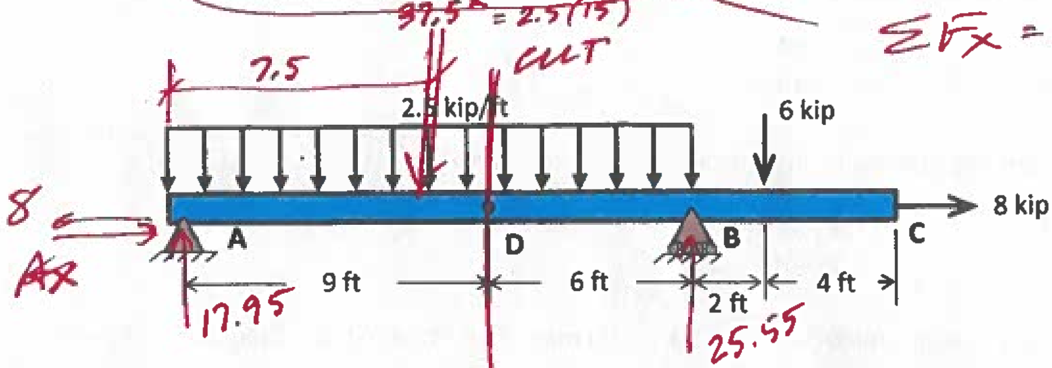
$\Sigma M_A = 0 = -37.5(7.5) + B_y(15) - 6(17)$ $B_y = \frac{383.25}{15} = 25.55$

Deformable Body Equilibrium

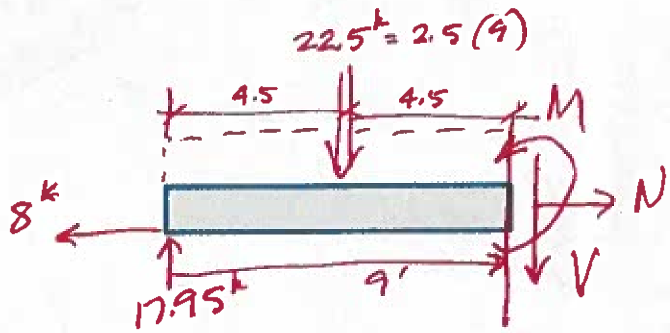
Beam loaded as shown. D is a point ~~within the beam (not a hinge).~~ Reactions $\mathbf{A_y}$ = 17.95 kips and $\mathbf{B_y}$ = 25.55 kips have been determined. **(Hint: There is another reaction at A.)**

$\Sigma F_x = 0 = A_x + 8$

$A_x = -8^k \leftarrow$



4) Draw a free body diagram for a cut at D. (5 pts)



5) Determine the internal shear force at D. (4 pts)

$+\uparrow \Sigma F_y = 0 = 17.95 - 22.5 - V$

$\boxed{V = -4.55^k}$

a. – 4.37 kip b. 0

c. – 4.55 kip d. -4.19 kip

6) Determine the internal normal force at D. (4 pts)

$\overset{+}{\rightarrow} \Sigma F_x = 0 = -8 + N$

$\boxed{N = +8^k\ (T)}$

a. 0 b. 6.00 kip (T)

c. 8.00 kip (T) d. 8.00 kip (C)

7) Determine the internal moment at D. (4 pts)

$\circlearrowleft \Sigma M_{cut} = 0 = -17.95(9) + 22.5(4.5) + M$

$\boxed{M = +60.3\ k\text{-}ft}$

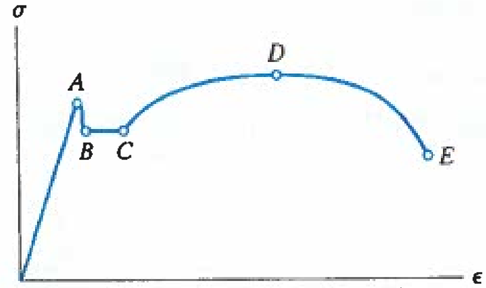
a. + 65.2 kip·ft b. + 62.7 kip·ft

c. + 67.8 kip·ft d. + 60.3 kip·ft

17

CHECK SOLUTION

Stress-Strain Relationships



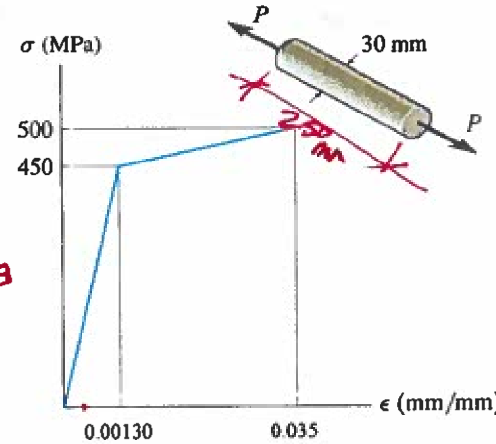
1) The point on the stress-strain diagram that represents ultimate stress is ___. (2 pts)

a. Point C
b. Point D
c. Point B
d. Point E

2) The point on the stress-strain diagram that represents yield stress is ___. (2 pts)

a. Point C
b. Point A
c. Point D
d. Point E

The specimen below has an unloaded length L = 250 mm. The Stress-Strain Diagram is shown.



3) Determine the modulus of elasticity, E, of the material. (2 pts)

a. 346 GPa
b. 360 GPa
c. 320 GPa
d. 333 GPa

$= \frac{\sigma}{\epsilon} = \frac{450}{0.00130} = 346.1\ GPa$

346154 MPa

4) Determine the normal strain of the specimen if it is loaded with a force P = 140 kN. (6 pts)

MC 5/5

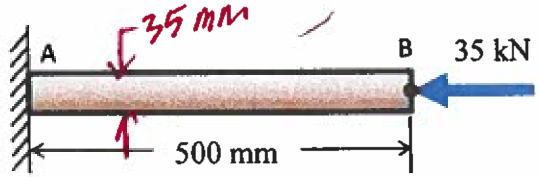
$\sigma = P/A \quad \epsilon = \sigma/E$

C $572(10^{-6})\frac{mm}{mm}$

$\epsilon = \sigma/E = \frac{P}{AE} = \frac{(140000)}{\frac{\pi}{4}(30)^2(346150)} = 0.000572$

Strain = $572.2\ E{-6}\ mm/mm$

The system is loaded and supported as shown. The solid shaft has a diameter, d = 35 mm.
E = 200 GPa



$\epsilon_{LONG} = \frac{P}{AE} = \frac{35000}{\frac{\pi}{4}(35)^2(200000)} = (0.0001819)$

5) Determine Poisson's Ratio if the measured change in diameter under the load is Δd = +2.40 (10^{-3}) mm. (10 pts)

$\epsilon = \frac{d}{D_0} = \frac{2.4E{-3}\ mm}{35\ mm} = 0.0000685$

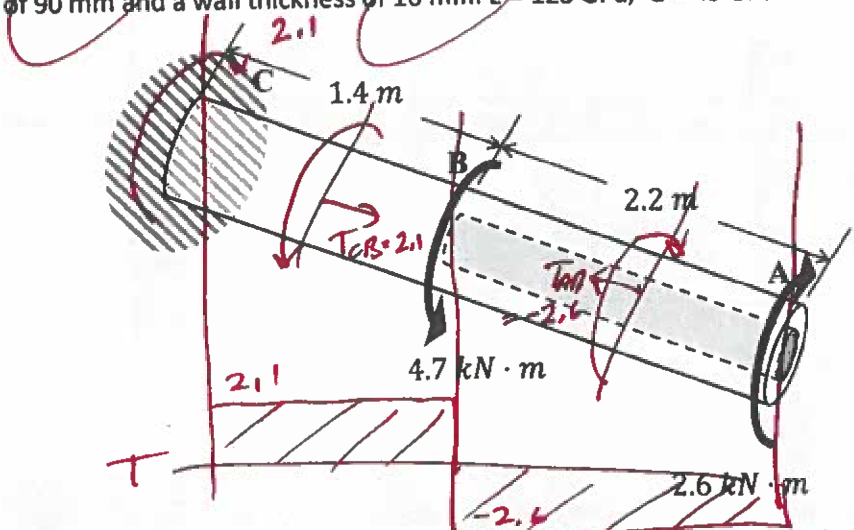
$\nu = -\frac{\epsilon_{LAT}}{\epsilon_{LONG}} = 0.3770$

ν = 0.377

6:00

Torsional Shear Stress & Angle of Twist

The bar is loaded as shown. Section BC is solid with ~~an outside~~ diameter of 90 mm. Section AB is a tube with an outside diameter of 90 mm and a wall thickness of 10 mm. E = 123 GPa, G = 45 GPa

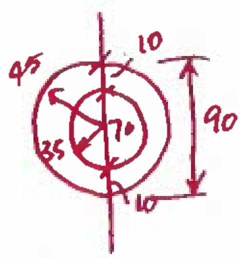


6) Determine the maximum shear stress in the section of the shaft between B and C. (8 pts)

MC

Maximum shear stress = 14.7 MPa

$$\tau = \frac{T\rho}{J} = \frac{2.1(1000)(1000)\left(\frac{90}{2}\right)}{6.441E6} = \boxed{14.67\ \frac{N}{mm^2}\ MPa}$$



$$J_{BC} = \frac{\pi}{2}\left(\frac{90}{2}\right)^4 = 6.441\ E6\ mm^4$$

$$J_{AB} = \frac{\pi}{2}\left(\left(\frac{90}{2}\right)^4 - \left(\frac{70}{2}\right)^4\right) = 4.084\ E6\ mm^4$$

7) Determine the angle of twist of point A relative to point B. (8 pts)

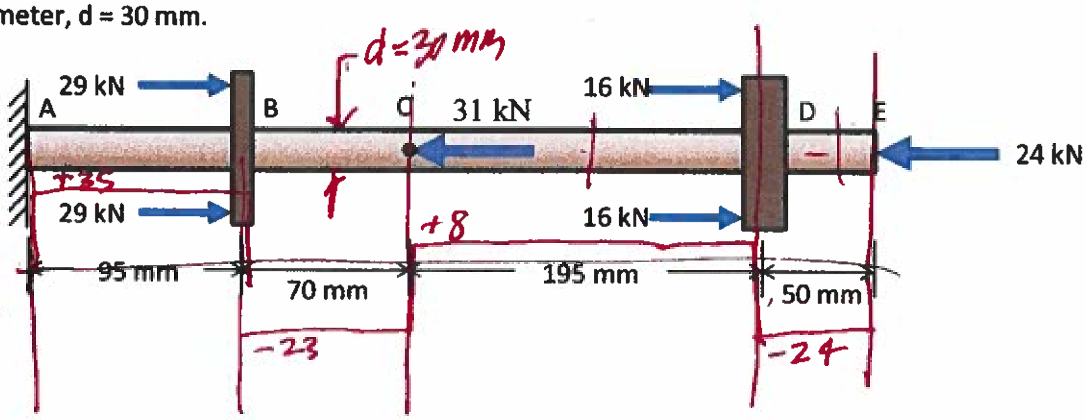
Angle of Twist = -0.0311 rad

$$\phi_{AB} = \frac{TL}{JG} = \frac{-2.6(1000)(1000)(2200)}{4.084E6\,(45000)} = \boxed{-0.03112\ rad}$$

5.00

Axial (Normal) Stress, Strain and Deformation

The system is loaded and supported as shown. The system is made of Titanium, E_T = 140 GPa. It is a solid shaft with diameter, d = 30 mm.



11) Calculate the axial normal stress in section BC, (6 pts)

$$\sigma = P/A = \frac{-23000}{\pi/4\,(30)^2} = \boxed{-32.54 \text{ MPa (C)}}$$

MC

Normal stress in section BC = -32.5 MPa (C)

12) Calculate the displacement of point B relative to point D. (10 pts)

$$\delta_{BC} = \frac{PL}{AE} = \frac{-23000(70)}{\pi/4(30)^2(140000)} = -0.01627 \text{ mm}$$

$$\delta_{CD} = \frac{+8000(195)}{\pi/4(30)^2(140000)} = +0.01576 \text{ mm}$$

$$-0.0005053 \text{ mm}$$

Displacement of B relative to D = -505E-6 mm

13) Calculate the axial normal strain in section DE. (4 pts)

$\varepsilon = \sigma/E$ $\quad \varepsilon = \sigma/E = P/AE$

$$= \frac{-24000}{\pi/4(30)^2(140000)}$$

$$= 242.5\text{E-}6 \text{ mm/mm}$$

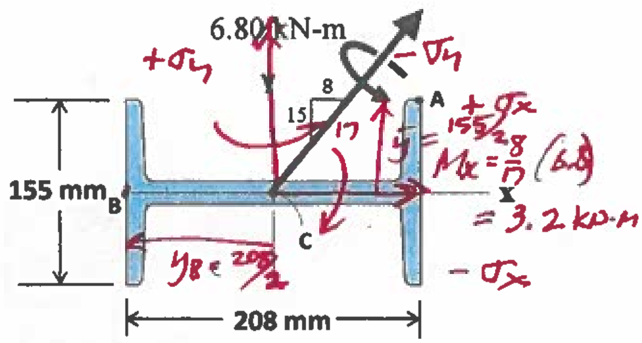
a. -252 (10^{-6}) mm/mm.
b. -243 (10^{-6}) mm/mm.
c. -262 (10^{-6}) mm/mm.
d. -233 (10^{-6}) mm/mm.

7:30

$M_y = \frac{15}{17}(6.8) = 6.0$ kN-m

Bending Stress and Flexure Formula

The cross section of a W200 x 36 is subjected the resultant internal bending moment as shown.
$I_x = 7.64\,(10^6)\text{ mm}^4$, $I_y = 34.4\,(10^6)\text{ mm}^4$



$+\sigma_y + \cancel{\sigma_x}$ 0

14) Determine the Direction of the bending stress at B. (2 pts)

a. + (Tension) b. – (Compression) c. No Stress

15) Calculate the magnitude of the bending stress at B. (4 pts)

a. 18.1 MPa b. 18.8 MPa
c. 0 d. 17.4 MPa

$$\sigma_B = \frac{M_y x}{I_y} = \frac{6.0(1000)(1000)\left(\frac{208}{2}\right)}{34.4E6} = \boxed{18.14\ N/mm^2}$$

16) Calculate the bending stress at A. (8 pts)

$$\sigma_A = +\sigma_x - \sigma_y$$
$$= +\frac{M_x y}{I_x} - \frac{M_y y}{I_y}$$
$$= \frac{3.2(1000)(1000)\left(\frac{155}{2}\right)}{7.64E6}$$
$$= 32.46\ MPa - 18.14\ MPa = +14.32\ MPa$$

Bending Stress @ A = $+14.3\ MPa$

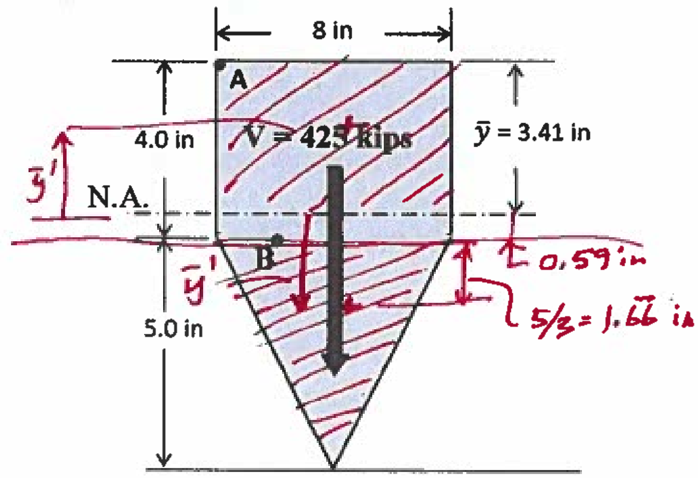
5:30

Transverse Shear

The cross-section has a moment of inertia about the neutral axis, $I = 236 \text{ in}^4$. Point B lies 4.0 inches below the top of the cross-section, 5.0 inches above the bottom of the cross-section.

17) Calculate the maximum transverse shear stress. (10 pts)

$$\tau = \frac{VQ}{It} = \frac{425\overbrace{\left(\frac{3.41}{2}\right)}^{\bar{y}'}\overbrace{(3.41)(8)}^{A'}}{(236)(8)} = \boxed{10.47 \text{ ksi}}$$



Maximum Shear Stress = 10.5 ksi

18) Calculate the transverse shear stress at point B. (4 pts)

a. 10.6 ksi   b. 11.0 ksi

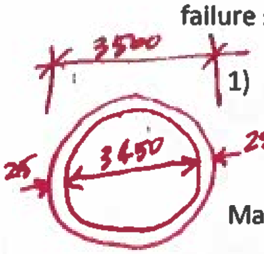
c. 10.2 ksi   d. 9.80 ksi

$$\tau_B = \frac{VQ}{It} = \frac{425\overbrace{\left(\frac{5}{3} + 0.59\right)}^{\substack{\bar{y}' \\ 2.256}}\overbrace{\left(\frac{1}{2}(5)(8)\right)}^{\substack{A' \\ 20}}}{236(8)} = \boxed{10.16 \text{ ksi}}$$

5:00

Thin Walled Pressure Vessels

A pressurized ***cylindrical*** tank with a diameter 3.5 m, made of 25 mm thick steel, E = 200 GPa. The failure stress of the steel is 250 MPa.



$\sigma = \frac{pr}{t} \quad p = \frac{\sigma t}{r} = \frac{250(25)}{3450/2} = \boxed{3.623 \text{ MPa}}$ N/mm² ans

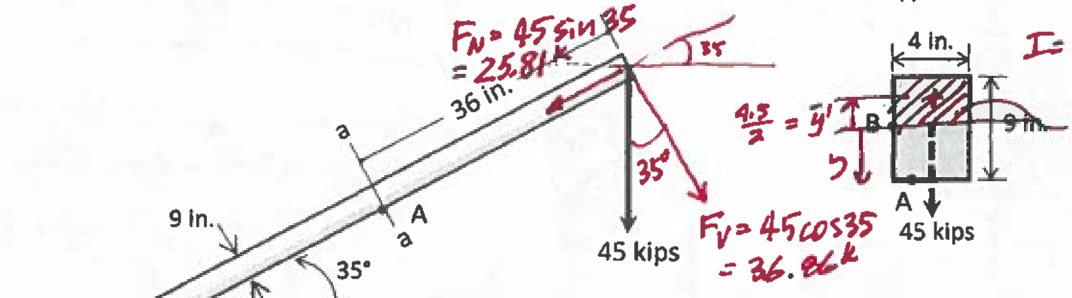
$\div 2.7 = \boxed{1.342 \text{ MPa}}$

1) Calculate the maximum pressure the tank can contain with a Factor of Safety of 2.7. (~~6~~ 5 pts)

Maximum Pressure = 1.34 MPa

Combined Loads and Stresses

The cantilever has a force applied at the center of the cross-section. This creates 3 types of stress at A.



Cross-section at a-a

2) Calculate the normal stress caused by the axial load at point A. (~~3~~ 5 pts) (45 sin 35)

a. -0.661 ksi
b. 0.747 ksi
c. 0.688 ksi
d. -0.717 ksi

$\sigma_{Ax} = \frac{P}{A} = \frac{25.81^k}{9(4)} = \boxed{0.7170 \text{ ksi}}$

3) Calculate the normal stress caused by the bending moment at point A. (~~3~~ 5 pts)

a. 27.6 ksi
b. -24.6 ksi
c. -25.6 ksi
d. 26.6 ksi

$\sigma_{bend} = \frac{My}{I} = \frac{-36.86(36)\left(\frac{9}{2}\right)}{243} = \boxed{-24.57 \text{ ksi}}$ (comp) (45 cos 35)

4) Calculate the transverse shear stress at point B (midheight of the cross-section). (~~3~~ 5 pts)

a. 1.54 ksi
b. 1.60 ksi
c. 1.70 ksi
d. 1.47 ksi

$\tau = \frac{VQ}{It} = \frac{36.86(4)\left(\frac{9}{2}\right)\left(\frac{4.5}{2}\right)}{243(4)} = \boxed{1.536 \text{ ksi}}$ (45 cos 35)

$= 1.5\frac{V}{A} = \frac{36.86}{9(4)}(1.5) = 1.536 \text{ ksi}$ ✓

5) Determine the combined normal stress at point A (3 pts)

$\sigma_A = -24.57 - 0.717 = \boxed{-25.29 \text{ ksi}}$

Normal Stress = - 25.3 ksi (C) means

23 ~~17~~ ~~18~~

Mohr's Circle (15 pts)

6) Draw Mohr's Circle for the Stress Element shown.
7) Show and give the stress values shown on the Stress Element (σ_x = 100 MPa, σ_y = 40 MPa, τ_{xy} = 50 MPa, signs shown on Stress Element),
8) Show and give the values of the 3 principal stresses,
9) Show and give their 3 angles of orientation,
10) Show and give the average normal stress (σ_{ave}),
11) Show and give the radius of Mohr's Circle (R)

$\sigma_x = +100 \quad \sigma_y = -40 \quad \tau_{xy} = 50$

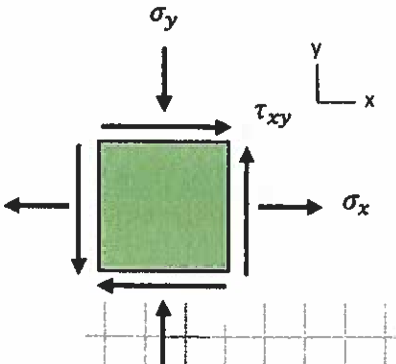
$C = \frac{\sigma_x + \sigma_y}{2} = \frac{100 + (-40)}{2} = \boxed{30}$

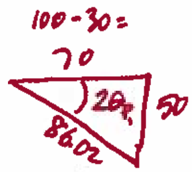
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{100 - -40}{2}\right)^2 + (50)^2} = \boxed{86.02}$

$\sigma_{max} = 30 + 86.02 = +116.0$ ksi

$\sigma_{MIN} = 30 - 86.02 = -56.02$ ksi

$\tau_{MAX} = 86.02$ ksi

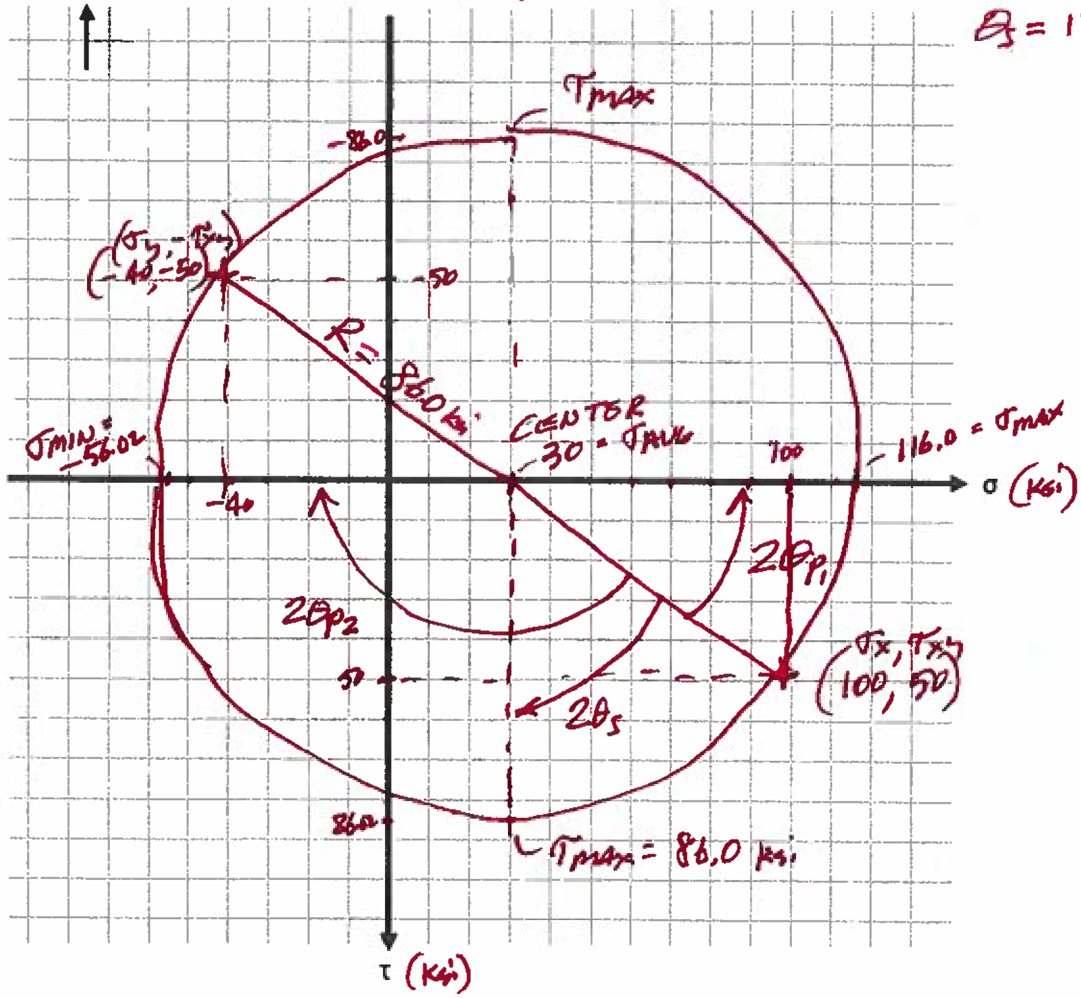




$2\theta_{p_1} = \tan^{-1}\frac{50}{70} = +35.54°$ $\quad \theta_{p_1} = +17.77°$ ccw

$\theta_{p_2} = 17.77 - 90 = -72.23°$ cw

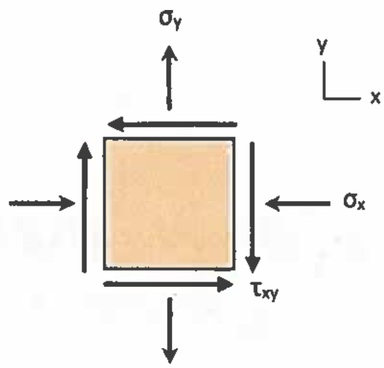
$\theta_s = 17.77 - 45 = -27.23°$



Stress Transformation

The magnitudes of the state of stress at a point on a member was recorded as σ_x = 140 MPa, σ_y = 70 MPa, and τ_{xy} = 30 MPa.

$\sigma_x = -140$ MPa
$\sigma_y = +70$ MPa
$\tau_{xy} = -30$ MPa



$$\sigma_{x'} = \frac{-140+70}{2} + \frac{-140-70}{2}\cos 2(-30) + (-30)\sin 2(-30)$$
$$(-35) + (-52.5) + (25.98)$$
$$= \boxed{-61.52 \text{ MPa } (C)} \quad -30°$$

12) Determine the normal stress acting in the x' direction if the element is oriented 30° clockwise from its original position. 3 (~~5~~ pts)

a. 59.1 MPa (C) c. 61.5 MPa (C)

b. 56.7 MPa (C) d. 63.9 MPa (C)

$$\sigma_{y'} = (-35) - (-52.5) - (25.98) = \boxed{-8.481 \text{ MPa } (C)}$$

13) Determine the normal stress acting in the y' direction if the element is oriented 30° clockwise from its original position. 3 (~~5~~ pts)

a. 9.17 MPa (T) c. 8.48 MPa (C)

b. 8.14 MPa (C) d. 8.82 MPa (T)

$$-90.93 \quad + \quad -15 \quad = \quad -105.9 \text{ MPa}$$
$$\tau_{x'y'} = -\frac{-140-70}{2}\sin 2(-30) + (-30)\cos 2(-30)$$

14) Determine the magnitude of the transformed in-plane shear stress if the element is oriented 30° clockwise from its original position. 3 (~~5~~ pts)

a. 114 MPa c. 110 MPa

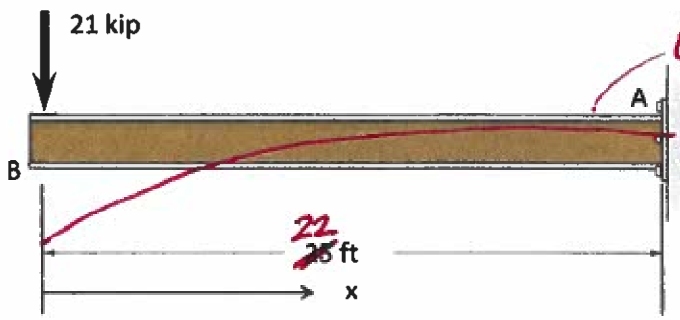
b. 102 MPa d. 106 MPa

9 15

Elastic Curve – Constants of Integration, Slope and Deflection

Given the beam loaded as shown with a concentrated Load at point B

$\theta = 0, \; v = 0$



The Moment Equation is: EI M(x) = -21x kip-ft

The Slope Equation is: EI θ(x) = $-10.5x^2 + C_1$ kip-ft^2

The Deflection Equation is: EI v(x) = $-3.5x^3 + C_1x + C_2$ kip-ft^3

$@x = 22, \; \theta = 0 = -10.5(22)^2 + C_1$

$C_1 = +5082$

$@x = 22, \; v = 0 = -3.5(22)^3 + 5082(22) + C_2$

$-37268 + 111804$

$C_2 = -74536$

Evaluate the Constants of Integration. (~~5~~ 4 pts each)

17) C_1 =

a. 5820 b. 5082

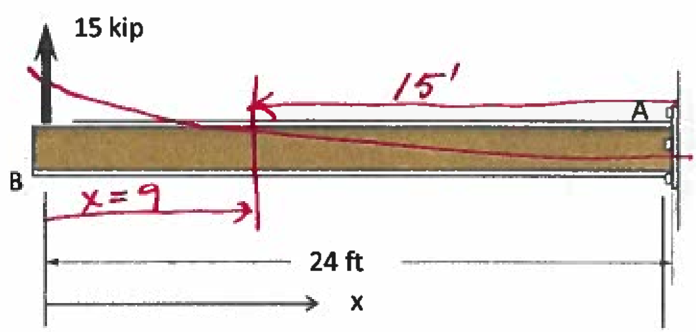
c. 0 d. -5280

18) C_2 =

a. -74536 b. -73654

c. -76543 d. -78635

Given the beam loaded as shown. E = 29,000 ksi, I = 1670 in.^{4}



The Deflection Equation is: EI v(x) = $2.5x^3 - 4320x + 69120$ kip-ft^3 $= 2.5(9)^3 - 4320(9) + 69120 =$

19) Calculate the deflection 15 ft left of point A. (~~10~~ 8 pts)

$= \dfrac{32062.5 \text{ k-ft}^3 \,(12^3 \text{ in}^3)}{(29000 \text{ k/in}^2)(1670 \text{ in}^4)}$

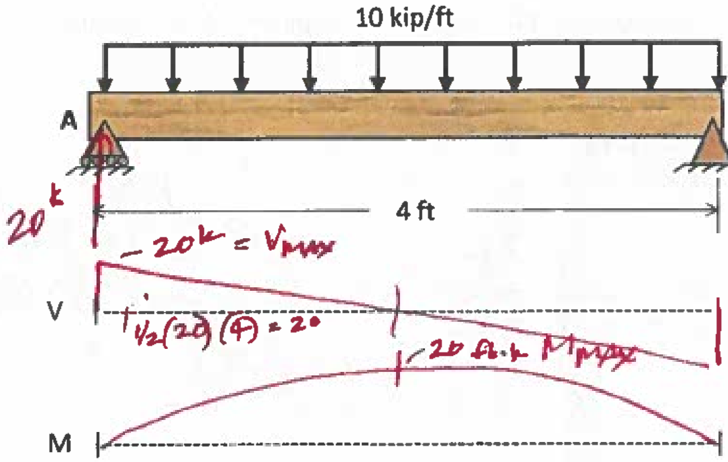
$= \boxed{+1.144 \text{ in}}$

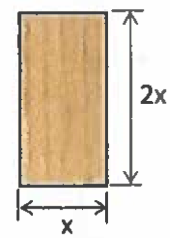
Deflection = +1.14 in ↑

16 20

Design of Beams

The simply supported beam is made of timber that has an allowable bending stress of σ_{allow} = 1.60 ksi and an allowable shear stress of τ_{allow} = 210 psi. (The architect wants the beam with proportions shown.) (Hint: Draw a Shear and Moment Diagram to find V_{max} and M_{max})





cross-section

$I = \frac{bh^3}{12}$ $S = \frac{I}{c}$

$S = \frac{bh^3/12}{h/2} = \frac{bh^2}{6}$

$= \frac{(x)(2x)^2}{6} = \frac{4}{6}x^3$

$= \frac{2}{3}x^3$

20) Calculate the required dimension, x, for bending stress. (~~9~~ 6 pts)

a. 6.32 in

b. 6.08 in

c. 6.58 in

d. ~~6.07~~ 5.84 in

$\sigma = \frac{M}{S}$ $S_{req'd} = \frac{M}{\sigma} = \frac{20(12)\ \text{k·in}}{1.60} = \boxed{150\ in^3}$

$= \frac{2}{3}x^3$ $x = \sqrt[3]{\frac{3}{2}(150)}$

$= \boxed{6.08\ in}$

21) Calculate the required dimension, x, for shear stress. (~~9~~ 6 pts)

a. 8.45 in

b. 8.79 in

c. 9.14 in

d. 8.11 in

rectangle $\tau = 1.5\frac{V}{A}$

$A = \frac{1.5V}{\tau} = \frac{1.5(20)\ k}{0.210\ k/in^2} = 142.86\ in^2$

$= 2x^2$ $x = \sqrt{\frac{142.86}{2}} = \boxed{8.452\ in}$

22) Rounding to the nearest inch, what dimension x should be used for the final beam design? (~~2~~ 3 pts)

a. 6.00 in

b. 7.00 in

c. 8.00 in

d. 9.00 in

9.0"

15 ~~18~~ 20

Column Buckling Load

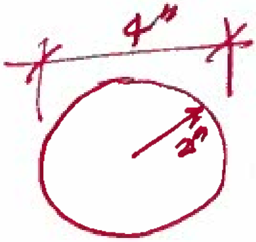
$I = \frac{\pi}{4} r^4 = \frac{\pi}{4}\left(\frac{4}{2}\right)^4 = 12.566 \text{ in}^4$

An A-36 steel rod with a 4 inch diameter (circular) cross section (I = ???? in.^{4}) is to be used as a column. (E_{st} = 29,000 ksi, σ_Y = 36 ksi)

23) Determine the maximum allowable axial load the column can support without buckling if the 15 foot long column is fixed at one end and pinned at the other. (~~10~~ 8 pts)

Fix-Pin $K = 0.7$

Maximum Axial Load = 227 kip



$$P_{cr} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2 \overset{k/in^2}{(29000)} \overset{in^4}{(12.566)}}{\left(0.7(15)(12)\right)^2_{in^2}}$$

$$\boxed{= 226.6 \text{ kip}}$$

24) Determine the maximum length of the column to support a 200 kip load without buckling. The column is pinned at both ends. (~~5~~ 4 pts) $K = 1.0$

Ⓐ ~~9.61~~ ft 11.2 Ⓒ ~~9.25~~ ft 12.0

Ⓑ ~~8.53~~ ft 11.6 Ⓓ ~~8.89~~ ft 10.8

$$(KL)^2 = \frac{\pi^2 E I}{P}$$

$$(1.0)^2 (L)^2 = \frac{\pi^2 \overset{k/in^2}{(29000)} \overset{in^4}{(12.566)}}{200 \text{ kip}} = (17984 \text{ in}^2)$$

$$L = \sqrt{} = 134.1 \text{ in}/12 = \boxed{11.18'}$$