UNIVERSITY OF HOUSTON
Cullen College of Engineering

Fundamentals of Engineering Spring 2005 Review

Mechanics of Materials

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problem distribution

- morning: 7% (8 out of 120 problems)
- afternoon: 8% (5 out of 60 problems)

subject areas

- 30% shear and bending stress (4)
- 23% shear & bending diagrams (3)
- 23% principal stresses (3)
- 8% axial stress and strain (1)
- 8% buckling (1) last
- 8% thermal stress (1)

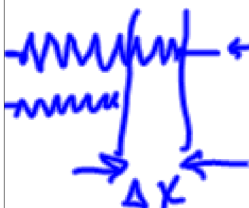
stress – has units of pressure
(solids pressure)

strain –
normalized deformation

axial stress and strain

deformation ∝ solids pressure

if linear, the elastic modulus is const. of propor.



$$\text{stress}\quad \boxed{\sigma = \frac{F}{A_0}}\qquad \text{strain}\quad \boxed{\varepsilon = \frac{\Delta L}{L_0}}\quad \boxed{\varepsilon_r = \frac{\Delta R}{R_0}}$$

force; area; change in length; initial length

Δx

$F = k\Delta x$

$\frac{F}{A} = \frac{k}{A}\Delta x$

$\sigma = \frac{k}{A}\Delta x \frac{L}{L}$

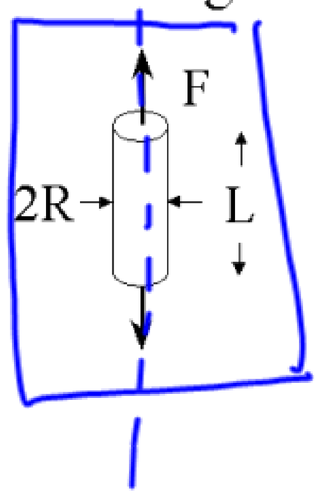
$\sigma = \underbrace{\frac{kL}{A}}_{E}\frac{\Delta x}{L}$

Tabulated

elastic modulus $E = \dfrac{\sigma}{\varepsilon}$

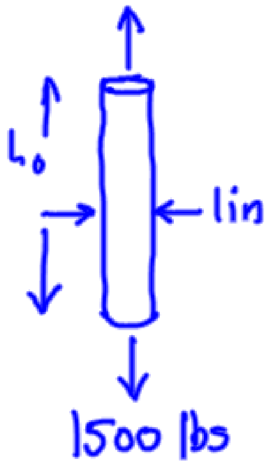
Poisson's ratio $\nu = -\dfrac{\varepsilon_r}{\varepsilon}$

Tabolated



2 ← used 2in. diameter in "numbers" below.

A ~~1~~ in. diameter aluminum rod becomes 0.007 in. longer due to 1500 lbf tensile load. What is the rod's initial length and change in radius?



$$\varepsilon = \frac{\Delta L}{L_o} \qquad \varepsilon_r = \frac{\Delta R}{R_o}$$

$$\Delta L = \varepsilon L_o$$

??

$$L_o = \frac{\Delta L}{\varepsilon}$$

$$L_o = \frac{0.007\,\text{in}}{4.77\cdot 10^{-5}} = 146\,\text{in}$$

$10\cdot 10^6$ psi (table)

$$E = \frac{\sigma}{\varepsilon}$$

$$E = \frac{\frac{F}{\pi R_o^2}}{\varepsilon}$$

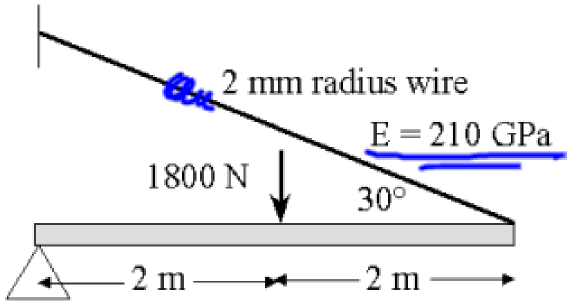
$\nu = 0.33$ (Table)

$$\varepsilon = \frac{F}{\pi R_o^2 E} = \frac{1500\,\text{lb}}{\pi (1\,\text{in})^2 \left(10\cdot 10^6 \frac{\text{lb}}{\text{in}^2}\right)} = 4.77\cdot 10^{-5}$$

$$\Delta R = -\underbrace{\frac{\varepsilon_r}{\varepsilon}}_{\nu}\cdot \varepsilon \cdot R_o$$

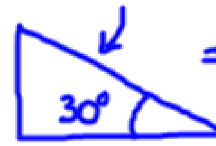
$$= -0.33\,(4.77\cdot 10^{-5})\,1\,\text{in} = -1.57\cdot 10^{-5}\,\text{in}$$

33. Determine the change in length of the steel wire.



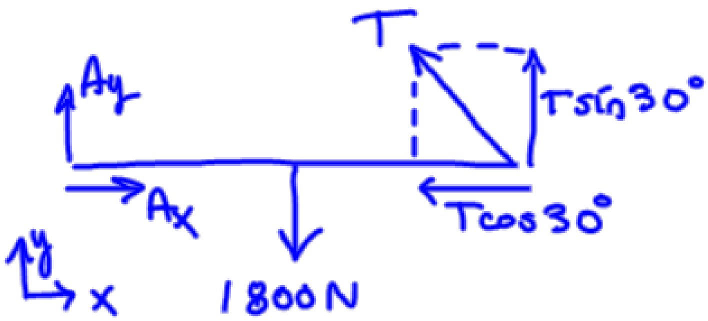
$210\cdot10^{9}\,\text{Pa}$

$L_o = \dfrac{4}{\cos 30^\circ} = 4.618\,\text{m}$



Geometry

① FBD



② Find forces

$$\circlearrowleft M_A = -1800(2) + T\sin 30(4)$$

$$T = \frac{1800(2)}{\sin 30(4)} = 1800\,\text{N}$$

③ $$\Delta L = L_o\,\varepsilon = L_o\frac{\sigma}{E} = \frac{L_o T}{E A_o} = \frac{(4.618\,\text{m})(1800\,\text{N})}{210\cdot10^{9}\frac{\text{N}}{\text{m}^2}\,\pi(0.002\,\text{m})^2}$$

$$= 3.149\cdot10^{-3}\,\text{m}$$

materials expand / contract with Δtemp.
- deformation is thermal strain.
- if free to deform, then no stress

thermal strain

change in temperature

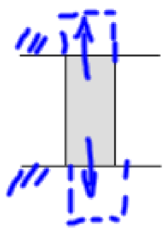
$$\varepsilon_T = \alpha(\underline{\underline{T - T_0}})$$

linear driving force model

coefficient of thermal expansion

change in length $\underline{\underline{\delta_T}} = \alpha L \underbrace{(T - T_0)}_{\Delta T}$

101. A steel rod (E = 210 GPa, $\alpha = 11.7\times10^{-6}\ {}^{\circ}C^{-1}$) is constrained and heated. What is the stress induced by a 40°C increase in temperature?



① Virtual work
② Combine

① Virtual work

(i) let expand

$$\Delta L = \alpha L_0 (T - T_0)$$

$$\frac{\Delta L}{L_0} = \underline{\underline{\varepsilon_T}} = \alpha (T - T_0)$$

(ii) Apply axial stress equivalent to the thermal deformation

$$\varepsilon_T E = \underline{\underline{\sigma}} = \frac{F}{A_0}$$

$$\sigma = \alpha (T - T_0) E = (11.7\cdot10^{-6}/{}^{\circ}C)(40^{\circ})(210\cdot10^{9}\,N/m^2)$$

$$= 98.2\cdot10^{3}\ kN/m^2 = 98.2\cdot10^{3}\ \underline{\underline{kPa}}$$

$$= 98.2\cdot10^{6}\ Pa$$

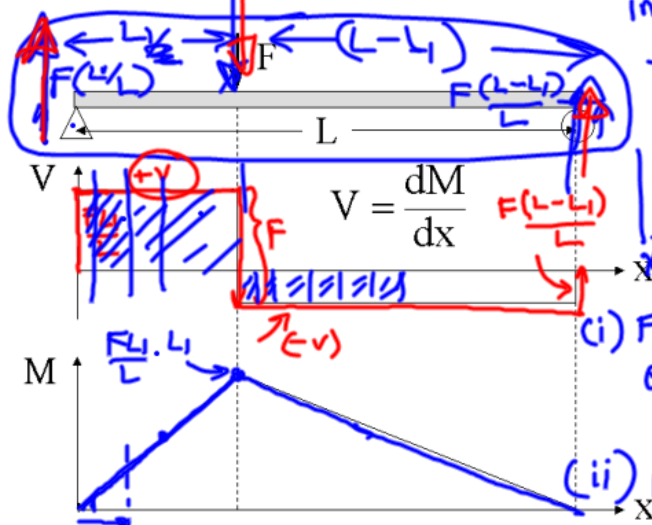
Beams internal effects
resisting
bending
shear
torsion

bending

shear

torsion

shear and bending diagrams



in-plane
- V, M only

action/reaction
V, M, T be equal & opposite at internal surface

(i) Find all external reactions

(ii) Isolate section equilibrium V & M

(iii) Move section bndry to find V & M at new location

(iv) Point loads & couples discontinuous change in V, M

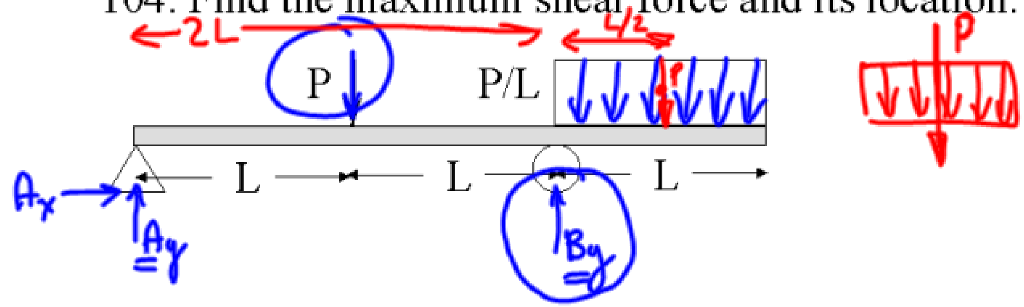
(v) distributed loads

$$\frac{d}{dx}\left(\frac{dM}{dx}\right) = -w(x)$$

$$\frac{dM}{dx} = \boxed{V = \int -w(x)\,dx} = -w(x)\cdot x + C$$

$$\boxed{M = \int V\,dx} = -\frac{w(x)\,x^2}{2} + Cx + D$$

104. Find the maximum shear force and its location.

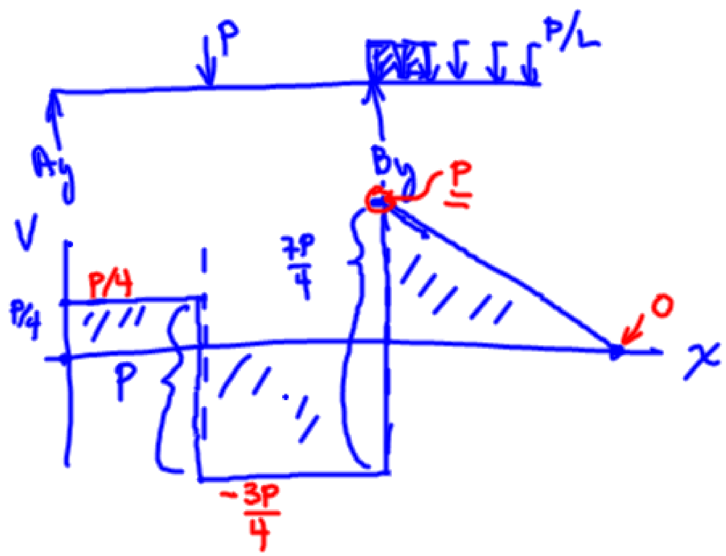


$$\sum M_A = 0 = -PL + 2B_yL - P\frac{5L}{2}$$

$$B_y = \frac{\frac{2PL}{2} + \frac{5PL}{2}}{2L} = \frac{7P}{4}$$

$$\Sigma F_y = 0 = A_y - 2P + B_y$$

$$A_y = 2P - \frac{7P}{4} = P/4$$

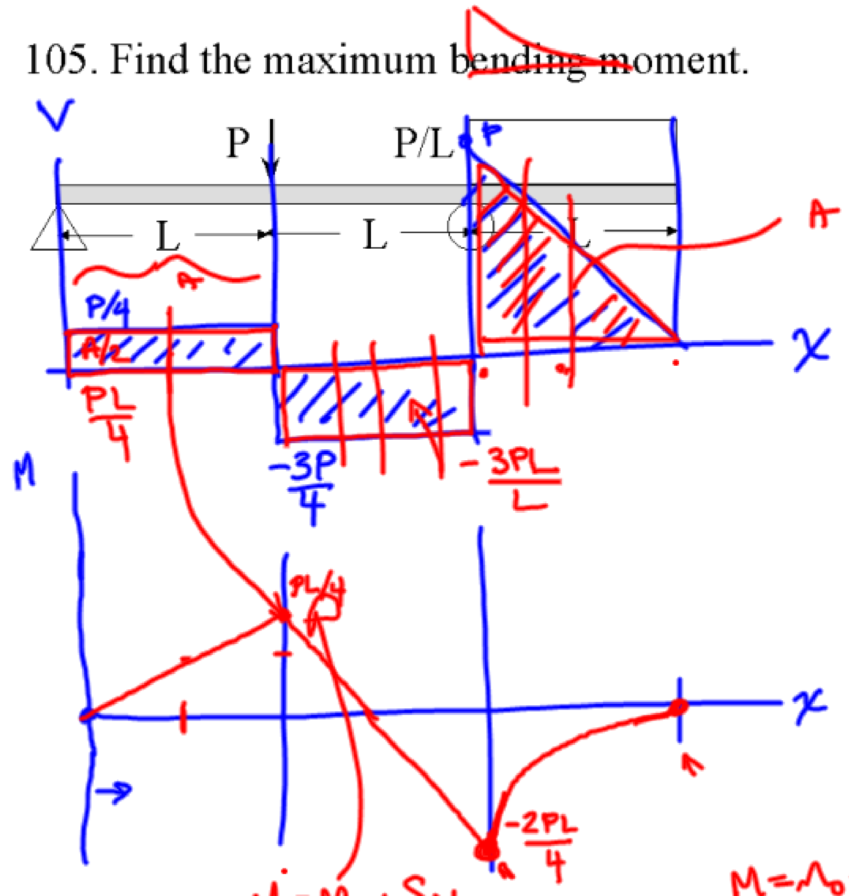


$$V = V_0 - \int w(x)\,dx$$

$V_{max} = P$

~~locate~~ located @ $x = 2L$ (2^{ND} support)

105. Find the maximum bending moment.



$$A = \frac{PL}{2} = \frac{2PL}{4}$$

$$M' = M_0 + SV$$

$$M = \frac{PL}{4} + \left(-\frac{3PL}{4}\right) = \frac{-2PL}{4}$$

$$M = M_0 + SV = -2PL/4 + \frac{2PL}{4}$$

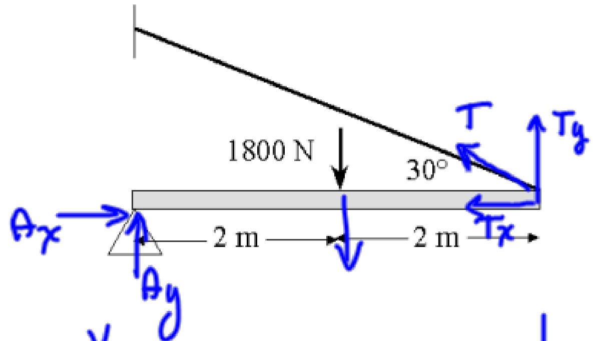
$|M'_{max}| = \frac{2PL}{4} = \frac{PL}{2}$ occurs at $x = 2L$ (2nd support)

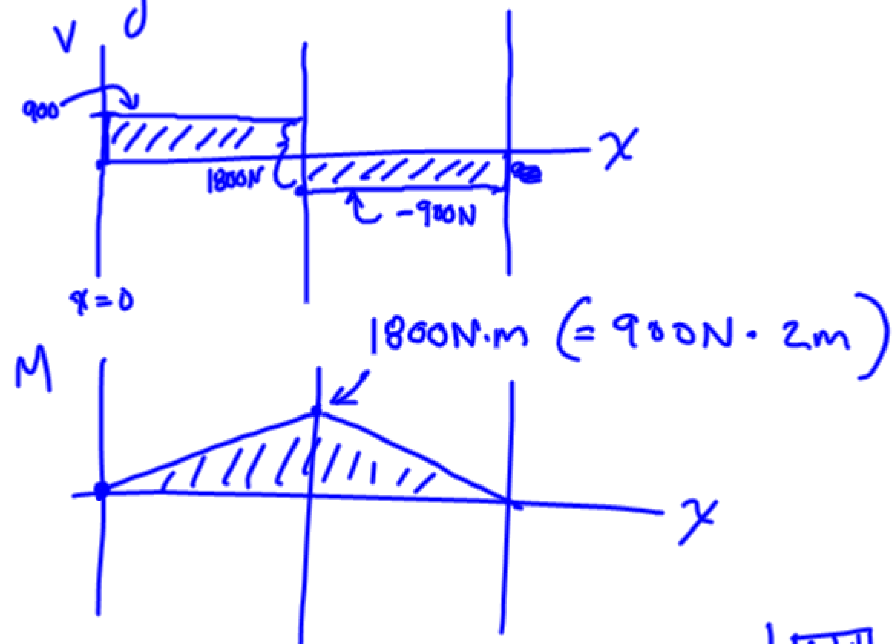
37. Sketch the shear and bending moment diagrams.

$\Sigma F_y = 0 = A_y + T_y - 1800$

$1800 = A_y + T_y$

$A_y = T_y = 900\,N$

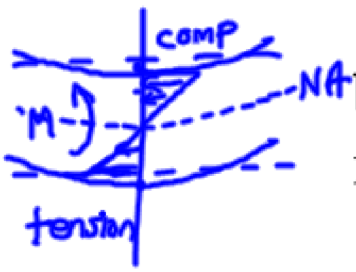




internal stress
in beam ~~sectn~~
section

relative to
neutral axis

bending stress



bending moment

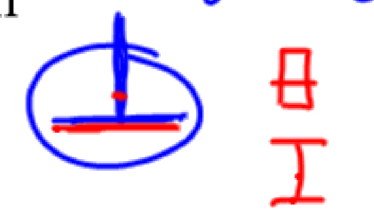
bending formula

$$\boxed{\sigma = -\frac{My}{I}} \;*$$

position

2nd moment of area

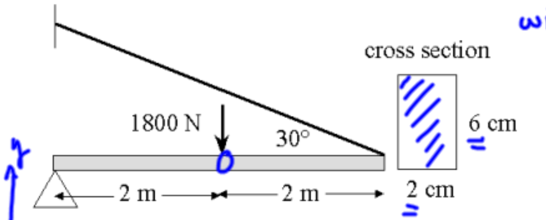
changes with
section geometry



- "y" is measured from the neutral surface (centroid)
- "I" is calculated about the neutral surface (centroid)

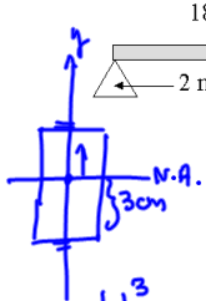
$I = I_c + Ad^2$ distance from
N.A. to centroid of I_c

34. Determine the maximum bending stress.



will occur where M is max.

$$\underline{\sigma} = \frac{-\underset{\text{load}}{M}\,\overset{\text{geometric}}{y}}{\underset{\text{geometric}}{I}}$$



$$I_c = \frac{bh^3}{12} = \frac{2(6)^3}{12} = \frac{1}{\cancel{6}}\cdot 6^{\cancel{3}\,2} = 36\,cm^4$$

$$\sigma(y) = -\frac{1800\,N\cdot m}{36\,cm^4}\,y$$

$$= \frac{-1800\,N\cdot 100\cancel{m}\,cm\cdot y}{36\,cm^4}$$

$$= \frac{-1800\,N\cdot 100\,cm\cdot(3\,cm)}{36\,cm^4}$$

$$= \frac{-\cancel{6}\cdot\cancel{3}\cdot 100\,N\cdot 100\,cm\;3\,cm}{2\;\cancel{6}\cdot\cancel{6}\,cm^4}$$

$$= \frac{30{,}000\,N\,cm^2}{2\,cm^4}$$

$$= \frac{15{,}000\,N}{cm^2}$$

convert to "m"

shear stress
~~fot~~ integral of bending stress

$$\tau_x(y) = \int \sigma_x(y)\,dy$$

shear stress

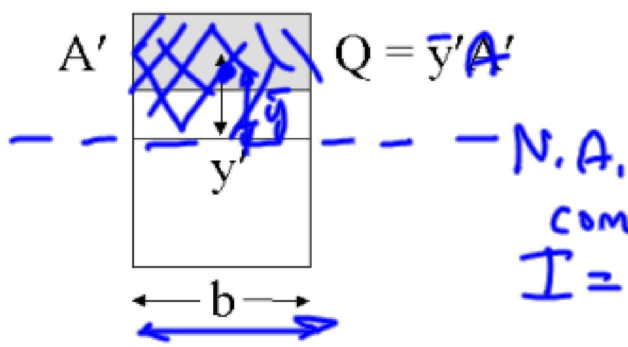
shear force

shear formula $\tau = \frac{VQ}{Ib}$ ← 1st moment of area

w

2nd moment of area

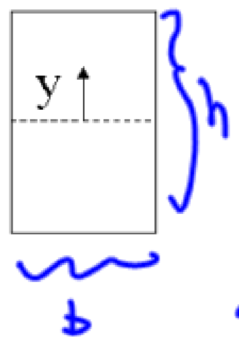
$\bar{y}A$ (~~sect~~ area above N.A.)



$$\tau = \frac{V \cdot \bar{y}A}{I \quad b}$$

composite
$$I = I_c + Ad^2$$

102. What is the largest shear stress in a beam with a rectangular cross section?

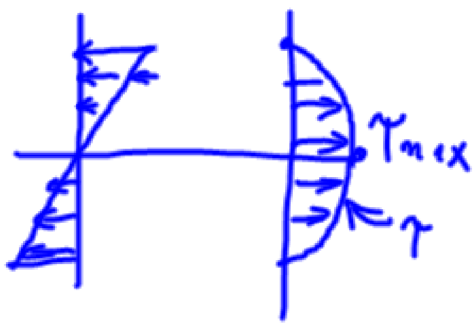


$$\tau = \frac{VQ}{Ib}$$

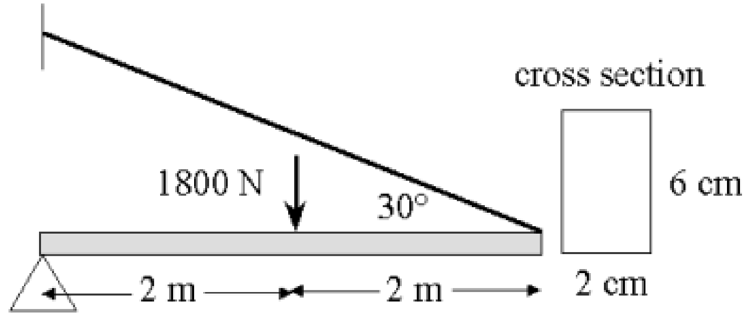
$$Q = \frac{h}{4}\,\frac{bh}{2} = \frac{bh^2}{8}$$

$$I = \frac{bh^3}{12} \leftarrow \text{Table}$$

$$\tau = \frac{V\,\frac{bh^2}{8}}{\frac{bh^3}{12}\cdot b} = \frac{3}{2}\,\frac{V}{bh}$$



35. Determine the maximum shear stress.



$$\tau = \frac{VQ}{Ib}$$

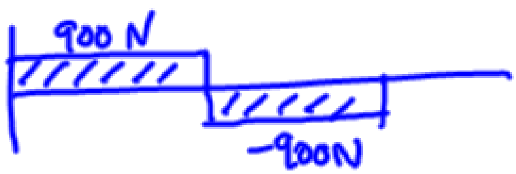
"load"

geometric

geometric

geometric

τ_{max} occurs at V_{max}



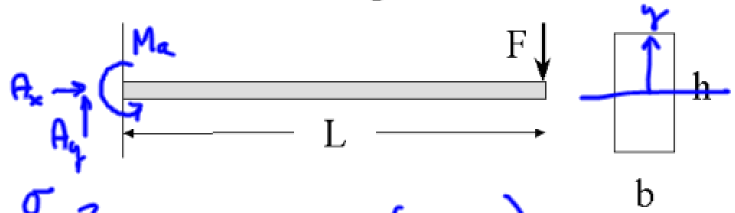
$$\tau = \frac{3}{2}\frac{V}{bh}$$

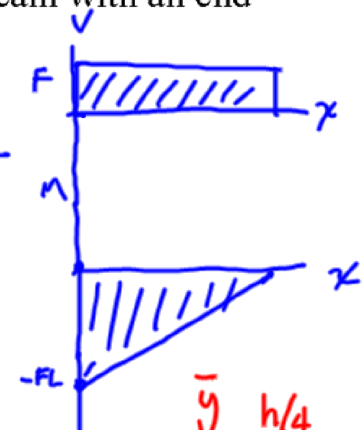
$$= \frac{3}{2}\frac{900N}{(0.02m)(0.06m)}$$

$$= 1.125 \cdot 10^6 \, N/m^2$$

1.125 MPa

103. What is the ratio of maximum bending stress to the maximum shear stress in a cantilever beam with an end load and a rectangular cross section?





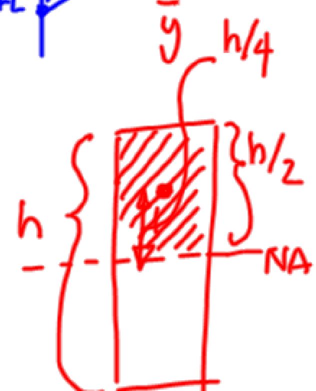
$$\frac{\sigma}{\tau} = ?$$

$$A_y = F \quad (\Sigma F_y = 0)$$

$$\underbrace{M_A = -FL}_{\text{Reactions}}$$

$$\sigma = -\frac{My}{I} = \frac{-(-FL)\,h/2}{\frac{bh^3}{12}} = \frac{6FL}{bh^2}$$

$$\tau = \frac{VQ}{Ib} = \frac{F(h/4)(bh/2)}{\frac{bh^3}{12}\cdot b} = \frac{F\ 12}{8\ bh}$$

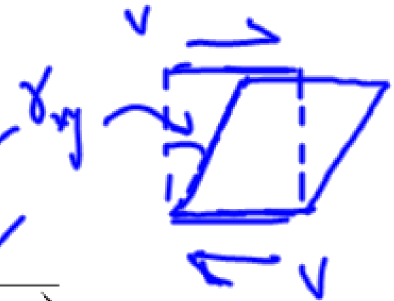


$A = b(h/2)$

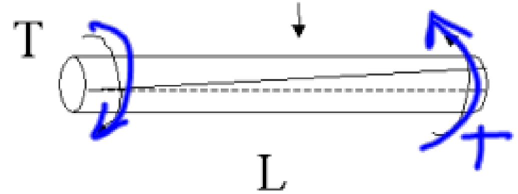
$$\bar{y}A = \boxed{\left(\frac{h}{4}\right)\left(\frac{bh}{2}\right)}$$

$$\frac{\sigma}{\tau} = \frac{6\cancel{F}L}{\cancel{bh^2}\,h}\cdot\frac{8\cancel{bh}}{12\cancel{F}} = \frac{48\,L}{12\,h} = \frac{4L}{h}$$

torsion



shear modulus $\boxed{G} = \frac{\tau}{\gamma} = \frac{E}{2(1+\nu)}$



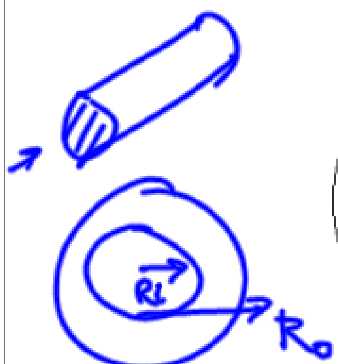
torsion formula $\tau = \frac{Tr}{J}$ ← polar moment of area

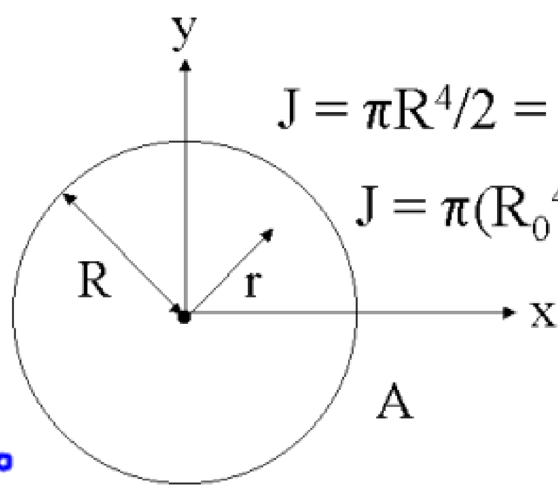
angle of twist $\boxed{\phi = \frac{TL}{JG}}$ ← material property

Moment of Inertia in polar coordinates

polar moment of area

$$J = \int r^2 dA = \int \left(x^2 + y^2\right) dA = I_x + I_y$$





$J = \pi R^4/2 = \pi D^4/32$ (solid)

$J = \pi(R_0{}^4 - R_i{}^4)/2$ (tube)

10.16 A 10 cm diameter shaft can tolerate up to a 140 MPa shear stress. What is the maximum torque (N·m)?

$$\tau = \frac{Tr}{J} \qquad \tau \le 140\text{ MPa}$$

$= 140\text{ MPa}$ 0.1m

$$T = \frac{\tau J}{r} = \frac{(140 \cdot 10^6\text{ Pa})(\pi D^4/32)}{r}$$

0.05m

$$= 140 \cdot 10^6 \frac{N}{m^2} \cdot \frac{\pi\ (0.1)^4}{32\ (0.05)} = 2.75 \cdot 10^4\ N \cdot m$$

principal stresses

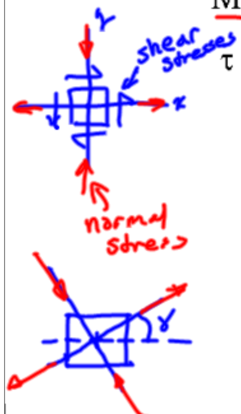
combined stresses

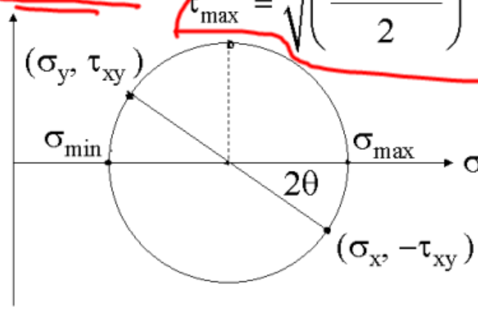
bending
torsion
axial load

$$\boxed{\sigma_{\substack{max\\min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}} \; * \; ✓$$

Mohr's circle

$$\boxed{\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}} \; ✓$$





plot on τ, σ plane

(σ_y, τ_{xy})

$(\sigma_x, -\tau_{xy})$

"draw" circle that contains the two points

– radius of that circle

τ_{max}, σ_{max}, σ_{min} by inspection

100. Determine the maximum shear and normal stresses given σ_x = 6 MPa, σ_y = 0 and τ_{xy} = 4 MPa.

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{6+0}{2} \pm \sqrt{\left(\frac{6}{2}\right)^2 + 4^2} = 3+5 = 8$$

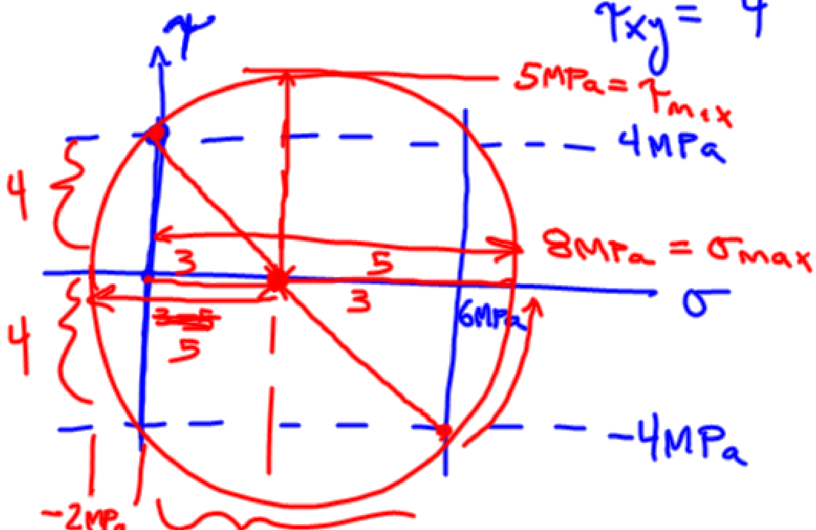
$\sigma_{max} = 8\,\text{mpa}$

$\sigma_{min} = 3-5 = -2\,\text{MPa}$

$\sigma_x = 6$

$\sigma_y = \cancel{3}\ 0$

$\tau_{xy} = 4$



$D = 10$

$R = D/2 = 5$

Determine the plane of normal stress in torsion.

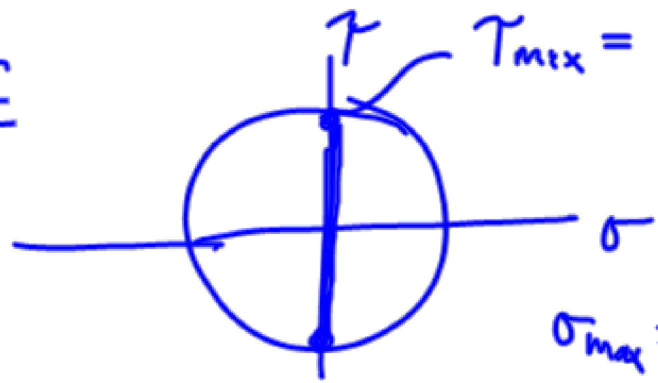
torsion only

$\sigma_x = \frac{F}{A} = 0$ (No axial force)

$\sigma_y = \frac{My}{I} = 0$ (No bending

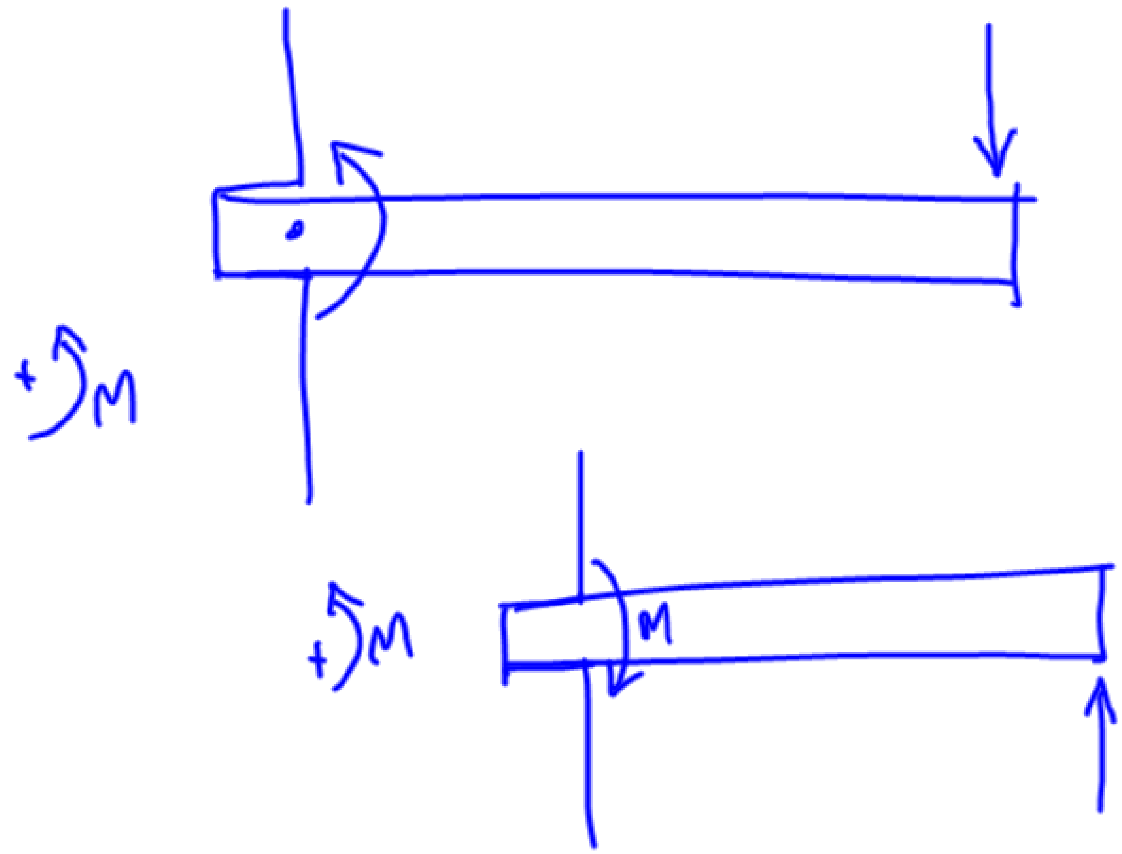
$\tau_{xy} = \pm \frac{Tr}{J}$

$\tau_{max} = \frac{Tr}{J}$

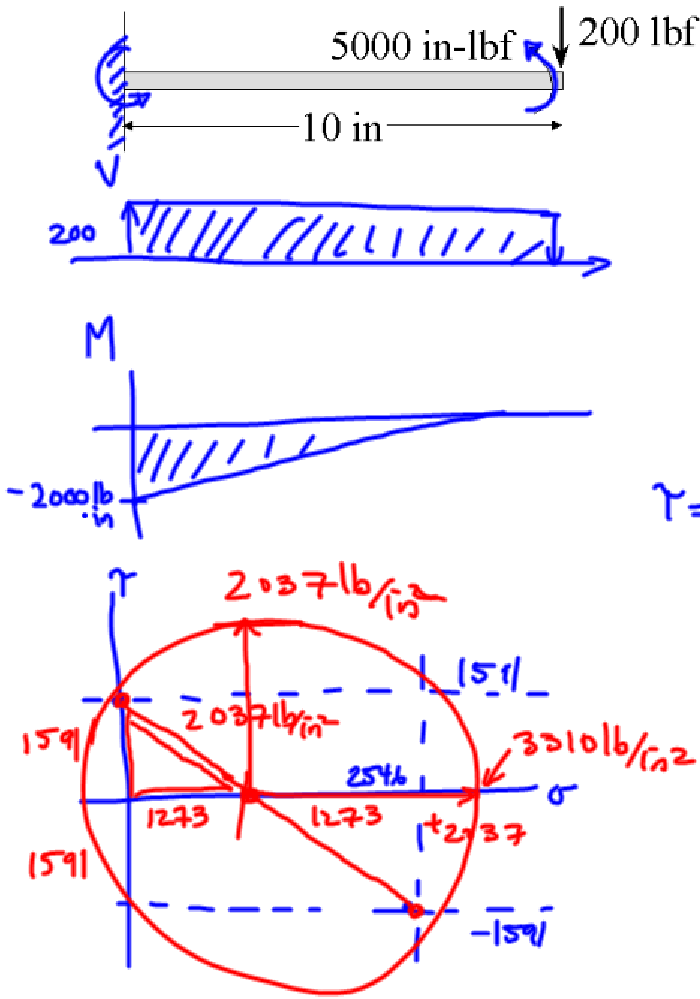


$\sigma_{max} = \sqrt{\tau_{xy}^2}$

$\sigma_{min} = -\sqrt{\tau_{xy}^2}$


+M
M
+M

107. What is the maximum shearing stress at the fixed end of the 2 in. diameter shaft?



$$\sigma_x = \frac{My}{I} = \frac{(2000\ lb\cdot in)(1 in)}{\frac{\pi (1 in)^4}{4}} = \frac{4(2000)(1)}{\pi} = 2546\ \frac{lb}{in^2}$$

$$\tau = \frac{Tr}{J} = \frac{5000\ lb\cdot in\ (1 in)}{\frac{\pi (2 in)^2}{32}} = 1591\ \frac{lb}{in^2}$$

elastic buckling

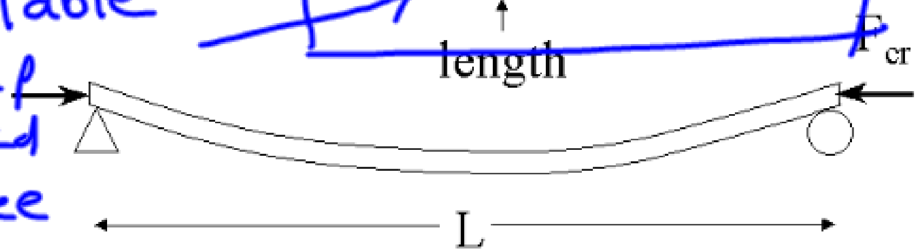
elastic modulus

critical load $\boxed{F_{cr} = \frac{\pi^2 EI}{k^2 L^2}}$ ← 2nd moment of area

length

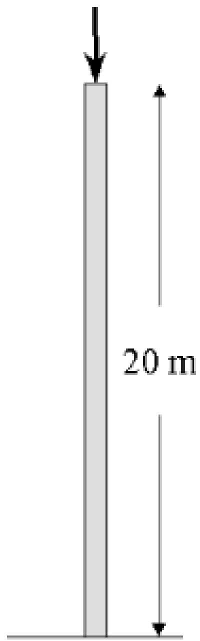
Table

fixed-fixed
pin-fixed
fixed-free



Use “2L” in place of “L” for a cantilever beam.

106. A 20 m flag pole is made of a 6 cm diameter steel (E = 210 GPa). What is the maximum axial load, using 2 as a factor of safety.



$$P_{cr} = \frac{\pi^2 E I}{k^2 L^2}$$

$$L = \frac{P_{cr}}{2}$$

$$\text{Load} = \frac{\pi^2 (210 \cdot 10^9 \text{ Pa})(0.03^4/4)\pi}{2 \cancel{1/2} \; 2^2 \cdot (20\text{m})^2}$$

"table" fixed-free

$k=2$

$= 412\text{N}$