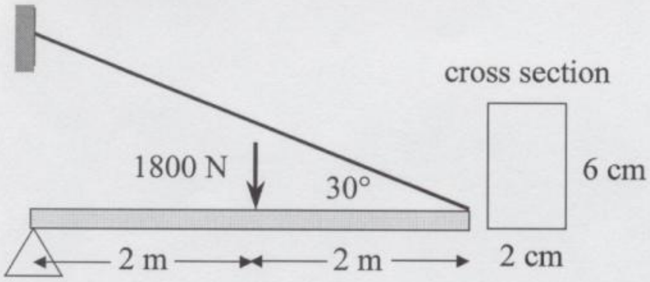
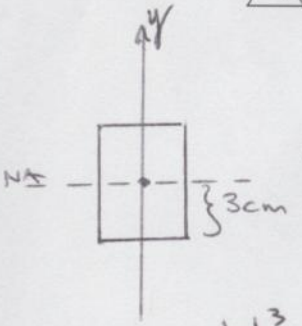
MECH-MATLS

34. Determine the maximum bending stress.



should occur at maximum moment value
(located where 1800N point load is applied)



$$\sigma(y) = -\frac{My}{I}$$

$$I_c = \frac{bh^3}{12} = \frac{2(6)^3}{12} = \frac{1}{6}\cdot 6^3 = 36\,cm^4$$

$$\sigma(y) = \frac{-1800\,N\cdot m}{36\,cm^4}\,y$$

$$= \frac{-1800\,N\cdot 1000\,cm}{36\,cm^4}\,y$$

$$= \frac{-1800\,N\cdot 1000\,cm\,(3\,cm)}{6\cdot 6\,cm^4}$$

$$= \frac{300{,}000\,N\,cm^2}{2\,cm^4}$$

$$= \frac{150\,kN}{cm^2}$$

shear stress is integral of bending stress

$$\tau_x(y) = \int \sigma_x(y)\,dy$$

shear stress

shear force

shear formula $\tau = \frac{VQ}{Ib_t}$ ← 1st moment of area

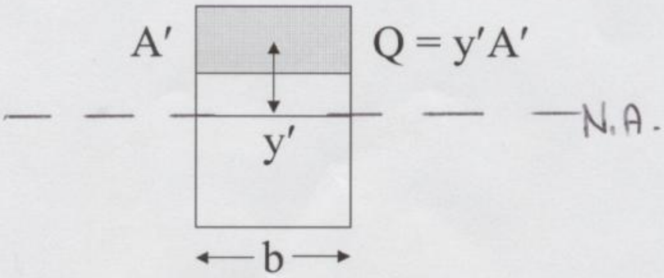
t section width

2nd moment of area

– Composite sections use parallel axis theorem

$$I = I_c + Ad^2$$

all measures relative to N.A.



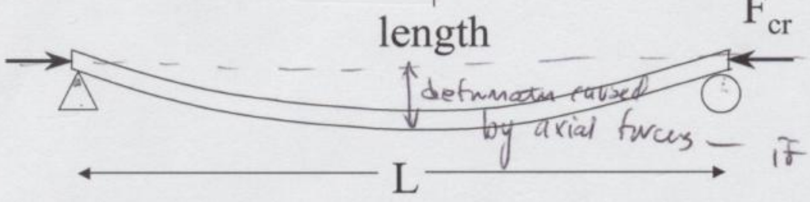
1

elastic buckling

elastic modulus

$$\text{critical load} \quad F_{cr} = \frac{\pi^2 EI}{L^2} \leftarrow \text{2}^{\text{nd}} \text{ moment of area}$$

length

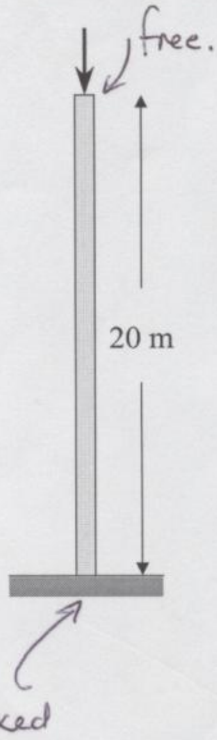


— if deformation is large enough, and forces stay constant axial beam will bend.

Use "2L" in place of "L" for a cantilever beam.

"k" in buckling formular.

106. A 20 m flag pole is made of a 6 cm diameter steel ← assume (E = 210 GPa). What is the maximum axial load, using 2 as a factor of safety.



$$P_{cr} = \frac{\pi^2 EI}{k^2 L^2}$$

Safety factor of 2 means we allow

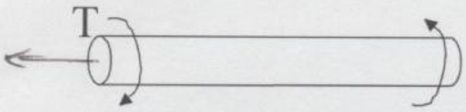
$$\text{Load} = \frac{P_{cr}}{2} = \frac{1}{2} \frac{\pi^2 EI}{k^2 L^2}$$

$$= \frac{\pi^2 (210 \cdot 10^9 \text{Pa})(0.03^4/4)\pi}{2 \cdot 2^2 \cdot 20 \text{m}^2}$$

$$= 412 \text{ N}$$

7

Determine the plane of normal stress in torsion.

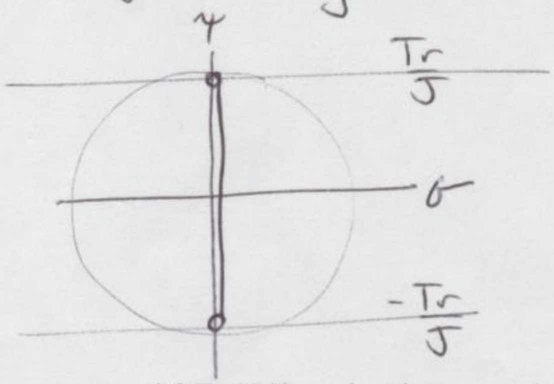


torsion only

$\sigma_x = \frac{F}{A} = 0$, No axial force

$\sigma_y = \frac{My}{I} = 0$, No bending moment

$\tau_{xy} = \pm \frac{Tr}{J}$

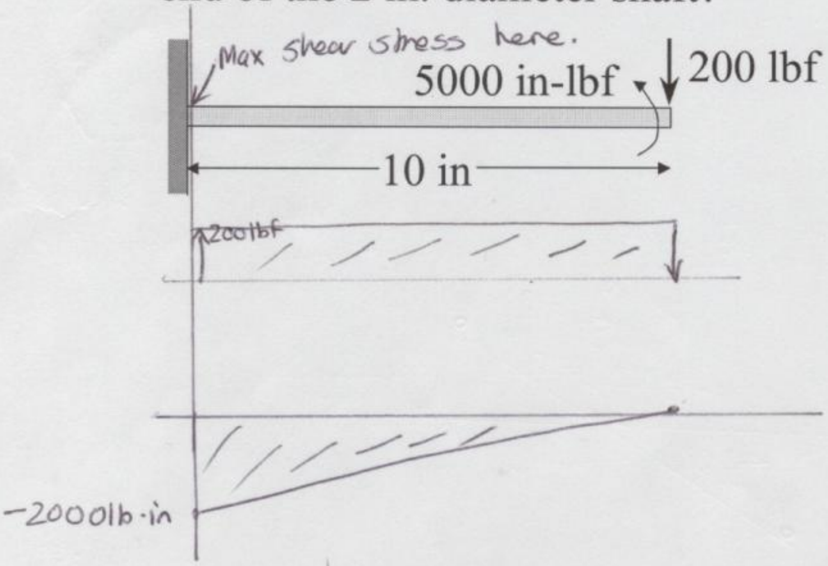


$\sigma_{max} = \sqrt{\tau_{xy}^2} = \frac{Tr}{J}$

$\sigma_{min} = -\sqrt{\tau_{xy}^2} = -\frac{Tr}{J}$

$\tau_{max} = \sqrt{\tau_{xy}^2} = \frac{Tr}{J}$

107. What is the maximum shearing stress at the fixed end of the 2 in. diameter shaft?



$\sigma_x = \frac{My}{I}$

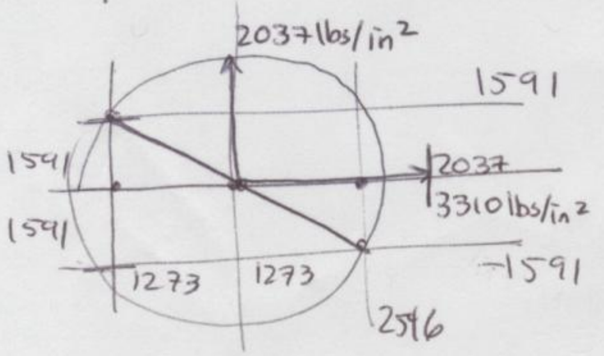
$= \frac{(2000 \text{ lb in})(1 \text{ in})}{\frac{\pi (1 \text{ in})^4}{4}} = \frac{4(2000)(1)}{\pi} =$

$= 2546.5 \ \frac{\text{lb in}^2}{\text{in}^4}$

$\tau = \frac{Tr}{J} = \frac{5000 \text{ in lb } (1 \text{ in})}{\pi (2 \text{ in})^4 / 32}$

$= \frac{32(5000)}{\pi (\cancel{32}\ 2\ \text{in}^4)} = \frac{5000}{\pi}$

$= 1591.5 \ \frac{\text{in}^2 \text{lbs}}{\text{in}^4}$



$d^2 = 1273^2 + 1591^2$

$d = 2037$

6

principal stresses

Used to [illegible]ate combined stresses

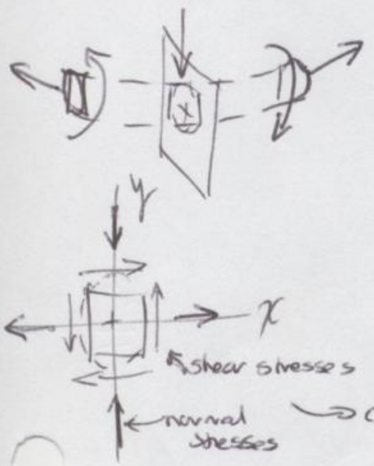
i.e.
bending, torsion &
compression/tension

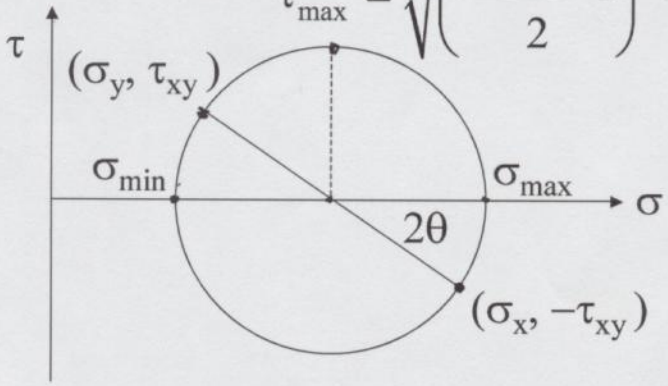
$$\sigma_{\substack{max \\ min}} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$

plot on
τ, σ plane
a ~~circle.~~ two ordered pairs
σ_y, τ_{xy}
σ_x, $-\tau_{xy}$
draw circle that contains these two points
τ_{max}
$\sigma_{min/max}$
by inspection

Mohr's circle

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\ 2}}$$



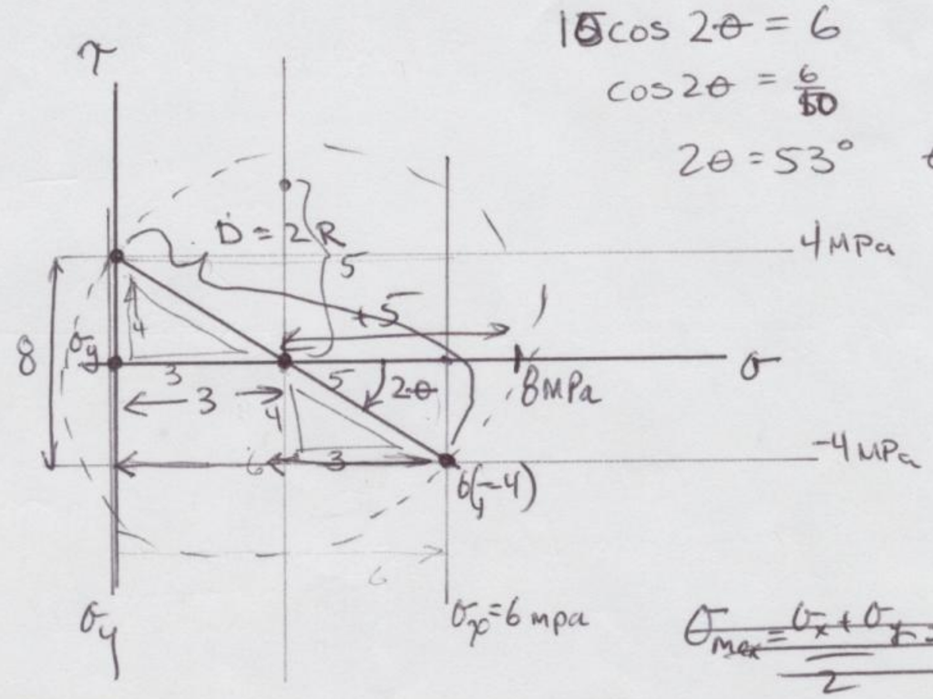


→ coordinate rotation (principal axis of stress)
normal stress is at max, min, shear "vanish")

100. Determine the maximum shear and normal stresses given $\sigma_x = 6$ MPa, $\sigma_y = 0$ and $\tau_{xy} = 4$ MPa.

$10 \cos 2\theta = 6$
$\cos 2\theta = \frac{6}{10}$
$2\theta = 53°$ $\quad \theta = 26.5°$

Mohr's circle is graphical solution to $\sigma_{max/min}$ τ_{max} equations.



$3 + \sqrt{9+16}$
$3 + \sqrt{25}$
$= 3 + 5 = 8$

~~$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} = \frac{6+0}{2} = 3$~~

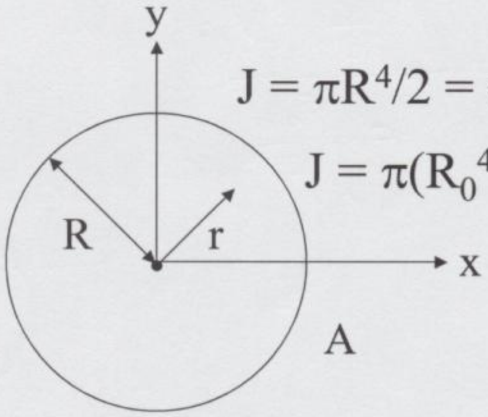
$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{2}}$$
$$= \frac{6+0}{2} \pm \sqrt{\left(\frac{6-0}{2}\right)^2 + 4^2}$$

$D^2 = 8^2 + 6^2$
$64 + 36 = 100$
$D = \sqrt{100} = 10 \quad R = 5$

Moment of inertia
polar coordinates
- used with cylinders

polar moment of area

$$J = \int r^2 dA = \int \left(x^2 + y^2\right) dA = I_x + I_y$$

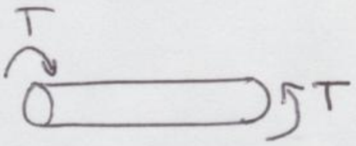


$J = \pi R^4/2 = \pi D^4/32$ (solid)

$J = \pi(R_0{}^4 - R_i{}^4)/2$ (tube)

$\tau = \dfrac{Tr}{J}$ torsonical shear stress

T= torque (couple causing moment)



10.16 A 10 cm diameter <u>shaft</u> can tolerate up to a 140 MPa shear stress. What is the maximum torque (N·m)?

$\tau = \dfrac{Tr}{J}$ $\tau \leq 140\,\text{MPa}$ solve for T

$\dfrac{\tau J}{r} = T$

0.10m

$\dfrac{(140 \cdot 10^6 \text{Pa})(\pi D^4/32)}{\cancel{0.05m}\ r} = 140 \cdot 10^6 \dfrac{N}{m^2} \cdot \dfrac{\pi\ (0.1)^4}{32\ (0.05)}$

0.05m

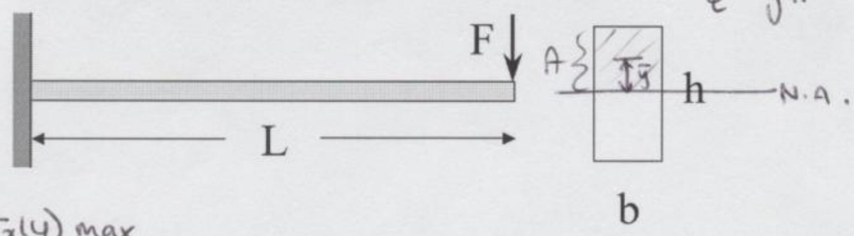
$= 2.748 \cdot 10^4$ N·m

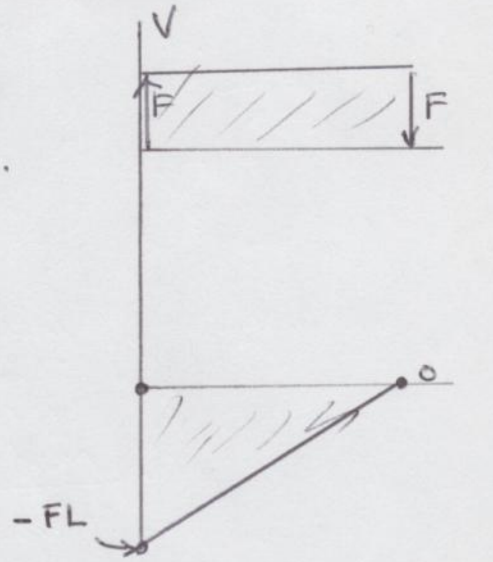
27.4 kN·m

4

103. What is the ratio of maximum bending stress to the maximum shear stress in a cantilever beam with an end load and a rectangular cross section?

$Q = \bar{y}A$





$$\frac{\text{Max bending } \sigma_x(y)_{max}}{\text{Max shear } \tau_x(y)_{max}}$$

$$\sigma = -\frac{My}{I} = \frac{-(FL)h/2}{\frac{bh^3}{12}} = +\frac{FLh}{bh^3}\frac{\cancel{12}}{\cancel{2}} = +\frac{6FL}{bh^2}$$

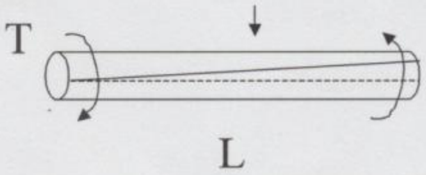
$$\tau = \frac{VQ}{Ib} = \frac{F(h/4)(bh/2)}{\frac{bh^3}{12}\cdot b} = \frac{F\cancel{b}h^{\cancel{2}}}{bh^{\cancel{3}}\cdot\cancel{b}}\frac{12}{8} = \frac{F\,12}{8bh}$$

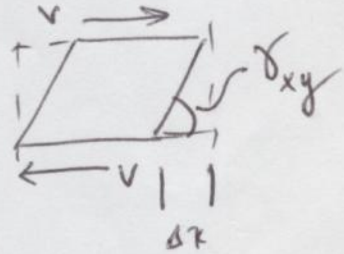
$$\frac{\sigma}{\tau} = \frac{+\cancel{6}\cancel{F}L}{\cancel{b}h^{\cancel{2}}}\cdot\frac{8\cancel{b}\cancel{h}}{\cancel{12}_{2}\,\cancel{F}} = \frac{4L}{h}$$

torsion

shear modulus $G = \frac{\tau}{\gamma} = \frac{E}{2(1+\nu)}$

represents deformation of material.





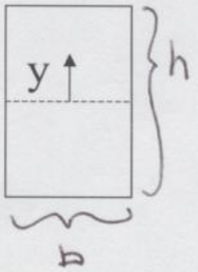
torsion formula $\tau = \frac{Tr}{J}$ ← polar moment of area

angle of twist $\phi = \frac{TL}{JG}$ ← Material property

Polar moment of area

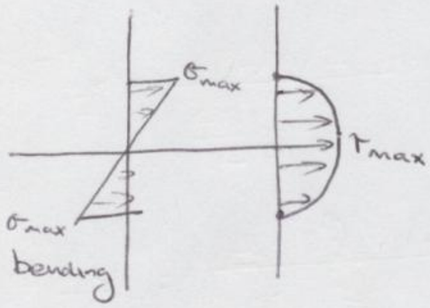
3

102. What is the largest shear stress in a beam with a rectangular cross section?

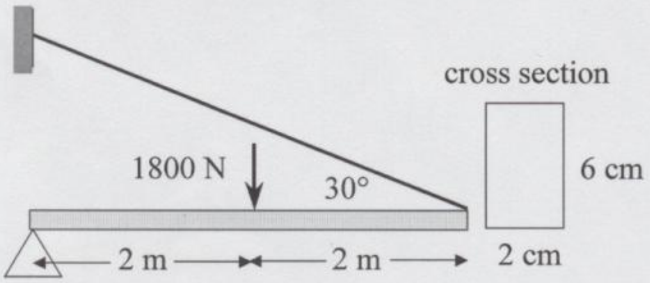


$$\tau = \frac{VQ}{Ib} \qquad Q = \frac{h}{4}\,\frac{bh}{2} = \frac{bh^2}{8} \qquad I = \frac{bh^3}{12}$$

$$\tau = \frac{V\,\frac{bh^2}{8}}{\frac{bh^3}{12}\cdot b} = \frac{V\,bh^2}{b^2h^3}\cdot\frac{12}{8} = \frac{3}{2}\,\frac{V}{bh}$$



35. Determine the maximum shear stress.



Q · geometric property
I geometric property
-uniform section

τ_{max} must occur at V_{max}, 900 (or -900)

$$\tau = \frac{3V}{2bh} = \frac{3}{2}\,\frac{900\,N}{0.06\,m\,(0.02)} = \cancel{22500}\ 1{,}125{,}000\ N/m^2$$

1.125 MPa

2