Problem 18: Mathematics
What is the inverse transform of the *z*-transform $F(z) = 1 - 2z/(1-z)$?

(A) $f(k) = \delta(k) - 2u(k)$
(B) $f(k) = \delta(k) + 2u(k)$
(C) $f(k) = \delta(k) + 2^k$
(D) $f(k) = \delta(k) - k^2$

Solution:

Rearrange the *z*-transform function.

$$F(z) = 1 - \frac{2z}{1-z} = 1 + \frac{2z}{z-1} = 1 + \frac{2}{1-z^{-1}}$$

Use the *z*-transform table.

$f(k) = \delta(k) + 2u(k)$

The answer is B.

Problem 19: Mathematics
Evaluate the following integral.

$\int \sqrt{x}\,(x^2 - 6)\,dx$

(A) $\frac{2}{7}x^{7/2} - 4x^{3/2} + C$

(B) $\frac{5}{2}x^{3/2} - 3x^{-1/2} + C$

(C) $\frac{5}{2}x^{7/2} - 3x^{3/2} + C$

(D) $\frac{2}{7}x^{7/2} - 4x^{3/2}$

Solution:

Use the distributive property of multiplication to change the integrand into the sum of two power functions.

$$
\begin{aligned}
\int \sqrt{x}\,(x^2 - 6)\,dx &= \int (x^{1/2}x^2 - 6x^{1/2})\,dx \\
&= \int (x^{5/2} - 6x^{1/2})\,dx \\
&= \int x^{5/2}dx - 6\int x^{1/2}dx \\
&= \frac{1}{1+\frac{5}{2}}x^{\frac{5}{2}+1} - (6)\left(\frac{1}{1+\frac{1}{2}}\right)x^{\frac{1}{2}+1} + C \\
&= \frac{2}{7}x^{7/2} - 4x^{3/2} + C
\end{aligned}
$$

The answer is A.

Problem 20: Mathematics
Evaluate the following integral.

$$\int \left(e^{2x} - 3\sin 4x\right)dx$$

(A) $\frac{1}{2}e^{2x} - \frac{3}{4}\cos 4x + C$
(B) $\frac{1}{2}e^{2x} + \frac{3}{4}\cos 4x + C$
(C) $2e^{2x} - 12\cos 4x + C$
(D) $e^{2x} + 3\cos 4x + C$

Solution:

$$\begin{aligned}\int \left(e^{2x} - 3\sin 4x\right)dx &= \int e^{2x}dx - 3\int \sin 4x\,dx \\ &= \frac{1}{2}e^{2x} - (3)\left(-\frac{1}{4}\cos 4x\right) + C \\ &= \frac{1}{2}e^{2x} + \frac{3}{4}\cos 4x + C\end{aligned}$$

The answer is B.

Problem 21: Mathematics [E0212030006]

Evaluate the following integral.

$$\int \frac{x^2}{4+x^2} dx$$

(A) $x + 2\tan^{-1}\frac{x}{2} + C$

(B) $x - 4\tan^{-1}\frac{x}{2} + C$

(C) $x - 2\tan^{-1}\frac{x}{2} + C$

(D) $x - \tan^{-1}\frac{x}{4} + C$

Solution:

Change the integrand into two parts and then use the following formula from the integral table.

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

(Note: As stated in the *NCEES Handbook*, $\frac{1}{a}\tan^{-1}\frac{x}{a} = \frac{1}{a}\arctan\frac{x}{a}$.)

$$\begin{aligned}
\int \frac{x^2}{4+x^2} dx &= \int \frac{4+x^2-4}{4+x^2} dx \\
&= \int \left(1 - \frac{4}{2^2+x^2}\right) dx \\
&= x - \frac{4}{2}\tan^{-1}\frac{x}{2} + C \\
&= x - 2\tan^{-1}\frac{x}{2} + C
\end{aligned}$$

Note: $a = 2$ for the second part of the integral.

The answer is C.