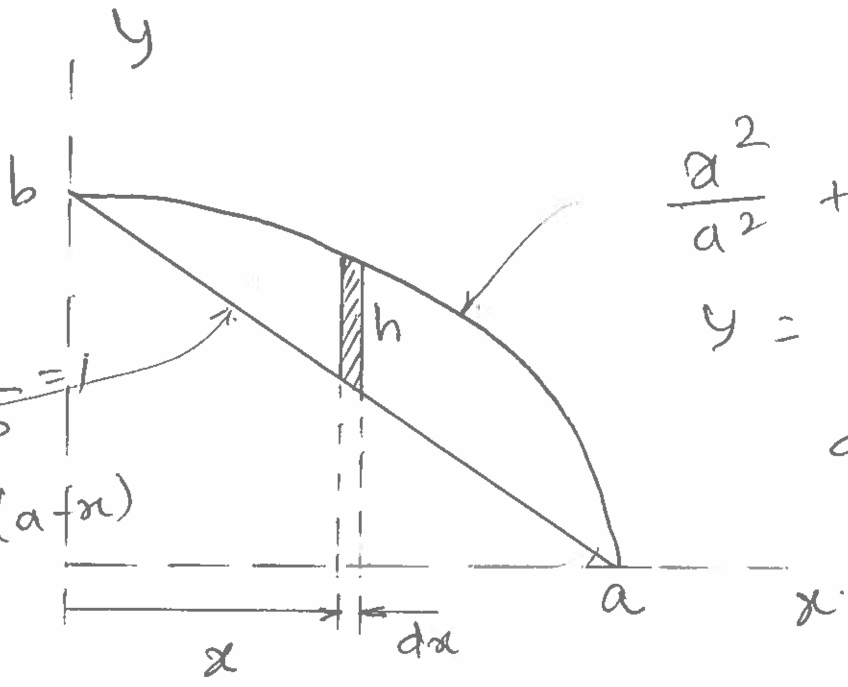
$$\theta = \tan^{-1}\frac{\bar{r}}{r} = \tan^{-1}\frac{2r/\pi}{r}$$

$$= \tan^{-1}\frac{2}{\pi}$$

$$\theta = 32.5^{\circ}$$

($\bar{r} = 2r/\pi$ from Sample. Prob 5/

Prob # 5.13



$$\frac{x}{a} + \frac{y}{b} = 1$$

$$y = \frac{b}{a}(a-x)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

$$dA = h\,dx.$$

$$h = \frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a-x) = \frac{b}{a}\left[\sqrt{a^2 - x^2} - (a-x)\right]$$

$$A\bar{x} = \int x \, dA$$

$$A = \int_0^a \frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right] dx.$$

$$= \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \left(ax - \frac{x^2}{2}\right)\right]_0^a$$

$$= \frac{ab}{2}\left[\frac{\pi}{2} - 1\right].$$

$$\int x \, dA = \int_0^a x \, \frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right] dx.$$

$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} - \left(\frac{ax^2}{2} - \frac{x^3}{3}\right)\right]_0^a = \frac{ba^2}{6}$$

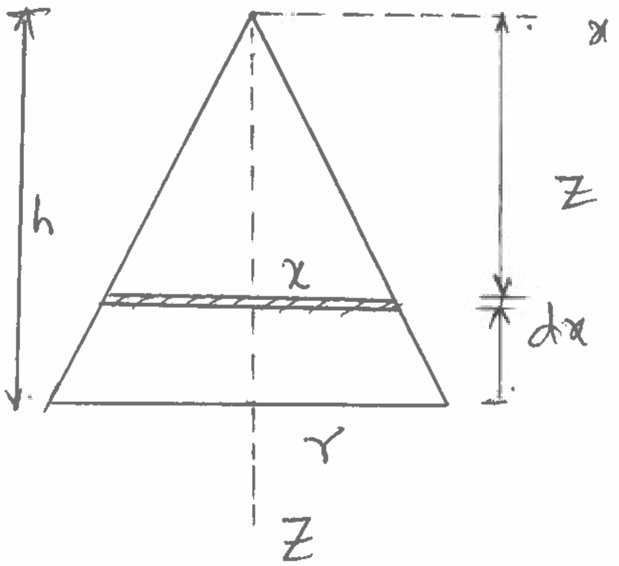
$$\bar{x} = \frac{\int x \, dA}{A} = \frac{ba^2/6}{\frac{ab}{2}\left(\frac{\pi}{2} - 1\right)} = \frac{a}{3\left(\frac{\pi}{2} - 1\right)}$$

Similarly $\bar{y} = \dfrac{b}{3\left(\frac{\pi}{2} - 1\right)}$

Prob # 5.16

$$x = \frac{r}{h} z, \quad dv = \pi x^2 dz$$

$$= \pi \frac{r^2}{h^2} z^2 dz.$$



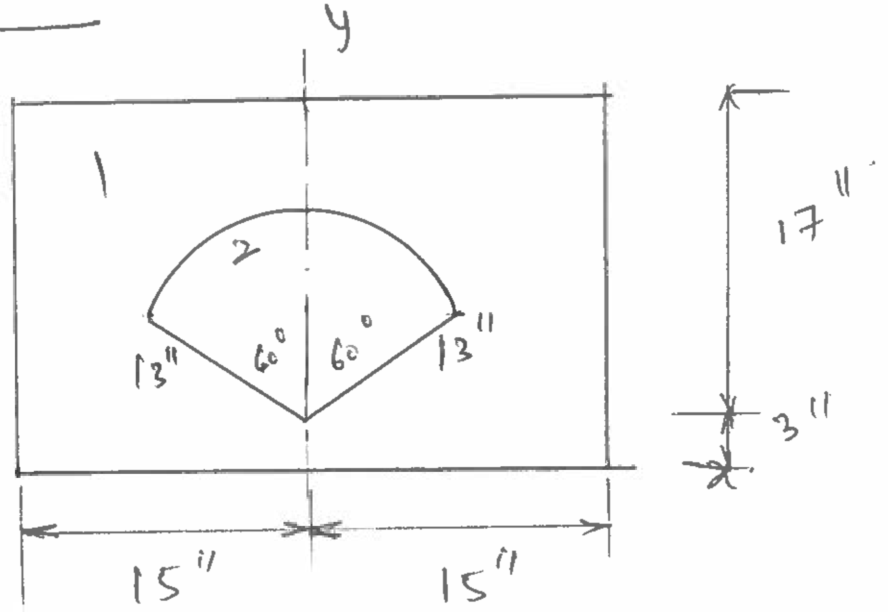
$$V = \frac{\pi r^2}{h^2} \int_0^h z^2 dz. = \frac{\pi r^2 h}{3.}$$

$$\int z \, dv = \frac{\pi r^2}{h^2} \int_0^h z^3 dz = \frac{\pi r^2 h^2}{4}$$

$$\bar{z} = \int z \, dv / v = 3h/4.$$

HW # 18

Prob # 5.46



Rectangle : $A_1 = 30(20) = 600 \text{ in}^2$; $\bar{y}_1 = 10 \text{ in}$

Circular Sector (prob 5/3) :

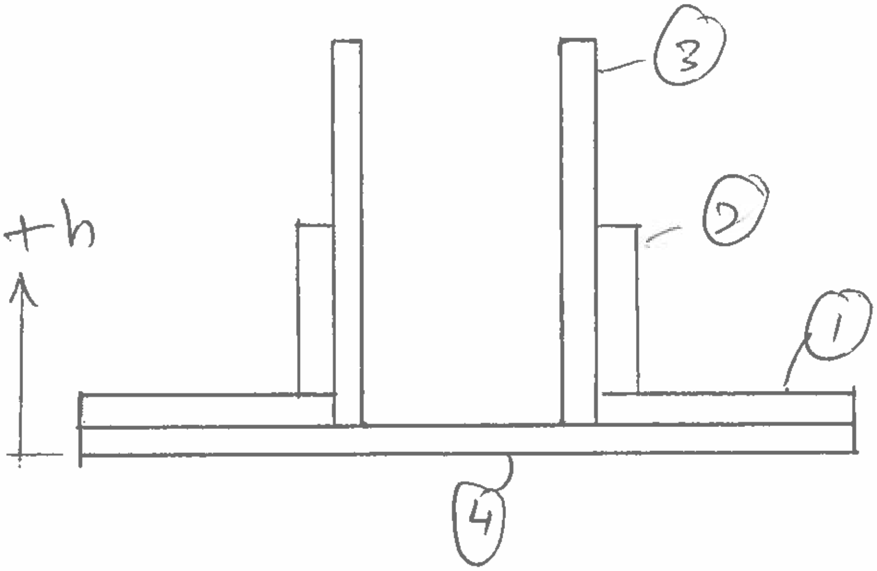
$$A_2 = -\frac{120}{360}\pi(13)^2 = -177.0 \text{ in}^2$$

$$\bar{y}_2 = 3 + \frac{2}{3}(13)\frac{\sin \pi/3}{\pi/3} = 10.17 \text{ in}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{600(10) - 177.0\,(10.17)}{600 - 177.0}$$

$$= 9.93 \text{ in}.$$

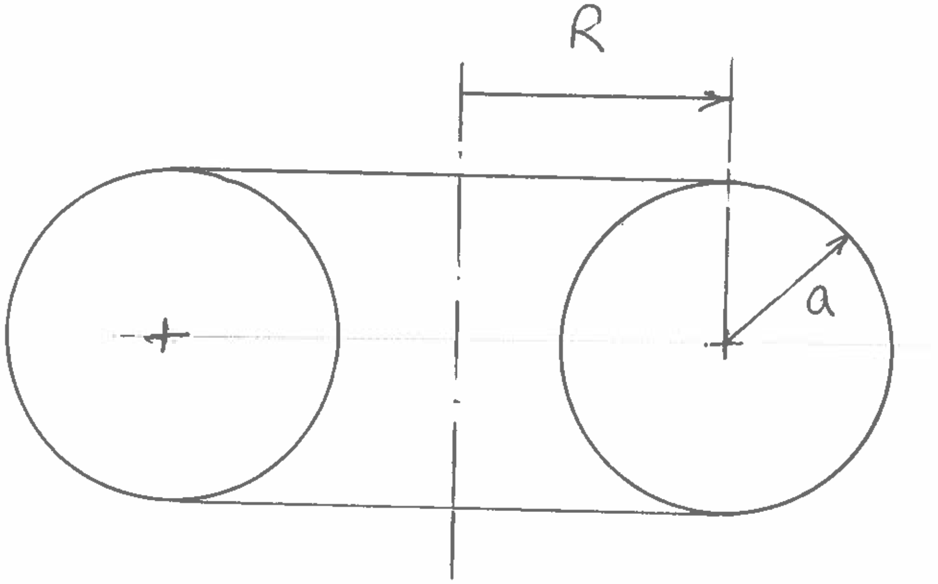
Prob # 5.51



Part	Size mm	A mm^2	$\bar{h}$ mm	$A\bar{h}$ mm^3
①	10 × 40	800	15	12000
②	10 × 40	800	40	32000
③	10 × 20	2400	70	168000
④	10 × 160	1600	5	8000
Σ's		5600		220,000.

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{220,000}{5600} = 39.3 \text{ mm}$$

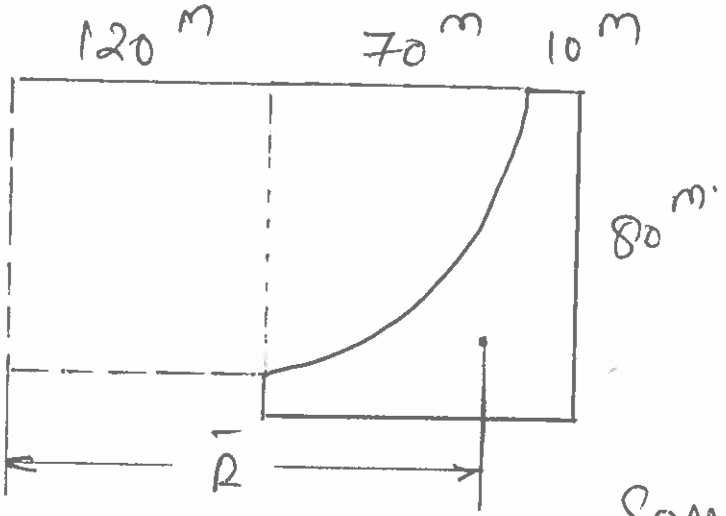
Prb #5.74



$$A = 2\pi\bar{r}L = 2\pi R(2\pi a) = 4\pi^2 Ra$$

$$V = 2\pi\bar{r}A = 2\pi R(\pi a^2) = 2\pi^2 Ra^2.$$

Prb # 5.92



Square $A = 80^2 = 6400 m^2$

$\frac{1}{4}$ Circle : $A = \frac{1}{4}\pi(70^2)$

$= 3848 \text{ m}^2$

Net Area $= 2552 \text{ m}^2$

$\bar{r}_{\text{square}} = 120 + 40 = 160 \text{ m}$

$\bar{r}_{1/4 \text{ Circle}} = 120 + \frac{4(70)}{3\pi} = 149.7 \text{ m.}$

$$\bar{R} = \frac{\Sigma A \bar{r}}{\Sigma A} = \frac{6400(160) - 3848(149.7)}{2552}$$

$= 175.5 \text{ m.}$

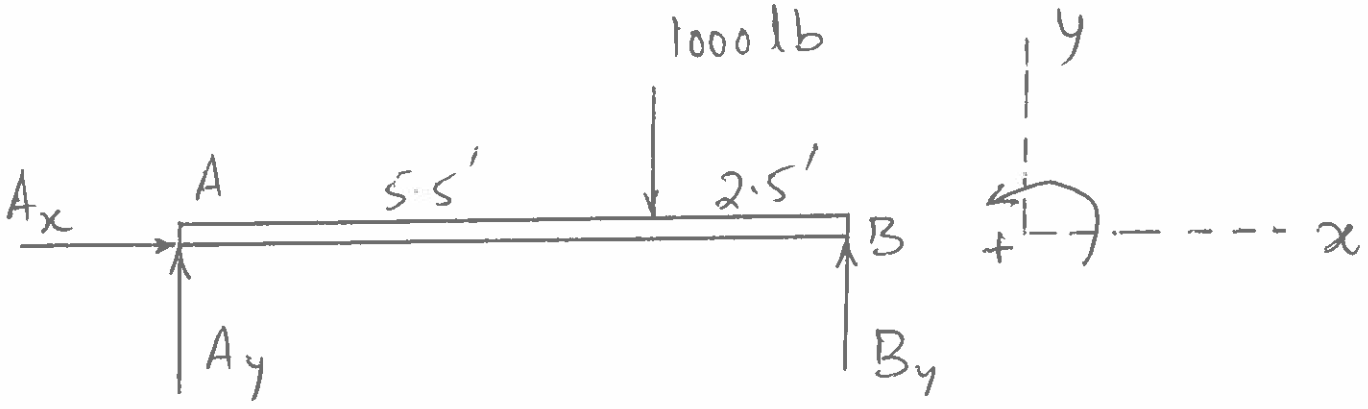
$V = \theta \bar{R} A = \frac{\pi}{3}(175.5)(2552) = 469\,000 \text{ m}^3$

$m = \rho V = 2.40(469000) = 1.126 \times 10^6$ M

CIVE 2330

HW # 19

Prob # 5.93

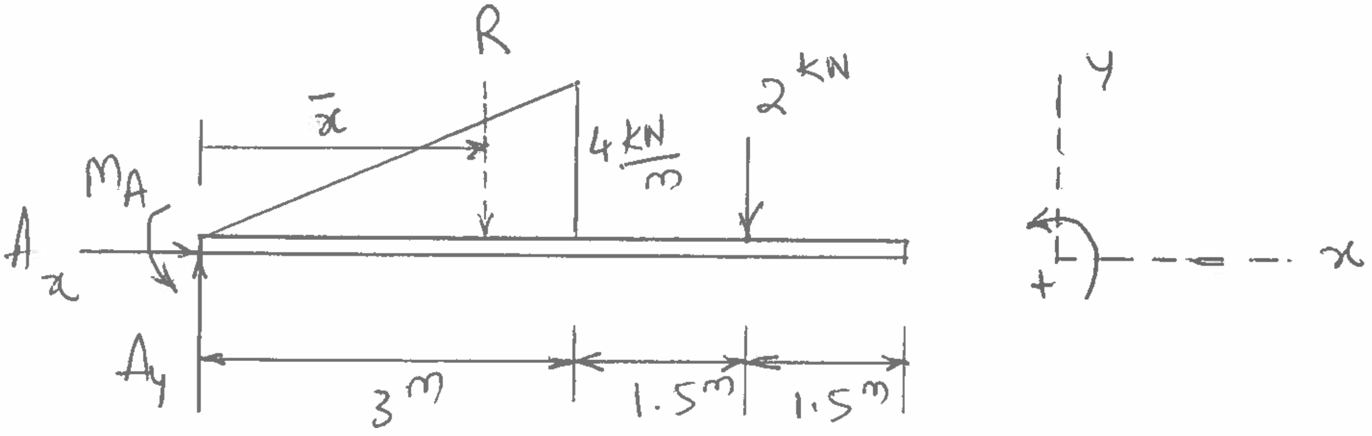


$\Sigma M_A = 0$: $B_y(8) - 1000(5.5) = 0$, $\underline{B_y = 688 \text{ lb}}$

$\Sigma F_y = 0$: $A_y + 688 - 1000 = 0$; $\underline{A_y = 312 \text{ lb}}$

$\Sigma F_x = 0$: $A_x = 0$.

Prob # 5.96



$R = \frac{1}{2}(3)(4) = 6 \text{ kN} @ \bar{x} = \frac{2}{3}(3) = 2^m$

$\Sigma M_A = 0 : M_A - 6(2) - 2(4.5) = 0$

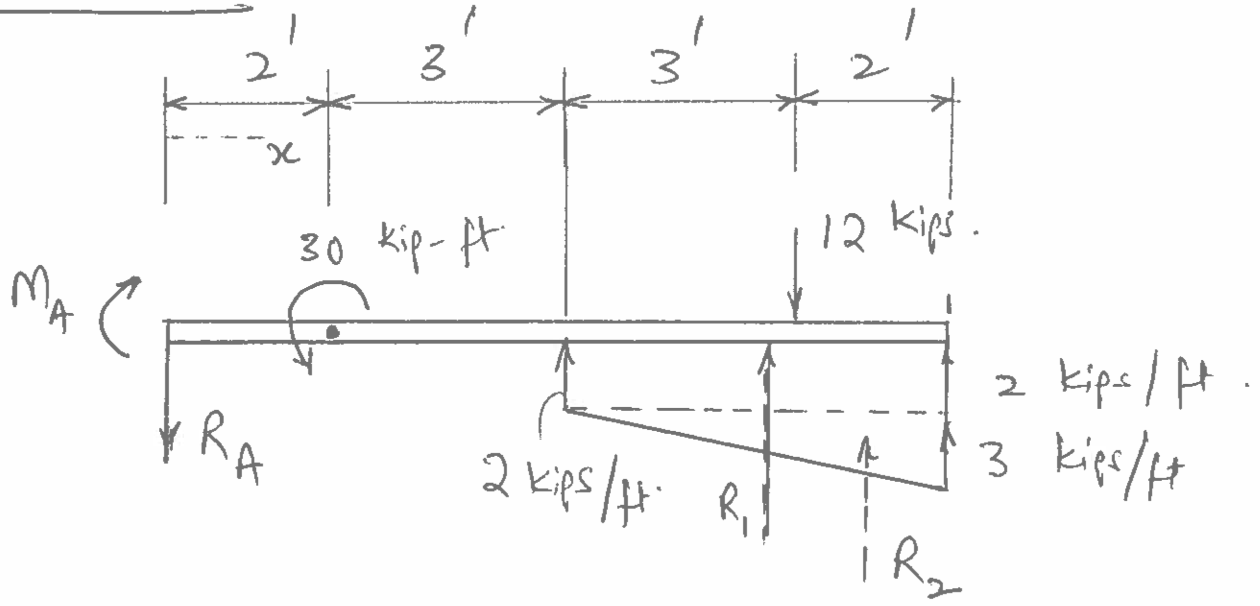
$\underline{M_A = 21 \text{ kN-m}}$

$\Sigma F_y = 0 : A_y - 6 - 2 = 0 ;$

$\underline{A_y = 8 \text{ KN}}$

$\Sigma F_x = 0 : \underline{A_x = 0}$

Prob # 5.102



$R_1 = 2(5) = 10$ kips @ $\bar{x}_1 = 7.5'$

$R_2 = \frac{1}{2}(3)(5) = 7.5$ kips @ $\bar{x}_2 = 8.33'$

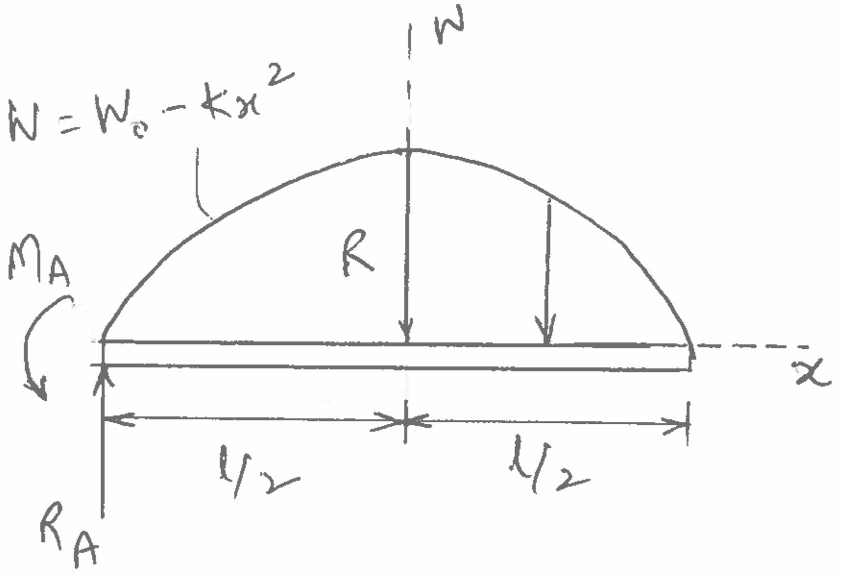
$\overset{+}{\circlearrowright}\ \Sigma M_A = 0: \quad M_A - 30 + 12(8) - 10(7.5) - 7.5(8.33) =$

$\underline{M_A = 71.5 \text{ kip-ft}}$

$+\uparrow \Sigma F = 0: \quad -R_A + 10 + 7.5 - 12 = 0,$

$\underline{R_A = 5.5 \text{ kips}}$

Prob 5.107



At $x = \frac{l}{2}$, $w = 0 = w_0 - k\left(\frac{l}{2}\right)^2$

$$\Rightarrow \quad k = \frac{4w_0}{l^2}$$

$$w = w_0\left(1 - \frac{4}{l^2}x^2\right)$$

$\bar{x} = 0$, by inspection.

$$R = \int w\,dx = 2\int_0^{l/2} w_0\left(1 - \frac{4}{l^2}x^2\right)dx.$$

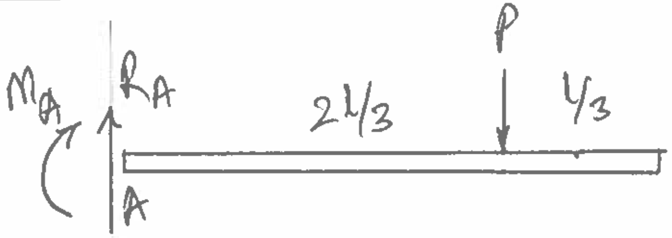
$$= 2w_0\left[x - \frac{4}{3l^2}x^3\right]_0^{l/2} = \frac{2}{3}w_0 l$$

$+\uparrow \sum F = 0$: $R_A - \frac{2}{3}w_0 l = 0$. $R_A = \frac{2}{3}w_0 l$

$\circlearrowleft + \sum M_A = 0$: $M_A - \frac{2}{3}w_0 l\left(\frac{l}{2}\right) = 0$, $M_A = \frac{1}{3}w_0 l^2$

HW # 20

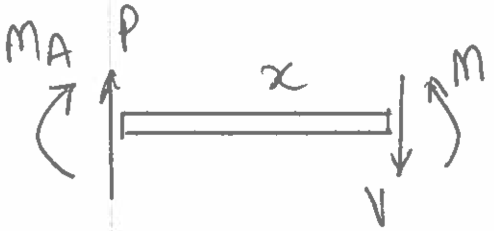
Prob # 5.113



$+\uparrow \sum F = 0 \Rightarrow R_A = P$

$\circlearrowleft + \sum M_A = 0: \ -M_A - P\left(\frac{2l}{3}\right) = 0$

$$M_A = \frac{-2Pl}{3}$$

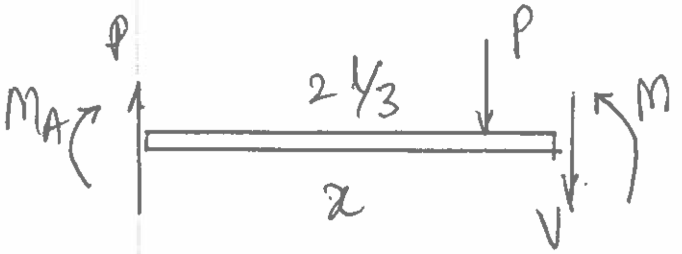


$0 < x < \frac{2l}{3}$:

$+\uparrow \sum F = 0 \Rightarrow V = P$

$\circlearrowleft + \sum M = 0: \ \frac{2Pl}{3} - Px + M = 0$

$$M = P\left(x - \frac{2l}{3}\right)$$

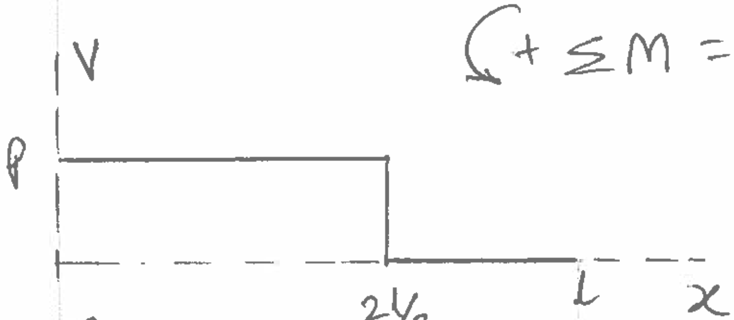


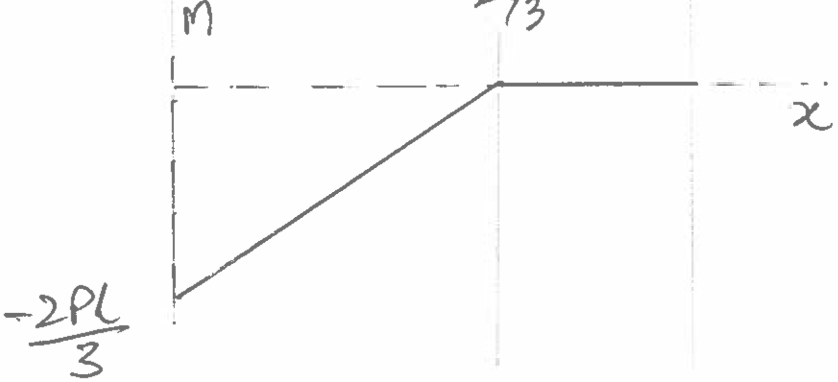
$\frac{2l}{3} < x < l$:

$+\uparrow \sum F = 0: \ P - P - V = 0, \ V = 0$

$\circlearrowleft + \sum M = 0: \ \frac{2Pl}{3} - P\left(\frac{2l}{3}\right) + M = 0$

$M = 0$





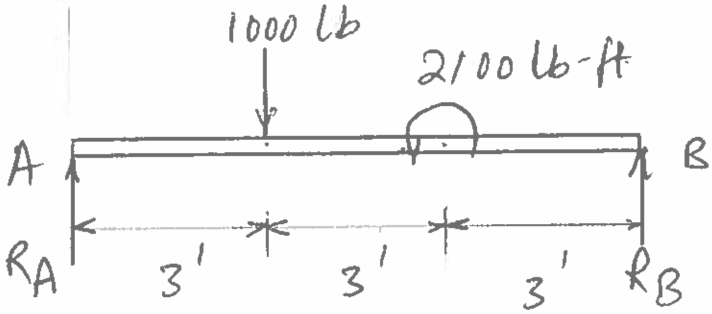
At $x = l/2$:

$V = P$

$$M = P\left(\frac{l}{2} - \frac{2l}{3}\right)$$

$$= -\frac{Pl}{6}$$

Prob 5.116

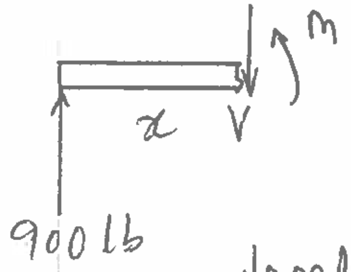


$\circlearrowleft + \sum M_A = 0: -1000(3) + 2100 + R_B(9) = 0$, $R_B = 100$ lb

$+\uparrow \sum F_y = 0:$

$R_A - 1000 + 100 = 0$

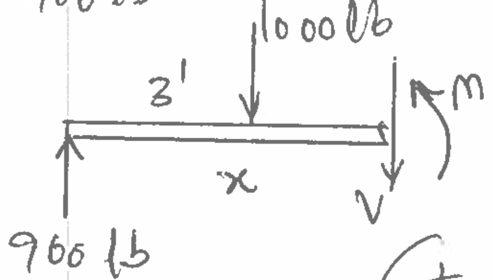
$R_A = 900$ lb.



$0 < x < 3$ ft.

$V = 900$ lb

$M = 900x$

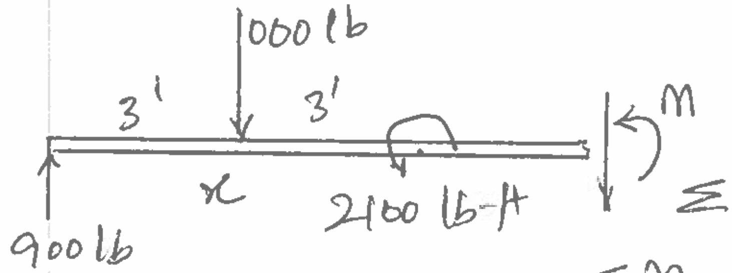


$3 < x < 6$ ft:

$+\uparrow \sum F = 0: 900 - 1000 - V = 0; V = -100$

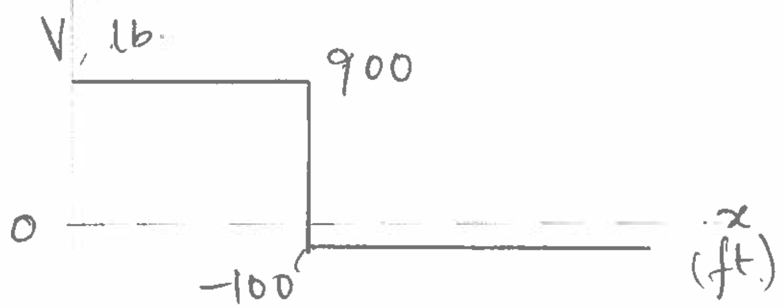
$\circlearrowleft + \sum M = 0: -900x + 1000(x-3) + M = 0$

$M = 3000 - 100x$.



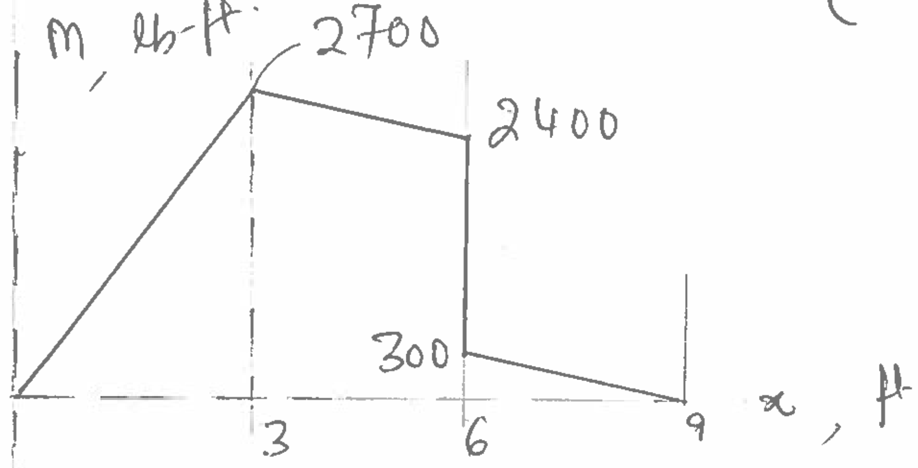
$\sum F = 0: 900 - 1000 - V = 0, V = -100$

$\sum M = 0: -900x + 1000(x-3) + 2100 + M = 0; M = 900 - 100x$

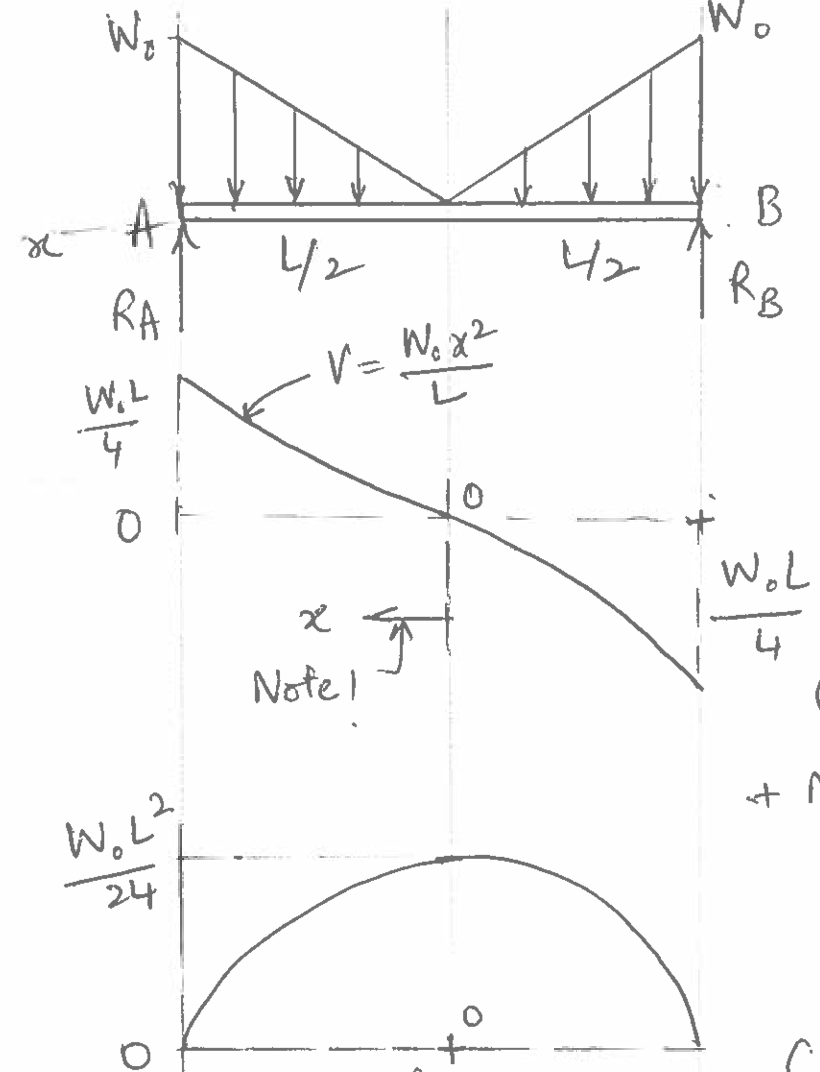


Values at middle:

$V = -100$ lb
$M = 2550$ lb-ft



Prob 5.121



$R_A = R_B = \frac{W_0 L}{4}$

For $x = L/2$, $M = 0$, $V = R_A$.

Element (lower left):

$+\sum F = 0$: $V - \frac{W_0 x^2}{L} = 0$

$V = \frac{W_0 x^2}{L}$

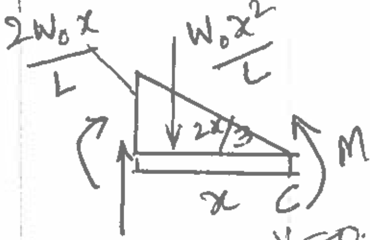
Consider $x = L/2$

$\curvearrowleft + \sum M_c = 0$:

$+M_0 - \frac{W_0 L}{4}\left(\frac{L}{2}\right) + \frac{W_0 (L/2)^2}{L}\left(\frac{2L}{3}\right) =$

$M_0 = \frac{W_0 L^2}{24}$

Consider arbitrary x:



$\curvearrowleft + \sum M_0 = 0$:

$V=0$. $-M - \frac{W_0 x^2}{L}x + \frac{W_0 x^2}{L}\frac{2x}{3} + \frac{W_0 L^2}{24} = 0$

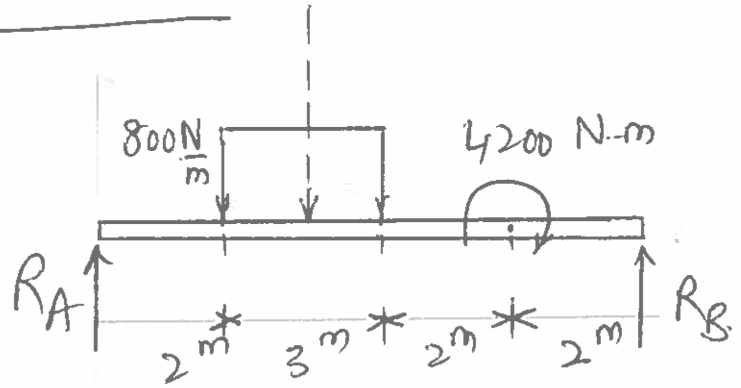
$M = \frac{W_0}{3L}\left(\frac{L^3}{8} - x^3\right)$

$M = M_{max}$ @ $x = 0$: $M_{max} = \frac{W_0}{3L}\left(\frac{L^3}{8} - 0\right)$

$= \frac{W_0 L^2}{24}$

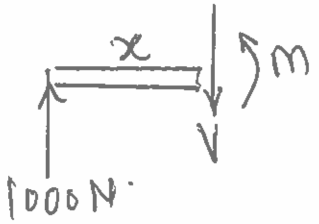
Prob 5.131

$R = 800(3) = 2400N$



$\Sigma M_A = 0: \ R_B(9) - 2400(3.5) - 4200 = 0, \ R_B = 1400 N$

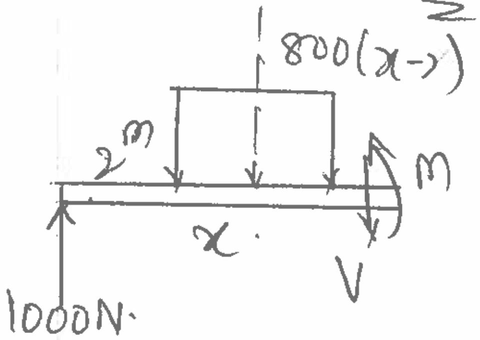
$\Sigma F = 0 \Rightarrow R_A = 1000N$



$0 < x < 2 \text{ m}:$

$\Sigma F = 0 \Rightarrow V = 1000N.$

$\Sigma M = 0 \Rightarrow M = 1000x.$



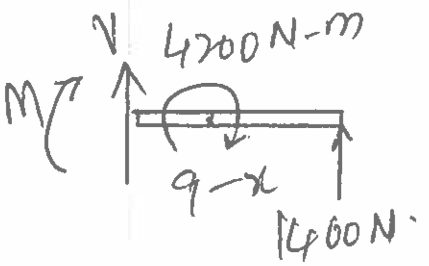
$2 < x < 5 \text{ m}:$

$\Sigma F = 0: \ 1000 - 800(x-2) - V = 0$

$V = 2600 - 800x$

$\Sigma M = 0: \ M + 800(x-2)\left(\frac{x-2}{2}\right) - 1000x = 0.$

$M = -400x^2 + 2600x - 1600.$

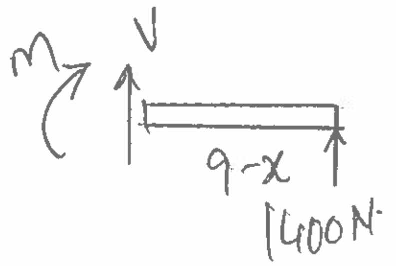


$5 < x < 7 \text{ m}:$

$\Sigma F = 0: \ V + 1400 = 0, \ V = -1400N$

$\Sigma M = 0: \ -M - 4200 + 1400(9-x) = 0$

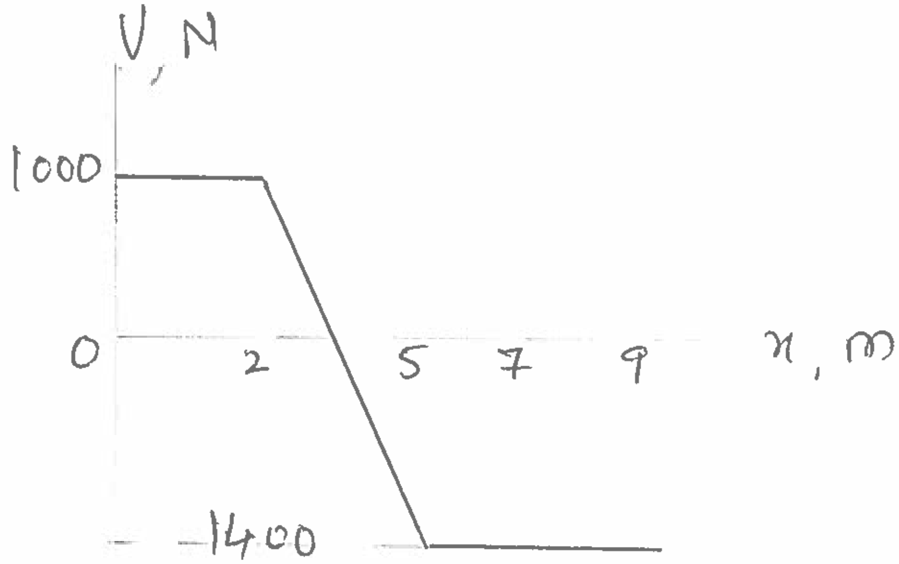
$M = 8400 - 1400x$



$7 < x < 9$ m:

$\Sigma F = 0 \Rightarrow V = -1400$ N.

$\Sigma M = 0 \Rightarrow M = 12600 - 1400x$

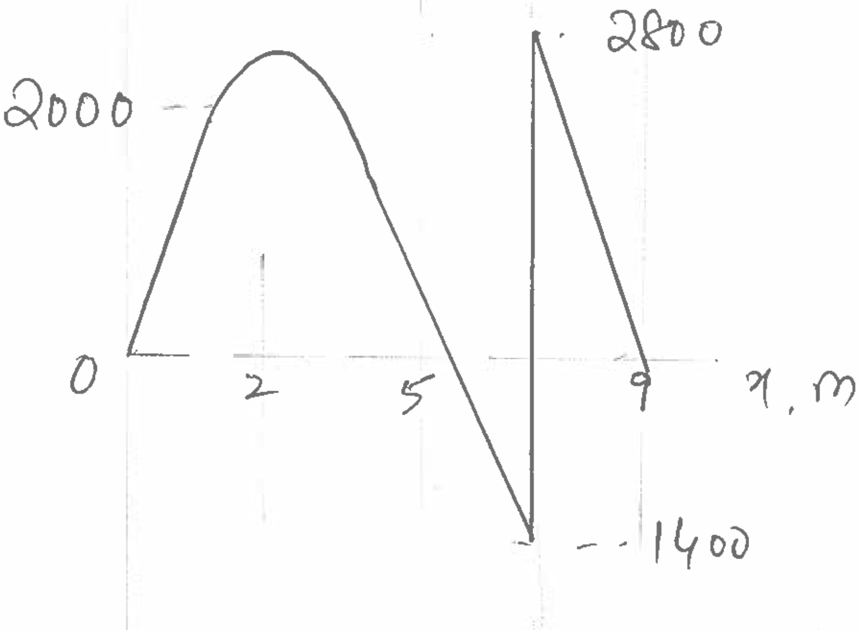


At $x = 6$ m,

$V = -1400$ N.

$M = 8400 - 1400(6)$

$= 0.$



For M_{max}, $\frac{dM}{dx} = 0$

$\frac{d}{dx}\left(-400x^2 + 2600x - 160\right.$

$= -800x + 2600 = 0$

$x = 3.25$ m

$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600$

$= 2625$ N·m

So $M_{max} = 2800$ N·m

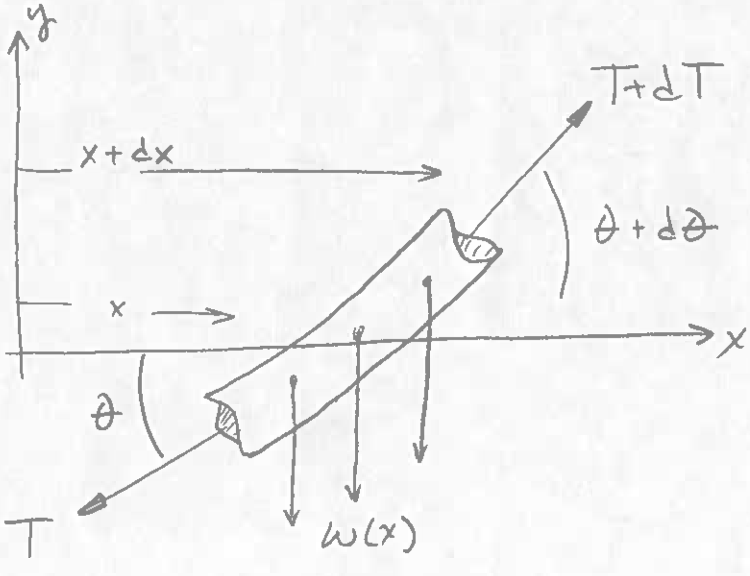
Cables

Cables are used in suspension bridges, transmission lines, etc.

It is assumed that a flexible cable has negligible bending resistance, so that force in cable is always in direction of cable.

External reactions at cable ends are found by applying equilibrium conditions to entire cable.

To calculate internal force, consider a small element



load on element $\approx$ $w(x)\,dx$ $= \dfrac{w(x+dx)\,dx + w(x)\,dx}{2}$

Total load $\quad L = \displaystyle\int_0^L w(x)\,dx$

Location of total Load

$$L\bar{x} = \int_0^L x\,w(x)\,dx$$

$$\therefore \bar{x} = \frac{\int_0^L x\,w(x)\,dx}{\int_0^L w(x)\,dx}$$

Cable is in equilibrium means each element is in equilibrium

$$\therefore \Sigma F_x = 0 \Rightarrow (T+dT)\cos(\theta+d\theta) - T\cos(\theta) = 0$$

$$\Sigma F_y = 0 \Rightarrow (T+dT)\sin(\theta+d\theta) - T\sin(\theta) - w(x)dx = 0$$

$$\cos(\theta+d\theta) = \cos\theta\cos d\theta - \sin\theta\sin d\theta$$

$$\sin(\theta+d\theta) = \sin\theta\cos d\theta + \sin d\theta\cos\theta$$

$\sin d\theta \approx \theta$, $\cos d\theta \approx 1$ for $d\theta \ll 1$

$$\Sigma F_x = 0 \Rightarrow (T+dT)(\cos\theta - \sin\theta\, d\theta) = T\cos\theta$$

$$T\cos\theta = T\cos\theta - T\sin\theta\, d\theta + dT\cos\theta - \underset{\substack{\text{Neglect} \\ dT\,d\theta \rightarrow \text{small}}}{\cancel{dT\sin\theta\, d\theta}}$$

$$0 = dT\cos\theta - T\sin\theta\, d\theta$$

from calculus

$$d(T\cos\theta) = dT\cos\theta + T\frac{d}{d\theta}(\cos\theta)$$

$$= dT\cos\theta - T\sin\theta\, d\theta$$

$\therefore$

$$d(T\cos\theta) = 0$$

$$\Rightarrow T\cos\theta = \text{constant} = T_0$$

(horizontal component of cable force is a constant)

$$\Sigma F_y = 0 \Rightarrow (T+dT)(\sin\theta + \cos\theta\, d\theta) - w\,dx = T\sin\theta$$

$$\cancel{T\sin\theta} + T\cos\theta\, d\theta + dT\sin\theta + \underset{\approx 0}{dT\cos\theta\, d\theta} - w(x)dx = \cancel{T\sin\theta}$$

$$T\cos\theta\, d\theta + dT\sin\theta - w(x)dx = 0$$

$$d(T\sin\theta) - w(x)dx = 0$$

$$\therefore \frac{d}{dx}(T\sin\theta) = w(x)$$

But $$T \sin\theta = T_0 \frac{\sin\theta}{\cos\theta} = T_0 \tan\theta = T_0 \frac{dy}{dx}$$

$\therefore$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{w(x)}{T_0}$$

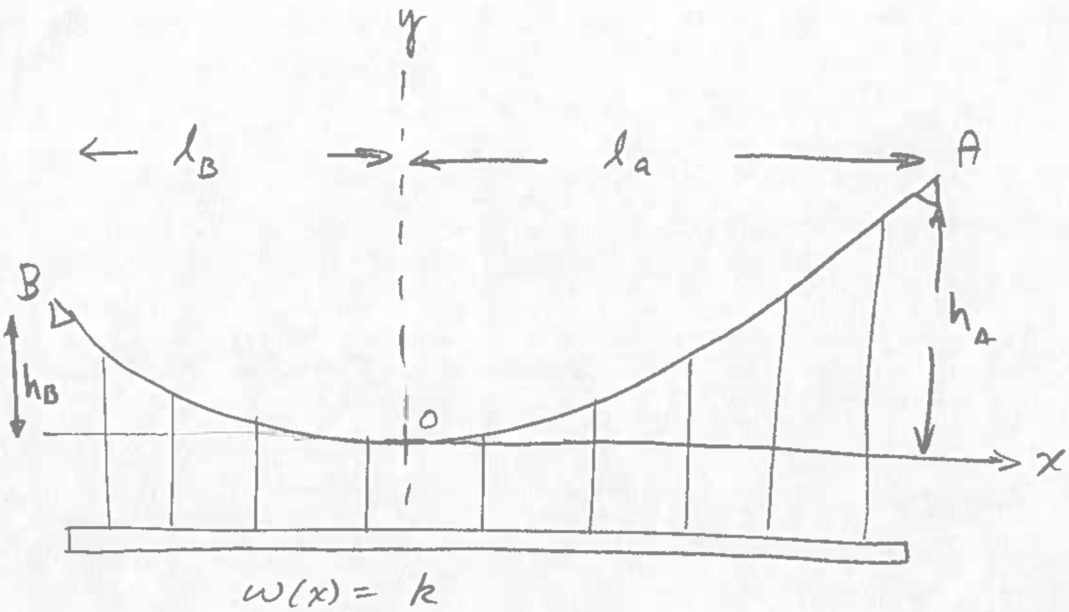
or

$$\underbrace{\frac{d^2y}{dx^2} = \frac{1}{T_0} w(x)}$$

differential equation for the cable. This relationship defines the shape of the cable for various loading conditions.

A solution, $y(x)$ must also satisfy boundary conditions at cable ends.

If $w(x)$ is a constant, the cable will have a parabolic shape



$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{k}{T_0}$$

$$\int d\left(\frac{dy}{dx}\right) = \int \frac{k\,dx}{T_0}$$

$$\frac{dy}{dx} = \frac{kx}{T_0} + c_1$$

$\frac{dy}{dx} = 0$ at $x = 0$ ∴ $c_1 = 0$

$$\int dy = \int \frac{kx}{T_0} + c_1 \; dx$$

Bandary conditions

$$y = \frac{kx^2}{2T_0} + c_1 x + c_2$$

$y = 0, \frac{dy}{dx} = 0$ at $x = 0$

$\Rightarrow c_1 = c_2 = 0$

$x = 0,$

∴

$$y = \frac{kx^2}{2T_0}$$

←

At $y = h_A$, $x = l_a$ $\quad T = T_0$

At $y = h_B$, $x = -l_B$ $\quad T = T_0$

$$\therefore \quad h_a = \frac{k l_a^2}{2T_0}, \quad h_B = \frac{k l_D^2}{2T_0}$$

(Solve for T_0

$$T_0 = \frac{k l_a^2}{2h_a} = \frac{k l_B^2}{2h_B}$$

$$\therefore \quad y = \frac{\cancel{2} h_a \cancel{k} x^2}{\cancel{2} \cancel{k} l_a^2} = \frac{h_a x^2}{l_a^2}, \quad = \frac{h_B x^2}{l_B^2}$$

From the FBD of the cable element,

$$T\cos\theta = T_0, \quad T\sin\theta = T_0 \frac{dy}{dx}$$

$$\therefore \quad T = \sqrt{T_0^2 + k^2 x^2}$$

T_{MAX} occurs when $x = l_a$ or $-l_B$

(at ends of cable).

Length of cable between A & B is found by

$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$$

$$= \sqrt{1 + \left(\frac{wx}{T_0}\right)^2}\ dx$$

$$\therefore\ S_A = \int_0^{l_a} \sqrt{1 + \left(\frac{kx}{T_0}\right)^2}\ dx$$

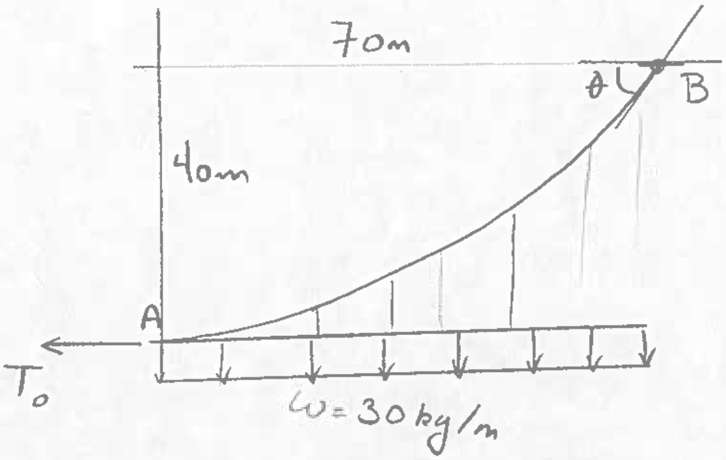
$$\simeq l_a\left(1 + \frac{2}{3}\left(\frac{h_a}{l_a}\right)^2 - \frac{2}{5}\left(\frac{h_a}{l_a}\right)^4\right)$$

Valid for $h_a/l_a < \frac{1}{2}$

$$S_B = \int_0^{l_B} \sqrt{1 + \left(\frac{kx}{T_0}\right)^2}\ dx$$

$$\simeq l_B\left(1 + \frac{2}{3}\left(\frac{h_B}{l_B}\right)^2 - \frac{2}{5}\left(\frac{h_B}{l_B}\right)^4\right)$$

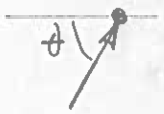
5-141 Find T_o & θ for cable shown.



$$w(x) = 30\frac{kg}{m}\left(\frac{9.8m}{s^2}\right) = 294N/m$$

$$T_o = \frac{294N/m \cdot (70m)^2}{2\,(40m)} = 18,007.5N \qquad 18kN$$

FBD @ B



$$T\cos\theta = T_o$$

$$T = \sqrt{T_o^2 + (294)^2 x^2}$$

$$= \sqrt{18,007.5^2 + (294)^2(70)^2}$$

$$T = 27346.05$$

$$\cos\theta = \frac{T_o}{T} = 0.658$$

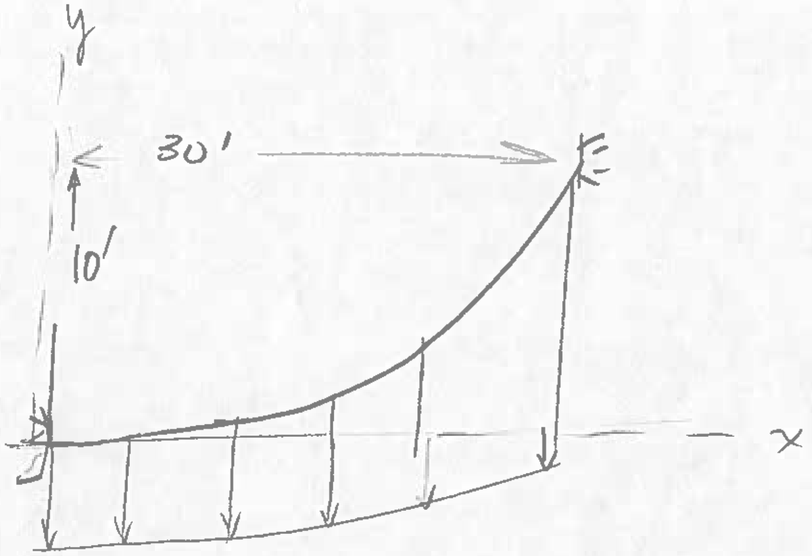
$$\theta = \cos^{-1}(0.658) = 48.8°$$

5-143 cable as shown. $\frac{dy}{dx} = 0$ at lower support.

$w(x) = 100$ ~~at~~ $x = 0$

40 at $x = 30$

decrease $\propto$ x^2. Find equation of cable



defn. of cable

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{w(x)}{T_0}$$

$w(x) = 100 - k x^2$

$w(30) = 100 - k(30)^2 = 40$

$-k\,900 = -60$

$k = \frac{60}{900} = 0.066$

$w(x) = 100 - 0.066x^2$

$$\int d\left(\frac{dy}{dx}\right) = \int \frac{100 - 0.66x^2}{T_0}\,dx$$

$$\frac{dy}{dx} = \left(100x - \frac{0.066}{3}x^3\right)\frac{1}{T_0} + c_1$$

at $x = 0$, $\frac{dy}{dx} = 0 \Rightarrow c_1 = 0$

$$dy = \frac{1}{T_0}\int 100x - \frac{0.066}{3}x^3\,dx$$

$$y = \frac{1}{T_0}\left(\frac{100x^2}{2} - \frac{0.066}{12}x^4\right) + c_2$$

$x = 0$ $y = 0$, $c_2 = 0$

$x = 30$, $y = 10$ solve for T_0

$$T_0 10 = \frac{100(30)^2}{2} - \frac{0.066}{12}(30)^4$$

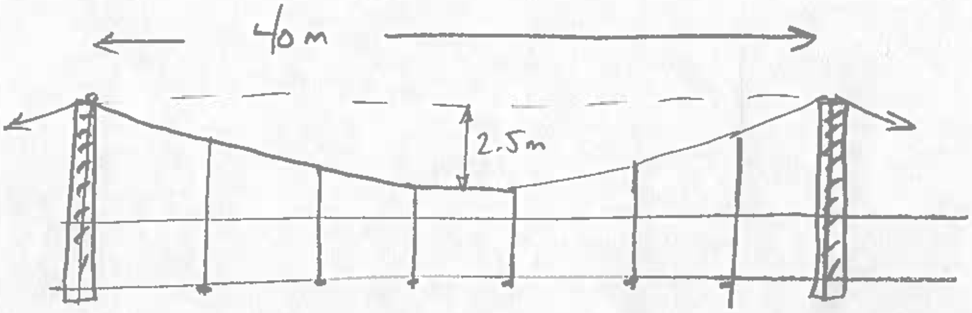
$$T_0 = \frac{10(30)^2}{2} - \frac{0.066}{120}(30)^4 = 4054.5$$

Substitute back into $y(x)$

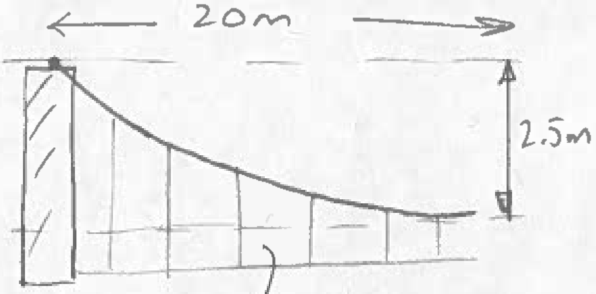
$$y(x) = 0.0123\,x^2 - 1.356\cdot 10^{-6} x^4 \quad (ft)$$

5-144 (350mm Water Pipe span 40m

Pipe weighs 1400 kg/m. Find compression C exherted by cable at each support



By symmetry, need only analyze ½ structure



$1400(9.8) N/m = 13720 N/m$

$$T = \sqrt{T_0 + (13720)^2 (x)^2}$$

$$= \sqrt{T_0^2 + (13720)^2 (20)^2} \quad N$$

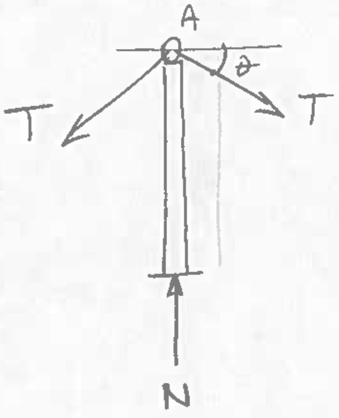
$$T_0 = \frac{13720 (20)^2}{2(2.5)} = 1,097,600 \ N$$

$$T = 1,131,380 N$$

$$\cos\theta = \frac{T_0}{T} = \frac{1097600}{1131380} = 0.970$$

$$\theta = \cos^{-1}(.970) = 14.03°$$

Diagram of support



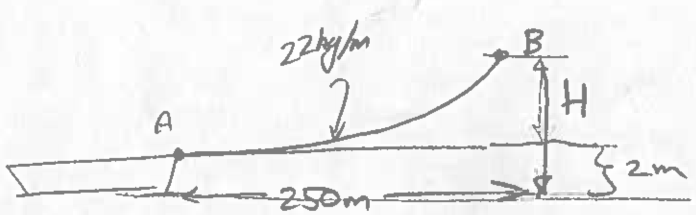
$\Sigma F_y = 0 = N - 2T\sin\theta$

$N = 2T\sin\theta$

$\therefore N = 2(1,131,380)\sin(14.03) = 548,800$

$\therefore$ Compressive force in support is $\underline{\underline{548 kN}}$ ←

5-148



$T_A = 300kN$ (required)

find H and length of cable

$$y = \frac{T_A}{N}\left(\cosh\left(\frac{Nx}{T_A}\right) - 1\right)$$ (catenary curve) eq 5-19

$$y = \frac{300(10^3)}{22(9.8)}\left(\cosh\frac{22(9.8)(250)}{300(10^3)} - 1\right)$$

$= 22.5m$

$H-2 = y \quad \therefore \quad y+2 = H \quad , \quad \underline{\underline{H = 24.5m}}$

$$S = \frac{T_A}{N}\sinh\frac{Nx}{T_A} = \frac{300(10^3)}{22(9.8)}\sinh\frac{22(9.8)250}{300(10^3)} = 251m$$

Recall:

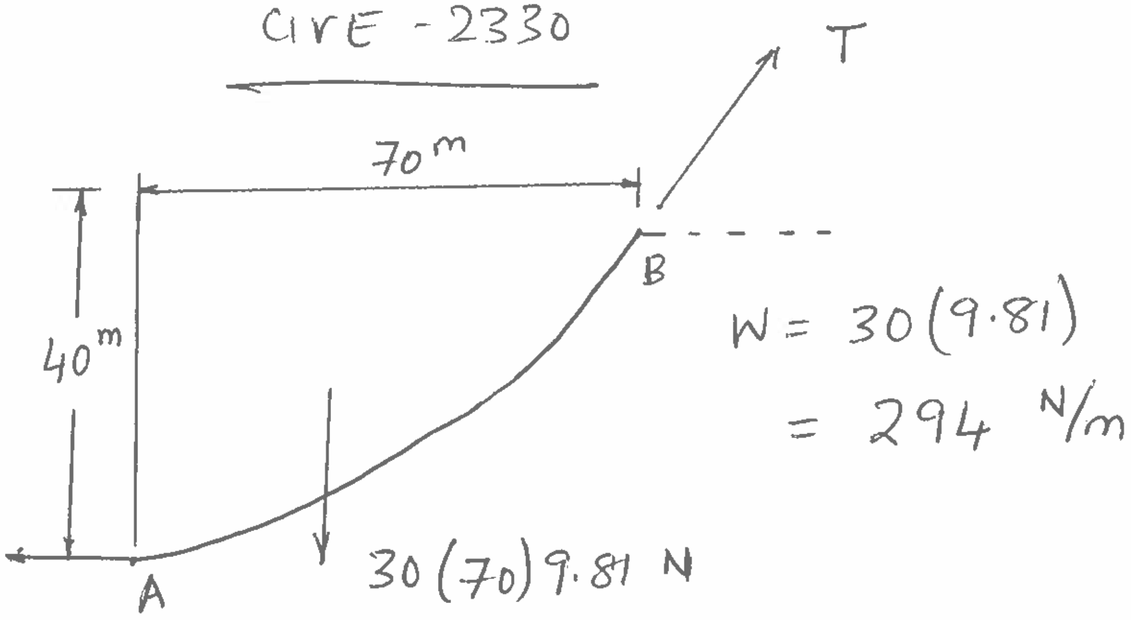
$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

HW # 21

CIVE - 2330

5.141



$W = 30(9.81)$

$= 294$ N/m

Eq: 5/14 : $y = \frac{Wx^2}{2T_0}$

$T_0 = \frac{Wx^2}{2y} = \frac{294(70)^2}{2(40)} = 18.03(10^3)$ N

or $\underline{18.03 \text{ kN}}$

$\tan\theta = \frac{dy}{dx} = \frac{Wx}{T_0}$

At B, $\theta = \tan^{-1}\left(\frac{294(70)}{18030}\right) = 48.8^\circ$

5.143

$W = 100 - Kx^2$; $40 = 100 - K(30)^2$

$K = \frac{1}{15}$ lb/ft^3, so

$$W = 100 - \frac{x^2}{15} \text{ lb/ft.}$$

From Eq 5/13: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{W}{T_0} = \frac{1}{T_0}\left(100 - \frac{x^2}{15}\right)$

$$\frac{dy}{dx} = \frac{1}{T_0}\int_0^x \left(100 - \frac{x^2}{15}\right)dx = \frac{1}{T_0}\left(100x - \frac{x^3}{45}\right)$$

$$y = \frac{1}{T_0}\int_0^x \left(100x - \frac{x^3}{45}\right)dx.$$

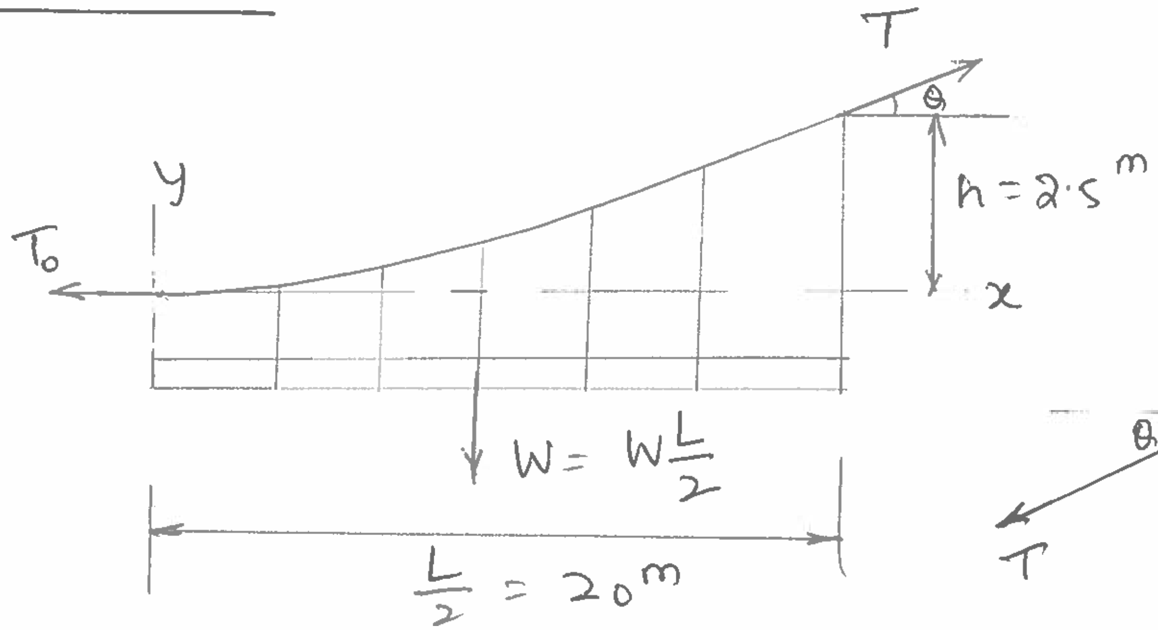
$$= \frac{1}{T_0}\left(50x^2 - \frac{x^4}{180}\right)$$

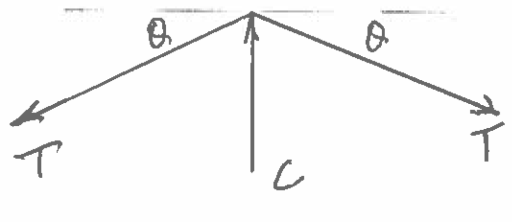
when $x = 30$ ft, $y = 10$ ft So

$10 = \frac{1}{T_0}\left(50 \times 30^2 - \frac{30^4}{180}\right)$, $T_0 = 4050$ lb.

So $y = (123.5x^2 - 0.01372x^4)(10^{-4})$ ft

Pr 5.144





$w = 1.400(9.81) = 13.73$ kN/m

Form Eq 5/15b. $T = \frac{wL}{2}\sqrt{1+(L/4h)^2}$

$= \frac{13.73(40)}{2}\sqrt{1+\left[\frac{40}{4(2.5)}\right]^2} = 1133$ kN

$T^2 = W^2 + T_0^2$, $T_0 = \sqrt{(1132)^2 - [(13.73)(20)]^2}$

$= 1099$ kN

$\frac{dy}{dx} = \tan\theta = \frac{w(L/2)}{T_0} = \frac{13.73(20)}{1099} = 0.250$,

$\underline{\theta = 14.04^\circ}$

$\Sigma F_y = 0$ at Support ; $2T\sin\theta - C = 0$

$C = 2(1133)\sin 14.04^\circ = 549$ kN

Prob 5.148

Eq 5/19 $$y = \frac{T_o}{\mu}\left(\cosh \frac{\mu x}{T_o} - 1\right)$$

So $$H - 2 = \frac{300(10^3)}{22(9.81)}\left(\cosh \frac{22(9.81)(250)}{300(10^3)} - 1\right)$$

$$H - 2 = 1390(\cosh 0.1798 - 1) = 1390(1.0162 - 1)$$

$$H - 2 = 22.5 \text{ m}$$

$$H = 24.5 \text{ m}$$

Eq 5/20 $$s = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$$

$$= \frac{300(10^3)}{22(9.81)} \sinh \frac{22(9.81)250}{300(10^3)}$$

$$= 1390(0.1808)$$

$$= 251 \text{ m}$$