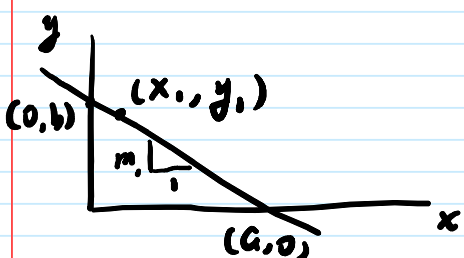
FE Math Review

Tuesday, September 30, 2025 8:01 PM

Geometry & Trigonometry

Straight line



general form: $Ax + By + C = 0$

standard form: $y = mx + b$

slope-intecept form

point-slope form:

$$y - y_1 = m(x - x_1)$$

slope for a straight line passing 2 points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Two lines are perpendicular if $m_1 = -\frac{1}{m_2}$

Straight line example:

A line goes through the point $(4, -6)$ and is perpendicular to the line $y = 4x + 10$. What is the equation of the line?

(A) $y = mx - 20$

(B) $y = -\frac{1}{4}x - 5$

(C) $y = \frac{1}{5}x + 5$

(D) $y = \frac{1}{4} + 5$

Given $m_1 = 4$

$\rightarrow m_2 = -\frac{1}{4}$

$(x_1, y_1) \rightarrow (4, -6)$

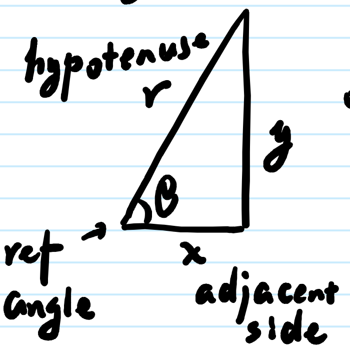
$y - (-6) = -\frac{1}{4}(x - 4)$

$y = -\frac{1}{4}x + 1 - 6 = -\frac{1}{4}x - 5$

Trigonometry

Right Triangle

Right Triangle



$\sin\theta = y/r$

$\cos\theta = x/r$

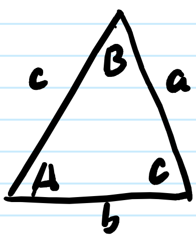
$\tan\theta = y/x$

Identities manual pg 38

e.g. $\tan\theta = \sin\theta/\cos\theta$

$\sin^2\theta + \cos^2\theta = 1$

General Triangle



law of sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

law of cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$A + B + C = 180° = \pi$ rad

Trigonometry example:

The vertical angle to the top of a flagpole from point A on the ground is observed to be $37°11'$. The observer walks 17 m directly away from point A and the flagpole to point B and finds the new angle to be $25°43'$. What is the approximate height of the flagpole?

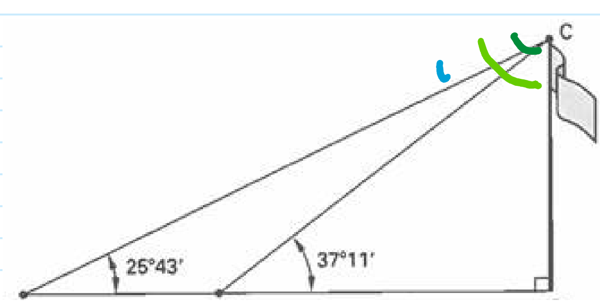
(A) 10 m
(B) 22 m
(C) 82 m
(D) 300 m

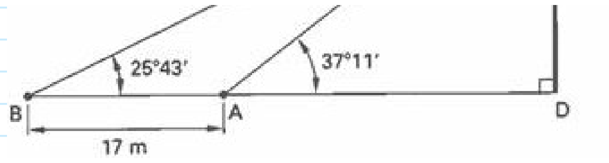
$\angle ACD = 180° - 90° - 37°11'$
$= 52°47'$

$\angle BCD = 180° - 90° - 25°43'$
$= 64°17'$

$\angle BCA = \angle BCD - \angle ACD$
$= 11°28'$

Triangle ABC:





Triangle ABC:

$$\frac{AB}{\sin\angle BCA} = \frac{AC}{\sin\angle CBA}$$

$\therefore AC = 37.106$ m

then Triangle ACD:

$$\frac{AC}{\sin\angle CDA} = \frac{CD}{\sin\angle CAD}$$

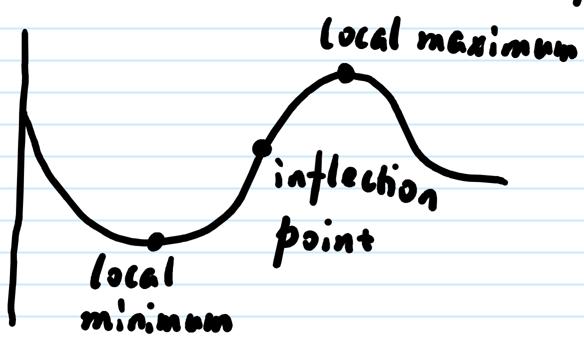
$CD = 22.426$ m

Calculus

Derivatives

↳ slope at point x of a continus function $f(x)$

↳ written $f'(x)$, $\frac{df(x)}{dx}$



$f'(a) = 0$ → slope $= 0$ at a

& $f''(a) > 0$ → concave up → minimum

$f''(a) < 0$ → concave down → maximum

if $f''(a) = 0$ → inflection point

Manual pg 48 derivative table

Partial Derivative

function of 2 independent varicbles $z = f(x, y)$

$$\frac{\partial z}{\partial x} = \frac{\partial f(x,y)}{\partial x}$$

Assume the other variable (y) is constant.

Derivative example

1. What is the maximum value of the following function on the interval $x \le 0$?

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$y = 2x^3 + 12x^2 - 30x + 10$

$y' = 6x^2 + 24x - 30$

let $y' = 0$

solve $x_1 = 1$, $x_2 = -5$

$\boxed{C}$

(A) -210
(B) -36
(C) -5
(D) 210

$y'' = 12x + 24$

$y''(-5) = -51 < 0 \leftarrow \checkmark$

$y''(1) = 36 > 0 \quad \times$

1. If $f(x, y) = x^2y^3 + xy^4 + sinx + \cos^2 x + \sin^3 y$, what is $\partial f/\partial x$?

(A) $(2x + y)y^3 + 3\sin^2 y\, cosy$
(B) $(4x - 3y^2)xy^2 + 3\sin^2 y\ \ cosy$
(C) $(3x + 4y^2)xy + 3sin^2y\, cosy$
(D) $(2x + y)y^3 + (1 - 2\sin x)\cos x$

$\frac{\partial f}{\partial x} = 2xy^3 + y^4 + \cos x + 2\cos x(-\sin x) + 0$

$= (2x + y)y^3 + (1 - 2\sin x)\cos x$

<u>Integration</u>

- Inverse operation of differentiation, when $f'(x) = h(x)$

definite integrals → independent variable range is specified

$\int_a^b h(x)\,dx = f(b) - f(a)$

indefinite integrals:

$\int h(x)\,dx = f(x) + C$

Integration table pg 49

Integration table pg 44

Integral Example

1. Determine the following indefinite integral.

$$\int \frac{x^3 + x + 4}{x^2} dx$$

$= \int x + x^{-1} + 4x^{-2}\, dx$

$= \frac{1}{2}x^2 + \ln|x| + (-1)(4)x^{-1} + C$

(A) $\frac{x}{4} + \ln|x| - \frac{4}{x} + C$

(B) $-\frac{x}{2} + \log(x) - 8x + C$

(C) $\frac{x^2}{2} + \ln|x| - \frac{2}{x^2} + C$

(D) $\frac{x^2}{2} + \ln|x| - \frac{4}{x} + C$

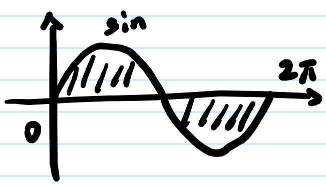
1. What is the approximate total area bounded by $y = \sin x$ and $y = 0$ over the interval $0 \le x \le 2\pi$? (x is in radians)

(A) 0

(B) $\pi/2$

(C) 2

(D) 4



$\int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} -\sin x\, dx$

$= 2\int_0^{\pi} \sin x\, dx$

$= -2 \cos x \Big]_0^{\pi}$

$= -2(-1-1) = 4$

Vector Operations

↳ has magnitude & direction

$\vec{A} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$

Addition & subtration.

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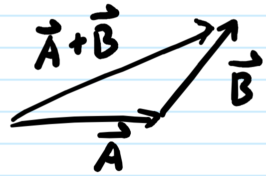
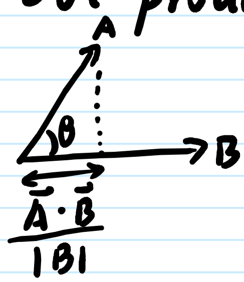


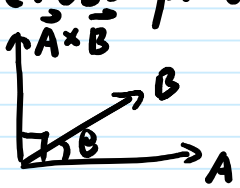
$$\vec{A}+\vec{B} = (a_x+b_x)\vec{i} + (a_y+b_y)\vec{j} + (a_z+b_z)\vec{k}$$
$$= \begin{pmatrix} a_x+b_x \\ a_y+b_y \\ a_z+b_z \end{pmatrix}$$
$$\vec{A}-\vec{B} = (a_x-b_x)\vec{i} + (a_y-b_y)\vec{j} + (a_z-b_z)\vec{k}$$

Dot product:



$$\vec{A}\cdot\vec{B} = a_x b_x + a_y b_y + a_z b_z$$
$$= |\vec{A}||\vec{B}|\cos\theta$$

Cross product:



$$\vec{A}\times\vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = |\vec{A}||\vec{B}|\vec{n}\sin\theta$$

unit vector $\perp$ plane of $\vec{A}$ & $\vec{B}$

Vector example

1. Given vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and C, what is the value of $(\boldsymbol{A}+\boldsymbol{B})\cdot(\boldsymbol{B}+\boldsymbol{C})$?

$\boldsymbol{A} = 8\boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$

$\boldsymbol{B} = 4\boldsymbol{i} + 2\boldsymbol{j} + 4\boldsymbol{k}$

$\boldsymbol{C} = 6\boldsymbol{i} + 8\boldsymbol{j} + 10\boldsymbol{k}$

$$\vec{A}+\vec{B} = \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix} \qquad \vec{B}+\vec{C} = \begin{pmatrix} 10 \\ 10 \\ 14 \end{pmatrix}$$

(A) 52
(B) 104
(C) 132
(D) 244

$$\begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}\cdot\begin{pmatrix} 10 \\ 10 \\ 14 \end{pmatrix} = 120 + 40 + 84 = 244$$

1. What is the angle between the two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$?

1. What is the angle between the two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$?

$$\boldsymbol{A} = 4\boldsymbol{i} + 12\boldsymbol{j} + 6\boldsymbol{k}$$

$$\boldsymbol{B} = 24\boldsymbol{i} - 8\boldsymbol{j} + 6\boldsymbol{k}$$

(A) $-84.32°$
(B) $84.32°$
(C) $101.20°$
(D) $122.36°$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\cdot|\vec{B}|} = \frac{36}{14\times 26} = 0.0989$$

$$\vec{A}\cdot\vec{B} = 4\times 24 - 12\times 8 + 6\times 6 = 36$$

$$|\vec{A}| = \sqrt{4^2 + 12^2 + 6^2} = 14$$

$$|\vec{B}| = \sqrt{24^2 + 8^2 + 6^2} = 26$$

$$\theta = \cos^{-1}(0.0989) = 84.320°$$