(Engineering Mechanics – Physics)

# STATICS

the study of

# FORCES

at

# EQUILIBRIUM

## using Vector Math

## +

*EVERY OTHER* kind of Math you have ever learned in your life

# Some Useful Equations for Quiz 1

Cartesian Form of a Vector: $\overline{F} = F_x\overline{i} + F_y\overline{j} + F_z\overline{k}$

Magnitude: $F = \left|\overline{F}\right| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

Direction Cosines For $\overline{F}$ : $\cos\alpha = \dfrac{F_x}{F}$ $\quad \cos\beta = \dfrac{F_y}{F}$ $\quad \cos\gamma = \dfrac{F_z}{F}$

Unit Vector: $\overline{u} = \left(\dfrac{F_x}{F}\right)\overline{i} + \left(\dfrac{F_y}{F}\right)\overline{j} + \left(\dfrac{F_z}{F}\right)\overline{k} = (\cos\alpha)\overline{i} + (\cos\beta)\overline{j} + (\cos\gamma)\overline{k}$

Directed Force Vector: $\overline{F} = F\overline{u}$

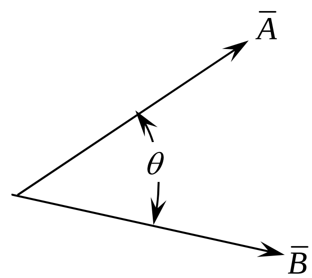
Absolute Position Vector: $\overline{r}_A = x_A\overline{i} + y_A\overline{j} + z_A\overline{k}$

Relative Position Vector: $\overline{r}_{B/A} = \left(x_B - x_A\right)\overline{i} + \left(y_B - y_A\right)\overline{j} + \left(z_B - z_A\right)\overline{k}$

Dot Product: $\overline{A}\cdot\overline{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

Projection of Vector along Line a-a: $A_{\mathrm{Proj}} = \overline{A}\cdot\overline{u}$

Angle between Two Vectors, $\overline{A}$ & $\overline{B}$ : $\theta = \cos^{-1}\left(\dfrac{\overline{A}\cdot\overline{B}}{AB}\right)$




Particle Equilibrium, Vector Formulation: $\sum\overline{F} = 0$

Particle Equilibrium, Scalar Formulation, 2D: $\sum F_x = 0 \quad \sum F_y = 0$

$\overline{A}$

Particle Equilibrium, Scalar Formulation, 3D: $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$

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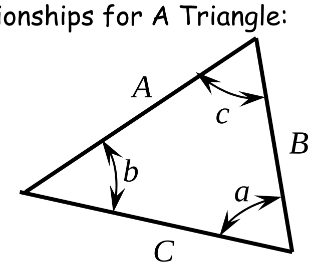
Geometric Relationships for A Triangle:




**Law of Cosines:**

$$A^2 = B^2 + C^2 - 2BC\cos a$$
$$B^2 = A^2 + C^2 - 2AC\cos b$$
$$C^2 = A^2 + B^2 - 2AB\cos c$$

**Law of Sines:**

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

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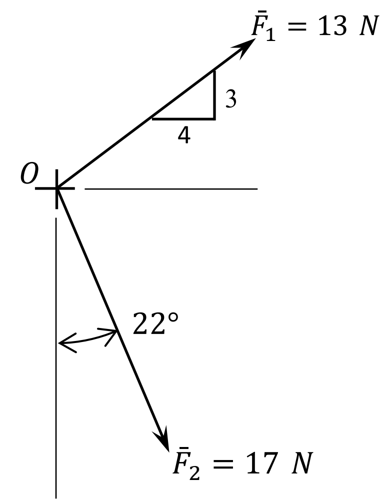
## Vector Addition




1. For vectors $\bar{F}_1$ and $\bar{F}_2$, determine the magnitude of the resultant $\bar{R} = \bar{F}_1 + \bar{F}_2$.

   A. 17.2 N B. 18.6 N

   C. 19.2 N D. 17.9 N

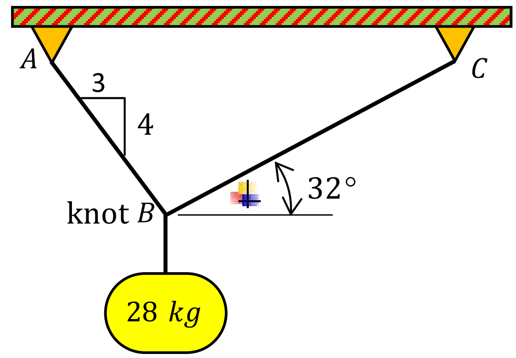
2. For vectors $\bar{F}_1$ and $\bar{F}_2$, determine the magnitude of the angle between the resultant $\bar{R} = \bar{F}_1 + \bar{F}_2$ and the horizontal reference line.

   A. $25.4^o$ B. $27.2^o$

   C. $24.1^o$ D. $22.6^o$

Particle Equilibrium in Two Dimensions – Part 1

11. Sketch a Free Body Diagram of knot B.




12. Determine the magnitude of the tension in cable segment AB.

A. 215 N B. 224 N

C. 243 N D. 234 N

13. Determine the magnitude of the tension in cable segment BC.

A. 165 N B. 159 N

C. 171 N D. 152 N

Some Useful Equations for Quiz 2

**Cross-Product**: $\overline{A} \times \overline{B} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

**Moment of a Force about a Point**: $\overline{M}_o = \bar{r} \times \overline{F}$

**Moment of a Force about a Point (Scalar):** $M = Fd$

**Moment of a Force about an Axis**: $M_{axis} = \bar{u} \bullet (\bar{r} \times \overline{F}) = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$

**Equivalent Force-Couple System at O**:

$$\overline{F}_R = \sum \overline{F}$$
$$\overline{M}_{R_O} = \sum \overline{M}_O$$

**Rigid Body Equilibrium (Vector Formulation)**: $\sum \overline{F} = 0 \qquad \sum \overline{M}_O = 0$

**Rigid Body Equilibrium (Scalar Formulation – 2D)**: $\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_O = 0$

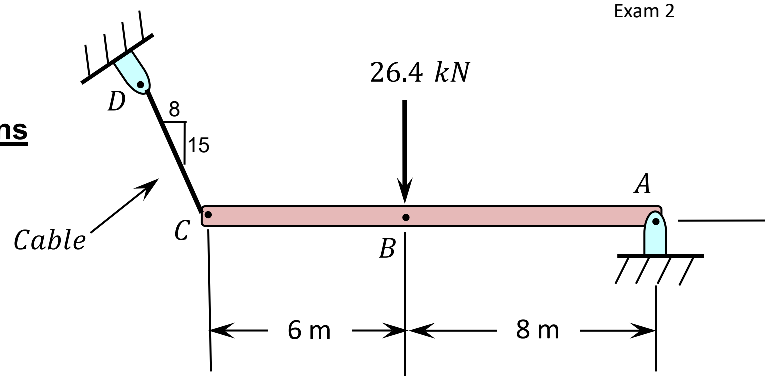
**Rigid Body Equilibrium (Scalar Formulation – 3D)**:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum \overline{F}_z = 0$$

$$\sum M_{O_x} = 0 \qquad \sum M_{O_y} = 0 \qquad \sum M_{O_z} = 0$$

## Rigid Body Equilibrium in Two Dimensions

11. Sketch a Free Body Diagram of rigid bar ABC.




12. Determine the magnitude of the tension in cable CD.
 a. 15.8 $kN$ b. 17.1 $kN$
 c. 15.2 $kN$ d. 16.5 $kN$

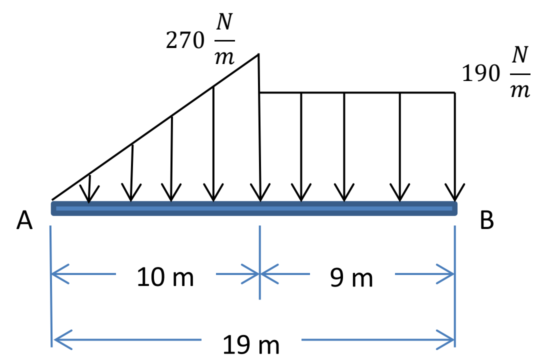
13. Determine the horizontal component of reaction at hinge A.
 a. 8.05 $kN$ → b. 8.68 $kN$ ←
 c. 8.36 $kN$ → d. 7.76 $kN$ ←

14. Determine the vertical component of reaction at hinge A.
 a. 11.8 $kN$ ↑ b. 10.9 $kN$ ↑
 c. 11.3 $kN$ ↑ d. 10.4 $kN$ ↑

### Distributed Loadings

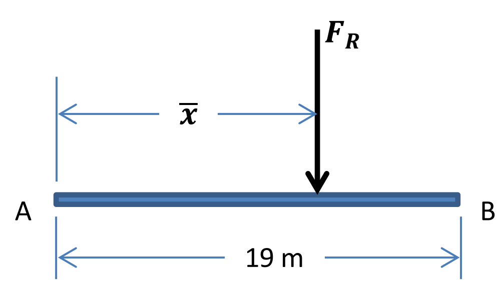
15. Determine the magnitude of the resultant force, $F_R$, equivalent to the distributed loading acting on bar AB.

 a. 3.29 kN b. 3.17 kN
 c. 2.94 kN d. 3.06 kN




16. Determine the location, $\bar{x}$, where the resultant force acts to have the same effect on bar AB as the distributed loading.

 a. 11.4 m b. 10.6 m
 c. 11.0 m d. 10.2 m



Some Useful Equations for Quiz 4

Area Formulae: $A = \int_A dA$ or $A = \sum A_i$

First Moments of Area: $\bar{x}A = \int_A \tilde{x} dA \quad \bar{y}A = \int_A \tilde{y} dA$ or $\bar{x}A = \sum \bar{x}_i A_i \qquad \bar{y}A = \sum \bar{y}_i A_i$

Fluid Pressure: $p = \gamma z$ (where "z" denotes depth and $\gamma = \rho g$ ; $\gamma_{water} = 62.4 \frac{lb}{ft^3}$ and $\rho_{water} = 1000 \frac{kg}{m^3}$

Centroidal Coordinates for Areas: $\bar{x} = \frac{\bar{x}A}{A} \qquad \bar{y} = \frac{\bar{y}A}{A}$

Area Moments of Inertia: $I_x = \int_A y^2 dA \qquad I_y = \int_A x^2 dA$ or (see parallel axis theorem)

$$J_O = \int_A \left(x^2 + y^2\right) dA$$

$$I_x = \Sigma(\bar{I}_{x_i'} + A_i d_{yi}^2) \qquad I_y = \Sigma(\bar{I}_{y_i'} + A_i d_{xi}^2)$$

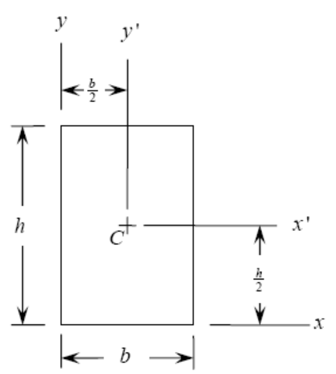
Radii of Gyration for Areas: $k_x = \sqrt{\frac{I_x}{A}} \qquad k_y = \sqrt{\frac{I_y}{A}}$

Products of Inertia for Areas: $I_{xy} = \int_A xy dA$ or $I_{xy} = \sum\left(\bar{I}_{x'y'} + A d_x d_y\right)$

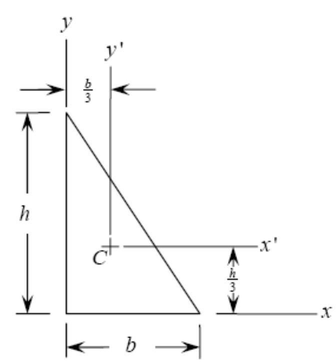
Parallel Axis Theorems for Areas: $I_x = \bar{I}_{x'} + A d_y^2 \qquad I_y = \bar{I}_{y'} + A d_x^2 \quad I_{xy} = \bar{I}_{x'y'} + A d_x d_y$

For Specific Areas:

Rectangle: $\bar{I}_{x'} = \frac{bh^3}{12} \qquad \bar{I}_{y'} = \frac{b^3 h}{12} \qquad I_x = \frac{bh^3}{3} \qquad I_y = \frac{b^3 h}{3}$




Right Triangle: $\bar{I}_{x'} = \frac{bh^3}{36} \qquad \bar{I}_{y'} = \frac{b^3 h}{36} \qquad I_x = \frac{bh^3}{12} \qquad I_y = \frac{b^3 h}{12}$



<u>Integration Formula</u>: $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C,\ n \neq -1$

$\int x^n dx = \ln|x| + C,\ n = -1$,

(C denotes a constant of integration that you need not worry about for definite integrals.)

<u>Rotation of Axes</u>:

$$I_u = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin 2\theta$$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) + I_{xy}\sin 2\theta$$

$$I_{uv} = \left(\frac{I_x - I_y}{2}\right)\sin(2\theta) + I_{xy}\cos 2\theta$$

$\tan(2\theta_p) = \dfrac{-2I_{xy}}{I_x - I_y}$ and $I_{\max,\min} = \left(\dfrac{I_x + I_y}{2}\right) \pm \sqrt{\left(\dfrac{I_x - I_y}{2}\right)^2 + (I_{xy})^2}$

Mass Formulae: $m = \int_m dm$ or $m = \sum m_i$

Location of Mass Center: $\bar{x} = \dfrac{\bar{x}m}{m} = \dfrac{\int_m x dm}{\int_m dm}$, $\bar{y} = \dfrac{\bar{y}m}{m} = \dfrac{\int_m y dm}{\int_m dm}$, $\bar{z} = \dfrac{\bar{z}m}{m} = \dfrac{\int_m z dm}{\int_m dm}$

Location of Mass Center for Composite Sections:

$$\bar{x} = \frac{\bar{x}m}{m} = \frac{\sum \bar{x}_i m_i}{\sum m_i},\ \bar{y} = \frac{\bar{y}m}{m} = \frac{\sum \bar{y}_i m_i}{\sum m_i},\ \bar{z} = \frac{\bar{z}m}{m} = \frac{\sum \bar{z}_i m_i}{\sum m_i}$$

Mass Moment of Inertia About an Axis: $I = \int_m r^2 dm = \int_V r^2 \rho dV$

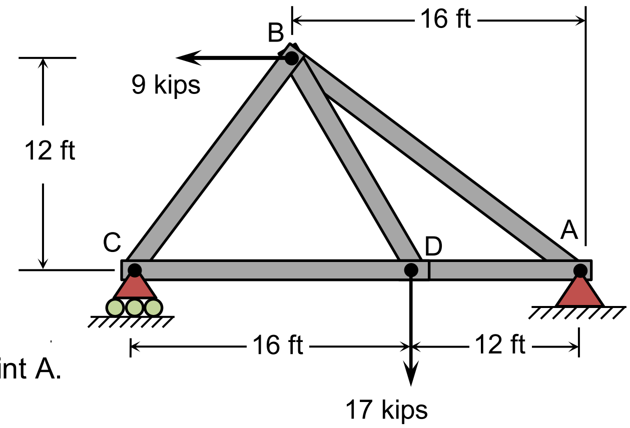
Parallel Axis Theorem for Mass Moments of Inertia: $I_O = \bar{I}_G + md^2$

Where $\bar{I}_G$ denotes the mass moment of inertia about an axis passing through the mass center of a body, $I_O$ denotes the mass moment of inertia about a parallel axis that passes through point O, and $d$ denotes the distance between the axes.

<u>Radii of Gyration for Masses</u>: $\bar{k}_G = \sqrt{\dfrac{\bar{I}_G}{m}}$ $\quad k_O = \sqrt{\dfrac{I_O}{m}}$

**Trusses – Method of Joints**

Truss ABCD is pin connected at all joints and is simply supported with a pin at A and a roller at C.




1. Determine the horizontal reaction at the pin at point A.

   a. 9.65 kip ← b. 8.68 kip ←

   c. 9.00 kip → d. 9.32 kip →

2. Determine the vertical reaction at the pin at point A.

   a. 5.86 kip ↑ b. 5.64 kip ↑

   c. 5.42 kip ↑ d. 6.00 kip ↑

3. Draw a Free Body Diagram of joint A.

4. Determine the force in member AB.

   a. 10.1 kip (T) b. 9.05 kip (T)

   c. 9.39 kip (C) d. 9.76 kip (C)

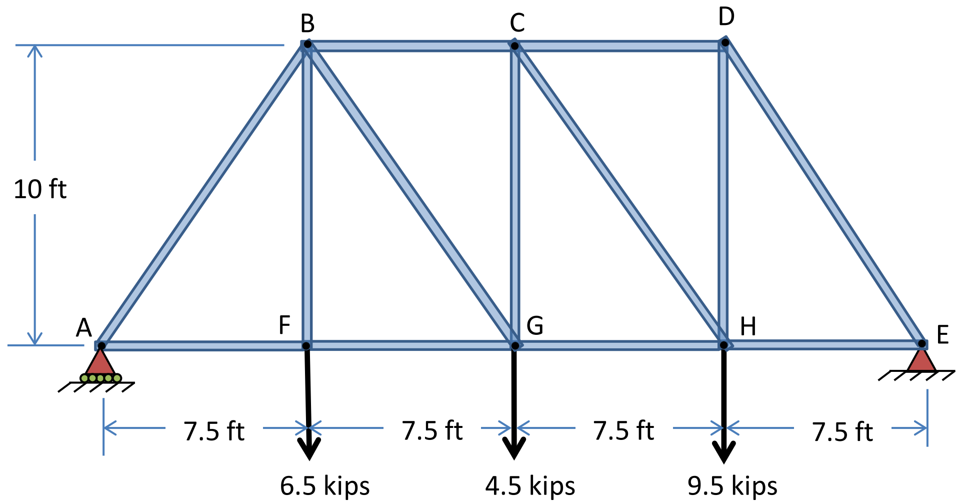
5. Determine the force in member AD.

   a. 16.8 kip (T) b. 15.5 kip (C)

   c. 14.9 kip (C) d. 16.2 kip (T)

**Trusses – Method of Sections**

The truss shown consists of members pinned at the joints, supported by a roller at A and a pin at E.




6. Sketch a free body diagram of a section of the truss that will allow you to find the member forces in members BC, FG and BG.

7. Determine the force in member BC.

   a. 9.36 kips (C)  b. 9.73 kips (T)

   c. 9.00 kips (C)  d. 8.66 kips (C)

8. Determine the force in member FG.

   a. 7.13 kips (T)  b. 7.67 kips (C)

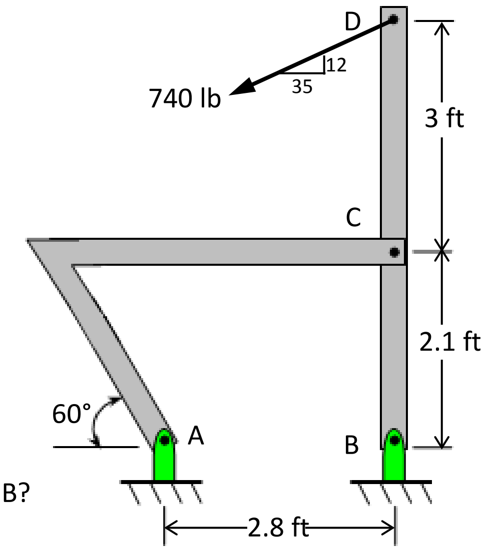
   c. 7.40 kips (T)  d. 7.96 kips (T)

9. Determine the force in member BG.

   a. 4.06 kips (C)  b. 3.75 kips (T)

   c. 3.60 kips (T)  d. 3.90 kips (C)

**Frames & Simple Machines**




15. What is the vertical component of the reaction at B?

a. 1.04 kip ↓ b. 1.28 kip ↑

c. 240 lb ↑ d. 1.52 kip ↓

16. What is the magnitude of the force acting on member BCD at pin C?

a. 1.59 kips b. 2.83 kips

c. 1.70 kips d. 2.13 kips

17. What is the horizontal component of the reaction at B?

a. 1.70 kips → b. 1.00 kip ←

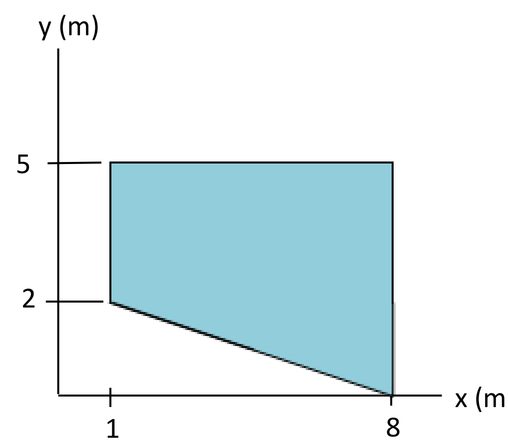
c. 575 lb ← d. 700 lb →

**Centroids by Integration**

18. What is area of the shaded region?

a. 40.0 $m^2$ b. 56.0 $m^2$

c. 37.3 $m^2$ d. 28.0 $m^2$




19. What is the x-coordinate of the centroid of the shaded region?

a. 4.50 m b. 4.79 m

c. 3.50 m d. 4.00 m

20. What is the y-coordinate of the centroid of the shaded region?

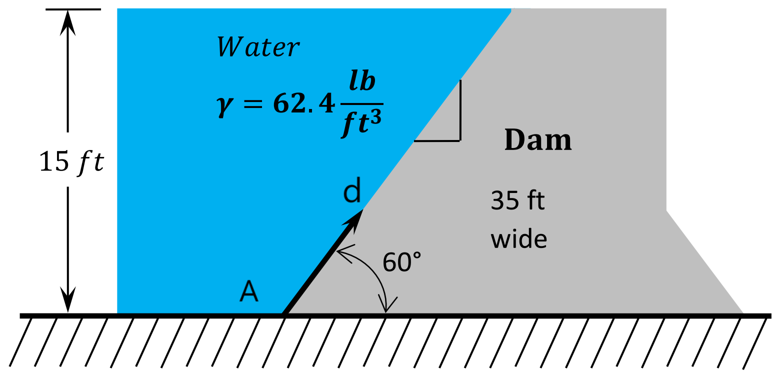
a. 2.04 m b. 2.50 m

c. 3.19 m d. 2.96 m

## Fluid Pressure

1. Determine the pressure the water exerts on point A at the bottom of the sloping upstream face of the dam .

   A $974\frac{lb}{ft^2}$  B $863\frac{lb}{ft^2}$
   C $899\frac{lb}{ft^2}$  D $936\frac{lb}{ft^2}$




2. Determine the fluid force the water exerts on a $1\ ft$ width of the dam.

   A $8.43\ kips$  B $8.77\ kips$

   C $8.11\ kips$  D $7.78\ kips$

3. Determine the total fluid force the water exerts on the dam.

   A $307\ kips$  B $284\ kips$

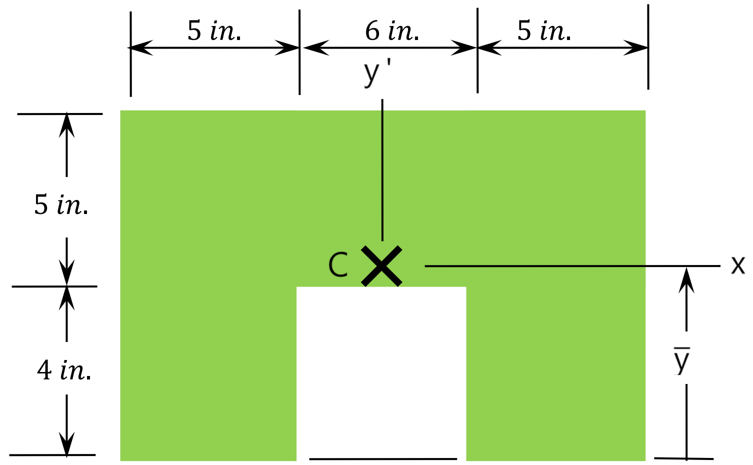
   C $295\ kips$  D $273\ kips$

4. Determine the distance, d, from point A at which the force acts, measured along the sloping upstream face of the dam.

   A $5.77\ ft$  B $5.00\ ft$

   C $5.53\ ft$  D $5.26\ ft$

## Moments and Products of Inertia for Composite Areas




14. The vertical distance from the base to the centroidal x-axis, $\bar{y}$, for the shaded area is:

A $4.80\ in.$ B $5.20\ in.$

C $5.00\ in.$ D $5.40\ in.$

15. The second moment of area about the centroidal x-axis, $\bar{I}_{x'}$, for the shaded area is:

A $760\ in.^4$ B $790\ in.^4$

C $730\ in.^4$ D $820\ in.^4$

16. The radius of gyration with respect to the centroidal x-axis, $\bar{k}_{x'}$, for the shaded area is:

A $2.32\ in.$ B $2.42\ in.$

C $2.22\ in.$ D $2.52\ in.$

17. The second moment of area about the y-axis, $\bar{I}_{y'}$, for the shaded area is:

A $3120\ in.^4$ B $3000\ in.^4$

C $3250\ in.^4$ D $3380\ in.^4$

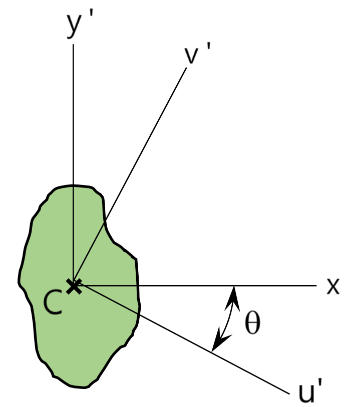
18. The product of inertia, $\bar{I}_{x'y'}$, for the shaded area with respect to the centroidal axes is:

A $4600\ in.^4$ B $0$

C $4800\ in.^4$ D $5000\ in.^4$

**Rotation of Axes for Moment of Inertia**

The shaded area has the following properties with respect to the centroidal x' and y' axes, $\bar{I}_{x'} = 830 in.^4$, $\bar{I}_{y'} = 270 in.^4$, $\bar{I}_{x'y'} = -150\ in.^4$ and $\theta = 25°$.




19. Determine the centroidal moment of inertia, $\bar{I}_{u'}$, with respect to the u' axis.

A 615 $in.^4$ B 575 $in.^4$

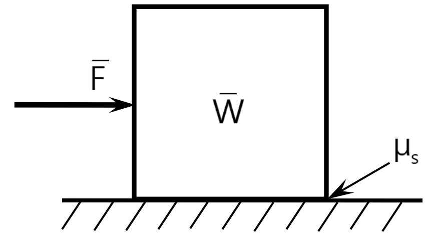
C 635 $in.^4$ D 595 $in.^4$

20. Determine the centroidal moment of inertia, $\bar{I}_{v'}$, with respect to the v' axis.

A 465 $in.^4$ B 505 $in.^4$

C 525 $in.^4$ D 485 $in.^4$

Dry Friction




46. Determine the magnitude of the friction force the rough plane exerts on the block if $\bar{W} = 100\ lb$, $\mu_s = 0.5$, and $\bar{F} = 40\ lb$.
    a. $50.0\ lb$ b. $40.0\ lb$
    c. $45.0\ lb$ d. $35.0\ lb$

47. Determine the smallest magnitude of force $\bar{F}$ for which motion of the block impends if $\bar{W} = 150\ lb$ and $\mu_s = 0.3$.
    a. $45.0\ lb$ b. $40.0\ lb$
    c. $50.0\ lb$ d. $60.0\ lb$

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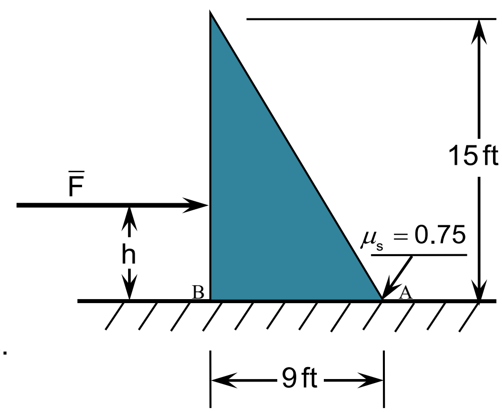
**Slipping or Tipping**: The uniform triangular object weighs 200 lb .




48. Determine the magnitude of the force $\bar{F}$ for which slipping impends provided it acts at height h sufficiently small so that tipping does not occur.
    a. 180 lb b. 75.0 lb
    c. 200 lb d. 150 lb

49. If force $\bar{F} = 120$ lb determine the height h at which tipping impends.
    a. $8.00\ ft$ b. $13.3\ ft$
    c. $10.0\ ft$ d. $15.0\ ft$

50. Determine the height h at which the force $\bar{F}$ acts to create slipping AND tipping at the same time.
    a. $8.00\ ft$ b. $13.3\ ft$
    c. $10.0\ ft$ d. $15.0\ ft$



Texas Tech University – David Spears @ 2024