Some Useful Equations for Quiz 1

Cartesian Form of a Vector: $\overline{F} = F_x\overline{i} + F_y\overline{j} + F_z\overline{k}$

Magnitude: $F = \left|\overline{F}\right| = \sqrt{F_x^2 + F_y^2 + F_z^2}$

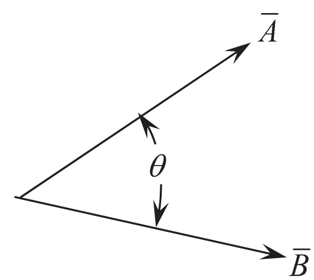
Direction Cosines For $\overline{F}$: $\cos\alpha = \dfrac{F_x}{F}$ $\quad \cos\beta = \dfrac{F_y}{F}$ $\quad \cos\gamma = \dfrac{F_z}{F}$

Unit Vector: $\overline{u} = \left(\dfrac{F_x}{F}\right)\overline{i} + \left(\dfrac{F_y}{F}\right)\overline{j} + \left(\dfrac{F_z}{F}\right)\overline{k} = (\cos\alpha)\overline{i} + (\cos\beta)\overline{j} + (\cos\gamma)\overline{k}$

Directed Force Vector: $\overline{F} = F\overline{u}$

Absolute Position Vector: $\overline{r}_A = x_A\overline{i} + y_A\overline{j} + z_A\overline{k}$

Relative Position Vector: $\overline{r}_{B/A} = (x_B - x_A)\overline{i} + (y_B - y_A)\overline{j} + (z_B - z_A)\overline{k}$



Dot Product: $\overline{A}\cdot\overline{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$

Projection of Vector along Line a-a: $A_{\mathrm{Proj}} = \overline{A}\cdot\overline{u}$

Angle between Two Vectors, $\overline{A}$ & $\overline{B}$: $\theta = \cos^{-1}\left(\dfrac{\overline{A}\cdot\overline{B}}{AB}\right)$

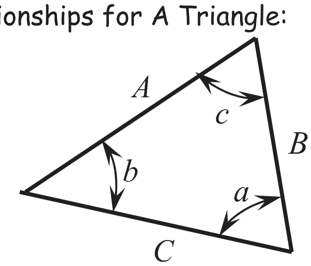
Particle Equilibrium, Vector Formulation: $\sum\overline{F} = 0$

Particle Equilibrium, Scalar Formulation, 2D: $\sum F_x = 0 \quad \sum F_y = 0$

$\overline{A}$

Particle Equilibrium, Scalar Formulation, 3D: $\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$

Geometric Relationships for A Triangle:



Law of Cosines:

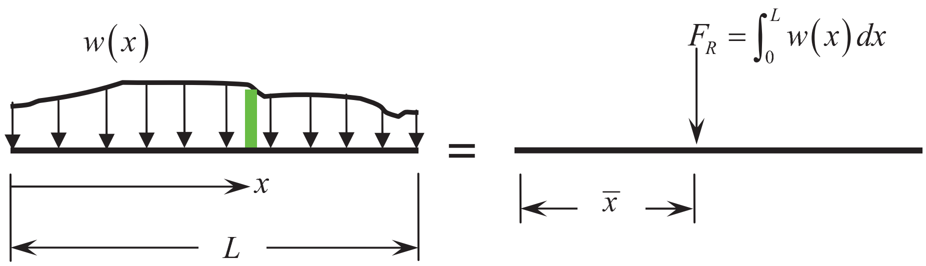
$$A^2 = B^2 + C^2 - 2BC\cos a$$
$$B^2 = A^2 + C^2 - 2AC\cos b$$
$$C^2 = A^2 + B^2 - 2AB\cos c$$

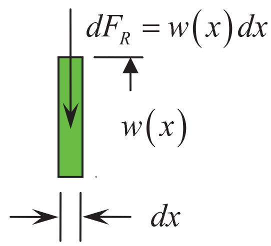
Law of Sines:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Some Useful Equations for Quiz 2 (cont'd.)

Distributed Loading:





$$\bar{x} = \frac{\int_0^L x w(x)\,dx}{\int_0^L w(x)\,dx} = \frac{\int_0^L x w(x)\,dx}{F_R}$$

Integration Formulae:

$$\int x^n\,dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C, & x \neq -1 \\ \ln(x) + C, & x = -1 \end{cases}$$

Some Useful Equations for Quiz 2

Cross-Product: $$\overline{A}\times\overline{B}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Moment of a Force about a Point: $$\overline{M}_o=\bar{r}\times\overline{F}$$

Moment of a Force about a Point (Scalar): $$M=Fd$$

Moment of a Force about an Axis: $$M_{axis}=\bar{u}\bullet(\bar{r}\times\overline{F})=\begin{vmatrix}u_x & u_y & u_z\\ r_x & r_y & r_z\\ F_x & F_y & F_z\end{vmatrix}$$

Equivalent Force-Couple System at O: $$\overline{F}_R=\sum\overline{F}$$ $$\overline{M}_{R_O}=\sum\overline{M}_O$$

Rigid Body Equilibrium (Vector Formulation): $$\sum\overline{F}=0 \qquad \sum\overline{M}_O=0$$

Rigid Body Equilibrium (Scalar Formulation – 2D): $$\sum F_x=0 \qquad \sum F_y=0 \qquad \sum M_O=0$$

Rigid Body Equilibrium (Scalar Formulation – 3D):

$$\sum F_x=0 \qquad \sum F_y=0 \qquad \sum\overline{F}_z=0$$

$$\sum M_{O_x}=0 \qquad \sum M_{O_y}=0 \qquad \sum M_{O_z}=0$$

Even Some More Useful Equations:

Area Formulae: $A = \int_A dA$ or $A = \sum A_i$

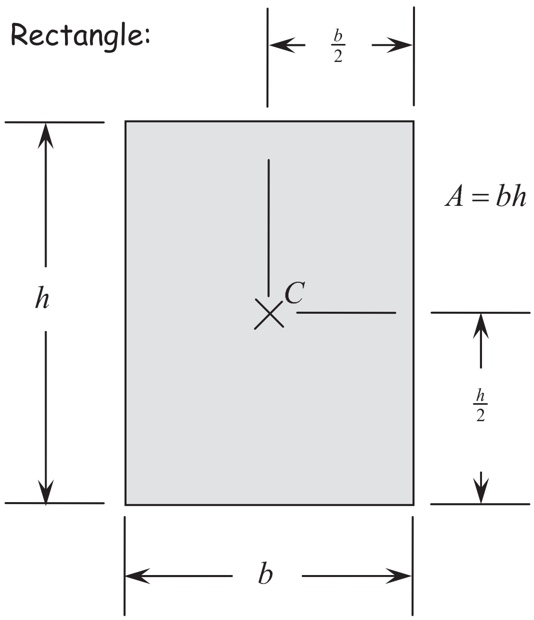
First Moments of Area: $\bar{x}A = \int_A \tilde{x} dA$ $\bar{y}A = \int_A \tilde{y} dA$ or $\bar{x}A = \sum \bar{x}_i A_i$ $\bar{y}A = \sum \bar{y}_i A_i$

Centroidal Coordinates for Areas: $\bar{x} = \dfrac{\bar{x}A}{A}$ $\bar{y} = \dfrac{\bar{y}A}{A}$

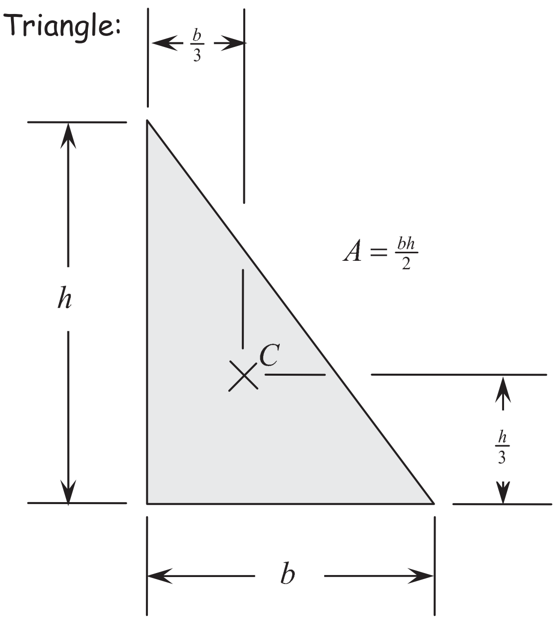
Theorems of Pappus & Guldinus: $A_{rev} = \theta \bar{r} L$ $V_{rev} = \theta \bar{r} A$

Properties of Very Common Shapes:

Rectangle:



Right Triangle:



Some Useful Equations for Quiz 4

Area Formulae: $A=\int_A dA$ or $A=\sum A_i$

First Moments of Area: $\bar{x}A=\int_A \tilde{x}dA$ $\bar{y}A=\int_A \tilde{y}dA$ or $\bar{x}A=\sum \bar{x}_i A_i$ $\bar{y}A=\sum \bar{y}_i A_i$

Fluid Pressure: $p=\gamma z$ (where "z" denotes depth and $\gamma=\rho g$; $\gamma_{water}=62.4\frac{lb}{ft^3}$ and $\rho_{water}=1000\frac{kg}{m^3}$

Centroidal Coordinates for Areas: $\bar{x}=\frac{\bar{x}A}{A}$ $\bar{y}=\frac{\bar{y}A}{A}$

Area Moments of Inertia: $I_x=\int_A y^2 dA$ $I_y=\int_A x^2 dA$ or (see parallel axis theorem)

$$J_O=\int_A\left(x^2+y^2\right)dA$$

$$I_x=\Sigma(\bar{I}_{x_i'}+A_i d_{yi}^2) \qquad I_y=\Sigma(\bar{I}_{y_i'}+A_i d_{xi}^2)$$

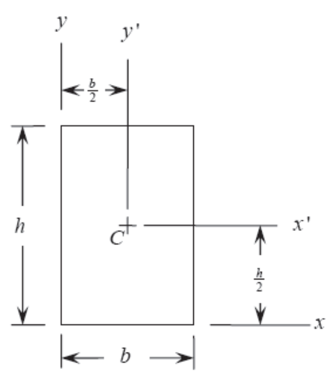
Radii of Gyration for Areas: $k_x=\sqrt{\frac{I_x}{A}}$ $k_y=\sqrt{\frac{I_y}{A}}$

Products of Inertia for Areas: $I_{xy}=\int_A xydA$ or $I_{xy}=\sum\left(\bar{I}_{x'y'}+Ad_x d_y\right)$

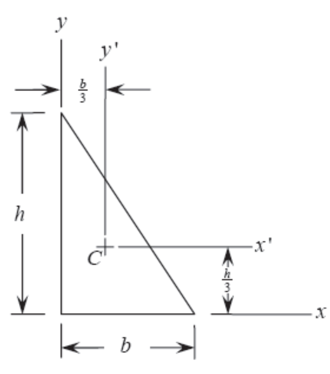
Parallel Axis Theorems for Areas: $I_x=\bar{I}_{x'}+Ad_y^2$ $I_y=\bar{I}_{y'}+Ad_x^2$ $I_{xy}=\bar{I}_{x'y'}+Ad_x d_y$

For Specific Areas:

Rectangle: $\bar{I}_{x'}=\frac{bh^3}{12}$ $\bar{I}_{y'}=\frac{b^3h}{12}$ $I_x=\frac{bh^3}{3}$ $I_y=\frac{b^3h}{3}$



Right Triangle: $\bar{I}_{x'}=\frac{bh^3}{36}$ $\bar{I}_{y'}=\frac{b^3h}{36}$ $I_x=\frac{bh^3}{12}$ $I_y=\frac{b^3h}{12}$



<u>Integration Formula</u>: $\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$

$\int x^n dx = \ln|x| + C,\ n = -1$,

(C denotes a constant of integration that you need not worry about for definite integrals.)

<u>Rotation of Axes</u>:

$$I_u = \left(\frac{I_x + I_y}{2}\right) + \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) - I_{xy}\sin 2\theta$$

$$I_v = \left(\frac{I_x + I_y}{2}\right) - \left(\frac{I_x - I_y}{2}\right)\cos(2\theta) + I_{xy}\sin 2\theta$$

$$I_{uv} = \left(\frac{I_x - I_y}{2}\right)\sin(2\theta) + I_{xy}\cos 2\theta$$

$\tan(2\theta_p) = \frac{-2I_{xy}}{I_x - I_y}$ and $I_{\max,\min} = \left(\frac{I_x + I_y}{2}\right) \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + (I_{xy})^2}$

Mass Formulae: $m = \int_m dm$ or $m = \sum m_i$

Location of Mass Center: $\bar{x} = \frac{\bar{x}m}{m} = \frac{\int_m x dm}{\int_m dm}$, $\bar{y} = \frac{\bar{y}m}{m} = \frac{\int_m y dm}{\int_m dm}$, $\bar{z} = \frac{\bar{z}m}{m} = \frac{\int_m z dm}{\int_m dm}$

Location of Mass Center for Composite Sections:

$$\bar{x} = \frac{\bar{x}m}{m} = \frac{\sum \bar{x}_i m_i}{\sum m_i},\ \bar{y} = \frac{\bar{y}m}{m} = \frac{\sum \bar{y}_i m_i}{\sum m_i},\ \bar{z} = \frac{\bar{z}m}{m} = \frac{\sum \bar{z}_i m_i}{\sum m_i}$$

Mass Moment of Inertia About an Axis: $I = \int_m r^2 dm = \int_V r^2 \rho dV$

Parallel Axis Theorem for Mass Moments of Inertia: $I_O = \bar{I}_G + md^2$

Where $\bar{I}_G$ denotes the mass moment of inertia about an axis passing through the mass center of a body, I_O denotes the mass moment of inertia about a parallel axis that passes through point O, and d denotes the distance between the axes.

<u>Radii of Gyration for Masses</u>: $\bar{k}_G = \sqrt{\frac{\bar{I}_G}{m}}$ $k_O = \sqrt{\frac{I_O}{m}}$

Some Friction Equations:

Maximum Friction Force: $F_{max} = \mu_s N$

Friction Angle: $\phi_s = \tan^{-1}(\mu_s)$

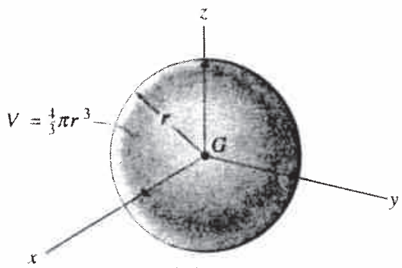
Lead Angle: $\theta = \tan^{-1}(\frac{L}{2\pi r})$

Screws Forces: $M = Wr\tan(\phi_s + \theta)$ (Tightening Screw, Lifting Weight)

$M = Wr\tan(\phi_s - \theta)$ (Loosening Screw, Lowering Weight)

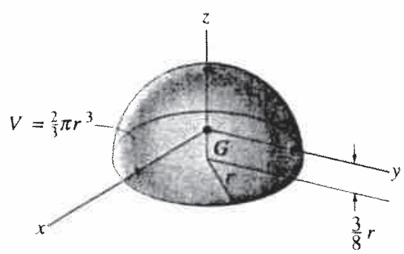
Flat Belts: $T_2 = T_1 e^{\mu\theta}$

Center of Gravity and Mass Moment of Inertia of Homogeneous Solids



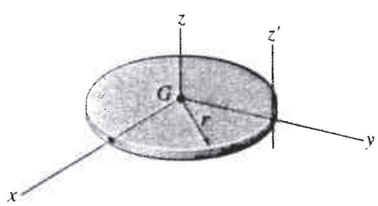
Sphere

$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} mr^2$



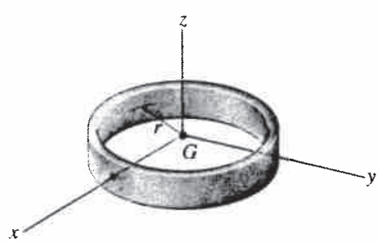
Hemisphere

$I_{xx} = I_{yy} = 0.259mr^2 \quad I_{zz} = \frac{2}{5}mr^2$



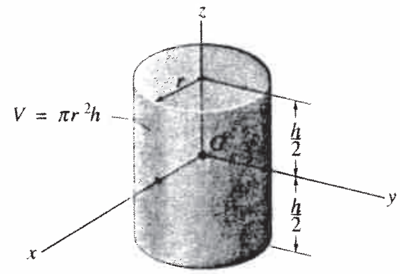
Thin Circular disk

$I_{xx} = I_{yy} = \frac{1}{4} mr^2 \quad I_{zz} = \frac{1}{2}mr^2 \quad I_{z'z'} = \frac{3}{2} mr^2$



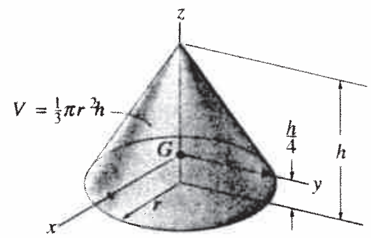
Thin ring

$I_{xx} = I_{yy} = \frac{1}{2} mr^2 \quad I_{zz} = mr^2$



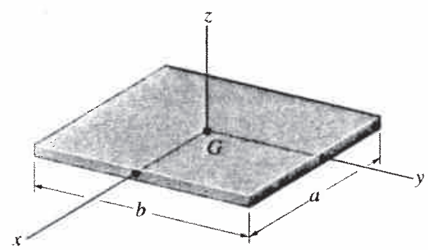
Cylinder

$I_{xx} = I_{yy} = \frac{1}{12} m(3r^2 + h^2) \quad I_{zz} = \frac{1}{2} mr^2$



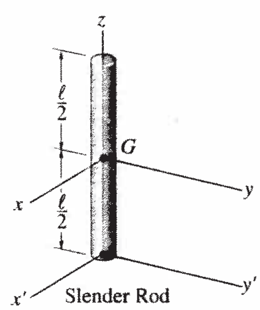
Cone

$I_{xx} = I_{yy} = \frac{3}{80} m (4r^2 + h^2) \quad I_{zz} = \frac{3}{10}mr^2$



Thin plate

$I_{xx} = \frac{1}{12} mb^2 \quad I_{yy} = \frac{1}{12} ma^2 \quad I_{zz} = \frac{1}{12} m(a^2 + b^2)$



Slender Rod

$I_{xx} = I_{yy} = \frac{1}{12} m\ell^2 \quad I_{x'x'} = I_{y'y'} = \frac{1}{3} m\ell^2 \quad I_{z'z'} = 0$