MECH-MATLS

A

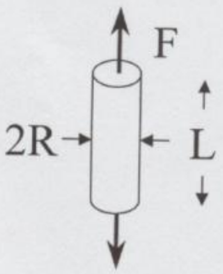
# axial stress and strain

stress $\sigma = \frac{F}{A_0}$ (force / area)

strain $\varepsilon = \frac{\Delta L}{L_0}$ (change in length / initial length) $\varepsilon_r = \frac{\Delta R}{R_0}$

elastic modulus $E = \frac{\sigma}{\varepsilon}$

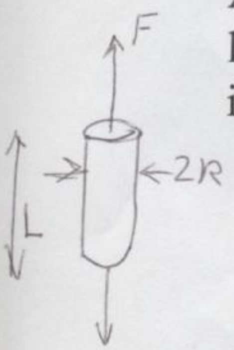
Poisson's ratio $\nu = -\frac{\varepsilon_r}{\varepsilon}$




A 1 in. diameter aluminum rod becomes 0.007 in. longer due to 1500 lbf tensile load. What is the rod's initial length and change in radius?

$E = 10 \cdot 10^6$ ~~ksi~~ psi

$\nu = 0.33$




$\Delta L = \varepsilon L_o$ $\quad \sigma = \varepsilon E$ $\quad L_o = \frac{\Delta L}{\varepsilon}$

$\Delta R = \varepsilon_r R_o$ $\quad 36.6\text{in} = \frac{0.007}{(1.909 \cdot 10^{-4})}$

$F = \sigma A_o$

$F = \varepsilon E A_o$ $\quad \Delta R = -\varepsilon \nu R_o$

$= \varepsilon E \, 2\pi R_o$

F =

$\frac{F}{\Delta L} = \frac{\sigma A_o}{\varepsilon L_o}$

geometry table

$F = A_o \sigma = A_o \varepsilon E$ $\quad -6.3025 \cdot 10^{-5} = - \Delta \, 0.33 \, (1\text{in})$

Need $\sigma$, $\varepsilon$ for aluminum

$F = \frac{\pi (1\text{in})^2}{4} \varepsilon \, (10 \cdot 10^6$ ~~ksi~~ psi$)$

$\varepsilon = \frac{1500\text{ lbs}}{10 \cdot 10^6 \text{ lbs/in}^2} \cdot \frac{4}{\pi (1\text{in})^2} = 0.000190986$

$1.909 \cdot 10^{-4}$

Verify

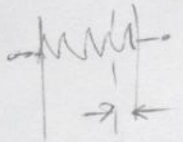
1

stress – has units of a "pressure" (solids pressure)

strain – normalized deformation

# axial stress and strain

deformation = f(solids pressure); if linear material then the elastic modulus is the constant of proportionality (generalized Hooke's Law)



$F = k\Delta x$

$\frac{F}{A} = \frac{k}{A}\Delta x$

$\frac{\sigma}{L} = \frac{K}{A}\frac{\Delta x}{L}$ $\varepsilon$

$\frac{\sigma}{L} = \frac{K}{AL}\Delta$

$\frac{KL}{A} = E$

$\sigma =$

Hooke's law analogy

stress $\sigma = \frac{F}{A_0}$ (force / area)

strain $\varepsilon = \frac{\Delta L}{L_0}$ (change in length / initial length) $\quad \varepsilon_r = \frac{\Delta R}{R_0}$

elastic modulus $E = \frac{\sigma}{\varepsilon}$

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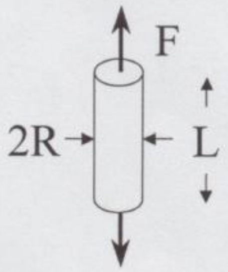


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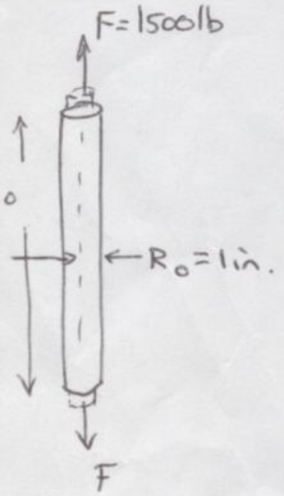
FBD




$\Delta L = \varepsilon L_o$

$\Delta R = \varepsilon_r R_o$

$L_o = \frac{\Delta L}{\varepsilon} = \frac{0.007 \text{ in}}{4.77\cdot 10^{-5}} = 146 \text{ in}$

$E = \sigma/\varepsilon = \frac{\frac{F}{\pi R_o^2}}{\varepsilon}$ solve for $\varepsilon$

$E = 10\cdot 10^6$ psi (table look-up)

$\varepsilon = \frac{F}{\pi R_o^2 E} = \frac{1500 \text{ lb}}{\pi (1 \text{ in})^2 (10\cdot 10^6 \text{ lb/in}^2)}$

$= 0.0000477$

$= 4.77\cdot 10^{-5}$

$\nu = 0.33$ (table look-up)

$\Delta R = -\varepsilon\nu\cdot R_o$

$= -4.77\cdot 10^{-5}(0.33)(1 \text{ in})$

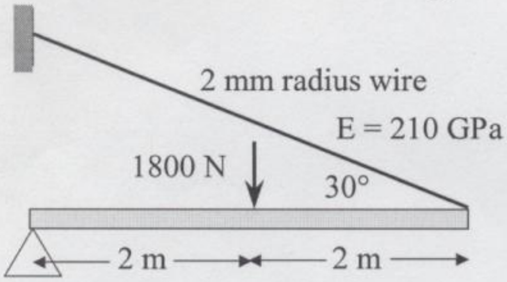
$= -1.57\cdot 10^{-5}$ in

B

2

A

33. Determine the change in length of the steel wire.




$L_o = \frac{4}{\cos 30} = 4.618$

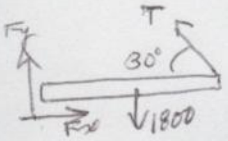
$\varepsilon, \sigma$ for steel

$T = \frac{1800(2)}{\sin 30(4)} = 1800$

$R_o =$

$10^9$ Pa

$E = 210 \cdot \cancel{10^6}\,kPa$




$\Delta L = L_o \varepsilon = L_o \frac{\sigma}{E}$

$$= \frac{\left[\frac{4}{\cos 30}\right]\left[\frac{1800(2)}{\sin 30(4)}\right]}{\frac{\pi(0.002)^2}{4}} \cdot \frac{1}{210 \cdot 10^6\,kPa}$$

1800

$-T\sin 30 + F_y = 0$

$F_y = -T\sin 30 + 1800$

$F_x - T\cos 30 = 0$

$F_x = T\cos 30$

$$\Delta L = (4.618)\left(\frac{1800}{\pi(0.002)^2}\right)\frac{1}{210 \cdot 10^{\cancel{6}\,9}\,\cancel{kPa}\,Pa} = \cancel{0.787\,m}\quad 3.15 \cdot 10^{-3}\,m$$

$\uparrow \Sigma M_A = T\sin 30(4) - 1800(2)$

solve for

$T = \frac{1800(2)}{\sin 30(4)}$

# thermal strain

change in temperature

$$\varepsilon_T = \alpha(T - T_0)$$

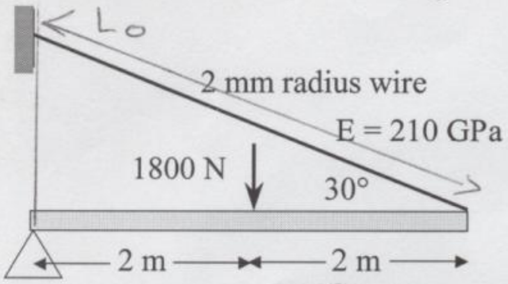
coefficient of thermal expansion

change in length $\delta_T = \alpha L(T - T_0)$

3

B

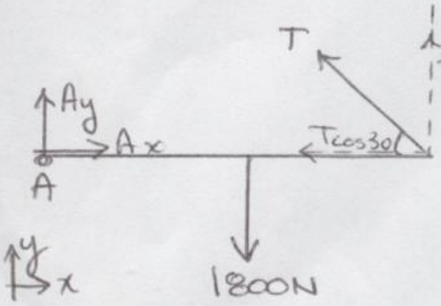
33. Determine the change in length of the steel wire.




$L_0 \cos 30 = 4$

$\therefore L_0 = \dfrac{4}{\cos 30} = 4.618$ } Geometry

① FBD




② Find reactions/tension

$\Sigma F_x = 0 = A_x - T\cos 30$

$\Sigma F_y = 0 = A_y + T\sin 30 - 1800\text{N}$

$\circlearrowright \Sigma M_A = 0 = T\sin 30(4) - 1800(2)$

$T = \dfrac{1800(2)}{\sin(30)(4)} = 1800$

③ $\Delta L = L_0 \varepsilon = L_0 \dfrac{\sigma}{E} = \dfrac{L_0 T}{A_0 E}$

$$\Delta L = \frac{(4.618\,\text{m})(1800\,\text{N})}{\pi(0.002)^2(210 \cdot 10^9\,\text{Pa})} = 3.149 \cdot 10^{-3}\,\text{m}$$

N/m²

# thermal strain

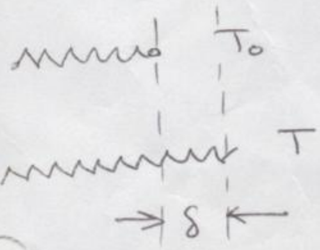
Materials expand & contract as a functn of temperature, ~~thermal~~ ~~internal~~ ~~forces~~ deformatn is called thermal strain.
- If material is free to deform, then no stress.

change in temperature

$$\varepsilon_T = \alpha(T - T_0)$$ } linear driving force model

coefficient of thermal expansion

change in length $\quad \delta_T = \alpha L(T - T_0)$




k

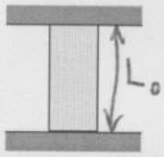
spring constant is temperatr dependent.
[ie. really cold steel shatters, really hot steel flows]



4

A

101. A steel rod ($E = 210$ GPa, $\alpha = 11.7\times10^{-6}\ {}^\circ C^{-1}$) is constrained and heated. What is the stress induced by a 40°C increase in temperature? (Virtual work concept)




① "Let rod deform", then determine force required to squeeze back into $L_0$ divide by area $= \sigma_T$

$\varepsilon_T = \alpha(T - T_0)$

$\sigma = \varepsilon_T E = (11.7\cdot10^{-6})(40^\circ C)\,210\cdot10^9\,Pa$ $= \cancel{2.1\cdot10^{11}\,Pa}$ $= \cancel{2.1\cdot10^{8}\,kPa}$

$= 98{,}280$

$= 98.2\,KpA$

$\delta_T = \alpha L_0(T - T_0)$

$\frac{\delta_T}{L_0} = \varepsilon = \alpha(T - T_0)$

$\varepsilon E = \sigma = \frac{F}{A_0}$

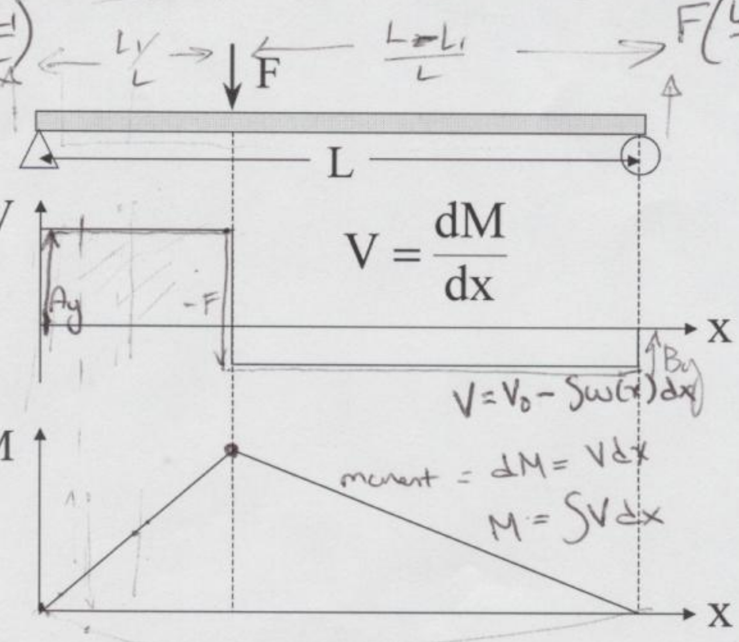
$\int d\left(\frac{dM}{dx}\right) = \int -w(x)\,dx$ $= V$

$\frac{dM}{dx} = -w(x)x$, $\int dM = \int -w(x)x\,dx$, $M = \int -w\frac{x^2}{2}\,dx + C$

# shear and bending diagrams

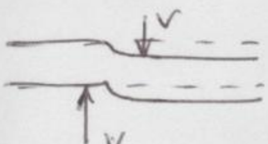
Beam internal effects.

Beams can resist: bending, shear & torsion.

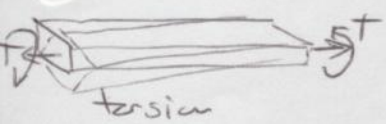




bending



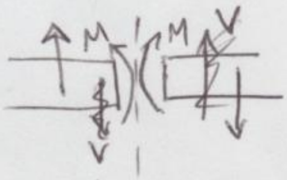

shear




torsion

reaction/action requires F & M on each side of internal surface must be equal & opposite

– In plane loading V & M only




by convention

– Usually let V & M be + in FBD & let algebra handel proper dire

① Find all external reacions

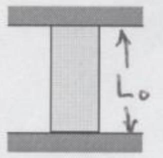
② Isolate secion of beam & apply equilibrium to find V & M

③ Move section boundary (2) to obtain V & M at new location

④ Concentrated load or couple causes discontinuity in V or M.

B

101. A steel rod (E = 210 GPa, $\alpha = 11.7\times10^{-6}\ ^\circ C^{-1}$) is constrained and heated. What is the stress induced by a 40°C increase in temperature?




① Virtual work

ⓐ Let rod expand

$\Delta L = \alpha L_o (T - T_o)$

ⓑ Normalize

$\frac{\Delta L}{L_o} = \varepsilon_T = \alpha(T - T_o)$

ⓒ Apply Force to create axial stress/deformation equivalent to thermal deformation

$\varepsilon_T E = \sigma = \frac{F}{A_o}$

Two approaches
① Virtual work
② Combine $E = \frac{\sigma}{\varepsilon_T}$

$\varepsilon_T = \alpha(T - T_o)$

$\sigma = \alpha(T - T_o)E$
$= (11.7\cdot10^{-6}/^\circ C)(40^\circ C)(210\cdot10^9 N/m^2)$
$= 98.2\cdot10^3\ kN/m^2 = 98.2\cdot10^3\ kPa$
$\approx (98.2\cdot10^6\ Pa)$

## shear and bending diagrams

Beam internal effects
- resist
  bending
  shear
  torsion

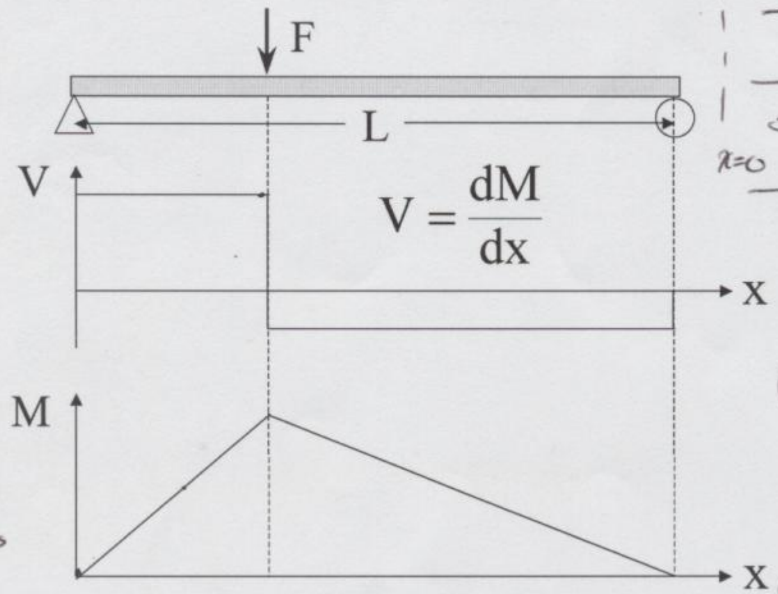
M bending M

shear V

torsion

action/reaction requires V, T, M to be equal & opposite at internal surface

- In-plane loading only have V & M




$V, M$ rest of beam

sign convention

$x=0$

(i) Find all external reactions

(ii) isolate section of beam & apply equilibrium to find V & M

(iii) Move section boundary to find V & M at new location

(iv) Concentrated load or couple causes discontinuous change

(v) dist. loads

$\frac{d}{dx}\left(\frac{dM}{dx}\right) = -w(x)$

$\frac{dM}{dx} = V = \int -w(x)\,dx$
$= -w(x)x + C$

$M = \int V\,dx = -\frac{w(x)x^2}{2} + C$

shear is integral of load prism



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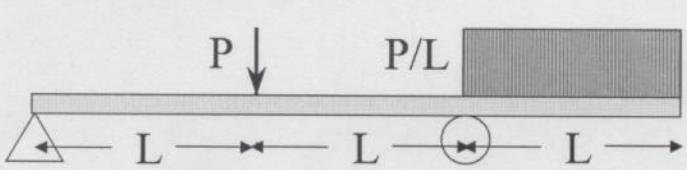
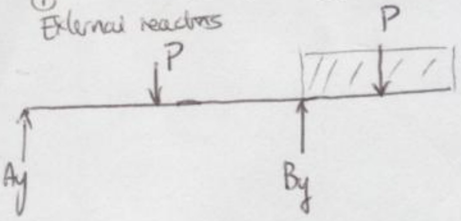
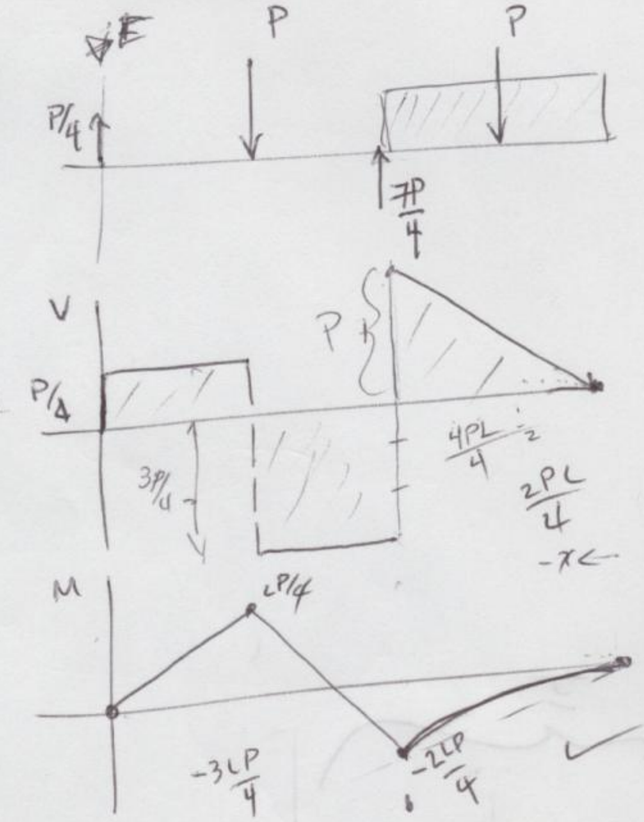
104. Find the maximum shear force and its location.




① External reactions




$\Sigma F_y = 0 = A_y + B_y - 2P$

$\Sigma M_A = 0 = -PL + B_y 2L - P2.5L$

$B_y = \frac{P(L+2.5L)}{2L} = P\frac{3.5\cancel{L}}{2\cancel{L}} = P\frac{7}{2}\cdot\frac{1}{2}$

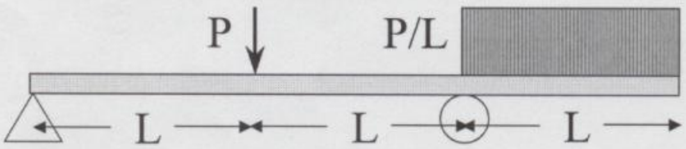
$= \frac{7P}{4}$

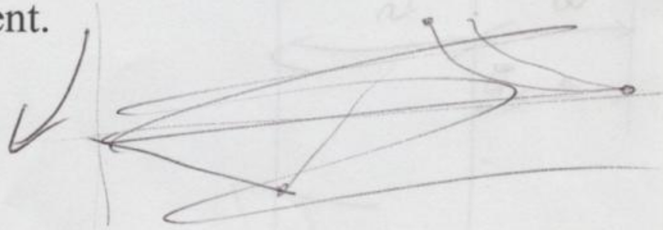
$B_y \quad \frac{7P}{4}$

$A_y = 2P - 7P/4$

$= \frac{8P-7P}{4} = P/4$

105. Find the maximum bending moment.





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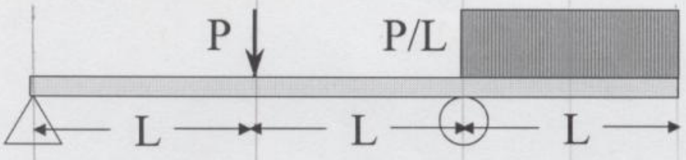
104. Find the maximum shear force and its location.




$\circlearrowleft \Sigma M_A = 0$

$= -PL + 2L B_y - P \frac{5}{2} L$

$B_y = \dfrac{\frac{2}{2}PL + \frac{5}{2}PL}{2L}$

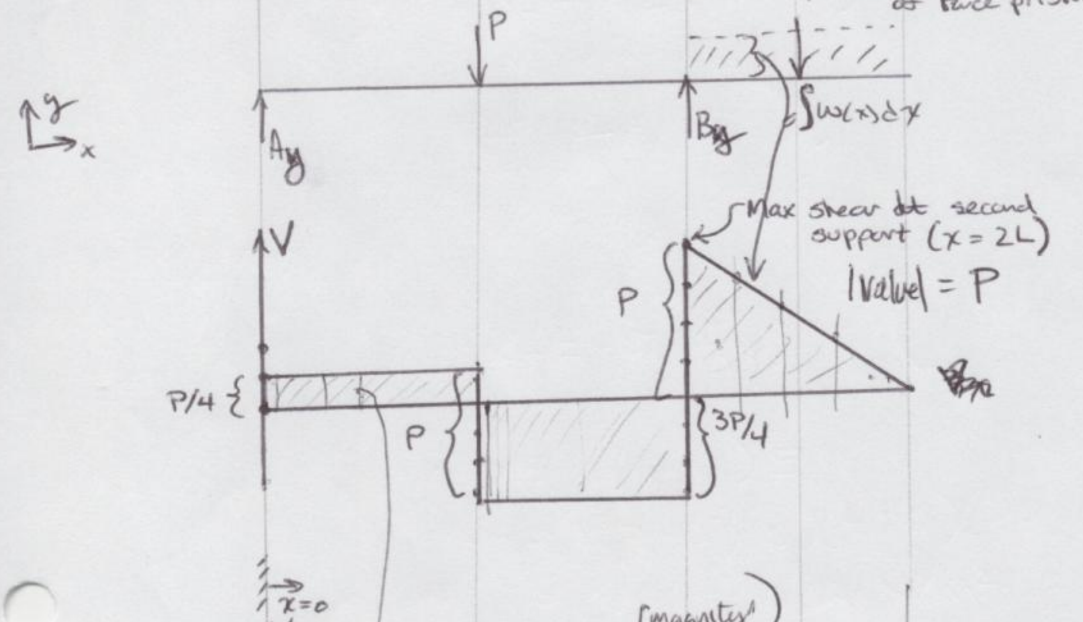
$= \dfrac{7P}{4}$

$\Sigma F_y = 0 = A_y - 2P + B_y$

$A_y = 2P - 7P/4$

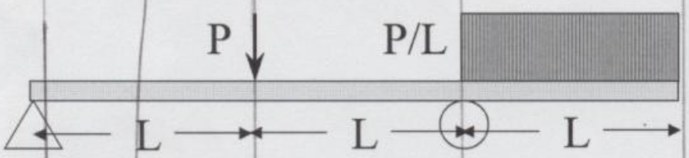
$= P/4$

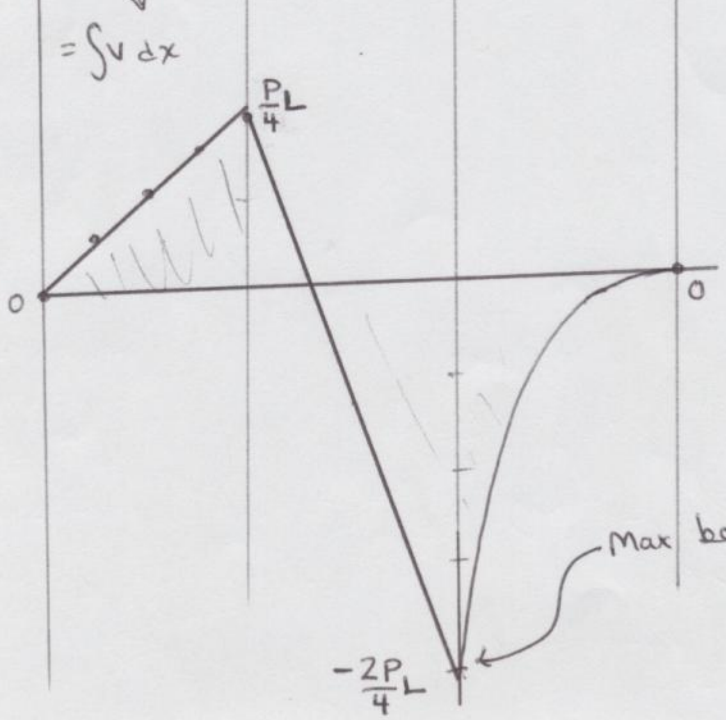
(i) external forces




(magnitude)

105. Find the maximum bending moment.




Max bending moment at this location, $|\text{value}| = \dfrac{2PL}{4} = \dfrac{PL}{2}$

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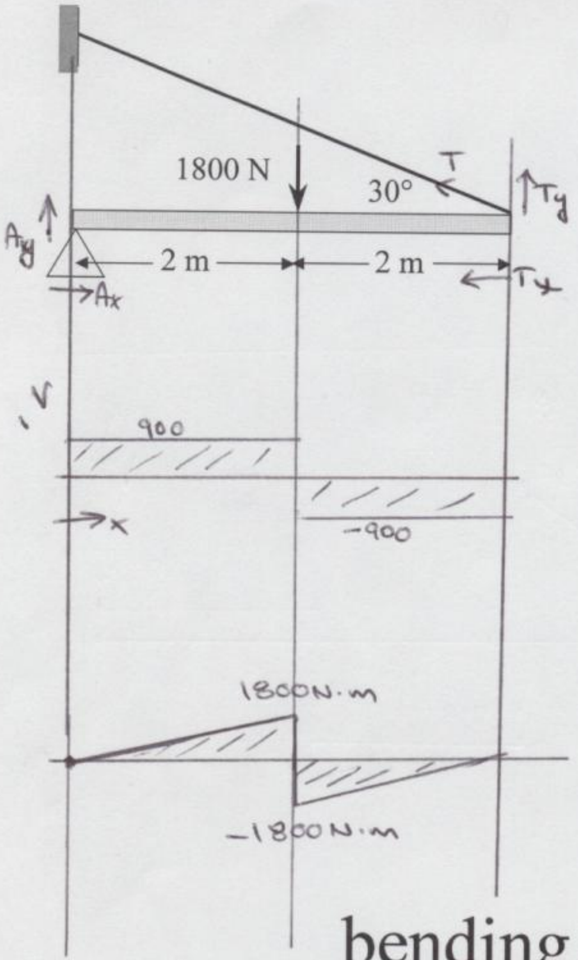
37. Sketch the shear and bending moment diagrams.




$\Sigma F_y = 0 = A_y + T_y - 1800$

$A_y + T_y = 1800$

$T_y = 900$ (prior result, or symetry)

# bending stress

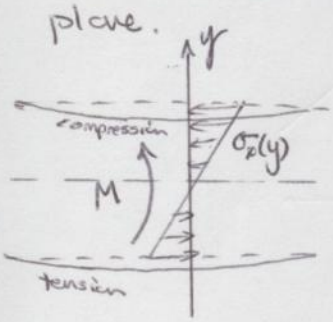
internal stress in beam section relative to neutral axis (centroid) relative to bending plane.

bending moment

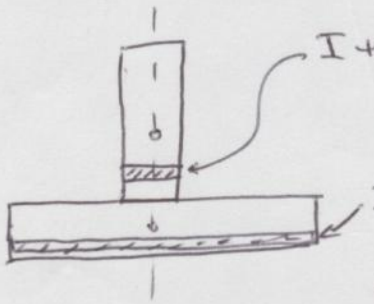
bending formula $\sigma = -\dfrac{My}{I}$ ← position

2nd moment of area

$I$ ← changes as cross section area changes (see ⊥ beam example)




- "y" is measured from the neutral surface
- "I" is calculated about the neutral surface



I this section is smaller than I

Use centroidal axis if possible for I +

$I = \bar{I} + Ad^2$

Uniform geometry the formula is relatively simple to apply.



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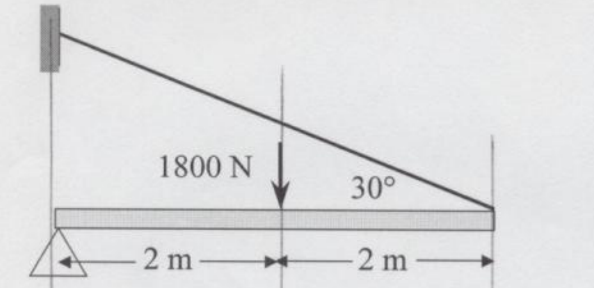
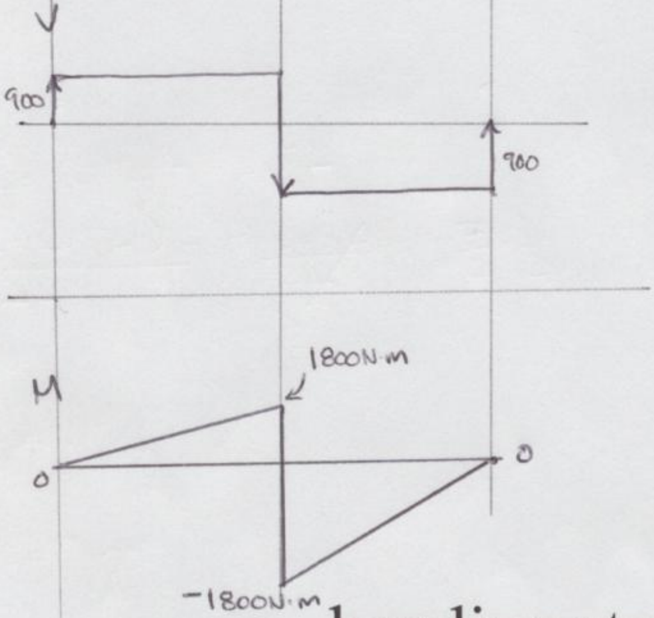
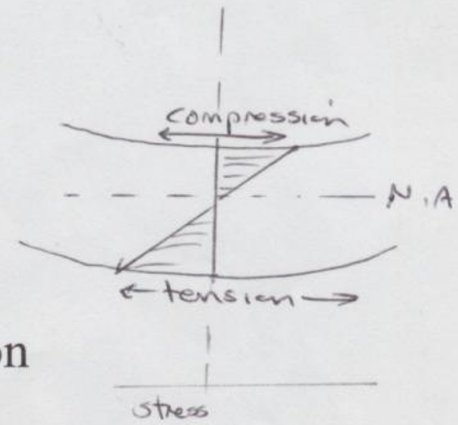
37. Sketch the shear and bending moment diagrams.




# bending stress




Quantity of internal stress in beam section relative to neutral axis

bending formula

$$\sigma = -\frac{My}{I}$$

bending moment (M)

position (y)

2nd moment of area (I) — will change as section geometry changes.

- "y" is measured from the neutral surface – use centroid as N.A.
- "I" is calculated about the neutral surface – use tables for I



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